Asset Pricing With Durable Goods: Potential Resolution of Some Asset Pricing Puzzles

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Abstract

When the intra-temporal consumption complementarity between nondurables and durables is high, and investors have a preference for early resolution of uncertainty, the implied risk premia on assets are high and time-varying. Stocks are risky not only because they co-vary with nondurable consumption growth but mainly because they tend to pay badly in times when the consumption basket is "out of whack" and because the uncertainty about their payoff resolves in the future. The model is able to explain the equity-premium puzzle, the risk-free rate puzzle and the small-minus-big and value-minus-growth spreads with a low coefficient of risk aversion. Furthermore, the time variation in the variety of the consumption basket over the business cycle naturally generates variation in expected returns on these benchmark portfolios.

JEL Classification: E21, E32, E44, G12

1 Introduction

Asset markets data pose a serious challenge to the traditional macroeconomic models, such as the Canonical Consumption-Based Asset Pricing model [Lucas (1978), Breeden (1979)]. Several stylized facts are particularly hard to justify. In fact, the return on common stocks averages about 6% in real terms and the real risk-free rate is very low and smooth [Hansen and Singleton (1982, 1983), Mehra and Prescott (1985), Weil (1989)]. Furthermore, expected returns on assets seem to vary over the business cycle [Fama and French (1989), Leroy and Porter (1981), Shiller (1981)]. As a result, valuation ratios, such as the price-dividend ratio, forecast long-horizon equity returns [cf. Campbell and Shiller (1988), Fama and French (1988)]. In addition, small stocks seem to earn higher average return than big stocks [Banz (1981)]. Similarly, value stocks significantly outperform growth stocks [cf. Fama and French (1992, 1993) and references therein]. Finally, both small-minus-big (SMB) and value-minus-growth (HML) spreads are time varying and predictable [cf. Cohen, Polk and Vuolteenaho (2001), Pakoš (2005)]. Mehra and Prescott (1988) offer the following guidelines for what they think would help solve the quantitative asset pricing puzzles: "Perhaps the introduction of some other preference structure will do the job...". Kocherlakota (1996) points out that it is important to consider if we can resolve these puzzles, considering possible alterations to preferences of the representative consumer and keeping the useful framework of complete and frictionless asset markets.

In light of these guidelines, I present a frictionless complete markets model that helps explain the above features of the money market and the common stock market. The gist of the model lies in the identification of two important *new* risk factors, in addition to the Lucas-Breeden nondurable consumption growth rate. These stem from a careful specification of investors' preferences which exhibit (i) high *consumption complementarity* across consumption goods and (ii) a *preference for early resolution of uncertainty*, coming from the non-expected utility framework.

Formally, I enrich the models of Eichenbaum and Hansen (1990), Heaton (1995), Ogaki and Reinhart (1998) and Yogo (2006). I justify the construct of a representative agent by the assumption of frictionless complete asset markets [Constantinides (1982)]. The model features a new, additional, consumption good, the service flow from consumer durables. In fact, in response to Shiller (1982) empirical analysis of the Canonical CCAPM, Hansen (1982a) argues that "... it is desirable to include the service flow from the stock of durable goods in the analysis." Following Becker (1965, 1993), Ghez and Becker (1975), Lancaster (1966), Stigler and Becker (1977), I impute this service flow by a constant-returns-to-scale household production function. Lucas (1993) uses a similar production function in his analysis of the economic growth. Greenwood and Hercowitz (1991) use CRS Beckerian household production function to study the cyclical allocation of capital and time between market and home activities. With multiple consumption goods, in our case, nondurables and the service flow from consumer durables, it becomes meaningful to talk about the variety of the consumption basket, and such a variety is desired by the convexity of the preferences. As a result, consumer durables introduce a new risk factor, the alteration of the variety of the consumption basket. Specifically, the consumption basket gets "out of whack" in recessions, which is costly for investors, and it is costlier the higher the consumption complementarity between the consumption goods.

Furthermore, I relax the highly restrictive assumption of the expected utility framework that investors are indifferent to the timing of uncertainty. In the realm of the non-expected utility, investors may exhibit preference for either early or late resolution of uncertainty [Kreps and Porteus (1978) and Epstein and Zin (1989, 19991)]. To facilitate the exposition, let us firstly focus on the case without consumer durables. The implied SDF is a combination of the standard Lucas-Breeden SDF, $(C_{t+1} \,/\, C_t)^{-1/\sigma}$, and a new factor, the return on the wealth portfolio that captures the attitudes toward the timing of uncertainty. Therefore, if the preference for early resolution of uncertainty is dominant in the market, stocks are risky not only because they co-vary with the consumption growth but also because the uncertainty about their payoff resolves late. This raises the intriguing question what role the timing of uncertainty plays in explaining the average returns across assets and over time.

There are two factors that combine together to determine jointly these attitudes toward the timing of uncertainty, the inter-temporal substitutability σ and the coefficient of risk aversion γ. The restriction $\sigma \times \gamma = 1$ corresponds to the case when indifference to the timing of uncertainty is exhibited, and we subsequently obtain the standard Expected Utility framework. The figure 2 graphically presents how the restriction toward the timing of uncertainty (i.e. $\kappa \equiv (1-\gamma)/(1-1/\sigma)$ greater or less than one) separates the (σ, γ) space into four regions. We see that as long as the inter-temporal substitutability is sufficiently large (i.e. σ large enough than zero), we may obtain a preference for early resolution of uncertainty with quite a low coefficient of risk aversion. Perhaps we can explain the quantitative asset pricing puzzles with a high inter-temporal substitutability and low risk aversion, implicitly imputing a fraction of the compensation for risk, the expected return, to the preference for early uncertainty. Empirically, it turns out that even a small preference for early resolution of uncertainty has a significant first-order effect upon expected returns. To document this fact empirically, Figure 3 plots the expected excess return on the value-weighted market portfolio as a function of σ and γ. As the right panel shows, the preference for early resolution of uncertainty is associated with a positive expected excess return. Furthermore, a small increase in the coefficient of the risk aversion leads to a large increase in expected returns. On the other hand, the left panel portrays the case where investors prefer late resolution of uncertainty to early one. Two facts emerge. Firstly, the expected return is negative in almost the whole region. Secondly, the expected return becomes positive only when the inter-temporal substitutability σ is so small that the consumption risk offsets the preference for late resolution of uncertainty, and this actually happens at quite a large coefficient of risk aversion, considered by many too extreme.

A large literature focuses on the estimation of the elasticity of intertemporal substitution σ. Hall (1988) presents estimates of the elasticity "... that are small. Most of them are also quite precise, supporting the strong conclusion that the elasticity is unlikely to be much above 0.1, and may well be zero." Using improved inference methods, Hansen and Singleton (1983) find that there is less precision and even obtain estimates that are negative. Using international data, Campbell (2002) and Yogo (2004) estimate the elasticity statistically and economically insignificant. However, these studies assume that the felicity function is separable over nondurables and durables. In response, Mankiw (1985) and Ogaki and Reinhart (1998) enrich the model by explicitly introducing the service flow from consumer durables. They argue that ignoring this consumption good induces a bias in favor of finding a low or even negative magnitude of intertemporal substitution. The real interest rate directly affects the user cost of consumer durables. For instance, a rise in the real interest rates leads to a higher user cost and consumers substitute toward nondurable goods. This channel is completely missing in the one good economy and hence the estimate of the intertemporal substitution is biased downward. Secondly, Mankiw (1985) finds that the service flow from consumer durables is itself more responsive to the interest rates and estimate a large elasticity of substitution for durable goods. Focusing on non-separability across goods, Ogaki and Reinhart find that there is quite a large inter-temporal substitutability when both nondurable and durable goods are considered. This result further underlines the need to introduce consumer durables if we want to estimate a large magnitude of intertemporal substitution, and thus obtain a preference for early resolution of uncertainty with low risk aversion.

One important criticism of the Ogaki-Reinhart empirical results[1], and in fact, of the models that use durable goods [cf. Hansen and Eichenbaum (1990), Mankiw (1985), Ogaki and Reinhart (1998), Yogo (2005)] is that they assume that the household "produces" [Becker (1965, 1993), Ghez and Becker (1975), Lancaster (1966), Stigler and Becker (1977)] the service flow from consumer durables using time- and state- independent linear household production function. The thesis of this paper is that the appropriate modeling of the returns-to-scale in the

[1]While I was working on this draft I learned about two recent papers by Okubo Masakatsu (2004a, 2004b) which also enrich the Ogaki and Reinhart's (1998) model. However, his specification does not allow for an easy interpretation of the preference parameters.

household sector is absolutely crucial to impute correct service flow from consumer durables, and in turn, to obtain an unbiased estimates of the magnitude of intra-temporal and inter-temporal substitutions. I follow Greenwood and Hercowitz (1991) and I find economically and statistically significant decreasing returns to scale in the consumer durables, *ceteris paribus*. Therefore, this critique is empirically relevant.

In addition, in contrast to the model with consumer durables, the one-good economy lacks a mechanism to generate time-variation in expected returns as the nondurables consumption growth rate is believed by economists to be i.i.d. [cf. Bansal and Yaron (2004) for a different model of consumption growth]. As may be seen in the stochastic discount factor, durable goods introduce a new state variable, the ratio of the service flow S_t over nondurables C_t, which varies over the business cycle [cf. Yogo (2006)] and naturally generates time-varying expected returns. Furthermore, the magnitude of this predictability is a function of the intra-temporal complementarity θ between service flow and nondurables, rising as θ diminishes. The intuition for this result is that service flow and nondurables get "out of whack" in the recession.

1.1 Household's Problem

1.1.1 Consumption and Portfolio Problem

The representative household faces the following consumption and portfolio problem: each time period t, it purchases in the market C_t units of nondurable consumption goods, $I_{D,\,t}$ units of new consumer durables and spends $I_{H,\,t}$ in order to augment its human capital. I choose the nondurable consumption as a numeraire. I denote $Q_{D,\,t}$ the relative price of the consumer durable goods in terms of nondurable consumption good and $Q_{H,\,t}$ the relative price of human capital investment. The nondurable good is immediately perishable but the stocks of consumer durables and human capital provide current and future services until they are fully depreciated. Their laws of motion are $D_t = (1-\delta_D)\,D_{t-1} + I_{D,t}$ and $H_t = (1-\delta_H)\,H_{t-1} + I_{H,t}$ where $\delta_D,\, \delta_H \in (0,1)$ stand for the depreciation rates. The budget constraint is standard and is relegated to Appendix A.

