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Learning in final-offer arbitration with multiple offers

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August 2, 2004

Abstract

Motivated by the price-setting process of water utilities in Chile, I study a final-offer arbitration game in which two parties simultaneously submits offers for each of the two or more units in which the item in dispute has been divided. The arbitrator is limited to choose one party's offer or the other for each unit. While the introduction of multiple offers allows the arbitrator to get closer to her ideal settlement it may prompt an arbitrarily large divergence between the parties' offers. The latter, however, does not affect the arbitrator's ability to learn from the offers.

Keywords: final-offer arbitration, price regulation, Nash equilibrium

JEL: L50, L90

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1 Introduction

Departing from the more traditional rate-of return and price-cap regulations, prices of public utilities in Chile are set using a particular form of yardstick regulation in which the benchmarking is based on a hypothetical efficient firm.[1] Under this price setting process—introduced first in the electricity sector in the early 1980s—both the regulator and the regulated firm have a very explicit interaction. Based on their own estimation for the long term costs of this hypothetical efficient firm, both parties propose the price to be charged by the regulated firm for the duration of the review period (4-5 years).[2] If parties cannot agree on the price, the disagreement is settled through an arbitration process.

Since 1999 this arbitration process takes a distinct form in the water sector. In order to prevent parties' offers to significantly diverge, as has occurred in the other regulated sectors, the legislation that norms the water sector considers a final-offer arbitration mechanism in which the arbitrator is constrained to choose one of the parties' offers as a settlement.[3] But because parties do no submit a single offer for the entire firm but rather an offer for each of the cost units in which the firm is divided,[4] the actual arbitration mechanism looks more like a hybrid between final-offer arbitration and conventional arbitration.[5]

While the division of the regulated firm in various units was aimed at introducing greater transparency into the regulatory process and avoiding subsidization across cost units, evidence on the first round of applying this price setting process for the different water utilities in the country has not been uncontroversial. As shown in Table 1, we observe in most cases an important divergence between the regulator's overall offer, p^r, and the firm's overall offer, p^f (to facilitate the exposition p^r has been normalize to 100).[6] And in five cases parties failed to negotiate the final price, p^s, and had instead resorted to final-offer arbitration (FOA).

INSERT TABLE 1 HERE OR BELOW

[1] See Vogelsang (2002) for an overview of the different regulatory approaches practiced over the last 20 years.

[2] In reality, each party constructs an efficient firm and announces the long term total cost that such firm would incur in providing the service during the review period. In this construction, parties may differ not only about unit costs but also about projections of future demand.

[3] The use final-offer arbitration is commonly seen in the settlement of labor disputes (with baseball as a classic example) but I am not aware of its explicit use elsewhere in a regulatory context.

[4] There are approximately 200 units including, for example, cost of raw water, cost of capital, cost of replacing pavement, etc. For more see Sánchez and Coria (2003).

[5] In conventional arbitration, the arbitrator is not constrained to any particular settlement. So, as the number of units goes large, final-offer arbitration would seem to approach conventional arbitration since the arbitrator is able to chose almost any settlement by using some combination of parties' offers.

[6] The numbers shown are based on parties' announcements of long term total costs.

The numbers in Table 1 naturally raise the empirical question about what are the factors that characterize the contract zone of Farber and Bazerman (1989), i.e., the range of settlements that both parties prefer to disagreement. The great divergence in parties' offers, however, have raised more fundamental questions. Some observers have challenged the advantages of the current regulatory mechanism over more conventional mechanisms, particularly price-caps as practiced in the UK, while others have questioned the privatization process itself arguing that the increase in information asymmetries have more than offset any productivity gains.[7] Rather than introducing radical changes in both the privatization program and the regulatory scheme, the authority is exploring ways in which the actual divergence in parties' offers could be diminished. In particular, it is proposing to substantially reduce the multiplicity of offers, i.e., the numbers of units in which the regulated firm is divided. Reducing the number of offers seems reasonable since it would make the arbitration process look less like the cheap-talk game associated to conventional arbitration.

With the purpose of better understanding agents' behavior in this price-setting process, in this paper I extend the (single-offer) final-offer arbitration models of Farber (1980) and Gibbons (1988) to the case in which parties simultaneously submit offers for each of the units that are part of the item in dispute and the arbitrator is limited to choose one party's offer or the other for each unit, so in principle, she is free to fashion a compromise by awarding some offers to one party and the rest to the second party. Despite this multi-dimensional variant of final-offer arbitration was already recognized by Farber in his article as "issue by issue" final-offer arbitration, there is no formal analysis of such problem in the literature. There is a seemingly related problem in the literature that is the analysis of split award auctions where it is possible for a buyer to split a production award between two or more suppliers (Anton and Yao, 1989 and 1992). Besides the multi-dimensional structure, these problems have little in common, however. While in split award auctions bidding parties seek to coordinate in high prices that would report positive profits for both, in final-offer arbitration parties have no incentives to coordinate in any particular outcome since they have opposing preferences (otherwise, there would be no reason to resort to arbitration).

Understanding the equilibrium properties of this arbitration game is not only relevant for the price-setting process that motivated this paper,[8] but more generally, for any final-offer

[7] See Gomez-Lobo and Vargas (2002) for a further discussion on the shortcomings of the current regulatory scheme.

[8] This arbitration scheme has also been proposed in place of the current mechanisms used to settle disputes

arbitration in which more than one issue is in dispute (e.g., a government and a contractor renegotiating a multi-part contract). The model of the paper is standard in that it is based on a one-period game that considers two parties (i.e., the firm and the regulator) with opposing preferences that simultaneously submit offers to an arbitrator whose ideal settlement is imperfectly known by both parties (recall that parties' uncertainty regarding the arbitrator's preferences is what leads to offers divergence).[9] Attending the spirit of the legislation, the arbitrator wants to choose efficient prices, i.e., prices that are closest to the long-term cost of the hypothetical efficient firm. But since the parties are much better informed about the true cost of this efficient firm than the arbitrator is (in part because they conduct detail studies before the price-setting process), I follow Gibbons (1988) in that the arbitrator may eventually learn a great deal from the parties' (equilibrium) offers about the true cost of this efficient firm.[10]

The results of the paper can be presented as the answers to three basic questions that I tackle in different sections of the paper. The first question is to what extent the introduction of multiple offers (whether two or more) affects the divergence between parties' overall offers (Section 3). I show that when parties have perfect knowledge about the arbitrator's ideal settlement, parties' offers exhibit, as in the single-offer game, perfect convergence. When parties are uncertain about the arbitrator's preferences, as it is usually the case, the division of the firm in just two cost units results in multiple equilibria implying that the divergence between parties' equilibrium offers is not unique but can be anything between that of the single-offer game and above.

