Estimating the Term Structure of Yield Spreads from Callable Corporate Bond Price Data[1]

Antje Berndt[2]

First Version: February 28, 2003
Current Version: December 16, 2004

[1] I am grateful to Darrell Duffie for extensive discussions. I thank Amir Dembo, Tze Leung Lai, Art Owen, Philip Protter, and Kenneth Singleton for valuable insights; and Eric Stone for a careful reading of this manuscript. The paper also profited by comments from seminar participants at Stanford University, the INFORMS Annual Meeting, the Stochastic Analysis in Finance and Insurance Meeting in Oberwolfach, at Deutsche Bank Frankfurt, the JSM Annual Meeting, and at Cornell University. I thankfully acknowledge financial support from the Moody's Investor Services Research Grant.

[2] **Correspondence Information:** School of Operations Research and Industrial Engineering, Cornell University, Ithaca, NY 14853, `mailto:antje.berndt@cornell.edu`.

Abstract

I extract credit pricing information from the prices of callable corporate debt, by disentangling the components of callable corporate bond prices associated with discounting at market interest rates, discounting for default risk, and optionality. The results include the first empirical analysis, in the setting of standard arbitrage-free term-structure models, of the time-series behavior of callable corporate bond yield spreads, explicitly incorporating the valuation of the American call options. As an application, I consider medium-quality callable issues of Occidental Petroleum Corporation, using a three-factor model for the term structures of benchmark (LIBOR-dollar) swap rates and for Occidental yield spreads.

JEL Codes: E43; G12; G13.

Keywords: Callable Corporate Bond; Credit Spread; American Option; Term Structure Model; Maximum Likelihood Estimation.

1 Introduction

This paper presents a methodology for extracting credit pricing information from the prices of callable corporate debt. Until recently, empirical research on corporate bond pricing has avoided a direct treatment of callable corporate bonds. In practice, however, callable debt is popular. As of April 2003, the Fixed Investment Securities Database (FISD (2002)) contained a total of about 23,950 fixed-rate U.S. corporate debentures, of which roughly 60% in number and 42% in offering amount were callable. In order to extract credit-quality information from yield spreads, one must treat the simultaneous effects of credit risk and optionality.

Figure 1 shows market prices of a Baa3-rated callable bond issued by Occidental over the time period from January 1990 through December 1995. Also plotted are the prices at which this bond would trade if it was noncallable and default-free. The reduction in price of the actual callable, defaultable bond relative to its noncallable, default-free equivalent is due to two factors: discounting for default risk, and callablity. The primary objective of my work is to disentangle these two components, thus identifying the values of both the embedded American call option and the noncallable (defaultable) bond. In practice, the problem of valuing the call option is often approached with a term-structure model of the default-free yields, possibly adjusted for default risk by adding yield spreads of noncallable bonds (see, for example, Fan, Haubrich, Ritchken, and Thomson (2003)). This method, however, reflects a somewhat superficial point of view, because the market value of the call option depends not only on uncertainty regarding market interest rates, but also on the risk of changes in the credit quality of the bond. For example, an upgrade in credit quality of the callable Occidental bond, holding default-free yields constant, would increase the value of the embedded call option. The challenge when estimating the term structure of callable corporate bond yield spreads stems from this interaction between call-free credit spreads and the prices of the call option, which calls for a simultaneous solution of both.

Previous approaches to describing the term structure of callable corporate bond yield spreads have relied on the price data for noncallable bonds of the same firm. By contrast, I propose a methodology that achieves the same goal using only the prices of the callable bond of interest. The benefits of this approach are twofold: first, it permits the term structure estimation of callable corporate bond yield spreads in the absence of a noncallable equivalent; and second, relative to existing methods, it dispenses with the stringent assumption that, for the same firm, both bond types have equal credit spreads.

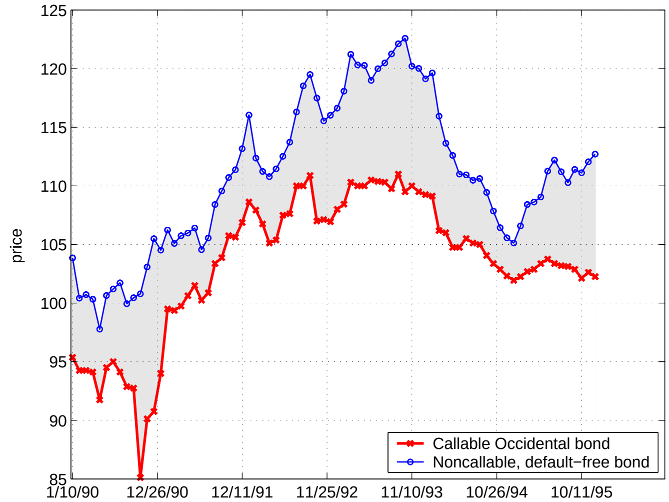

Figure 1: Prices of a callable Occidental bond and its noncallable, default-free equivalent. Source: Datastream.

Significantly, the method I propose respects the fact that a single firm may issue multiple bonds whose yield spreads incorporate issue-specific default or liquidity characteristics.

