Restructuring Risk in Credit Default Swaps: An Empirical Analysis*

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Abstract

This paper estimates the price for restructuring risk in the U.S. corporate bond market during 1999-2005. Comparing quotes from default swap (CDS) contracts with a restructuring event and without, we find that the average premium for restructuring risk represents 6% to 8% of the swap rate without restructuring. We show that the restructuring premium depends on firm-specific balance-sheet and macroeconomic variables. And, when default swap rates without a restructuring event increase, the increase in restructuring premia is higher for low-credit-quality firms than for high-credit-quality firms. We propose a reduced-form arbitrage-free model for pricing default swaps that explicitly incorporates the distinction between restructuring and default events. A case study illustrating the model's implementation is provided.

Keywords: credit default swaps, restructuring credit event, reduced-form credit risk modeling

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1 Introduction

Since their emergence in the late 1990s, credit default swap (CDS) markets have grown exponentially, to an estimated outstanding notional value of 17.1 trillion dollars in 2005.[1] This phenomenal growth is due to the fact that CDS provide an essential tool for hedging credit risk in financial markets. CDS are financial instruments that provide insurance against a credit event destroying value in an entity's (usually a corporation's) debt. The insurer of the credit event is paid a premium (usually quarterly) over a fixed time period to provide the insurance. And, the insured gets reimbursed for any losses in the value of the entity's debt, if a credit event occurs over the contract's life. Various different types of CDS trade, differentiated with respect to: (i) the maturity of the contract, (ii) whether the reimbursement procedure requires physical delivery of the debt issue or not, and (iii) the definition of a credit event. This paper concentrates on the last provision.

In the definition of a credit event, the crucial distinction is between default and financial restructuring. Default occurs when the borrower violates the debt contract's covenants (e.g. a failure to pay required interest or principal on time), and financial restructuring occurs when the financial liabilities of the borrowing entity are changed. Default clearly destroys the value of an entity's debt. But, a financial restructuring could also destroy existing debt value, even if the entity does not default. For example, the restructuring could change the debt contract's subordination, reducing its priority in the event of default. An open question is the importance of this "restructuring event" in the market pricing of CDS. This paper provides both an empirical and theoretical investigation of this issue.

First, we present an empirical investigation of the restructuring credit risk premium for the U.S. corporate bond market during 1999-2005. Comparing default swap contracts that include restructuring and those that do not, we find that the average premium for restructuring risk represents 6% to 8% of the swap rate without restructuring. This is an economically significant risk premium. Everything else constant, we find restructuring premia to be highest in the Telephone, Service & Leisure and Railroad sectors, and lowest in the Oil and Gas industry and for Gas utilities. And, when default swap rates without a restructuring clause increase, the increase in the restructuring premium is higher for high-yield CDS and lower for investment-grade firms.

Next, we fit a regression model to identify the determinants of CDS rates after controlling for the restructuring clause, the time to maturity of the contract, and changing International Swaps and Derivatives Association (ISDA) regulations. Key explanatory variables include the distance to default (a proxy for default risk), the level and slope of the default-free forward rate curve, a stock-market volatility index, Moody's Baa corporate bond yield, and the spread between Moody's Aaa yield and

[1] According to the International Swaps Dealer Association (ISDA), see www.isda.org/statistis.

the 20-year Treasury yield. The model fits the data well, with an R^2 of almost 60% for a linear model, and over 71% when using a logarithmic model.

Last, we provide a reduced-form arbitrage-free model for CDS pricing that explicitly takes into account the restructuring clause. We incorporate both default and restructuring as separate events, where restructuring (if it occurs and default has not) causes a jump in the default intensity. The jump size can be positive or negative, and possibly random. A negative jump is interpreted as a successful restructuring, while a positive jump is interpreted as an unsuccessful restructuring. Our model formulation extends the primary-secondary framework of Jarrow and Yu (2001), where a primary firm's default causes a jump in a secondary firm's default intensity.

The remainder of this paper is organized as follows. Section 2 gives a brief description of the CDS restructuring rules. Section 3 describes the CDS data used in this study. Section 4 provides a panel regression to estimate a model for restructuring risk premia. Section 5 develops a reduced-form model for pricing CDS under different restructuring clauses, and it applies this model as a case study for Ford Motor Co. Finally, Section 6 concludes.

2 Credit Default Swap Restructuring Rules

To keep this paper self-contained, we provide a brief description of the restructuring rules embedded in credit default swaps. As mentioned in the introduction, a CDS provides insurance against the default of a reference entity's debt. In our data, the reference entity is a firm. The insurance is for a fixed maturity. The buyer of protection pays periodic (usually quarterly) premia. If the credit event occurs, the insurer compensates the buyer for the loss in the debt's value. This compensation can be through physical delivery or cash delivery. For physical delivery, the face value of the security is exchange for the debt contract itself. For cash delivery, the difference between the par value of the bond and the post-default market value is paid in cash. Physical delivery is the more common contract, and the one we concentrate on below. The premium, called the at-market CDS rate, is measured as a fraction of the face value the debt. The premium is that quantity that sets the market value of the CDS contract equal to zero at initiation.

According to the ISDA definitions, a contractually defined credit event includes: bankruptcy, failure to pay, repudiation/moratorium, obligation acceleration, obligation default and restructuring. By the 2003 ISDA Credit Derivatives Definitions, a restructuring credit event occurs if there is: (i) a reduction in the interest rate or in the amount of principal, (ii) a postponement or other deferral of dates for the payment of interest, principal, or premium, (iii) a change in the ranking in priority of payment of any obligation that causes subordination of it to other obligations, and (iv) any change in the currency or composition of any payment of interest or principal.

ISDA provides four choices for contracting with respect to restructuring:

Table 1: Restructuring Maturity Limitations on Deliverable Obligations in the Case of Physical Settlement. T and $\bar{T}$ denote the maturity of the CDS contract and the maturity of the deliverable obligation, respectively.

Restructuring Clause	Deliverable Obligations
FR	Any bond with maturity of up to 30 years.
MR	$T \leq \bar{T} < (T + 30 \text{ months})$
MMR	Allows an additional 30 months for the restructured bond. For other obligations, same as MR.

1. full restructuring (**FR**), based on the ISDA 1999 Definition,
2. modified restructuring (**MR**), based on the ISDA 2001 Supplement Definition,
3. modified-modified restructuring (**MMR**), based on the ISDA 2003 Definition,
4. no restructuring (**XR**).

With respect to physical delivery, each restructuring rule has different restrictions regarding the maturities of the deliverable obligations, as summarized in Table 1. As seen, FR allows delivery of any bond with maturity up to 30 years. MR restricts the bond to have a maturity within 30 months of the CDS contract's maturity, while MMR is similar to MR, except that it allows an additional 30 months for only the restructured bond. More details on the contractual terms regarding restructuring can be found in FitchRatings (2003) and Packer and Zhu (2005). Our paper investigates the impact of the FR, MR, MMR and XR rules on the market pricing of CDS.

3 Data

Our at-market CDS rate quotes are obtained from ValuSpread Credit Data (provided by Lombard Risk Systems), from July 1999 to June 2005. For a given date and reference firm, the database reports a composite at-market CDS rate. This composite rate is derived from the mid-market quotes contributed by up to 25 investment banks and default-swap brokers. Besides quotes, the database includes for each date and named firm, the seniority (senior or subordinated) and the currency of the underlying debt, the maturity of the CDS contract (1, 3, 5, 7, or 10 years), the standard deviation of the mid-market quotes, and the restructuring clause applied in the contract. Also reported is the average, across sources, recovery rate used by the quote providers. Interestingly, the frequency of observations has increased over time. The database contains only month-end quotes between 1999 and 2001, biweekly information from January 2002 to June 2002, weekly data from July 2002 to May 2003, and daily quotes starting May 15, 2003.

The standard deviation of the mid-market rates can be interpreted as a reliability measure of the associated composite CDS quotes. Indeed, a large reported standard deviation indicates a wide intra-day dispersion across the contributed CDS quotes, whereas a small standard deviation might indicate that only one or two sources contributed to the composite quote. In an effort to limit our exposure to outliers and small-sample bias, we therefore filtered out observations with standard deviations of less than 1% or greater than 20%.

Industry information for each reference name was obtained from the Fixed Investment Securities Database (FISD). Among the 2,781 tickers listed in ValueSpread, we were able to identify the industry information and CUSIP numbers for 1,521 tickers, of which 929 are U.S. names, 532 are non-U.S. names, and 60 are CDS indices such as TRAC-X and iBoxx. The number of identified tickers in each industry, both for U.S. and non-U.S. tickers, are available from the authors upon request.[2]

Table 2 reports the number of CDS quotes by restructuring clause. We partition the time periods by changes to the ISDA definitions. Table 7 in Appendix D provides the number of quotes per industry for U.S. firms. As seen in Table 2, the majority of U.S. market transactions are according to MR. Contrary to the European credit market, MMR is the least popular in the U.S.

Table 3 compares the use of restructuring clauses across investment and non-investment grade debt. Restructuring is excluded as a covered credit event more for high-yield CDS contracts than for investment-grade entities. In particular, 36.2% of the quotes for speculative-grade firms are under the XR rule, whereas for investment-grade firms, XR applies to only 24.1% of the quotes. Also notice that investment-grade firms comprise 83.4% of the quotes provided.

For analysis, we focus on CDS contracts for senior, U.S. dollar-denominated debt. Let $c_\Delta^{\cdot R}$ denote the annualized Δ-year CDS rate under restructuring rule $\cdot R$, where $\cdot R \in \{XR, MR, MMR, FR\}$.[3] The *restructuring premium RP* of $\cdot R$ over some base rule BR is defined as

$$RP_\Delta^{\cdot R, BR} = c_\Delta^{\cdot R} - c_\Delta^{BR}.$$

The *relative restructuring premium RRP* is defined as

$$RRP_\Delta^{\cdot R, BR} = \frac{c_\Delta^{\cdot R} - c_\Delta^{BR}}{c_\Delta^{BR}}.$$

Descriptive statistics for three pairs of restructuring rules are summarized in Table 8 in Appendix D. Using 5-year CDS rates, we find that the average premium for restructuring risk represents 6% to 8% of the swap rate without restructuring. This

[2] Note that the number of reference names is about 2,100 which is less than the number of tickers. This is because tickers may change over time even though the company name does not change, for example due to mergers and acquisitions.