1.1.2 Household's Intra-Period Preference Specification

Many empirical implementations of the consumption-based asset pricing models implicitly use durable goods by assuming that nondurable consumption and the service flow from durables enter the felicity function separably [cf. Hansen and Singleton (1982b, 1983)]. Eichenbaum and Hansen (1990) find empirical evidence in favor of non-separability. Therefore, the intra-period utility function, defined over the flow of nondurable goods C_t and the service flow S_t from consumer durables D_t, is specified as a constant-elasticity-of-substitution (CES) function

$$u\left(C_t, S_t\right) = \left\{(1-a)^{1-\frac{1}{\theta}} C_t^{1-\frac{1}{\theta}} + a^{1-\frac{1}{\theta}} S_t^{1-\frac{1}{\theta}}\right\}^{\frac{\theta}{\theta-1}} \tag{1}$$

where $a \in (0,1)$ is the preference weight given to the service flow, and $\theta \geq 0$ is the elasticity of intra-temporal substitution between the flow of nondurable goods and the service flow from consumer durables. Furthermore, observe that the consumption index $u(C,S)$ is homogenous of degree one. This will become important for the derivation of the first-order conditions, which require homogenuity. Dunn and Singleton (1986) and Pakoš (2000) assume that the consumption index $u(C_t, S_t)$ is Cobb-Douglas; their implied $\theta = 1$. Eichenbaum and Hansen (1990), Ogaki and Reinhart (1998) and Yogo (2005) relax the restriction $\theta = 1$.

1.1.3 Imputation of Service Flow From Consumer Durables

Becker (1965, 1993), Ghez and Becker (1975), Lancaster (1966), Stigler and Becker (1977), and others, provide a new foundation for the theory of household behavior. According to this new view, the household purchases "goods" in the market and combines them with time and human capital in a "household production function" to produce "commodities". These commodities rather than the goods are the arguments of the household's utility function. For example, consumers do not derive utility from the stock of household durables D_t. Rather, they "produce" the services flow X_t using human and household capital. As a result, I assume that the representative household uses the household capital D_t and human capital H_t to "produce" the services flow X_t using the constant-returns-to-scale household production function

$X_t = F(H_t, D_t)$. Becker (1962, 1964) emphasizes the role of human capital. The specification features constant returns to scale in both the human capital H_t and household capital D_t. However, the marginal product of the durables D_t is diminishing, holding human capital fixed. Lucas (1993) uses a similar production function in his analysis of the economic growth. Greenwood and Hercowitz (1991) use a similar Beckerian household production function to study the cyclical allocation of capital and time between market and home activities.

There is a large exciting literature using durable goods [cf. Mankiw (1985), Eichenbaum and Hansen (1990), Heaton (1995), Ogaki and Reinhart (1998), Yogo (2006)]. These studies do not estimate the returns to scale in the household sector empirically. In fact, their implicit household production function is linear in the household capital D_t. This specification is similar to the "AK" endogenous growth models literature [cf. Harrod (1939) and Domar (1946)] in that the marginal product of capital is constant. However, as we will see in Empirical Section, careful modeling of the returns to scale in the household sector is crucial to obtain a good measure of the magnitudes of the inter-temporal and intra-temporal substitutions. Forcing counter-factually constant marginal product of the household capital D_t, as the aforementioned studies in fact do, delivers a biased estimate of the services flow X_t and therefore biased estimates of the elasticities. In contrast, I determine the returns to scale of the household technology empirically, and find that there are statistically and economically significant diminishing returns to scale of the household technology, holding human capital fixed.

1.1.4 Household's Inter-Temporal Preference Specification

Following Giovannini and Weil (1989), Giovannini and Jorian (1989), Epstein and Zin (1989, 1991), Kreps and Porteus (1978), Yogo (2006) and Weil (1989, 1990), I assume that the representative agent's inter-temporal utility function is specified by the recursive form

$$V_t = U\left\{u(C_t, S_t), \mathbf{E}_t[V_{t+1}]\right\} \tag{2}$$

with the Koopmans' (1960) aggregator function taking the constant-elasticity-of-substitution form

$$U(x,y) \;=\; \left\{(1-\delta)\,x^{1-1/\sigma} \;+\; \delta\,y^{(1-1/\sigma)/(1-\gamma)}\right\}^{(1-\gamma)/(1-1/\sigma)} \tag{3}$$

where σ is the elasticity of inter-temporal substitution, $\delta \in (0,1)$ is the consumer's subjective discount factor, and γ is the coefficient of risk aversion.

Kreps and Porteus (1978) identify the curvature of $U\{x,\bullet\}$ as the determinant of attitudes toward the timing of uncertainty, with convexity (concavity) corresponding to a preference for early (late) resolution. With our parametrization, consumers prefer early resolution if $\kappa \equiv (1-\gamma)\,/\,(1-1/\sigma)$ is less than one. We can depict this restriction graphically as in Figure 2, where the positive quadrant is divided into four regions, with regions I and IV corresponding to late, and regions II and III early, resolutions of uncertainty. When the elasticity of inter-temporal substitution σ is the inverse of the coefficient of risk aversion, $\sigma \times \gamma = 1$, consumers are indifferent to the timing of uncertainty and we obtain the Von Neumann-Morgenstern expected utility model.

1.2 First-Order Conditions

There are three first-order conditions which I derive using simple microeconomic arguments and relegate the formal analysis to the Appendix. Firstly, there is the intra-temporal first-order condition which states that the marginal utility per last dollar spent is the same across all consumption goods. Specifically, suppose we buy an additional unit of nondurables at price one[2]. The marginal utility per last dollar spent is $u_C(C_t,S_t)\,/\,1$. On the other hand, suppose we consume an additional unit of the services flow. For that we need to rent $1/F'_D(H_t,D_t)$ units of household capital at a price $RC_{D,t} \times 1/F'_D(H_t,D_t)$. Therefore, the marginal utility per dollar spent is $F'_D(H_t,D_t)\,u_S(C_t,S_t)\,/\,RC_{D,t}$. In equilibrium, it must be true that the

[2]Recall that nondurables are numeraire and therefore have price one.

marginal utility per dollar spent is the same across all goods and thus

$$\frac{F'_D(H_t, D_t)\, u_S(C_t, S_t)}{RC_{D,t}} \;=\; \frac{u_C(C_t, S_t)}{1} \tag{4}$$

Upon re-arranging, we obtain the intra-temporal first-order condition

$$RC_{D,t} \;=\; \frac{F'_D(H_t, D_t)\, u_S(C_t, S_t)}{u_C(C_t, S_t)} \tag{5}$$

We find the rental cost of consumer durables $RC_{D,t}$ by the following no-arbitrage argument. Suppose we buy one unit of durables at price $Q_{D,t}$, which after one period depreciates to $1 - \delta_D$. We can sell it for $(1 - \delta_D)\, Q_{D,t+1}$. In equilibrium, the rental cost $RC_{D,t}$ must be the net present value of this transaction

$$RC_{D,t} \;\equiv\; Q_{D,t} \;-\; (1 - \delta_D)\, \mathbf{E}_t \left\{ M_{t+1}\, Q_{D,t+1} \right\} \tag{6}$$

Secondly, a similar first-order condition holds for the human capital

$$RC_{H,t} \;=\; \frac{F'_H(H_t, D_t)\, u_S(C_t, S_t)}{u_C(C_t, S_t)} \tag{7}$$

where

$$RC_{H,t} \;\equiv\; Q_{H,t} \;-\; (1 - \delta_H)\, \mathbf{E}_t \left\{ M_{t+1}\, Q_{H,t+1} \right\} \tag{8}$$

Thirdly, the primary testable restrictions on asset prices are the set of Euler equations, which I derive using a simple variational argument. Suppose we decrease the nondurable consumption C_t by one unit $dC_t = 1$ by purchasing $1/P_{it}$ units of an asset i, where I denote P_{it} the price of the asset i. The change in the utility at time t is $U_{1t}\, u'_C(C_t, S_t)$. Next period, we collect the extra dividend and sell the asset. In total, we can raise the nondurable consumption dC_{t+1} by $R_{it+1} \;=\; (P_{it+1} + DIV_{it+1}) \,/\, P_{it}$. The change in the marginal utility next period is $\mathbf{E}_t \left[U_{2t}\, U_{1t+1}\, u'_C(C_{t+1}, S_{t+1})\, R_{it+1} \right]$. In equilibrium, we cannot raise the overall welfare of the consumer and so

$$U_{1t}\, u_C'(C_t, S_t) \;=\; \mathbf{E}_t \left[U_{2t}\, U_{1t+1}\, u_C'(C_{t+1}, S_{t+1})\, R_{it+1}\right] \tag{9}$$

Let us define the stochastic discount factor as

$$M_{t+1} \;=\; \frac{U_{2t}\, U_{1t+1}}{U_{1t}} \times \frac{u_C'(C_{t+1}, S_{t+1})}{u_C'(C_t, S_t)} \tag{10}$$

Then, we obtain the inter-temporal first-order condition, the Euler equation,

$$\mathbf{E}_t \left[M_{t+1}\, R_{t+1}\right] \;=\; 1 \tag{11}$$

where R_{t+1} is a (gross) return on a test asset.

Formal analysis in Appendix A follows Giovannini and Weil (1989), Epstein and Zin (1991), Yogo (2006) and Weil (1989). They need homogeneity of the household problem to solve explicitly for the inter-temporal marginal rate of substitution (IMRS). My preference specification is homothetic, intra-period function is homogenous of degree one, and the household production function is also homogenous of degree one (constant-returns-to-scale). As a result, the value function is homogenous of degree $1-\gamma$. Appendix A then shows that the stochastic discount factor equals the intertemporal marginal rate of substitution (IMRS)

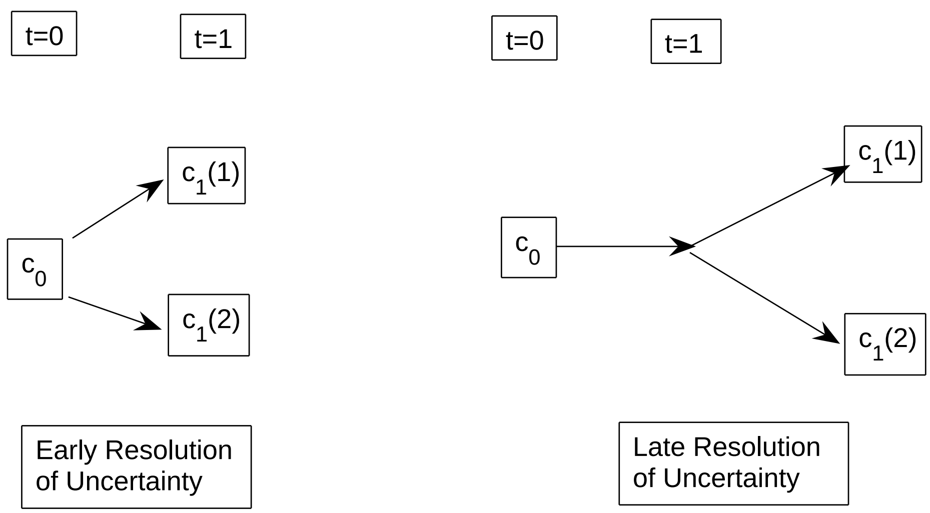
$$M_{t+1} \;=\; \left[\delta \left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma} \left(\frac{\psi(S_{t+1}/C_{t+1})}{\psi(S_t/C_t)}\right)^{1/\theta - 1/\sigma} R_{W,t+1}^{1-1/\kappa}\right]^{\kappa} \tag{12}$$

where the auxiliary function

$$\psi\left(\frac{S}{C}\right) \;=\; \left[(1-a)^{1-\frac{1}{\theta}} \,+\, a^{1-\frac{1}{\theta}} \left(\frac{S}{C}\right)^{1-1/\theta}\right]^{\theta/(\theta-1)} \tag{13}$$

The constant $\kappa \;=\; (1-\gamma)\,/\,(1-1/\sigma)$, and $R_{W,t+1}$ is the return on wealth from the optimal portfolio.