This paper includes a model for pricing callable bonds that accounts for both default and American callability, and allows for compensation for illiquidity. I develop a methodology that is designed to disentangle the components of callable corporate bond prices associated with *(i)* discounting at market interest rates, *(ii)* discounting for default and illiquidity risk, and *(iii)* callability. I build on the framework of Duffie and Singleton (1999), who show that the cash flows promised by a corporate bond can be priced using a default-adjusted short-term discount rate that reflects the mean arrival rate of a credit event and the associated loss in market value upon arrival. Additionally, one can adjust the discount rate by a mean fractional cost rate to compensate for illiquidity effects. The model incorporates the dependency of the value of the call option on both interest rate risk and the risk of changes in credit quality.

The valuation of callable corporate bonds requires an assumption about the call policy of the issuer. For the purpose of this paper, I will assume that a callable bond is called so as to minimize its market value, which is justified under the assumption of perfect capital markets and absence of other motives including the impact of early redemption on the pricing of other corporate securities. In practice, however, firms may exercise bond calls, or fail to exercise them, for many different reasons. For example, the desire to change the firm's capital structure or the need to eliminate restrictive covenants are two reasons why a company might decide to call in the debt even though its market value is below the strike price. Liquidity constraints, on the other hand, could defer calls. In addition, for the issuer of a portfolio of corporate liabilities, in order to minimize the portfolio's market value, it may not be optimal to call in a particular bond so as to minimize that bond's market value, because of joint default risk and signals that may go to the market. To the extent that firms do not call optimally, the implied credit spreads of the callable bond would be corrupted, and should be interpreted only as a benchmark.

I evaluate corporate bonds within the class of multi-factor affine term-structure models, where the short-term discount rate is modeled as affine with respect to a multi-dimensional Markov process. This process is in turn modeled as a regular affine process (Duffie, Filipovic, and Schachermayer (2003)), meaning that its characteristic function is exponential-affine in the present state. In this context, I use the popular Least-Squares Method (LSM) (Longstaff and Schwartz (2001)[1]) to price the American-style options embedded in callable bonds. The LSM is a simulation-based algorithm that solves for the optimal stopping rule. The key to this approach is to use least-squares regression to estimate the conditional expected value of the bond if not called. Uglum (2001) suggests an "override" condition that suppresses approximate call exercise whenever it is more valuable at present to commit to exercise at some future date. I find that the LSM algorithm, when accompanied by this override feature, achieves a high degree of both accuracy and robustness for a wide range of parameterizations.

The results of this work include the first empirical analysis, in the setting of standard arbitrage-free term-structure models, of the time-series behavior of callable corporate bond yield spreads, explicitly incorporating valuations of the American call options. As an application, I consider a Baa3-rated callable issue of Occidental Petroleum Corporation, using a three-factor model for the term structures of benchmark (LIBOR-dollar) swap rates and for Occidental yield spreads. Applying an approximate-maximum-likelihood estimator, I estimate a model of the term structure of noncallable credit spreads, using as data the prices of the callable bond issued by Occidental.

Using the parameters and implied noncallable spreads, I examine some implications of the estimated model for the current market practice of pricing callable corporate debt, and study the correlations of these spreads with various macroeconomic and firm-specific time series, including a U.S. chemicals index and Occidental's leverage ratio. Given a simple model of recovery at default and for the mean fractional liquidity cost rate, one is then in a position to estimate the implied risk-neutral probability of default from corporate bond prices. The actual probability of default can finally be estimated on the basis of the estimated risk premia (see, for example, Driessen (2003) and Huang and Huang (2003)).

The remainder of this paper is organized as follows. Section 2 gives a summary of the related literature. Section 3 presents my valuation framework for callable corporate bonds, and Section 4 discusses how to approximate the optimal strategy for exercising the American call option. In Section 5, I describe the estimation strategy applied. Section 6 presents the empirical results for Occidental's bond data. Finally, Section 7 summarizes the results presented in this paper and provides some concluding remarks.

2 Related Literature

This work draws on four different lines of literature, namely *(i)* the theory of valuing callable, defaultable bonds, *(ii)* simulation-based American option pricing methods in the setting of multi-factor term-structure models, *(iii)* empirical methods for time-series modeling of credit spreads, and *(iv)* estimation techniques for latent-factor term-structure models. A recent discussion of the literature on the latter subject can be found in Umantsev (2001).

2.1 Valuation of Callable, Defaultable Coupon Bonds (Theory)

Two major building blocks of any valuation framework for callable, defaultable bonds are the treatments of discounting for default risk and of callability. Corporate default risk has previously been captured by a variety of models. Recent methods are based on either a structural or a reduced-form model. Structural models are based on a model of the firm's value, as a stochastic process, and on the assumption that default is triggered when the firm's value falls below some critical value, related to liabilities. The structural approach was pioneered by Black and Scholes (1973), and much of the literature is based on Merton (1974).[2] Models that are reduced form, meaning that they are based on an assumed form of default intensity, generally treat default as the arrival of a counting process with a (stochastic) intensity process. See, for example, Pye (1974), Das and Tufano (1995), Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), Madan and Unal (1998), Duffie and Singleton (1999), and Collin-Dufresne, Goldstein, and Hugonnier (2004). Treatments of loss given default vary, both among reduced-form models and among structural models.