[3] CDS rates are quotes in basis points.

Table 2: Number of quotes by restructuring clause and period for U.S. firms whose industry information was verified using FISD data. The periods are divided based on the publication months of the 2001 ISDA supplements and the 2003 ISDA definitions.

Period	Restructuring clause	Number of quotes
1999 - April 2001	FR	8,562
May 2001 - Jan 2003	XR	5,767
	MR	41,498
	FR	47,232
Feb 2003 - present	XR	112,520
	MMR	2,436
	MR	435,027
	FR	64,251

documents that restructuring premia are a significant component of CDS rates. Relative to XR, the average and median premia are positive for all three restructuring clauses (results reported for FR and MR only) and all time horizons. We also find that FR has positive mean and median spreads over both MR and MMR (results reported for MR only). The variations in restructuring premia are quite large, however, considering the magnitude of the average quotes. This suggests that the median restructuring premia are the more reliable summary statistics.

Although the average and median restructuring premia of FR, MR and MMR over XR are all positive, negative premia are occasionally observed for all three restructuring rules. In theory, however, the existence of negative premia is possible if investors believe that a restructuring credit event will cause a default event afterwards, and if recovery rates are higher under restructuring than under default. Nevertheless, conversations with market participants lead us to believe that these occurrences are more likely due to the differences between quotes by default-swap brokers and investment banks. To be conservative, we removed negative restructuring premia observations from our sample.

4 A Regression Model for Restructuring Premia

This section provides a simple and robust regression model for CDS risk premia, including the restructuring clause as an explanatory variable. The analysis uses 10,020 paired 5-year c_5^{XR} and c_5^{MR} observations from May 2002 through December 2004, taking into account all U.S. firms that belong to either the Industrial or the Utilities sectors as listed in Table 7 in Appendix D. The estimations are summarized in Table 4.

We run three regressions, controlling for different explanatory variables. In the

Table 3: Number of 5-year CDS rate quotes for U.S. firms by rating status. For both investment-grade (IG) and speculative-grade (SG) firms, for each restructuring clause, we report the number of quotes, the percentage of total number of quotes, the row percentage, and the column percentage.

	Restructuring clause				Total
	FR	MM	MR	XR	
IG	42,228	733	152,212	61,869	257,042
	13.7	0.2	49.4	20.1	83.4
	16.4	0.3	59.2	24.1	
	87.9	98.0	85.0	76.9	
SG	5,818	15	26,921	18,581	51,335
	1.9	0.0	8.7	6.0	16.7
	11.3	0.0	52.4	36.2	
	12.1	2.0	15.0	23.1	
Total	48,046	748	179,133	80,450	308,377
	15.6	0.2	58.1	26.1	100.0

first regression, the restructuring premium increases on average by 5.1 basis points for each 100 basis points increase in the non-restructuring CDS rate. The associated coefficient of determination is 51.8%. The estimate of the intercept is 0.537 basis points, meaning that the price of protection against restructuring risk is almost zero for high-credit-quality firms. The fact that the intercept is statistically different from zero (12 times its standard deviation) could be due to the omission of liquidity effects, or to a mis-specification of the linear model. The scatter plot of $c_5^{MR} - c_5^{XR}$ over c_5^{XR} (not shown) also reveals substantial heteroscedasticity, which casts additional doubt on the linear model specification.[4]

Our second and third regressions control for investment-grade (IG) or speculative-grade (SG) status, and for changes in restructuring premia across industries, respectively. The estimation results are listed in columns 4 and 6 of Table 4. Both the differences in the level and slope effect for speculative-grade firms are significant at the 1% level. When default swap rates without a restructuring clause increase, the increase in the restructuring premium is higher for high-yield CDS and lower for investment-grade firms. Holding the value of a non-restructuring CDS contract constant, sectoral differences in the modified restructuring premia are relative small, about 1 basis point. They are highest in the Telephone, Service & Leisure and Railroad sectors, and lowest in the Oil and Gas industry and for Gas utility firms.

In our next set of regressions, we want to control for the relevant economic characteristics of the firm underlying the CDS contract. To decide on which variables to

[4] We also experimented with a linear log-log specification that reduced the heteroscedasticity, but the coefficient of determination was lowered to 0.437.

Table 4: Results of OLS regression of the modified restructuring premium, $c_5^{MR} - c_5^{XR}$, on the CDS rate under no restructuring, c_5^{XR}, and on credit-quality and sector fixed effects. The reference firm has IG status and belongs to the manufacturing sector. Results for full restructuring risk premia are available upon request.

	estimate	SD	estimate	SD	estimate	SD
Intercept	0.537	0.045	0.748	0.059	0.683	0.069
c_5^{XR}	0.051	0.000	0.050	0.001	0.049	0.001
SG			-2.010	0.162	-2.093	0.164
SG $\times\, c_5^{XR}$			0.006	0.001	0.007	0.001
Media & Comm					0.397	0.132
Oil & Gas					-0.167	0.142
Railroad					0.678	0.424
Retail					-0.040	0.113
Service & Leisure					0.641	0.115
Transportation					0.197	0.190
Telephone					0.668	0.198
Electric					0.099	0.146
Gas					-0.210	0.265
R^2	0.518		0.521		0.522	
no obs	25,814		25,814		25,814	

include, we need to better understand the economic theory for financial restructurings. The restructuring event in CDS contracts can be considered as a soft version of private workouts. "Soft" because it is restricted to debt restructuring prior to any violation of the contract. Should the firm violate contractual terms, the event would be classified as a default. Consequently, the literature on the choice between private workouts and legal bankruptcy proceedings provides us with variables that might be effective in capturing the relative likelihood of out-of-court debt restructuring (see, John (1993), Chatterjee et al. (1996), and Chen (2003)). For our analysis, we focus on the market and balance-sheet variables considered in the later two articles. In addition, we include the 5−year constant maturity Treasury rate as well as Moody's seasoned Baa corporate bond yield. These two variables are intended to control for the state of the economy and credit markets, respectively. (Descriptive statistics for the firm-specific and macro-economic variables are available upon request.)

In financial distress, if a firm is still economically viable, then it is optimal to restructure its debt and continue operations. Although the debt restructuring could be processed under Chapter 11 bankruptcy laws, Chatterjee et al. (1996) show that economically viable firms prefer private workouts. To capture the economic viability of a firm, we use the ratio of operating income to total liabilities, as well as the average stock return over the past twenty business days.

Private workouts require voluntary coordination among debtors and creditors. If coordination problems are severe, bankruptcy would be the only alternative. It is reasonable to believe that coordination costs are higher the larger the firm's size and the more complex its debt structure. We proxy the size of the firm using total assets, total sales, and total liabilities. A preliminary analysis shows, however, that these variables are highly correlated, causing a multi-collinearity problem. Consequently, we only use the logarithm of total sales in the subsequent regressions. We also consider the ratio of subordinated debt to total liabilities, and the ratio of secured debt to total liabilities, to measure the complexity of the debt structure of the firm.

Additionally, information asymmetry between debtors and creditors may cause coordination costs to increase. We include the logarithm of the number of employees to proxy for labor coordination costs. As in Chen (2003), in order to capture information asymmetry, we include a dummy variable for the auditor's opinion with respect to the level of information disclosure.[5] It is 1 if the auditor's opinion is an "unqualified opinion" (highest disclosure) and 0 otherwise. Chen (2003) also uses stock return volatility, a variable that is not considered here because its high correlation with the likelihood of default, as predicted by the Merton (1974) model. We have verified that the sample correlation between the stock return volatility and the base CDS rate is around 50%, and that the volatility coefficient is statistically insignificant when included in our regression analysis.

Next, the "cheapest-to-deliver" option inherent in the different restructuring clauses could be an important determinant of CDS rates. The higher the value of the option to the protection buyer, the higher the restructuring premia. Because the cheapest debt is often the debt with the longest maturity (and the lowest coupon rate), we include the ratio of debt maturing in more than five years to long-term debt, as a proxy for the "cheapest-to-deliver" option value. Recall that under the modified restructuring clause, the deliverable obligations should mature between 5 to 7.5 years after initiation of a 5-year CDS contract.

Table 5 shows the result of the regressions of the modified restructuring premia on contemporaneous non-restructuring credit spreads, after replacing the sectoral dummy variables in Table 4 by the firm-specific and macro-economic parameters discussed above. In summary, we find that even though the coefficients of the covariates are statistically significant, they have only limited power in explaining restructuring premia above the CDS rate itself. The R^2 increases from 51.8%, when regressing on the base CDS rate only, to 54.6% when including all covariates.

Next, using a panel-regression setting, we examine the impact of the restructuring clause and other potential determinants on CDS rates. Recent empirical work on the determinants of CDS rates include Aunon-Nerin et al. (2002), Benkert (2004), Berndt et al. (2005), Cao et al. (2005) and Ericsson et al. (2004), and on the determinants of

[5]Compustat annual data provides the auditor's opinion information for non-banks which consists of six categories: unaudited, adverse opinion, qualified opinion, no opinion, unqualified opinion with explanatory language, and unqualified opinion. The "unqualified opinion" represents the highest level of accounting transparency. See the Compustat User's Guide for more details.

Table 5: Results of OLS regression of the modified restructuring premium, $c_5^{MR} - c_5^{XR}$, on the CDS rate under no restructuring, c_5^{XR}, as well as credit-quality, firm-specific accounting data, and macro-economic variables. Results for the full restructuring risk premia are available upon request.

	estimate	SD	estimate	SD	estimate	SD
Intercept	-4.980	0.791	-10.495	1.113	-9.005	1.118
c_5^{XR}	0.046	0.001	0.045	0.001	0.045	0.001
SG	-1.779	0.162	-4.062	0.227	-4.391	0.225
SG $\times c_5^{XR}$	0.009	0.001	0.019	0.001	0.019	0.001
Gov5yr	-1.865	0.119	-2.274	0.155	-2.281	0.155
Baa	1.899	0.135	2.399	0.175	2.383	0.175
EBITDA/TtlDebt			6.783	1.439	7.778	1.354
StockRet20days			128.519	11.677	124.144	11.763
log(sales)			0.428	0.065	0.390	0.066
log(no employee)			-0.311	0.056	-0.335	0.057
SubDebt/TtlDebt			11.338	1.191	12.822	1.228
SecDebt/TtlDebt			3.340	0.640	4.011	0.649
AuditorOp			0.354	0.104	0.259	0.106
Intangible/TtlAsset			2.542	0.287		
Collateral/TtlAsset					-1.079	0.221
Deliverable			0.198	0.037	0.165	0.037
R^2	0.527		0.546		0.546	
no obs	25,814		14,539		14,495	

corporate bond yield spreads or changes include Duffee (1998), Collin-Dufresne et al. (2001) and Elton et al. (2001), among others.

