ASSET PRICING WITH HOME CAPITAL

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ABSTRACT. I analyze a stylized consumption-based asset pricing model that features heterogeneous agents and household capital, and discover a novel recession risk factor related to the cross-sectional second moments of the corresponding *investments* into such home capital. In order to fully isolate the orthogonal effects at work, I completely shut off the well-known mechanism of Constantinides and Duffie (1996) by explicitly stipulating homoscedastic cross-sectional distribution of nondurable goods and services.

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I. INTRODUCTION

This paper presents a consumption-based asset pricing model that supplies a novel link between the real and financial sectors of the macroeconomy. This new channel arises suddenly, but naturally, once the quite restrictive assumption of complete markets is lifted, and, at the same time, consumption is conceived more broadly than just nondurable goods and services. The outcome of the analysis is the surprising discovery that the Euler equations of consumption depend also on the cross-sectional second moments of the consumer durables *investments*, such as purchases of cars, furniture or houses, a notably volatile and inherently countercyclical component of National Income, in particular if one considers the respective high-end market, luxurious, varieties.

In my endevour, I build upon the prominent heterogeneous-agents models in the asset-pricing literature from the pen of Mankiw (1986) and Constantinides and Duffie (1996). I resist the temptation to experiment with the manner that market incompleteness is introduced. Rather than being a sign of the model's frailty, such modus operandi only helps to illuminate the asset-pricing implications of durable goods within an otherwise well-accepted framework. I therefore partially track the previous literature, and stipulate the labor income stream to be uninsurable, persistent and heteroscedastic. However, I do challenge the canonical idea of consumption by explicitly introducing the flow of services from the stock of durable goods. Although often ignored, perhaps due to increased difficulty of dealing with extra time-nonseparabilities in household preferences, it is without question an asset that by far outstrips, for example, the value of producer durables in the United States (Eisner 1988, Greenwood and Hercowitz 1991).

Ingenious though it may be, Constantinides and Duffie's (1996) model demands an arguably implausible variation in the cross-sectional moments of the nondurable goods and services. This is problematic especially by recognizing the fact that these consumption goods are necessary economic goods (Costa 2001), and one naturally does not expect large enough countercyclical swings in the cross-sectional distribution of necessary goods to (fully) account for the dramatic price fluctuations in financial markets.

The related literature is growing. Yogo (2006) explores the ability of durable goods to explain the cross-sectional variation in expected returns on common stocks. Piazzesi, Schneider and Tuzel (2007) introduce aggregate housing in asset pricing. Both papers feature complete markets. In an extension of the endogeneously incomplete markets model, Lustig and Nieuwerburgh (2005) evaluate how the collaterability of housing influences risk sharing, and hence asset prices.

II. MODEL

A. **Households' Consumption-Portfolio Problem.**

a. *Primitives.* Consider an incomplete-market frictionless exchange economy populated by a continuum of households, indexed by $i \in [0,1]$. Each household i has standard von-Neumann Morgenstern preferences U^i, defined over the final consumption good C_{it}

$$U^i = \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} C_{it}^{1-\gamma} \mid \mathcal{F}_0\right\}, \tag{II.1}$$

where such final consumption flow C_{it} is "produced" using the constant-returns-to-scale household production function over the home capital d_{it} and the nondurable goods flow c_{it} as

$$C_{it} \;=\; C\left[c_{it},\, d_{it}\right]. \tag{II.2}$$

In a related paper, Yogo (2006) uses a CES production function to study the pricing of common stocks in a representative-agent complete-markets framework. Greenwood and Hercowitz (1991) use a similar production function to study the cyclical allocation of capital and time between market and home activities.

As consumer durables are a *stock*, we have a law of motion analogous to the one from the capital theory, that is,

$$d_{it+1} \;=\; (1-\delta_d)\, d_{it} \;+\; I^d_{it+1} \tag{II.3}$$

In words, next-period stock of durables d_{it+1} equals the stock from last period d_{it} minus the depreciation[1] $\delta_d\, d_{it}$ plus the consumer durables investment I^d_{it+1}.

