

Financial Leverage and the Leverage Effect - A Market and Firm Analysis *

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ABSTRACT

We quantify the effect of financial leverage on stock return volatility in a dynamic general equilibrium economy with debt and equity claims. The effect of financial leverage is studied both at a market and a firm level where the firm is exposed to both idiosyncratic and market risk. In a benchmark economy with both a constant interest rate and constant price of risk, financial leverage generates little variation in stock return volatility at the market level but significant variation at the individual firm level. In an economy that generates time-variation in interest rates and the price of risk, there is significant variation in stock return volatility at the market and firm level. In such an economy, financial leverage has little effect on the dynamics of stock return volatility at the market level. Financial leverage contributes more to the dynamics of stock return volatility for a small firm.

Keywords: stock volatility, leverage effect, corporate debt, general equilibrium

JEL Classification: G10, G12

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1. Introduction

The leverage effect is that stock volatility is negatively correlated to stock returns—stock volatility tends to increase when stock prices drop. There are two common economic explanations for the leverage effect. The first explanation is based on the relationship between volatility and expected returns. When volatility rises, expected returns tend to increase, leading to a drop in the stock price. As a consequence, volatility and stock returns are negatively correlated. The second explanation is based on financial leverage. When stock prices fall, financial leverage increases, leading to an increase in stock return volatility.

No consensus exists on the size of the effects of financial leverage on stock volatility. Empirical studies that quantify the effect have produced mixed results, but these studies face several difficulties. Any study of the effect of financial leverage on volatility should use market debt valuations, which are difficult to obtain in practice. The empirical literature also lacks a theoretical benchmark for asset volatilities consistent with many features of asset pricing data in the presence of both debt and equity claims. We provide such a benchmark.

The hypothesis that financial leverage can explain the leverage effect was first discussed by Black (1976) and Christie (1982). Christie (1982) provides empirical evidence—based on a sample of large firms—for the negative relationship between stock returns and volatility induced by financial leverage. Duffee (1995) argues that such a relationship does not hold when small firms are also included in the sample. Schwert (1989) shows empirically that financial leverage cannot fully account for the observed variation in market volatility. Figlewski and Wang (2000) document a stronger leverage effect in down markets than in up markets. Campbell and Hentschel (1992) and Bekaert and Wu (2000) provide econometric models of asymmetric volatility. None of these empirical studies use market debt values to compute financial leverage.

Wu (2001) builds a partial equilibrium model to study the sources of asymmetric volatility. Tauchen (2005) builds a general equilibrium model with an endogenous volatility asymmetry. In both models, the leverage effect is mainly determined by the relationship between volatility and expected returns since neither of these theoretical models include corporate debt. We provide a benchmark general equilibrium economy with debt to study the effects of financial leverage on the

dynamics of stock volatility.

Our work explores the dynamics of stock volatility at both a market and a firm level. We characterize the economic channels through which financial leverage drives the dynamics of stock volatility, and quantify the leverage effect. The assumptions in Black (1976) and Christie (1982) are a constant asset return volatility and a constant riskless debt return. Both assumptions are inconsistent with realistic asset prices. We therefore study two different economies. In both economies, the cash flows generated by a firm's assets are specified exogenously, have a constant volatility, and are split into an exogenously specified riskless debt service and a dividend stream to equity holders. We derive the equilibrium prices and dynamics of all financial claims. We identify the economic forces behind the dynamics of stock volatility and quantify the effect of financial leverage on the dynamics of stock volatility.

The first economy we study is consistent with the assumptions in Black (1976) and Christie (1982). Here, macroeconomic conditions are fixed and financial leverage is the only driving force behind the dynamics of stock volatility. A firm's financial leverage is solely driven by innovations to the firm's cash flows. Financial leverage generates little variation at the market level where cash flow volatility is low, and significant variation at the firm level where cash flow volatility is higher. When financial leverage is the only factor affecting the dynamics of stock volatility, the leverage effect hypothesis holds at the firm level although stock volatility does not vary enough to be consistent with empirical evidence.

The second economy has a representative agent with habit persistent preferences similar to Campbell and Cochrane (1999). Such preferences lead to more realistic asset prices than in the first economy. The driving force in this economy is counter-cyclical risk aversion caused by external habit formation in the representative agent's preferences. The model is calibrated to several features of empirical asset prices, including the level and variation of the equity premium, the riskless rate, and the market price of risk. We simulate the economy and explore the time-series behavior of a firm's stock returns and volatility allowing for both debt and equity. At both the daily and monthly frequencies, we find at most a weak relationship between returns and volatility. We conclude that financial leverage has little economic significance at the market level when macroeconomic

conditions lead to realistic dynamics for the riskless rate and the market price of risk. But at the firm level, financial leverage can partially explain variations in stock return volatility.

2. Model

2.1. The Economy

We consider a continuous-time pure-exchange economy with a finite horizon T . Uncertainty in the economy is represented by the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, with $\mathcal{F} \equiv \mathcal{F}_T$, on which is defined a two-dimensional Brownian motion $W = (W_a, W_i)$. The filtration $\{\mathcal{F}_t\}$ is generated by W , augmented by the null sets. The uncorrelated components of the Brownian motion W represent an aggregate consumption shock W_a and an idiosyncratic shock W_i . The idiosyncratic shock is only used when studying the asset pricing of a small firm embedded in the economy. All stochastic processes introduced are assumed to be progressively measurable, all (in)equalities involving random variables hold \mathcal{P} -a.s., and all stochastic differential equations are assumed to have solutions without explicitly stating the required regularity.

The exogenous aggregate output in the economy $\delta(t)$ is assumed to follow a geometric Brownian motion with dynamics given by

$$d\delta(t) = \delta(t) [\mu_\delta dt + \sigma_\delta dW_a(t)], \quad \delta_0 > 0, \quad (1)$$

where μ_δ and σ_δ are scalars.

Preferences are modeled in the economy by a representative agent with external habit formation preferences as in Campbell and Cochrane (1999). The representative agent maximizes expected lifetime utility given by

$$E \left[\int_0^T e^{-\rho t} \frac{(C(t) - X(t))^{1-\gamma}}{1-\gamma} dt \right], \quad (2)$$

where $C(t)$ is the representative agent's consumption, $X(t)$ is the habit level, γ captures the risk aversion with respect to surplus consumption, and ρ is the subjective discount factor. When $X(t) = 0$ for all t , the preferences are identical to a CRRA utility representation with coefficient of

relative risk aversion equal to γ .

In equilibrium, the representative investors consumes aggregate output:

$$C(t) = \delta(t). \quad (3)$$

To complete our description of preferences, the process governing the habit $X(t)$ must be defined. Following Campbell and Cochrane (1999), we assume that the habit is external; agents do not optimize over $X(t)$. It is convenient to define the habit in terms of the surplus-consumption ratio defined as

$$S(t) = \frac{C(t) - X(t)}{\delta(t)} = \frac{\delta(t) - X(t)}{\delta(t)}, \quad (4)$$

with the second equality from the equilibrium condition (3).

The evolution of the habit level, $X(t)$, is characterized by the mean-reverting log surplus-consumption ratio,

$$s(t) \triangleq \ln(S(t)). \quad (5)$$

Unless noted otherwise, lower case letters denote the natural logarithms of the variables. The dynamics of $s(t)$ are

$$ds(t) = \phi(\bar{s} - s(t)) dt + \lambda(s(t))\sigma_\delta dW_a(t), \quad s(0) \in (0, s_{max}), \quad (6)$$

where ϕ is the mean-reversion rate, \bar{s} is the steady-state level, and σ_δ is the volatility of the aggregate consumption growth with \bar{S} and s_{max} defined as

$$\bar{S} = \sigma_\delta \gamma \sqrt{\frac{1}{\phi\gamma - B}}, \quad (7)$$

$$s_{max} = \bar{s} + 0.5 \times (1 - \bar{S}^2), \quad (8)$$

with B a constant. As in Wachter (2006), our model is an extension of Campbell and Cochrane (1999) to incorporate stochastic interest rates. The parameter B determines the sensitivity of interest rates to the output shock. If $B = 0$, our model is the same as in Campbell and Cochrane

(1999) with constant interest rates.

The sensitivity of the surplus-consumption ratio to aggregate consumption shocks, $\lambda(s)$, is

$$\lambda(s) = \frac{1}{\bar{S}} \sqrt{1 - 2(s - \bar{s})} - 1. \quad (9)$$

Given $\lambda(s) > 0$, the habit and aggregate consumption move together as any intuitive definition of habit would require.

The surplus-consumption ratio $S(t)$ captures the state of the economy. A low surplus-consumption ratio implies that the economy has been hit by a succession of bad shocks; current consumption is low compared to the historical average which is captured by the habit. Alternatively, a high $S(t)$ is an indicator of a good state of the economy, in which the agent enjoys higher consumption levels compared to his habit. The local curvature of the utility function, $\frac{\gamma}{S(t)}$, varies counter-cyclically with $S(t)$ and generates a counter-cyclical market price of risk in our model.

The representative investor finances his optimal consumption stream by trading in a locally riskless money market with price $B(t)$ and the market portfolio with price $V(t)$. The market portfolio is a claim to aggregate consumption; the cash flow from the market portfolio is split into debt and equity. The money market is in zero net-supply, while the market portfolio is in positive net-supply, normalized to one. The posited dynamics of the two securities are

$$dB(t) = B(t)r(t)dt, \quad B(0) > 0, \quad (10)$$

$$dV(t) + \delta(t)dt = V(t) [\mu_V(t)dt + \sigma_{V,a}(t)dW_a(t)], \quad V(T) = 0, \quad (11)$$

where the price system $(r, \mu_V, \sigma_{V,a})$ is determined in equilibrium.

Given our main goal is to explore the volatilities of debt and equity claims to V as well as a small firm exposed to both aggregate and idiosyncratic risk, it is convenient to summarize the endogenous price system in terms of a state price density process $\xi(t)$ with dynamics given by

$$d\xi(t) = -\xi(t) [r(t)dt + \kappa(t)dW_a(t)], \quad \xi(0) = 1, \quad (12)$$

where $\kappa(t) \triangleq \sigma_{V,t}^{-1} [\mu_V(t) - r(t)]$ is the market price of risk in the economy.

The representative agent's complete markets dynamic opportunity set from trading the money market and the market is summarized by a static budget constraint using standard martingale techniques (Karatzas et al. (1987), Cox and Huang (1989)):

$$E \left[\int_0^T \xi(t) C(t) dt \right] \leq V(0), \quad (13)$$

where the representative agent is initially endowed with the claim V .