Figure 1:



1.3 Timing of Uncertainty and Expected Returns

1.3.1 Motivation

This section is an adaptation of the results in Chew and Epstein (1989). Let us consider a two-period economy where the investor has preferences described by the functional U. I denote $\delta[c, m]$ the measure which assigns all mass to (c,m). There are two risky assets, with the relevant payoffs portrayed in Figure 1.3.1. I assume that $c_1(1) > c_1(2)$. The first asset pays *after* a coin is flipped at time $t = 0$. In contrast, the second asset pays c_0 in t=0, *then* the coin is flipped and $t = 1$ payoff c_1 is determined.

Suppose the coin flipped is biased, with the probabilities of α and $1 - \alpha$. The utility from buying the first asset is[3]

$$U\left(\alpha\,\delta[c_0, c_1(1)] + (1-\alpha)\delta[c_0, c_1(1)]\right) \tag{14}$$

Let us define the probability β in such a way that the investor is indifferent between buying either asset, that is,

$$U\left(\alpha\,\delta[c_0, c_1(1)] + (1-\alpha)\delta[c_0, c_1(2)]\right) \;=\; U\left(\delta[c_0, \beta c_1(1) + (1-\beta)c_1(2)]\right) \tag{15}$$

The interpretation is that in equilibrium the investor must hold both assets. Chew and Epstein (1989, p. 109) prove that if the investor prefers early to late resolution of uncertainty, the probability β must be larger than α. In other words, the expected return on the second asset must be larger than the expected return on the first one

$$\mathbf{E}[R_1] \equiv \frac{\alpha c_1(1) + (1-\alpha)c_1(2)}{c_0} \;<\; \frac{\beta c_1(1) + (1-\beta)c_1(2)}{c_0} \equiv \mathbf{E}[R_2] \tag{16}$$

This simple example suggests that risk premia on assets are also driven by the way the uncertainty about their payoffs unfolds.

1.3.2 Preference for Early Resolution of Uncertainty: Epstein-Zin Preferences

To facilitate the exposition, let us focus on the case without consumer durables. The Epstein-Zin model, building upon the work of Kreps and Porteus (1978), distinguishes between the preference for early and late resolution of uncertainty. The implied SDF is a combination of the standard Lucas-Breeden SDF, $(C_{t+1}\,/\,C_t)^{-1/\sigma}$, and a new factor, the return on the wealth portfolio that captures the attitudes toward the timing of uncertainty. Therefore, if the preference for early resolution of uncertainty is dominant in the market, stocks are risky not only because they co-vary with the consumption growth but also because the uncertainty about their payoff resolves late.

[3]The notation purposefully follows Chew and Epstein (1989)

There are two factors that combine together to determine jointly these attitudes toward the timing of uncertainty, the inter-temporal substitutability σ and the coefficient of risk aversion γ. The restriction $\sigma \times \gamma = 1$ corresponds to the case when indifference to the timing of uncertainty is exhibited, and we subsequently obtain the standard Expected Utility framework. Figure 2 graphically presents how the restriction toward the timing of uncertainty (i.e. $\kappa \equiv (1-\gamma)/(1-1/\sigma)$ greater or less than one) separates the (σ, γ) space into four regions. We see that as long as the inter-temporal substitutability is sufficiently large (i.e. σ large enough than zero), we may obtain a preference for early resolution of uncertainty with quite a low coefficient of risk aversion. Empirically, it turns out that even a small preference for early resolution of uncertainty has a significant first-order effect upon expected returns. To document this fact empirically, Figure **??** plots the expected excess return on the value-weighted market portfolio as a function of σ and γ. As the right panel shows, the preference for early resolution of uncertainty is associated with a positive expected excess return. Furthermore, a small increase in the coefficient of the risk aversion leads to a large increase in expected returns. On the other hand, the left panel portrays the case where investors prefer late resolution of uncertainty to early one. Two facts emerge. Firstly, the expected return is negative in almost the whole region. Secondly, the expected return becomes positive only when the inter-temporal substitutability σ is so small that the consumption risk offsets the preference for late resolution of uncertainty, and this actually happens at quite a large coefficient of risk aversion, considered by many too extreme.

Unfortunately, Weil (1989) shows in a calibrated economy that considering the timing premium along with the nondurable consumption growth is not sufficient to account for the equity premium puzzle. I strongly confirm his findings empirically. The thesis of this paper is that the combination of the timing premium for early resolution of uncertainty *and* consumption complementarity across nondurable and durable goods is necessary.

1.4 The Role of Consumption Complementarity

Consumption complementarity plays several roles in the model. Firstly, it raises the consumption risk compared to the Epstein-Zin-Weil model. Secondly, it allows us to estimate a large magnitude of the elasticity of intertemporal substitution. Thirdly, it generates a time variation in average returns on assets.

In detail, the representative consumer preferences are convex and thus investors have a preference for consumption variety, that is, a particular mix of nondurable consumption C_t and the service flow S_t from the household capital D_t. However, as Yogo (2006) shows, nondurables and the service flow from consumer durables get out of whack in recessions. This raises the marginal utility of nondurable consumption and gives rise to a new risk factor. This effect is particularly pronounced when the substitutability θ between nondurables and durables is small.

Secondly, a large literature focuses on the estimation of the elasticity of intertemporal substitution σ. In a seminal paper, Hall (1988) presents estimates of the elasticity "... that are small. Most of them are also quite precise, supporting the strong conclusion that the elasticity is unlikely to be much above 0.1, and may well be zero." Using improved inference methods, Hansen and Singleton (1983) find that there is less precision and even obtain estimates that are negative. Using international data, Campbell (2003) and Yogo (2004) estimate the elasticity statistically and economically insignificant. However, these studies assume that the felicity function is separable over nondurables and durables. In response, Mankiw (1985) and Ogaki and Reinhart (1998) enrich the model by explicitly introducing the service flow from consumer durables. They argue that ignoring this consumption good induces a bias in favor of finding a low or even negative magnitude of intertemporal substitution. The real interest rate directly affects the user cost of consumer durables. For instance, a rise in the real interest rates leads to a higher user cost and consumers substitute toward nondurable goods. This channel is completely missing in the one good economy and hence the estimate of the intertemporal substitution is biased downward. Secondly, Mankiw (1985) finds that the service flow from consumer durables is itself more responsive to the interest rates. He estimates a large elasticity of substitution for durable goods. Focusing on non-separability across goods, Ogaki and Reinhart find that there is quite a large inter-temporal substitutability when both nondurable and durable goods are considered.

One important criticism of the Ogaki-Reinhart empirical results[4], and in fact, of all models that use durable goods [cf. Hansen and Eichenbaum (1990), Mankiw (1985), Ogaki and Reinhart (1998), Yogo (2006)] is that they assume that the household "produces" [Becker (1965, 1993), Ghez and Becker (1975), Lancaster (1966), Stigler and Becker (1977)] the service flow from consumer durables using time- and state- independent linear household production function. The thesis of this paper is that the appropriate modeling of the returns-to-scale in the household sector is absolutely crucial to impute correct service flow from consumer durables, and in turn, to obtain an unbiased estimates of the magnitude of intra-temporal and inter-temporal substitutions. Empirically, I find economically and statistically significant decreasing returns to scale in the consumer durables, *ceteris paribus* and therefore this critique is empirically relevant.

Thirdly, in contrast to the model with consumer durables, the one-good economy lacks a mechanism to generate time-variation in expected returns as the nondurables consumption growth rate is believed by economists to be i.i.d. [cf. Bansal and Yaron (2004) for a different model of consumption growth]. As may be seen in the stochastic discount factor, durable goods introduce a new state variable, the ratio of the service flow S_t over nondurables C_t, which varies over the business cycle [cf. Yogo (2006)] and naturally generates time-varying expected returns. Furthermore, the magnitude of this predictability is a function of the intra-temporal complementarity θ between service flow and nondurables, rising as θ diminishes. The intuition for this result is that service flow and nondurables get "out of whack" in the recession.

[4] While I was working on this draft I learned about two recent papers by Okubo Masakatsu (2004a, 2004b) which also enrich the Ogaki and Reinhart's (1998) model. However, the specification does not allow for an easy interpretation of the preference parameters.

2 Empirical Section

Quarterly consumption data is from the U.S. *National Income and Product Accounts (NIPA).* Following convention, I classify as nondurables the NIPA nondurable consumption and services. These are seasonally adjusted at annual rates (SAAR) and I correct this by dividing the data by four. The quarterly data on durables stock were kindly provided to me by Motohiro Yogo. The advantage of his data is that they precisely match the BEA annual estimates and correctly imbed the variation in the depreciation rate δ_D. Of course, all consumption data have been converted to per-capita basis by dividing by the population size at the end of the quarter. The relative price of durables, denoted $Q_{D,t}$ hereafter, is computed as the ratio of the price index for PCE on durable goods to the price index for PCE on nondurable goods and services. Although quarterly consumption data is available since 1947, the period immediately after the war is associated with unusually high durable consumption growth due to the rapid restocking of durable goods. As a result, I follow Ogaki and Reinhart (1998) and Yogo (2006), and I use quarterly data 1951.I - 2001.IV. Figure 4 portrays the time series of the ratio of the stock of consumer durables over the flow of nondurable consumption and the relative price $Q_{D,t}$. The data on Baa and Aaa bond yields, and the number of civilians unemployed less than 5 weeks, were obtained from the St. Louis Fed. The risk-free rate is the three-month Treasury bill rate, converted to ex-post real returns by the implicit price deflator for the total consumption, and the market return is the return on the value-weighted portfolio of NYSE, AMEX and NASDAQ firms, both obtained from Professor Ken French's web site. The price-dividend ratio for the aggregate market is obtained from Professor Shiller's web site.

2.1 Estimation the Elasticity of Intra-Temporal Substitution and Expenditure Elasticities: Co-Integration Approach

2.1.1 Methodology

In order to empirically implement the model I need to specify the functional form of the household production function $F(H_t, D_t)$. Specifically,

$$F(H_t, D_t) = B\, H^{1-\eta}\, D^{\eta} \tag{17}$$

where $B \in \mathbb{R}^+$. Furthermore, the human capital is unobservable. Therefore, I normalize $B\, H^{1-\eta} \equiv 1$. I test the null hypothesis that the series $c_t = \log(C_t)$, $d_t = \log(D_t)$ and $q_t = \log(Q_{D,t})$ are difference stationary against the alternative of trend stationarity. Using Phillips-Perron test and including a constant and a linear time trend, I cannot reject the hypothesis that the data are difference stationary. Stationary bootstrap test of Parker, Paparoditis and Politis (2005) agrees with this conclusion. Table 1 summarizes the results. Therefore, the marginal rate of substitution M_{t+1} and the ratio $Q_{D,t+1} / Q_{D,t}$ are stationary, and hence the conditional expectation $E_t \left\{ M_{t+1} \frac{Q_{D,t+1}}{Q_{D,t}} \right\}$ is stationary as well.