Contrary to the single-offer game, in which parties' equilibrium offers are unique (Farber, 1980), the multiplicity of equilibria raises a second question that is to what extent the arbitrator's ability to learn from the parties' offers is hampered by the introduction of multiple offers (Section 4). As in the single-offer game, in which the arbitrator perfectly recovers parties' cost information from the average of parties' offers (Gibbons, 1988), I find that the introduction of multiple offers does not necessarily affect the arbitrator's ability to learn from the parties' offers. This is because in (separating Bayesian) equilibrium the arbitrator does not learn from the absolute value of the individual or overall offers submitted by the parties but from the

over regulated prices in the electricity and telecommunication sectors in Chile.

[9] As in Farber (1980) and the literature that has followed, I do not include a previous stage in which parties bargain over the final price before going to arbitration, so I do not intent to explain what makes parties more likely to reach an agreement rather than end in arbitration. For more see Farber and Bazerman (1989).

[10] More generally, empirical studies of arbitrator behavior indicate that arbitrators do use parties' offers to compute their ideal settlement (e.g., Farber and Bazerman, 1986; Ashenfelter and Bloom, 1984).

relationship that these offers exhibit in equilibrium; a relationship that remains regardless of the divergence between parties' equilibrium offers.

If the introduction of multiple offers does not affect learning, despite parties' offers can exhibit substantial divergence, the remaining question deals with welfare gains or losses from introducing multiple offers (Section 5). Intuitively, one would argue that multiple offers provide the arbitrator with more flexibility to put together a settlement closer to her ideal settlement (i.e., the true cost of the efficient firm) by combining offers from both parties. Although one can construct examples where the arbitrator is further away from her ideal choice, I show that in equilibrium, the parties' offers are structured in such a way that it is always possible for the arbitrator to choose a final price (which combines offers from both parties) that is expected to be closer to her ideal settlement than in the single-offer case.

The model developed in this paper provides us with results that have important implications for the design of final-offer arbitration mechanisms. In particular, they indicate that the introduction of multiple offers is likely to enhance welfare, despite the increase in the divergence between parties' offers. Before proceeding, however, I should emphasize that this paper is by no means an attempt to discuss the merits of the regulatory approach under study over alternative approaches such as price caps but rather understand the effect of regulatory design on parties behavior. With that objective in mind, the rest of the paper is organized as follows. In Section 2, I introduce the model using the single-offer game. In Sections 3, 4 and 5, I extend the model to two offers and use it to address, respectively, the three questions raised above. Concluding remarks are in Section 6.

2 The single-offer arbitration model

Let start with the single-offer arbitration game. In this case parties are asked to submit a single offer for the entire firm and the arbitrator is constrained to choose one of the parties' offers as a settlement. The parties' offers are denoted by p^f and p^r.

2.1 Preferences and information

The arbitrator is characterized by the parameter z, which describes the arbitrator's most preferred settlement. If the actual settlement is p, the arbitrator's utility is $v_a(p, z) = -(p - z)^2$. Since the spirit of the legislation is to charge (efficient) prices to consumers that just cover the long term costs of an hypothetical efficient firm, we assume that the arbitrator's ideal price settlement is directly related to the cost of this efficient firm, which we denote by c. In particular, I assume that $z(c) = c$. This assumption is also consistent with the idea the arbitrator would like to be rehired.[11]

Unlike the arbitrator, the firm and the regulator are assumed to be risk-neutral.[12] As in Farber (1980) and Gibbons (1988), both parties are assumed to have strictly opposed preferences: the firm seeks to maximize the arbitrator's expected settlement, while the regulator seeks to minimize it. It may seem odd that preferences are totally disconnected from the cost of the hypothetical efficient firm. While little problematic for a firm that faces an inelastic demand,[13] it is unlikely that the regulator would only care about consumer surplus and put no weight on firm's profits. As shown in Montero (2003), however, the results do not qualitatively change if the regulator puts some weight on firm's profits because parties incentives work basically the same as long as their preferences are not perfectly aligned. Accordingly, I maintain the assumption that parties have strictly opposed preferences in order to keep the analysis simple.

Neither the arbitrator nor the parties have perfect information about the true cost of the hypothetical efficient firm (which is not necessarily the same as the actual firm) but they do not necessarily share the same perceptions about this cost. Following Gibbons' (1988) information structure (I also follow Gibbons' notation very closely), let the arbitrator's perception about the true cost c be summarized by the noisy signal

$$c^a = c + \varepsilon^a \tag{1}$$

where c is normally distributed with mean m and precision h, and ε^a is normally distributed with zero mean and precision h^a. The parameters m and h are common knowledge and can be interpreted as the publicly observable facts relevant for the regulation of the firm. Note that as h^a grows infinitely large (i.e., variance of ε^a goes to zero), the arbitrator can perfectly infer the cost c.

[11] For more on arbitrator behavior see Ashenfelter and Bloom (1984).

[12] The introduction of risk-aversion complicates the algebra without producing a qualitative change in the results. See Montero (2003).

[13] A sufficiently low price elasticity ensures that, in equilibrium, the firm will never submitt a price offer above its monopoly price.

Similarly, let the parties' knowledge about the true cost c be summarized by the noisy signal

$$c^p = c + \varepsilon^p \tag{2}$$

where ε^p is normally distributed with zero mean and precision h^p. It is important to emphasize that this information structure assumes that the parties –the firm and the regulator– share the same perception about the true cost c. While letting $h^p > h^a$ this information structure captures the idea that both parties are considerably better informed than the arbitrator, it is not so clear that both parties should share the exact same perception about c. Since c is not the cost of the firm that is currently providing the service (although it is related), the firm is likely to be better informed about site specificities while the regulator, making use of information collected from all the other regulated water firms, may be better informed about some of the parameters that are common across firms (e.g., labor productivity).[14] It would certainly add more realism to the analysis the introduction of asymmetric information via different random shocks with different levels of precision, but that has not been done for the single-offer case, much less so for the case of multiple offers. I return to this point in the last section of the paper.