There are K financial assets traded, with their ex-dividend prices p_{jt}, paying a dividend div_{jt}. Let us define the price vector $p_t = (p_{1t}, \ldots, p_{Kt})$ and the dividend vector $div_t = (div_{1t}, \ldots, div_{Kt})$. The budget constraint takes the standard form

$$\text{(II.4)} \qquad c_{it} \,+\, q^d_t\, I^d_{it} \,+\, \theta_{it} \cdot p_t \,=\, \theta_{it-1} \cdot (p_t \,+\, div_t) \,+\, w_{it}$$

where θ_{it} is the household's trading strategy, w_{it} is the labor income, and q^d_t is the relative price of consumer durables.

The information structure is modeled by a filtration $\{\phi_t\}_{t\in\mathbb{N}}$, which includes the aggregate labor income history, the aggregate consumer durables' price histories, the financial assets' dividend and price histories, and any additional information available to an econometrician at time t. Furthermore, the information set $\mathcal{F}_t$ available to households consists of ϕ_t plus the history of the disaggregated labor income $\{w_{is} : i \in [0,1],\, 0 \le s \le t\}$.

b. *First-Order Conditions.* I derive the first-order conditions by means of a simple variational argument. Suppose the household decreases its consumption of nondurables by one unit, that is, $dc_{it} = 1$, and uses such proceeds to purchases

[1]The parameter δ_d is the depreciation rate and without loss of generality it is assumed constant across consumers.

$1/p_{jt}$ shares of an asset $j \in \{1, ..., K\}$. Next period, the additional shares pay $p_{jt+1} + div_{jt+1}$ (per share), or in total

$$(p_{jt+1} + div_{jt+1})\,/p_{jt} \;\equiv\; R_{jt+1} \tag{II.5}$$

The change in lifetime utility is

$$\delta U^i \;=\; -C_{it}^{-\gamma}\,\frac{\partial C_{it}}{\partial c_{it}} \;+\; \mathbb{E}\left\{\beta\, C_{it+1}^{-\gamma}\,\frac{\partial C_{it+1}}{\partial c_{it+1}} R_{jt+1}\,|\,\mathcal{F}_t\right\}. \tag{II.6}$$

In equilibrium, there cannot exist a trading strategy that would raise lifetime utility, and therefore one of the first-order conditions is that the variation $\delta U^i = 0$. Let us define household i's marginal rate of substitution m_{it+1} as

$$m_{it+1} \;=\; \beta\left(\frac{C_{it+1}}{C_{it}}\right)^{-\gamma}\,\frac{\partial C_{it+1}/\partial c_{it+1}}{\partial C_{it}/\partial c_{it}} \tag{II.7}$$

We may rewrite the Euler equation in the familiar form

$$1 \;=\; \mathbb{E}\left\{m_{it+1}\, R_{jt+1}\,|\,\mathcal{F}_t\right\}, \quad i \in [0,1],\ j \in \{1, ..., K\} \tag{II.8}$$

There is also an intra-temporal first-order condition which states that the marginal utility per last dollar spent is the same across all consumption goods. Specifically, suppose we buy an additional unit of nondurable goods at price one[2]. The marginal utility per last dollar spent is $C_{it}^{-\gamma}\,\frac{\partial C_{it}}{\partial c_{it}}\,/\,1$. On the other hand, suppose we rent an additional unit of durable goods for one period. The marginal utility per last dollar spent is $C_{it}^{-\gamma}\,\frac{\partial C_{it}}{\partial d_{it}}\,/\,rc_t^d$, where rc_t^d is the rental cost of durables. In

[2]Recall that nondurables are numeraire and therefore have price one.

equilibrium, it must be true that the marginal utility per dollar spent is the same across all goods, and thus

$$rc_t^d \; = \; \frac{\partial C_{it}/\partial d_{it}}{\partial C_{it}/\partial c_{it}} \tag{II.9}$$

