Habit Formation, Time-Varying Risk Aversion and Cross-Section of Expected Returns [1]

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Abstract

We identify three common risk factors in the returns on common stocks: the nondurable consumption growth rate, the change in the term premium, and the change in the default premium. We motivate these interesting factors by a linearized version of the Campbell-Cochrane external habit model where the last two term structure factors proxy for changes in the aggregate risk aversion. Our three-factor model successfully explains the average returns of the 25 size and book-to-market sorted portfolios, and the temporal variation in the conditional first moments of the market, SMB, and HML portfolios.

JEL: G12, G10

Keywords: Risk Aversion, Default Premium, Term Premium, Size, Book-to-Market, Cross Section

1 Introduction

It is already a well-established fact in the empirical finance literature that stocks of small market capitalizations and high book-to-market ratios earn higher average returns compared to their counterparts of large market capitalizations and small book-to-market ratios. In an influential paper, Fama and French (1992, 1993) develop a three-factor model to account for these stylized facts. However, their model is empirically motivated and thus lacks a structural explanation.

The thesis of this paper is to propose and estimate a structural consumption-based asset pricing model whose empirical success would be comparable to Fama and French (1993). We take as our benchmark the Campbell and Cochrane (1999) consumption-based external habit model which implies that assets are risky because they co-vary with the intertemporal marginal rate of substitution IMRS of the representative agent. The IMRS itself is functionally dependent upon nondurable consumption growth rate and the change in investors' relative risk aversion RRA. We map the unobservable RRA coefficient to observables. In particular, we argue that the term-structure factors - term and default premia - are proxies for the level in the aggregate risk aversion; the term premium picks up the variation of aggregate risk aversion closely related to business cycles, and the default premium tracks the variation in aggregate risk aversion that transcends the NBER-designated business cycles. These observations are motivated by the results in Fama and French (1989), who discover that the term premium displays a strong business-cycle variation, and the default premium variation transcends the business cycles.

The empirical results are easy to communicate. Our structural model successfully replicates the results in Fama and French (1993). In fact, that we are able to reproduce apparently well the cross-section of the 25 size- and book-to-market- sorted portfolio returns may be seen in Figure 1, bottom right panel: the fitted returns of the linear Campbell-Cochrane model line up along the 45-degree line in a manner that is remarkably similar to the Fama-French three-factor model in the top-right panel. The model has a straightforward economic interpretation:

stocks are risky not only because they co-vary positively with consumption, but also because they tend to pay badly in times when risk aversion is particularly high relative to the past. In addition, we also investigate the ability of the model to account for the temporal variation in the conditional first moments on the market, small-minus-big (SMB) and value-minus-growth (HML) portfolios using a rich set of instrumental variables (including price/dividend ratio, value spread etc.), and find strong empirical support for the model.

We find the intuition behind the results compelling. The change in the default premium is a barometer of the aggregate risk aversion that transcends the NBER-designated business cycle recessions [Fama and French (1989)], and its mimics the risk factors related to market capitalization and the book-to-market ratio. In fact, both value and small stocks are sensitive to innovations in this component of the aggregate risk aversion. Furthermore, the change in the term premium mimics the risk factor related to the book-to-market ratio. This component of the risk aversion, which is closely related to the business cycles, contributes toward explaining the average returns on value stocks relative to growth stocks. Overall, it turns out that small and value stocks do particularly poorly when the risk aversion is high relative to the past, after controlling for the consumption growth rate. As an example, small stocks did exceptionally poorly in the 1980s. Our explanation is that the risk aversion, as proxied by the default premium, was also extraordinarily high during this period.

In a classical paper, Chen, Roll and Ross (1986) also explore the role of time-varying risk aversion. They construct the innovation in risk aversion as a simple difference between default premium and term premium. Their results indicate that the model is capable to account for the size premium. Another closely related paper is Petkova (2006), who empirically evaluates the performance of the Merton's (1973) ICAPM. Her state variables are the dividend-price ratio, default premium, term premium, short-term Treasury Bill, HML and SMB. Following Campbell (1996) she uses a triangularized VAR(1) to extract the innovations in these variables, and then uses these surprise series to price the monthly cross-section of 25 Fama-French portfolios. In contrast, both our story and the data are different. In specifying the pricing kernel we fall back on the structural consumption-based Campbell-Cochrane model. Our measure of the *level* of risk-aversion is proxied by the level of the default premium and term premium. The fact that we use *changes* in default premium and term premiums in our empirical exercise is dictated by the pricing kernel of the the model. Note, however, that Petkova's innovations to default premium and term premium, and our simple changes in the default and the term are significantly different time series. Whereas our most significant priced factor is the change in default premium, in Petkova it is the surprise in the term premium that captures the most of the cross-sectional variation, with the surprise in default premium not being priced. Our paper is also related to Lettau and Ludvigson (2001) who explore a conditional version of the consumption CAPM. Their focus is, however, not on role of the aggregate risk aversion in asset pricing neither the interaction between the bond and stock market.

A related literature examines the link between stock market and bond market. Fama and French (1993) find that portfolios constructed to mimic risk factors related to market capitalization, book-to-market, and two term-structure factors, the *level* of the term and default premiums, capture strong common variation in common stock and bond returns. They conclude that "... there are at least three stock-market factors and two term-structure factors in returns. Stock returns have shared variation due to the three stock-market factors, and they are linked to bond returns through shared variation in the two term structure factors." However, the explanatory power of the term structure factors is marginal once stock market factors are included in the regression. Gebhardt, Hvidkjaer and Swaminathan (2005) focus on the level of the default and term premiums as risk factors to price to the cross-section of corporate bond returns.

The rest of the paper is organized as follows: Section 2 establishes the theoretical framework and our empirical proxies. Section 3 covers our empirical results. Section 4 concludes.

2 Theoretical Framework and Empirical Proxies

2.1 Linear Campbell-Cochrane (1999) External Habit Model

Building on the work of Abel (1990), Constantinides (1990), Heaton (1995) and Sundaresan (1989), Campbell and Cochrane (1999), [hereafter CC], have developed a representative agent asset pricing model in which investors' preferences exhibit an external habit formation. CC specify the felicity function as a difference between consumption C_t and the habit X_t, which generates a time-varying risk-aversion. Formally,

$$u(C_t, X_t) \;=\; \frac{1}{1-\gamma}\left(C_t - X_t\right)^{1-\gamma}. \tag{1}$$

The utility functional is

$$U\left(\{C_t\}_{t\geq 0}\right) \;=\; \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t\, u(C_t, X_t), \tag{2}$$

where β is the subjective discount factor. In complete markets equilibrium, investors equalize their marginal rates of substitution across states and this common value is the stochastic discount factor

$$M_{t+1} \;=\; \beta\, \frac{u_C(C_{t+1}, X_{t+1})}{u_C(C_t, X_t)}. \tag{3}$$

For purposes of clear exposition, CC define the so-called consumption surplus ratio, denoted S_t, as an excess of current consumption relative to the habit expressed as a fraction of the current consumption. Small consumption ratio, for instance, indicates that the economy is in recession where consumption is particularly low relative to the habit level. In equations,

$$S_t \;=\; \frac{C_t - X_t}{C_t}. \tag{4}$$

We may express the discount factor M_{t+1} as

$$M_{t+1} \;=\; \beta \left(\frac{C_{t+1}}{C_t} \frac{S_{t+1}}{S_t} \right)^{-\gamma}. \tag{5}$$

Intuitively, stocks are risky because they co-vary either with consumption growth C_{t+1} / C_t or with the consumption surplus growth S_{t+1} / S_t. It is illuminating to express the discount factor in terms of the RRA coefficient. To do that, recall that RRA is defined as

$$RRA_t \;\equiv\; -\frac{\partial \log J_W(t)}{\partial \log W_t} \;=\; -\frac{\partial \log u_C(t)}{\partial \log C_t} \times \frac{\partial \log C_t}{\partial \log W_t} \;=\; \eta_t \times MPC_t, \tag{6}$$

where MPC_t is the marginal propensity to consume out of wealth and the parameter η_t is the curvature of the felicity function, defined as

$$\eta_t \;=\; -\frac{C_t \, u_{CC}(C_t, X_t)}{U_C(C_t, X_t)} \;=\; \frac{\gamma}{S_t}. \tag{7}$$