Divide the intra-temporal first-order condition by the durables price Q_t

$$\frac{F'_D(D_t)\, u_S(C_t, S_t)}{Q_{D,t}\, u_C(C_t, S_t)} = 1 - (1 - \delta_D)\, E_t \left\{ M_{t+1} \frac{Q_{D\, t+1}}{Q_{D,t}} \right\} \tag{18}$$

I now use Cooley and Ogaki's (1996) co-integration Euler equation approach. First, notice that the left hand side of the intra-temporal first-order condition is stationary[5]. Second, substitute the functional forms of the felicity function $u(C_t, S_t)$ and the household production $S_t = D_t^{\eta}$ to get $\eta \frac{a^{1-\frac{1}{\theta}}}{(1-a)^{1-\frac{1}{\theta}}} \frac{D^{\eta-1}\, D^{-\eta/\theta}}{Q_{D,t}\, C_t^{-1/\theta}}$. Third, take logs on both sides and multiply by θ to obtain

$$c_t = intercept + \theta\, q_t + (\theta + \eta - \eta\, \theta)\, d_t + \varepsilon_t \tag{19}$$

[5]The stationarity implies that the durables price $Q_{D,t}$ and the rental cost $RC_{D,t}$ are co-integrated with the co-integrating vector $[1, -1]$.

If the variables c_t, d_t and q_t are co-integrated, we may run the regression (in levels)

$$c_t = \delta_0 + \delta_1 q_t + \delta_2 d_t + \varepsilon_t \tag{20}$$

and estimate the preference parameter θ and the household production parameter η super-consistently as $\hat{\theta} = \hat{\delta}_1$ and $\hat{\eta} = \left(\hat{\delta}_2 - \hat{\delta}_1\right) / \left(1 - \hat{\delta}_1\right)$.

Omitting significantly decreasing returns to scale in the household capital, ceteris paribus, may bias both estimates of the magnitude of inter-temporal and intra-temporal substitutions. Using the same preference specification but assuming linear household production function, Ogaki and Reinhart (1998) cannot reject the null hypothesis $H_0 : \theta > 1$. As argued before, imposing counter-factually constant returns to scale $\eta = 1$ biases upward the estimate of the elasticity of substitution. This observation is strongly confirmed empirically hereafter.

I test for co-integration using likelihood ratio test[6] [Johansen (1988, 1991]. Firstly, the likelihood ratio test of the null hypothesis of no cointegration versus the alternative of three cointegrating vectors is $LR = 30.899$, which is greater than the 5% critical value of 21.279. Secondly, the likelihood ratio test of the null hypothesis of no cointegration versus the alternative of one cointegrating vectors is $LR = 24.587$, with the 5% critical value of 21.279. Thirdly, the likelihood ratio test of the null hypothesis of one cointegrating vector versus the alternative of three cointegrating vectors is $LR = 6.313$, less than the 5% critical value of 14.595. Fourthly, the likelihood ratio test of the null hypothesis of one cointegrating vector versus the alternative of two cointegrating vectors is $LR = 6.308$, less than the the 5% critical value. I conclude that there exists a unique cointegrating vector.

Stock and Watson (1993) and Wooldridge (1991) suggest to augment the regression (20) with leads and lags of the right hand side variables to correct for the correlation between the inno-

[6] I use 2th order VAR for likelihood ratio test and AR(2) for the co-integrating residual to create confidence intervals and t-stats. The results seem robust to higher lags.

vations in d_t and q_t and the cointegrating residual ε_t. This is important for the construction of confidence intervals and hypothesis testing. I therefore estimate the preference parameters θ and η by running the dynamic least squares regression[7]

$$c_t = \delta_0 + \delta_1 q_t + \sum_{s=-p}^{p} \delta_{1,s} \Delta q_{t-s} + \delta_2 d_t + \sum_{s=-p}^{p} \delta_{2,s} \Delta d_{t-s} + \varepsilon_t \tag{21}$$

2.1.2 Interpretation of the Empirical Results

Table 2, row 2, reports the point estimates of the elasticity of substitution $\hat{\theta} = 0.0084$ with $s.e.(\hat{\theta}) = 0.1339$. The estimate has the correct sign but although it is super-consistent, the asymptotic confidence interval is quite large[8] [0, 0.268]. Still, compared to previous studies, the economic magnitude is significantly smaller. I attribute the difference to the careful modeling of the returns to scale in the household sector. I formally test the hypothesis of zero substitutability between nondurables and services flow $H_0 : \theta = 0$. The t-statistics $t = 0.063$ and thus I am unable to statistically reject the hypothesis that the consumption index $\Omega(C_t, S_t)$ is Leontief at 5% significance level. However, extremely small intratemporal substitution (i.e. $\theta \simeq 0$) has the unfortunate implication that it gives rise to a volatile implied rental cost of capital, which would be counterfactual. I therefore conclude that the economically plausible magnitude of intra-temporal substitution must be roughly around 0.1 to 0.3, the right tail of the asymptotic CI. Hereafter, I show that $\theta = 0.249$, within the asymptotic CI but smaller than estimates in the related literature, generates about the right volatility of the durables price. Furthermore, intratemporal substitutability indirectly affects the estimates of the intertemporal substitutability [cf. Pakoš (2006)]. As real interest rates rise, the rental cost of consumer durables rises and consumers substitute from durables to nondurables compared to one-good economy. Therefore, we expect higher intertemporal substitutability *ex ante*. This hunch is confirmed in the data in the subsequent section.

[7] The number of leads/lags is $p = 4$.

[8] I do not report the economically meaningless negative magnitudes of the elasticity of intratemporal substitution θ.

Table 2, row 3, reports the point estimates of the returns to scale η in the household sector, *ceteris paribus*. The super-consistent estimate is $\hat{\eta} = 0.566$ with $s.e.(\hat{\eta}) = 0.065$. The estimate is statistically and economically significant and has the correct sign. It suggests that the marginal product of consumer durables declines as consumers augment their stock, *ceteris paribus*. Formally, I test the linear household production function specification $H_0 : \eta = 1$; the t-statistics $t = -5.91$ and I thus reject the null hypothesis at conventional significance levels.

In a related paper, Masakatsu Okubo (2004) introduces non-homothetic addilog-type utility function to Ogaki-Reinhart (1998) model and imputes the service flow from consumer durables using state- and time- independent household production function. He uses both NIPA and Gordon's (1990) data, sample periods 1947.I to 1983.IV and 1951.I to 1983.IV. Unfortunately, it is not possible to identify simultaneously the non-homotheticity measure and the curvature of the household production function, which is one reason I work with homothetic preferences[9]. His parameter $1 / \alpha$ corresponds in my model to the elasticity of intra-temporal substitution θ. For comparison, he estimates in Table 2 the parameter θ between 0.28, with standard error[10] about 0.06 (Gordon's data) to 0.44, with standard error 0.14 (NIPA data). His results may be attributed to different data (Gordon's data) and/or different sample period, but his and my asymptotic CI_θ overlap. Furthermore, his ratio of the parameters γ / α correspond in my model to the curvature of the household production function η. He estimates in Table 2 the parameter η from 0.41 (Gordon's data) to 0.54 (NIPA data), with the standard errors from 0.03 to 0.06. Therefore, my estimates of the household production function curvature η are remarkably close to his estimates, and certainly within the asymptotic confidence interval for the NIPA data.

[9] Another reason is that I need homogeneity of the consumer's problem to solve for the Epstein-Zin-type stochastic discount factor.

[10] I have to assume that the Okubo's estimates $1/\gamma$ and α/γ are uncorrelated.

2.1.3 Robustness Check: Bootstrapping the Co-Integrating Regression

It is well-known that the co-integrating vector $(1, -\theta, -\eta)$ is super-consistent but biased. This is important especially in small samples. I therefore apply the sieve bootstrap to the co-integrating regression [Chang, Park and Song (2005)] and construct percentile confidence intervals. The empirical distributions based on 40,000 Monte Carlo simulations are displayed in Figure 2. The mean of the distribution for the elasticity of intratemporal substitution θ is -0.008. It has the wrong sign but the 5% symmetric percentile confidence interval is quite wide, $CI_\theta = [-0.262, 0.249]$. As with the asymptotic Wald test, the null hypothesis of Leontief preferences $H_0 : \theta = 0$ still cannot be rejected at 5% level, but the qualification regarding extremely small estimates (i.e. $\theta \simeq 0$) with respect to the volatility of durables price applies. In my subsequent work, I choose[11] $\theta = 0.249$.

Furthermore, the mean of the distribution for η is 0.563, slightly lower than the point estimate in Table 2. The 5% symmetric percentile confidence interval is $CI_\eta = [0.356, 0.732]$. As before, we again are unable to accept the null hypothesis of the constancy of the marginal product of consumer durables in the household sector (i.e. the AK model).

2.2 Estimation of the Rest of the Parameters: Euler Equations Approach

2.2.1 Methodology

A naive approach to estimating the preference parameter vector $p = (\theta, \eta, \sigma, \beta, \gamma, a)$ is to totally disregard any co-integrating relationships and use one grand GMM. Why is this wrong? Firstly, there is a huge advantage to using the information in trends of the consumption and price variables as much as possible [cf. Ogaki (1988, 1992), Cooley and Ogaki (1996), Ogaki and Reinhart (1998)]. The estimates are super-consistent and therefore by definition have *smaller* standard errors than those obtained from GMM. In addition, and even more impor-

[11] Magnitudes of θ between 0.2 and 0.3 do not affect the results significantly.

tantly, the method is *robust* to possible adjustment costs[12]. As Ogaki and Reinhart put it, "... it is robust to various specifications of adjustment costs, relying on the co-integration properties between the observed and the desired stock of durables in the presence of adjustment costs, which is discussed in Caballero (1993)." In other words, the inference based on the co-integrating regressions yields consistent estimates as long as adjustment costs do not affect the long-run behavior of durable good consumption. Furthermore, it can be shown that the Euler equation for nondurable consumption is robust to various forms of adjustment costs for durable good consumption. Unfortunately, many exciting studies such as Dunn and Singleton (1986) or Hansen and Eichenbaum (1990), among others, do not allow implicitly nor explicitly in their estimation for adjustment costs. Piazzesi, Schneider and Tuzel (2006) and Yogo (2006) are one of the few recent and related exceptions.

Secondly, Stock and Wright (2000), Stock, Wright and Yogo (2004) and Yogo (2004) address the important issue of weak identification meaning that the instruments are only weakly correlated with the relevant first-order condition so that the parameters are poorly identified. In this respect, what is the point of running a grand GMM when the S sets [see Stock and Wright (2000) for definition] for the preference vector p covers the bulk of the parameter space? Compared to this scenario, co-integration gives us relatively tight confidence intervals.