The information structure can be summarized as follows: the arbitrator observes c^a, the parties both observe c^p, no one observes c, and m, h, h^p, and h^a are common knowledge. In addition, the three random variables c, ε^a, and ε^p are assumed to be independent of each other, which facilitates the computation of the Bayesian updating following the arrival of new information (e.g., signals, parties' offers). For example, the conditional distribution of c given c^j, where $j = a, p$, is normal with mean $M^j(c^j)$ and precision H^j, where

$$M^j(c^j) = \frac{hm + h^j c^j}{h + h^j} \tag{3}$$

and

$$H^j = h + h^j \tag{4}$$

Similarly, the conditional distribution of c given c^a and c^p is normal with mean $M^{ap}(c^a, c^p)$ and precision H^{ap}, where

$$M^{ap}(c^a, c^p) = \frac{hm + h^a c^a + h^p c^p}{h + h^a + h^p} \tag{5}$$

[14] See Teeples and Glyer (1987) for a discussion on differences in production efficiency across water utilities.

and

$$H^{ap} = h + h^a + h^p \tag{6}$$

I will make use of these definitions of beliefs updating in the models that follow.

2.2 Arbitration without learning

Let consider first the case in which the arbitrator only pays attention to its noisy signal c^a in constructing her ideal settlement. Ignoring parties' offers may not be sequentially rational, as we discussed later, but it is a useful starting point to understand the implications of learning and the multiplicity of offers on the equilibrium of this arbitration game.

Acknowledging that the arbitrator ignores their offers, the parties will form the common belief that the arbitrator's ideal settlement z is randomly distributed according to some cumulative distribution (to be determined below) function $F(z)$, with density $f(z)$. Since the arbitrator is constrained to choose one of the parties' offers as the settlement, she will choose the offer that is closer to her ideal settlement z. Assuming for the moment that in equilibrium the regulator's offer, p^r, will be smaller than the firm's offer, p^f, the arbitrator will choose the regulator's offer if and only if $z < \overline{p}$, where $\overline{p} = (p^r + p^f)/2$; hence, the probability that p^r is picked by the arbitrator is $F(\overline{p})$.

The timing of the final-offer arbitration game is as follows. First, the regulator and the firm simultaneously submit their offers to the arbitrator.[15] Second, the arbitrator chooses the offer that maximizes his utility function $v_a(p, z)$ as the settlement. The parties' Nash equilibrium offers (p^f and p^r) maximize their expected payoffs, so they are found by simultaneously solving

$$\max_{p^f} p^r F(\overline{p}) + p^f [1 - F(\overline{p})] \tag{7}$$

and

$$\min_{p^r} p^r F(\overline{p}) + p^f [1 - F(\overline{p})] \tag{8}$$

The first-order conditions for this optimization problem are[16]

$$1 - F(\overline{p}) = (p^f - p^r) f(\overline{p})/2 \tag{9}$$

[15] As in Farber (1980) and subsequent papers I do not explicitly model a first stage where parties can bargain before going to arbitration. We can think of p^r and p^f as the last offers during the bargaining period.

[16] Note that the convexity of the arbitrator's utility function assures the existence of equilibrium.

and

$$F(\overline{p}) = (p^f - p^r) f(\overline{p})/2 \tag{10}$$

that rearranged yields

$$F(\overline{p}) = 1/2 \tag{11}$$

and

$$p^f - p^r = 1/f(\overline{p}) \tag{12}$$

Eqs. (11) and (12) summarize Farber's (1980) Nash equilibrium: parties' offers are centered around the mean of the parties' belief about the arbitrator's ideal settlement (i.e., $E_p[z]$, where $E[\cdot]$ is the expected value operator) and the distance between the equilibrium offers decreases as this belief becomes more precise (i.e., higher $f(\cdot)$). Notice that in equilibrium $p^f > p^r$, as previously assumed. In deciding about their offers, each party must balance a trade-off between making a more aggressive offer and reducing the probability that the offer will be chosen by the arbitrator. In the limit, when there is no uncertainty about the arbitrator's preferences (h infinitely large), both parties submit the arbitrator's ideal settlement, that is $p^r = p^f = z$.

The equilibrium values of p^r and p^f depend on $F(z)$. Parties know from (3) that the arbitrator's ideal settlement (in the absence of learning) would be

$$z(c^a) = M^a(c^a) = \frac{hm + h^a c^a}{h + h^a} \tag{13}$$

Given c^p, parties know that $F(z(c^a))$ is a normal distribution with mean m' ($E_p[z] = m'$) and precision h', where

$$m' = \frac{hm + h^a M^p(c^p)}{h + h^a} \tag{14}$$

and

$$h' = \frac{(h + h^p)(h + h^a)}{h^a(h + h^a + h^p)} \tag{15}$$

which imply that the equilibrium offers reduce to

$$p^f = m' + \sqrt{\frac{\pi}{2h'}} \tag{16}$$

and

$$p^r = m' - \sqrt{\frac{\pi}{2h'}} \tag{17}$$

Note that c^p has an effect on the parties' equilibrium offers not because it improves their knowledge about c but because it affects parties' belief about the arbitrator's ideal settlement.

2.3 Learning

As explained by Gibbons (1988), it is not sequentially rational for the arbitrator to ignore parties' offers because she can learn from them. In fact, the average of the offers is m', so from (3) and (14), the arbitrator can obtain a point estimate of c^p, that is $c^p(m')$. Sequential rationality then requires that the arbitrator's ideal settlement be not $M^a(c^a)$ but $M^{ap}(c^a, c^p(m'))$, which, from (5), is given by

$$z(c^a, p^f, p^r) = \frac{hm + h^a c^a + h^p c^p(m')}{h + h^a + h^p} \tag{18}$$

In this way, the parties' offers help the arbitrator to have a more precise estimate, in statistical terms, of c. Knowing that the arbitrator may learn from their offers, each party now takes also into account the effect that his/her offer can have on the arbitrator's inference about the ideal settlement. Gibbons (1988) demonstrates that there exists a separating perfect Bayesian equilibrium in which the arbitrator perfectly infers c^p from the average of the parties' offers. Despite parties consider the gain from misleading the arbitrator when choosing their offers, in equilibrium parties find it optimal not to do so. To save on space, I leave the development of the learning equilibrium for the multiple-offers case (Section 4).

3 Multiple offers without learning

An important difference between Farber's and Gibbon's models and the regulatory scheme studied in this paper is that parties do not submit a single offer but multiple offers. Consider then the case in which the regulated firm is divided in two units or production centers: 1 and 2 (e.g., water production and water distribution).[17] Note that the possibility of submitting multiple offers only affect parties' strategy space but not the actual operation of the water utility (the firm will minimize costs regardless the price chosen for each unit), so both parties and the arbitrator only care about the overall offer $p = p_1 + p_2$ (i.e., about the final price to be paid by consumers) and not about the price of each individual unit.

I retain the information structure from the single-offer case in that $c = c_1 + c_2$, $\varepsilon^a = \varepsilon_1^a + \varepsilon_2^a$, and $\varepsilon^p = \varepsilon_1^p + \varepsilon_2^p$ are independent random variables with mean and precision as before. I do not

[17] The case with three or more offers yields same results (Montero, 2003).

impose, however, any particular correlation between c_1 and c_2 and between ε_1^j and ε_2^j, where $j = a, p$.