This allows us to re-write the RRA coefficient in terms of the consumption surplus ratio as

$$RRA_t \;=\; \frac{\gamma}{S_t} \times MPC_t. \tag{8}$$

Or equivalently,

$$S_t \;=\; \frac{\gamma}{RRA_t} \times MPC_t. \tag{9}$$

We can thus eliminate the consumption surplus ratio S_t from the stochastic discount factor M_{t+1}

$$M_{t+1} \;=\; \beta \left(\frac{C_{t+1}}{C_t} \frac{RRA_t}{RRA_{t+1}} \frac{MPC_{t+1}}{MPC_t} \right)^{-\gamma} \;=\; e^{\log(\beta) - \gamma \, \Delta \, c_{t+1} + \gamma \, \Delta \, rra_{t+1} - \gamma \, \Delta \, mpc_{t+1}} \tag{10}$$

Lower-case variables are in logs. We subsequently use the approximation $e^x \cong 1 + x$

$$M_{t+1} \;=\; 1 \,+\, \log(\beta) \,-\, \gamma\,\Delta\,c_{t+1} \,+\, \gamma\,\Delta\,rra_{t+1} \,-\, \gamma\,\Delta\,mpc_{t+1}. \tag{11}$$

The primary testable restrictions of the model are the set of Euler equations

$$\mathbf{E}_t\left[M_{t+1}\,R^e_{t+1}\right] \;=\; 0, \tag{12}$$

where R^e is an excess return. Using the linearization and demeaning the factors, we may rewrite the Euler equation

$$\mathbf{E}_t\left[R^e_{t+1}\right] = \gamma\,cov_t\left[\Delta c_{t+1}, R^e_{t+1}\right] - \gamma\,cov_t\left[\Delta rra_{t+1},\, R^e_{t+1}\right] + \gamma\,cov_t\left[\Delta mpc_{t+1}, R^e_{t+1}\right]. \tag{13}$$

The model identifies three sources of risk. The first one is the growth rate in nondurable consumption. This risk factor is well-understood. Stocks that pay badly when people already consume little are risky. The other two risk factors are recession factors. In the model, recessions are not only periods of low consumption growth but also of high growth rate of risk-aversion and low (even negative) growth of the marginal propensity to consume. Holding the consumption growth constant, stocks that pay badly when the risk-aversion is rising are bad. They let investors down exactly at the wrong time. Similarly, stocks that pay badly in times when the marginal propensity to consume out of wealth is falling are risky.

Because marginal propensity to consume Δmpc_{t+1} is unobservable, we drop it in equation (11) and have the following linear factor asset pricing model

$$M_{t+1} \;=\; b_0 \,+\, b_1\,\Delta\,c_{t+1} \,+\, b_2\,\Delta\,rra_{t+1}, \tag{14}$$

with two restrictions $b_1 < 0$ and $b_2 \;=\; -b_1$. We argue that dropping the marginal propensity to consume out of wealth is innocuous as there is only one state variable in addition to the financial wealth, the habit level X_t, and therefore only one independent risk factor in addition to the consumption growth rate. We refer to this model hereafter as the linear Campbell-Cochrane model.

2.2 Empirical Proxies

Our yardstick of aggregate risk aversion employs two term-structure related variables: the term premium and the default premium. Our motivation is based on the general decomposition

$$Expected\ Return = Price\ of\ Risk \times Quantity\ of\ Risk \tag{15}$$

In Campbell-Cochrane model, price of risk corresponds to the aggregate risk aversion. In fact, in an extension of Campbell-Cochrane model, Wachter (2005) models the real and nominal term structure of interest rates. She finds that the excess return on nominal bonds depends on the quantity of risk such as covariance of inflation with consumption growth, and the price of risk, the aggregate risk aversion. We can therefore back out a measure of the aggregate risk aversion from the term structure of interest rates. One may argue that the same argument follows for equities. In fact, this is exactly in the spirit of the influential Lettau and Ludvigson's (2001) model. Using the budget constraint, they construct a new interesting predictive variable, denoted cay_t, the deviation from the stochastic trend between asset wealth a, labor income y and nondurable consumption c. Using the linearized Campbell-Cochrane (1999) model, they then obtain an interesting conditional asset pricing model. However, the advantage of backing out the risk aversion from the yield curve is that default and term premiums are observable in real time. Moreover, the quantity of risk for bonds is smaller than that for equities and therefore our measure of risk aversion is arguably less noisy. Secondly, we do not want the error in measuring the risk aversion to be correlated with stock returns, which might be more likely if we back it out from equity returns.

Furthermore, in an interesting paper, Fama and French (1989) show that the time-series behavior of the term premium TERM is closely related to business cycles as identified by the NBER. TERM tends to be particularly low near business cycle peaks and particularly high at troughs. In other words, TERM tracks the variation in expected returns on stocks and bonds in response to short-term variation in business conditions. Therefore, it is a good proxy for the changes in aggregate risk aversion over the business cycle.

If bond portfolios are priced rationally, the default spread DEF reflects the business cycle state of the U.S. macro-economy. DEF displays some business cycle variation but most of its movement transcends the business cycles as measured by the NBER. In the post-war period, DEF is particularly high during the early 80s. As a result, DEF picks up the variation in aggregate risk aversion that goes beyond the business cycles. Therefore, we map the model-implied coefficient of relative risk-aversion toward wealth bets by employing two proxies, the term spread TERM and the default spread DEF.

3 Empirical Investigation

To test our model, we first test the unconditional moment restrictions of the model and then the conditional moment restrictions. We find that our model performs well in both tests.

3.1 Data

The nominal nondurable consumption and services are obtained from the *National Income and Product Accounts (NIPA)*, period 1947:Q1 - 2004:Q4, and are seasonally adjusted. They are converted to real per-capita series by dividing by the consumer price index and the quarterly U.S. population.

Asset data except Baa and Aaa yields are from Ken French's web site. The risk-free rate is the three-month Treasury bill rate, the market return is the return on the value-weighted portfolio of NYSE, AMEX and NASDAQ firms. The 25 Fama-French portfolios, which are constructed at the end of each June, are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t. BE/ME for June of year t is the book equity for the last fiscal year end in $t-1$ divided by ME for December of $t-1$. The BE/ME breakpoints are NYSE quintiles. The portfolios for July of year t to June of $t+1$ include all NYSE, AMEX, and NASDAQ stocks for which Fama and French have market equity data for December of $t-1$ and June of t, and (positive) book equity data for $t-1$. The yields on the portfolio of the corporate bonds with Moody's ratings Aaa and Baa are obtained from the Federal Reserve Bank of St. Louis.

Following Fama and French (1989, 1993), we define two bond factors based on the term structure of interest rates for corporate bonds. Firstly, we define the term spread, $TERM$, as a simple difference between time t yield-to-maturity on the Aaa bond portfolio and the 3-month Treasury bill rate. Using the decomposition that the expected return = price of risk $\times$ quantity of risk, TERM is a measure of the price of risk, that is to say, the aggregate yardstick of risk aversion. Fama and French (1989) show that TERM displays a clear business cycle pattern, rising in troughs and falling in peaks. Therefore, TERM picks the variation in aggregate risk aversion that is closely related to the NBER-designated recessions. Furthermore, we define the default spread DEF as the difference between time t yield-to-maturity on the Baa bond portfolio minus time t yield-to-maturity on the Aaa bond portfolio. Fama and French (1989) show that DEF exhibits variation that seem to transcend the business cycle frequencies. For example, DEF is particularly high in '80s when small stocks did surprisingly poorly. Table 1 reports the summary statistics.

3.2 Empirical Methods

We use two well-established empirical methodologies to evaluate the presented models. Firstly, in order to be easily comparable with the previous finance literature, we use Fama-MacBeth methodology [Fama and MacBeth (1973)]. The primary testable asset pricing implications of linear factor models are the β-representation [Ross (1978), Dybvig and Ingersoll (1982)]

$$\mathbf{E}\left[R^e_{i,t+1}\right] = \beta_i{}'\lambda, \quad i = 1, ..., 25 \tag{16}$$

where λ is a $K \times 1$ vector of the market prices of risk corresponding to a $T \times K$ vector of risk factors F_{t+1}. We test these implications of the economic theory as follows. We run the time series regression to estimate the betas

$$R^e_{i,t+1} = a + \beta_i{}' F_{t+1} + \varepsilon_{i,t+1}, \quad i = 1, ..., 25 \tag{17}$$

We then run the cross-sectional regression

$$\mathbf{E}_T\left[R^e_{i,t+1}\right] = \beta_0 + \beta_i{}'\lambda + \alpha_i, \quad i = 1, ..., 25 \tag{18}$$

to estimate λ, where we use the Hansen's (1982) notation $\mathbf{E}_T = T^{-1}\sum_{t=1}^{T}$. In the cross-sectional regression, a well-specified asset-pricing model produces intercept β_0 that is indistinguishable from zero [Merton (1973)]. The estimated intercept provides a simple return metric and a formal test of how well different sets of the risk factors account for the cross-section of expected returns. Moreover, judging asset-pricing models on the basis of the intercept in excess return regressions imposes a stringent standard. Competing models are asked to explain the three-month Treasury Bill rate as well as the excess returns on 25 Fama-French portfolios. Furthermore, note that the α_is represent the pricing errors of a particular model. We therefore evaluate the empirical performance of the model using the asymptotic J_T test, described below. This test is a direct analog of the Gibbons, Ross and Shanken (1989) test.