Regarding optimal exercise strategies for the embedded call options, several approaches have been developed. In particular, one can distinguish between models using partial differential equation (PDE) techniques and the martingale approach. Regarding the PDE-based literature, models with default were advocated by Merton (1974), who argued that prices of a callable, defaultable bond solve a PDE subject to boundary conditions that describe default and call events. Closed-form solutions do not exist, and finite-difference methods are applied. Kim, Ramaswamy, and Sundareasan (1993) extend this work by allowing for stochastic interest rates. In Sarkar (2000), imperfections in the capital structure (refunding costs, taxes, bankruptcy costs) alter the optimal call policy. Martingale methods simplify the calculation of prices of bonds with embedded options. For example, Duffie and Singleton (1999) rely on the martingale approach to price callable, defaultable bonds assuming that the issuer calls the bond so as to minimize its market value, which is optimal assuming perfect capital markets and the absence of other motives to exercise bond calls or to postpone exercise. Acharya and Carpenter (2002) model call and default options as American options written on a noncallable, default-free bond with fixed continuous coupons. In a related paper, Guntay (2002) proposes a double-hazard framework to price callable, defaultable coupon bonds. He models call risk and default risk as two correlated hazard processes, while allowing taxes and refunding costs to affect the arrival rate of the call, and firm characteristics to influence the arrival of default. Peterson and Stapleton (2003), in a recent contribution to the pricing of options on credit-sensitive bonds, build a three-factor model for the term structures of default-free yields and for correlated credit spreads. They price Bermudan-style options on defaultable bonds using a recombining log-binomial tree methodology.

2.2 Simulation-Based American Option Pricing in Multi-Factor Term Structure Models

In general, there are no analytic expressions for the prices of a callable bond.[3] Monte Carlo (MC) simulation is an appropriate method in higher-dimensional settings or with stochastic parameters. Introductory and review papers on MC simulation for derivative pricing are Boyle (1977), Bossaerts (1989), Tilley (1993), and Boyle, Broadie, and Glasserman (1997). The literature on simulation-based methods embraces the parameterization of the decision boundary (Li and Zhang (1996), Andersen (2000), Picazo (2000), Garcia (2002)), dimensionality reduction or nonparametric representation of the early exercise region (Barraquand and Martineau (1995), Clewlow and Strickland (1998), Carr and Yang (2001), Duan (2002)), and approximation of the value function. Value-function approximation can be based on decision trees (Broadie and Glasserman (1997)), stochastic-mesh methods (Broadie and Glasserman (2004)), regression methods (Carrière (1996), Tsitsiklis and Roy (1999, 2001), Longstaff and Schwartz (2001), Clément, Lamberton, and Protter (2002), Kusuoka (2003), Lai and Wong (2003), Moreno and Nava (2003)), or dual methods (Rogers (2002), Andersen and Broadie (2004), Haugh and Kogan (2004)). Fu, Laprise, Madan, Su, and Wu (2001) empirically test and compare the performance of some of these simulation-based algorithms.

2.3 Empirical Estimation of Credit Spreads

Existing empirical literature on the estimation of credit spreads of corporate bonds, such as Duffee (1999), Elton, Gruber, Agrawal, and Mann (2001), or Driessen (2003), investigates expected returns and risk premia of noncallable bonds only. Fan, Haubrich, Ritchken, and Thomson (2003) calibrate the term structure of credit spreads for five large banks to subordinated noncallable bond data, and use the parameter estimates to generate theoretical prices of puttable coupon bonds. Duffie, Pedersen, and Singleton (2003) estimate a time-series model of the term structure of yield spreads for noncallable Russian dollar-denominated bonds. To my knowledge, however, the existing empirical literature on estimating the term structure of credit spreads using as data the prices of callable bonds is limited to this paper and recent work by Jarrow, Li, Liu, and Wu (2003). Whereas I assume that a callable bond is exercised so as to minimize its market value, the latter authors employ a reduced-form representation of the call's exercise. They apply an extended Kalman filter technique to estimate a four-factor model for the term structure of default-free yields and for the default and the call probability characterized by a default-arrival and a call-arrival intensity process, respectively.

3 Methodology

I introduce a probability space with measure $\mathcal{P}$ (actual or data-generating measure) and an increasing family $\{\mathcal{F}_t : t \geq 0\}$ of information sets[4] defining the resolution of information over time. I consider the price P_t, at any time t before default, of a corporate security that promises to pay a single, possibly random, amount Z at some stopping time $\tau \geq t$. I assume that Z is $\mathcal{F}_\tau$-measurable so that the payment can be made based on information that is currently available. I take as given a short-rate process r and an equivalent martingale measure $\mathcal{Q}$.[5] Harrison and Kreps (1979) and Delbaen and Schachermayer (1999) show that the existence of such a "risk-neutral" measure $\mathcal{Q}$ and the absence of arbitrage are "essentially" equivalent. With respect to this risk-neutral measure, prices are discounted expected cash flows, as specified in more detail in what follows. This means that a default-free claim to Z at time τ has a price at t of $E_t^{\mathcal{Q}}\left[e^{-\int_t^\tau r_s\,ds} Z\right]$, where $E_t^{\mathcal{Q}}$ denotes $\mathcal{F}_t$-expectation with respect to $\mathcal{Q}$.

Consider a bond issue that defaults at the first arrival of a doubly-stochastic counting process with intensity h (with respect to $\mathcal{Q}$). That is, h_t is the risk-neutral default intensity. This means, under technical regularity conditions, that the $\mathcal{F}_t$-conditional risk-neutral probability of default between t and $t+\Delta$, given that default has not occurred by t, is approximately $h_t\Delta$, in the limit as Δ goes to zero, and moreover that the probability of no default by t is $E^{\mathcal{Q}}\left[e^{-\int_0^t h_s\,ds}\right]$. It is also assumed that the short rate r_t is in the subfiltration of $\{\mathcal{F}_t\}$ that "drives" the doubly-stochastic arrival of default.