We characterize asset prices using general equilibrium restrictions to identify the equilibrium state price density process. The equilibrium consumption rules, asset prices, and asset holdings must clear both the consumption and the financial markets, as well as satisfy the necessary and sufficient conditions for optimality of the representative agent's consumption-portfolio problem. The martingale formulation of the representative agent's optimal consumption-investment problem provides us with a direct way to construct the equilibrium (Karatzas et al. (1990)). The state price density process implied by the marginal utility of the representative agent evaluated at consumption market clearing: $C(t) = \delta(t)$ gives equilibrium asset prices that clear the financial markets. The equilibrium is characterized by solving for the unique state price density process. The necessary and sufficient conditions for the representative agent's optimization problem implies the equilibrium state price density is

$$\xi_t = e^{-\rho t} \left(\frac{S(t)C(t)}{S(0)\delta(0)} \right)^{-\gamma}. \quad (14)$$

The endogenous equilibrium parameters $r(t)$ and $\kappa(t)$ are easily determined in terms of economic primitives by computing the dynamics of equation (12),

$$r(t) = \rho + \gamma \left(\mu_\delta - \frac{\sigma_\delta^2}{2} \right) - \frac{\phi\gamma - B}{2} - B(s(t) - \bar{s}), \quad (15)$$

$$\kappa(t) = \gamma(1 + \lambda(s(t)))\sigma_\delta. \quad (16)$$

In particular, $r(t)$ and $\kappa(t)$ both time vary as they are functions of the log surplus-consumption ration $s(t)$.

Additionally, the equilibrium state price density values any marketed consumption stream. In particular, the value of the aggregate consumption stream is given by

$$V(t) = \frac{1}{\xi(t)} E \left[\int_t^T \xi(s) \delta(s) ds | \mathcal{F}_t \right]. \quad (17)$$

2.2. The Representative Firm

To evaluate the determinants of the return volatility at the aggregate level, we derive the equilibrium dynamics of the market portfolio. In our model, the market portfolio is a representative firm that pays out the aggregate consumption stream, $\delta(t)$, to its outside claimants. To explore leverage and time-varying risk premium effects on aggregate volatility dynamics, the firm's payouts are split the into two financial claims.

The representative firm's capital structure consists of debt and equity. All financial claims issued by the firm are held by the representative agent in the economy. Since the representative agent holds all claims to the firm in equilibrium, his incentives are aligned with total firm value maximization. In such a first-best scenario, bankruptcy due to debt service payment defaults never occurs along the equilibrium path, and the representative agent covers the debt service by injecting capital into the firm when needed. This assumption can be justified by an infinitesimal, but positive, bankruptcy cost. As a consequence, debt claims are not subject to default risk. Hence, the payout claimed by the equity holders is equal to $\delta_E(t) = \delta(t) - \delta_D(t)$, where $\delta_D(t)$ is the required debt service at time t . The specific terms of the debt service are described later. The total payout generated by the firm is invariant to the capital structure of the firm. We abstract from frictions such as security issue costs, taxation at both the corporate and personal level, and bankruptcy costs.

The equilibrium price of the debt claim issued by the representative firm, $D(t)$, is computed using the equilibrium state price density (14), and is

$$D(t) = \frac{1}{\xi(t)} E \left[\int_t^{T_d} \xi(s) \delta_D(s) ds | \mathcal{F}_t \right], \quad (18)$$

where $T_d \in [0, T]$ is the maturity of the firm's debt. The equilibrium debt dynamics are

$$dD(t) + \delta_D(t)dt = D(t) [(r(t) + \kappa(t)\sigma_{D,a}(t)) dt + \sigma_{D,a}(t)dW_a(t)], \quad (19)$$

for $t \in [0, T_d]$, with $D(T_d) = 0$, and where $\sigma_{D,a}(t)$ is the endogenous debt return volatility. As long as the equilibrium interest rate $r(t)$ is time-varying, the volatility of the debt contract will be non-zero even when the debt service $\delta_D(t)$ is deterministic as the contract is exposed to interest rate risk.

The equilibrium price of the equity claim of the representative firm $E(t)$ is

$$E(t) = V(t) - D(t). \quad (20)$$

Equity dynamics are

$$dE(t) + \delta_E(t)dt = E(t) [(r(t) + \kappa(t)\sigma_{E,a}(t)) dt + \sigma_{E,a}(t)dW_a(t)], \quad (21)$$

with $E(T) = 0$, and where $\sigma_{E,a}(t)$ is the endogenous equity return volatility.¹

Equity is a claim to the residual cash flows given the current debt service structure. Since the Miller and Modigliani Theorem (1958) holds in the model, all future capital structure adjustments will be zero NPV transactions, and have no consequences on current equity prices and dynamics. The next lemma formalizes the argument.

Lemma 2.1 *Current equity prices and equity return dynamics are invariant to future capital structure adjustments.*

The equilibrium dynamics of the debt and equity claims are fully characterized by equations (19) and (21), once the conditional volatilities are determined. The procedure followed to numerically obtain the endogenous asset dynamics is described in Appendix 6.

¹Throughout our analysis, we consider market-wide debt recapitalization policies that guarantee positive market equity valuations.

2.3. A Small Firm

In addition to the representative firm, we also consider a small firm in the economy facing market and idiosyncratic risk. We take the firm's investment decisions as given, implying they are unaffected by its financing decisions. The firm is modeled as an EBIT-generating machine, with an exogenous cash flow process

$$d\delta^f(t) = \delta^f(t) \left[\mu_\delta^f dt + \sigma_{\delta,a}^f dW_a(t) + \sigma_{\delta,i}^f dW_i(t) \right], \quad \delta^f(0) > 0, \quad (22)$$

where μ_δ^f is the instantaneous growth rate of the cash flows, $\sigma_{\delta,a}^f$ is the factor loading on the aggregate shock, and $\sigma_{\delta,i}^f$ is the factor loading on the firm-specific shock. The total value of the firm is

$$V^f(t) = \frac{1}{\xi(t)} E \left[\int_t^T \xi(s) \delta^f(s) ds | \mathcal{F}_t \right]. \quad (23)$$

The posited dynamics of the firm value process are:

$$dV^f(t) + \delta^f(t) dt = V^f(t) \left[\mu_V^f(t) dt + \sigma_{V,a}^f(t) dW_a(t) + \sigma_{V,i}^f(t) dW_i(t) \right], \quad (24)$$

with $V^f(T) = 0$.

The firm's exogenous capital structure consists of debt and equity claims. The firm's debt service is a deterministic stream of cash flows $\delta_D^f(t)$ with maturity $T_d \in [0, T]$. The debt's cash flows are riskless. To ensure this, we assume that the debt holders take control of the firm when the value of the EBIT-generating machine falls to the debt's value. At this point, debt holders can costlessly replace their claim on the firm with a riskless security from the capital markets. Since the equity holders do not have a bankruptcy timing option, the firm is able to issue debt with riskless cash flows. However, the debt's market value is risky due to its exposure to interest rate risk.

The equilibrium value of the firm's outstanding debt is

$$D^f(t) = \frac{1}{\xi(t)} E \left[\int_t^{T_d} \xi(s) \delta_D^f(s) ds | \mathcal{F}_t \right], \quad (25)$$

with its equilibrium dynamics given by

$$dD^f(t) + \delta_D^f(t)dt = D^f(t) \left[\left(r(t) + \kappa(t)\sigma_{D,a}^f(t) \right) dt + \sigma_{D,a}^f(t)dW_a(t) + \sigma_{D,i}^f(t)dW_i(t) \right], \quad (26)$$

for $t \in [0, T_d]$, with $D^f(T_d) = 0$, and where $\sigma_{D,a}^f(t)$, and $\sigma_{D,i}^f(t)$ are endogenous factor loadings on the aggregate and firm-specific shocks, respectively.

The firm's equity is defined as a claim to the firm's residual cash flow as long as the firm remains solvent. Define the stopping time T_b :

$$T_b = \inf\{t : V^f(t) = D^f(t)\}. \quad (27)$$

The value of equity is positive for $t \leq T_B$ and is given by

$$E^f(t) = V^f(t) - D^f(t), \quad (28)$$

for $t \leq T_B$ and zero otherwise. Equilibrium dynamics are

$$dE^f(t) + \delta_E^f(t)dt = E^f(t) \left[\left(r(t) + \kappa(t)\sigma_{E,a}^f(t) \right) dt + \sigma_{E,a}^f(t)dW_a(t) + \sigma_{E,i}^f(t)dW_i(t) \right], \quad (29)$$

with $E^f(T_b) = 0$, and where $\sigma_{E,a}^f(t)$, and $\sigma_{E,i}^f(t)$ are the endogenous factor loadings on the aggregate and the firm-specific shocks, respectively.

The equilibrium dynamics of the debt and equity claims are fully characterized by equations (26) and (29), once the factor loadings are determined. A summary of the procedure used to obtain the asset return dynamics is given in the Appendix.

3. Calibration and Simulation

We calibrate the exogenous model parameters to U.S. post-war data. The output process parameters are chosen to fit the annual growth rate of U.S. consumption data; we set $\mu_\delta = 0.019$ and $\sigma_\delta = 0.015$. The parameters that govern the dynamics of the surplus-consumption ratio are chosen

to fit the empirical moments of asset prices as summarized in the righthand column of Table 1. The model is broadly consistent with the observed moments of asset prices for $\phi = 0.1385$ and $B = 0.015$. Other model parameters are specified as $\rho = 0.09$ and $\gamma = 1.8$. The first two columns of Table 1 summarize the equilibrium asset pricing implications of our calibrated model. The first column assumes there is no aggregate debt in the economy, while the second column assumes the aggregate debt in the market takes the form of an annuity with a maturity of $T_d = 10$ and a constant debt service of 60% of current total payouts. With such a capital structure, the unconditional leverage ratio of the model roughly matches the U.S. mean aggregate leverage ratio from 1952-1998 as documented in Korajczk and Levy (2003).

Equations (9), (15), and (16) imply that the equilibrium riskless rate $r(t)$ and the market price of risk $\kappa(t)$ are solely driven by the surplus-consumption ratio. Figure 1 plots the riskless rate, the market price of risk, and the market's price-dividend ratio in different states of the economy, as well as the stationary distribution of $S(t)$. Both the riskless rate and market price of risk move counter-cyclically. At the extremely bad states of the economy, both variables rise dramatically. The stochastic process of $S(t)$; however, never attains a value of zero and the equilibrium asset prices are well-defined in all states of the economy. Since the riskless rate fluctuates with the state of the economy, all financial claims to future cash flows are subject to interest rate risk. Similarly, the counter-cyclical variation in the market price of risk generates time-variation in the price-dividend ratios of risky claims as can be seen in the bottom panel of Figure 1.

To explore the relationship between stock returns, stock volatility, and financial leverage, we simulate the economy for 2,000 years at a daily frequency. Monthly data is then constructed using the daily observations. The cash flows of the representative firm are given by the aggregate consumption stream in the economy. We simulate two small firms with different cash flow risk compositions. In particular, both small firms have identical expected cash flow growth rates $\mu_\delta^f = 0.019$ and identical idiosyncratic cash flow risk volatilities $\sigma_{\delta,i}^f = 0.2$, whereas their aggregate cash flow risk volatilities are different. We set $\sigma_{\delta,a}^f = 0.015$ for the firm we denote firm AI and $\sigma_{\delta,a}^f = 0.0$ for the firm we denote firm I. Therefore, the former has both systematic and idiosyncratic cash flow risk, while the other firm has only idiosyncratic cash flow risk.