We may find the rental cost of consumer durables rc_t^d by the following no-arbitrage argument. Suppose we buy one unit of durables at the "cum-dividend" price q_t^d, which after one period depreciates to $1 - \delta_d$. We can sell it for $(1 - \delta_d)\, q_{t+1}^d$. In equilibrium, the rental cost rc_t^d must be the net present value (NPV) of this transaction

$$rc_t^d \; \equiv \; q_t^d \; - \; (1 - \delta_d)\, \mathbb{E}\left\{ m_{it+1}\, q_{t+1}^d \mid \mathcal{F}_t \right\}, \tag{II.10}$$

c. *Household Production Function.* In order to make an analytical headway, I stipulate the household production function to be of Cobb-Douglas form

$$C_{it} \; \propto \; c_{it}^{\lambda}\, d_{it}^{1-\lambda} \tag{II.11}$$

thereby restricting the intratemporal elasticity of substitution to one. The parameter λ controls the share of nondurable consumption in total within-period expenditures.

B. A**bstract Harrison-Kreps** (1978) **Pricing Kernel.** Suppose that the financial market and the market for consumer durables are jointly free from arbitrage. Then, under certain technical conditions[3], a strictly positive pricing kernel $\{M_t,\, \phi_t\}_{t\in\mathbb{N}}$ exists and it prices all assets, *including* the consumer durables. In detail, for each financial asset $j \in \{1, ..., K\}$ it must be true that

$$p_{jt} \;=\; \mathbb{E}\left\{\frac{M_{t+1}}{M_t}\,(p_{jt+1} + div_{jt+1}) \mid \phi_t\right\} \tag{II.12}$$

Furthermore, no arbitrage dictates that the rental costs of consumer durables must satisfy the following net-present-value (NPV) condition

$$rc^d_t \;=\; q^d_t \;-\; (1-\delta_d)\mathbb{E}\left\{\frac{M_{t+1}}{M_t}\,q^d_{t+1} \mid \phi_t\right\} \tag{II.13}$$

The following proposition links the abstract Harrison-Kreps pricing kernel M_{t+1} to the economic fundamentals.

Theorem 1. There exists $\mathbb{R}_+$-valued stochastic process $\{x_t\}_{t\in\mathbb{N}}$, adapted to $\{\phi_t\}_{t\in\mathbb{N}}$, such that the pricing kernel M_{t+1} is given by the equation

$$\begin{aligned}\frac{M_{t+1}}{M_t} \;&=\; \beta\left(\frac{c_{t+1}}{c_t}\right)^{\lambda(1-\gamma)-1}\left(\frac{d_{t+1}}{d_t}\right)^{(1-\lambda)(1-\gamma)} \times\\ &\times\;\exp\left\{\frac{1}{2}\,\kappa\,(\kappa-1)\,x^2_{t+1}\right\}\end{aligned}$$

where the parameter $\kappa \;=\; (1-\lambda)(1-\gamma)$.

Proof. See the Appendix. □

[3]These may be inferred from the work of Harrison and Kreps (1979), who show that the absence of arbitrage implies the existence of a strictly positive, but not necessarily unique, stochastic discount factor M_{t+1} that prices all payoffs.

C. **Construction of No-Trade Equilibrium.** Consider an uninsurable idiosyncratic shock to the household i's labor income, denoted u_{it}. I conjecture that the individual nondurable consumptions c_{it} and household capital stock d_{it} are related to the market aggregates as

$$c_{it} = c_t \tag{II.14}$$

$$d_{it} = u_{it}\, d_t \tag{II.15}$$

where $\int_{[0,1]} u_{it}\, di = 1$ to ensure that the sum of the individual stocks of durable goods equals the aggregate one. Further, define the share u_{it} as

$$\ln\left(\frac{u_{it+1}}{u_{it}}\right) = \eta_{it+1}\, x_{t+1} - \frac{1}{2}\, x_{t+1}^2 \tag{II.16}$$

for yet to be defined x process. The intuition for the quadratic term in the formula comes from the properties of log-normal distribution $E\left[e^{\xi - \frac{1}{2}\xi^2}\right] = 1$ for $\xi \sim \mathcal{N}(0,1)$. This means that we can invoke the law of large numbers and get $\int_{[0,1]} u_{it}\, di = 1$.

