Optimal Dispatch in Electricity Markets

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Abstract

The problem of calculating the optimal dispatch and prices in a single-period electricity auction in a wholesale electricity market is considered here. The novel necessary and sufficient conditions of optimality for this problem are derived and computational algorithms for solving these conditions are constructed.

Keywords: Optimal dispatch; Electricity market; Nonlinear programming; Non-convex problems; Dynamic programming.

Introduction

Economic dispatch problem for electricity generation has been the focus of numerous studies (eg [1], [5]) for a long time. Recent market deregulation led to its reformulation. Calculating market-based economic dispatch still requires solving non-linear programming problem but this problem became qualitatively different. Firstly, its objective function became the "cost of generation", defined as suppliers’ price-volume bid functions integrated over their dispatched volumes. In most markets these bids are step-wise functions. This yields discontinuities of gradients and reduces the efficiency of standard numerical methods. Secondly, because negative price steps are commonly used in bids the market-based dispatch problem is non-convex and may have multiple solutions. These are the distinguishing characteristics of dispatch problem in a deregulated electricity market which make it important to develop algorithms especially tailored to solve them efficiently.
In this paper we develop novel necessary and sufficient conditions for dispatch and pricing problem in a deregulated electricity market and construct computational methods, based on these conditions, capable of efficiently calculating dispatch and prices.

**Dispatch in Australian electricity market**

We consider normal trading only, did not consider any trading in auxiliary markets, reserves etc. Electricity market is described as a network of scalar flows, which is standard in modelling of economic electricity dispatch. The model considered in this paper is used in practice to dispatch the Australian market, NEM see [3], [4].

Thus, we consider electricity market that includes \( n \) regional markets connected by a network of interregional connectors, which transfer electricity between them, (Fig. 2). Each participant (generator or consumer) is located in one of the regional markets. All participants in one regional market are paid or pay the same regional price for electricity they sell or consume. That is, each regional auction is a single-price auction. As a rule, regional prices in different regions are different. Every trading day is divided into a sequence of identical single-period auctions. A single period auction is in fact a number of linked regional auctions that take place in all regions simultaneously. The results of a single period auction depends on generators’ price-volume bids, the state of the market (regional generations and inter-regional flows) before the auction, and the regional demands. These auction results determine regional dispatches - the amounts of electricity sold by every generators in the region, power flows between regions and the regional prices.

Generators submit price-volume bids to the market operator. These bids are step-wise functions that show how much power they are prepared to supply for a particular price (Fig. 1). Price steps \( P^k_i \) in price bids could be (and indeed often are in practice) negative. That is, the producer offers to pay a consumer who agrees to buy its electricity. Trading day is divided into sequence of equal-time periods. Market operators calculates dispatch for each of this period by running single period electricity auction, using the following information:

- Combined regional demands by all consumers \( d_i \);
- Combined regional price-bids \( P_i(q_i) \) by all generators,
- Combined regional generations before the auction \( q_i(0) \),
- Inter-regional flows before the auction \( g_{ij}(0) \).
Functions $P_i(q_i)$ are continuous from the right and have left limits $\lim_{q_i \rightarrow Q_i^-} P_i(q_i) = P_i'$. Regional generations are range-constrained $q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}}$. Inter-regional energy flows $g_{ij}$ from the $i$-th to $j$-th region are also range-constrained, $g_{ij}^{\text{min}} \leq g_{ij} \leq g_{ij}^{\text{max}}$. These constraints are due to technological and dynamical reasons. Note that $g_{ij} = -g_{ji}$.

We assume that when the power $g_{ij}$ is transferred from $i$-th to the $j$-th regional markets some of it is lost and this loss is described by the function $L_{ij}(g_{ij})$. This function is a given continuous function of $g_{ij}$ such that $L_{ij}(g_{ij}) = L_{ji}(-g_{ij}) = L_{ii}(x) \equiv 0$, $L_{ij}(0) = 0$, $L_{ij} = L_{ji}$, $L_{ij} > 0$ for $g_{ij} \neq 0$ and $\frac{d^2 L_{ij}}{dg_{ij}} > 0$ for $\forall g_{ij}$. Characteristic dependence $L_{ij}(g_{ij})$ on $g_{ij}$ is shown in Fig. 3. The loss $L_{ij}$ is divided between corresponding markets in a fixed proportion giving additional demand $\alpha_{ij}L_{ij}$ in the $i$-th region and $\alpha_{ji}L_{ij}$ in the $j$-th region, Fig. 4. $\alpha_{ij}$ are constants, $0 \leq \alpha_{ij} \leq 1$, $\alpha_{ji} = 1 - \alpha_{ij}$, $\alpha_{ii} = 0$.