4. Results - Constant Asset Pricing Moments

Before exploring our fully calibrated economy, it is useful to consider a case fully consistent with the leverage hypothesis where asset return volatilities are constant. Such a benchmark is easily constructed by setting the habit level to zero or equivalently by setting $\phi = 0$, $\bar{S} = 1$, and $S_0 = 1$. The model then collapses to the representative agent having standard power utility preferences. With power utility preferences and *i.i.d.* consumption growth, the riskless rate and market price of risk in the economy are both constant. This leads to unrealistic asset pricing implications, but is consistent with the assumptions underlying the leverage effect hypothesis. This is a natural place to start, and a useful benchmark for quantifying the leverage effect.

In this setting, closed-form solutions for the prices of all financial are easily obtained. Modeling the debt service as a constant annuity of T_d years with a constant cash flow equal to δ_D , the market values of the representative firms's total assets and debt are

$$V(t) = \frac{\delta(t)}{r - \mu_\delta + \gamma\sigma_\delta^2} \left[1 - e^{-(r - \mu_\delta + \gamma\sigma_\delta^2)(T-t)} \right], \quad (30)$$

$$D(t) = \frac{\delta_D}{r} \left[1 - e^{-r(T_d-t)} \right], \quad (31)$$

where the constant riskless rate is given by $r = \rho + \gamma \left(\mu_\delta - \frac{\sigma_\delta^2}{2} \right) - \frac{\gamma^2 \sigma_\delta^2}{2}$. The value of the firm's equity is then given by $E(t) = V(t) - D(t)$ as long as $E(t) \geq 0$.

For the representative firm, the debt volatility is zero since there is no interest rate risk. The stock volatility is equal to

$$\sigma_{E,a}(t) = \left(1 + \frac{D(t)}{E(t)} \right) \sigma_\delta. \quad (32)$$

A similar expression is obtained for the volatility of a small firm's stock. Hence, in this economy, stock volatility is driven only by the underlying cash flow volatility and the variation in the market value of financial leverage.

To quantify the impact of financial leverage on the dynamics of stock volatility, we simulate our economy.² In our simulations for the levered aggregate firm, we assume that the firm rebalances

²We use the same parameters as in the previous section with $X(t) = 0$ for all t .

its debt payments to a target level of 0.3 of total firm cash flows every 5 years. Table 2 shows some descriptive statistics from the simulated market level data. When the representative firm is unlevered, its stock volatility is identical to the dividend volatility, and shows no variation. When the representative firm is levered, its stock volatility rises, and shows some variation over time. With a realistic level of market leverage around 0.3; however, both the level and the variation in market volatility is much lower than those observed in the data.

Table 3 and 4 provide similar descriptive statistics for the two small firms. Since both small firms have larger cash flow volatilities than the representative firm, their stock return volatilities are both higher on average. Since the debt's value is constant for a given debt service, more volatile stock returns lead to more volatile market leverage, which in turn generates further variation in the stock return volatility.

While the CRRA preferences leads to a significant variation in stock volatility at the small firm level, the stock volatility dynamics at the market level are quite unrealistic. Additionally, the model does not generate a realistic spread between the cash flow volatility and the stock volatility.³ While CRRA preferences imply a constant riskless rate and a constant price of risk, which are consistent with the financial leverage hypothesis as in Christie (1982), several important characteristics of asset prices are not obtained in this equilibrium.

5. Results - Time-Varying Asset Pricing Moments

We now calibrate our economy to the observed moments of asset prices. In contrast to the special case studied in the previous subsection, the assumptions of the leverage effect hypothesis are not satisfied in our calibrated economy. Because of the time-variation in riskless rate and market price of risk, the value of a firm's assets will have a time-varying volatility and the value of debt contracts will be exposed to interest rate risk.

³Shiller (1981) shows stock returns are at least five times more volatile than their underlying cash flows.

5.1. Market Level Analysis

We consider two different debt service structures for the representative firm: a 10-year constant annuity and a constant perpetuity.⁴ While a 10-year debt maturity is realistic, the perpetual maturity debt is widely used in the corporate finance literature for technical and computational simplicity.⁵ Given we are not solving for optimal capital structures, our framework enables us to analyze both cases and study the effect of debt maturity on equity return volatility. With either maturity structure, we adjust the level of the coupon payment such that the debt claim comprises 30% of the total value of the firm initially.

The total value of the representative firm is driven by the instantaneous cash flows $\delta(t)$ and the surplus-consumption ratio $S(t)$. The value of the outstanding debt is similarly driven by $\delta^d(t)$ and $S(t)$. Both the total firm value and the debt value are proportional to their underlying cash flows. This implies that the equity value is proportional to its underlying cash flows and depends only on $\delta^e(t)$ and the surplus-consumption ratio $S(t)$.

To see the impact of the macro factors on market volatility, Figure 4 plots market volatility at different states of the economy keeping the ratio of debt service to total cash flows constant. The figure makes clear that even the unlevered firm has significant variation in its stock volatility. In good states of the economy, i.e. when S is high, stock volatility is low. As the riskless rate and the market price of risk increases in the economy, stock volatility also rises. In extremely bad states of the economy, the state variable S has very low volatility to keep it positive, leading to a decrease in stock volatility. In most states of the economy, however, the inverse relationship between S and stock volatility obtains.

While both the 10-year annuity and the perpetuity specifications are constructed with a 30% financial leverage, their implications on the dynamics of market volatility are different. The impact of financial leverage on stock volatility is higher when debt maturity is lower. The interest rate risk exposure of the debt contracts depend on their duration. When debt has a low duration, its value is less sensitive to increasing interest rates in bad times. Hence, market leverage increases faster as

⁴In our analysis, we approximate a perpetuity by a 100-year constant annuity. Perturbing the maturity of this annuity led to similar quantitative results.

⁵See for example Fischer et al. (1989); Leland (1994); Leland and Toft (1996); Leland (1998); Goldstein et al. (2001).

S decreases when debt duration is lower, leading to a higher stock volatility in bad states of the economy.

We simulate our economy to see how financial leverage affects the variation in stock volatility. We take the debt service as a 10-year annuity with an initial debt service to total cash flow ratio of 74%, leading to 30% market leverage on average. We assume that the firm continuously issues 10-year discount bonds to keep the debt maturity constant at every point in time. We also assume that the firm readjusts its debt service to 60% of the total cash flows every 5 years. Figure 5 plots a 100 year window of the simulated market volatility and returns series.

Table 5 provides descriptive statistics on asset returns from the simulated data. Financial leverage has a mean of 0.29 and a standard deviation of 0.06 for the market. When the market is levered, both the mean and the standard deviation of realized returns increase. The average market volatility rises from 13.95% to 19.01%, and the standard deviation of market volatility rises from 3.95% to 6.25%. Thus, financial leverage has increased both the level and the variation of market volatility in our simulated data. The increased variation in market volatility, however, does not necessarily imply that the leverage effect hypothesis holds true. The variation in financial leverage that leads to variation in market volatility can also be driven by changing market conditions.

An implication of the leverage effect hypothesis is high realized returns are associated with low market volatility. Thus, the probability density of realized returns should be negatively skewed. In our simulated data, however, leverage leads to a small increase in the skewness of market returns. The skewness of market returns increase from 0.07 to 0.09 in the daily data, and from 0.18 to 0.28 in the monthly data. To see whether skewness is indeed increased by leverage, we plot the frequency distributions of the volatility and returns. To visualize the impact of leverage on the skewness of market returns, Figure 6 plots the frequency distributions of conditional volatility of equity returns, realized returns, and expected returns, computed from the daily data. The figure makes clear that there no observable change in skewness in market returns when leverage is introduced.

To further explore the relationship between market returns and market volatility caused by leverage further, we replicate the regression exercises presented by the related empirical literature using our simulated data. Following Christie (1982), Duffee (1995), and Schwert (1989), we consider

the following linear regression models:

$$\log\left(\frac{\sigma_{t+1}}{\sigma_t}\right) = a_0 + b_0 R_t + \epsilon_{t+1,0} \quad (33)$$

$$\log(\sigma_{t+1}) = a_1 + b_1 R_t + \epsilon_{t+1,1} \quad (34)$$

$$\log(\sigma_t) = a_2 + b_2 R_t + \epsilon_{t,2} \quad (35)$$

$$\log(\sigma_{t+1}) = a_3 + b_3 \frac{D_t}{E_t} + \epsilon_{t+1,3} \quad (36)$$

where σ represents the volatility of stock returns, R represents the equity return, and $\frac{D_t}{E_t}$ represents the market value of the firm's leverage. Regression (33) was studied by Christie (1982) with $b_0 < 0$ consistent with the financial leverage hypothesis. Duffee (1995) argued that the source of a negative coefficient on b_0 was a positive contemporaneous relation between firm equity returns and firm equity volatility by estimating the two regressions (34) and (35). Schwert (1989) estimated a variation of regression (36); $b_3 > 0$ is consistent with the financial leverage hypothesis.

To control for the time-variation in the market price of risk, we also consider an augmented version of regression equation (36):

$$\log(\sigma_{t+1}) = a_4 + b_4 \frac{D_t}{E_t} + c_4 \kappa_t + \epsilon_{t+1,4} \quad (37)$$

Table 6 summarizes the results of the regressions which should be interpreted as characterizing the population moments of our model.⁶ In the daily data, the regression models (33), (34) and (35) have no explanatory power regardless of the leverage. Model (36) has a high R^2 and a significant coefficient for market leverage. Model (37) makes clear that model (36) has an important omitted variable: the market price of risk. When the market price of risk is introduced as an independent variable, the sign of the coefficient of leverage changes. The results of the regressions using the monthly data is similar except for the significant explanatory power of Model (33). The significant negative relationship between realized returns and innovations in market volatility is consistent with the financial leverage hypothesis. But, the financial leverage hypothesis implies that the b

⁶We also ran kernel regressions to detect any non-linear relationships between returns and volatility. As no significant non-linearities were detected, we only present the results for the linear regressions.

coefficient should be lower with increased financial leverage. The explanatory power in model (33) is therefore not driven by effect of financial leverage.

Our analysis of the dynamics of market volatility shows that most of the variation in market volatility is driven by the variation in market conditions. Financial leverage does not lead to significant skewness in realized returns. The regressions we ran on our simulated data do not provide support that financial leverage is a major driver of stock volatility.

5.2. Small Firm

While the aggregate consumption process restricts the calibration of the representative firm total cash flow process, we are able to assume different risk loadings on the small firm's cash flows. In this section, we explore the impact of financial leverage on the dynamics of stock volatility at the individual firm level, and in the presence of idiosyncratic cash flow risk.

We consider two different firms: a firm with both aggregate and idiosyncratic risk (firm AI), and a firm with only idiosyncratic risk (firm I). For firm AI, we set $\sigma_{\delta,a} = 0.015$ and $\sigma_{\delta,i} = 0.2$, while we set $\sigma_{\delta,a} = 0.0$ and $\sigma_{\delta,i} = 0.2$ for firm I. We assume that the levered firm restructures its debt service to a target debt service to total cash flow ratio every 5 years.