Secondly, we follow the stochastic discount methodology [Cochrane (1996)] and test the over-identifying restriction [Hansen (1982), Hansen, Heaton and Yaron (1996), Hansen and Singleton

(1982)]. In all models considered, the stochastic discount factor takes the linear form

$$M_{t+1} \;=\; 1 \,+\, b'\, F_{t+1} \tag{19}$$

where F_{t+1} is a $T \times K$ vector of risk factors and b is a $K \times 1$ vector of constant factor loadings[1]. The primary testable asset pricing implications of the models are the set of Euler equations

$$\mathbf{E}_t \left[M_{t+1}\, R^e_{i,t+1} \right] \;=\; 0, \quad i \;=\; 1, ..., 25 \tag{20}$$

where $R^e_{i,t+1}$ are excess returns on 25 Fama-French portfolios. We condition down the models by taking unconditional expectation of the equation (29) and form the 25×1 vector of Euler equations (pricing) errors

$$e^i(b) \;=\; \mathbf{E}_T \left[\, M_{t+1}\,(b)\;\, R^e_{i,t+1} \right]$$

We follow Hansen, Heaton and Yaron (1996) and estimate the unknown vector of factor loadings $\hat{b}$ by making the pricing errors close to zero in the sense of minimizing the quadratic form

$$\hat{b} \;=\; \arg\min_{b}\; J_{T,S} \;=\; e^i\,(b)'\; S^{-1}(b)\, e^i\,(b)$$

where S is the spectral density matrix at frequency zero. We report only the results[2] of the test of the over-identifying restriction $J_{T,S}$ [Hansen (1982)]. Furthermore, Hansen and Jagannthan (1997) suggest to perform model comparison using the weighting matrix $\mathbf{E}[RR']$, the inverse of the second moments of asset returns, and computing the HJ-distance $J_{T,HJ}$. The advantage is that the weighting matrix is invariant across models and thus suitable for model comparison. Finally, we also report the test of over-identifying restriction for the identity weighting matrix, the so-called first-stage GMM, which is equivalent to running OLS cross-sectional regression. The advantage is that the test focuses on economically interesting portfolios, that is, those

[1]Note that without loss of generality, the coefficient b_0 is normalized to one. The reason for this is that we use excess returns, and we want all parameters to be identified.

[2]Detailed results, including the code, are available from authors upon request.

sorted based on the market capitalization and the accounting book-to-market ratio. Last but not least, it allows to compare the pricing errors across models, knowing that we did not blow up the spectral density matrix.

3.3 Unconditional Tests

In addition to the linear Campbell-Cochrane model advocated in the paper, we also consider three additional familiar linear factor pricing models as benchmarks. We evaluate the power of all these models to account for the cross-section of expected equity returns using excess returns on the 25 size and book-to-market sorted portfolios [Fama and French (1993)]. Table 3 presents the results of estimating the β-representation

$$\mathbf{E}\left[R^e_{i,t+1}\right] = \beta' \lambda, \quad i = 1, ..., 25 \tag{21}$$

for the Sharpe-Lintner (CAPM) model [row 1], the Fama-French three-factor model [row 2], the canonical Consumption-Based CAPM of Lucas (1978) and Breeden (1979) [row 3], and the linear Campbell-Cochrane model [row 4]. The table reports the estimated coefficients of market prices of risk λ, Shanken-corrected and uncorrected t-statistics for these coefficients, unadjusted and adjusted R^2s for the cross-sectional regression, and the Hansen's (1982) J_T statistics corresponding to (i) the identity weighting matrix [$J_{T,W}$], (ii) Hansen-Jagannathan's (1997) distance matrix [$J_{T,HJ}$], and (iii) the efficient weighting matrix of continuous-updating GMM [$J_{T,S}$], all to formally test the model.

Perhaps the most familiar unconditional linear factor pricing model is the static Sharpe-Lintner model (CAPM). We implement this model by using the CRSP value-weighted excess market return as a proxy for the unobservable market portfolio return. The cross-sectional implication of this model is

$$\mathbf{E}\left[R^e_{i,t+1}\right] = \beta_w \lambda_w \tag{22}$$

The results are presented in the first row of table 3. The t-statistics of the market price of risk λ_w is statistically insignificant, indicating that the beta is unable to account for the cross-sectional variation in average returns. Moreover, it has the wrong sign according to Sharpe (1964) and Lintner (1965). This failure is summarized by (i) very low cross-sectional R^2, about 1% [see also Fama and French (1992)] and (ii) large $J_{T,W}$, $J_{T,HJ}$ and $J_{T,S}$ statistics, rejecting the model formally[3].

Table 3, row 2, presents the results for the influential Fama-French three-factor model [Fama and French (1992, 1993)] given by

$$\mathbf{E}\left[R^e_{i,t+1}\right] = \beta_w \, \lambda_w + \beta_{SMB,i} \, \lambda_{SMB} + \beta_{HML,i} \, \lambda_{HML} \tag{23}$$

where SMB is the small-minus-big portfolio and HML is the value-minus-growth portfolio. In stark contrast to the Sharpe-Lintner Capital Asset Pricing Model, the three-factor FF model is able to explain about 78% of cross-sectional variation in average returns, and the Shanken-corrected t-statistics on the HML factor is statistically significant[4]. However, the intercept comes out statistically significant as well, with t statistics 3.310, contrary to the economic theory. The model is statistically rejected as the test of the over-identifying restriction $J_{T,S} = 47.498$, with p-value$=$ 0.000, but Fama and French (1993) correctly point out that the model is estimated too precisely, and therefore the pricing errors have "too" small standard errors (see also the section Euler Equation Errors below).

Table 3, row 3, presents the results for the canonical Consumption-based CAPM [Lucas (1978), Breeden (1979)]. The cross-sectional implication of this model is given by

$$\mathbf{E}\left[R^e_{i,t+1}\right] = \beta_{\Delta\, c} \, \lambda_{\Delta\, c} \tag{24}$$

[3] Jagannathan and Wang (1996) augment the canonical CAPM with human capital and obtain more encouraging results.

[4] The likelihood ratio test [Newey and West (1987a)] yields J_T(restricted)-J_T(unrestricted)$=$ $60.063 - 47.498 = 12.565 > \chi^2_{95\%}(1) = 3.842$.

where Δc is the (log) growth rate in real nondurable consumption and services. As it is well-known [cf. Breeden, Gibbons and Litzenberger (1989), Hansen and Singleton (1983)], the model does not perform very well. Although the sign of the estimated market price of risk $\lambda_{\Delta c}$ is positive, the t-statistics is statistically insignificant. The model can explain a poor 19% of cross-sectional variation in average returns. Furthermore, the intercept comes out statistically insignificant, in accordance with the economic theory. The test of over-identifying restriction based on the efficient weighting matrix does not reject the model statistically at conventional significance levels as there is a lot of noise. In fact, the first-stage GMM tests $J_{T,W}$ and $J_{T,HJ}$ clearly reject the model. In other words, the failure to reject is indicative of large pricing errors having even larger standard errors, probably blowing up the spectral density matrix, exactly opposite of what happened to the three factor Fama-French model.