The net energy balance for the whole network of regional markets is

$$\sum_{i=1}^{n} q_i = \sum_{i=1}^{n} d_i + \frac{1}{2} \sum_{i,j=1,i \neq j}^{n} L_{ij}(g_{ij}).$$

Since $g_{ij} = -g_{ji}$ and $\alpha_{ij} + \alpha_{ji} = 1$, $\forall i, j = 1, \ldots, n$ this balance holds if the
Figure 3: Characteristic dependence of power loss $L_{ij}$ on the power flow $g_{ij}$.

Figure 4: Two markets linked by inter-connector.

Regional energy balances

$$q_i = d_i + \sum_{j=1, j\neq i}^n (g_{ij} + \alpha_{ij}L_{ij}(g_{ij})), \quad i = 1, \ldots, n$$  \hspace{1cm} (2)

hold. The cost of generation in the $i$-th market is defined as

$$C_i(q_i) = \int_0^{q_i} P_i(x) dx.$$  \hspace{1cm} (3)

The optimal dispatch is determined by minimizing the combined cost of generation of all generators in all regional markets, $I$, on $q_i$, $g_{ij}$

$$I(d_1, \ldots, d_n, q_1, \ldots, q_n, g_{ij}) = \sum_{i=1}^n C_i(q_i) \rightarrow \min_{q_i, g_{ij}}$$  \hspace{1cm} (4)

subject to range constraints

$$q_i^{\min} \leq q_i \leq q_i^{\max}, \quad i = 1, \ldots, n$$  \hspace{1cm} (5)
\[ g_{ij}^{\text{min}} \leq g_{ij} \leq g_{ij}^{\text{max}} \quad j = 1, \ldots, n, \quad i = 1, \ldots, n \] (6)

and regional markets’ energy balances (2). We denote its solution as \( q^*_i, g^*_{ij} \) and the value of the objective function (the minimal cost of supply) as

\[
I^*(d_1, \ldots, d_n) = \min_{q, g} I(d_1, \ldots, d_n, q_1, \ldots, q_n, g_{ij}) = I(d_1, \ldots, d_n, q^*_1, \ldots, q^*_n, g^*_{ij}).
\] (7)

The cost of generation \( I \) and the minimal cost of generation \( I^* \) both depend not only on \( d_i \) but also on the bids \( P_i(q_i) \) and pre-auction state of the market \( q_i(0), g_{ij}(0) \) (via range constraints on \( q_i \) and \( g_{ij} \)).

The unknown variables in the optimal dispatch problem (4), (5), (25), (2) are the regional generations and inter-regional flows after the auction. If network contains connections between all regional markets then the number of unknown variables here is \( \frac{n^2 - n}{2} + n = \frac{n(n+1)}{2}, \quad n \geq 0 \). The first term is the number of unknown exchange flows \( g_{ij} \) and the second term is the number of generated regional powers \( q_i \). Since these variables obey \( n \) balance equations (2) the number of free unknowns in the dispatch problem is equal to the number of inter-regional flows, \( \frac{n(n-1)}{2} \) for a network where all regional markets are connected. For \( n = 1 \) there is one unknown, for \( n = 3 \) - three, etc.

Note that the problem (4-25) is not easy to solve, it has both range constraints on its independent variables and constraints on its dependent variables \( q_i \).