The joint distribution of the random shock η_{it} is assumed to be Gaussian, as follows

$$\eta_{it} \sim \mathcal{N}(0,1) \tag{II.17}$$

and it is independent of all other variables in the economy.

The implications are as follows. First, the durables investment I^d_{it} is defined implicitly as

$$I^d_{it+1} = d_{it+1} - (1-\delta_d)\, d_{it} \tag{II.18}$$

$$= u_{it+1} d_{t+1} - (1-\delta_d)\, u_{it} d_t \tag{II.19}$$

Note that the sum of individual consumer durables investments I^d_{it} equals the aggregate consumer durables investment

$$\begin{aligned} \int_{[0,1]} I^d_{it+1}\, di &= \int_{[0,1]} u_{it+1}\, d_{t+1}\, di - (1-\delta_d) \int_{[0,1]} u_{it}\, d_t\, di \\ &= d_{t+1} - (1-\delta_d)\, d_t \\ &= I^d_t \end{aligned}$$

Second, the individual household i's labor income is defined implicitly as a function of the aggregates and the share u_{it}

$$w_{it} = c_t + q^d_t \left[\, u_{it+1} d_{t+1} - (1-\delta_d)\, u_{it} d_t \right] - \tag{II.20}$$

$$- \sum_{j=1}^{K} div_{jt} \tag{II.21}$$

It remains to show that the first-order conditions hold.

To that end, I interpret c_{it}, d_{it}, and I^d_{it} as *post-trade* non-durable consumption, the stock of durable goods, and the durables investment flow. To prove this hypothesis, it is sufficient to prove that the first-order conditions are satisfied.

Theorem 2. Given the process for the share u_{it}, no household i chooses to trade.

Proof. See the Appendix. □

D. **Economic Interpretation.** The share process u_{it} is driven by the normal random shock η_{it}, but in addition depends on the process x_{t+1}. It is important to discuss its economic meaning as it is the essential part of the pricing kernel under incomplete markets.

From the equations in the previous section, we obtain the joint cross-sectional distribution of durable goods growth rate as

$$\ln\left(\frac{d_{it+1}/d_{t+1}}{d_{it}/d_t}\right) \sim \mathcal{N}\left[-\frac{1}{2}x_{t+1}^2,\, x_{t+1}^2\right]$$

Define the cross-sectional mean of an auxiliary variable ξ_{it} as

$$\mathbb{E}^*\left(\xi_{it}\right) \;=\; \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\xi_{it} \tag{II.22}$$

and the cross-sectional variance as

$$var^*\left(\xi_{it}\right) \;=\; \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\left[\xi_{it}-\mathbb{E}^*(\xi_{it})\right]^2 \tag{II.23}$$

Clearly, x_{t+1}^2 is the cross-sectional variance of the durables growth rate

$$x_{t+1}^2 \;=\; var^*\left[\ln\left(\frac{d_{it+1}/d_{t+1}}{d_{it}/d_t}\right)\right] \tag{II.24}$$

It is insightful to decompose this cross-sectional variance as follows. The law of motion for the stock of the durable goods of household i is

$$\frac{d_{it+1}}{d_{it}} = (1-\delta_d) + \frac{I^d_{it+1}}{d_{it}} \tag{II.25}$$

and for the aggregate is

$$\frac{d_{t+1}}{d_t} = (1-\delta_d) + \frac{I^d_{t+1}}{d_t} \tag{II.26}$$

Suppose the depreciation rate is economically negligible[4], formally, $\delta_d = 0$. Then, it may be shown that[5] that

$$\ln\left(\frac{d_{it+1}/d_{t+1}}{d_{it}/d_t}\right) = \frac{I^d_{t+1}\,/\,d_t}{1 + I^d_{t+1}\,/\,d_t}\left[\ln\left(\frac{I^d_{it+1}}{I^d_{t+1}}\right) - \ln\left(\frac{d_{it}}{d_t}\right)\right]$$