In light of this, I follow Cooley and Ogaki (1996), Hansen and Singleton (1982), Ogaki (1988, 1992), Ogaki and Reinhart (1998), Stock and Wright (2000), Yogo (2006) and Zhang (2006). I use the output of the co-integrating regression to find such a value of the elasticity of intra-temporal substitution that seem to match the volatility of the durables price. Secondly, due to weak identification[13] of the preference parameter a, I follow Ogaki and Reinhart again. I empirically construct the rental cost of capital $RC_{D,t}$ and invert the intra-temporal first-order condition to solve for a. I then calibrate the marginal rate of substitution IMRS with these

[12] It is well-known that adjustment costs play a crucial role in the consumption of durable goods [cf. Bernanke (1984), Lam (1989), Eberly (1994)].

[13] I actually was estimating the preference weight a. The S-set turned out to be (0,1). The parameter was not identified at all!

three parameters, θ, η and a, and estimate the rest of preference parameters σ, β and γ using continuous-updating GMM of Hansen, Heaton and Yaron (1996).

In detail, the primary testable asset pricing implications of the model are the set of Euler equations

$$E_t \left[M_{t+1} \, R_{i,t+1} \right] \; = \; 1 \tag{22}$$

where M_{t+1} is the marginal rate of substitution, given in eq. (12), and $R_{i,t+1}$ is the gross return on an asset i. Let z_t be a vector of variables in the information set I_t. Using the components of z_t as instruments, I form the function

$$g_T(p) \; = \; \frac{1}{T} \sum_{t=1}^{T} M_{t+1} \left(p \right) \; R_{i,t+1} \, \otimes \, z_t \; - \; z_t$$

The vector $g_T(p)$ is a consistent estimator of $E[M_{t+1} \, R_{i,t+1} \otimes z_t - z_t]$. I calibrate the parameters θ, η and a based on the intratemproal first-order condition. I estimate the rest of the preference parameter vector $p \; = \; (\sigma, \, \beta, \, \gamma)$ by the choice of p in the admissible parameter space that makes $g_T(p)$ close to zero in the sense of minimizing the quadratic form

$$\min_p \; g'_T \left(p \right) \; S_T^{-1} \, g_T \left(p \right)$$

where S_T^{-1} is the consistent estimate of the spectral density matrix at frequency zero [Hansen, Heaton and Yaron (1996)].

My set of instruments contains the well-known predictors of consumption and asset returns. Firstly, as predictors of consumption, I use nondurables and durable stock growth rates in addition to the number of civilians unemployed less than 5 weeks. Secondly, as predictors of asset returns, in particular the value-weighted market return, I use (i) the price-dividend ratio [Campbell and Shiller (1988) and Fama and French (1988)], (ii) small-minus-big SMB and value-minus-growth HML spreads [Cohen Polk, Vuolteenaho (2003)], and (iii) default (Baa yield - Aaa yield) and term (Aaa yield - three month Treasury Bill rate) premiums [Fama and French (1989)]. All instrumental variables are lagged twice to take care of the time-aggregation [Hall (1988)]. Furthermore, Ogaki (1988) shows that the additional lag is consistent with the information structure of a monetary economy with cash-in-advance constraints.

2.2.2 Selection of the Elasticity of Intra-Temporal Substitution Based on the Sieve Bootstrap Confidence Interval

Although the co-integration restriction gives us a super-consistent estimate of the elasticity of intra-temporal substitution θ, both its asymptotic and sieve bootstrap confidence intervals are quite large[14], [0, 0.268] and [0, 0.249], respectively[15]. This raises a question as to which magnitudes should we use in the second-step, the Euler equations approach. I address this issue as follows. I choose such a magnitude of θ, based on the sieve bootstrap CI interval, that best matches the volatility of the empirical and the imputed relative prices of durables. In sample period 1951.I-2001.IV, the quarterly volatility of the (log) growth rate of the price of durables is about 0.7%. The model-implied volatility of the durables price is monotonically decreasing function of the elasticity of substitution θ, which measures the concavity of the indifference curves between nondurables and service flow. As we decrease θ, we raise the concavity of the indifference curves and not surprisingly, generate volatile durables price. This effect is most pronounced for $\theta \simeq 0$, the case of Leontief preferences.

Based on this microeconomic intuition, I choose $\theta = 0.249$, the right end of the sieve CI [0, 0.249]. This choice is also consistent with the implied estimates of Okubo (2004). The model implied quarterly volatility of the durables price is about 2.3%, 1.6% higher than in the data[16]. The difference may be attributed to adjustment costs. It is well-known that adjustment costs play a crucial role in the consumption of durable goods [cf. Bernanke (1984), Lam (1989), Eberly (1994)]. I conjecture that a proper modeling of realistically large adjustment

[14] I do not report the part of CI that yields the economically meaningless negative magnitudes.
[15] Though still smaller than the asymptotic GMM confidence interval.
[16] This is an excellent fit as no model matches all sample moments exactly to the last decimal point.

costs would bring these two numbers even closer. In conclusion, with the parametrization $\theta = 0.249$ and $\eta = 0.566$, the model does not generate durables price puzzle, that is to say, extremely wild price fluctuations.

2.2.3 Estimating the Preference Weight

In this section I closely follow Ogaki and Reinhart (1998). The rental cost of service flow from consumer durables is defined by equation (8). I make the simplifying assumption[17] and drop the covariance terms to obtain

$$E_t\left[M_{t+1}\, Q_{D,\,t+1}\right] \;=\; E_t\left[M_{t+1}\right]\, E_t\left[Q_{D,\,t+1}\right] \tag{23}$$

Clearly, $E_t\left[M_{t+1}\right]$ equals one over the gross real interest rate $1\,/\,R_t^f$. I use 4th-order autoregressive process[18] AR(4) to obtain the expected value $E_t\left[Q_{D,\,t+1}\right]$. I then estimate the rental cost $RC_{D,\,t}$ as

$$\widehat{RC}_{D,\,t} \;=\; Q_t \;-\; 0.94\, E_t\left[Q_{D,\,t+1}\right]\,/\,R_t^f \tag{24}$$

In the co-integration step, we cannot obtain a consistent estimate for the preference parameter a. However, we may estimate[19] it by using the constructed rental cost of capital $\widehat{RC}_t$ and invoking the intratemporal condition

$$\frac{\eta\, a^{1-\frac{1}{\theta}}}{(1-a)^{1-\frac{1}{\theta}}} \times \frac{D^{\eta-1-\eta/\theta}}{C_t^{-1/\theta}} \;=\; RC_{D,\,t} \tag{25}$$

Specifically, I solve for the preference weight a by taking the sample average

$$\frac{a}{1-a} \;=\; \left(\frac{1}{\eta}\,\mathbf{E}_T\left[\exp\left\{\widehat{rc}_t \;-\; \frac{1}{\theta}\,c_t \;+\; \left(1-\eta+\frac{\eta}{\theta}\right)\,d_t\right\}\right]\right)^{\frac{\theta}{\theta-1}} \tag{26}$$

[17] This step differs from Ogaki and Reinhart (1998) equation (7) because I think that the approximation is better.

[18] In fact, Bayesian Information criterion (BIC) recommends 4 lags.

[19] Careful reader may naturally inquire why not estimate the preference weight using GMM with the rest of the parameter vector. However, the parameter a is weakly identified. Estimating a using GMM delivers the S-set for a equal to (0,1)!

Small letters are in logs. I obtain the plausible magnitude for the preference weight of the service flow $a = 0.671$. This suggests that durables indeed play a crucial role in the model.

2.2.4 GMM Results: Potential Resolution of Asset Pricing Puzzles

Bansal and Yaron (2004), Epstein and Zin (1989, 1991) and Weil (1989), among others, consider the model with nondurable consumption and Epstein-Zin recursive preferences, hereafter referred to Epstein-Zin CCAPM. To be easily comparable with this important literature, I also furnish estimates for this model. Specifically, Table 3 estimates the Epstein-Zin CCAPM when the test assets are the value-weighted market return, the risk-free rate, the small-minus-big SMB and value-minus-growth portfolios HML. Row 1 presents the estimates for the Epstein-Zin CCAPM without consumer durables. The subjective discount factor is estimated a bit larger than one but this may still be consistent with equilibrium if there is a growth in the economy [Kocherlakota (1990)]. However, the estimate of the inter-temporal substitutability σ is extremely small, 0.02, and the coefficient of risk aversion γ is a large 185. As the figure 2 shows, the representative investor has actually a preference for late resolution of uncertainty[20], which has a tendency to decreases the premia on risk assets. The only way to fit the average returns on the test assets is to have huge consumption risk of the stock market, that is, small inter-temporal substitutability σ. Overall, the $J_T(1)$ doesn't reject the model statistically. However, the 95% confidence S-set [Stock and Wright (2000)], obtained by concentrating out the parameters σ and β suggests that we need a risk aversion over 94 to explain the average returns. This is a manifestation of the equity premium puzzle and it is consistent with the findings in Weil (1989).

Table 3, Row 2, presents the estimated preference parameters for the Epstein-Zin CCAPM with consumer durables. As in row 1, the subjective discount factor is estimated a bit larger than 1 [as before cf. Kocherlakota (1990)]. The inter-temporal substitutability σ is estimated to be 1.002 and quite precisely. This suggests that the consumption risk of the stock-market is

[20] The parameter $\kappa = (1 - 185)/(1 - 1/0.02) = 3.8 > 1$.

not extreme, but we still need the preference for early resolution of uncertainty to fit the average returns. Indeed, the restrictions (i) $\sigma \times \gamma > 1$ and (ii) $\sigma > 1$ hold , and thus investors do have preference for early resolution of uncertainty. The coefficient of risk aversion is estimated about 1.6 and the 95% confidence S-set is quite small $(0.010, 0.82) \cup (1.52, 11.04)$. The model is not statistically rejected at conventional significance levels. I conclude that the Epstein-Zin CCAPM with consumer durables can account for the average returns on the value-weighted market portfolio, and match the risk-free rate, and the SMB and HML return spreads, with plausible risk aversion.

It is well-known that expected returns are time-varying [Campbell and Shiller (1987), Fama and French (1988), among others]. Therefore, it is not sufficient to explain the magnitude of asset returns (including the risk-free rate). A good model should also capture the variation in expected returns of benchmark assets over time. As a result, and despite the high coefficient of risk aversion for Epstein-Zin CCAPM without consumer durables [Table 3, Row 1], I re-estimate this model using 4 sets of instruments. The test assets are the value-weighted market return and the risk-free rate. Table 4, rows 1-4, present the estimates for each set of instrumental variables. Column 5 reports the estimated coefficient of risk aversion together with the asymptotic CI. Based on asymptotics, the models are rejected at conventional significance levels for all instrument sets. Furthermore, it is worrying to see how the estimate of the coefficient of risk aversion changes across different assets, that is, from the previous table, and across instrumental variables (across rows 1-4). This suggests that the coefficient of risk aversion is weakly identified [Stock and Wright (2000)] and blind reliance on asymptotic confidence intervals may be misleading. In fact, as Column 7 betrays, the 95% S-set for γ is large. It seems that we need γ at least 14 (row 2). Moreover, because these S-sets are not empty, we actually do not reject the model[21]. Figure 6 displays the implied SDF calibrated based on row 2. It is clear that although we statistically cannot reject the model, the stochastic discount

[21] As Stock and Wright (2000), p. 1064 explain "The S-sets consist of parameter values at which one fails to reject the joint hypothesis that [the estimate equals the true parameter] and the over-identifying conditions are valid."

factor is meaningless for γ as low as 14.87, the low bound for the S-set in row 2, column 7. I conclude that the Epstein-Zin CCAPM with no durables explains neither the magnitude nor the time variation in the equity premium and the risk-free rate.