This paper advocates a linear Campbell-Cochrane model, where the risk-aversion is proxied by the default and term premia. The results are presented in row 4 of table 3. The cross-sectional implication of this model is given by

$$\mathbf{E}\left[R^e_{i,t+1}\right] = \beta_{\Delta c}\,\lambda_{\Delta c} + \beta_{\Delta def}\,\lambda_{\Delta def} + \beta_{\Delta term}\,\lambda_{\Delta term} \tag{25}$$

Consistent with the economic theory, the intercept comes out statistically insignificant. In addition, the default and term factors are statistically significant, with the uncorrected t-statistics -3.335 and -2.101, respectively. The likelihood ratio test of Newey and West (1987a) confirms this result as J_T(restricted)-J_T(unrestricted)$= 35.715 - 29.283 = 6.432 > \chi^2_{95\%}(2) = 5.992$. The growth rate of nondurable consumption Δc is not statistically significant, but its market price of risk has the correct sign. The model is able to account for nearly 80% of cross-sectional variation in average returns. Furthermore, the test of the over-identifying restriction does not reject the model at conventional significance levels ($J_{T,S} = 29.283$ with p-value$=$ 0.137). Finally, based on equation (14) the model predicts that the factor loadings b_1 and b_2 on Δc and Δrra_{t+1} are equal to γ. Campbell and Cochrane (1999) calibrate $\gamma = 2$. Because we proxy the risk aversion with default and term premia, which are noisy yardsticks, we relax the restriction on the term-structure factors but impose the restriction $b_1 \equiv -\gamma = -2$. The test of over-identifying restriction yields[5] $J_{T,S} = 31.376$ with p-value$= 0.114$. We are still unable to reject the model at conventional significance levels.

3.3.1 Factor βs

Table 2 reports the average excess returns on 25 Fama-French portfolios sorted by size and the book-to-market ratio. For each book-to-market quintile, the average return tends to rise as size gets smaller. Similarly, for each size quintile, the average return tends to rise as the book-to-market ratio of the portfolios increases. This table, therefore, compactly summarizes the size and book-to-market effects uncovered by Banz (1981), Basu (1977), Fama and French (1992, 1993) and others.

The challenge for rational asset pricing is to develop credible models that can successfully account for these two phenomena. We claim in this paper that the linear Campbell-Cochrane model does exactly that. In fact, Table 4 reports the default betas. They tend to come out negative as the market price of risk $\lambda_{\Delta def}$ is negative as well. The small value portfolio has the largest magnitude of beta and is therefore most sensitive to changes in the default premium. In general, the default beta tends to fall in magnitude across size quintiles, holding the book-to-market ratio of portfolios constant, except the two smallest book-to-market quintiles. Similarly, the default beta falls[6] across book-to-market quintiles, holding the market equity of the portfolios fixed. As a result, the innovation in default premium is a good proxy for the risk factors uncovered by Fama and French (1992, 1993) as it picks both the size and book-to-market effects.

A careful reader may have noticed that the individual t-statistics for the default factor βs are seemingly indistinguishable from zero. To clear up the scepticism, we perform two types

[5] Details available from authors upon request.
[6] Note that the market price of risk of Δdef is negative.

of additional tests. Firstly, we use the asymptotic Wald test from the 25 seemingly unrelated regressions (SUR) and find that the $\beta_{\Delta def}$s are jointly significant across the 25 time-series regressions [Table 4, last column]. Secondly, in each time-series regression we perform a "single equation" F-test for the joint significance of both $\beta_{\Delta def}$ and $\beta_{\Delta term}$. The results are reported in the right bottom of Table 4. Apparently, except the small growth portfolio $S1B1$, we cannot accept the null hypothesis that both βs are zero. We interpret the seeming statistical insignificance of $\beta_{\Delta def}$ as a multi-collinearity problem.

Table 4, Panel C, reports the term premia betas. It seems that the betas vary across book-to-market quintiles but not much across size quintiles. We interpret the innovation in term premium as an additional risk factor that mimics the risk factors related to the book-to-market ratio.

In conclusion, as we mentioned before, DEF tracks the variation in risk aversion that transcends the business cycle. Both small and value stocks are very sensitive to the variation in this component of aggregate risk aversion. For example, DEF was particularly high during the 1980s when small stocks did exceptionally poorly. In contrast, $TERM$ picks the variation in risk aversion that is related closely to business cycles. It seems that value stocks, but not small stocks, are particularly sensitive also to this business-cycle related vicissitude in the risk aversion.

3.3.2 Euler Equations Errors

Figure 1 plots the Euler equation pricing errors e_{t+1}, or α's as they are better known in the investment literature, especially on CAPM. Formally, Euler equations errors are defined

$$e^i = \mathbf{E}\left[m_{t+1}R^e_{i,t+1}\right], \quad i = 1, ..., 25 \tag{26}$$

where R^e_{it} is ith portfolio out of 25 Fama-French portfolios in excess of the risk-free rate. A simple manipulation shows that

$$\frac{e^i}{\mathbf{E}[m_{t+1}]} = \mathbf{E}\left[R^e_{i,t+1}\right] - \left[-cov(m_{t+1}, R^e_{i,t+1})\right] = Average\ Return - Fitted\ Return \quad (27)$$

Because $\mathbf{E}[m_{t+1}]$ is close to one, we refer to the Euler equation errors as *pricing errors.* Cochrane (1996, 2001) derives the asymptotic distribution for these pricing errors.

Two models stick out as potentially successful descriptions of the cross-section of expected returns based on the empirical results yielded by the Fama-MacBeth regressions: (i) the Fama-French three-factor model and (ii) the linear Campbell-Cochrane model that we advocate in this paper. Firstly, the last one is not formally rejected in the data, whereas Fama and French (1993) interpret the statistical rejection of their model as small pricing errors having even smaller standard errors. Figure 1 provides a visual impression of the relative empirical performance of each model we investigate. For a given empirical specification, we portray the fitted expected returns for each of the 25 portfolios against their realized average returns. For reference, the pricing errors along with their standard errors for these plots are given in Table 5, Panels B and C, for the two aforementioned models.

Figure 1, top left panel, graphically portrays the failure of the Sharpe-Lintner model, which explains virtually no variation in average returns. The pricing errors reported in Table 5, Panel A, are large, with the mean absolute error MAE above 70 basis points per quarter. Figure 1, bottom left panel, shows that the Canonical CCAPM explains some, but generally small, variation in average returns on these portfolios. The mispricing of the smallest quintiles portfolios is particularly evident. To be specific, portfolios 11 (small growth), 14 and 15 (small value), lie particularly far from the 45 degree line. The mean absolute pricing error for the Canonical CCAPM is $MAE = 58.6$ basis points per quarter.

When the fitted returns from the linear Campbell-Cochrane model are compared with those of the Fama-French three factor model, it is clear that the linear Campbell-Cochrane model does about as well as, if not better than, the Fama-French. Both models have difficulty pricing portfolios 11 (small growth) and 15 (small value). Interestingly, the linear Campbell-Cochrane model does significantly better than Fama-French in pricing 51 (big growth) portfolio. The mean absolute pricing error, based on Table 5, Panel B and C, is $MAE = 40$ basis points per quarter for Fama-French and $MAE = 34.62$ basis points per quarter for the linear Campbell-Cochrane.

We conclude that the linear Campbell-Cochrane model, advocated in this paper, produces the lowest pricing errors and the test of the over-identifying restriction does not reject the model statistically.

3.3.3 Including Characteristics

This section investigates whether there are any residual effects of firm characteristics in the linear Campbell-Cochrane model evaluated above. Kan and Zhang (1999) argue that "useless" factors can appear statistically significant in the Fama-MacBeth Methodology, when the model being tested is misspecified. Berk (1995) and Jagannathan and Wang (1998) show that this misspecification can be tested for by including firm characteristics as additional explanatory variables in cross-sectional asset pricing tests. In fact, Jagannathan and Wang, in Theorem 6, prove that a useless factor cannot drive out a firm characteristics in the cross-sectional (second-pass) regression, and Berk demonstrates that if the model is misspecified, including (log) market equity of a firm picks up the effects of the missing factors and the coefficient in the cross-sectional regression should be negative. Because of Berk's result and the fact that the portfolios are sorted based on market size and book-to-market ratio, we examine the model misspecification using two characteristics: (log) market equity and (log) book-to-market ratio. In detail, we include portfolio size, measured as the time-series average of the log of market equity for each portfolio, and the portfolio book-to-market, again gauged as the time-series average of the book-to-market ratio for each portfolio. A large t-statistic on the characteristics term suggests that the model may be misspecified. We present these results in table 6. Panel A reports the results when the characteristic included in the cross-sectional regression is size. Panel B reports the results when book-to-market is included.