We first analyze the equity return volatility dynamics of firm AI. Figure 7 plots the equity return volatility in different states of the economy. The figure makes clear that the impact of financial leverage on the dynamics of stock volatility depends on the firm's debt duration. With shorter duration debt, market leverage rises rapidly in the bad states of the world, leading to increased equity volatility. For the long term debt, leverage increases the level of the equity volatility, but leverage has little impact on the dynamics of the equity volatility.

Table 7 provides descriptive statistics on asset returns from the simulated data. For the levered firm, financial leverage has a mean of 0.318 and a standard deviation of 0.117. Leverage increases the average daily returns from 0.019% to 0.029%. Leverage increases the average equity volatility from 24.47% to 37.02%. The impact of leverage on the variation in equity volatility is quite significant. Leverage increases the standard deviation of equity volatility from 2.07% to 12.85%. On the other hand, leverage increases the skewness of the equity returns. Figure 8 plots the

frequency distributions of equity volatility and returns, and clearly shows that leverage does not induce negative skewness to equity returns.

Table 8 summarizes the results of the regressions for firm AI. We find no significant relationship between realized returns and equity volatility in the daily data. In the monthly data, changes in market volatility is negatively related to realized returns, as predicted by the leverage effect hypothesis. The predictive power of this regression, model (33), is quite weak with an R^2 of around 12%. Results of regressions (36) and (37) show that leverage has predictive power on stock volatility even when the market price of risk is included in the regressions as an independent variable. Thus, our regression analysis provide at least weak evidence supporting the leverage effect hypothesis.

Next, we analyze a small firm when all cash flow risk is idiosyncratic, firm I. Figure 9 plots the equity return volatility in different states of the economy. Since all financial claims to the cash flows of the firm are only subject to idiosyncratic shocks, the effect of changing market conditions on equity volatility is only through the interest rate risk channel. As before, when debt has a low duration relative to equity, market leverage increases rapidly in bad times, leading to higher stock volatility.

Table 9 provides descriptive statistics on asset returns from the simulated data. As before, leverage increases the level and the variation in stock volatility significantly. The variation in volatility is lower compared to firm AI. The skewness of the return distribution does not seem to decrease with leverage.

Table 10 summarizes the results of the regressions for the firm I. Model 33 provides no support for the leverage effect hypothesis in both the daily and monthly data. Model 36 provides a significant link between leverage and stock volatility. The predictive power of leverage on stock volatility is strong even when the market price of risk is included as an independent variable.

Overall, our analysis provides some support that financial leverage drives the dynamics of stock volatility at the firm level. This feature is driven by idiosyncratic risk influencing the firm's equity value and not the firm's debt value. Hence, the firm's financial leverage can move independent of market conditions in contrast to our market-wide analysis. Time-varying market conditions are still important determinants of even firm I's equity volatility. Given the firm's debt value is still

driven by systematic interest rate risk, variations in financial leverage are still partially explained by systematic risk which ultimately feeds into the variation in the firm's equity volatility.

6. Conclusion

We explore the relationship between financial leverage and the dynamics of stock volatility in an economy with realistic interest rate and market price of risk dynamics. We show that for the market as a whole, financial leverage increases the level of equity volatility, but the dynamics of equity volatility are mostly driven by a time-varying interest rate and a time-varying market price of risk. Financial leverage contributes more to the dynamics of stock volatility for a small firm exposed to both idiosyncratic risk and market risk. But in both cases, the variation in interest rates and the market price of risk is the main force behind the dynamics of stock volatility.

Understanding why stock volatility moves over time is crucial for many financial applications. Equilibrium asset pricing models with realistic asset pricing implications can be used to explore the dynamics of financial claims issued by firms. Our analysis relies on one of many such models, the habit formation model of Campbell and Cochrane (1999), and assumes a simple riskless debt cash flow structure. Future work is needed to extend this analysis to other asset pricing frameworks, and explore the implications of risky debt cash flows to the dynamics of stock volatility.

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Appendix

Computational Procedure

In this section, we describe the details of the numerical solution procedure used to compute the asset return dynamics. We describe the details for the small firm analysis, since the market level analysis is a special case with no idiosyncratic risk.

Since the cash flow growth is an *i.i.d.* process, the total firm value is characterized by two state variables, i.e. $V^f(t) = V^f(S(t), \delta^f(t))$. A quick observation of equation (23) yields $V(S(t), \delta^f(t)) = \delta^f(t) \times V(S(t), 1)$. Hence, to obtain the firm value, we first compute $V(S(t), 1)$ for all $S(t) \in (0, S_{max}]$.

Numerically calculating $V(S(t), 1)$ in the Campbell and Cochrane (1999) model is problematic; see for example Chen, Collin-Dufresne, and Goldstein (2003), Cosimano, Chen, and Himonas (2003), and Wachter (2005). A convenient way to obtain $V(S(t), 1)$ is to compute equation (23) using a risk-neutral valuation. Under the risk-neutral measure, we can rewrite (23) as

$$V^f(S(t), 1) = \tilde{E} \left[\int_t^T e^{-\int_t^s r(s_u) du} \delta^f(s) ds | \mathcal{F}_t \right]. \quad (38)$$

Then, by Monte Carlo simulating $r(t)$ and $\delta^f(s)$ under the risk-neutral measure, and using numerical integration methods, we evaluate (38). Precise estimates are obtained by using 20,000 paths and 10,000 steps where $T = 100$.

To obtain the dynamics of the firm value, we apply Ito's Lemma on $V^f(S(t), \delta^f(t))$. The diffusion terms of equation (24) are obtained as

$$\sigma_{V,a}^f(t) = \frac{S(t)\lambda(S(t))}{V^f(S(t), 1)} \frac{\partial V^f(S(t), 1)}{\partial S(t)} \sigma_\delta + \sigma_{\delta,a}^f, \quad (39)$$

$$\sigma_{V,i}^f(t) = \sigma_{\delta,i}^f. \quad (40)$$

Given a deterministic debt service, the value of the debt claims are also given by $D(S(t), \delta_D^f(t)) = \delta_D^f(t) \times D(S(t), 1)$. Hence, we obtain the debt value by computing $D(S(t), 1)$ using Monte Carlo simulations under the risk-neutral measure. The diffusion terms of equation (26) are obtained by

applying Ito's Lemma on $D(S(t), \delta_D^f(t))$.

$$\sigma_{D,a}^f(t) = \frac{S(t)\lambda(S(t))}{D^f(S(t), 1)} \frac{\partial D^f(S(t), 1)}{\partial S(t)} \sigma_\delta, \quad (41)$$

$$\sigma_{D,i}^f(t) = 0. \quad (42)$$

As the residual claim, equity value is given by $E(t) = \delta^f(t)V^f(S(t)) - \delta_D^f(t)D(S_T, 1)$. The diffusion terms of equity return dynamics (29) are given by

$$\sigma_{E,a}^f(t) = (1 + L(t)) \sigma_{V,a}^f(t) - L(t)\sigma_{D,a}^f(t), \quad (43)$$

$$\sigma_{E,i}^f(t) = (1 + L(t)) \sigma_{V,i}^f(t). \quad (44)$$

Proofs

Proof of Lemma 2.1. Consider a firm whose assets pay the cash flow stream $\delta(t)$ and has a required debt service $\delta_D(t)$. The value of the firm's equity at time s is

$$E(s) = \frac{1}{\xi(s)} E \left[\int_s^T \xi(t) (\delta(t) - \delta_D(t)) dt | \mathcal{F}_s \right]. \quad (45)$$

Consider a capital structure adjustment where the firm issues new debt contracts that require a service $\delta_D(t)$ for $t \in (T_a, T_b]$, where $s < T_a \leq T_b \leq T$. The value of the newly issued debt D^n is paid out to the shareholders on date T_a . The new equity price for the existing shareholders $E^n(t)$ is given by

$$E(s) = \frac{1}{\xi(s)} E \left[\int_s^{T_a} \xi(t) (\delta(t) - \delta_D(t)) dt + \xi(T_a) D^n(T_a) | \mathcal{F}_s \right] \quad (46)$$

$$+ \frac{1}{\xi(s)} E \left[\int_{T_a}^{T_b} \xi(t) (\delta(t) - \delta_D(t) - \delta_D^n(s)) dt | \mathcal{F}_s \right] \quad (47)$$

$$+ \frac{1}{\xi(s)} E \left[\int_{T_b}^T \xi(t) (\delta(t) - \delta_D(t)) dt | \mathcal{F}_s \right]. \quad (48)$$

Given no financial frictions when issuing the new debt, its value at time T_a is

$$D^n(T_a) = \frac{1}{\xi(T_a)} E \left[\int_{T_a}^{T_b} \xi(t) \delta_D^n(t) dt | \mathcal{F}_{T_a} \right]. \quad (49)$$

Substituting (49) into (46) and applying the law of iterated expectations,

$$E(s) = \frac{1}{\xi(s)} E \left[\int_s^T \xi(t) (\delta(t) - \delta_D(t)) dt | \mathcal{F}_s \right]. \quad (50)$$

implying that the equity price and dynamics are invariant to the future capital structure. ■

Table 1. Key properties of equilibrium asset prices. The levered market has a debt maturity of $T_d = 10$ and constant debt service at 60% of current total payouts.

	Model (Unlevered)	Model (Levered)	Data
Expected Excess Market Returns	6.7%	8.5%	8.1%
Std. Deviation of Market Returns	13.7%	17.2 %	15.6 %
Expected Riskfree Rate	0.7%	0.7%	0.9%
Std. Deviation of Riskfree Rate	0.7%	0.7%	< 1.7%
Expected Market Price of Risk	0.43	0.43	0.39
Leverage	0	0.30	0.30

Table 2. Descriptive statistics of simulated equilibrium market returns with CRRA preferences.

Unlevered Market						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.034%	0.079%	-0.338%	0.404%	0.006	0.009
Monthly Returns	1.040%	0.436%	-0.508%	3.055%	0.008	0.041
Conditional Volatility	1.500%	0.0%	1.500%	1.500%	N.A.	N.A.
Levered Market with Readjustment Every 5 Years						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.035%	0.111%	-0.490%	0.558%	0.006	0.012
Monthly Returns	1.041%	0.613%	-1.134%	3.830%	-0.009	0.047
Conditional Volatility	2.106%	0.030%	1.997%	2.176%	-0.490	-0.312
Market Leverage	0.288	0.010	0.249	0.311	-0.552	-0.208

Table 3. Descriptive statistics of simulated equilibrium small firm with both systematic and idiosyncratic cash flow risk equity returns with CRRA preferences.

Unlevered Market						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.030%	1.056%	-4.609%	5.193%	0.033	0.017
Monthly Returns	0.905%	5.886%	-19.01%	24.99%	0.171	0.067
Conditional Volatility	20.06%	0.0%	20.06%	20.06%	N.A.	N.A.
Levered Market with Readjustment Every 5 Years						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.028%	1.709%	-19.32%	19.98%	0.047	1.817
Monthly Returns	0.857%	9.508%	-38.76%	74.56%	0.326	1.762
Conditional Volatility	31.42%	8.148%	22.53%	152.5%	3.837	24.36
Market Leverage	0.335	0.113	0.110	0.868	1.131	1.457

Table 4. Descriptive statistics of simulated equilibrium small firm with fully idiosyncratic cash flow risk equity returns with CRRA preferences.

