4. ABOUT A NONLINEAR TWO-PARAMETER PREDICTION MODEL USED FOR INVESTIGATING THE DISTRIBUTION OF CO2 EMISSION IN EUROPE

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Abstract

The growth rate of different economies in Europe depends strongly on the production and, especially, on the consumption of energy resources. As an empirical rule, the development of such economies is closely interrelated with a consistent CO2 emission into atmosphere. However, this aspect has a negative impact on the quality of life of the people all over the world.

In the present article we propose a methodology to improve the estimator accuracy of a nonlinear two-parameter prediction model used in Albu (2007) for studying the impact of the greenhouse effect on the macroeconomic growth. In order to prove the validity of the methodological proposals, a stochastic Monte Carlo simulation technique was applied.

Keywords: nonlinear prediction model, parameter estimation, stochastic Monte Carlo simulation, the distribution of CO2 emission

JEL Classification: C46, C51, C53, O11, Q53

1. Introduction

Many forecasting studies concerning the adequate activities for the environment protection have as a main target to establish, for different countries, the appropriate expression of the dependence relationship between the CO2 emission into atmosphere and the consumption of the energy resources in those countries [Albu (2007); Zhihong, Dongxiao, Wang (2008)].

The prediction analysis is concentrated on the impact of the greenhouse effect against the economic growth in the future [Albu (2007)].

For modeling the process of climate change and its economic consequences it is very important to define the real problems, to establish the goals and how to approach the

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problem under prediction uncertainty, to evaluate the efforts and the limitations of the forecasting model, to suggest political advises [Schroder, Lingner (1999)].

The main difficulties in the climate forecasting arise from a huge dimensionality of the modeling system and from the accuracy estimation of the prediction model [Palmer (2003)].

As a rule, a complex mathematical calculus is used to evaluate the likeness of a proposed theoretical model to the reality. We mention here especially the nonlinear and the linear regression predictions, more complex dispersion studies or the Fourier analysis of the dynamic time series, all suggested hypothesis being finally validated by applying different statistical tests [Pisnichenko, Tarasova (2007)].

Albu (2007) established high correlations between the level of economic development, the energy consumption and the CO2 emission into atmosphere.

Thus, for a long-time forecasting, the existence for the European countries of some robust nonlinear dynamic relations was proved, having the following form:

$$y = g(x; a, b) \tag{1}$$

where:

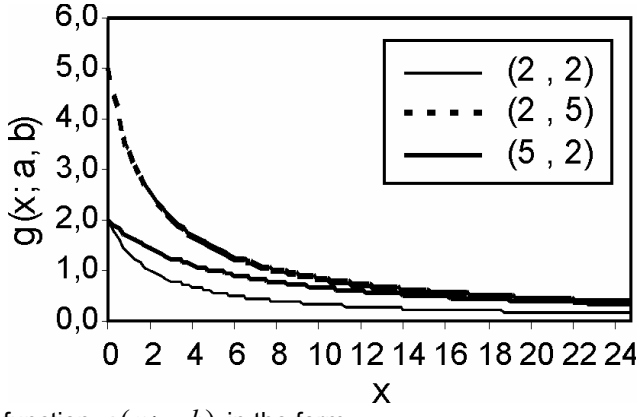
$$g(x; a, b) = \frac{ab}{a + x}, \quad x \geq 0 \tag{2}$$

y being, for example, the CO2 emission per unit of GDP and the explanatory variable x signifies the GDP per capita [Albu (2007)].

The specific expression (2) was imposed by the inhibitor role of the GDP per inhabitant variable x [Albu (2007)].

The shape of the forecasting nonlinear function (2) depends on the concrete values of the parameters $a > 0$ and $b > 0$ (see Graph G1).

G1. The graph of the function g(x ; a , b) for different values of the parameters (a , b)



Rewritting the function $g(x; a, b)$ in the form

$$g(x;a,b)=\frac{b}{1+x/a},\quad x\geq 0 \tag{3}$$

we interpret the parameter a as a scale factor.