**Necessary conditions of optimality**

The optimal dispatch problem (4), (5), (2) can be rewritten as a non-linear programming problem

\[
\sum_{i=1}^{n} C_i \left[ d_i + \sum_{j=1,i\neq j}^{n} \left( g_{ij} + \alpha_{ij} L_{ij}(g_{ij}) \right) \right] \rightarrow \max_{g_{ij}}
\] (8)

subject to constraints

\[
q_i^{\text{min}} \leq d_i + \sum_{j=1,i\neq j}^{n} \left( g_{ij} + \alpha_{ij} L_{ij}(g_{ij}) \right) \leq q_i^{\text{max}}.
\] (9)

and range constraints

\[
g_{ij}^{\text{min}} \leq g_{ij} \leq g_{ij}^{\text{max}} \quad i, j = 1, \ldots, n.
\] (10)
The independent unknown variables now are the flows $g_{ij}$. Regional genera-
tions are calculated from balances (2).
We define reduced price of the region $i$ with respect to region $j$ as

$$
\tilde{P}_{ij}(g_{ij}) = P_i(d_i + \sum_j (g_{ij} + \alpha_{ij} L_{ij}(g_{ij}))) \left[ 1 + \alpha_{ij} \frac{dL_{ij}(g_{ij})}{dg_{ij}} \right],
$$

(11)
and for $j$-th with respect to $i$-th it is

$$
\tilde{P}_{ji}(g_{ij}) = P_j(d_j + \sum_i (-g_{ij} + \alpha_{ji} L_{ij}(g_{ij}))) \left[ 1 - \alpha_{ji} \frac{dL_{ij}(g_{ij})}{dg_{ij}} \right].
$$

We also define their left and right limits

$$
\tilde{P}_{ab}^+ = \lim_{\epsilon \to 0} \tilde{P}_{ab}(g_{ij} + \epsilon), \quad \tilde{P}_{ab}^- = \lim_{\epsilon \to 0} \tilde{P}_{ab}(g_{ij} - \epsilon).
$$

The necessary conditions of optimality for dispatch problem is then given by the following

**Theorem.** If $\{g_{ij}^*\}$ is an optimal solution of the problem (8)–(10) then one of the following conditions holds

- **Non-binding range constraints**

  $$
  g_{ij}^{\text{min}} < g_{ij}^* < g_{ij}^{\text{max}},
  \quad q_i^{\text{min}} < d_i + \sum_j (g_{ij}^* + \alpha_{ij} L_{ij}(g_{ij}^*)) < q_i^{\text{max}},
  \quad (12)
  $$

  $$
  q_j^{\text{min}} < d_j + \sum_i (-g_{ij}^* + (1 - \alpha_{ij}) L_{ij}(g_{ij}^*)) < q_j^{\text{max}},
  \quad \tilde{P}_{ij}(g_{ij}^*) \geq \tilde{P}_{ji}^-(g_{ji}^*), \quad \tilde{P}_{ij}^-(g_{ij}^*) \leq \tilde{P}_{ji}^+(g_{ji}^*),
  $$

- **Binding flow constraints**

  $$
  g_{ij}^* = g_{ij}^{\text{max}},
  \quad q_i^{\text{min}} < d_i + \sum_j (g_{ij}^* + \alpha_{ij} L_{ij}(g_{ij}^*)) < q_i^{\text{max}},
  \quad (13)
  $$

  $$
  q_j^{\text{min}} < d_j + \sum_i (-g_{ij}^* + (1 - \alpha_{ij}) L_{ij}(g_{ij}^*)) < q_j^{\text{max}},
  \quad \tilde{P}_{ij}(g_{ij}^*) \leq \tilde{P}_{ji}^+(g_{ji}^*),
  $$

  or

  $$
  g_{ij}^* = g_{ij}^{\text{min}},
  \quad q_i^{\text{min}} < d_i + \sum_j (g_{ij}^* + \alpha_{ij} L_{ij}(g_{ij}^*)) < q_i^{\text{max}},
  \quad (14)
  $$

  $$
  q_j^{\text{min}} < d_j + \sum_i (-g_{ij}^* + (1 - \alpha_{ij}) L_{ij}(g_{ij}^*)) < q_j^{\text{max}},
  \quad \tilde{P}_{ij}^-(g_{ij}^*) \geq \tilde{P}_{ji}^-(g_{ji}^*),
  $$

or

$$
\tilde{P}_{ij}^+(g_{ij}^*) \leq \tilde{P}_{ji}^-(g_{ji}^*),
$$

or

$$
\tilde{P}_{ij}^+(g_{ij}^*) \geq \tilde{P}_{ji}^+(g_{ji}^*),
$$

or

$$
\tilde{P}_{ij}^-(g_{ij}^*) \leq \tilde{P}_{ji}^-(g_{ji}^*),
$$

or

$$
\tilde{P}_{ij}^+(g_{ij}^*) \geq \tilde{P}_{ji}^+(g_{ji}^*).$$
• Binding generation constraint (A)