Table 5 presents the estimates for Epstein-Zin CCAPM with consumer durables for three increasing sets of test assets, namely, (i) the value-weighted market return and the risk-free rate (Panel A), (ii) the value-weighted market return, the risk-free rate and the small-minus-big portfolio SMB (Panel B), and lastly (iii) the value-weighted market return, the risk-free rate, the small-minus-big portfolio SMB and the value-minus-growth portfolio HML (Panel C). In all cases, the inter-temporal substitutability σ is estimated large, above one, although quite imprecisely. Recall that the constraint $\sigma > 1$ allows us to have quite a small coefficient of risk aversion γ and still obtain a preference for early resolution of uncertainty. Indeed, across all three panels, the point estimates of γ range from slightly above 1 to nearly 4. Furthermore, except the Panel C results, the 95% confidence S-sets [Stock and Wright (2000)] tend to be disconnected and include the plausible magnitudes of risk aversion between one to five. All models in rows 1-4 in Panels A and B are not statistically rejected using both the asymptotic J_T statistics or the S-sets. Panel C seems to present a challenge for the model in terms of capturing the time variation in the value-minus-growth portfolio HML. The point estimates of the risk aversion γ tend to be larger and estimated less precisely. The 95% confidence S-sets also have higher lower bounds (i.e. 4 in row 2 and even 8 in row 4). However, although the asymptotic J_T statistics rejects the model, the S-sets are non-empty and thus we do *not* reject the model (see the previous footnote and Stock and Wright (2000)). Finally, a careful reader may object that many of the S-sets contain also the highly implausible magnitudes of risk aversion. To address this criticism, I calibrate IMRS with estimates in Table 5, Panel A, Row 4, and choose[22] $\gamma \in \{5, 50\}$. Figure 7 shows that although we cannot statistically reject the hypothesis that γ may be large, 50, even 250, the implied marginal rate of substitution is

[22] In fact, any number above 5 would do.

highly implausible and contains several dominat spikes[23]. I therefore conclude that the S-sets are somewhat misleading in that they are unable to discriminate between a plausible IMRS and highly implausible one. I conclude that S-sets do not provide convincing evidence that risk aversion above 5, perhaps 10, is consistent with the over-identifying restrictions.

3 Conclusion

This paper empirically investigates the simultaneous role of intra-temporal consumption complementarity and the attitudes toward the timing of uncertainty on risk premia of important benchmark portfolios. It asks two specific questions. Firstly, can we account for (i) the average returns on the value-weighted market return, the three-month Treasury Bill Rate, the small-minus-big SMB portfolio and the value-minus-growth HML portfolio, and for (ii) temporal variation in expected returns on these benchmark assets over the business cycle, all with a low risk aversion? Secondly, what role do the intra-temporal consumption complementarity and the preference for early resolution of uncertainty play in this exercise?

To this end, I construct a complete markets frictionless model. The investors are endowed with the non-expected utility preferences of Kreps and Porteus (1978) and Epstein and Zin (1989, 1991). The model features a two-sector economy, the market sector and the household sector. In the market sector, investors purchase goods to consume and to augment their stock of household capital - durable goods. In the household sector, they "produce" the service flow of this household capital [Becker (1965, 1993), Lancaster (1966), Stigler and Becker (1977)]. The investors' felicity function is defined over nondurable consumption and the service flow from these consumer durables.

The main findings have implications for both economics and finance. With respect to finance, the intra-temporal consumption complementarity and the preference for early resolution of

[23] Although Yogo (2006) estimated $\gamma \simeq 200$, his estimates of $\sigma \simeq 0.03$ and therefore his IMRS does not exhibit the spikiness.

uncertainty are significant factors that combine to account for the average returns on the benchmark assets with a *low* coefficient of risk aversion. Firstly, the consumption risk of the stock market is dual. It consists of the nondurable consumption growth but also of the vicissitude of the consumption basket variety, where the latter effect is directly contingent on the elasticity of intra-temporal substitution. Intuitively, the tighter the complementarity, the costlier it is for investors if their consumption basket gets "out of whack". Secondly, stocks are risky not only because they co-vary with consumption but also because the *uncertainty* about their payoff resolves late. Furthermore, changes in the variety of the consumption happen over the business cycle and thus the model gives occasion to time-varying expected returns, again with low risk aversion.

With respect to the implications for economics, I find that there are significantly diminishing returns to scale, both economically and statistically, in the household capital, ceteris paribus. This allows me to get an unbiased estimate of the unobservable service flow from consumer durables. In contrast to the previous literature, I estimate much higher intra-temporal complementarity between the service flow and nondurables. Furthermore, Mankiw (1986) and Ogaki and Reinhart (1998) explain that introduction of durable goods has a potential to raise the otherwise small estimate of the inter-temporal substitutability. Using the unbiased imputation of the service flow, I find quite a strong evidence for economically and statistically significant magnitude of the elasticity of the inter-temporal substitution.

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A Derivation of First-Order Conditions

The representative investor's budget constraint takes the form

$$\sum_{i=1}^{K} X_{i,\,t} \;=\; A_t \;-\; C_t \;-Q_{D,\,t}\,I_{D,\,t} \;-\; Q_{H,\,t}\,I_{H,\,t} \tag{27}$$

where A_t denotes the financial wealth. The laws of motion for the household capital D_t (i.e. consumer durables) and the human capital H_t are

$$D_t \;=\; (1-\delta_D)\,D_{t-1} \;+\; I_{D,t} \tag{28}$$

$$H_t \;=\; (1-\delta_H)\,H_{t-1} \;+\; I_{H,t} \tag{29}$$

Following Bansal, Tallarini and Yaron (2004), Cuoco and Liu (2000) and Yogo (2006), I simplify the household's problem. Let us define

$$X_{K+1,\,t} \;=\; Q_{D,\,t}\,D_t \tag{30}$$

$$X_{K+2,\,t} \;=\; Q_{H,\,t}\,H_t \tag{31}$$

Define the full wealth including the household capital and the human capital as

$$W_t \;=\; A_t \;+\; (1-\delta_D)\,Q_{D,\,t}\,D_{t-1} \;+\; (1-\delta_H)\,Q_{H,\,t}\,H_{t-1} \tag{32}$$

The budget constraint may be re-written as

$$\sum_{i=1}^{K+2} X_{i,\,t} \;=\; W_t \;-\; C_t \tag{33}$$

The total wealth in the next period is

$$W_{t+1} \;=\; \sum_{i=1}^{K} X_{i,\,t}\,R_{i,\,t+1} \;+\; (1-\delta_D)\,Q_{D,\,t+1}\,D_t \;+\; (1-\delta_H)\,Q_{H,\,t+1}\,H_t \tag{34}$$

Treating human capital and durables as assets yields

$$W_{t+1} \;=\; \sum_{i=1}^{K+2} X_{i,\,t}\, R_{i,\,t+1} \tag{35}$$

where the "return" on household and human capital is

$$\begin{aligned} R_{K+1} &= (1-\delta_D)\,\frac{Q_{D,\,t+1}}{Q_{D,\,t}} && (36)\\ R_{K+2} &= (1-\delta_H)\,\frac{Q_{H,\,t+1}}{Q_{H,\,t}} && (37) \end{aligned}$$

Define the portfolio share ω_{it} as

$$\omega_{it} \;=\; X_{it}\,/\,(W_t \;-\; C_t) \tag{38}$$

The obvious restriction is that

$$\sum_{i=1}^{K+2} \omega_{it} \;=\; 1 \tag{39}$$

This allows us to re-write the original budget constraint as

$$W_{t+1} \;=\; (W_t \;-\; C_t) \sum_{i=1}^{K+2} \omega_{i,\,t}\, R_{i,\,t+1} \tag{40}$$

Therefore, the original constraints are summarized by the equations (32), (39) and (40). Furthermore, we can "solve" for D_t and H_t as follows

$$\begin{aligned} D_t &= \frac{\omega_{K+1,\,t}(W_t \;-\; C_t)}{Q_{D,\,t}} && (41)\\ H_t &= \frac{\omega_{K+2,\,t}(W_t \;-\; C_t)}{Q_{H,\,t}} && (42) \end{aligned}$$

The household production $S_t \;=\; F\,(H_t,\, D_t)$ is CRS and thus may expressed as

$$S_t \;=\; F\left[\frac{\omega_{K+2,\,t}}{Q_{H,\,t}},\, \frac{\omega_{K+1,\,t}}{Q_{D,\,t}}\right]\,(W_t \;-\; C_t) \tag{43}$$

The problem of the representative consumer is to choose a stream of nondurable consumption C_t and the portfolio shares $\omega_{i,\,t}$ to maximizes his lifetime utility. This problem can be easily re-cast as a dynamic programming as follows

$$V_t(W_t) \;=\; \max_{C_t,\,\{\omega_{i,\,t}\}_{i=1}^{K+2}} \; U\left[u(C_t, S_t),\, \mathbf{E}_t\, V_{t+1}(W_{t+1})\right] \tag{44}$$

subject to (32), (39) and (40) and (43) - with the Koopman's aggregator function given in (**??**).