The significance of the parameter b is imposed by the relation $b=g(0;a,b)$, that is the intersection point of the graph $y=g(x;a,b)$ with the Oy axis.

In the following, we will estimate the unknown values of the parameters a,b when n independent observations (x_i,y_i), $1\leq i\leq n$, about a (X,Y) random vector are given.

More exactly, the quantity y_i will be interpreted as an independent variate of the random variable Y, when the variable X has taken the value x_i, $1\leq i\leq n$.

2. The Linearization of the Prediction Model

When the values x_i are known, we will predict for the random variable Y the quantities $\hat{y}_i=g(x_i;a,b)$.

Then, from (2) we have

$$\frac{1}{\hat{y}_i}=\frac{1}{g(x_i;a,b)}=\frac{a+x_i}{ab},\quad 1\leq i\leq n$$

and hence

$$\frac{1}{\hat{y}_i}=c+dx_i,\quad 1\leq i\leq n \tag{4}$$

where:

$$c=\frac{1}{b}\qquad d=\frac{1}{ab} \tag{5}$$

Approximating the value $1/y_i$ by the quantity $1/\hat{y}_i$ we get an error e_i,

$$e_i=\frac{1}{y_i}-\frac{1}{\hat{y}_i}=\frac{1}{y_i}-(c+dx_i),\quad 1\leq i\leq n$$

In this case, the global error $h^{\#}(c,d)$ is given by the expression

$$h^{\#}(c,d)=\frac{1}{n}\sum_{i=1}^{n}e_i^2=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{y_i}-c-dx_i\right)^2 \tag{6}$$

The minimum value $h^{\#}(c^{\#},d^{\#})$ for the expression $h^{\#}(c,d)$ is attained when [Papoulis (1990)]:

$$d^{\#} = \frac{\overline{x/y} - (\bar{x}).\left(\overline{1/y}\right)}{\overline{x^2} - (\bar{x})^2} \qquad c^{\#} = \overline{1/y} - d^{\#}\,\bar{x} \tag{7}$$

with

$$\overline{1/y} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i} \qquad \overline{x/y} = \frac{1}{n}\sum_{i=1}^{n}\frac{x_i}{y_i}$$
$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i \qquad \overline{x^2} = \frac{1}{n}\sum_{i=1}^{n}x_i^2 \tag{8}$$

Taking into consideration the relations (5), the estimates $a^{\#}, b^{\#}$ for the parameters a, b are given by the expressions

$$a^{\#} = \frac{c^{\#}}{d^{\#}} \qquad b^{\#} = \frac{1}{c^{\#}} \tag{9}$$

3. Solving the Nonlinear Prediction Model

In fact, the model (1) asserts that for an x_i value of the explicative variable X we will forecast for the random variable Y the quantity $\hat{y}_i = g(x_i; a, b)$.

The prediction errors ε_i have in this approach the form

$$\varepsilon_i = y_i - \hat{y}_i = y_i - g(x_i; a, b), \quad 1 \le i \le n \tag{10}$$

Using the ordinary least squares principle, we shall obtain the estimates (a^*, b^*) for the unknown parameters (a, b) of the nonlinear model (2) [Bard (1974); Papoulis (1990)]. More precisely, $h^*(a^*, b^*)$ is just the minimum value for the function $h^*(a, b)$ which characterizes the mean square error of the statistical model, that is

$$h^*(a,b) = \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \frac{ab}{a + x_i}\right)^2 \tag{11}$$

The vector (a^*, b^*) is the solution of the following nonlinear system [Bard (1974); Demidovich, Maron (1981)]

$$\begin{cases} \dfrac{\partial h^*(a,b)}{\partial a} = 0 \\ \dfrac{\partial h^*(a,b)}{\partial b} = 0 \end{cases} \tag{12}$$

Since we have the restrictions $a > 0$, $b > 0$, the previous system is equivalent to

$$\begin{cases} h_1(a,b) = 0 \\ h_2(a,b) = 0 \end{cases} \tag{13}$$

where:

$$h_1(a,b)=\sum_{i=1}^{n}\frac{x_i y_i}{(a+x_i)^2}-a.b\sum_{i=1}^{n}\frac{x_i}{(a+x_i)^3}$$

$$h_2(a,b)=\sum_{i=1}^{n}\frac{y_i}{a+x_i}-a.b\sum_{i=1}^{n}\frac{1}{(a+x_i)^2} \quad (14)$$