In our analysis, firm-specific covariates include: the distance to default (DD), Merton's default probability (Φ (DD)), and leverage (Lev), the level and slope of the risk-free term structure (Level, Slope), a stock-market volatility index (VIX), Moody's Baa corporate yield (Baa), and the spread between Moody's Aaa yield and 20-year Treasury yield (Spread). A detailed description of these covariates is given in Appendix A.

As an extension to the existing literature, we further take into account the following dummy variables:

1. **Restructuring Rule Dummy ($\cdot$ R).** We include dummy variables for each restructuring clause: XR, MR, and MMR.

2. **Period Dummy (ISDAyr).** These dummy variables are used to capture possible structural shifts due to changes in the ISDA credit definitions. In particular, we consider the 2001 ISDA supplements issued in April 2001, and the January 2003 ISDA definitions introducing MMR. Due to some time lag in

the market's adjustment to these changes, we set ISDA99 to be 1 if the date is before June 30, 2001, and 0 otherwise; ISDA01 is 1 if the date is between July 1, 2001 and May 31, 2003, and 0 otherwise; and ISDA03 equals 1 if the date is after June 1, 2003, and 0 otherwise.

3. **Industry Dummy (INDj).** The default intensity and the recovery rate are also affected by the industry-specific environment. Following Chava and Jarrow (2004), we categorize the industry as other industries (IND1), manufacturing and oil and gas (IND2), transportation, media and communications, and utility (IND3), and finance (IND4).

4. **Maturity Dummy (Tyr).** The CDS rate depends on the time to maturity. The distance to default may not be sufficient to capture the whole shape of the term structure. Maturity dummies are also included to capture different levels of liquidity premia for CDS with different maturities.

Table 9 shows the regression results including both the distance to default and the restructuring clause dummies, yielding an R^2 of 54.5%. The R^2 increase to 59.8% after accounting for the macroeconomic variables, the maturity and sectoral effects. Similar tables where we substitute the DD measure with leverage or Merton default probabilities are available from the authors upon request.

Table 10 shows similar results, but with an higher R^2, when using the logarithm of the CDS rate as the dependent variable. Here we achieve a R^2 of 71.1% when using distance to default, and 41.3% and 42.9% for leverage and the Merton default probabilities, respectively.[6] We also experimented with using the CDS rate divided by the reported loss given default, or the logarithm thereof, as the dependent variable. The regression results did not change noticeably, due to the fact that the reported recovery rates move little across our sample period. (The results are available upon request.)

Finally, Table 11 in Appendix D reports the results from regressing the loss given default on the distance to default in order to gain intuition about the relationship between recovery estimates and expected default frequencies. We find that the DD measure and the restructuring clause dummies explain up to 33.2% of the variation in loss given default as reported in the ValuSpread data. The coefficients for the first three powers of distance to default are significant and they indicate a positive relationship between default probabilities and loss given default as reported by Lombard Risk Systems.

As a benchmark, throughout the regression analysis, we use 5-year CDS rates under no restructuring, in the ISDA03 period, for firms belonging to the finance industry. The proxies for default probability used in the tables are the 1-year distance

[6] Our analysis suggests that Merton default probabilities predict CDS rates better in a linear fashion, with a resulting R^2 of 46.9%, since they themselves are approximately exponential functions of the DD measure.

to default (DD1), Merton's 1-year default probability (NDD1), and leverage (Lev). The regressions using T-year DD and NDD where T is the corresponding CDS maturity are not reported here, but are available upon request. Although not all estimates are significant, both the signs and magnitudes of the coefficients of the restructuring rule dummies and the maturity dummies all coincide with our expectations. In each time period between changes to ISDA regulations, CDS rates are, on average, highest under full restructuring and lowest under no restructuring.

5 A Reduced-Form Pricing Model with Different Restructuring Clauses

In this section, we develop a reduced-form arbitrage-free model for pricing default swaps that explicitly includes restructuring clauses. To keep the notation simple, we will distinguish between two categories of credit events, restructuring and default, where default includes bankruptcy and a material failure by the obligor to make debt payments.

5.1 The Basic Model

We suppose that the restructuring of a given firm occurs at the first event time of a (non-explosive) counting process N^R, relative to a probability space with measure $\mathbb{P}$ (called the physical measure) and an increasing family $\{\mathcal{F}_t\}_{t\geq 0}$ of information sets that satisfy the usual conditions (see, for example, Protter (2005)). Assuming arbitrage-free and frictionless markets, Harrison and Kreps (1979) and Delbaen and Schachermayer (1999) show under mild technical conditions that there exists a risk-neutral measure $\tilde{\mathbb{P}}$ (called an equivalent martingale measure) under which the time-t price P_t of a security paying a random amount Z at a stopping time $\tau > t$ is

$$P_t = \tilde{E}_t\left(e^{-\int_t^\tau r_s\,ds} Z\right),$$

where $\tilde{E}_t$ is expectation under $\tilde{\mathbb{P}}$ conditional on $\mathcal{F}_t$, and r is the default-free spot rate process.[7] We do not require markets to be complete, so the martingale measure $\tilde{\mathbb{P}}$ need not be unique. We suppose, however, that the measure is determined uniquely by the market being in equilibrium.

We assume that the counting process N^R has a risk-neutral restructuring intensity process λ^R under $\tilde{\mathbb{P}}$ for which the doubly-stochastic property applies. The doubly-stochastic, or Cox-process assumption implies that the probability $s^R(t,T)$ that the

[7] r is progressively measurable with $\int_0^t |r_s|\,ds < \infty$ $\tilde{\mathbb{P}}$-almost surely, and $\tilde{E}(e^{-\int_0^t r_s\,ds}) < \infty$ for all t (see Protter (2005)).

obligor will not restructure on or before time T, given no restructuring by time t, is

$$s^R(t,T) = \tilde{\mathbb{P}}\left(\tau^R > T|\mathcal{F}_t\right) = \tilde{E}_t\left(e^{-\int_t^T \lambda_s^R\, ds}\right).$$

Similarly, we assume that a default occurs at the first time τ^D of a (non-explosive) counting process N^D, with a risk-neutral default intensity process h^D. We extend the doubly stochastic arrival of credit events under the risk-neutral measure to include h^D. As a first approximation, we let

$$h_t^D \;=\; \lambda_t^D + k_1\, 1_{\{t\geq\tau^R\}} + k_2\, \lambda_t^D\, 1_{\{t\geq\tau^R\}}, \tag{1}$$

where k_1 and $k_2 > -1$ are random variables, and λ^D is a non-negative process that can be interpreted as the pre-restructuring default intensity.

The model specification in (1) allows for both upward and downward jumps in the risk-neutral default intensity, capturing the possibility for both unsuccessful and successful debt restructurings. This formulation extends the primary-secondary framework of Jarrow and Yu (2001), where a primary firm's default causes the secondary firm's default intensity to jump. The primary-secondary structure violates the standard Cox process framework in Lando (1998). However, as discussed in Collin-Dufresne et al. (2004), the no-jump condition in Duffie and Singleton (1999) is still satisfied. This enables us to utilize the standard pricing machinery, because the standard relation between the conditional survival probability and the default intensity still holds. That is,

$$s^D(t,T) \;=\; \tilde{\mathbb{P}}(\tau^D > T|\mathcal{F}_t) = \tilde{E}_t\left(e^{-\int_t^T h_s^D\, ds}\right), \tag{2}$$

where $s^D(t,T)$ denotes the conditional risk-neutral survival probability with regard to default events.

The overall conditional risk-neutral probability of survival until time T, given that a credit event (including both restructuring and default) did not occur by time t, is given by

$$s(t,T) \;=\; \tilde{\mathbb{P}}(\tau > T|\mathcal{F}_t) = \tilde{E}_t\left(e^{-\int_t^T \lambda_s^D + \lambda_s^R\, ds}\right), \tag{3}$$

where $\tau \equiv \tau^D \wedge \tau^R \equiv \min\left\{\tau^D, \tau^R\right\}$. In our doubly-stochastic setting, conditional on the paths of the intensities, the probability that both restructuring and default events happen at the same time is zero.

Equation (2) can be rewritten as

$$
\begin{aligned}
s^D(t,T) &= \tilde{\mathbb{P}}(\tau > T|\mathcal{F}_t) + \tilde{\mathbb{P}}(\tau^D > T, \tau^R \le T|\mathcal{F}_t) \\
&= \tilde{E}_t\left(e^{-\int_t^T \lambda_s^D + \lambda_s^R\, ds}\right) \\
&\quad + \tilde{E}_t\left(e^{-\int_t^T \lambda_s^D\, ds} \int_t^T e^{-\left(k_1(T-v) + k_2 \int_v^T \lambda_s^D\, ds\right)} \lambda_v^R e^{-\int_t^v \lambda_s^R\, ds}\, dv\right) \\
&= \tilde{E}_t\left[e^{-\int_t^T \lambda_s^D ds} \underbrace{\left(e^{-\int_t^T \lambda_s^R\, ds} + \int_t^T e^{-\left(k_1(T-v)+k_2\int_v^T \lambda_s^D\, ds\right)} \lambda_v^R e^{-\int_t^v \lambda_s^R\, ds}\, dv\right)}_{RF}\right],
\end{aligned} \tag{4}
$$

where RF can be interpreted as an adjustment factor due to restructuring risk. It equals 1 if a restructuring event has no direct impact on the default intensity, i.e., when $k_1 = k_2 = 0$. If the jump size is positive ($k_1 + k_2\lambda^D > 0$), the restructuring adjustment factor RF falls between 0 and 1, implying a decrease in the risk-neutral survival probability $s^D(t,T)$. In case the jump size is negative, RF will exceed 1 and lead to an increase in $s^D(t,T)$.