In Panel A, column *size* is consistent with Berk (1995) in that all coefficients on size are negative. Specifically, row 1 reports the well-known result for the Sharpe-Lintner model, namely, the coefficient on size is strongly significant, and the R^2s rises from 1% to nearly 78% when it is included as the explanatory variable. In addition, the market price of risk, though still negative, becomes statistically significant. Rows 2-3 report the results for the Fama-French three-factor model and the canonical CCAPM of Lucas (1978) and Breeden (1979). In all cases, the coefficient on size comes out negative and strongly significant. In particular, the results for the influential Fama-French model indicate in Berk's (1995) words that some factor(s) is still missing. In contrast, as row 4 reports, the results for the linearized Campbell-Cochrane model are encouraging: the coefficient on size is negative but statistically insignificant. The characteristic does not drive out our risk factors in the cross-sectional regression, and the overall fit is roughly the same regardless of whether or not size is included in the regression.

Panel B, row 1, shows another failure of Sharpe-Lintner model: the coefficient on the book-to-market ratio is strongly significant and the R^2 rises by more than 70% once the book-to-market ratio is included in the Sharpe-Lintner model. In contrast to size, the Fama-French model reported in row 2 does not have difficulty eliminating the residual book-to-market effects. The Canonical CCAPM in row 3 is unable to eliminate the residual book-to-market effects which is indicative that it is perhaps misspecified. The linear Campbell-Cochrane model, row 5, fares very well: the coefficient on the book-to-market ratio is statistically insignificant, and the R^2 practically doesn't change once the book-to-market ratio is included in the model.

In summary, the linear Campbell-Cochrane model performs better in explaining the cross-section of returns than the other familiar models considered, as portfolio characteristics do not show up as significant explanatory variables.

3.4 Conditional Test of the Model

In this section, we test the time-series implications of the linear Campbell-Cochrane model. Following Cochrane (1996), the analysis is conducted in the language of the stochastic discount factor. The primary testable asset pricing implications of the model are the set of Euler equations, repeated here for convenience,

$$\mathbf{E}_t \left[M_{t+1} \, R^e_{i,t+1} \right] \; = \; 0 \tag{28}$$

In order to keep the total number of moments manageable[7], we use a small number of assets and add instruments informative of the state of the U.S. economy. In our estimation, we center on the three portfolios: value-weighted market return in excess of the three-month Treasury Bill, the small-minus-big SMB portfolio and the value-minus-growth HML portfolio. Fama and French (1993) discover that these portfolios pick up the common variation in returns across the 25 Fama-French portfolios.

We condition down the model using 7 well-known instruments in addition to a constant. In detail, we use variables known to predict excess returns well: (i) the price-dividend ratio [Campbell and Shiller (1988), Fama and French (1988), Cochrane (1994)], (ii) Lettau and Ludvigson's (2001) co-integrating residual $\widehat{cay}$, (iii) term and default premia [Fama and French (1989)], (iv) size (SMB) and value (HML) spreads [Cohen, Polk and Vuolteenaho (2003)], (v) nondurable consumption growth rate and (vi) a constant. All instruments are lagged twice to account for the time aggregation in consumption data [cf. Hall (1988)]. Furthermore, Ogaki (1988) shows that the additional lag is consistent with the information structure of a monetary

[7] Hansen and Singleton (1982) warn that large number of moment conditions may affect the small sample properties of GMM.

economy with cash-in-advance constraints. The scaled Euler equations, after conditioning down, take the form

$$\mathbf{E}\left[M_{t+1}\,R^e_{i,t+1}\otimes Z_t\right] \;=\; 0 \tag{29}$$

where Z_t is a 1×7 vector of instrumental variables.

To form a basis for comparison, we begin by presenting results from a series of familiar models that we discussed in the previous sections. Table 7, row 1, reports the results for the static Sharpe-Lintner model

$$M_{t+1} \;=\; 1\;+\;b_1\,R_{W,t+1} \tag{30}$$

The factor loading b_1 on the value-weighted market return comes out significant and with a correct, negative sign. The model however is able to capture neither the average excess returns nor any time variation in the conditional expected return on the benchmark portfolios, namely, excess market return, SMB and HML; the asymptotic J_T statistics is above 50 no matter what weighting matrix we use (i.e. identity matrix W for $J_{T,W}$, Hansen-Jagannathan's (1997) distance matrix for $J_{T,HJ}$ and the efficient weighting matrix for $J_{T,S}$), and the model is statistically rejected. This result is not surprising given that the Sharpe-Lintner model already had trouble pricing the average, unconditional, returns on these benchmark assets. The results for the Canonical CCAPM

$$M_{t+1} \;=\; 1\;+\;b_1\,\Delta\,c_{t+1} \tag{31}$$

are reported in row 3 and the conclusion is practically the same as for the Sharpe-Lintner model - there is an apparent inability to capture the time variation in the conditional expected returns on the benchmark assets.

Table 7, row 2, presents the results for the three factor Fama-French model [Fama and French (1992, 1993)], which in the stochastic discount factor language is

$$M_{t+1} \;=\; 1\;+\;b_1\,R_{W,t+1}\;+\;b_2\,SMB_{t+1}\;+\;b_3\,HML_{t+1} \tag{32}$$

All factor loadings come out with a correct sign and except for b_2, all are statistically significant. The model appears to track successfully the temporal variation in conditional expected returns on the benchmark portfolios. In fact, the asymptotic $J_{T,S}$ statistic based on the efficient weighting matrix comes out slightly above 32, with p-value 0.07. The row 2, column 7, reports the Newey-West's (1987) ΔJ_T test of whether the incremental ability of SMB and HML is significant. The result is not surprising given Fama and French (1992, 1993). However, the Hansen-Jagannathan's (1997) distance metric as measured by $J_{T,HJ}$ is statistically significant. We interpret this result as saying that the misspecification of the stochastic discount proxy M_{t+1} is important and the maximum pricing errors in the payoff space are large.

Table 7, row 4, presents the estimated factor loadings for the linearized Campbell-Cochrane model

$$M_{t+1} = 1 + b_1 \Delta c + b_2 \Delta def + b_3 \Delta term \tag{33}$$

According to the original Campbell-Cochrane (1999) calibration, $\gamma = 2$. We therefore impose the theoretical restriction from the linearization (see the theoretical section) $b_1 = -\gamma = -2$. Furthermore, although the theory has implications also for the factor loading on the coefficient of risk aversion Δrra, we hesitate to impose them as the default and term spreads are fairly noisy measures of aggregate risk aversion. Thus, in row 4 we keep b_2 and b_3 unrestricted in the estimation. According to the linearization, the factor loading(s) on the factor Δrra should come out positive. Indeed, we estimate b_2 and b_3 with the correct sign and statistically significant. The test of the over-identifying restriction does not reject the model for (i) identity weighting matrix [$J_{T,W} = 8.356$, p-value = 0.996], (ii) Hansen-Jagannathan's (1997) distance metric [$J_{T,HJ} = 25.961$, p-value = 0.253], and (iii) the Hansen's (1982) efficient weighting matrix [$J_{T,S} = 27.318$, p-value 0.199]. These results indicate that the model produces small Euler equation errors rather than blowing up the spectral density matrix. We conclude that the linearized Campbell-Cochrane model successfully tracks the time-variation in the conditional expected returns on the equity premium, and the small-minus-big (SMB) and the value-minus-growth (HML) spreads. Furthermore, as the Newey-West's (1987) ΔJ_T test reported in column 9 indicates, the incremental ability of SMB and HML is not statistically significant. In row 5 we impose the tight theoretical restriction $-b_1 = b_2 = b_3$ and still do not reject the model.

4 Conclusion

This paper makes empirical use of the structural consumption-based Campbell-Cochrane (1999) model to explain (i) the cross-sectional differences in average returns, and (ii) temporal variation in conditional expected returns, on 25 Fama-French size- and book-to-market- sorted portfolios. Based on the model's intertemporal marginal rate of substitution, we propose a three-factor linear pricing model. Our risk factors are the nondurable consumption growth rate and the change in aggregate risk aversion. We map the unobservable risk aversion to two interesting term-structure related proxies - default and term spreads. We discover that our structural model is capable of accounting for the average returns on 25 Fama-French portfolios with slightly smaller pricing errors than the empirically-based three-factor Fama-French model. Furthermore, the linear factor structure successfully captures the average, unconditional, moments and the time variation in the first conditional moments of the equity premium, and SMB and HML spreads.