Thus, the cross-sectional variance of the growth rate in consumer durables

$$x^2_{t+1} = var^*\left[\ln\left(\frac{d_{it+1}/d_{t+1}}{d_{it}/d_t}\right)\right] \tag{II.27}$$

can be decomposed into the cross-sectional variance of the share of a household's durables investment in the aggregate one

$$x^2_{1,t+1} = \left(\frac{I^d_{t+1}\,/\,d_t}{1 + I^d_{t+1}\,/\,d_t}\right)^2 var^*\left[\ln\left(\frac{I^d_{it+1}}{I^d_{t+1}}\right)\right] \tag{II.28}$$

[4]Yogo (2006) estimates the depreciation rate for durable goods (excluding houses) to be around 0.06 per quarter.

[5]The law of motion for the stock of consumer durables allows us to write

$$\ln\left(d_{it+1}/d_{it}\right) = \ln\left(1 + I^d_{it+1}/d_{it}\right) = \ln\left(1 + \exp\left\{\ln I^d_{it+1} - \ln d_{it}\right\}\right)$$

Linearizing around $\ln I^d_{t+1} - \ln d_t$ yields the results.

plus the cross-sectional variance of the share of a household's durables stock in the aggregate one

$$\text{(II.29)} \qquad x^2_{2,t+1} \;=\; \left(\frac{I^d_{t+1} \,/\, d_t}{1 \,+\, I^d_{t+1} \,/\, d_t}\right)^2 \; var^* \left[\ln\left(\frac{d_{it}}{d_t}\right)\right]$$

minus their cross-sectional covariance $x_{12,t+1}$. That is to say,

$$x^2_{t+1} \;=\; x^2_{1,t+1} \,+\, x^2_{2,t+1} \,-\, x_{12,t+1}$$

This allows us to interpret the expression

$$\frac{1}{2}\,\kappa\,(\kappa - 1)\,x^2_{t+1}$$

as an affine function of the second moment of the cross-sectional distribution of durable goods, and the corresponding investments. This suggests a potentially significant role in asset pricing of not aggregate household investment but rather the second moments of the cross-sectional distribution of households' investments.

E. Asset-**Pricing Implications.** Theorem 1 decomposes the incomplete markets pricing kernel M_{t+1} into complete markets Lucas-Breeden one and the following correction for market incompleteness

$$\exp\left\{\frac{1}{2}\,\kappa\,(\kappa - 1)\,x^2_{t+1}\right\}$$

The conditional expected return $\mathbb{E}\left[R_{t+1} \mid \phi_t\right]$ on an asset satisfies

$$\mathbb{E}\left[R_{t+1} \mid \phi_t\right] - R_t^f = -\frac{\operatorname{cov}\left[M_{t+1}, R_{t+1} \mid \phi_t\right]}{\mathbb{E}\left[M_{t+1} \mid \phi_t\right]}$$

Assets are risky because they co-vary either with the Lucas-Breeden discount factor - aggregate nondurable consumption growth rate, or aggregate durables growth rate. Or, under imperfect consumption insurance, with the second moments of the cross-sectional distribution of durable goods, in particular, the cross-sectional variance of households' *investments* into their home capital. The last source of risk is not present in complete markets framework because consumers equalize their marginal rates of substitution state by state and share the risk perfectly. It is the introduction of such home capital and consumer heterogeneity, coupled with the market incompleteness, which appears significantly to enrich the asset-pricing implications of the complete-market's framework.

III. Conclusion

This article predicts a novel recession risk factor, a cross-sectional variance of investment into home capital, such as cars, furniture, but also yachts and jewellery, by analyzing asset-pricing implications of a stylized heterogeneous-agent incomplete-market economy. In order to isolate the orthogonal effect of home capital, the well-known mechanism of

Mankiw (1986) and Constantinides and Duffie (1996) is completely shut off.

The model has not been taken to data due to a lack of availability of a detailed enough panel, which would have to contain the information on the *stock* of consumer durables. Unfortunately, all panels that I am at this point aware of are too short to even construct iteratively these stocks, using the law of motion for the home capital.