Applying the classical Newton-Raphson method [Demidovich, Maron (1981), p.156-157; Bard (1974)], the solution (a^*,b^*) of the nonlinear system (13) can be obtained as a limit of the vector string (a_j,b_j), $j\in N$, which result after an iterative process

$$a^*=\lim_{j\to\infty} a_j \qquad b^*=\lim_{j\to\infty} b_j \quad (15)$$

Thus, primarily the starting point (a_1,b_1) is chosen in a neighborhood of the unknown solution (a^*,b^*). After that, the vectors (a_j,b_j) will be "adjusted" successively with the quantities (γ_j,δ_j), that is

$$a_{j+1}=a_j+\gamma_j \qquad b_{j+1}=b_j+\delta_j \quad (16)$$

The "corrections" (γ_j,δ_j) are the solution of the next linear system

$$\begin{cases} h_{11}(a_j,b_j)\gamma_j+h_{12}(a_j,b_j)\delta_j=-h_1(a_j,b_j) \\ h_{21}(a_j,b_j)\gamma_j+h_{22}(a_j,b_j)\delta_j=-h_2(a_j,b_j) \end{cases} \quad (17)$$

where the functions $h_{11},h_{12},h_{21},h_{22}$ have the expressions

$$h_{11}(a,b)=\frac{\partial h_1(a,b)}{\partial a}=-2\sum_{i=1}^{n}\frac{x_i y_i}{(a+x_i)^3}-b\sum_{i=1}^{n}\frac{x_i(x_i-2a)}{(a+x_i)^4}$$

$$h_{12}(a,b)=\frac{\partial h_1(a,b)}{\partial b}=-a\sum_{i=1}^{n}\frac{x_i}{(a+x_i)^3}$$

$$h_{21}(a,b)=\frac{\partial h_2(a,b)}{\partial a}=-\sum_{i=1}^{n}\frac{y_i}{(a+x_i)^2}-b\sum_{i=1}^{n}\frac{x_i-a}{(a+x_i)^3}$$

$$h_{22}(a,b)=\frac{\partial h_2(a,b)}{\partial b}=-a\sum_{i=1}^{n}\frac{1}{(a+x_i)^2} \quad (18)$$

The stopping rule of the iterative process (16) after k steps is defined by the following restrictions

$$\begin{cases} |h_1(a_k, b_k)| < \varepsilon_{1*} \\ |h_2(a_k, b_k)| < \varepsilon_{2*} \end{cases} \tag{19}$$

In fact, the quantities $\varepsilon_{1*} > 0$, $\varepsilon_{2*} > 0$ characterize the accuracy of the approximation (a_k, b_k) which will be used instead of the real solution (a^*, b^*) of the nonlinear system (13).

4. Comparing the Estimates ($a^{\#}$, $b^{\#}$), (a*,b*)

We will apply the stochastic Monte Carlo simulation in order to establish the accuracy of both estimations $(a^{\#}, b^{\#})$ and (a^*, b^*) of the unknown parameters (a, b) which define the forecasting model (1) [see Gentle (1998)].

For this reason, in the subsequent the values of the parameters a, b will be fixed.

If the quantities x_i are given, $1 \le i \le n$, we can get by simulation the corresponding sample $\{y_i\}_{1 \le i \le n}$, from the random variable Y. Thus, the observations y_i are generated by using the formula

$$y_i = \frac{ab}{a + x_i} + \sigma z_i, \qquad 1 \le i \le n \tag{20}$$

where: $z_1, z_2, z_3, \ldots, z_n$ are independent variates of the random variable Z which has a univariate standard normal distribution. A statistical interpretation of the model (20) is discussed in Papoulis (1990), p. 402-411.