In this multiple-offer game, the regulator and the regulated firm submit simultaneously price offers for each of the two units. The regulator's individual offers are denoted by p_1^r and p_2^r and the firm's individual offers are denoted by p_1^f and p_2^f. The arbitrator's task is to choose a price offer for each unit following a final-offer arbitration procedure. The arbitrator will choose prices p_1 and p_2 that maximize its utility $v_a(p_1, p_2, z) = -(p_1 + p_2 - z)^2$. Then, there will be four possible offer combinations for the arbitrator to choose from: $\{p_1^r, p_2^r\}$, $\{p_1^f, p_2^r\}$, $\{p_1^r, p_2^f\}$ and $\{p_1^f, p_2^f\}$. In this section I study the case of no learning and leave for the next section the case in which the arbitrator uses the parties' offers to obtain a better estimate of c.

3.1 Certainty about the arbitrator's preferences

I start by studying the game in which both parties know the arbitrator's ideal settlement (i.e., $\varepsilon_k^a = \varepsilon_k^p = 0$, where $k = 1, 2$) because it helps to illustrate equilibrium properties that carry over to the case in which parties are uncertain about the arbitrator's ideal settlement. Parties' action space and arbitrator's ideal settlement z are depicted in Figure 1. More specifically, parties' offers for units 1 and 2 are in the horizontal and vertical axis, respectively. For example, point A represents a regulator's offer consisting of ${}^A p_1^r$ for the first unit and ${}^A p_2^r$ for the second unit. The line z, on the other hand, contains those combinations of p_1 and p_2 that add up to z. The arbitrator is indifferent between any two combinations that lie on this line.

As in the one-offer case, an obvious equilibrium of the game is for each party i to submit a pair $\{p_1^i, p_2^i\}$ where $p^i \equiv p_1^i + p_2^i = z$. We know that if party i submits an overall offer of $p^i = z$, party $-i$'s best response is not constrained to any offer because the arbitrator would pick p^i regardless his offer. But for $p^i = z$ to be a best response to party $-i$'s offer, we must necessarily have $p^{-i} \equiv p_1^{-i} + p_2^{-i} = z$.

Let us explore now whether a pair of offers equally distant from the line z, such as A and B in Figure 1 ($\overline{OA} = \overline{OB}$), could also constitute an equilibrium of the game . If this were the case, we could observe offers divergence in equilibrium but with the same settlement outcome as above. In fact, the arbitrator would be indifferent between the pairs $\{{}^A p_1^r, {}^B p_2^f\}$ and $\{{}^B p_1^f, {}^A p_2^r\}$ because both yield z; her ideal settlement. However, this is not a suitable equilibrium candidate. If the regulator is playing A, the firm's best response is not playing B but playing C, where $\overline{O'C'} = \overline{O''C''} = \overline{OA} - \epsilon$ and ϵ is a very small positive number. This play leaves the arbitrator

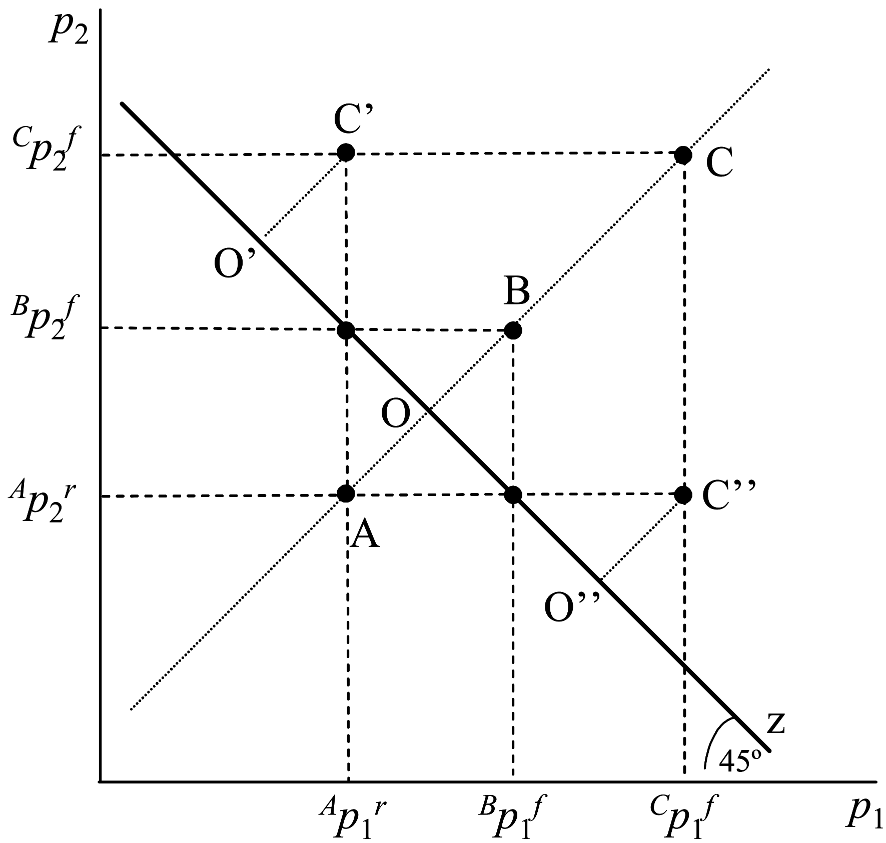

Figure 1: Two-offers game under certainty

indifferent between $C' = \{^{A}p_1^r, ^{C}p_2^f\}$ and $C'' = \{^{C}p_1^f, ^{A}p_2^r\}$ with a price settlement of $z + \overline{AO}$ $-\epsilon > z$).[18] And following the same logic, we know that A cannot be the best response to C but something further apart (more precisely, three times larger than $\overline{OC}$). As this illustration shows, there is no best-response correspondence off the z-line. To summarize

Proposition 1 *If both parties know the arbitrator's preference z, the Nash equilibria of the two-offers game are $p^i \equiv p_1^i + p_2^i = z$ for $i = r, f$.*

This proposition indicates that the introduction of multiple offers (as many as the number of units in which the firm has been divided) does not affect the perfect convergence of parties' offers when there is certainty about the arbitrator's preferences. Although it has only been formally shown for the two-offers case, it should be clear that Proposition 2 extends to the case

[18] If for any reason the regulator's offer is to the north-east of the line z, the firm's best response is to play any pair equally or further distant from z in the north-east direction.

of three or more offers.[19] This is an interesting result because one would think that as the number of offers increase the arbitration process would converge to conventional arbitration in the sense that the arbitrator can impose almost any settlement she wishes by choosing the right combination of parties' offers. But in conventional arbitration we know that in equilibrium we can observe either any offers (as in any cheap-talk game) or maximum differentiation if the arbitrator is believed to split differences.