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APPENDIX A. SUFFICIENCY OF THE FIRST-ORDER CONDITIONS

This appendix concisely demonstrates that the intertemporal and intratemporal first-order conditions characterize optimality. For that purpose, I assume the technical condition that the value of every zero-coupon bond converges to zero as the maturity rises. Formally, $\lim_{T\to\infty} \mathbb{E}[M_T] = 0$. Next, I shall call a process $\alpha_t = (\alpha_{ct}, \alpha_{dt})$, $\alpha_{d0} = 0$ a budget-feasible deviation for consumer i from the optimum consumption processes (c_{it}, d_{it}) if there exists a budget-feasible strategy of the form $(\theta, c_i + \alpha_c, d_i + \alpha_d)$. The aim is to verify that, for any such α, the total utility $u\left(C\left[c_i + \alpha_c, d_i + \alpha_d\right]\right)$ is less than equal to $u\left(C\left[c_i, d_i\right]\right)$. Let $u(C) = C^{1-\gamma}/(1-\gamma)$ denote the felicity function. Its concavity implies that

$$u(C[c_{it} + \alpha_{ct}, d_{it} + \alpha_{dt}]) \leq u(C[c_{it}, d_{it}]) + \frac{\partial u}{\partial C}\left[\frac{\partial C}{\partial c_{it}}\alpha_{ct} + \frac{\partial C}{\partial d_{it}}\alpha_{dt}\right] \quad \text{(A.1)}$$

and hence

$$
\begin{aligned}
& U(c_i+\alpha_c, d_i+\alpha_d) - U(c_i, d_i) \\
&= \mathbb{E}\left\{\sum_{t=0}^{\infty} u(C[c_{it}+\alpha_{ct}, d_{it}+\alpha_{dt}]) - u(C[c_{it}, d_{it}])\right\} \\
\leq \Delta(\alpha) \equiv\ & \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t \left(u(C[c_{it}, d_{it}]) + \frac{\partial u}{\partial C}\left[\frac{\partial C}{\partial c_{it}}\alpha_{ct} + \frac{\partial C}{\partial d_{it}}\alpha_{dt}\right]\right)\right\}
\end{aligned}
$$

It suffices to show that $\Delta(\alpha) = 0$ for any budget-feasible deviation α. Let $\pi_t = \beta^t \frac{\partial u}{\partial C}(C[c_{it}, d_{it}]) \frac{\partial C}{\partial c_{it}}(c_{it}, d_{it})$ denote the shadow price process for non-durable consumption. The inter-temporal Euler equation for an asset (i.e. equity or a bond) is as follows

$$
\text{(A.2)} \qquad \mathbb{E}_t\left[\pi_{t+1}\left(p_{t+1} + div_{t+1}\right)\right] = \pi_t\, p_t \qquad \mathbb{P} - a.s.
$$

and the intratemporal first-order condition is

$$
\text{(A.3)} \qquad \pi_t \frac{\partial C_{it} \,/\, \partial d_{it}}{\partial C_{it} \,/\, \partial c_{it}} \;=\; \pi_t\, q_t^d \;-\; (1-\delta_d)\,\mathbb{E}_t\left\{\pi_{t+1}\, q_{t+1}^d\right\}
$$

In order to obtain budget-feasible processes for nondurable consumption $c_{it}+\alpha_{ct}$, and the stock of consumer durables $d_{it}+\alpha_{dt}$, the consumer must deviate from the no-trade portfolio strategy by (i) some risk-asset strategy φ, satisfying $\varphi_{-1} = 0$, for all t,

$$
\text{(A.4)} \qquad \alpha_{ct} \;+\; q_t^d\, \delta I_{it}^d \;=\; \varphi_{t-1}\left(p_t \;+\; div_t\right) \;-\; \varphi_t\, p_t
$$

and (ii) strategy for the durables investment δI_{it}^d, dictated by the laws of motion for the stocks of durables,

$$
\text{(A.5)} \qquad \delta I_{it}^d \;=\; \alpha_{dt} - (1-\delta_d)\alpha_{dt-1}
$$