For the case of a single credit event, Duffie and Singleton (1999) show that for pricing purposes, under technical regularity conditions, one can treat the issue's promised cash flow Z as default-free, and allow for default risk by replacing the discount rate r with the default-adjusted discount rate $r + hL$, where L is the risk-neutral expected fractional loss in market value in the event of default. Under the assumption that illiquidity of the security can be captured by a fractional cost rate of l, where l is a predictable process, the total mean loss rate of the security due to default and illiquidity is

$$s = hL + l. \tag{1}$$

I call the process s the *short spread*. The *default- and liquidity-adjusted short-rate process* R associated with the bond is defined as

$$R = r + s. \tag{2}$$

Duffie and Singleton (1999) show that, under technical conditions,

$$P_t = E_t^{\mathcal{Q}}\left(e^{-\int_t^\tau R_u\,du} Z\right). \tag{3}$$

I evaluate corporate bonds within the class of multi-factor Affine Term-Structure Models (ATSMs). The default- and liquidity-adjusted short rate R is modeled as affine with respect to the state vector $X \in \mathbb{R}^3$, which is modeled as a regular affine process under both the data-generating measure $\mathcal{P}$ as well as under the equivalent martingale measure $\mathcal{Q}$. (See Duffie, Filipovic, and Schachermayer (2003) for a complete definition.)

3.1 Valuation of Callable Corporate Bonds

Suppose that a corporate bond that matures at time T pays a coupon of size c_i, as a fraction of face value, at T_i for an increasing sequence $T_1 < T_2 < \cdots < T_{L+1} = T$ of times. If the bond is noncallable, the market value (ex-coupon) at any time t before maturity, as a fraction of face value, is, from (3),

$$V_t(T) \quad = \quad E_t^{\mathcal{Q}} \left[\sum_{t<T_i\leq T} c_i \, e^{-\int_t^{T_i} R_u \, du} + e^{-\int_t^{T} R_u \, du} \right].$$

I have implicitly assumed the same model for coupons and principal. One could, in principle, have adopted a different specification of the short spread s in (2) for each coupon.

Now, suppose the bond is callable, possibly only after some initial lock-out period, $\underline{T}$, and let us denote the exercise price at time t, as a fraction of face value, by $\bar{V}_t$, where $\bar{V}_T = 1$ (often, $\bar{V}_t$ is the principal ("par") plus accrued interest). Throughout, I assume that a callable bond is called so as to minimize its market value. For any time t before maturity, let $\mathcal{T}(t,T)$ denote the set of feasible call policies on or after t ($\{\mathcal{F}_t\}$-stopping times with outcomes in $[t,T]$). Provided default has not occurred by time t, the ex-coupon market value at t, as a fraction of face value, is

$$V_t \quad = \quad \operatorname*{ess\,inf}_{\tau\in\mathcal{T}(t,T)} V_t(\tau), \tag{4}$$

where, from (3),

$$V_t(\tau) \quad = \quad E_t^{\mathcal{Q}} \left[\sum_{t<T_i\leq \tau} c_i \, e^{-\int_t^{T_i} R_u \, du} + \bar{V}_\tau \, e^{-\int_t^{\tau} R_u \, du} \right], \tag{5}$$

which can be shown by standard arguments for nondefaultable securities.[6] The pricing relation (4) applies under technical regularity conditions discussed, for example, in Duffie and Singleton (1999). According to Bellman's principle of optimality, at each time t, the issuer minimizes the market value of the liability by exercising the option to call in the bond if and only if its market value, if not called, is higher than the call price.

I conclude this section by stating that, in general, it is suboptimal to redeem a callable bond at inter-coupon times. This argument assumes that the short-rate process R associated with the corporate security is nonnegative and is standard. I will ignore minor adjustments due to market conventions for quoting accrued interest on corporate bonds.

3.2 Parametric Model of the Adjusted Short Rate

I rely on the term structure of U.S.-dollar LIBOR-quality swap yields as the reference curve based on the short-rate process r. Duffie, Pedersen, and Singleton (2003) discuss advantages of using swap yields over Treasury yields as a benchmark term structure against which to measure corporate yield spreads, and propose a two-factor affine model for the reference term structure which describes the dynamics of the short-rate process r and the process v driving its volatility. Specifically,

$$\begin{aligned} d\begin{bmatrix} v_t \\ r_t \end{bmatrix} &= \begin{bmatrix} K^{vv} & 0 \\ K^{rv} & K^{rr} \end{bmatrix} \left(\begin{bmatrix} \theta^v \\ \theta^r \end{bmatrix} - \begin{bmatrix} v_t \\ r_t \end{bmatrix} \right) dt \\ &+ \begin{bmatrix} 1 & 0 \\ \Sigma^{rv} & \Sigma^{rr} \end{bmatrix} \begin{bmatrix} \sqrt{v_t} & 0 \\ 0 & \sqrt{v_t} \end{bmatrix} \begin{bmatrix} dW_t^v \\ dW_t^r \end{bmatrix}, \end{aligned} \tag{6}$$

where $(W^v, W^r)'$ is a two-dimensional standard Brownian motion under the actual measure. The distribution of the Brownian motion $(W^v, W^r)'$ under the risk-neutral measure is specified by