3.2 Uncertainty about the arbitrator's preferences

Let us now turn to the more realistic case in which the parties are uncertain about the arbitrator's preferences but let maintain the assumption, for now, that the arbitrator ignores parties' offers in constructing her ideal settlement. To estimate the probability that the arbitrator choose a particular offer combination we need first to understand some regularities that prevail in equilibrium. From the certainty case we know that if the regulator plays something like A, the firm's best response will lie somewhere along the line ABC depending on the value of z (if by any chance the z-line falls to the south-west of A, the firm will pick A). This implies that in equilibrium we must have $p_k^f > p_k^r$ for $k = 1, 2$,[20] which, in turn, assures that $p^f > p^r$ in equilibrium.

Since p_1 and p_2 are perfect substitutes, we can adopt the convention that in equilibrium $p_2^i \geq p_1^i$ for $i = r, f$, which leads to $p_2^f - p_1^f \geq p_2^r - p_1^r$. The probabilities can then be found by dividing the z space in four different regions, each supporting the election of one particular offer combination. Depending on the parties' offers there will be values $z_1 < z_2 < z_3$ such that if z falls in the region $(-\infty, z_1)$, the arbitrator will choose $\{p_1^r, p_2^r\}$, if z falls in the region $[z_1, z_2)$ the arbitrator will choose $\{p_1^f, p_2^r\}$, if z falls in the region $[z_2, z_3)$ the arbitrator will choose $\{p_1^r, p_2^f\}$, and if z falls in the region $[z_3, +\infty)$ the arbitrator will choose $\{p_1^f, p_2^f\}$.

As before, the parties' Nash equilibrium offers maximize their expected payoffs so are found

[19] A simple example should be enough here. Consider a three-offers game in which the arbitrator's ideal settlement is $z = \$10$. If the regulator submits the offer $p^r = \{1, 2, 3\}$, which is \$4 off the z-plane, the firm's best response is not to play a symmetrically distant offer such as $p_a^f = \{3, 5, 6\}$ but to play $p_b^f = \{8.99, 9.99, 10.99\}$, where 0.01 is the smallest possible number, say, a penny. By submitting the latter the firm assures itself a settlement of 13.99. Since p^r is, by the same arguments, not the regulator's best response to p_b^f, we cannot have an equilibrium with parties' offers located off the z-plane.

[20] It is a strict inequality because in this uncertainty environment there will be at least one z-line to the north-east of A.

by simultaneously solving

$$\max_{p_1^f, p_2^f} (p_1^r + p_2^r)F(z_1) + (p_1^f + p_2^r)[F(z_2) - F(z_1)] \\ + (p_1^r + p_2^f)[F(z_3) - F(z_2)] + (p_1^f + p_2^f)[1 - F(z_3)] \quad (19)$$

$$\min_{p_1^r, p_2^r} (p_1^r + p_2^r)F(z_1) + (p_1^f + p_2^r)[F(z_2) - F(z_1)] \\ + (p_1^r + p_2^f)[F(z_3) - F(z_2)] + (p_1^f + p_2^f)[1 - F(z_3)] \quad (20)$$

where

$$z_1 = (p_1^r + 2p_2^r + p_1^f)/2 \quad (21)$$

$$z_2 = (p_1^r + p_2^r + p_1^f + p_2^f)/2 \quad (22)$$

$$z_3 = (p_1^r + p_1^f + 2p_2^f)/2 \quad (23)$$

and $F(z)$ is a cumulative normal distribution with mean and precision given, respectively, by (14) and (15).[21]

The first-order conditions for this optimization problem are[22]

$$[p_1^f] : 1 - F(z_1) + F(z_2) - F(z_3) + (p_1^r - p_1^f)[f(z_1) - f(z_2) + f(z_3)]/2 + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (24)$$

$$[p_2^f] : 1 - F(z_2) + (p_1^r - p_1^f)[-f(z_2)/2 + f(z_3)] + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (25)$$

$$[p_1^r] : F(z_1) - F(z_2) + F(z_3) + (p_1^r - p_1^f)[f(z_1) - f(z_2) + f(z_3)]/2 + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (26)$$

$$[p_2^r] : F(z_2) + (p_1^r - p_1^f)[f(z_1) - f(z_2)/2] + (p_2^r - p_2^f)f(z_2)/2 = 0 \quad (27)$$

Although the solution involves multiple equilibria as in the certainty case (any of the four equations is a linear combination of the other three; in particular $[p_1^f] + [p_1^r] = [p_2^f] + [p_2^r]$ where $[p_k^i]$ denotes the first-order condition for p_k^i), they all must satisfy the conditions above that rearranged leads to

[21] Note also that $z_3 - z_2 = z_2 - z_1 = p_2^f - p_2^r > 0$ and that $z_2 = \overline{p}$.

[22] Identical FOCs will be obtained if we adopt the alternative convention that in equilibrium $p_1^i \geq p_2^i$ for $i = r, f$.

Proposition 2 *The parties' overall offers* $p^r = p_1^r + p_2^r$ *and* $p^f = p_1^f + p_2^f$ *are centered around* $E_p[z]$ *and the distance between them can be anywhere between the distance in the single-offer case and above.*

Proof. Let first prove that the parties' offers are centered around $E_p[z]$, i.e., $F(z_2 = \overline{p}) = 1/2$. Combine (24) with (26) and (25) with (27) to obtain, respectively

$$F(z_2) = F(z_1) + F(z_3) - 1/2 \tag{28}$$

$$F(z_2) = 1/2 + (p_1^f - p_1^r)[f(z_1) - f(z_3)]/2 \tag{29}$$

In addition, we know that

$$z_3 - z_2 = z_2 - z_1 \tag{30}$$

Given the perfect colinearity between first-order conditions (which implies that we have 3 equations for 4 unknowns), we can make an unrestricted selection for one of the 4 offers, or alternatively, for $\Delta \equiv p_1^f - p_1^r \geq 0$. Furthermore, any particular value of Δ leads to a unique equilibrium given the parties' objective functions (including the arbitrator's) that we are considering here.[23] And since $f(z_1) = f(z_3)$ and $F(z_2) = 1/2$ is an equilibrium candidate in that solves the system (28)–(30) for any $\Delta \geq 0$ and a symmetric density function such as the normal distribution, uniqueness implies that $z_2 = E_p[z]$. On the other hand, to find an expression for the distance between parties' offers add (24) and (26) and rearrange to obtain

$$p^f - p^r = \frac{1}{f(z_2)} - (p_1^f - p_1^r)\left[\frac{f(z_3) + f(z_1) - 2f(z_2)}{f(z_2)}\right] \tag{31}$$

where $p^f = p_1^f + p_2^f$ and $p^r = p_1^r + p_2^r$. Replacing $f(z_3) = f(z_1)$ and $z_2 = \overline{p} = E_p[z]$, eq. (31) can be re-written as

$$p^f - p^r = \frac{1}{f(\overline{p})} - 2(p_1^f - p_1^r)\left[\frac{f(z_1) - f(\overline{p})}{f(\overline{p})}\right] \tag{32}$$