References

[1] Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics*, Vol. 9, Issue 1, Pages 3-18

[2] Basu, S., 1977, The Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis, *Journal of Finance* 32, 663-682

[3] Berk, Jonathan B., 1995, A Critique of Size-Related Anomalies, *Review of Financial Studies* 8, 275-286

[4] Breeden, Douglas T., 1979, An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities, *Journal of Financial Economics* 7, 265-296

[5] Breeden, Douglas T., Michael R. Gibbons, and Robert H. Litzenberger, 1989, Empirical Tests of the Consumption-Oriented CAPM, *Journal of Finance* 44, 231-262

[6] Campbell Y. John and Robert J. Shiller (1988), Stock Prices, Earnings, and Expected Dividends, *Journal of Finance*, Vol. 43, No. 3, Papers and Proceedings of the Forty-Seventh Annual Meeting of the American Finance Association, Chicago, Illinois, 661-676

[7] Campbell Y. John (1996), Understanding Risk and Return, *Journal of Political Economy*, Vol. 104, No. 2, 298-345

[8] Campbell, John Y., and John H. Cochrane (1999), By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior, *Journal of Political Economy* 107, 205-251

[9] Chen, Nai-Fu, Richard Roll, and Stephen A. Ross (1986), Economic Forces and the Stock Market, *Journal of Business*, Vol. 59, No. 3, 383-403

[10] Cochrane H. John (1994), Permanent and Transitory Components of GNP and Stock Prices, *Quarterly Journal of Economics*, Vol. 109, No. 1, 241-265

[11] Cochrane, John H., 1996, A Cross-Sectional Test of an Investment-Based Asset Pricing Model, *Journal of Political Economy* 104, 572-621

[12] _____ , 2001, *Asset Pricing*, Princeton, N.J.: Princeton University Press

[13] Cohen B. Randolph, Christopher Polk and Tuomo Vuolteenaho (2003), The Value Spread, *Journal of Finance*, Vol. 58, No. 2, 609-640

[14] Constantinides, George M., 1990, Habit Formation: A Resolution of the Equity Premium Puzzle, *Journal of Political Economy* 98, 519-543

[15] Dybvig and Ingersoll (1982), Mean-Variance Theory in Complete Markets, *Journal of Business*, Vol. 55, No. 2, 233-251

[16] Fama, Eugene F., and Kenneth R. French, 1988, Dividend Yields and Expected Stock Returns, *Journal of Financial Economics* 22, 3-25

[17] _____ , 1989, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics* 25, 23-49

[18] _____ , 1992, The Cross-Section of Expected Returns, *Journal of Finance* 47, 427-465

[19] _____ , 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3-56

[20] Fama, Eugene F., and James D. MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy* 81, 607-636

[21] Gebhardt R. William, Soeren Hvidkjaer and Bhaskaran Swaminathan (2005), The Cross Section of Expected Corporate Bond Returns: Betas or Characteristics?, *Working Paper*, Johnson Graduate School of Management, Cornell University

[22] Hall E. Robert (1988), Intertemporal Substitution in Consumption, *Journal of Political Economy*, Vol. 96, No. 2, 339-357

[23] Hansen, Lars Peter (1982), Large Sample Properties of Generalized Method of Moments Estimators, *Econometrica* 50, 1029-1054

[24] Hansen, Lars Peter, John Heaton, and Amir Yaron (1996), Finite-Sample Properties of Some Alternative GMM Estimators, *Journal of Business and Economic Statistics* 14, 262-280

[25] Hansen, Lars Peter, and Ravi Jagannathan (1997), Assessing Specification Errors in Stochastic Discount Factor Models, *Journal of Finance* 52, 557-590

[26] Hansen, Lars Peter, and Singleton J. Kenneth (1982), Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica* 55, 1269-1286

[27] _____ , 1983, Stochastic Consumption, Risk Aversion and the Temporal Behavior of Asset Returns, *Journal of Political Economy* 91, 249-265

[28] Heaton John (1995), An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications, *Econometrica*, Vol. 63, No. 3, 681-717

[29] Jagannathan, Ravi, and Zhenyu Wang (1996), The Conditional CAPM and the Cross-Section of Expected Returns, *Journal of Finance* 51, 3-53

[30] _____ (1998), An Asymptotic Theory for Estimating Beta-Pricing Models Using Cross-Sectional Regression, *Journal of Finance* 53, 1285-1309

[31] Kan, Raymond, and Chu Zhang (1999), Two-Pass Tests of Asset Pricing Models with Useless Factors, *Journal of Finance* 54, 203-235

[32] Lettau, Martin, and Sydney Ludvigson, 2001a, Consumption, Aggregate Wealth and Expected Stock Returns, *Journal of Finance* 56, 815-849

[33] Lettau, Martin, and Sydney Ludvigson, 2001b, Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying, *Journal of Political Economy*, 109(6), 1238-1287

[34] Lintner, John, 1965, Security Prices, Risk and Maximal Gains from Diversification, *Journal of Finance* 20, 587-615

[35] Merton, Robert C., 1973, An Intertemporal Capital Asset Pricing Model, *Econometrica* 41, 867-886

[36] Newey, Whitney K., and Kenneth D. West, 1987a, Hypothesis Testing with Efficient Method of Moments Estimation, *International Economic Review* 28, 777-787

[37] _____, 1987b, A Simple, Positive Semi-definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703-708

[38] Ogaki Masao (1988), Learning about Preferences from Time Trends, *Ph.D. Dissertation*, University of Chicago

[39] Petkova Ralitsa (2006), Do the Fama-French factors proxy for innovations in predictive variables?, *Journal of Finance*, Vol. 61, 581-612

[40] Ross, Stephen A. (1978), A Simple Approach to the Valuation of Risky Streams, *Journal of Business* 51, 453-475

[41] Shanken Jay (1992), On the Estimation of Beta-Pricing Models, *Review of Financial Studies* 5(1), 1-33

[42] Sharpe, William F. (1964), Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance* 19, 425-42

[43] Sundaresan, Suresh M. (1989), Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth, *Review of Financial Studies*, 2(1), 73-89

Table 1: Descriptive Statistics

Variable	Mean (%)	S.D. (%)	Autocorrelation	Correlation							
				Mkt	SMB	HML	Nondurables	DEF	TERM	$\Delta\, def$	$\Delta\, term$
Market	1.537	8.671	0.027								
SMB	1.080	6.144	-0.019	0.495							
HML	1.064	6.258	0.137	-0.370	-0.108						
Nondurables	0.528	0.478	0.360	0.172	0.079	0.007					
DEF	1.012	0.440	0.903	0.147	0.072	-0.002	-0.261				
TERM	2.314	1.558	0.843	0.144	0.112	0.073	0.069	0.203			
$\Delta\, def$	1.313	17.319	-0.010	0.060	-0.052	-0.133	-0.254	0.226	-0.139		
$\Delta\, term$	0.404	6.542	-0.022	-0.325	-0.032	-0.084	0.011	-0.185	-0.339	-0.222	

NOTE - The table reports the mean, standard deviation, and first-order autocorrelation of excess market return Mkt, SMB return, HML return, real nondurable consumption growth, default premium DEF = the yield spread between Baa and Aaa bonds, term premium $TERM$ = spread between yields on Aaa bonds and 3-month Trasury Bill rate, and their respective (log) growth rates $\Delta\, def$ and $\Delta\, term$. It also reports the correlations among these variables. Sample period 1964,1-2004,4.

Table 2: Average Returns on 25 Fama-French Portfolios

	Book-to-Market Equity					Book-to-Market Equity				
Size	Low	2	3	4	High	Low	2	3	4	High
	Average Return					t-statistics				
Small	1.189	2.837	2.954	3.588	3.939	0.895	2.522	2.990	3.815	3.831
2	1.545	2.235	2.985	3.145	3.423	1.319	2.277	3.452	3.727	3.749
3	1.539	2.405	2.345	2.789	3.349	1.446	2.775	3.038	3.617	3.930
4	1.804	1.689	2.337	2.689	2.811	1.887	2.112	3.200	3.681	3.445
Big	1.325	1.473	1.533	1.726	1.824	1.803	2.219	2.592	2.899	2.698

NOTE - The table reports the mean returns on 25 Fama-French portfolios and the t-statistics for the mean. Sample period 1964,1-2004,4.