Unlevered Market						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.030%	1.053%	-4.598%	5.151%	0.032	0.015
Monthly Returns	0.902%	5.871%	-18.89%	24.73%	0.171	0.084
Conditional Volatility	20.00%	0.0%	20.00%	20.00%	N.A.	N.A.
Levered Market with Readjustment Every 5 Years						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.028%	1.172%	-18.40%	23.82%	0.055	2.150
Monthly Returns	0.850%	9.548%	-39.56%	74.65%	0.344	2.081
Conditional Volatility	31.44%	8.454%	22.54%	176.2%	4.127	28.66
Market Leverage	0.336	0.114	0.113	0.886	1.137	1.504

Table 5. Descriptive statistics of simulated equilibrium market returns.

Unlevered Market						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.021%	0.764%	-4.110%	4.769%	0.073	0.678
Monthly Returns	0.621%	4.198%	-17.21%	23.47%	0.184	0.692
Conditional Volatility	13.95%	3.953%	1.798%	18.31%	-0.836	-0.288

Levered Market with Readjustment Every 5 Years						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.028%	1.054%	-6.099%	7.285%	0.089	0.961
Monthly Returns	0.849%	5.816%	-25.50%	36.62%	0.280	1.043
Conditional Volatility	19.01%	6.254%	2.23%	28.84%	-0.448	-0.752
Market Leverage	0.289	0.062	0.188	0.640	1.299	2.082

Table 6. Estimates of the regression equations at the market level. Quantities in parenthesis are standard errors.

	Daily Data					
	Levered Market		Adjusted R^2	Unlevered Market		Adjusted R^2
	a	b		c	a	
Model (33)	-1.02×10^{-6} (2.1×10^{-5})	4.83×10^{-4} (2.0×10^{-3})	-1.3×10^{-6}	-9.02×10^{-7} (2.0×10^{-5})	-6.16×10^{-4} (2.6×10^{-3})	-1.3×10^{-6}
Model (34)	-1.73 (0.0005)	-0.33 (0.046)	6.8×10^{-5}	-2.02 (0.0004)	-0.45 (0.06)	8.6×10^{-5}
Model (35)	-1.73 (0.0005)	-0.33 (0.046)	6.8×10^{-5}	-2.02 (0.0004)	-0.45 (0.06)	8.6×10^{-5}
Model (36)	-2.54 (0.0001)	1.93 (0.0002)	0.43			
Model (37)	-2.15 (0.0005)	-2.98 (0.003)	0.92			
	Monthly Data					
	Levered Market		Adjusted R^2	Unlevered Market		Adjusted R^2
	a	b		c	a	
Model (33)	0.004 (0.0005)	-0.46 (0.008)	0.124	0.003 (0.0004)	-0.55 (0.01)	0.10
Model (34)	-2.97 (0.003)	-0.32 (0.05)	0.002	-3.26 (0.002)	-0.44 (0.055)	0.003
Model (35)	-2.97 (0.003)	0.14 (0.045)	0.0004	-3.26 (0.002)	0.11 (0.055)	0.0001
Model (36)	-3.76 (0.006)	1.89 (0.013)	0.46			
Model (37)	-3.39 (0.003)	-2.80 (0.018)	0.88			

Table 7. Descriptive statistics of simulated asset prices for the small firm with systematic risk.

Unlevered						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.019%	1.295%	-6.062%	6.632%	0.054	0.105
Monthly Returns	0.565%	7.146%	-23.90%	38.04%	0.222	0.091
Conditional Volatility	24.47%	2.068%	20.08%	27.11%	-0.387	-1.133
Levered with Readjustment Every 5 Years						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.029%	2.068%	-62.31%	41.25%	0.121	9.817
Monthly Returns	0.847%	11.38%	-70.49%	178.42%	0.832	8.902
Conditional Volatility	37.02%	12.85%	22.86%	491.96%	7.080	100.42
Market Leverage	0.318	0.117	0.091	0.953	1.216	2.126

Table 8. Estimates of the regression equations for the small firm with systematic risk. Quantities in parenthesis are standard errors.

		Daily Data						
		<u>Levered Firm</u>		<u>Unlevered Firm</u>				
		a	b	c	Adjusted R^2	a	b	Adjusted R^2
Model (33)		-3.9×10^{-6} (2.2×10^{-5})	0.01 (1.1×10^{-3})		2.7×10^{-4}	-5.2×10^{-8} (5.2×10^{-6})	-2.9×10^{-4} (4.0×10^{-4})	0.0
Model (34)		-1.03 (0.0004)	-0.24 (0.02)		0.0004	-1.41 (0.0001)	-0.03 (0.01)	1.5×10^{-5}
Model (35)		-1.03 (0.0004)	-0.25 (0.02)		0.0004	-1.41 (0.0001)	-0.03 (0.01)	1.5×10^{-5}
Model (36)		-1.25 (0.0003)	0.41 (0.0004)		0.75			
Model (37)		-1.42 (0.0003)	0.34 (0.0003)	0.49 (0.0008)	0.88			
		Monthly Data						
		<u>Levered Firm</u>		<u>Unlevered Firm</u>				
		a	b	c	Adjusted R^2	a	b	Adjusted R^2
Model (33)		0.001 (0.0005)	-0.18 (0.004)		0.12	2.8×10^{-4} 1.2×10^{-4}	-0.05 (0.002)	0.07
Model (34)		-2.27 (0.002)	-0.24 (0.02)		0.01	-2.65 (0.001)	-0.03 (0.011)	0.0004
Model (35)		-2.27 (0.002)	-0.06 (0.02)		0.001	-2.65 (0.001)	0.03 (0.011)	0.0003
Model (36)		-2.45 (0.002)	0.33 (0.003)		0.56			
Model (37)		-2.64 (0.003)	0.27 (0.002)	0.54 (0.006)	0.73			

Table 9. Descriptive statistics of simulated asset prices for the small firm with no systematic risk.

Unlevered						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.017%	1.243%	-5.656%	6.294%	0.050	0.093
Monthly Returns	0.498%	6.859%	-23.22%	35.67%	0.213	0.076
Conditional Volatility	23.51%	1.815%	20.00%	25.95%	-0.279	-1.227
Levered with Readjustment Every 5 Years						
	Mean	Std.Dev.	Minimum	Maximum	Skewness	Kurtosis
Daily Returns	0.025%	1.961%	-55.38%	26.78%	0.033	5.780
Monthly Returns	0.732%	10.74%	-77.40%	100.67%	0.456	3.021
Conditional Volatility	35.50%	11.32%	22.66%	470.42%	6.798	106.56
Market Leverage	0.319	0.115	0.092	0.952	1.191	1.963

Table 10. Estimates of the regression equations for the small firm with no systematic risk. Quantities in parenthesis are standard errors.

		Daily Data						
		Levered Firm		Unlevered Firm				
		a	b	c	Adjusted R^2	a	b	Adjusted R^2
Model (33)		-2.2×10^{-6} (2.0×10^{-5})	6.0×10^{-3} (1.0×10^{-3})		9.0×10^{-5}	-0.1×10^{-7} (4.6×10^{-6})	-2.8×10^{-4} (3.8×10^{-4})	0.0
Model (34)		-1.07 (0.0004)	-0.25 (0.019)		0.0004	-1.45 (0.0001)	-0.017 (0.011)	0.00001
Model (35)		-1.07 (0.0004)	-0.25 (0.019)		0.0004	-1.45 (0.0001)	-0.017 (0.011)	0.00001
Model (36)		-1.30 (0.0003)	0.43 (0.0004)		0.78			
Model (37)		-1.45 (0.0003)	0.37 (0.0003)	0.42 (0.0007)	0.89			
		Monthly Data						
		a	b	c	Adjusted R^2	a	b	Adjusted R^2
Model (33)		-2.2×10^{-6} (-2.0×10^{-5})	0.006 (0.001)		0.0001	-5.5×10^{-8} (4.6×10^{-6})	-2.8×10^{-4} (3.7×10^{-4})	0.0
Model (34)		-1.07 (0.0004)	-0.25 (0.019)		0.0004	-1.45 (0.0001)	-0.02 (0.01)	4.7×10^{-6}
Model (35)		-1.07 (0.0004)	-0.25 (0.019)		0.0005	-1.45 (0.0001)	-0.02 (0.01)	4.4×10^{-6}
Model (36)		-1.30 (0.0003)	0.43 (0.0004)		0.78			
Model (37)		-2.68 (0.002)	0.36 (0.002)	0.42 (0.002)	0.82			

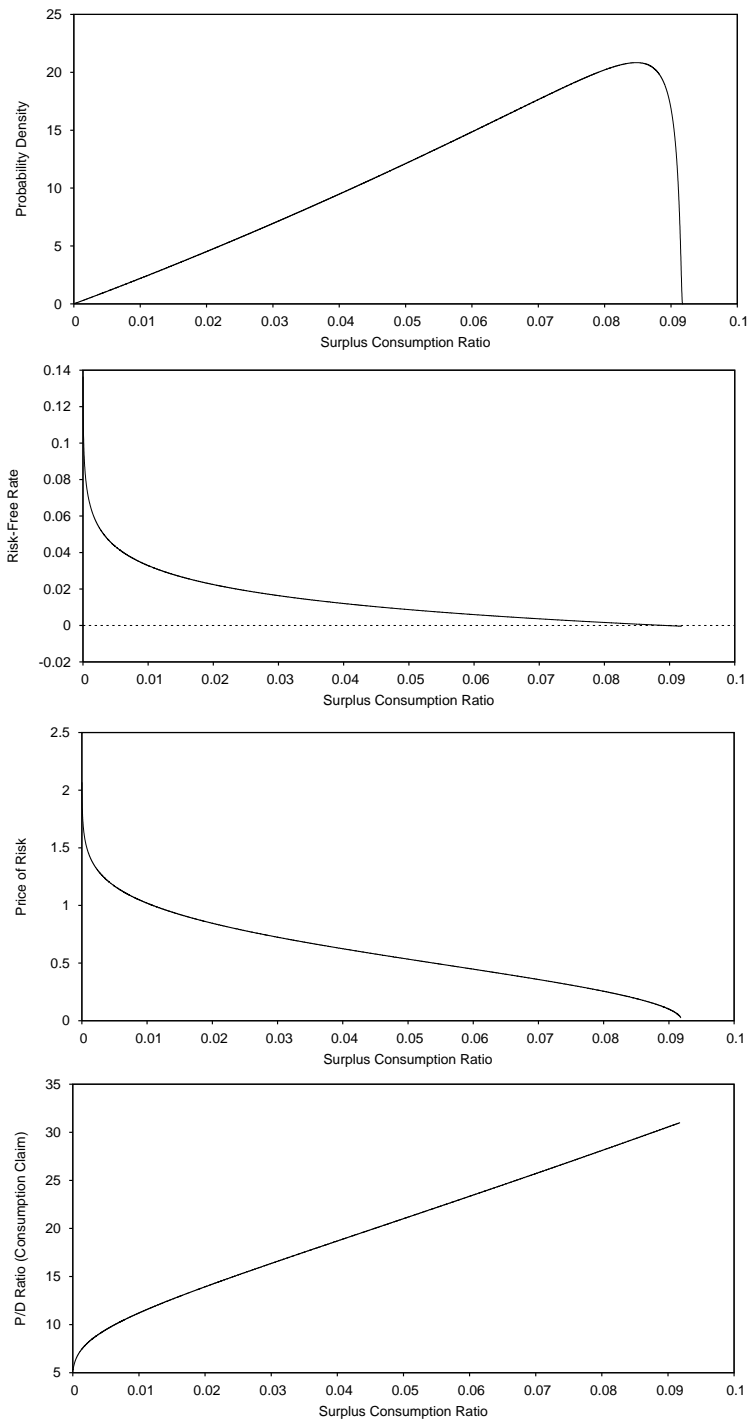


Figure 1. Equilibrium properties of the pricing kernel and the market portfolio's price-dividend ratio at different states of the economy.