For a default swap contract with maturity T, we assume that the risk-neutral expected fractional loss of notional in the event of a time-t restructuring equals $L_t^R = (1-\delta^R)\, p(t,T)$, where $\delta^R \in (0,1)$ and $p(t,T)$ is the time-t price of a risk-free zero-coupon bond with maturity T. And, in default, it is given by $L_t^D = (1-\delta^D)\, p(t,T)$, where $\delta^D \in (0,1)$. This is known as the "Recovery of Treasury" assumption (see, for example, Jarrow and Turnbull (1995)). Our motivation for choosing the recovery of Treasury assumption over the recovery of market value or the recovery of face value stems from the fact it better describes corporate bond data, see Bakshi et al. (2001).

5.2 Pricing CDS

We now derive the pricing formula for default swaps under the different restructuring clauses. The derivation is an extension of the existing literatures such as Duffie (1999), Hull and White (2000), and Jarrow and Yildirim (2002). For simplicity, we assume a continuous payment structure for default swaps where the protection seller receives a fixed payment flow of c dollars per unit time until maturity T, or until a credit event occurs.

Let c^{RR} denote the continuous at-market CDS rate when restructuring is included, and c^{XR} if it is not. The instantaneous risk-free rate r is assumed to be independent of the default times τ^D and τ^R under $\tilde{\mathbb{P}}$.[8] Also, for computational simplicity, we

[8]This assumption can be relaxed, as in Jarrow and Yildirim (2002). Duffie (1999) shows, however, that the CDS rate is not much affected by this dependency.

assume that

$$\lambda_t^R \;=\; m\,\lambda_t^D,$$

where m is a positive constant. Similarly, the recovery ratio for a restructuring versus a default event is denoted by $n = \delta^R/\delta^D$.

In the event of restructuring or default before maturity of a CDS with restructuring, the risk-neutral fractional loss of notional is given by $(1-\delta)p(\tau,T)1_{\{\tau\le T\}}$, where

$$\delta\,1_{\{\tau\le T\}} \;=\; \delta^D\,1_{\{\tau^D\le T,\,\tau^D<\tau^R\}} + \delta^R\,1_{\{\tau^R\le T,\,\tau^R<\tau^D\}}. \tag{5}$$

The default swap rate is given in Proposition 1. Proofs for all the propositions can be found in Appendix B.

Proposition 1 *If restructuring is a credit event, then the time-t at-market rate for a default swap with maturity T is given by*

$$c_{t,T}^{RR} \;=\; \frac{p\,(t,T)\left(1-\frac{1+mn}{1+m}\,\delta^D\right)\left(1-\tilde{E}_t\left[e^{-\int_t^T(1+m)\lambda_s^D ds}\right]\right)}{\int_t^T p\,(t,v)\,\tilde{E}_t\left[e^{-\int_t^v(1+m)\lambda_s^D\,ds}\right]\,dv}. \tag{6}$$

If restructuring is not a credit event, then the CDS can be computed as in Proposition 2. Notice that c_{T-t}^{XR} depends on the likelihood of restructuring unless both k_1 and k_2 are equal to zero.

Proposition 2 *If restructuring is not a credit event, then the time-t at-market rate for a default swap with maturity T is given by*

$$c_{t,T}^{XR} \;=\; \frac{p\,(t,T)\left(1-\delta^D\right)\left(1-\tilde{E}_t\left[1_{\{\tau^D>T\}}\right]\right)}{\int_t^T p\,(t,v)\,\tilde{E}_t\left[1_{\{\tau^D>v\}}\right]dv}, \tag{7}$$

where $\tilde{E}_t\left[1_{\{\tau^D>v\}}\right]$, $t<v\le T$, is given in (4).

5.3 Simulation Study

This section investigates how the (relative) restructuring premium is affected by the parameters m, n, and $k_1 = k$. To facilitate intuition, we set $k_2 = 0$ and assume that the restructuring intensity, the default intensity, and the risk-free rate are constants. Extensions to the stochastic setting are straightforward, and can be easily implemented using Monte Carlo simulations. A preliminary analysis confirmed, however, that the main conclusions will be similar to the ones drawn from this simpler scenario.

From (6), we have

$$c_{t,T}^{RR} \;=\; \frac{p\,(t,T)\left(1-\frac{1+mn}{1+m}\delta^{D}\right)\left(1-e^{-(1+m)\lambda^{D}(T-t)}\right)}{\int_t^T p\,(t,v)\,e^{-(1+m)\lambda^{D}(v-t)}\,dv}. \tag{8}$$

$c_{t,T}^{XR}$ is given by (7). Using (4), we have

$$\tilde{E}_t\left[1_{\{\tau^D>v\}}\right]=\begin{cases}\frac{1}{k-m\lambda^D}\left(ke^{-(1+m)\lambda^D(v-t)}-m\lambda^D e^{-\left(\lambda^D+k\right)(v-t)}\right); & \text{if } \lambda^D\neq\frac{k}{m}\\ (1+k(v-t))\,e^{-\left(\lambda^D+k\right)(v-t)}; & \text{if } \lambda^D=\frac{k}{m}\end{cases}$$

for all $t<v\leq T$. Note that for a constant risk-free rate r, both $c_{t,T}^{RR}$ and $c_{t,T}^{XR}$ are available in closed form.

As mentioned previously, the Lombard Risk Systems ValuSpread database reports the recovery rate as a fraction of notional value for each restructuring clause. In addition, Varma and Cantor (2005) report average recovery rates as a fraction of notional by initial credit event. We use this information to calibrate δ^D and δ^R. For simplicity, we set $\tau^D\approx(T+t_0)/2$ and $\tau^R\approx(T+t_0)/2$ if the credit event occurs after the time of initiation t_0 of the CDS contract but before its maturity date T. Let $\bar{\delta}^D$ and $\bar{\delta}^R$ denote the reported recovery rates for restructuring and non-restructuring default events, respectively. Then

$$\delta^D \;\approx\; 1-\left(1-\bar{\delta}^D\right)\frac{p\,(t_0,(T+t_0)/2)}{p\,(t_0,T)} \tag{9}$$

and

$$\delta^R \;\approx\; 1-\left(1-\bar{\delta}^R\right)\frac{p\,(t_0,(T+t_0)/2)}{p\,(t_0,T)}. \tag{10}$$

The ValuSpread database shows that from May 2001 to December 2004, the median $\bar{\delta}^D$ is 0.40 and the median 5-year c_5^{XR} is 49.88 basis points (bps). From this we calibrate λ^D to be $c_5^{XR}/(1-\bar{\delta}^D)=83.13$ bps. The risk-free interest rate r is set equal to 1.63%, the average 3-month Treasury rate during the same period. The estimate of m can be obtained from the Moody's annual and monthly surveys of global corporate defaults and recovery rates, see Table 12 in Appendix D. Since most of our CDS observations are from 2003 and 2004, we set $m=0.173$, the relative frequency of distressed exchanges with respect to other credit events during that time. From Varma and Cantor (2005) we obtain a rough estimate of $n=1.51$.

The Jump Parameter Figure 1 shows the effect of k, the expected change in the default intensity at restructuring, on the relative restructuring premium. Note that the CDS rate with restructuring, c_{Δ}^{RR}, is not affected by k (see (8)). The restructuring premium decreases as k increases.

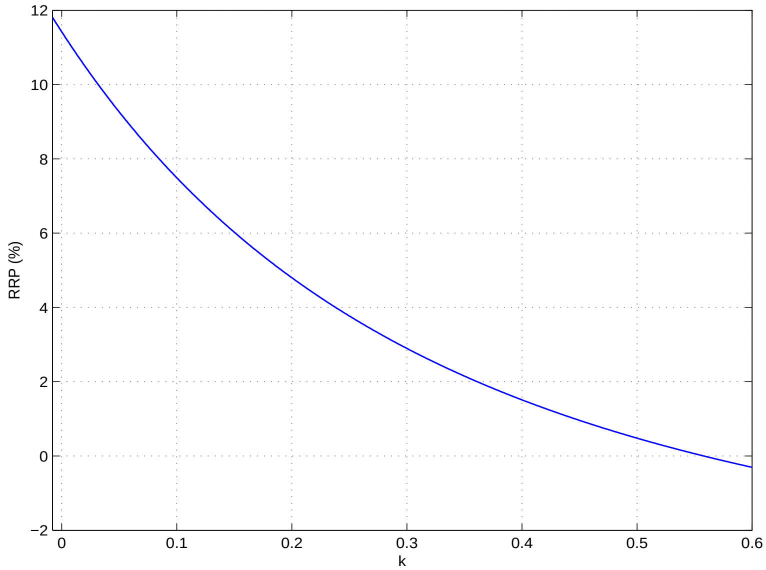

Figure 1: Relative restructuring premium with respect to k, the expected change in default intensity at restructuring event. Other parameters are fixed at $\lambda^D = 83.13$ bps, $\bar{\delta}^D = 0.40$, $m = 0.173$, $n = 1.51$, $\Delta = 5$, and $r = 1.63\%$.

The median RRP, based on our data, is 6.3%, which corresponds to a k of roughly 0.13 according to Figure 1. This implies that investors expect that a restructuring event will be unsuccessful. Also note that the restructuring premium can possibly be negative for high levels of k. In our example, it becomes negative when k exceeds 0.56.

The Default Intensity Figure 2 shows that the restructuring premium increases as the default intensity increases, almost linearly, regardless of the sign of k. The relationship between the relative restructuring premium and the default intensity can be both positive or negative depending on k. For k equal to -0.004 they show a negative relationship, but as k increases above zero, the sign becomes positive. This sign change in the slope can provide a testable hypothesis as to when the market expects the restructuring event to be successful.

The Restructuring Intensity and Recovery Rate Next, we investigate how (relative) restructuring premia are affected by the ratio of restructuring to default intensities, m, and the ratio of recovery rates, n. If the restructuring is expected to be successful ($k < 0$), there exists a positive relationship between both the restructuring premium and m and the relative restructuring premium and m, for all levels of n. This is because as m increases, the likelihood of a restructuring event increases. This implies that the overall likelihood of default, and thereby c_Δ^{XR}, decreases since a restructuring event will lower h^D. On the other hand, c_Δ^{RR} always increases with m.