Table 3: Fama-MacBeth Regressions Using 25 Fama-French Portfolios: λ_j Coefficient Estimates on Betas in Cross-Sectional Regression

		Factors$_{t+1}$						R^2	$J_{T,W}$	$J_{T,S}$	$J_{T,HJ}$
Row	Constant	R_w	SMB	HML	$\Delta\, c$	$\Delta\, def$	$\Delta\, term$	$\{\bar{R}^2\}$	(p-value)	(p-value)	(p-value)
1	0.028	-0.0034						0.010	95.932	70.171	96.907
	(2.982)	(-0.305)						(0.200)	(0.000)	(0.000)	(0.000)
	{2.979}	{-0.262}						{-0.033}		[0.000]	
2	0.039	-0.023	0.008	0.015				0.785	67.211	47.498	64.585
	(3.310)	(-1.729)	(1.668)	(2.757)				(0.000)	(0.000)	(0.001)	(0.000)
	{3.043}	{-1.448}	{1.141}	{1.930}				{0.754}		[0.002]	
3	0.006				0.005			0.194	59.746	35.715	60.052
	(0.797)				(1.753)			(0.000)		(0.058)	(0.000)
	{0.563}				{1.233}			{0.159}		[0.114]	
4	-0.009				0.002	-0.119	-0.044	0.797	43.312	29.283	48.576
	(-0.828)				(0.795)	(-3.335)	(-2.101)	(0.000)	(0.009)	(0.137)	(0.002)
	{-0.538}				{0.515}	{-2.113}	{-1.348}	{0.767}		[0.097]	

NOTE - The table presents λ estimates from cross-sectional Fama-MacBeth regressions using excess returns of 25 Fama-French portfolios: $\mathbf{E}[R^e_{i,t+1}] = \beta'\lambda$. The individual λ_j estimates (from the second-pass cross-sectional regression) for the beta of the factor listed in the column heading are reported. In the first stage, the time-series betas β are computed in one multiple regression of the portfolio returns on the factors. The term R_w is the excess return on the value-weighted CRSP index, SMB and HML are the Fama-French mimicking portfolios related to size and book-to-market equity ratios, $\Delta\, c$ is the (log) growth rate in nondurable consumption and services, $\Delta\, def$ is the (log) growth rate in the default premium, computed as the yield spread on Baa and Aaa corporate bonds, and $\Delta\, term$ is the (log) growth rate in the term premium, defined as the spread between the Aaa corporate bond yields and the 3-month Treasury Bill rate. The table reports the Fama-McBeth cross-sectional regression coefficient; in parentheses is the Fama-MacBeth t-statistics, and in curly brackets Shanken correction t-statistics. The term R^2 denotes unadjusted cross-sectional R^2 statistics, in brackets below is the p-value for the F-test for the significance of the whole cross-sectional regression, and in curly brackets $\{\bar{R}^2\}$ adjusts for the degrees of freedom. The term $J_{T,W}$ provides the test of the over-identifying restriction based on the identity weighting matrix, $J_{T,HJ}$ is based on the Hansen-Jagannathan's (1997) distance matrix (pseudo-inverted due to singularity), and $J_{T,S}$ is based on the continuous-updating GMM. Asymptotic p-values are below in parentheses, and bootstrapped (based on 10,000 simulations of two-stage GMM) p-values are in brackets [cf. Hall and Horowitz (1996)].

Table 4: Factor Betas for the 25 Fama-French Portfolios

	Book-to-Market Equity					Book-to-Market Equity					Wald Test
Size	Low	2	3	4	High	Low	2	3	4	High	(p-value)
			$\beta_{\Delta c}$					$t_{\Delta def}$			
Small	4.867	5.907	4.544	4.642	5.195	2.046	2.904	2.597	2.628	2.822	81.497
2	4.221	3.755	4.221	3.964	5.122	1.915	2.054	2.660	2.577	3.174	(0.000)
3	3.677	3.589	3.660	3.638	4.441	1.915	2.054	2.660	2.577	3.174	
4	3.387	3.541	3.061	3.237	4.620	1.666	2.434	2.219	2.488	2.833	
Big	3.288	2.316	3.553	2.697	3.924	2.231	1.769	3.365	2.462	3.541	
			$\beta_{\Delta def}$					$t_{\Delta def}$			
Small	-3.095	-4.412	-6.259	-7.313	-9.489	-0.577	-0.904	-1.502	-1.892	-2.138	102.056
2	1.609	-2.314	-2.189	-4.239	-7.575	0.344	-0.540	-0.603	-1.228	-2.011	(0.000)
3	2.958	-0.357	-1.523	-2.461	-5.839	0.703	-0.104	-0.509	-0.789	-1.710	
4	4.528	0.290	-0.851	-2.184	-3.127	1.308	0.100	-0.334	-0.801	-0.958	
Big	3.078	2.226	-0.007	-1.507	-0.396	1.235	0.917	-0.004	-0.717	-0.190	
			$\beta_{\Delta term}$					$t_{\Delta term}$			
Small	-3.910	-4.343	-4.658	-4.457	-4.840	-2.130	-3.010	-3.360	-3.430	-3.623	95.554
2	-4.143	-4.978	-5.215	-5.676	-5.357	-2.595	-3.593	-4.082	-4.760	-4.682	(0.000)
3	-4.762	-5.170	-5.313	-5.501	-5.538	-2.595	-3.593	-4.082	-4.760	-4.682	
4	-4.021	-5.074	-5.232	-5.743	-5.791	-2.976	-4.296	-5.032	-4.995	-4.852	
Big	-3.822	-4.179	-4.314	-4.814	-4.801	-3.053	-4.073	-4.822	-5.909	-5.090	
			R^2s in %					Single Equation F-test			
Small	4.059	7.709	9.411	10.528	11.436	0.139	0.026	0.003	0.001	0.001	
2	5.233	8.566	12.509	14.912	14.629	0.055	0.003	0.000	0.000	0.000	
3	7.516	11.528	15.173	16.099	15.377	0.007	0.000	0.000	0.000	0.000	
4	7.854	13.222	15.624	18.574	17.279	0.006	0.000	0.000	0.000	0.000	
Big	11.144	13.042	18.720	19.603	17.566	0.001	0.000	0.000	0.000	0.000	

NOTE - The table reports the estimated factor loadings $\beta_{\Delta c}$, $\beta_{\Delta def}$ and $\beta_{\Delta term}$, and the corresponding HAC Newey-West t-statistics (5 Bartlett lags), time-series R^2s and the asymptotic F-test that $\beta_{\Delta def}$ and $\beta_{\Delta term}$ are jointly significant. The regression we run is

$$R^e_{it} = \beta_{i,0} + \beta_{i,\Delta c}\Delta c + \beta_{i,\Delta def}\Delta def + \beta_{i,\Delta term}\Delta term + \varepsilon_{i,t} \tag{34}$$

The last column reports the asymptotic Wald test and its corresponding p-value from an SUR system, testing the joint significance of the each of the 25 factor loadings across the 25 regressions simultaneously. Sample period 1963,1-2004,4.

Table 5: Euler Equations Errors

Size Quintile	Book-to-Market Equity (BE/ME) Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Panel A. Sharpe-Lintner Model									
	Pricing Errors (%)					Standard Errors (%)				
Small	-2.233	-0.130	0.366	1.149	1.354	0.046	0.025	0.025	0.026	0.029
2	-1.684	-0.458	0.571	0.858	1.046	0.032	0.016	0.018	0.022	0.027
3	-1.461	-0.090	0.199	0.704	1.155	0.036	0.014	0.018	0.024	0.030
4	-0.950	-0.593	0.257	0.636	0.651	0.038	0.020	0.020	0.024	0.026
Big	-0.819	-0.464	-0.097	0.118	0.100	0.043	0.030	0.027	0.030	0.038
MAE	72.57 bp									
	Panel B. Three-Factor Fama-French Model									
	Pricing Errors (%)					Standard Errors (%)				
Small	-1.240	-0.288	-0.149	0.327	-0.103	0.024	0.017	0.016	0.017	0.016
2	-0.622	-0.558	0.028	-0.227	-0.494	0.017	0.017	0.016	0.017	0.017
3	-0.165	-0.150	-0.464	-0.393	-0.391	0.017	0.017	0.016	0.015	0.018
4	0.528	-0.642	-0.265	-0.166	-0.539	0.023	0.018	0.015	0.020	0.021
Big	0.424	-0.183	-0.268	-0.566	-0.820	0.021	0.015	0.017	0.015	0.024
MAE	40.00 bp									
	Panel C. Linear Campbell-Cochrane Model									
	Pricing Errors (%)					Standard Errors (%)				
Small	-0.953	-0.449	-0.427	0.247	-0.209	0.036	0.014	0.023	0.019	0.020
2	0.088	-0.010	-0.348	-0.094	-0.154	0.025	0.030	0.020	0.024	0.026
3	-0.091	-0.232	-0.346	-0.352	0.184	0.022	0.021	0.018	0.021	0.034
4	0.495	-0.299	-0.329	-0.241	-0.468	0.020	0.021	0.019	0.022	0.032
Big	-0.219	-0.283	-0.638	-0.590	-0.909	0.046	0.028	0.045	0.026	0.048
MAE	34.62 bp									

NOTE - This table reports the Euler equations pricing errors $e^i = \mathbf{E}[m_{t+1}R^e_{i,t+1}]$ (in percent per quarter) from the first-stage GMM with the identity weighting matrix along with their standard errors [cf. Cochrane (1996)]. The term S1 refers to the portfolios with the smallest firms, and S5 includes the largest firms. Similarly, B1 includes firms with the lowest book-to-market ratio and B5 the highest. The model is estimated using data from 1963:Q4 to 2004:Q4.