\[ g_{ij}^{\text{min}} < g_{ij}^* < g_{ij}^{\text{max}}, \quad \tilde{P}_{ij}^- \leq \tilde{P}_{ji}^+, \]  
\[ q_i^{\text{min}} = d_i + \sum_j (g_{ij}^* + \alpha_{ij} L_{ij}(g_{ij}^*)) , \quad \text{or} \quad q_j^{\text{max}} = d_j + \sum_j (-g_{ij}^* + (1 - \alpha_{ij}) L_{ij}(g_{ij}^*)). \]  

or (B)

\[ g_{ij}^{\text{min}} < g_{ij}^* < g_{ij}^{\text{max}}, \quad \tilde{P}_{ij}^+ \geq \tilde{P}_{ji}^-, \]  
\[ q_i^{\text{max}} = d_i + \sum_j (g_{ij}^* + \alpha_{ij} L_{ij}(g_{ij}^*)) , \quad \text{or} \quad q_j^{\text{min}} = d_j + \sum_j (-g_{ij}^* + (1 - \alpha_{ij}) L_{ij}(g_{ij}^*)). \]

**Proof.** We derive the proof by examining the first variation of the function \( I \). The particular form of problem’s constraints allows us to simplify the problem by solving some of its constraints explicitly.

First we consider the case were both range constraints (9), (10) are not binding and \( P_i \) and \( P_j \) are continuous at \( g_{ij} \). Differentiation leads to the following expression for the variation of the objective function

\[ \Delta I = (\tilde{P}_{ij} - \tilde{P}_{ji}) \Delta g_{ij}. \]

Since here both positive and negative variations of \( \Delta g_{ij} \) are feasible, the current arguments are where the bid functions are continuous, \( \tilde{P}_{ij}^+ = \tilde{P}_{ij}^- = \tilde{P}_{ij} \) and \( \tilde{P}_{ji}^+ = \tilde{P}_{ji}^- = \tilde{P}_{ji} \), the necessary condition of optimality (condition of non-improvement of \( I \) by a feasible infinitesimal variation of \( g_{ij} \)) coincides with (12).

Let us consider the general case of possible discontinuous points of \( P_i \) and/or \( P_j \). Since \( P_i \) is a non increasing and \( P_j \) is a non increasing functions of the same flow \( g_{ij} \), from (8) it follows that \( \Delta I \) can be written as

\[ \Delta I = \left[(P_{ij}^+ - P_{ij}^-) - (P_{ji}^+ - P_{ji}^-)\right]\delta(\Delta g_{ij})(\Theta_+(\Delta g_{ij}) - \Theta_-(\Delta g_{ij})) + \\
(\tilde{P}_{ij}^+ - \tilde{P}_{ji}^-)\Theta_+(\Delta g_{ij}) + (\tilde{P}_{ij}^- - \tilde{P}_{ji}^+)\Theta_-(\Delta g_{ij}) \]  

(17)

where \( \delta(x) \) is the Dirac function, \( \Theta_+(x) = 1, \text{ if } x > 0 \) and 0 otherwise. The first term here describes the net effect of two jumps in bids.

Suppose all range constraints are not binding. Then \( \Delta g_{ij} \) are not constrained and can have arbitrary sign. Therefore, the condition \( \Delta I \Delta g_{ij} \geq 0 \) that guarantees that \( g_{ij} \) is not improvable by infinitesimal variation of \( \Delta g_{ij} \) become (12).

Finally, if at least one of the following conditions \( g_{ij} = g_{ij}^{\text{max}} \) or \( q_i^{\text{max}} = d_i + \sum_j (g_{ij} + \alpha_{ij} L_{ij}(g_{ij})) \) hold, then only negative variations \( \Delta g_{ij} \) are feasible. In this case the condition \( \Delta I \Delta g_{ij} \geq 0 \) takes the form (16). The same derivations for the case, where only positive \( \Delta g_{ij} \) are feasible conclude the proof. \( \square \)
Long-chain conditions of optimality