Since $\Delta \equiv p_1^f - p_1^r \geq 0$ and $f(z_1) \leq f(\overline{p})$, the distance between offers cannot be smaller than in the single-offer case. ∎

[23] Uniqueness can be easily proved using the results from the certainty case. If the regulator's offer is, say, the pair A of Figure 1, the firm's best response for a given value of z is unique and equal to the pair C of Figure 1 (if for some value of z the pair A falls to the north-east of the z-line, the firm's best response is A). And since the firm's best response is a non-decreasing function of z (strictly increasing if A is to the south-west of the z-line), the firm's best response to A is unique when z distributes according to $F(z)$.

Provided that in the absence of learning $F(\cdot)$ is a normal distribution with mean m' and precision h', the parties' (overall) equilibrium strategies satisfy

$$p^f = m' + \sqrt{\frac{\pi}{2h'}} + \gamma \tag{33}$$

and

$$p^r = m' - \sqrt{\frac{\pi}{2h'}} - \gamma \tag{34}$$

where $p^f = p_1^f + p_2^f$, $p^r = p_1^r + p_2^r$, and γ is an arbitrary non-negative value that corresponds to the last term of (32).

Unlike in the single-offer game, these equilibrium offers show that the introduction of uncertainty regarding parties' perception about the arbitrator's preferences has significant implications in the multiple-offers game. If parties are fully certain about the arbitrator's ideal settlement, the equilibrium of the game shows perfect convergence but if parties are just a bit uncertain, divergence between parties' offers can be arbitrarily large.

This likely increase in offers divergence raises the key question that to what extent the use of multiple offers prevents the arbitrator to improve her knowledge about the cost c. One may find hard to believe that the arbitrator can learn the same about c regardless whether parties' offers are close to each other or very far apart. I turn to this issue in the following section.

4 Multiple offers with learning

We now turn to the central model of the paper. Since we have already seen that is not sequentially rational for the arbitrator to ignore parties' offers, the objective of this section is to show, as in Gibbons' single-offer game, that there exists a separating perfect Bayesian equilibrium in this multiple-offer final-offer arbitration game. Suppose that the arbitrator believes that $\overline{p}$, the average of the parties' overall offers, perfectly reveals c^p, both on and off the equilibrium path. This means that for any pair of multiple-offers, $\mathbf{p}^f = \{p_1^f, p_2^f\}$ and $\mathbf{p}^r = \{p_1^r, p_2^r\}$, the arbitrator computes the point estimate $c^p = c^p(\overline{p} = (p_1^f + p_2^f + p_1^r + p_2^r)/2)$.[24] From (5), the arbitrator's

[24] Gibbons mentions that other separating equilibria may exist in which a different function of $\mathbf{p}^f$ and $\mathbf{p}^r$ reveals c^p to the arbitrator. I see this as a reasonably possibility in the multiple-offers case with random variables that are not normally distributed because in such a case the parties' equilibrium offers are no longer center around $E_p[z]$ but they can be above or below $E_p[z]$ (Montero, 2003).

ideal settlement is then

$$z(c^a, \mathbf{p}^f, \mathbf{p}^r) = \frac{hm + h^a c^a + h^p c^p(\overline{p})}{h + h^a + h^p} \tag{35}$$

As in the no-learning case, depending on the parties' offers there will be cut-off values $z_1 < z_2 < z_3$ such that if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $(-\infty, z_1)$, the arbitrator will choose $\{p_1^r, p_2^r\}$ as the settlement, if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $[z_1, z_2)$, she will choose $\{p_1^f, p_2^r\}$, if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $[z_2, z_3)$ she will choose $\{p_1^r, p_2^f\}$, and if $z(c^a, \mathbf{p}^f, \mathbf{p}^r)$ falls in the region $[z_3, +\infty)$, she will choose $\{p_1^f, p_2^f\}$, where z_1, z_2 and z_3 are given by (21), (22) and (23), respectively.

Using (35), we can then express the event that the arbitrator chooses $\{p_1^r, p_2^r\}$ as $c^a < C_1(z_1, z_2)$, that she chooses $\{p_1^f, p_2^r\}$ as $C_1(z_1, z_2) \leq c^a < C_2(z_2)$, that she chooses $\{p_1^r, p_2^f\}$ as $C_2(z_2) \leq c^a < C_3(z_2, z_3)$, and that she chooses $\{p_1^f, p_2^f\}$ as $C_3(z_2, z_3) < c^a$, where (recall that $z_2 = \overline{p}$)

$$C_1(z_1, z_2) = \frac{h^a z_1 + h(z_1 - m) + h^p(z_1 - c^p(z_2))}{h^a} \tag{36}$$

$$C_2(z_2) = \frac{h^a z_2 + h(z_2 - m) + h^p(z_2 - c^p(z_2))}{h^a} \tag{37}$$

$$C_3(z_2, z_3) = \frac{h^a z_3 + h(z_3 - m) + h^p(z_3 - c^p(z_2))}{h^a} \tag{38}$$