It is easy to verify that the value function is homogenous of degree $1-\gamma$ in wealth, and may be written as

$$V_t(W_t) \;=\; \Phi_t\, W_t^{1-\gamma} \tag{45}$$

Following Epstein and Zin (1989, 1991), Giovannini and Jorian (1989), Giovannini and Weil (1989), Yogo (2006), Weil (1989, 1990), it may be shown that

$$\Phi_t \;=\; \left[(1-\delta)\,(1-a)^{1-\frac{1}{\theta}}\,\widehat{\psi}\left(\frac{C_t^*}{W_t},\,\omega^*_{K+1,\,t},\,\omega^*_{K+2,\,t}\right)\right]^{(1-\gamma)/(1-1/\sigma)}\;\left(\frac{C_t^*}{W_t}\right)^{(1-\gamma)/(1-\sigma)} \tag{46}$$

where the function $\widehat{\psi}$ is defined as follows

$$\widehat{\psi} \;=\; \left\{(1-a)^{1-\frac{1}{\theta}} \;+\; a^{1-\frac{1}{\theta}}\,\left(F\left[\frac{\omega_{K+2,\,t}}{Q_{H,\,t}},\,\frac{\omega_{K+1,\,t}}{Q_{D,\,t}}\right]\left(\frac{W_t}{C_t}-1\right)\right)^{1-\frac{1}{\theta}}\right\}^{\frac{\theta}{\theta-1}} \tag{47}$$

Let us denote $R^*_{W,t+1} \;=\; \sum_{i=1}^{K+2}\,\omega^*_{i,\,t}\,R_{i,\,t+1}$ the gross return on the optimal portfolio and define the stochastic discount factor

$$M^*_{t+1} \;=\; \left[\delta\,\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-1/\sigma}\;\left(\frac{\psi(S^*_{t+1}/C^*_{t+1})}{\psi(S^*_t/C^*_t)}\right)^{1/\theta-1/\sigma}\;R^{*1-1/\kappa}_{W,t+1}\right]^{\kappa} \tag{48}$$

After a bit of unpleasant algebra, the first-order condition with respect to C_t is

$$\mathbf{E}_t\left[M^*_{t+1}\,R^*_{W,\,t+1}\right] \;=\; \left(1 \;-\; F\left[\frac{\omega_{K+2,\,t}}{Q_{H,\,t}},\,\frac{\omega_{K+1,\,t}}{Q_{D,\,t}}\right]\,\frac{u_{S,\,t}}{u_{C,\,t}}\right) \tag{49}$$

The first-order condition with respect to $\omega_{i,t},\ i = 1, ..., K$ is

$$\mathbf{E}_t \left[M^*_{t+1} \; (R_{i,\,t+1} \; - \; R_{1,\,t+1}) \right] \; = \; 0 \tag{50}$$

The first-order condition with respect to $\omega_{K+1,\,t}$, which is the fraction of household capital in full wealth W_t, is

$$\mathbf{E}_t \left[M^*_{t+1} \; (R_{K+1,\,t+1} \; - \; R_{1,\,t+1}) \right] = -\frac{F'_D \, u_{S,\,t}}{Q_{D,\,t} \, u_{C,\,t}} \times \left(1 \; - \; F \left[\frac{\omega_{K+2,\,t}}{Q_{H,\,t}}, \; \frac{\omega_{K+1,\,t}}{Q_{D,\,t}} \right] \; \frac{u_{S,\,t}}{u_{C,\,t}} \right)^{k-1} \tag{51}$$

and the first-order condition with respect to the fraction of human wealth $\omega_{K+2,\,t}$ is

$$\mathbf{E}_t \left[M^*_{t+1} \; (R_{K+2,\,t+1} \; - \; R_{1,\,t+1}) \right] = -\frac{F'_H \, u_{S,\,t}}{Q_{H,\,t} \, u_{C,\,t}} \times \left(1 \; - \; F \left[\frac{\omega_{K+2,\,t}}{Q_{H,\,t}}, \; \frac{\omega_{K+1,\,t}}{Q_{D,\,t}} \right] \; \frac{u_{S,\,t}}{u_{C,\,t}} \right)^{k-1} \tag{52}$$

Using the normalization

$$R_{W,t+1} \; = \; \left(1 \; - \; F \left[\frac{\omega_{K+2,\,t}}{Q_{H,\,t}}, \; \frac{\omega_{K+1,\,t}}{Q_{D,\,t}} \right] \; \frac{u_{S,\,t}}{u_{C,\,t}} \right)^{-1} \times \; R^*_{W,\,t+1} \tag{53}$$

$$M_{t+1} \; = \; \left(1 \; - \; F \left[\frac{\omega_{K+2,\,t}}{Q_{H,\,t}}, \; \frac{\omega_{K+1,\,t}}{Q_{D,\,t}} \right] \; \frac{u_{S,\,t}}{u_{C,\,t}} \right)^{1-k} \times \; M^*_{t+1} \tag{54}$$

delivers the first-order conditions in the main text.

Table 1: Test for the Null of Difference Stationarity

	Phillips-Perron Test		Stationary Bootstrap Test
	z_ρ	z_t	p-value
C_t	-6.66	-2.04	0.95
D_t	5.77	3.77	0.94
Q_t	-11.71	-2.56	0.99

NOTE - Critical value for z_ρ is -20.7 (5% level) and -17.5 (10% level), z_t is -3.45 (5% level) and -3.15 (10% level). The number of lags used in the Phillips-Perron test is four. The stationary bootstrap test is based on 10,000 Monte Carlo simulations of the test developed by Parker, Paparoditis and Politis (2005, Journal of Econometrics); p-value is for the one-sided test of the null hypothesis of unit root. Sample period is 1951.I-2001.IV.

Table 2: Estimated Cointegrating Vector

	Point Estimate	Asymptotic Standard Error	Bootstrap Confidence Intervals
Const	-0.0342	(0.0626)	N/A
θ	0.0084	(0.1339)	[-0.262,0.249]
η	0.5664	(0.0654)	[0.421,0.716]

NOTE - The table reports the estimated co-integrated vector and the asymptotic standard errors. The empirical distribution is constructed using sieve bootstrap of Chang, Park and Song (2005); 40,000 Monte Carlo simulations were used. The last column shows the 5% symmetric percentile confidence intervals. Sample period 1951.I-2001.IV.

Table 3: GMM Results for Epstein-Zin CCAPM: Fitting Unconditional Moments for Value-Weighted Market, Risk-Free Rate, Small-Minus-Big SMB and Value-Minus-Growth HML Portfolios

		Sample Period 1951.I-2001.IV				
Row	Model	σ	β	γ	J_T	95% S-set for γ
1	EU-CCAPM with	0.031	1.230		19.949	
	Durables; $\theta = 1.05$	(0.098)	(0.838)		(0.000)	
2	Epstein-Zin CCAPM	0.020	1.132	185.317	1.186	$(94.44, 250.01)$
	without Durables	(0.009)	(0.104)	(54.482)	(0.276)	
3	Epstein-Zin CCAPM	1.002	1.259	1.578	1.280	$(0.010, 0.82) \cup (1.52, 11.04)$
	with Durables	(0.003)	(0.000)	(0.516)	(0.258)	

NOTE - The table presents estimates of the elasticity of inter-temporal substitution σ, the subjective discount factor β and the coefficient of risk aversion γ for Epstein-Zin Consumption-based CCAPM, with and without consumer durables. The last column reports the 95% confidence S-set [Stock and Wright (2000)] obtained by concentrating out the parameters σ and β. Continuous-updating GMM of Hansen, Heaton and Yaron (1996) was used. Asymptotic HAC standard errors and p-values are in parentheses. Multiple local minima were encountered.

Table 4: GMM Results for Epstein-Zin CCAPM Without Consumer Durables: Fitting Conditional Moments for Value-Weighted Market and the Risk-Free Rate

Row	Instruments	Sample Period 1951.I-2001.IV				
		σ	β	γ	J_T	95% S-set for γ
1	Const, CG, DG	1.675	0.986	2.716	18.671	$(21.90, 250)$
	P-D, HML, SMB	(4.747)	(0.009)	(7.203)	(0.028)	
2	Const, CG, DG,	1.399	0.986	2.228	21.199	$(14.87, 250)$
	P-D, HML, SMB, TERM	(2.248)	(0.006)	(4.895)	(0.031)	
3	Const, CG, DG, P-D,	1.831	0.986	3.243	24.846	$(22.47, 250)$
	HML, SMB, TERM, U	(3.900)	(0.006)	(5.652)	(0.024)	
4	Const, CG, DG, P-D,	1.919	0.986	3.939	25.664	$(22.50, 250)$
	HML, SMB, TERM, U, DEF	(4.433)	(0.007)	(7.269)	(0.019)	

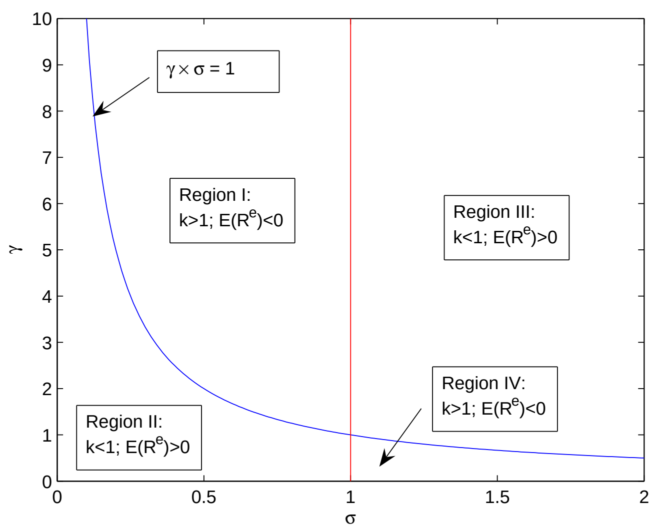
NOTE - The table presents estimates of the elasticity of inter-temporal substitution σ, the subjective discount factor β and the coefficient of risk aversion γ for the Epstein-Zin Consumption-based CCAPM without consumer durables. The last column reports the 95% confidence S-set [Stock and Wright (2000)] obtained by concentrating out the parameters σ and β. The instruments are lagged twice to take account of time aggregation. Variable definitions: CG = nondurable consumption growth rate, DG = durables stock growth rate, P-D = aggregate price-dividend ratio, HML = value-minus-growth spread, SMB = small-minus-big spread, TERM = Aaa bonds YTM minus three-month Treasury Bill, DEF = Baa bonds YTM - Aaa bonds YTM, U = number of civilians unemployed less than 5 weeks. Continuous-updating GMM of Hansen, Heaton and Yaron (1996) was used. Asymptotic HAC standard errors and p-values are in parentheses. Multiple local minima were encountered.

Table 5: GMM Results for Epstein-Zin CCAPM With Consumer Durables: Fitting Conditional Moments for Value-Weighted Market, the Risk-Free Rate, Small-Minus-Big SMB and Value-Minus-Growth HML Portfolios

		Sample Period 1951.I-2001.IV				
Row	Instruments	σ	β	γ	J_T	95% S-set for γ
		Panel A.: Test Assets are Scaled Market and Risk-Free Rate				
1	Const, CG, DG	1.265	0.988	1.809	16.809	$(1.59, 2.28) \cup (12.33, 232.44)$
	P-D, HML, SMB	(2.691)	(0.009)	(6.538)	(0.052)	
2	Const, CG, DG,	1.417	0.987	2.254	18.020	$(1.87, 3.11) \cup (15.97, 250)$
	P-D, HML, SMB, TERM	(2.382)	(0.006)	(5.036)	(0.081)	
3	Const, CG, DG, P-D,	1.049	0.989	1.211	21.559	$(1.17, 1.31) \cup (3.39, 52.9)$
	HML, SMB, TERM, U	(1.235)	(0.006)	(5.040)	(0.063)	
4	Const, CG, DG, P-D,	1.709	0.989	3.100	25.734	$(2.78, 3.32) \cup (19.67, 250)$
	HML, SMB, TERM, U, DEF	(2.792)	(0.005)	(4.812)	(0.041)	
		Panel B.: Test Assets are Scaled Market, Risk-Free Rate and SMB				
1	Const, CG, DG	1.011	0.987	1.044	24.171	$(1.04, 13.03)$
	P-D, HML, SMB	(1.642)	(0.008)	(6.677)	(0.0620)	
2	Const, CG, DG,	1.293	0.986	1.949	26.295	$(1.66, 2.64) \cup (11.42, 249.67)$
	P-D, HML, SMB, TERM	(1.633)	(0.005)	(4.092)	(0.093)	
3	Const, CG, DG, P-D,	1.002	0.987	1.011	28.540	$(0, 3.21)$
	HML, SMB, TERM, U	(0.583)	(0.002)	(2.637)	(0.125)	
4	Const, CG, DG, P-D,	1.968	0.984	3.978	38.864	$(3.91, 3.98) \cup (20.55, 250)$
	HML, SMB, TERM, U, DEF	(3.322)	(0.005)	(5.189)	(0.028)	
		Panel C.: Test Assets are Scaled Market, Risk-Free Rate, SMB and HML				
1	Const, CG, DG	1.160	0.985	3.518	48.915	$(7.26, 152.12)$
	P-D, HML, SMB	(2.192)	(0.009)	(29.792)	(0.000)	
2	Const, CG, DG,	1.090	0.983	2.503	62.689	$(4.79, 90.97)$
	P-D, HML, SMB, TERM	(1.373)	(0.008)	(21.013)	(0.000)	
3	Const, CG, DG, P-D,	1.196	0.981	3.709	66.529	$(7.96, 178.08)$
	HML, SMB, TERM, U	(1.541)	(0.007)	(17.769)	(0.000)	
4	Const, CG, DG, P-D,	1.201	0.981	2.512	85.299	$(8.17, 181.83)$
	HML, SMB, TERM, U, DEF	(1.240)	(0.005)	(7.737)	(0.000)	