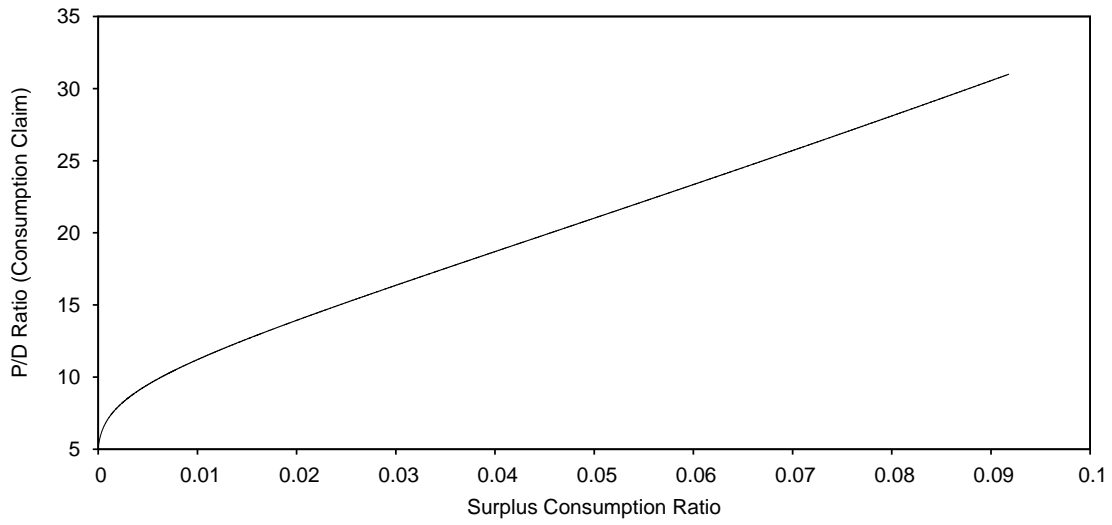


Figure 2. The dividend-price ratio of the market portfolio in different states of the economy.

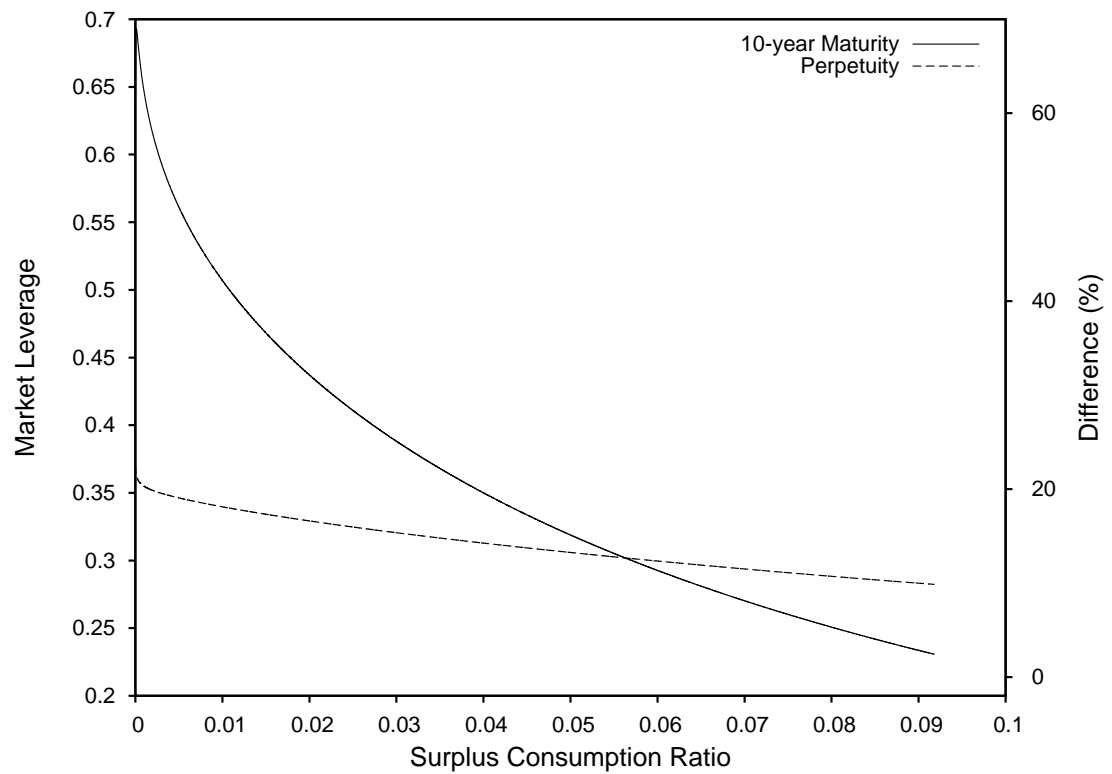


Figure 3. Market leverage for two different debt service maturity specifications.

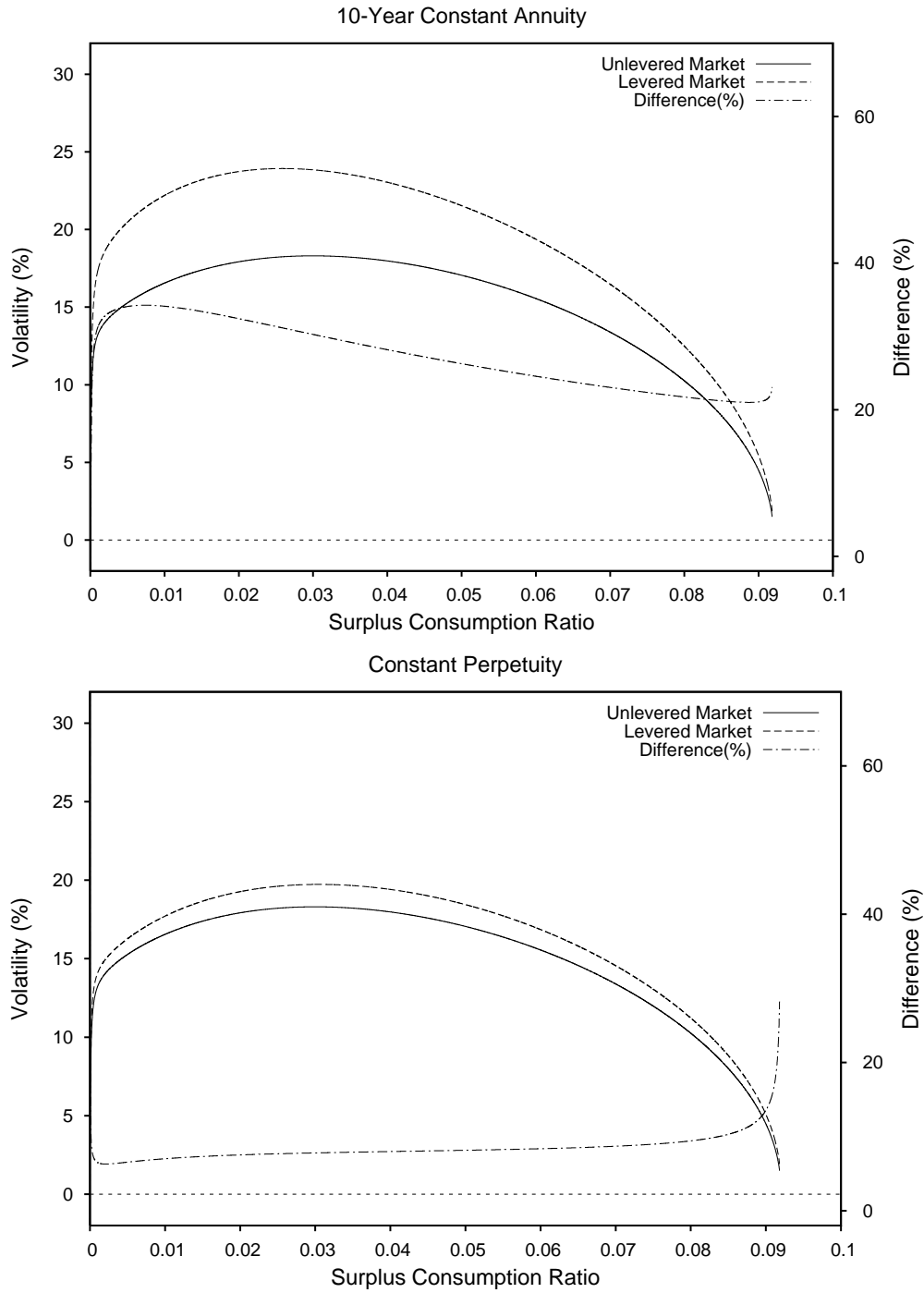


Figure 4. Conditional volatility of equity returns of the representative firm at different states of the economy, for a given debt service structure.