Multiplying the equation (A.4), and using equation (A.5) yields

$$
\text{(A.6)} \qquad \pi_t\, \alpha_{ct} \;+\; q_t^d\, \pi_t\left[\alpha_{dt} - (1-\delta_d)\alpha_{dt-1}\right] \;-\; (1-\delta_h)\alpha_{ht-1}] \;=
$$

$$
\text{(A.7)} \qquad = \pi\left[\varphi_{t-1}\left(p_t \;+\; div_t\right) \;-\; \varphi_t\, p_t\right]
$$

Law of iterated expectations along with the intratemporal first-order conditions and the initial conditions $\alpha_{d0} = \alpha_{d,\,-1} = 0$ gives us

$$\text{(A.8)} \qquad \mathbb{E}\left\{\sum_{t=0}^{\infty} \pi_t \, \frac{\partial C_{it} \,/\, \partial d_{it}}{\partial C_{it} \,/\, \partial c_{it}} \alpha_{dt}\right\} \quad =$$

$$\text{(A.9)} \qquad = \mathbb{E}\left\{\sum_{t=0}^{\infty} \pi_t \, q_t^d \left[\alpha_{dt} - (1-\delta_d)\alpha_{dt-1}\right]\right\}$$

In view of this results, summing the equation (A.6) and taking expectations yields that

$$\text{(A.10)} \qquad \Delta(\alpha) \;=\; \mathbb{E}\left\{\sum_{t=0}^{\infty} \pi_t \left[\varphi_{t-1}\left(p_t \,+\, div_t\right) \,-\, \varphi_t \, p_t\right]\right\}$$

$$\text{(A.11)} \qquad = \lim_{T\to\infty} \mathbb{E}\left\{\sum_{t=0}^{T} \pi_t \left[\varphi_{t-1}\left(p_t \,+\, div_t\right) \,-\, \varphi_t \, p_t\right]\right\}$$

Denote $V_t \;=\; \pi_t \, \varphi_{t-1}\left(p_t \,+\, div_t\right)$. The intertemporal first-order condition implies that V_t satisfies

$$\text{(A.12)} \qquad \mathbb{E}_t\left[V_{t+1}\right] \;=\; \varphi_t \, p_t$$

Using this result, and the law of iterated expectations gives

$$\text{(A.13)} \qquad \Delta(\alpha) = -\lim_{T\to\infty} \mathbb{E}\left\{V_{T+1}\right\}$$

It may be shown, invoking the fact that the trading strategy is bounded so that no more than n of any assets is ever held long or short, and the technical assumption $\lim_{T\to\infty} \mathbb{E}[M_T] = 0$ that $\lim_{T\to\infty} \mathbb{E}\left\{V_{T+1}\right\} = 0$. This proves that any budget-feasible strategy cannot raise consumer's well-being. ■

Appendix B. Proof of No-trade for Euler Equations

The objective of this appendix is to show in a concise way that the proposed equilibrium exhibits a no-trade property. In order to do that, I first show that the intertemporal Euler equation holds so that the private marginal valuations equal the market prices. Thereafter, I demonstrate that the intra-temporal first-order condition holds as well.

Take the proposed processes for an individual $i \in [0,1]$, namely, non-durable consumption flow c_{it}, and the stock of consumer durables d_{it}. Define the auxiliary function

$$\Psi(x,y) \;=\; \beta\, x^{\lambda(1-\gamma)-1}\, y^{(1-\lambda)(1-\gamma)} \tag{B.1}$$

The market price for an asset j satisfies

$$\mathbb{E}\left\{\frac{M_{t+1}}{M_t}\,(p_{jt+1}+div_{jt+1}) \;|\; \phi_t\right\} \tag{B.2}$$

whereas the private valuation is

$$\mathbb{E}\left\{\Psi\left(\frac{c_{it+1}}{c_{it}},\, \frac{d_{it+1}}{d_{it}}\right)\,(p_{jt+1}+div_{jt+1}) \;|\; \mathcal{F}_t\right\} \tag{B.3}$$