$$\begin{bmatrix} dW_t^v \\ dW_t^r \end{bmatrix} = -\begin{bmatrix} \sqrt{v_t} & 0 \\ 0 & \sqrt{v_t} \end{bmatrix} \begin{bmatrix} 0 \\ \lambda^r \end{bmatrix} dt + \begin{bmatrix} d\tilde{W}_t^v \\ d\tilde{W}_t^r \end{bmatrix}, \tag{7}$$

where $(\tilde{W}^v, \tilde{W}^r)'$ is a two-dimensional standard Brownian motion under $\mathcal{Q}$. Duffie, Pedersen, and Singleton (2003) use weekly data on two- and ten-year swap rates for the period January 1987 through July 1999 to estimate the model parameters ($K^{vv} = 0.0047, K^{rv} = -0.027, K^{rr} = 0.34, \theta^v = 107.40, \theta^r = 5.68, \Sigma^{rv} = 0.044, \Sigma^{rr} = 0.11, \lambda^r = -0.076$) using approximate-maximum-likelihood estimation. For the purpose of my empirical analysis, I will treat these parameter estimates as the true parameters.

With regard to the short spread s of a particular bond, I model the joint behavior of the benchmark term structure and the process s as

$$s_t = \alpha + \beta^v v_t + \beta^r r_t + u_t, \tag{8}$$

where u is a Vasicek-type (Ornstein-Uhlenbeck) process under both the data-generating and the risk-neutral measure. The dynamics of the process u are assumed to be given by

$$du_t = -K^{uu} u_t \, dt + \Sigma^{uu} \, dW_t^u, \tag{9}$$

where $W^{vru} = (W^v, W^r, W^u)'$ is a standard three-dimensional Brownian motion under $\mathcal{P}$. I specify that, under $\mathcal{Q}$, $\tilde{W}^{vru} = (\tilde{W}^v, \tilde{W}^r, \tilde{W}^u)'$ is a standard Brownian motion in $\mathbb{R}^3$ with

$$dW_t^u = -\left(\lambda_0^u + \lambda_1^u u_t\right) dt + d\tilde{W}_t^u. \tag{10}$$

As discussed in Duffie, Pedersen, and Singleton (2003), the fact that the short spread can take on negative values is not necessarily inconsistent with the proposed theoretical model, due to the possibility of a negative liquidity factor. Moreover, as illustrated in Figure 2, even for issues below LIBOR-quality negative yield spreads can be observed in the market.

If the issuer has issued several bonds, each bond's short spread may incorporate an idiosyncratic component specific to this issue (like special default or liquidity characteristics), and not necessarily to a maturity segment of that firm's yield curve. I select one bond as a "benchmark," and model its short spread s as in (8) through (10). For any non-benchmark bond i, the short-spread process is assumed to be given by $s^i = s + \eta^i$, where

$$d\eta_t^i = \kappa^i \left(\vartheta^i - \eta_t^i\right) dt + \sigma^i \, d\xi_t^i, \tag{11}$$

and ξ^i is a standard Brownian motion independent of $\{W^{vru}, \xi^j : j \neq i\}$. The distribution of the idiosyncratic Brownian motion ξ^i under $\mathcal{Q}$ is specified as

$$d\xi_t^i = -\left(\lambda_0^i + \lambda_1^{i,u} u_t + \lambda_1^{i,\eta} \eta_t\right) dt + d\tilde{\xi}_t^i, \tag{12}$$

where, with respect to the risk-neutral measure, $\tilde{\xi}^i$ is a standard Brownian motion independent of $\left\{\tilde{W}^{vru}, \tilde{\xi}^j : j \neq i\right\}$. In Appendix A, I show that the parameterizations of the market price of spread risk for s and η^i in (10) and (12) do indeed generate an equivalent martingale measure, in the sense that Girsanov's theorem applies.

The model setup in (8) through (12) represents a trade-off between the aims of capturing important empirical features of corporate bond yield spreads, while maintaining a setting that allows to estimate the model parameters in a feasible fashion. As will become more evident in Sections 4 and 5, for callable bonds, the estimation procedure is numerically intensive, and requires time-consuming attention by the user in order to obtain a reasonable fit. This leaves us with the vector

$$\Theta \;=\; \left(\alpha, \beta^v, \beta^r, K^{uu}, \Sigma^{uu}, \lambda_0^u, \lambda_1^u, \{\vartheta^i\}, \{\kappa^i\}, \{\sigma^i\}, \{\lambda_0^i\}, \{\lambda_1^{i,u}\}, \{\lambda_1^{i,\eta}\}\right)$$

of unknown parameters, governing the stochastic behavior of corporate bond yield spreads, to be estimated from observed bond prices.

In Appendix A, I show that for the given parametrization of R, the value of the short spread s_t at time t can be implied from the observed (callable) bond price V_t at time t, given the current states of the reference curve, that is v_t and r_t, and the parameter vector Θ governing the stochastic behavior of s.

4 American Option Pricing by LSM

In general, there is no closed-form solution available for the price (4) of a callable bond. The underlying dynamic optimization problem is solved numerically, for example by dynamic programming. I now suppose, as is often the case in practice, that the bond is callable at par with first call date at $\underline{T}$, and that the times of "callability" include all coupon payment dates thereafter. If the bond is callable at some inter-coupon time, and is not called before that date, the issuer maximizes the value of the call option embedded in the bond (and hence minimizes the market value of the callable bond) by not exercising it until the next coupon payment date. This argument assumes that the short-rate process R associated with the corporate security is nonnegative. I will ignore minor adjustments due to market conventions for quoting accrued interest on corporate bonds. The model assumptions for R in (2) and in (6) through (10) allow for negative values of R. Throughout my empirical analysis, however, the (risk-neutral) probability of such occurrences is small. I therefore make the simplifying assumption that the issue is callable only at coupon dates, that is, $\mathcal{T}(t,T) = \{T_1, \ldots, T_{L+1} \,:\, T_j \geq \max(t, \underline{T})\}$ for all t before maturity.