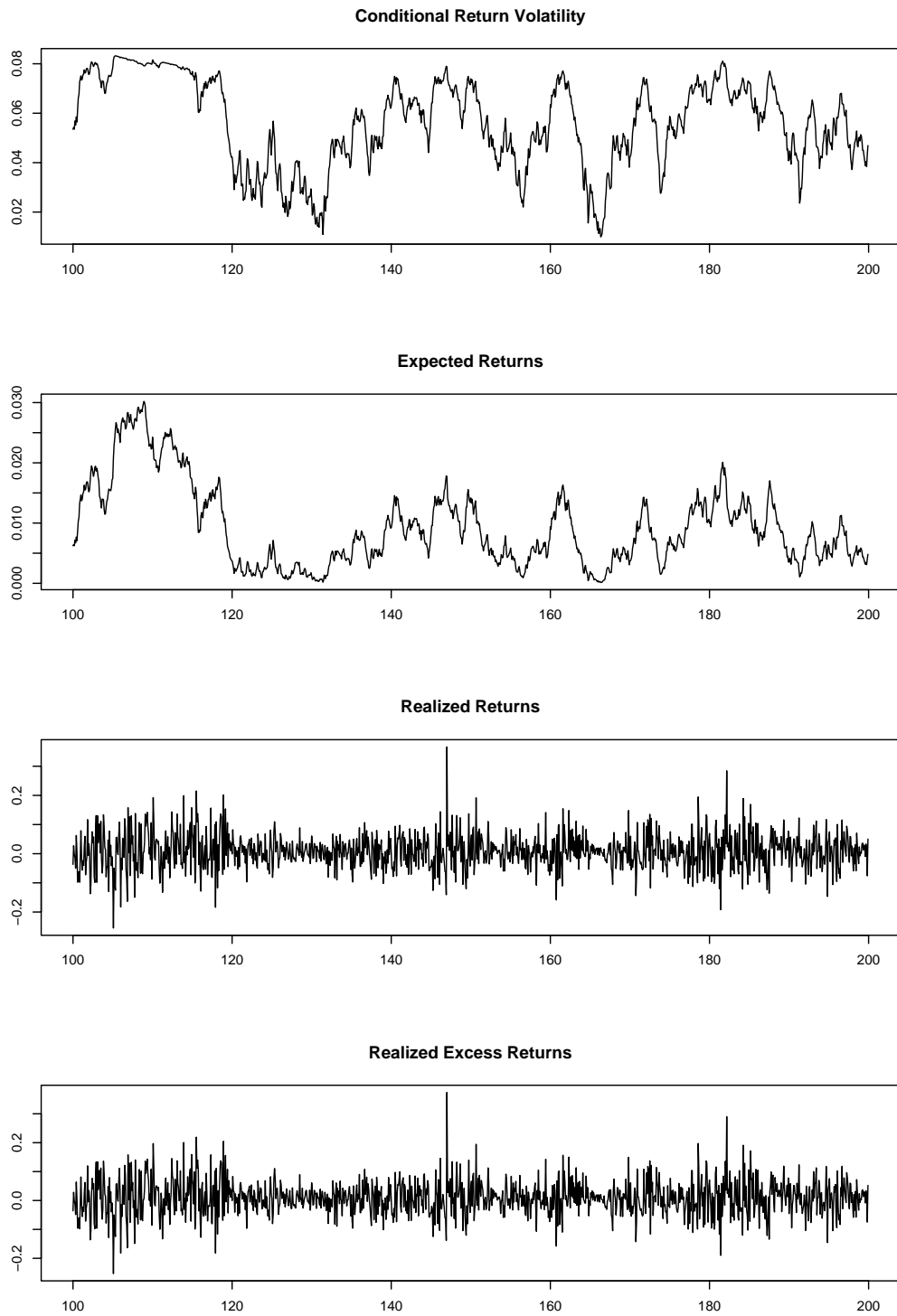


Figure 5. A 100 year window of observations from the simulated data at the market level.

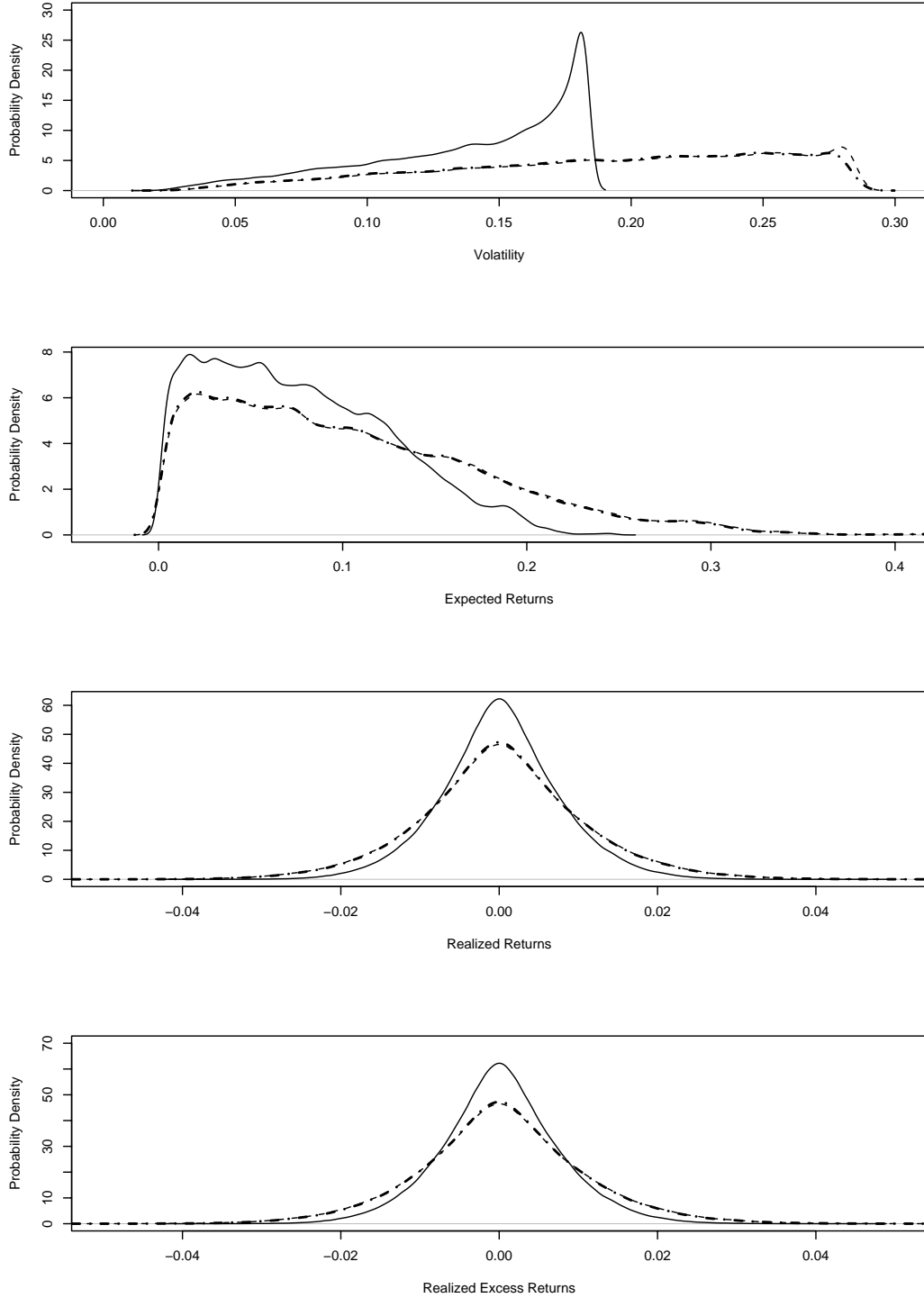


Figure 6. Probability density estimates of daily observations from the simulated data. The solid line denotes an unlevered economy. The dashed line denotes an economy with a 10 year annuity debt structure. The dash-dot line denotes an economy with a perpetual debt structure.

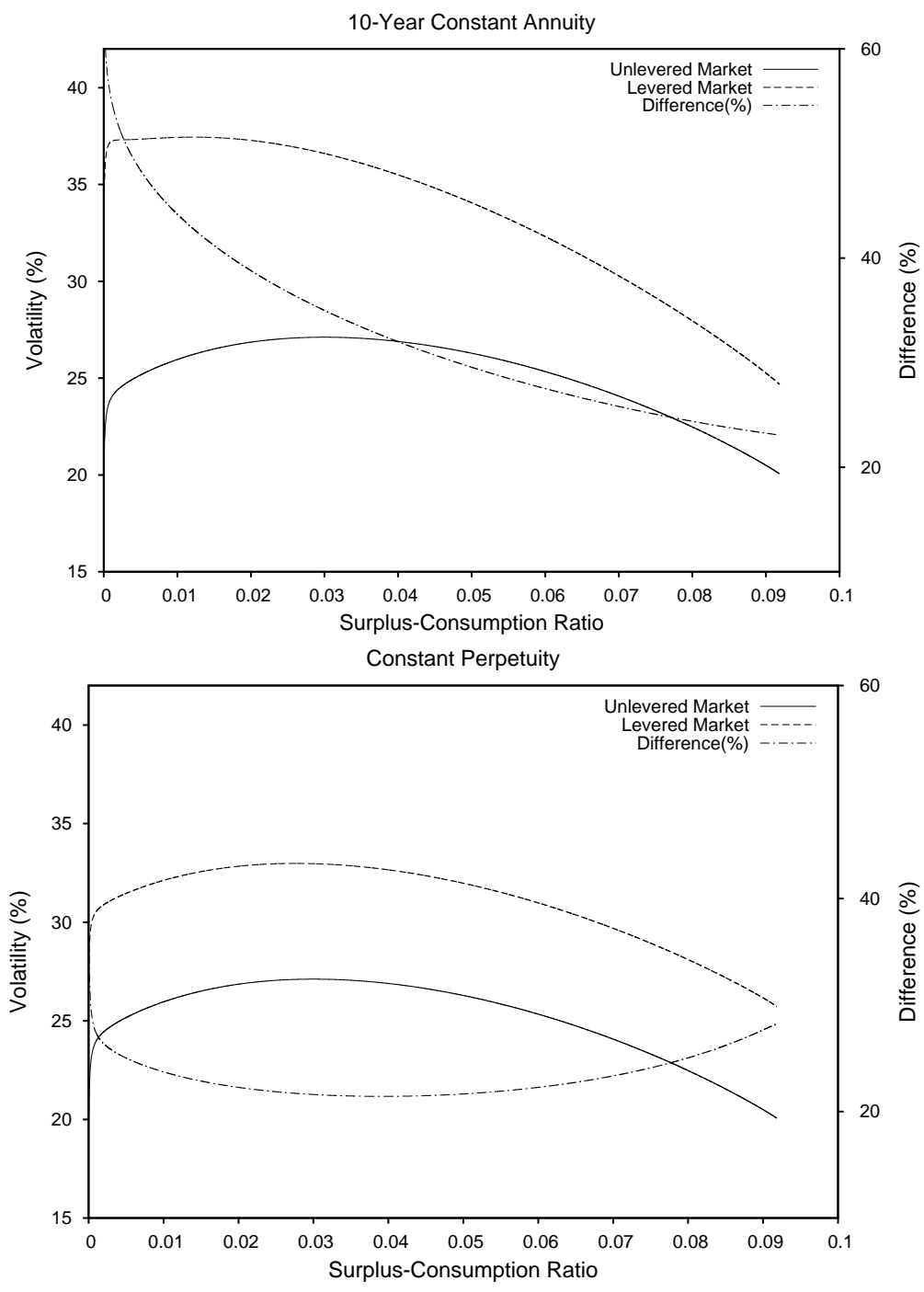


Figure 7. Conditional volatility of equity returns of the small firm with both systematic and idiosyncratic cash flow risk at different states of the economy, for a given debt service structure.