Remark 1. In general, the computer software has specialized routines for generating independent observations from a normal random variable that has a zero mean and a variance equal to one [Gentle (1998)].

The expression (20) allows us to produce the data y_i, $1 \le i \le n$, used in the model (1), by considering normal distributed errors with a null mean and a dispersion equal to σ^2.

For analyzing the statistical characteristics of the estimators $(a^{\#}, b^{\#})$, (a^*, b^*), the results obtained from $t = 30$ stochastic simulations were used. In the subsequent, we denoted by s, $1 \le s \le t$, the index of the current simulation.

The stopping rule (19) for the iterative estimation process (16) is characterized by the thresholds $\varepsilon_{1*} = \varepsilon_{2*} = 10^{-7}$, the variable k designating the number of iterations to obtain (a^*, b^*).

Practically, to get the n values y_i of the predicted random variable Y (formula (19)) we operated with : $a = 3$, $b = 50$, $n = 100$, $x_i = i - 1$, $1 \le i \le n$, $\sigma \in \{0.05; 0.1; 0.15; 0.2; 0.25; 0.3\}$.

Table 1

The estimates ($a^{\#}$, $b^{\#}$), (a* , b*) for the parameters (a , b)

(a = 3 , b = 50 , σ = 0.1 , t = 30 , n = 100).

s	$a^{\#}$	$b^{\#}$	a*	b*	k
1	2.98850	49.76794	2.99435	50.04658	2
2	3.21397	47.29605	3.01439	49.95521	3
3	2.54977	58.10341	3.00014	50.05816	4
4	2.77021	53.76374	3.00265	49.88341	3
5	3.17905	47.66534	3.01499	49.85243	3
6	3.21948	46.84254	2.99839	50.03502	3
7	3.32547	45.24840	2.98959	50.00302	3
8	3.08390	48.74467	2.99009	50.08879	3
9	2.42182	60.59708	2.99181	49.99430	4
10	3.07461	48.75707	2.98444	50.10936	3
11	1.83516	79.19920	3.00252	49.96051	5
12	2.89003	51.52567	2.97698	50.19892	3
13	2.80303	52.73853	2.98511	50.07161	3
14	2.56380	57.85376	3.01657	49.92045	4
15	2.48452	58.96065	3.00003	49.97555	4
16	3.11193	48.48708	3.00728	49.93429	3
17	2.79961	53.50813	3.01296	50.00346	3
18	3.46323	43.86223	3.00266	49.90776	4
19	3.19100	47.34065	3.00010	49.99464	3
20	3.68131	41.62699	3.01275	49.94651	4
21	2.72500	54.43240	3.00236	49.88515	3
22	3.72142	41.09663	3.00442	50.00420	4
23	1.77943	80.97946	2.99094	50.06170	5
24	3.21984	46.82018	3.00152	49.98713	3
25	2.54722	58.15209	3.00922	49.91311	4
26	2.82978	52.46293	3.00949	49.92842	3
27	3.40784	44.95617	3.02006	50.00120	4
28	2.68467	55.16939	2.98250	50.00517	3
29	2.84962	52.26876	3.00234	49.89335	3
30	1.51649	94.56732	3.00723	49.98230	5

All the $t = 30$ stochastic simulation results obtained for $\sigma = 0.1$ are listed together in Table 1.

The graphs *G*1 and *G*2 illustrate the fluctuations of the estimates $(a^{\#}, a^*)$ and $(b^{\#}, b^*)$ when $a = 3$ and $b = 50$ were considered in (2).

Remark 2. The iterative process (16) is convergent to the solution (a^*, b^*) of the system (13) if the starting point (a_1, b_1) is in a close neighbor of (a^*, b^*) (to pursuit the explanations from Bard (1974) and Demidovich, Maron (1981)).