Table 6: Fama-MacBeth Regressions Including Characteristics

Panel A. λ_j Estimates on Betas in Cross-Sectional Regressions Including Size

		Factors$_{t+1}$							R^2
Row	Constant	R_w	SMB	HML	$\Delta\, c$	$\Delta\, def$	$\Delta\, term$	Size	$\bar{R}^2$
1	0.090	-0.031						-0.005	0.779
	(4.743)	(-2.498)						(-3.844)	0.770
2	0.070	0.006	-0.017	0.011				-0.007	0.825
	(4.046)	(0.373)	(-1.672)	(2.151)				(-3.001)	0.800
3	0.039				0.000			-0.003	0.343
	(2.810)				(0.124)			(-2.097)	0.315
4	0.006				0.000	-0.092	-0.045	-0.001	0.829
	(0.308)				(0.290)	(-2.201)	(-2.202)	(-0.858)	0.804

Panel B. λ_j Estimates on Betas in Cross-Sectional Regressions Including Book-to-Market Ratio

		Factors$_{t+1}$						Book-to-	R^2
Row	Constant	R_w	SMB	HML	$\Delta\, c$	$\Delta\, def$	$\Delta\, term$	-Market	$\bar{R}^2$
1	0.008	0.019						0.014	0.723
	(0.767)	(1.414)						(3.761)	0.711
2	0.043	-0.021	0.007	0.001				0.008	0.804
	(3.669)	(-1.523)	(1.494)	(0.104)				(1.834)	0.776
3	0.013				0.004			0.009	0.664
	(1.530)				(1.360)			(2.589)	0.649
4	-0.009				0.002	-0.119	-0.044	-0.0001	0.797
	(-0.658)				(0.901)	(-2.404)	(-2.145)	(-0.014)	0.767

NOTE - See notes to table 3. This table presents estimates of cross-sectional Fama-MacBeth regressions using the excess returns on 25 Fama-French portfolios:

$$E[R^e_{i,t+1}] = \beta'\,\lambda + d\,\Theta_i$$

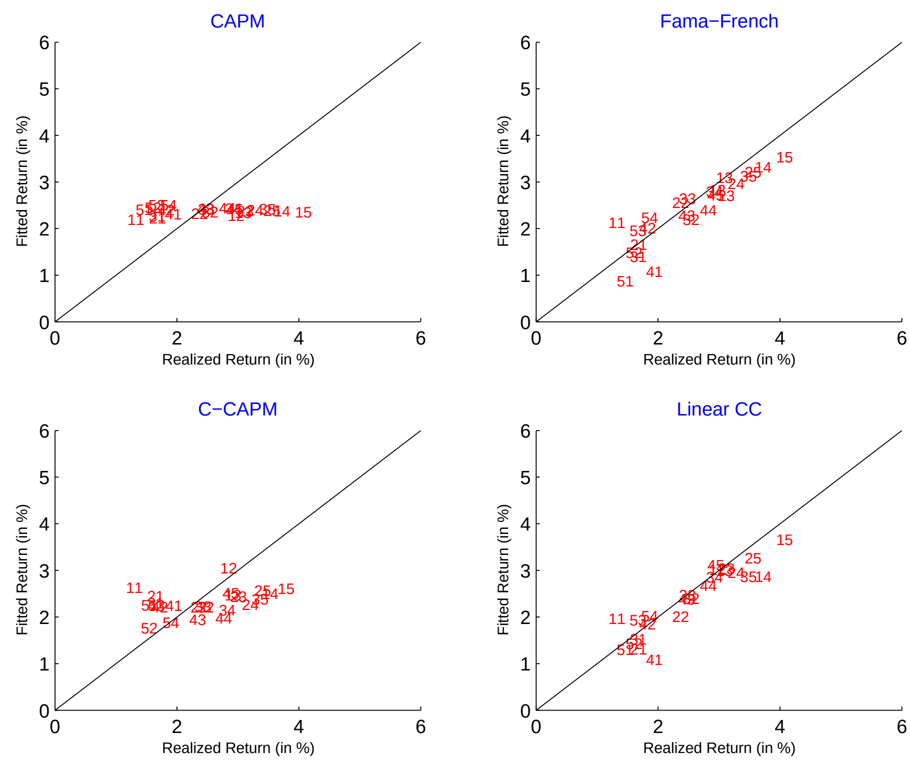
where Θ_i denotes a characteristic variable: Θ_i is either the log of the portfolio size (in panel A) or the log of the portfolio book-to-market ratio (in panel B).

Table 7: GMM Results: Time-Series Test

Row	Model	SDF Loadings b_1	b_2	b_3	$J_{T,W}$ (p-value)	$J_{T,HJ}$ (p-value)	$J_{T,S}$ (p-value)	$\Delta J_{T,S}$
1	CAPM	-5.042			57.401	57.174	56.211	
		(0.692)			(0.000)	(0.000)		
2	FF	-6.132	-2.429	-12.532	49.109	43.218	32.279	23.932
		(1.318)	(1.325)	(1.605)	(0.001)	(0.004)	(0.073)	(0.000)
3	CCAPM	-930.209			53.082	45.794	38.211	6.047
		(144.854)			(0.000)	(0.017)	(0.002)	(0.049)
4	Linear CC	-2	34.158	19.159	8.356	25.961	27.318	2.707
		(calibrated)	(5.906)	(6.180)	(0.996)	(0.253)	(0.199)	(0.258)
5	Linear CC	-15.046	15.046	15.046	16.914	13.702	29.961	1.797
		(40.154)	(2.263)	(2.263)	(0.768)	(0.912)	(0.119)	(0.407)

NOTE - The table reports the factor loadings b_is in the stochastic discount factor for the four models (for the efficient GMM case). In the linearized Campbell-Cochrane model, b_1 corresponds to the minus the utility curvature parameter γ. Campbell and Cochrane (1999) calibrate $\gamma = 2$. Therefore, in row 4 we restrict $b_1 = -2$ in the estimation. In row 5 we impose the theoretical restriction that $b_1 = -b_2 = -b_3$. The test assets are the value-weighted market portfolio (in excess of the three-month Treasury Bill rate), SMB portfolio and HML portfolio. The instruments are second lags (due to time aggregation) of nondurable consumption growth rate, dividend-price ratio, size spread, value spread, term spread, default spread, $\widehat{cay}_t$ of Lettau and Ludvigson (2001), and a constant. We report the tests of over-identifying restrictions for (i) the 1st stage GMM case (identity weighting matrix, column 6), (ii) 1st stage GMM with Hansen-Jagannathan weighting matrix $J_{T,HJ}$ (column 7), and (iii) the continuous-updating GMM of Hansen, Heaton and Yaron (1996), the test of the over-identifying restriction is denoted $J_{T,S}$ (column 8) . HAC standard errors and the p-values are in parentheses. Column 9 reports Newey-West's (1987) ΔJ_T test of whether SMB and HML contain incremental ability to explain asset prices. HAC standard errors and the p-values are in parentheses. Sample size 1964:Q1 - 2004:Q4.

Figure 1: Realized vs. Predicted Excess Returns for the 25 Fama-French Portfolios



NOTE - Realized vs. fitted excess returns: 25 Fama-French portfolios. The top left panel corresponds to Sharpe-Linter (CAPM) model, the top right panel corresponds to the Three-Factor Fama-French Model, the bottom left panel corresponds to the Canonical Consumption-Based CAPM, and the bottom right panel to the linearized Campbell-Cochrane model. The figure shows the pricing errors for each of the 25 Fama-French portfolios for the four models. Each two-digit number represents one portfolio. The first digit refers to the size quintiles (1 indicating the smallest firms, 5 the largest), and the second digit refers to book-to-market quintiles (1 indicating the portfolio with the lowest book-to-market ratio, 5 with the highest). The pricing errors are generated using the Fama-MacBeth regressions in Table 3.