Given the probability that the parties assign to each of these four events occurring, a derivation analogous to that leading to the first order conditions (24)–(27) results in the following equilibrium conditions

$$\begin{aligned}[p_1^f] : 1 - F(C_1(z_1, z_2)|c^p) + F(C_2(z_2)|c^p) - F(C_3(z_2, z_3)|c^p) + (p_2^r - p_2^f) f(C_2|c^p) \frac{\partial C_2}{\partial p_1^f} \\ + (p_1^r - p_1^f) \left[f(C_1|c^p) \frac{\partial C_1}{\partial p_1^f} - f(C_2|c^p) \frac{\partial C_2}{\partial p_1^f} + f(C_3|c^p) \frac{\partial C_3}{\partial p_1^f} \right] = 0\end{aligned} \tag{39}$$

$$[p_2^f] : 1 - F(C_2|c^p) + (p_2^r - p_2^f) f(C_2|c^p) \frac{\partial C_2}{\partial p_2^f} + (p_1^r - p_1^f) \left[f(C_1|c^p) \frac{\partial C_1}{\partial p_2^f} - f(C_2|c^p) \frac{\partial C_2}{\partial p_2^f} + f(C_3|c^p) \frac{\partial C_3}{\partial p_2^f} \right] = 0 \tag{40}$$

$$\begin{aligned}[p_1^r] : F(C_1|c^p) - F(C_2|c^p) + F(C_3|c^p) + (p_2^r - p_2^f) f(C_2|c^p) \frac{\partial C_2}{\partial p_1^r} \\ + (p_1^r - p_1^f) \left[f(C_1|c^p) \frac{\partial C_1}{\partial p_1^r} - f(C_2|c^p) \frac{\partial C_2}{\partial p_1^r} + f(C_3|c^p) \frac{\partial C_3}{\partial p_1^r} \right] = 0\end{aligned} \tag{41}$$

$$[p_2^r]: F(C_2|c^p)+(p_2^r-p_2^f)f(C_2|c^p)\frac{\partial C_2}{\partial p_2^r}+(p_1^r-p_1^f)\left[f(C_1|c^p)\frac{\partial C_1}{\partial p_2^r}-f(C_2|c^p)\frac{\partial C_2}{\partial p_2^r}+f(C_3|c^p)\frac{\partial C_3}{\partial p_2^r}\right]=0 \tag{42}$$

where $F(\cdot)$ is now the distribution of c^a conditional on c^p, which is normal with mean $M^p(c^p)$ given by (3) and precision

$$H' = \frac{(h+h^p)h^a}{h+h^a+h^p} \tag{43}$$

As in the no-learning case, the first-order conditions (39)—(42) do not lead to a unique equilibrium because any of the four conditions is a linear combination of the other three. A derivation analogous to that leading to (28) and (29) then yields the equilibrium conditions

$$F(C_2(z_2)|c^p) = F(C_1(z_1,z_2)|c^p) + F(C_3(z_2,z_3)|c^p) - 1/2 \tag{44}$$

$$F(C_2(z_2)|c^p) = \frac{1}{2} + \frac{(p_1^f-p_1^r)(f(C_1(z_1,z_2)|c^p) - f(C_3(z_2,z_3)|c^p))}{2} \cdot \left(\frac{\partial C_3}{\partial p_2^f} - \frac{\partial C_1}{\partial p_2^f}\right) \tag{45}$$

Since $\partial C_3/\partial p_2^f > \partial C_1/\partial p_2^f$, from the arguments leading to Proposition 2 we know that these two conditions imply that $C_2(z_2) = M^p(c^p)$.

To compute $C_2(z_2)$ (and also $C_1(z_1,z_3)$ and $C_3(z_2,z_3)$) we make use of the properties that $c^p(z_2)$, the rule the arbitrator uses to infer the value of c^p from the parties' offers, must satisfy in equilibrium. If the equilibrium value of $\overline{p}$ $(= z_2)$ is to reveal c^p, it must hold that $c^p(z_2) = c^p$ in equilibrium, so substituting $c^p(z_2)$ for c^p in $M^p(c^p)$ and using $C_2(z_2) = M^p(c^p)$ yields

$$c^p(\overline{p} = z_2) = \frac{(h+h^p)\overline{p} - hm}{h^p} \tag{46}$$

Replacing (46) into (36)—(38) yields $C_1(z_1,z_2) = z_1 - (z_2-z_1)(h+h^p)/h^a$, $C_2(z_2) = z_2$ and $C_3(z_2,z_3) = z_3 + (z_3-z_2)(h+h^p)/h^a$. The results imply both that parties' offers are center around the mean of the parties' belief about the arbitrator's ideal settlement ($\overline{p} = E_p[z] = M^p(c^p)$) and that, by arguments analogous to those leading to (32), the distance between the parties' offers is given by

$$p^f - p^r = \frac{1}{f(\overline{p})} - 2(p_1^f - p_1^r)\left[\frac{f(C_1(z_1,\overline{p}) - f(\overline{p})}{f(\overline{p})}\right] \tag{47}$$

where $p_1^f - p_1^r \geq 0$.

Provided that $F(\cdot)$ is a normal distribution with mean $M^p(c^p)$ and precision H' given by

(43), the parties' (overall) equilibrium strategies satisfy

$$p^f = M^p(c^p) + \sqrt{\frac{\pi}{2H'}} + \Gamma \tag{48}$$

and

$$p^r = M^p(c^p) - \sqrt{\frac{\pi}{2H'}} - \Gamma \tag{49}$$

where $p^f = p_1^f + p_2^f$, $p^r = p_1^r + p_2^r$, and Γ is an arbitrary non-negative value that corresponds to the last term of (47). These results can be summarized in the following proposition

Proposition 3 *The parties' offers strategies in (48) and (49) and the arbitrator's decision strategy based on her ideal settlement (35) and inference rule (46) constitute a separating perfect Bayesian equilibria of the multiple-offers final-offer arbitration game. As in Gibbons (1988), in this equilibrium the arbitrator's ideal settlement can be written as* $z = \alpha\overline{p} + (1-\alpha)c^a$*, where* $\alpha = (h + h^p)/(h + h^a + h^p)$*.*

In this separating equilibrium, the arbitrator infers c^p from the average of the parties' overall offers (not from the absolute value of the offers submitted to each cost unit) according to (46), then uses this value in (35) to compute her "Bayesian-updated" ideal settlement, and finally chooses the combination of individual offers that is closer to this ideal settlement.[25] Anticipating this, parties find it optimal not to mislead the arbitrator and submit offers satisfying (48) and (49). As the precision of the parties' signal about the true cost c increases relative to that of the arbitrator' signal, the arbitrator puts more weight on the information coming from the parties' offers than on her own signal in constructing her ideal settlement.

One of the main implications of Proposition 3 is that the multiplicity of offers does not affect the arbitrator's ability to learn from the parties' offers despite they may exhibit great divergence. The reason for this is that the regulator does not learn from the absolute value of individual offers but rather from the way offers are related. Since the multiplicity of offers does not remove the regularities that the parties' offers must exhibit in equilibrium, the arbitrator uses these regularities (eqs. (44) and (45)) to correctly infer parties' private information from their offers.

[25] Note that the arbitrator uses the same inference rule (46) regardless whether parties' offers are on or off the equilibrium path.

5 Flexibility from multiple offers

If the introduction of multiple offers does not affect learning, despite parties' offers can exhibit substantial divergence, one could argue that the use of multiple offers is socially desirable as long as it provides the arbitrator with more flexibility to put together a settlement closer to her ideal settlement (i.e., the true cost of the efficient firm) by combining offers from both parties.