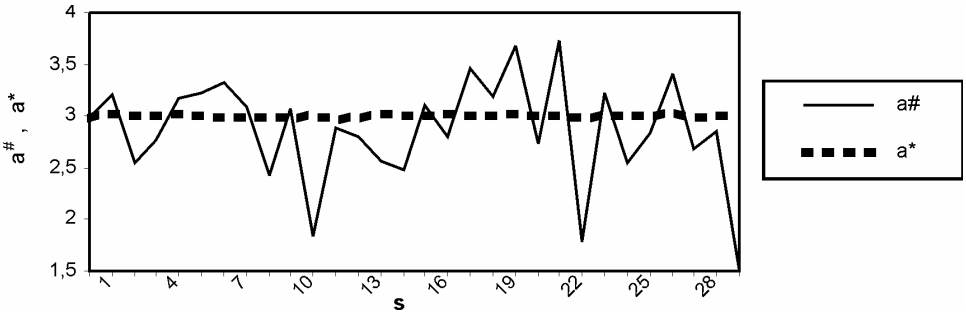
We strongly recommend to take $a_1 = a^{\#}$, $b_1 = b^{\#}$ as a starting point for the iterative algorithm (16), since the estimation $(a^{\#}, b^{\#})$ deduced after the linearization of the forecasting function (2) is a good approximation for (a^*, b^*) (to compare the corresponding values in Table 1).

The convergence of the iterative process (16) after less than $k = 10$ steps is assured if we choose $(a^{\#}, b^{\#})$ as starting point (Table 1). In this case, the corrections (γ_j, δ_j) obtained from (17) become very small, practically zero (to compare in Table 1 the estimations (a^*, b^*) with the real values (a, b); always it resulted in $k \leq 5$ iterations).

Remark 3. If a data sample of size $2n$ is taken by adding to the initial data (x_i, y_i), $1 \leq i \leq n$, the same set of data, then the values of the functions $h_1, h_2, h_{11}, h_{12}, h_{21}, h_{22}$ will be multiplied by two (see the form of the expressions (14) and (18)). Thus, in order to avoid large coefficients for the linear system (17) all the functions h_p, h_{pq} will be adjusted with the multiplicative factor $1/n$.

Taking into account all the results deduced after $t = 30$ stochastic simulations (Table 1), we conclude that for every Monte Carlo simulation s, $1 \leq s \leq t$, the estimates (a_s^*, b_s^*) in k_s iterations are always more accurate than the estimates $(a_s^{\#}, b_s^{\#})$ obtained from a linearization procedure of the forecasting function (2).

G2. The estimates a# , a* deduced at the simulations s (Table 1)



This aspect is also pointed out in Tables 2 and 3, which listed the averages $\bar{a}^{\#}, \bar{b}^{\#}, \bar{a}^*, \bar{b}^*, \bar{k}$ and the dispersions $\tilde{\bar{a}}^{\#}, \tilde{\bar{b}}^{\#}, \tilde{\bar{a}}^*, \tilde{\bar{b}}^*, \tilde{\bar{k}}$ of the values $a_s^{\#}, b_s^{\#}, a_s^*, b_s^*, k_s$, $1 \leq s \leq t$, resulted after $t = 30$ Monte Carlo simulations. More exactly, for any string $w_1, w_2, w_3, \ldots, w_t$ of t real values the following notations were used:

$$\overline{w} = \frac{1}{t}\sum_{s=1}^{t} w_s \qquad \overline{\widetilde{w}} = \frac{1}{t}\sum_{s=1}^{t} w_s^2 - \overline{w}^2 \tag{21}$$

Tables 2 and 3 present the fluctuations of the indicators $\overline{w}, \overline{\widetilde{w}}$ (formulas (21)), which depend on the dispersion σ^2 of the residuals from the simulation model (20), where

$$\overline{w} \in \left\{\overline{a}^{\#}, \overline{b}^{\#}, \overline{a}^{*}, \overline{b}^{*}, \overline{k}\right\} \quad , \quad \overline{\widetilde{w}} \in \left\{\overline{\widetilde{a}}^{\#}, \overline{\widetilde{b}}^{\#}, \overline{\widetilde{a}}^{*}, \overline{\widetilde{b}}^{*}, \overline{\widetilde{k}}\right\}.$$

We observe that usually an increase in the dispersion σ^2 implies the decrease in the precision of the estimators $(a^{\#}, b^{\#})$ or (a^*, b^*).