Consider a linear subnetwork of three regional markets. Suppose that
\[ \tilde{P}_{ij} < \tilde{P}_{ji}, \quad \tilde{P}_{jk} < \tilde{P}_{kj}, \quad q_j^* = q_j^{\text{max}}. \]

The flows is directed from the \( i \)-th to the \( j \)-th and then \( j \)-th to the \( k \)-th markets. The range constraint on the generation in the \( j \)-th market is active. This prohibits positive variation of \( g_{ij} \). However, two simultaneous positive variations \( \delta g_{ij} = \delta g_{jk} > 0 \) are feasible. Analysis identical to a single connector analysis above yields the following conditions of optimality for non-binding range constraints for \( q_i \) and \( q_k \) on the intervals where \( P_i(\cdot) \) and \( P_k(\cdot) \) are continuous

\[ P_i(1 - \alpha_{ij} L'_{ij})(1 - \alpha_{jk} L'_{jk}) = P_k(1 + \alpha_{ij} L'_{ij})(1 + \alpha_{jk} L'_{jk}), \quad (18) \]

and the general condition of optimality for discontinuous points

\[ P_i^+(1 - \alpha_{ij} L'_{ij})(1 - \alpha_{jk} L'_{jk}) \leq P_k^-(1 + \alpha_{ij} L'_{ij})(1 + \alpha_{jk} L'_{jk}), \quad (19) \]
\[ P_i^-(1 - \alpha_{ij} L'_{ij})(1 - \alpha_{jk} L'_{jk}) \geq P_k^+(1 + \alpha_{ij} L'_{ij})(1 + \alpha_{jk} L'_{jk}), \quad (20) \]

For binding range constraints these condition of long-range optimality is exactly the same as for short-range optimality above - but for long-range reduced prices (lhs and rhs of (18)) instead of short-range reduced prices (11).

They state that for the optimal dispatch two regional markets connected by a network of generation-constrained intermediate markets are different, can have different long-range reduced prices if and only if at least one of the flows between them is constraint or the generation in one these two markets is range constrained.

\[ \begin{array}{ccc}
& q_j = q_j^{\text{max}} & \\
\tilde{P}(q_i) & g_{ij} & P_j(q_j) \\
\rightarrow & \rightarrow & \rightarrow \\
\tilde{P}(q_i) & g_{jk} & P_k(q_k)
\end{array} \]

Figure 5: 3-market linear fragment.

Numerical methods

In practice dispatch and prices in most of the markets now is solved by linearizing dispatch problem and then solving it numerically using classical linear programming algorithms (see [3], [4]).
The derived condition of optimality require that reduce prices between linked regional markets be equal unless flow constraint or generation constraint becomes binding. They can be used to find the dispatch optimization numerically.

As an initial point this algorithm requires a feasible set of flows $g_{ij}$ such that range constraints (25) and (9) hold. This solution is then improved iteratively by applying the following elementary computational operation:

1. calculate the reduced prices difference $\Delta \tilde{P}_{ij} = \tilde{P}_{ij} - \tilde{P}_{ji}$ for every pair of regions connected by inter-connector;
2. mark all pair where one of the optimality conditions holds;
3. for each unmarked pair if $\tilde{P}_{ij} > \tilde{P}_{ji}^+$ then $g_{ij}^{\text{min}} < g_{ij} < g_{ij}^{\text{max}}$, $q_i^{\text{max}} < q_i(g_{ij}) < q_i^{\text{max}}$, $q_j^{\text{max}} < q_j(g_{ij}) < q_j^{\text{max}}$ then $g_{ij}$ is increased, until one of these inequalities becomes equality. This increase continues across jumps of the price-volume bids, provided that the reduced $i$-th price at these new steps is still lower that reduced price $\tilde{P}_{ji}$. Similarly, if $\tilde{P}_{ij} < \tilde{P}_{ji}^+$ then $g_{ij}$ is reduced until either reduced prices equalize or one of range constraints becomes binding. The improved solution is again submitted as an input to step (2).

The iterations will converge because variation of $g_{ij}$ increase one of the prices $P_i$, $P_j$ and decreases another. After iterations converge, one can use its output as an initial solution for another iterative algorithm which uses the improvement operation for 3-market substrings instead of 2-market ones, then for 4-market etc.

The conditions of maximal equalization of reduced prices between all regions across the network can be maintained by an automatic feedback control system. This would allow to replace the sequence of single-auctions with a single continuous real-time auction and will approve dispatch dramatically.