Initially, I fix some time t before maturity. Let $\tau^* \in \mathcal{T}(t, T)$ denote an optimal stopping time, characterized by $V_t = V_t(\tau^*)$ in (4) and (5). Given τ^*, V_t can be evaluated accurately by straightforward MC simulation. An optimal exercise strategy τ^* is usually not explicitly known, however, and must be approximated. In general, an optimal stopping time will depend in a complicated way on the discount factors $B(T_i, T_j)$, where

$$B(t_0, t_1) \;=\; E_{t_0}^{\mathcal{Q}} e^{-\int_{t_0}^{t_1} R(s)\,ds}, \quad t_0 < t_1,$$

for all feasible call dates T_i and T_j after t. As an optimal stopping time depends on too many variables to be feasibly determined within a standard MC setting, I rely, therefore, on the popular Least-Squares Method (LSM, Longstaff and Schwartz (2001)) to price the American-style options embedded in callable bonds.[7]

Longstaff and Schwartz (2001) find an approximate exercise strategy through a recursive, simulation-based algorithm that proceeds backwards in time and solves for the stopping rule that minimizes the value of the callable bond at each time point along each path. The key to the approach is that, at each exercise date, least-squares regression is used to estimate the conditional expected value of the bond, if not exercised (value of continuation), as a function in polynomials of the underlying state variables X (here, $X = (v, r, u)$), and possibly of other nonlinear transformations of X.[8]

Convergence results for the LSM algorithm are available (see, for example, Tsitsiklis and Roy (2001), Clément, Lamberton, and Protter (2002), and Kusuoka (2003)). The arguments build on the fact that, under technical conditions, the conditional risk-neutral expected value of continuation is an element of the Hilbert space of square-integrable functions relative to the risk-neutral measure. This Hilbert space has a countable orthonormal basis, hence the conditional expectation can be represented as a linear combination of the basis elements. Clément, Lamberton, and Protter (2002) distinguish between two types of approximations in the LSM algorithm: *(i)* replace the conditional expected values of continuation by projections on a finite set of basis functions, and *(ii)* use MC simulations and least-squares regression to compute the value function of the first approximation. They prove, under fairly general conditions, the almost-sure convergence of both approximations.[9] In a recent related article, Kusuoka (2003) gives an even more rigorous justification of the LSM procedure, and shows that the complete algorithm converges. The author provides explicit error bounds for the estimated prices of Bermudan-style options.[10]

Longstaff and Schwartz (2001) argue that, for many applications, a moderate number of basis functions suffices. In a recent study on the robustness of the LSM algorithm, Moreno and Nava (2003) analyze the impact of different basis functions on option prices. They show that, for American put options, LSM is quite robust to the choice of basis functions, and that for more complex derivatives the choice can affect options prices slightly. In my empirical applications, I use the first three powers of all unmatured discount bond prices with final maturity up to and including maturity, as well as products of the unmatured discount bond prices with the shortest and the longest remaining maturities. In order to obtain an estimator of the decision boundary that is smooth in the state and the parameter vector, s and Θ, I use all sample paths in the regression step, and not only the in-the-money paths as recommended by Longstaff and Schwartz (2001).[11]

Uglum (2001) suggests an override, avoiding exercise whenever the value of current exercise is dominated by the value of exercising at a later deterministic time. To protect against misspecification of the decision boundary associated with my implementation of the LSM method, I implement Uglum's override condition. Extensive numerical tests indicate that the LSM algorithm, accompanied by this override feature, achieves a high degree of accuracy and is robust for a wide range of parameterizations of the term structure of callable corporate bond yield spreads.

5 Estimation Strategy

The general estimation setting in this paper is typical for state-space models of the term structure, in that the short spread s_t that determines the current price of the corporate bond is not directly observable. (Remember that I consider the present state of the reference curve, $(v_t, r_t)'$, as given.) Instead, at every time t, I record the observed prices of d corporate securities $V_t^1, \ldots, V_t^d$ which, according to (4) and (5), are deterministic functions of $s_t^1, \ldots, s_t^d$, given $(v_t, r_t)'$. Throughout this work, the number of observables, d, equals the dimension of the state vector $(s^1, \ldots, s^d)'$, and exact inversion of the model is possible. Once the model is inverted and the path $\{s_t^\Theta\}$ of the state vector is inferred, the estimation can proceed in the usual fashion, that is, either by maximizing the likelihood of the state vector's transition density (corrected by the Jacobian) or by matching a set of model-implied and actual moments of the state variables. One could also adopt an estimation strategy that doesn't require model inversion.[12] In the remainder of this section, I outline the procedure used for estimating the parameter vector Θ associated with the short spread of a callable bond from its observed market prices, using the valuation framework established in Sections 3 and 4. The estimation strategy is based on standard maximum likelihood estimation techniques for latent-factor affine models of the term structure as discussed above and, for example, by Duffie, Pedersen, and Singleton (2003).