Moreover, for any dispersion value σ^2, the accuracy of the estimates (a^*, b^*) is clearly better, by comparing with the $(a^{\#}, b^{\#})$ values (to analyze the quantities listed in Tables 2 and 3).

G3. The estimates $b^{\#}$, b^* deduced at the simulation s (Table 1)

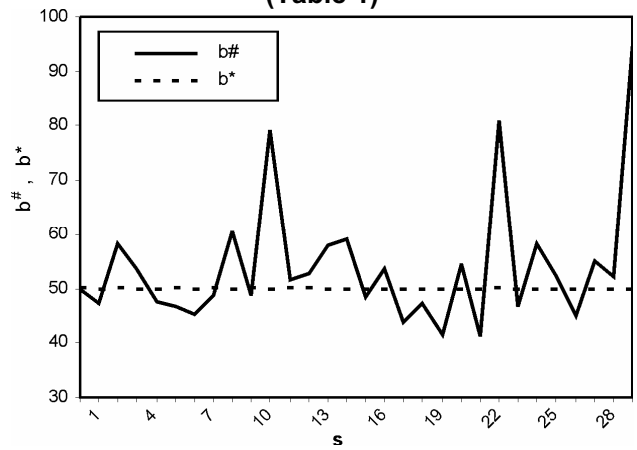


Table 2

The average $\overline{w}$ for t = 30 values w_s (formula (21))

σ^2	$a^{\#}$	$b^{\#}$	a*	b*	k
0.05	2.9074	51.7978	3.0001	49.9993	2.80
0.10	2.8644	54.0931	3.0009	49.9867	3.47
0.15	3.0655	49.9380	2.9994	50.0048	3.47
0.20	2.8470	54.8027	3.0013	50.0251	3.77
0.25	2.1935	145.6521	2.9983	50.0461	5.03
0.30	1.9056	119.9814	3.0024	49.9603	6.03

Table 3

The dispersion $\bar{\tilde{w}}$ for t = 30 values w_s (formula (21))

σ^2	$a^{\#}$	$b^{\#}$	a*	b*	k
0.05	0.05744	18.1812	0.000022	0.00217	0.4414
0.10	0.26799	139.8853	0.000119	0.00584	0.5333
0.15	0.21610	55.1619	0.000312	0.02192	0.3954
0.20	0.41925	145.4728	0.000573	0.04381	0.5989
0.25	1.43813	66757.0458	0.000702	0.04746	4.9299
0.30	1.87744	63197.3733	0.000945	0.08996	4.7230

Obviously, if we use very distinct estimated values $(a^{\#}, b^{\#})$ and (a^*, b^*), significant forecasting differences result. Concretely, considering the estimates obtained at the simulation $s = 30$ in Table 1, that is

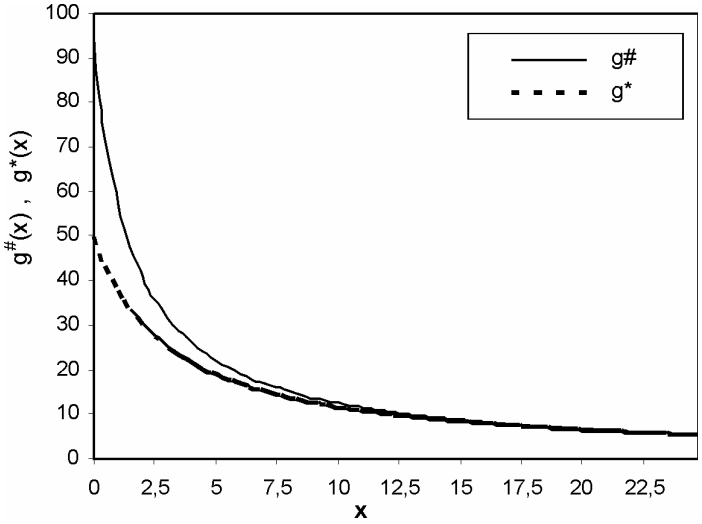
$$(a^{\#}, b^{\#}) = (1.52, 94.57) \qquad (a^*, b^*) = (3.01, 49.98)$$

we get the prediction functions $g^{\#}(x)$ and $g^*(x)$, $x \geq 0$,

$$g^{\#}(x) = g(x; a^{\#}, b^{\#}) = \frac{a^{\#} b^{\#}}{a^{\#} + x} \qquad g^*(x) = g(x; a^*, b^*) = \frac{a^* b^*}{a^* + x}$$