I show that the private valuation equals the market price. First, note that

$$\Psi\left(\frac{c_{it+1}}{c_{it}},\, \frac{d_{it+1}}{d_{it}}\right) = \Psi\left(\frac{c_{t+1}}{c_t},\, \frac{d_{t+1}}{d_t}\right) \times A_{it+1} \tag{B.4}$$

where

$$A_{it+1} \;=\; \exp\left\{\kappa\,\Delta\log u_{it+1}\right\} \tag{B.5}$$

$$\;=\; \exp\left\{\kappa\,\eta_{it+1}\,x_{t+1} - \frac{\kappa}{2}\,x_{t+1}^2\right\} \tag{B.6}$$

and $\kappa = (1-\lambda)(1-\gamma)$. Thus, the private valuation of an asset j at time t is

$$\mathbb{E}\left\{\Psi\left(\frac{c_{t+1}}{c_t},\, \frac{d_{t+1}}{d_t}\right)\,A_{it+1}\,(p_{jt+1}+div_{jt+1}) \;|\; \mathcal{F}_t\right\} \tag{B.7}$$

Invoking the law of iterated expectations, using the moment generating function for the Gaussian random variable $\Delta\log u_{it+1}$, and the fact that the shock η_{it+1} is independent of $\mathcal{F}_t$, it is immediate that

$$\mathbb{E}\left\{A_{it+1} \mid \mathcal{F}_t \cup \{x_{t+1}\}\right\} = \mathbb{E}\left\{A_{it+1} \mid \{x_{t+1}\}\right\}, \tag{B.8}$$

$$= \exp\left\{\frac{1}{2}\,\kappa\,(\kappa-1)\,x_{t+1}^2\right\} \tag{B.9}$$

Note that by definition

$$\frac{M_{t+1}}{M_t} = \Psi\left(\frac{c_{t+1}}{c_t}, \frac{d_{t+1}}{d_t}\right) \exp\left\{\frac{1}{2}\,\kappa\,(\kappa-1)\,x_{t+1}^2\right\} \tag{B.10}$$

Therefore, the private valuation equals the market price

$$\mathbb{E}\left\{\frac{M_{t+1}}{M_t}\,(p_{jt+1} + div_{jt+1}) \mid \mathcal{F}_t\right\} \tag{B.11}$$

as the information sets ϕ_t and $\mathcal{F}_t$ differ only in variables that are irrelevant for computing these expectations.

Second, I have to demonstrate that the market for consumer durables is in equilibrium. No arbitrage implies that the rental cost of consumer durables satisfies the intratemporal first-order condition

$$rc_t^d = q_t^d - (1-\delta_d)\,\mathbb{E}\left\{\frac{M_{t+1}}{M_t}\,q_{t+1}^d \mid \phi_t\right\} \tag{B.12}$$

. We presently show that the marginal willingness to rent a unit of consumer durables equals the market rental price, that is,

$$rc_t = q_t^d - (1-\delta_d)\,\mathbb{E}\left\{\Psi\left(\frac{c_{it+1}}{c_{it}}, \frac{d_{it+1}}{d_{it}}\right) q_{t+1}^d \mid \mathcal{F}_t\right\} \tag{B.13}$$

$$= q_t^d - (1-\delta_d)\,\mathbb{E}\left\{\frac{M_{t+1}}{M_t}\,q_{t+1}^d \mid \phi_t\right\}. \tag{B.14}$$

Thus, it suffices to show that

$$\mathbb{E}\left\{\Psi\left(\frac{c_{it+1}}{c_{it}}, \frac{d_{it+1}}{d_{it}}\right) q_{t+1}^d \mid \mathcal{F}_t\right\} = \mathbb{E}\left\{\frac{M_{t+1}}{M_t}\,q_{t+1}^d \mid \phi_t\right\}, \tag{B.15}$$

but this easily follows the same steps as for the intertemporal Euler equation. ■