As before, I fix some time t before maturity. Given the states v_t and r_t of the reference curve, the bond value in (4) is a deterministic function, here denoted by $G_t(\cdot;\cdot)$, of the current short spread s_t and the parameter vector Θ governing its stochastic behavior. That is,

$$V_t \quad = \quad G_t(s_t;\Theta).$$

Hence, the estimation problem is similar to that for latent-factor models because one directly observes the bond prices V_t but not the short spreads s_t which determine the current and near-future term structure of the corporate bond yield spreads.

Suppose that I observe the prices of a callable corporate bond at times t_0 through t_N. Standard change-of-variable arguments lead to the log-likelihood function for the observed bond price vector $\underline{V} = (V_{t_0}, V_{t_1}, \ldots, V_{t_N})$ given by

$$l\left(\underline{V}\,|\,\Theta\right) \quad = \quad \frac{1}{N}\sum_{n=1}^{N}\left[\log\mathcal{P}\left(s_n^{\Theta}|s_{n-1}^{\Theta};\Theta\right) - \log\left|DG_{t_n}\left(s_n^{\Theta};\Theta\right)\right|\right],$$

where $s_n^{\Theta} = G_{t_n}^{-1}\left(V_{t_n};\Theta\right)$, and where $DG_t\left(\cdot;\Theta\right)$ denotes the partial derivative of $G_t\left(\cdot;\Theta\right)$ with respect to s_t. In Appendix A, I show that, for the short-spread model in (8) through (10), $G_{t_n}(\cdot;\Theta)$ is indeed invertible for each Θ. A MLE for Θ is given by

$$\hat{\Theta} \quad \in \quad \arg\max_{\Theta}\, l\left(\underline{V}|\Theta\right). \tag{13}$$

Faced, however, with the problem of estimating the parameter vector Θ from the observed prices of corporate bonds with an embedded American-style call option, one encounters several additional challenges, and (13) holds only in an approximate sense. First, the optimal exercise strategy in (4) is, for most cases, not explicitly known. Hence, I approximate it using the LSM algorithm together with Uglum's override condition. Second, given an (approximate) stopping rule, often there is no explicit formula at hand to calculate the callable bond prices and MC simulation is employed. And third, computing the likelihood function for the time series of observed callable corporate bond prices

Table 1: Contractual features of two bonds issued by Occidental. Source: FISD.

Security	Callable 10yr note	Straight 12yr note
Issue date	7/1/89	11/15/89
Maturity	7/1/99	11/15/01
Amount issued (MM)	$300	$330
Coupon	9.625%	10.125%
Credit rating (Moody's)	Baa3	Baa3/Baa2
Seniority	Senior/Unsecured	Senior/Unsecured
First call date	7/1/96	–
Redemption	7/1/96 at par	–

involves both inverting the value function $G_{t_n}(\cdot;\Theta)$ in s_{t_n} and calculating the sensitivity $DG_{t_n}(\cdot;\Theta)$ of the bond prices relative to the short spread. Again, numerical methods are applied.[13]

6 Case Study: Occidental Petroleum

In this section, I investigate the behavior of the term structure of credit spreads of bonds issued by Occidental Petroleum Corporation. Headquartered in Los Angeles, California, Occidental is a large, multinational company with worldwide interests in oil and gas exploration and production, as well as the manufacturing of chemicals. The principal operations of the company are conducted through the company's subsidiaries, Occidental Oil and Gas Corporation and Occidental Chemical Corporation.

The contractual characteristics of a callable bond issue are summarized in the second column of Table 1. The callable ten-year note was issued on 7/1/89 with an initial size of $300 million and a semi-annual coupon at an annual rate of 9.625%. The issue was rated Baa3 by Moody's on 3/17/94, and neither downgraded nor upgraded by that rating agency thereafter. The notes were redeemable on or after 7/1/96, at Occidental's option, at a redemption price equal to 100% of the principal. The entire issue was called on 7/1/96, its first call date. Figure 10 in Appendix B displays the yields to maturity and the yields to first call of the callable Occidental issue. The yield to first call exceeds the yield to maturity throughout 1990, until January 1991. This pattern is reversed, however, after May 1991, and from that time the option of calling the issue at the first possible date consistently moves deeper into the money.

The empirical analysis will address the pricing of callable bonds relative to noncallable bonds. Hence, I collect data on a straight Occidental bond with the same credit rating and with similar features as the callable issue. For example, on 11/15/89, Occidental issued a straight twelve-year note with an initial size of $330 million and a semi-annual coupon at an annual rate of 10.125%. On 3/17/94, Moody's assigned a Baa3 credit rating to this bond, which was upgraded one notch to Baa2 on 12/16/96, but again downgraded to Baa3 on 2/2/99.[14] Neither bond had a sinking fund provision nor any variation over time in promised coupon payments. Both notes were senior-unsecured, nonputtable, nonconvertible, and nonexchangable. They did not default prior to maturity or redemption. (See, for example, FISD (2002).)