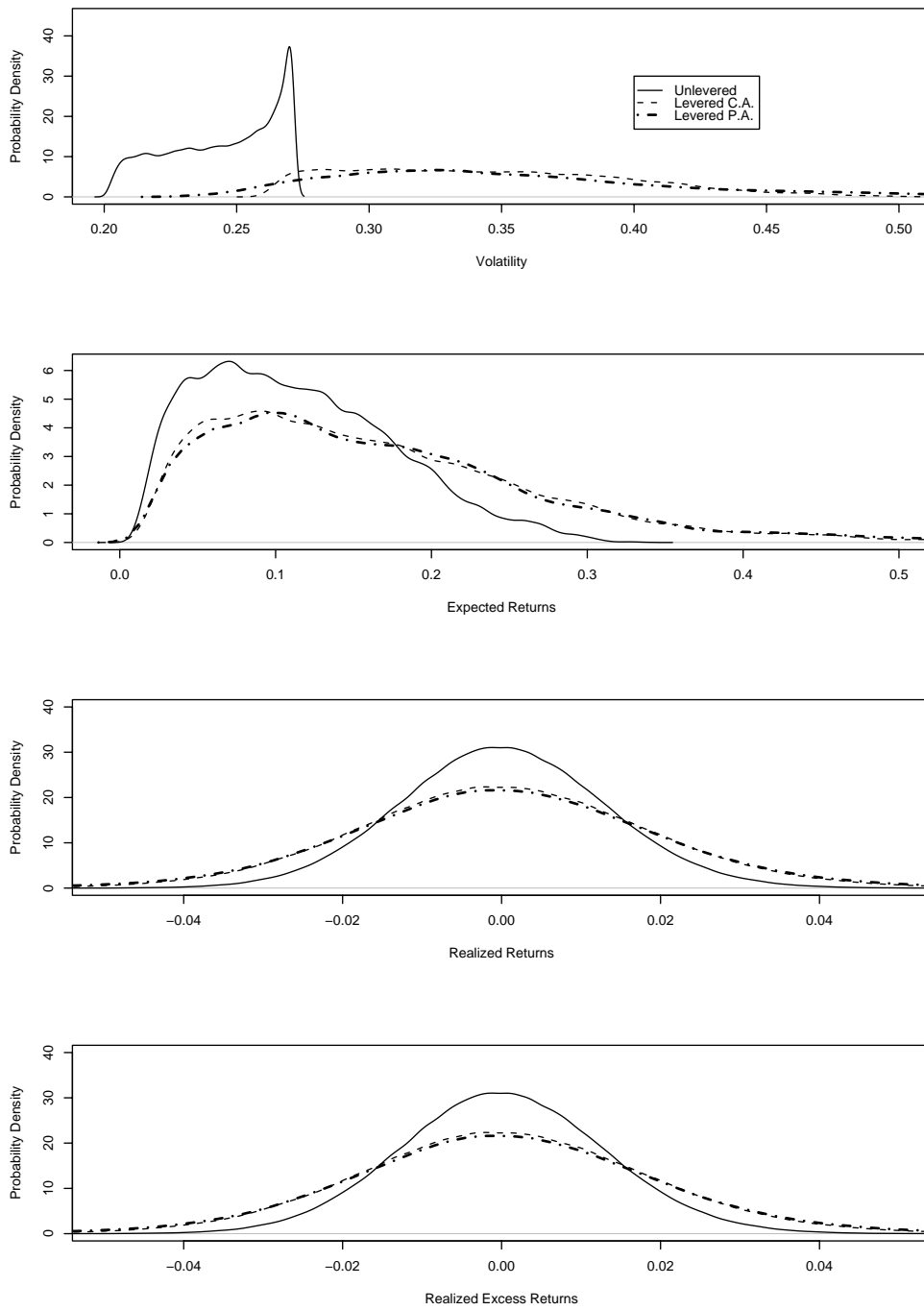


Figure 8. Probability density estimates of daily observations from the simulated data for the small firm with both systematic and idiosyncratic cash flow risk.

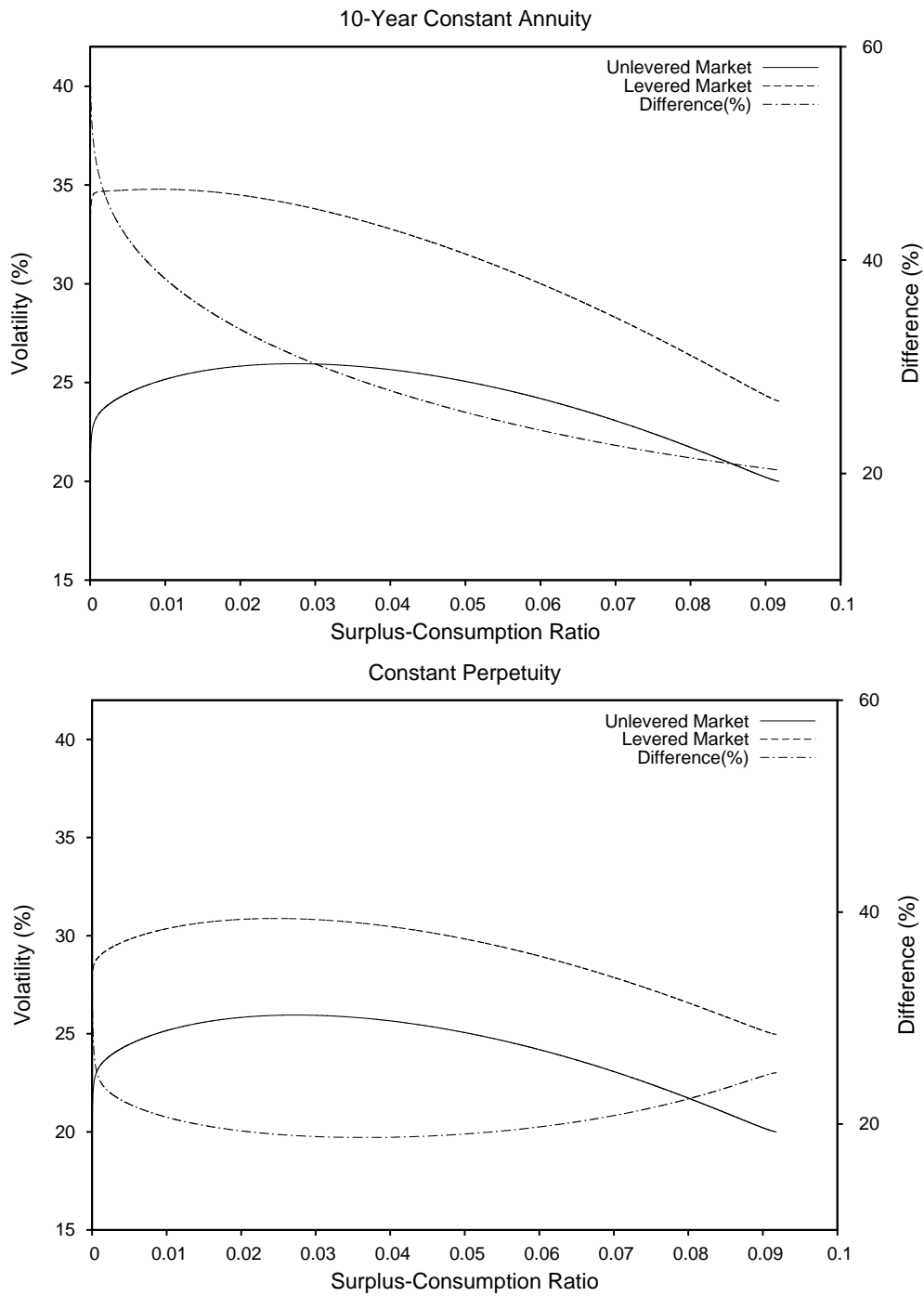


Figure 9. Conditional volatility of equity returns of the small firm with idiosyncratic cash flow risk at different states of the economy, for a given debt service structure.

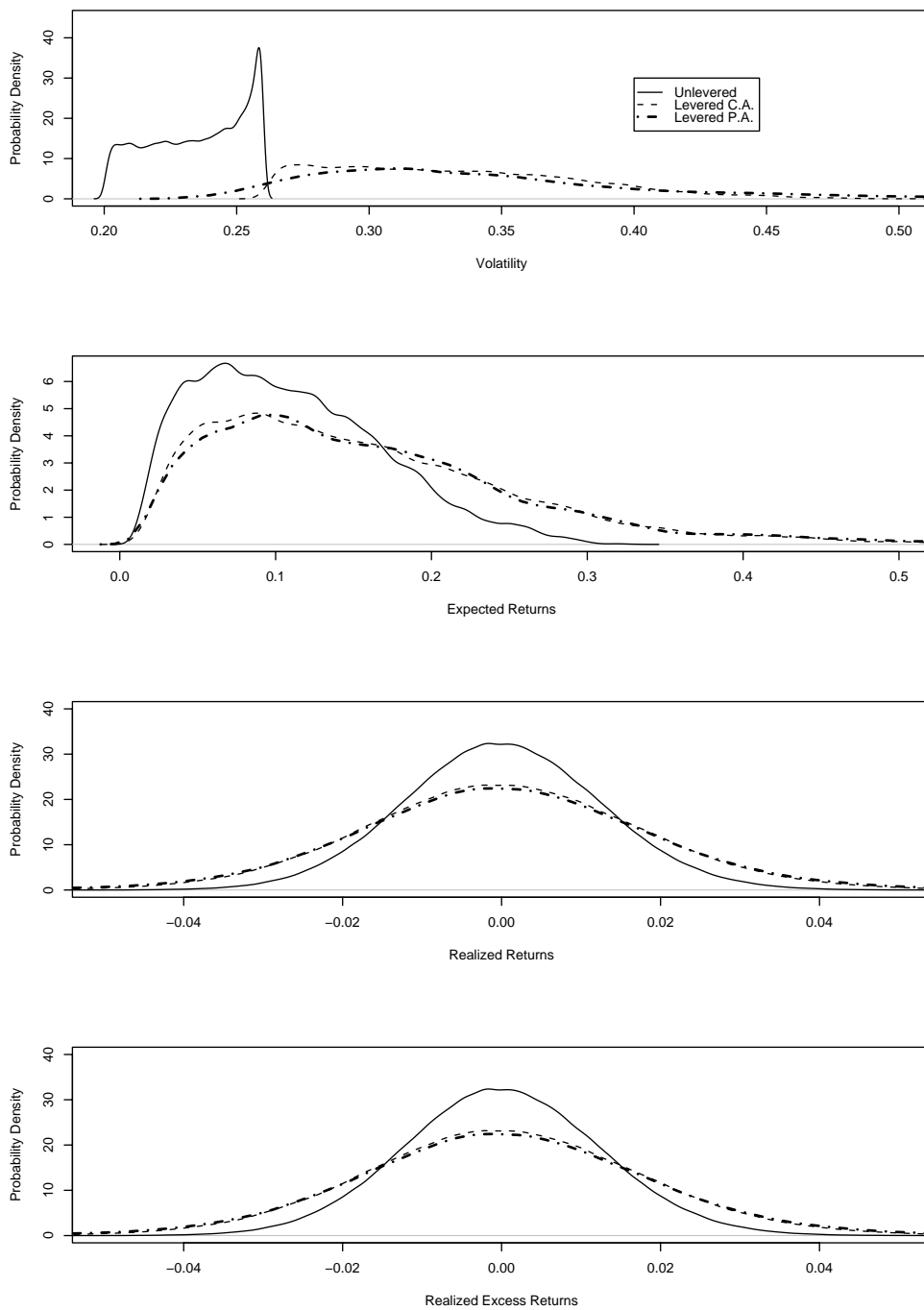


Figure 10. Probability density estimates of daily observations from the simulated data for the small firm with fully idiosyncratic cash flow risk.