To explore such possibility, consider the following example in which parties' expectation about the arbitrator's ideal settlement in the single-offer arbitration game is 20 and that the equilibrium offers of such game are $p^r = 10$ and $p^f = 30$. Since the multiplicity of offers does not impaired arbitrator's learning possibilities, parties' expectation about the arbitrator's ideal settlement is also 20 in the two-offers game. Consider now two equilibrium candidates of the two-offers game: (i) $p_1^r = 1$, $p_2^r = 2$, $p_1^f = 18$, and $p_2^f = 19$; and (ii) $p_1^r = 1$, $p_2^r = 2$, $p_1^f = 3$, and $p_2^f = 34$. Both equilibrium candidates and the single-offer equilibrium satisfy the condition that the average of parties' overall offers is 20. But clearly, candidate (i) is, in expected terms, the most attractive to the arbitrator since it allows her to impose 20 as the final price by either choosing $\{p_1^f, p_2^r\}$ or $\{p_1^r, p_2^f\}$. Candidate (ii), on the other hand, is the least attractive to the arbitrator because she is expected to be off by 15 from her ideal choice.

The example seems to suggest that the welfare effects from introducing multiple offers depend to a large extent on the way parties' offers are structured in equilibrium. As it turns out, candidate (ii) is not a suitable equilibrium because it fails to satisfy equilibrium condition (47). It is then possible to establish

Proposition 4 *By combining offers from both parties, the two-offers game provides the arbitrator with flexibility to construct a settlement that is expected to be closer to her ideal settlement than the single-offer game does.*

Proof. Since overall offers $p^r = p_1^r + p_2^r$ and $p^f = p_1^f + p_2^f$ are as least as close to each other than the offers in the single-offer game are, we need to demonstrate that either $p^{fr} = p_1^f + p_2^r$, $p^{rf} = p_1^r + p_2^f$ or both (where $p^{rf} > p^{fr}$) are expected to be closer to the arbitrator's ideal settlement than the offers in the single-offer game are. From Proposition 3, we know that $(p^{fr} + p^{rf})/2 = \overline{p} = E_p[z]$. In addition, rearranging (47) yields

$$p^{rf} - p^{fr} = \frac{1}{f(\overline{p})} - 2(p_1^f - p_1^r)\frac{f(C_1(z_1, \overline{p})}{f(\overline{p})} < \frac{1}{f(\overline{p})}$$

which finishes the proof. ■

6 Concluding remarks

Motivated by the price-setting process in the water sector in Chile, I have developed a multiple-offers final-offer arbitration model. The main result of the paper is that despite the increase in divergence between parties' offers, the use of multiple offer helps the arbitrator to establish a final price closer to her ideal settlement (i.e., the long-run cost of a hypothetical efficient firm) without affecting her ability to learn from the parties' offers about the true cost of the efficient firm.

Since moving from a single-offer scheme to a multiple-offers scheme, whether with two or more offers, can increase the divergence between the parties' overall offers by an arbitrarily large amount in equilibrium, one of the practical implications of the paper is that authority's proposal that call for a reduction in the number of cost units from something around 200 to 50 offers (or down to two offers for that matter) would make little difference, if any, in its effort to lower the divergence between parties' offers. I found, however, that failing to reduce divergence is less of a concern because divergence does not affect the arbitrator's ability to learn from the parties' offers.

Part of our results depend on the information assumption that parties have symmetric information about the cost of the efficient firm. It is likely, instead, that each of the parties will be better informed about some aspects of the efficient firm than the other party. As mentioned by Gibbons (1988), it is possible that such information asymmetry may influence both the means and the substance of the parties' communication with the arbitrator. The effect can be even larger in multiple-offer arbitration if the arbitrator has a good idea that such party is better informed about that aspect of the efficient firm than the other party. This is an interesting, although difficult, direction for further research.

Another question that deserves future work is why parties came to be in arbitration in the first place. The data summarized in Table 1 provide some insights. Ownership status seems to explain, at least in part, why some parties are more likely to reach agreement than others. In fact, for 3 of the 6 privately-owned companies,[26] prices were determined through arbitration while for only 2 of the 9 state-owned companies, prices were determined in such a way. Firm

[26] With the exception of Aguas Cordillera, these companies have gone private only recently: 1-2 years before the price reviews.

size, which may serve as a proxy for firm's complexity and uncertainty about the arbitrator' preferences, also seems relevant (although the largest two firms also happens to be in private hands). Given the small sample size, however, there is no much else that can be said.

If we believe that negotiated settlements are valuable from a policy standpoint because it allows parties more discretion in negotiating their own settlement (Farber, 1980), it is also relevant to understand whether and how a reduction (or increase) in the number of offers affect the likelihood of parties ending up in arbitration. Empirical and experimental work comparing conventional and single-offer final-offer arbitration shows that it is not clear whether dispute rates (i.e., number of negotiations that end in arbitration) and distance between parties' offers are greater in conventional arbitration than in final-offer arbitration (Farber and Bazerman, 1986 and 1989; and Ashenfelter et al., 1992).

Finally, there is the question about the overall optimality of the regulatory approach studied in this paper relative to alternative approaches such as cost-of-return and price-cap schemes. Perhaps more realistic within the existing regulatory scheme, it is to ask for ways in which the construction of the hypothetical efficient firm could be improved. Following the yardstick regulatory scheme practiced in the water sector in the UK, one possibility it is to require, at least partially, the use of actual costs from previous review periods and from other water utilities.

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Table 1. Firms' characteristics, parties' offers and settlements

Firm	Location	Size	Ownership	p^r	p^f	p^s	FOA
ESSAT	I	3.3	state	100	148	118	yes
ESSAN	II	3.3	state	100	110	106	no
EMSSAT	III	1.9	state	100	112	102	no
ESSCO	IV	4.1	state	100	128	108	no
ESVAL	V	12.9	private	100	184	141	yes
SMAPA	MR	4.7	state	100	125	107	no
Aguas Cordillera	MR	2.7	private	100	156	113	no
Aguas Andinas	MR	37.2	private	100	256	139	yes
ESSEL	VI	4.3	private	100	137	109	no
ESSAM	VII	4.7	state	100	131	113	yes
ESSBIO	VIII	10.8	private	100	115	104	no
ESSAR	IX	4.4	state	100	127	112	no
ESSAL	X	3.9	private	100	146	117	yes
EMSSA	XI	0.6	state	100	137	108	no
ESMAG	XII	1.2	state	100	119	109	no

source: Superintendencia de Servicios Sanitarios (Agency of Water Services).

Size is the fraction of consumers served from the total number of consumers