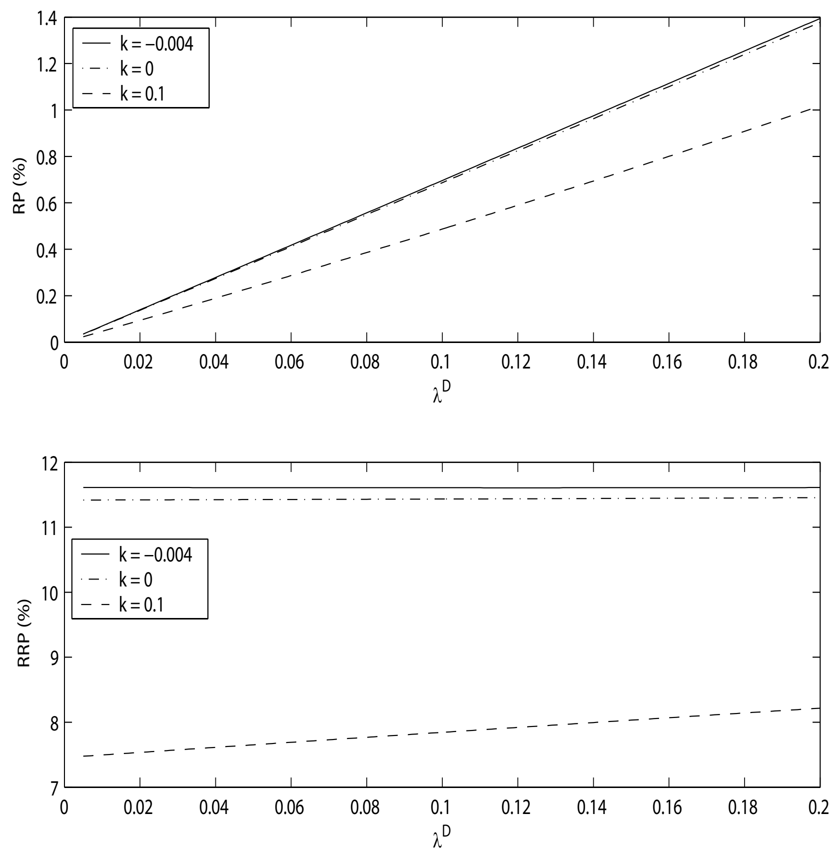

Figure 2: Restructuring premium and relative restructuring premium with respect to the default intensity λ^D for various k. Other parameters are fixed at $\bar{\delta}^D = 0.40$, $m = 0.173$, $n = 1.51$, $\Delta = 5$, and $r = 1.63\%$.

If the restructuring is expected to be unsuccessful ($k > 0$), then the marginal impact of m on c_Δ^{XR} is positive. The relationship between m and the restructuring premium will generally be positive, but the effect of m on c_Δ^{RR} becomes less for higher ratios of recovery rates, n. As n increases, the restructuring loss rate increases relative to the default loss rate, and the marginal impact of m on the (relative) restructuring premium might eventually turn negative.

Note that n and the (relative) restructuring premium are always negatively related, since n has no effect on c_Δ^{XR}, while its marginal impact on c_Δ^{RR} is negative. These results are illustrated in Figures 3 and 4.

The Time to Maturity Figure 5 shows how the relative restructuring premium changes with the time to maturity of the CDS contract. If we ignore the effect of restructuring events on default risk by setting k equal to 0, then c_Δ^{XR} is overestimated when the true value of k is negative. The opposite is true for positive k. Since this effect is amplified for longer maturities Δ, the difference in CDS rates computed using

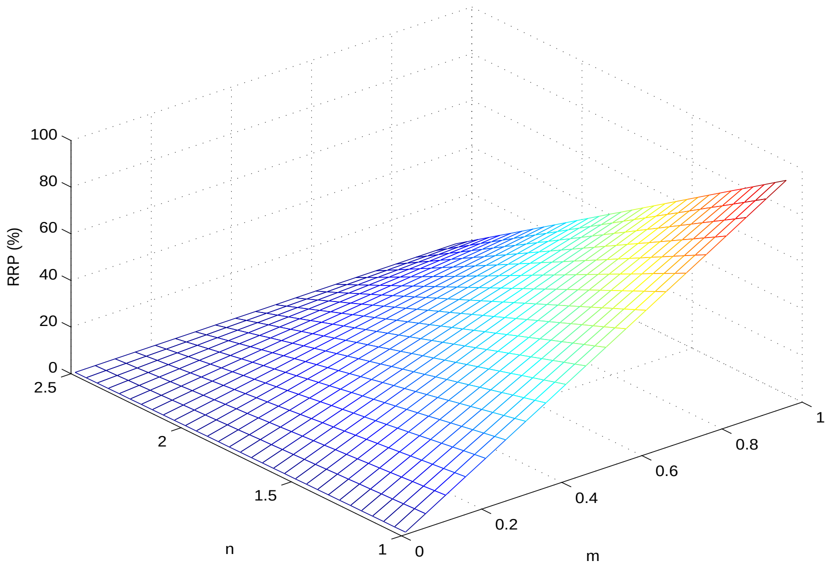

Figure 3: Relative restructuring premium with respect to m and n for $k = -0.003$. Other parameters are fixed at $\lambda^D = 83.13$ bps, $\bar{\delta}^D = 0.40$, $\Delta = 5$, and $r = 1.63\%$.

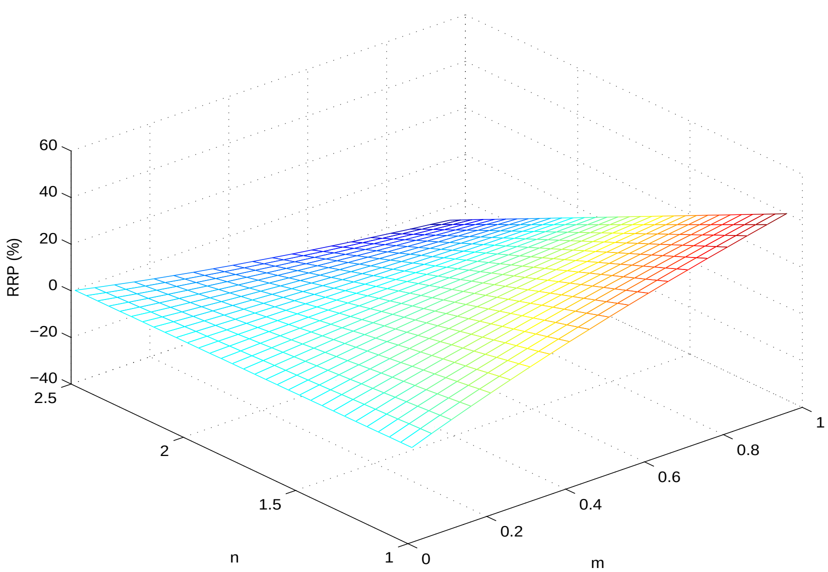

Figure 4: Relative restructuring premium with respect to m and n for $k = 0.2$. Other parameters are fixed at $\lambda^D = 83.13$ bps, $\bar{\delta}^D = 0.40$, $\Delta = 5$, and $r = 1.63\%$

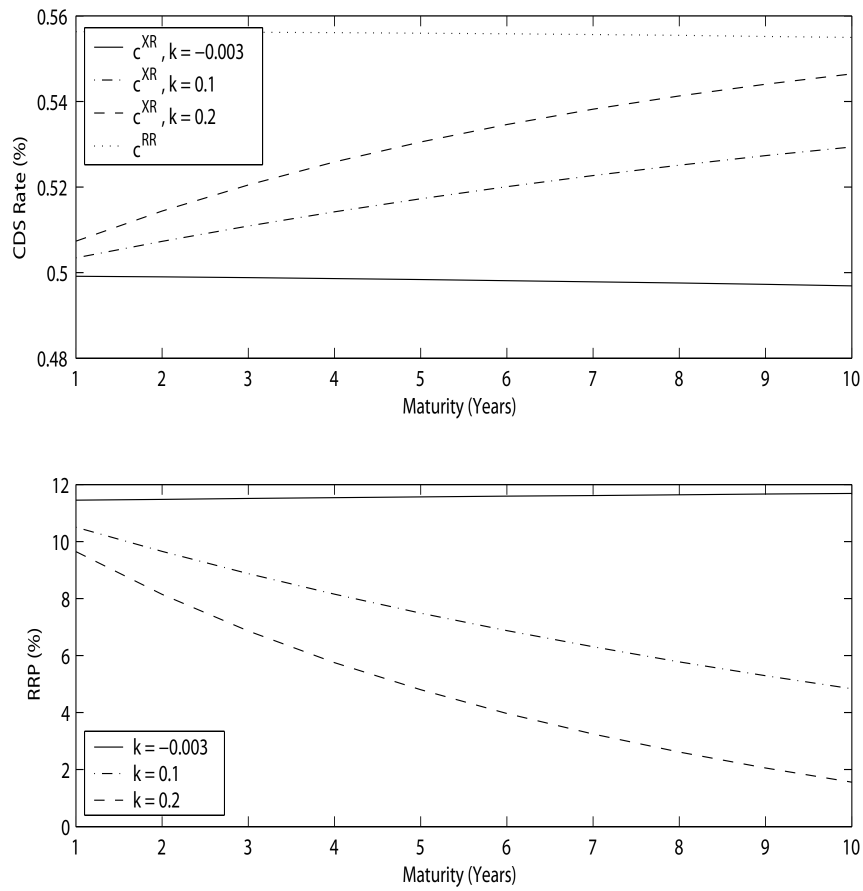

Figure 5: Relative restructuring premium with respect to Δ for various k. Other parameters are fixed at $\lambda^D = 83.13$ bps, $\bar{\delta}^D = 0.40$, $m = 0.173$, and $r = 1.63\%$.

different values for k increase as time to maturity increases. A similar observation holds for the relationship between the relative restructuring premium and Δ, as a function of k.