The graphs of the functions $g^{\#}(x)$, $g^*(x)$ are very different, especially for small values of the explanatory variable X (compare the shape of the two graphs in G4).

G4. Graphs of the functions g#(x) , g*(x)
(Table 1, s = 30)



5. Conclusions

Taking into consideration the features of the CO2 emission process, Albu (2007) proposed a forecasting function of type (2).

The accuracy of the prediction process depends essentially on the statistical qualities of the estimators for (a,b) parameters. These parameters ensure the flexibility of the statistical model (1) (compare the graphs G1 for different (a,b) vector values). In fact, a is a scale parameter for the explanatory variable X (formula (3)) and the significance of the parameter b is suggested by the relation $b = g(0;a,b)$.

Applying the linearization transformation (4) to the nonlinear regression (1) we deduced the estimations $(a^{\#},b^{\#})$ defined by formulas (7)-(9). These estimates are enough accurate (to compare with the data listed in Table 1).

Other estimation (a^*,b^*) for (a,b) parameters was deduced by solving directly the nonlinear system (13). The approximation vector (a^*,b^*) of the real solution (a,b) for the system (13) resulted iteratively by applying the classical Newton-Raphson method. Imposing the stopping rule (19) with $\varepsilon_{1*} = \varepsilon_{2*} = 10^{-7}$, the convergence of the iterative algorithm is ensured in less than 6 steps (to pursue the k index values in Table 1).

If we consider $(a^{\#},b^{\#})$ as starting point, the iterative process (16) converges very fast. The stochastic Monte Carlo simulations proved how important is to select an adequate starting point (a_1,b_1) (see also Remark 2). Choosing an arbitrary starting point (a_1,b_1), the convergence speed of the Newton-Raphson algorithm (16) became very slow (more than 200-300 steps) and, sometimes, non-convergence situations resulted, too. Thus, we suggest $(a_1,b_1) = (a^{\#},b^{\#})$.

Applying a Monte Carlo simulation technique we determined the accuracy of the estimates $(a^{\#},b^{\#})$ and (a^*,b^*). The simulation process, defined by the statistical model (20), was always repeated $t = 30$ times.

In all the simulated cases, we obtained a very high precision for the estimates (a^*,b^*) as compared to the initial evaluations $(a^{\#},b^{\#})$ of the parameters (a,b) (see Table 1 and the graphs G2 and G3).

The differences between the forecasting quantities $g^{\#}(x)$ and $g^*(x)$ are higher for small values of x, the predicted values becoming closer when x increases (graph G4).

The precision of the estimations $(a^{\#},b^{\#})$ decreases substantially when the dispersion σ^2 of the errors in the stochastic simulation model (20) is increased. This

aspect is illustrated in Tables 2 and 3 if we pursue the statistics concerning the estimations $a^{\#}, b^{\#}$ (to compare these evaluations with $a = 3$ and $b = 50$).

In addition, a larger dispersion σ^2 for the residuals in the prediction model (2) imposed more iterations to the Newton-Raphson algorithm (see the fluctuations of the variable k in Tables 2 and 3).

In conclusion, we recommend using first the linearization procedure (4) to get the estimates $(a^{\#}, b^{\#})$ for (a, b) parameters (the formulas (7)-(9)). After that, the initial vector $(a^{\#}, b^{\#})$ will be adjusted successively by applying the Newton-Raphson procedure (the formulas (16)) until the stopping condition (19) becomes true. Finally, it results in a very good estimate (a^*, b^*) for the unknown parameters (a, b) which ensures the flexibility of the forecasting model (2).

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