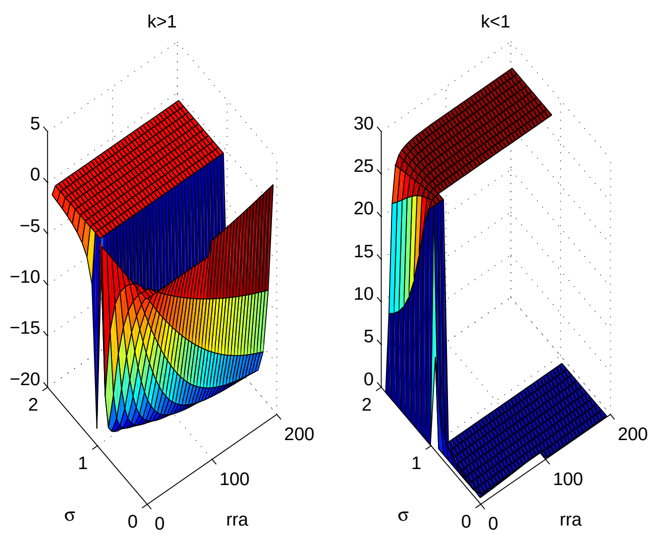
NOTE - The table presents estimates of the elasticity of inter-temporal substitution σ, the subjective discount factor β and the coefficient of risk aversion γ for the Epstein-Zin Consumption-based CCAPM with consumer durables. The last column reports the 95% confidence S-set [Stock and Wright (2000)] obtained by concentrating out the parameters σ and β. The instruments are lagged twice to take account of time aggregation. Variable definitions: CG = nondurable consumption growth rate, DG = durables stock growth rate, P-D = aggregate price-dividend ratio, HML = value-minus-growth spread, SMB = small-minus-big spread, TERM = Aaa bonds YTM minus three-month Treasury Bill, DEF = Baa bonds YTM - Aaa bonds YTM, U = number of civilians unemployed less than 5 weeks. Continuous-updating GMM of Hansen, Heaton and Yaron (1996) was used. Asymptotic HAC standard errors and p-values are in parentheses. Multiple local minima were encountered.

Figure 2: Consumption Substitutability, Risk Aversion and Timing of Uncertainty



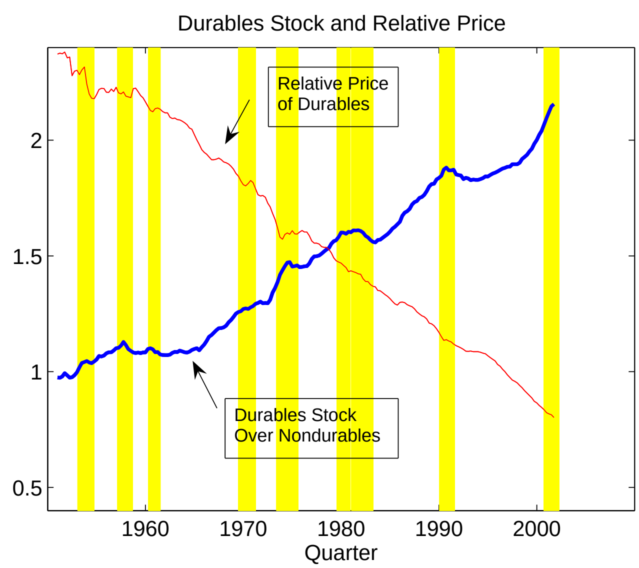
NOTE - The figure presents 4 regions corresponding to the preference for early ($\kappa < 1$) and late ($\kappa > 1$) resolution of uncertainty in terms of the inter-temporal consumption substitutability σ and risk aversion γ. Formally, $\kappa \equiv (1-\gamma)/(1-1/\sigma)$. If consumption risk is small (i.e. σ greater enough than zero), expected returns on assets are positive when investors have a preference for early resolution of uncertainty and vice versa.

Figure 3: Expected Excess Return on the Value-Weighted Market Portfolio as a Function of Uncertainty Timing



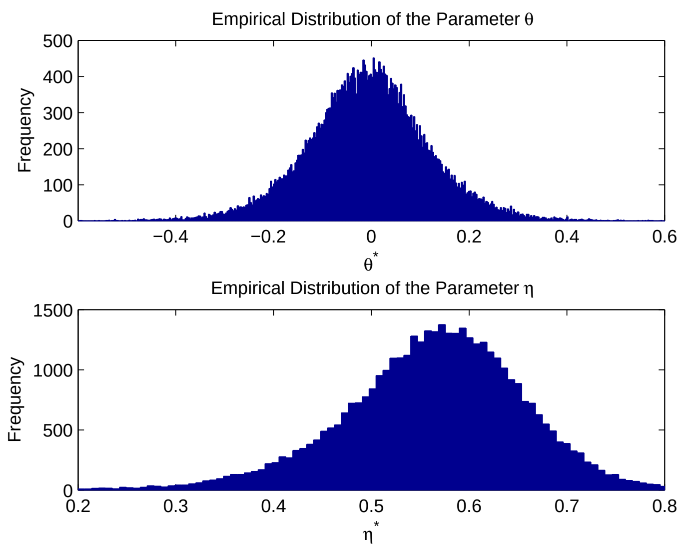
NOTE - The figure presents the expected return on the value-weighted market return as a function of the attitudes toward timing of uncertainty. The case $\kappa < 1$ corresponds to the preference for early resolution of uncertainty and vice versa.

Figure 4: Time Series Behavior of the Durables Stock and the Relative Price in the Post-War U.S. Economy



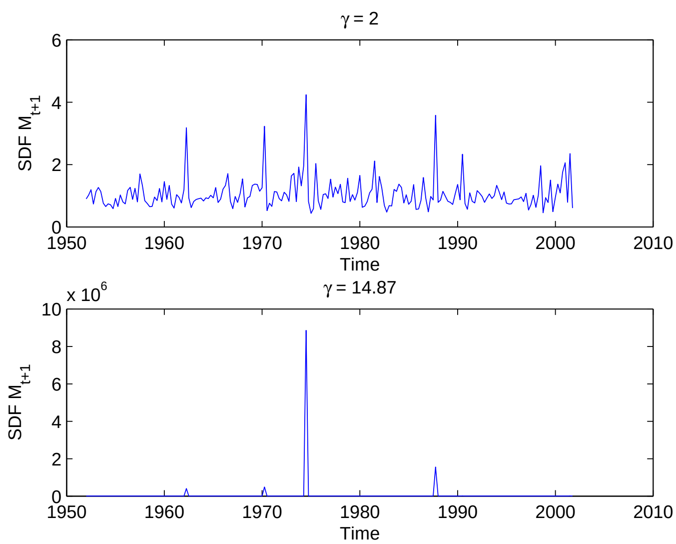
NOTE - The plot portrays the quarterly time-series of the ratio of the constructed durables stock over non-durables and services, and the relative price of durables in terms of nondurables. Bars represent recessions as classified by the National Bureau of Economic Research (NBER). Sample period 1951.I - 2001.IV.

Figure 5: Empirical Distributions of the Preference Parameters θ and η Based on the Sieve Bootstrap of the Intra-Temporal First-Order Condition



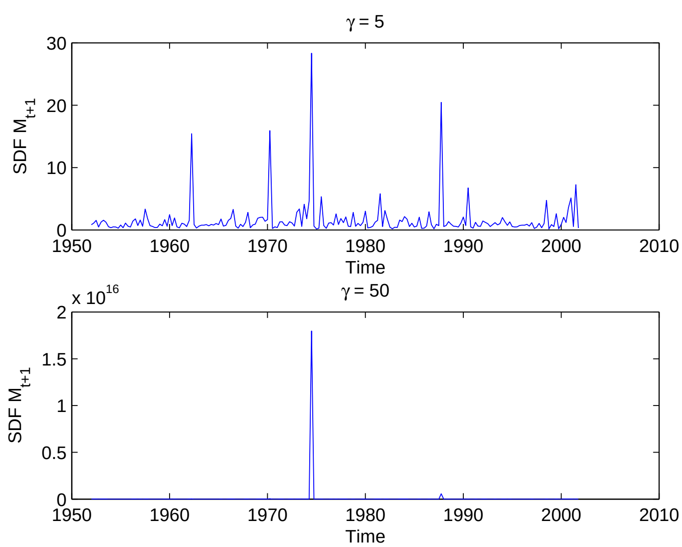
NOTE - The picture displays the empirical distribution of the parameters θ and η obtained by 40,000 Monte Carlo simulations of the Sieve bootstrap [Chang, Park and Song (2005)] applied to the co-integrating regressions implied by the intra-temporal first-order condition.

Figure 6: Stochastic Discount Factor for Epstein-Zin CCAPM with No Durables As a Function of Risk Aversion When Inter-temporal Substitutability is Greater Enough Than Zero.



NOTE - The figure portrays the stochastic discount factor M_{t+1} for the Epstein-Zin Consumption-based CAPM without consumer durables when the inter-temporal consumption substitutability $\sigma = 1.39$, the subjective discount factor $\beta = 0.99$ and the coefficient of risk aversion $\gamma \in \{2,\ 14.87\}$.

Figure 7: Stochastic Discount Factor for Epstein-Zin CCAPM with Durables As a Function of Extreme Risk Aversion When Inter-temporal Substitutability is Greater Enough Than Zero.



NOTE - The figure portrays the stochastic discount factor M_{t+1} for the Epstein-Zin Consumption-based CAPM with consumer durables when the inter-temporal consumption substitutability $\sigma = 1.70$, the subjective discount factor $\beta = 0.99$ and the coefficient of risk aversion $\gamma \in \{5, 50\}$.