5.4 Model Specification

This section provides a time-series model for pricing CDS with and without restructuring risk. The risk-neutral restructuring intensity and the pre-restructuring default intensity are modeled as functions of a state variable X_t, which follows a square-root process

$$dX_t = (a - bX_t)\,dt + \sigma\sqrt{X_t}\,dW_t,\ X_0 > 0, \tag{11}$$

where W_t is a standard Brownian motion with respect to the physical measure $\mathbb{P}$, and a, b and $\sigma > 0$ are constants. We assume that the boundary non-attainment condition $2a > \sigma^2$ holds to ensure that X_t stays positive $\mathbb{P}$-almost surely. Under the risk-neutral measure $\tilde{\mathbb{P}}$, we have

$$dX_t = \left(\tilde{a} - \tilde{b}X_t\right) dt + \sigma\sqrt{X_t}\, d\tilde{W}_t, \tag{12}$$

where $\tilde{W}_t$ is a standard Brownian motion under $\tilde{\mathbb{P}}$. The market-price-of-default-risk process Λ_t is given by $dW_t = -\Lambda_t\, dt + d\tilde{W}_t$, where

$$\Lambda_t = \frac{a - \tilde{a}}{\sigma\sqrt{X_t}} - \frac{b - \tilde{b}}{\sigma}\sqrt{X_t}. \tag{13}$$

For classical affine term-structure models, $a - \tilde{a}$ is restricted to be zero (see, for example, Dai and Singleton (2000)). However, Cheridito et al. (2005) show the existence of the equivalent martingale measure $\tilde{\mathbb{P}}$ under the more general specification (13), as long as the boundary non-attainment condition $2\tilde{a} > \sigma^2$ also holds under $\tilde{\mathbb{P}}$. We assume that this condition is satisfied. A desirable feature of this "essentially" affine specification proposed by Duffee (2002) is that Λ_t does not approach zero, even if the volatility of X_t approaches zero. Also, Λ_t can switch signs over time.[9]

We assume that the risk-neutral restructuring and pre-restructuring default intensities are given by

$$\lambda_t^R = \frac{m}{1+m} X_t \quad \text{and} \quad \lambda_t^D = \frac{1}{1+m} X_t.$$

From (5), (9), and (10) it follows that the risk-neutral expected fractional loss of notional for CDS with restructuring at time t is given by $(1 - \delta^{RR})\, p(t, T)$, where

$$\begin{aligned}
\delta^{RR} &= \frac{\delta^D + m\delta^R}{1+m} \\
&\approx 1 - \left(1 - \frac{\bar{\delta}^D + m\bar{\delta}^R}{1+m}\right) \frac{p\left(t_0, (T+t_0)/2\right)}{p\left(t_0, T\right)} \\
&\triangleq 1 - \left(1 - \bar{\delta}^{RR}\right) \frac{p\left(t_0, (T+t_0)/2\right)}{p\left(t_0, T\right)}.
\end{aligned} \tag{14}$$

As before, t_0 and T are the time of initiation and the maturity of the CDS contract, respectively. $\bar{\delta}^{RR}$ can be interpreted as the recovery rate as a fraction of notional for the CDS with restructuring as anticipated at the time t_0, assuming that $\tau \approx (T + t_0)/2$. Similarly, $\delta^R = n\delta^D$ implies $\bar{\delta}^R = n\bar{\delta}^D + (1-n)(1-1/x)$, where $x =$

[9] The correct sign of Λ_t can be determined from the expected return on defaultable bonds, as in Yu (2002). In our model, given risk-averse investors, Λ_t is negative.

$p(t_0,(T+t_0)/2)/p(t_0,T)$. From (14), we have

$$m = \frac{\delta^{RR}-\delta^{D}}{\delta^{R}-\delta^{RR}} = \frac{\bar{\delta}^{RR}-\bar{\delta}^{D}}{\bar{\delta}^{R}-\bar{\delta}^{RR}}. \tag{15}$$

Estimates of $\bar{\delta}^{RR}$ and $\bar{\delta}^{D}$ are available, on a firm-by-firm basis, from Lombard's ValuSpread database. Given a value for n, (15) then allows us to obtain benchmark values for m for each firm. Appendix C shows that closed-form approximations for pricing CDS are available for our model specification.

5.5 Ford Motor: A Case Study

This section estimates the credit risk parameters in form of a case study. To estimate the parameters driving the term structure of credit spreads, we first need to obtain the time series of the risk-free term structure. Zero-coupon bond prices are stripped from the constant-maturity Treasury rate curve by assuming a piecewise linear forward rate curve.

We follow a two-step procedure to estimate the intensity parameters. In a first step, we estimate the parameters for the state variable process X_t introduced in (11) and (12) from observed CDS rates with restructuring. We assume that 5-year CDS rates are priced without errors so that we can invert the pre-restructuring default intensity λ_t^D from the CDS rate at t, given (firm-specific) estimates for m, n, and $\bar{\delta}^D$.

The 1-year and 10-year CDS rates are assumed to be measured with the noise processes u_t^1 and u_t^{10}, respectively. The measurement error is defined as

$$u_t^h \equiv c_{t,t+h}^{RR} - \bar{c}_{t,t+h}^{RR},$$

for $h = 1, 10$. Here, $c_{t,t+h}^{RR}$ is the observed annualized h-year at-market CDS rate with restructuring, and $\bar{c}_{t,t+h}^{RR}$ denotes its "true" counterpart. $\{u_t^h\}_t$ are assumed to be independently and normally distributed with mean zero and standard deviation σ^h, for $h = 1, 10$. This leaves us with the parameter vector $\Theta = \{a, b, \sigma, \tilde{a}, \tilde{b}, \sigma^1, \sigma^{10}\}$ to be estimated in this first step. We employ maximum-likelihood estimation to obtain an estimate $\hat{\Theta}$.

Given estimates for m, n, $\bar{\delta}^D$ and Θ, the CDS rate without restructuring is a function of the jump parameter k only. In the second stage of our estimation procedure, k can therefore be determined in the sense of the best least-squares fit.

We conclude this section by investigating Ford Motor Company. The maximum-likelihood estimate $\hat{\Theta}$, based on observed default swap rates with modified restructuring, are reported in Table 6. Here, we set $m = 0.173$ and $n = 1.51$ as discussed in Section 5.3. While the median relative restructuring premium for Ford Motor during

Table 6: Parameter estimates for Ford Motor in nominal terms.

	σ	a	b	$\tilde{a}$	$\tilde{b}$	σ^1	σ^{10}
Estimate	0.0090	0.0003	0.0062	0.0022	-0.0057	0.0164	0.0019
Std. Dev.	0.0003	0.0003	0.0050	0.0004	0.0048	–	–

our sample period is 0.77%, observed RRP values are quite volatile. We believe this can be attributed, at least in part, to substantial measurement errors. To calibrate k, we therefore use the assumption that relative risk premia are constant at 0.77%, by replacing CDS rates without restructuring accordingly. We find $\hat{k}$ to be 1.56. These results, although obtained in a simplified setting, indicate that investors that bought default protection for Ford Motor in recent years expected default to become more likely, risk-neutrally, once a restructuring event occurred.

6 Conclusion

This paper presents both an empirical and theoretical investigation of restructuring risk in credit default swap markets. Estimates for the price of restructuring risk in the U.S. corporate bond market during 1999-2005 are obtained by comparing quotes for default swap contracts with restructuring as a covered credit event and without. Here, we find that the average premium for restructuring risk represents 6% to 8% of the CDS rate without restructuring. This is a significant percentage of the swap rate spread. We also show that when default swap rates without restructuring increase, the increase in restructuring premia is higher for low-credit-quality firms than for high-credit-quality firms. Next, we fit a regression model to identify the determinants of CDS rates. Key explanatory variables include the distance to default (a proxy for default risk), the level and slope of the default-free forward rate curve, a stock-market volatility index, Moody's Baa corporate bond yield, and the spread between Moody's Aaa yield and the 20-year Treasury yield. The model fits the data well, with an R^2 of almost 60% for a linear model, and over 71% when using a logarithmic model.

On the theory side, we provide a reduced-form arbitrage-free model for CDS pricing that explicitly takes into account the restructuring clause. We incorporate both default and restructuring as separate events, where restructuring (if it occurs and default has not) causes a jump in the default intensity. The jump size can be positive or negative, and possibly random. A negative jump is interpreted as a successful restructuring, while a positive jump is an unsuccessful restructuring. We simulate the model using calibrated data and provide a case study of Ford Motor to illustrate the feasibility of estimating and implementing the model.

A Control Variables

1. **Distance to Default (DD).** This measure is based on Merton (1974) and is, roughly, the number of standard deviations of asset growth by which a firm's market value of assets exceeds a liability measure. It is the primary firm-specific variable in our regression models. A detailed description of the construction of distances to default with respect to the physical measure can be found in Appendix A of Duffie et al. (2005). The results in Berndt et al. (2005) indicate this distance to default measure is a significant determinant of CDS rates. Note that here we use the risk-neutral distance to default by replacing the mean rate of asset growth with the risk-free rate.

2. **Merton Default Probability (MDP).** The annualized T-year risk-neutral Merton default probability is defined as

$$\tilde{\pi}_M(T) \;=\; 1 - \Phi\left(DD_T\right)^{1/T},$$

where DD_T is the T-year risk-neutral distance to default and Φ denotes the normal cumulative distribution function.

3. **Leverage (Lev)** As in Collin-Dufresne et al. (2001), leverage is defined as the ratio of book value of total debt divided by the sum of market value of equity plus book value of total debt.

4. **Level and Slope of Risk-Free Term Structure (Level, Slope).** We use the 2-year Treasury yield for the level, and the difference between 10-year and 2-year Treasury yields for the slope variable. Daily data of constant-maturity Treasury rates is available from the Federal Reserve Bank of St. Louis.

5. **VIX Index (VIX).** The VIX Index measures the implied volatility of S&P 500 index options. Daily data can be downloaded from the Chicago Board Options Exchange (CBOE) website.

6. **Moody's Baa Corporate Bond Yield (Baa).** This variable captures the state of the corporate bond market which is closely related to the CDS market. The time series of Moody's seasoned Baa (and Aaa) corporate bond yields is available from the Federal Reserve Bank of St. Louis.

7. **Market Spread (Spread).** Spread is defined as the difference between the Moody's seasoned Aaa corporate bond yields and the 20-year Treasury yields. It is used to capture the illiquidity of the corporate bond market, given that Aaa-rated corporate bonds are almost free of default risk.

8. **Firm Size (Size).** As in Duffie et al. (2005), firm size is measured as the logarithm of the firm's total assets (Compustat item 44).