Non-convexity of the dispatch problem when bids have negative price steps

We illustrate non-convexity by plotting the cost of generation $I(g)$ as a function of $g$ in Figure 7 for the market with two regional markets, regional bids $P_1(q)$ and $P_2(q)$ shown in Figure 6, and losses $L(g) = 0.14g^2$ equally divided between regional markets. The graph clearly shows that $I(g)$ has two minima. The left minimum corresponds to $I^* = -269.137$, $g^* = -11.21$ and dispatches $q_1^* = 37.59$ and $q_2^* = 60$.

The link between non-convexity of dispatch problem and negative prices in bids can be traced using the following analysis. Each regional cost of generation (integrated price-volume bids) $C_i(q_i)$ is a convex, piece-wise linear function. Therefore, the total cost of generation - the sum of regional costs
of generation - is also convex on \( q_i \). But after we express these variables in terms of inter-regional flows \( g_{ij} \) using energy balances (2) the objective function of the single-period auction problem becomes non-convex. Indeed, the second derivative of \( C_i(g_{ij}) \) with respect to \( g_{ij} \) is

\[
\frac{d^2}{dg_{ij}^2} C_i = \frac{d}{dg_{ij}} \left( \frac{dC_i}{dq_i} \frac{dL_{ij}}{dg_{ij}} \right) = \sum_j P_i \frac{dL_{ij}}{dg_{ij}^2}
\]  

(21)

Since \( \frac{dL_{ij}}{dg_{ij}} > 0 \) the sign of \( \frac{d^2}{dg_{ij}^2} C_i \) coincides with the sign of the price-volume bid (current price step). Thus, the use of negative price steps in bids is the causes of non-convexity.

Thus, if one solves the necessary conditions of optimality derived above or uses a direct search to minimize the cost of supply, then there is no guarantee the the solution found is one of the local minima. That is why it is important
to verify if the solution found is local or global minimum and it is local then how close the local minimum is from the global one, how much improvement in terms of the cost of supply can be achieved if search for optimal dispatch continues from another initial point.

Globally optimal dispatch.

Consider relaxation of the optimal dispatch problem (4), (5), (25), (2) by deleting range constraints on the flows (25) and regional energy balances (2). The generalized optimal dispatch problem then becomes

\[ I(d_1, \ldots, d_n, q_1, \ldots, q_n, g_{ij}) = \sum_{i=1}^{n} C_i(q_i) \rightarrow \min_{q_i} \]  
subject to

\[ \sum_i q_i = M \]  
\[ q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}}, \quad i = 1, \ldots, n \]  

\( M \) here is the parameter that describes inter-connectors losses and which must obey the range constraints

\[ \sum_i Q_i^{\text{min}} \leq \sum_i d_i \leq M \leq \sum_i q_i^{\text{max}}. \]

Bellman function ([6]) is defined using the following recurrent equation

\[ \phi_1(x_1) = C_1(x_1), \quad \phi_2(x_2) = \min_{q_2^{\text{min}} \leq q_2 \leq q_2^{\text{max}}} \left[ C_2(q_2) + \phi_1(x_2 - q_2) \right], \]
\[ \phi_\nu(x_\nu) = \min_{q_\nu^{\text{min}} \leq q_\nu \leq q_\nu^{\text{max}}} \left[ C_\nu(q_\nu) + \phi_1(x_\nu - q_\nu) \right] \]  

By construction

\[ \phi_n(M) = \min q_1, q_2, \ldots, q_n \sum_i C_i(q_i^*(M)) \]

gives global minimum and corresponding network losses are given by the network energy balance

\[ \sum_i d_i + \frac{1}{2} \sum_{ij} L_{ij}(g_{ij}) = M \]

Suppose we used numerical algorithm described in the section above and obtained \( g_{ij}^*, q_i^*, I^* \) and \( M^* = \sum_i q_i^* \). \( \phi_n(M^*) \) then gives the lower bound on
the cost of generation $I$. If it turns out that $I^* = \phi_n(M^*)$ then $g^*_{ij}, q^*_i$ is the
global minimum. If $|I^* - \phi_n(M^*)|$ is small enough then one may reasonably
decide to stop search and use the current local minimum instead of global
one.