B Proofs

Proof of Proposition 1. To keep notation simple, we assume that a T-year CDS with restructuring is initiated at time 0. The market value of the payments by the buyer of protection at time 0 is given by

$$
\begin{aligned}
c_T^{RR}\tilde{E}\left[\int_0^T e^{-\int_0^v r_s\,ds} 1_{\{\tau>v\}}\,dv\right] &= c_T^{RR}\int_0^T p(0,v)\,\tilde{E}\left[1_{\{\tau>v\}}\right]dv \\
&= c_T^{RR}\int_0^T p(0,v)\,\tilde{E}\left[e^{-\int_0^v (1+m)\lambda_s^D ds}\right]dv.
\end{aligned}
$$

Recall that in Section 5.2 we assume risk-neutral independence between the risk-free rate r and the default time τ. The market value of the potential payment by the seller of protection is

$$
\begin{aligned}
\tilde{E}\left[e^{-\int_0^\tau r_s\,ds}(1-\delta)\,p(\tau,T)\,1_{\{\tau\le T\}}\right] &= p(0,T)\,\tilde{E}\left[(1-\delta)\,1_{\{\tau\le T\}}\right] \\
&= p(0,T)\Big\{\left(1-\delta^D\right)\tilde{E}\left[1_{\{\tau^D\le T,\tau^D\le\tau^R\}}\right] \\
&\quad + \left(1-\delta^R\right)\tilde{E}\left[1_{\{\tau^R\le T,\tau^R\le\tau^D\}}\right]\Big\}.
\end{aligned}
$$

The risk-neutral probability of a default event occurring prior to both maturity and restructuring can be calculated as

$$
\begin{aligned}
\tilde{E}\left[1_{\{\tau^D\le T,\tau^D\le\tau^R\}}\right] &= \int_0^T \tilde{E}\left[\tilde{E}\left(1_{\{\tau=\tau^D\}}|\tau=v\right)1_{\{\tau=v\}}\right]dv \\
&= \tilde{E}\left[\int_0^T \frac{\lambda_v^D}{\lambda_v^D+\lambda_v^R}\left(\lambda_v^D+\lambda_v^R\right)e^{-\int_0^v\left(\lambda_s^D+\lambda_s^R\right)ds}\,dv\right] \\
&= \frac{1}{1+m}\left(1-\tilde{E}\left[e^{-\int_0^T(1+m)\lambda_s^D ds}\right]\right).
\end{aligned}
$$

Similarly, the risk-neutral probability of restructuring occurring prior to both maturity and default is

$$
\begin{aligned}
\tilde{E}\left[1_{\{\tau^R\le T,\tau^R\le\tau^D\}}\right] &= \tilde{E}\left[\int_0^T \frac{\lambda_v^R}{\lambda_v^D+\lambda_v^R}\left(\lambda_v^D+\lambda_v^R\right)e^{-\int_0^v\left(\lambda_s^D+\lambda_s^R\right)ds}\,dv\right] \\
&= \frac{m}{1+m}\left(1-\tilde{E}\left[e^{-\int_0^T(1+m)\lambda_s^D ds}\right]\right).
\end{aligned}
$$

The initial at-market CDS rate is that choice for c_T^{RR} at which the market values of the payments by the buyer and seller of protection are equal. ■

The proof of Proposition 2 is similar to that of Proposition 1, and therefore omitted.

C Closed-Form Approximations of CDS Rates

In this appendix, we provide closed-form approximations of the default swap rates derived in (6) and (7), using the model specification in Section 5.4.

To calculate the CDS rates derived in (6) and (7), we need to compute expectations of the form

$$
\begin{aligned}
E1 &= \tilde{E}\left[e^{-\int_0^T (1+m)\lambda_s^D\, ds}\right] \quad \text{and} \\
E2 &= \tilde{E}_t\left[e^{-\int_t^T \lambda_s^D ds} \int_t^T e^{-\left(k_1(T-v)+k_2\int_v^T \lambda_s^D\, ds\right)} \lambda_v^R e^{-\int_t^v \lambda_s^R\, ds}\, dv\right].
\end{aligned}
$$

The first term $E1$ is of the form

$$
E1 = e^{\alpha_1(0,T)+\beta_1(0,T)\lambda_0^D},
$$

where $\alpha_1(0,T)$ and $\beta_1(0,T)$ are available in closed form (see, for example, Duffie et al. (2000)).

To approximate the second expectation $E2$ for constants k_1 and k_2, we assume that credit events occur only at discrete time intervals $\Delta, 2\Delta, \ldots, T = n\Delta$. We have

$$
\begin{aligned}
E2 &= \tilde{E}_t\left[e^{-\int_t^T \lambda_s^D ds} e^{-\left(k_1(T-\tau^R)+k_2\int_{\tau^R}^T \lambda_s^D\, ds\right)} 1_{\{\tau^R\le T\}}\right] \\
&= \sum_{j=1}^n \tilde{E}\left[e^{-\int_t^T \lambda_s^D ds} e^{-\left(k_1(T-\tau^R)+k_2\int_{\tau^R}^T \lambda_s^D\, ds\right)} 1_{\{(j-1)\Delta<\tau^R\le j\Delta\}}\right] \\
&\approx \sum_{j=1}^n \tilde{E}\left[e^{-\int_t^T \lambda_s^D\, ds} e^{-\left(k_1(T-j\Delta)+k_2\int_{j\Delta}^T \lambda_s^D\, ds\right)} 1_{\{(j-1)\Delta<\tau^R\le j\Delta\}}\right].
\end{aligned}
$$

By the iterative conditioning, the last term can be written as

$$
\begin{aligned}
E2 &= \sum_{j=1}^n \tilde{E}\left[e^{-\int_t^{j\Delta} \lambda_s^D\, ds}\, e^{-k_1(T-j\Delta)} \left(e^{-\int_0^{(j-1)\Delta} \lambda_s^R\, ds} - e^{-\int_0^{j\Delta} \lambda_s^R\, ds}\right)\right. \\
&\qquad \left. \tilde{E}_{j\Delta}\left(e^{-(1+k_2)\int_{j\Delta}^T \lambda_s^D\, ds}\right)\right] \\
&= \sum_{j=1}^n e^{-k_1(T-j\Delta)}\, \tilde{E}\left[e^{-\int_t^{(j-1)\Delta}\left(\lambda_s^D+\lambda_s^R\right) ds} \left(1-e^{-\int_{(j-1)\Delta}^{j\Delta} \lambda_s^R\, ds}\right) e^{\alpha_2^j+\beta_2^j\lambda_{j\Delta}^D}\right] \\
&= \sum_{j=1}^n e^{-k_1(T-j\Delta)}\, \tilde{E}\left[e^{-\int_t^{(j-1)\Delta}\left(\lambda_s^D+\lambda_s^R\right) ds}\, \tilde{E}_{(j-1)\Delta}\left\{e^{\alpha_2^j+\beta_2^j\lambda_{j\Delta}^D}\right\}\right] \\
&\quad -\sum_{j=1}^n e^{-k_1(T-j\Delta)}\, \tilde{E}\left[e^{-\int_t^{(j-1)\Delta}\left(\lambda_s^D+\lambda_s^R\right) ds}\, \tilde{E}_{(j-1)\Delta}\left\{e^{-\int_{(j-1)\Delta}^{j\Delta} m\lambda_s^D\, ds}\, e^{\alpha_2^j+\beta_2^j\lambda_{j\Delta}^D}\right\}\right] \\
&= \sum_{j=1}^n e^{-k_1(T-j\Delta)}\, \tilde{E}\left[e^{-\int_t^{(j-1)\Delta}(1+m)\lambda_s^D\, ds} \left(e^{\alpha_3^j+\beta_3^j\lambda_{(j-1)\Delta}^D} - e^{\alpha_4^j+\beta_4^j\lambda_{(j-1)\Delta}^D}\right)\right],
\end{aligned}
$$

where $\alpha_2^j = \alpha_2^j(j\Delta, T)$ and $\beta_2^j = \beta_2^j(j\Delta, T)$, $\alpha_3^j = \alpha_3^j((j-1)\Delta, j\Delta)$ and $\beta_3^j = \beta_3^j((j-1)\Delta, j\Delta)$, and $\alpha_4^j = \alpha_4^j((j-1)\Delta, j\Delta)$ and $\beta_4^j = \beta_4^j((j-1)\Delta, j\Delta)$ are available in closed form, for all j.

Finally, $E2$ can be approximated by

$$E2 \;=\; \sum_{j=1}^{n} e^{-k_1(T-j\Delta)} \left(e^{\alpha_5^j + \beta_5^j \lambda_0^D} - e^{\alpha_6^j + \beta_6^j \lambda_0^D} \right),$$

where $\alpha_5^j = \alpha_5^j(0, (j-1)\Delta)$, $\beta_5^j = \beta_5^j(0, (j-1)\Delta)$, $\alpha_6^j = \alpha_6^j(0, (j-1)\Delta)$, and $\beta_6^j = \beta_6^j(0, (j-1)\Delta)$ are also available in closed form. (Again, see Duffie et al. (2000)) for details.)

D Additional Tables and Background Statistics

Table 7: Number of quotes by industry for U.S. names with industry information verified using the FISD database.

FISD Industry code	Number of observations				
	Total	XR	FR	MR	MMR
Industrial					
10 Manufacturing	258,355	40,648	46,008	170,240	1,459
11 Media/Communications	48,693	12,192	7,682	28,819	0
12 Oil & Gas	38,429	5,972	6,269	26,188	0
13 Railroad	1,961	395	247	1,319	0
14 Retail	58,842	10,551	10,855	37,424	12
15 Service/Leisure	21,358	12,184	7,855	1,319	0
16 Transportation	20,301	3,613	3,950	12,738	0
32 Telephone	14,738	3,457	1,474	9,807	0
Finance					
20 Banking	30,990	3,391	7,528	20,071	0
21 Credit/Financing	28,256	5,130	6,193	16,933	0
22 Financial Services	38,567	4,323	6,283	27,961	0
23 Insurance	41,358	5,957	4,474	30,927	0
24 Real Estate	26,256	2,397	4,478	18,416	965
25 Savings & Loan	137	0	0	137	0
26 Leasing	1,629	273	108	1,248	0
Utility					
30 Electric	39,685	6,019	4,731	28,935	0
31 Gas	7,148	1,356	1,069	4,723	0
33 Water	0	0	0	0	0
Government					
40 Foreign Agencies	0	0	0	0	0
41 Foreign	0	0	0	0	0
42 Supranational	835	0	593	242	0
43 U.S. Treasuries	0	0	0	0	0
44 U.S. Agencies	2,151	429	248	1,474	0
45 Taxable Municipal	0	0	0	0	0
Miscellaneous					
60 Miscellaneous	0	0	0	0	0
99 Unassigned	0	0	0	0	0
Total	679,689	118,287	120,045	438,921	2,436

Table 8: Summary statistics for (relative) restructuring premia.

Restructuring Premium of FR over XR										
Variable	N	Mean	StdDev	Max	P99	Q3	Med	Q1	P1	Min
RP1Y	9,819	5.89	63.27	1465.33	102.50	8.50	2.56	-1.03	-54.35	-3356.00
RP3Y	17,046	5.72	93.00	1429.63	83.54	7.67	3.13	0.49	-25.10	-7170.33
RP5Y	19,719	6.89	64.99	1641.67	70.31	7.75	3.86	1.70	-19.25	-5080.50
RP7Y	16,538	7.65	39.70	1760.18	58.15	8.67	4.54	1.86	-14.50	-1277.90
RP10Y	13,534	8.18	58.84	1856.02	63.18	10.57	5.34	1.75	-18.63	-3193.70
RRP1Y	9,819	10.03	28.07	497.53	120.92	19.26	8.22	-3.57	-43.06	-72.20
RRP3Y	17,046	9.20	16.64	792.62	51.11	15.03	8.31	1.51	-18.26	-46.90
RRP5Y	19,719	8.43	8.81	212.50	32.54	12.21	7.95	4.02	-11.37	-48.90
RRP7Y	16,538	8.85	9.77	166.97	41.03	12.74	7.93	3.73	-12.07	-50.61
RRP10Y	13,534	8.69	12.30	252.24	52.56	13.05	7.56	2.86	-17.11	-49.38

Restructuring Premium of MR over XR										
Variable	N	Mean	StdDev	Max	P99	Q3	Med	Q1	P1	Min
RP1Y	27,209	1.72	46.45	3813.34	74.35	4.50	1.19	-2.54	-62.00	-1658.44
RP3Y	52,460	3.16	31.67	1773.00	55.26	5.13	2.20	0.30	-45.18	-1604.66
RP5Y	56,952	3.78	33.86	2343.18	47.50	5.06	2.66	1.12	-34.68	-1898.73
RP7Y	46,978	3.97	34.02	4470.15	42.15	4.75	2.38	0.67	-20.97	-1065.70
RP10Y	45,558	4.12	33.76	2750.03	46.30	5.93	2.66	0.37	-31.85	-951.45
RRP1Y	27,209	4.34	23.92	468.29	89.77	12.40	4.01	-6.81	-47.02	-96.01
RRP3Y	52,460	5.90	11.64	262.31	44.16	10.47	5.70	0.78	-23.30	-94.87
RRP5Y	56,952	5.69	8.91	194.65	37.78	7.91	5.19	2.42	-16.21	-93.56
RRP7Y	46,978	4.86	9.76	249.30	37.39	6.93	4.07	1.26	-14.96	-93.20
RRP10Y	45,558	4.68	10.99	357.09	47.90	7.07	3.93	0.55	-18.55	-92.89

Restructuring Premium of FR over MR										
Variable	N	Mean	StdDev	Max	P99	Q3	Med	Q1	P1	Min
RP1Y	15,322	3.86	70.34	1923.82	105.70	6.53	1.30	-2.28	-78.34	-3058.68
RP3Y	24,752	3.37	47.80	2672.97	72.06	4.27	0.97	-1.45	-46.52	-1691.49
RP5Y	27,355	3.95	54.49	2964.89	55.90	3.51	1.14	-0.45	-25.99	-3088.89
RP7Y	22,944	4.42	53.72	3219.04	57.28	5.40	2.07	-0.21	-24.26	-3088.89
RP10Y	18,784	3.61	58.31	3322.50	50.92	6.80	2.26	-1.30	-34.73	-3088.89
RRP1Y	15,322	9.64	44.29	966.18	139.98	17.71	4.12	-7.67	-47.44	-78.74
RRP3Y	24,752	4.36	20.13	528.63	62.65	9.21	2.59	-3.75	-25.60	-75.66
RRP5Y	27,355	3.72	14.30	334.21	49.97	6.26	2.35	-0.92	-19.52	-71.79
RRP7Y	22,944	4.95	13.86	246.94	58.01	8.04	3.67	-0.36	-20.70	-80.65
RRP10Y	18,784	4.48	15.63	449.46	59.75	8.58	3.30	-1.86	-26.61	-73.90

Table 9: Regression results for CDS rates.

Variable	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
Intercept	1049.376	5.909	1045.541	6.180	934.285	6.718	1282.206	27.764
MR	8.631	2.378						
FR	17.903	2.378						
XR01			-1.627	4.396	3.528	4.243	-70.194	6.354
MR01			15.815	4.396	20.971	4.243	-52.752	6.354
FR01			33.931	4.396	39.086	4.243	-34.636	6.354
MR03			6.569	2.640	6.569	2.543	6.569	2.483
FR03			13.772	2.640	13.772	2.543	13.772	2.483
IND1					122.343	3.719	81.347	4.171
IND2					115.944	3.518	82.545	3.873
IND3					173.073	3.996	140.239	4.205
T1					-18.771	3.247	-18.404	3.173
T3					0.347	2.718	-0.738	2.655
T7					10.552	2.701	12.828	2.641
T10					11.904	2.870	11.488	2.805
DD	-445.985	3.189	-444.991	3.200	-443.209	3.116	-446.207	3.062
DD^2	60.703	0.546	60.688	0.545	59.530	0.533	59.052	0.522
DD^3	-2.553	0.028	-2.555	0.028	-2.471	0.027	-2.433	0.027
Size							-18.908	1.002
Level							1.118	0.065
Slope							0.710	0.119
Baa							-0.999	0.063
Spread							3.148	0.119
Obs.	25266		25266		25266		25266	
R^2	0.545		0.545		0.578		0.598	
adj R^2	0.545		0.545		0.578		0.597	

Table 10: Regression results for logarithm of CDS rates.

	Leverage		**Merton Default Prob**		**Distance to Default**	
Variable	estimate	SD	estimate	SD	estimate	SD
Intercept	3.9864	0.1184	3.5491	0.1163	9.8921	0.0894
XR01	-0.0571	0.0291	-0.0840	0.0287	-0.2535	0.0205
MR01	0.0336	0.0291	0.0067	0.0287	-0.1628	0.0205
FR01	0.1111	0.0291	0.0842	0.0287	-0.0854	0.0205
MR03	0.0790	0.0114	0.0790	0.0112	0.0790	0.0080
FR03	0.1585	0.0114	0.1585	0.0112	0.1585	0.0080
IND1	0.5811	0.0191	0.3934	0.0188	0.4059	0.0134
IND2	0.3125	0.0178	0.0376	0.0174	0.3061	0.0125
IND3	0.6429	0.0192	0.3813	0.0190	0.5467	0.0135
T1	-0.2817	0.0145	-0.2376	0.0143	-0.3293	0.0102
T3	-0.0743	0.0122	-0.0521	0.0120	-0.0917	0.0086
T7	0.0556	0.0121	0.0424	0.0119	0.0931	0.0085
T10	0.1518	0.0128	0.1175	0.0127	0.1736	0.0090
Lev	2.2968	0.0232				
MDP			0.0011	0.0000		
DD					-1.0496	0.0099
DD^2					0.1009	0.0017
DD^3					-0.0035	0.0001
Size	-0.1848	0.0048	-0.0405	0.0045	-0.1295	0.0032
Level	-0.0013	0.0003	0.0015	0.0003	0.0094	0.0002
Slope	-0.0011	0.0005	-0.0003	0.0005	0.0047	0.0004
Baa	0.0008	0.0003	-0.0004	0.0003	-0.0088	0.0002
Spread	0.0110	0.0005	0.0123	0.0005	0.0200	0.0004
Obs.	25,266		25,266		25,266	
R^2	0.413		0.429		0.711	
adj R^2	0.412		0.429		0.710	

Table 11: Regression results for loss given default as a fraction of notional.

Variable	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
Intercept	67.450	0.110	68.082	0.113	66.740	0.125	74.630	0.505
MR	-0.534	0.044						
FR	2.939	0.045						
XR01			-0.402	0.080	-0.336	0.079	1.004	0.116
MR01			-2.640	0.079	-2.566	0.078	-1.224	0.116
FR01			2.338	0.079	2.415	0.078	3.764	0.116
MR03			-0.130	0.048	-0.123	0.047	-0.146	0.045
FR03			2.988	0.049	3.003	0.048	3.023	0.046
IND1					1.234	0.069	0.942	0.075
IND2					1.166	0.065	0.946	0.070
IND3					2.046	0.074	1.721	0.076
T1					0.259	0.060	0.160	0.057
T3					0.124	0.051	0.080	0.048
T7					0.170	0.051	0.103	0.048
T10					0.172	0.054	0.169	0.051
DD	-3.890	0.059	-4.026	0.058	-3.984	0.058	-3.991	0.055
DD^2	0.591	0.010	0.593	0.010	0.579	0.010	0.558	0.009
DD^3	-0.026	0.001	-0.026	0.001	-0.025	0.001	-0.024	0.000
Size							-0.112	0.018
Level							0.025	0.001
Slope							0.067	0.002
Baa							-0.047	0.001
Spread							0.069	0.002
Obs.	23910		23910		23910		23910	
R^2	0.332		0.361		0.381		0.441	
adj R^2	0.332		0.360		0.381		0.441	

Table 12: This table shows the number of initial credit events of Moody's rated U.S. bonds from 2000 to 2004. It is constructed from the Moody's annual and monthly surveys of global corporate defaults and recovery rates from 2000 to 2004. m is calculated as the number of restructurings (here, distressed exchanges) divided by the total number of other credit events.

Year	Failure to Pay	Bankruptcy	Distressed Exchange	Total	m
2000	83	40	2	125	0.016
2001	91	44	7	142	0.052
2002	55	22	11	88	0.143
2003	28	22	8	58	0.160
2004	17	8	5	30	0.200
2000-2004	274	136	33	443	0.080
2003-2004	45	30	13	88	0.173

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