**Entrepreneurial inter-connectors**

Entrepreneurial lines represent another new feature of modern electricity
markets. Physically they are no different to the standard inter-regional con-
nectors that are controlled by market operator. But entrepreneurial lines
operate differently. Before trading starts they submit price-volume bids,
that are identical to generators price-volume bids. They submits these bids
for two regional markets connected by their line. These bids are the offers
to flow energy, which are taken into account when single-period auction is
calculated.

Let us consider for simplicity two regional market $i$-th and $j$-th linked
by an entrepreneurial line, see Fig. 8. We denote their price-volume bids
as $E_i(x)$ and $E_j(x)$, the flow and losses in the entrepreneurial line as $e$ and
$LE(e)$, the coefficient that shows how these losses are apportioned to the
$i$-th and $j$-th regions as $\alpha_e$ and the contribution of the line to the net energy
balances in the $i$-th and $j$-th regions (amount flown in/out of the region)
as $q_e_i$ and $q_e_j$. The dispatch is then found by solving the extended optimal
dispatch problem (4), (5), (25), (2),

$$I(d_1, \ldots, d_n) = C_i(q_i) + C_j(x_j) + \frac{\text{sign}(e) + 1}{2} \int_0^{q_e_i} E_i(x)dx -$$

$$- \frac{\text{sign}(e) - 1}{2} \int_0^{q_e_j} E_j(x)dx \rightarrow \min_{q_i, g_{ij}, q_e_i, q_e_j, e}$$

subject to regional energy balances

$$q_i = d_i - \sum_j (g_{ij} - \alpha_{ij} L_{ij}(g_{ij})) + e + \alpha_e LE(e), \quad (30)$$

range constraints (25), and

$$e^{\text{min}} \leq e \leq e^{\text{max}} \quad (31)$$

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Furthermore balances for the entrepreneurial line

\[ qe_i = e + \alpha e LE(e), \quad qe_j = -e + (1 - \alpha e) LE(e), \quad qe_i + qe_j = LE(e) \quad (32) \]

In (29) \( \text{sign}(s) = 1 \) if \( s \geq 0 \) and \( \text{sign}(x) = -1 \) if \( x < 0 \). This reflects the nature of operations by entrepreneurial line - it flows energy from one regional market to another. The cost of generation now includes the cost of supply for entrepreneurial line itself. If flow is directed from \( i \) to \( j \) then the extra term corresponding to this cost is \( C(qe_i) = \int_0^{qe_i} E_i(x) dx \), if the energy is transferred from \( j \) to \( i \) then this added term is \( C(qe_j) = \int_0^{qe_j} E_j(x) dx \). For the positive \( e \) the variation of \( I \) is

\[ \delta I = \left[ P_i(1 + \alpha e \frac{d}{de}(LE)) + P_j(-1 + (1 - \alpha e) \frac{d}{de}(LE)) + E_i(e)(1 + \alpha e \frac{d}{de}(LE)) \right] \delta e \quad (33) \]

If \( e \) belongs to an interval where function \( E_i(.) \) is continuous, then the following necessary condition of optimality take the form

\[ E_i^* = (P_j^* - P_i^*) + \frac{\frac{d}{de}(LE)}{1 + \alpha e \frac{d}{de}(LE)} P_j^* \quad (34) \]

Figure 8: Entrepreneurial inter-connectors.
That is, the highest price band dispatched via the entrepreneurial line is determined by the difference between spot prices in these two markets plus correctional term that depends on the network losses in entrepreneurial line. In algorithmic terms if

\[ P_i(1 + \alpha_e LE') + P_j(-1 + (1 - \alpha_e)LE') < E_i(e)(1 + \alpha_e LE') \] (35)

then one needs to increase \( e \) until this condition is no longer true.

**Conclusions**

The dispatch and pricing problem for single-period electricity auction in a network of regional markets is considered. Its conditions of optimality are derived and solved. It is shown these conditions require that the values of some well-defined function, called reduce price, in connected regional markets be as close to each other as possible. Computational algorithms for solving these conditions numerically are constructed. It is shown that dispatch problem is non-convex if negative prices are used in bids. The dynamic programming based algorithm for calculation of the lower bound on the cost of generation is constructed. It is shown how it can be used to verify if the obtained solution is global.

**References**