Datastream provides weekly (each Wednesday) market price information for both issues, for the period 1/10/90 to 12/6/95. Figure 2 shows the corresponding (four-weekly) yield spreads relative to the U.S.-dollar swap curve.[15] Of particular interest is the surge in yield spreads during the Fall of 1990. The callable note lost 9% of its market value between 8/1/90 and 10/10/90, while the value of the straight issue dropped 12% over the same period of time. Both issues recovered quickly and were back at August 1990 levels by mid-January 1991. The economic background pertinent to these observations is the Gulf War. After Iraq's invasion of Kuwait in August 1990, anxiety about Mideast stability caused oil prices to jump from $17 a barrel in July to $36 by October 1990. On the other hand, confidence in the U.S. economy faltered and consumer spending fell off, which forced the U.S. economy into a recession in the Fall of 1990. In general, higher oil prices mean good news to oil producing companies (given they have sufficient reserves or crude-oil production), boosting their stock and bond prices. So why did Occidental's bond (and stock) prices fall, instead of rise? Contrary to popular perception, Occidental at that time was more like a chemical company than an oil and gas producer, as illustrated in Figure 11 in Appendix B. Hence, I attribute the sharp rise in Occidental's yield spreads to a drastic softening in the U.S. chemical markets just prior to the Gulf War.

In Figure 2, one further observes very low yield spreads for the callable bond from December 1994 forward, and even occasional trading through LIBOR (periods of negative yield spreads). One possible explanation for the observed overpricing when approaching the first call date in July 1996 is that investors had assigned a (significant) positive probability to the event that Occidental would not redeem this issue at the first possible call date. Interest rates had fallen substantially since the initiation of the bond, hence investors would have profited considerably from Occidental's decision to postpone redemption of the debt beyond the "optimal" date. Another observation is that the callable bond yield spreads appear to be more volatile. While taking into account the

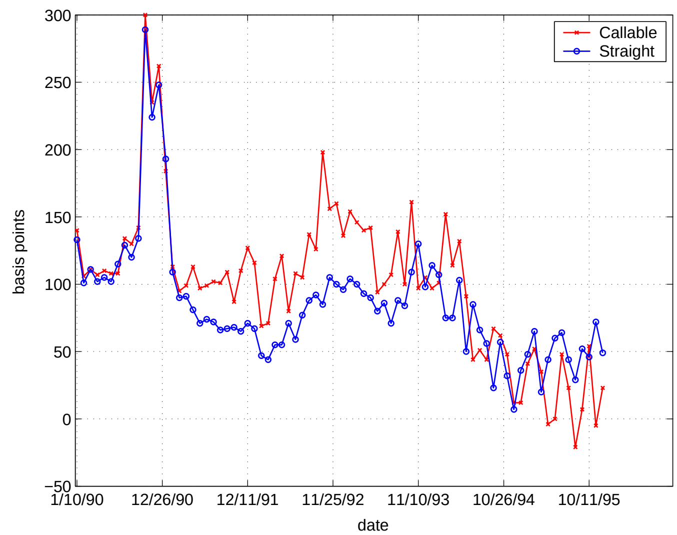

Figure 2: Yield spreads (relative to Dollar swap curve) of the callable and straight Occidental bonds. Source: Datastream.

shorter effective duration of the callable note, I suspect this to be due, at least partly, to issue-specific illiquidity risks caused by different clientele trading patterns, or asymmetrically informed traders, or noisy data. Near the optimal call date, the implied yield spread (and short spread) is of course extremely sensitive to the measured price.

6.1 Call-Corrected Short Spreads

Estimates of the parameters in (8) through (10) for the callable Occidental bond are displayed in Table 2,[16] together with their MC distribution.[17] I impose three over-identifying restrictions in order to reduce the dimension of the parameter space. First, I set the coefficient β^v equal to zero. Equation (8) still allows for correlation between the instantaneous reference rate and the short spread. I have verified that, for levels of the volatility-driving process v estimated for the sample period 1/10/90 to 12/6/95, the role of the risk-neutral correlation between v and s is negligible for the purpose of pricing the callable Occidental bond. Second, in preliminary empirical investigations I found the mean-reversion parameter of the process u under the risk-neutral measure, $\tilde{K}^{uu}$, to be negative, but to have relatively small absolute value. (This is in line with the results for noncallable debt found, for instance, by Duffee (1999) for many issuers.) To ensure, however, that the one-year-ahead distribution of the short spread under $\mathcal{Q}$ is negative with only a reasonably small probability, I set $\tilde{K}^{uu} = 0$, which guarantees that the process

Table 2: Parameter estimates for the callable Occidental issue, and their MC distribution.

Parameter	Estimate	MC distribution mean	std. dev.
α	1.849	1.645	0.718
β^v	0.000	–	–
β^r	−0.187	−0.210	0.115
K^{uu}	1.767	1.889	0.566
Σ^{uu}	1.201	1.197	0.105
λ_0^u	0.000	–	–
λ_1^u	−1.471	–	–

u is not risk-neutrally "explosive." From (9) and (10) we have $\tilde{K}^{uu} = K^{uu} + \lambda_1^u \Sigma^{uu} = 0$, which translates to $\lambda_1^u = -\frac{K^{uu}}{\Sigma^{uu}}$. Lastly, since $\tilde{K}^{uu} = 0$, the market price of spread risk factor λ_0^u is not identified by the model specification in (9) and (10), and can be set to zero without loss of generality.

The estimate of the constant term α is 1.849, and the estimate for β^r implies that a 100 basis point incline in the reference rate r lowers s by 18.7 basis points. To get a sense of how both these terms affect the yield spreads, assume that the reference curve r is at its sample average, that is, $\bar{r} = 5.11\%$. Then, on average, these estimates imply that the yield spread on a near-zero-maturity zero-coupon bond is roughly 89.3 basis points. I note that the estimate of the coefficient β^r documents a negative correlation between the short spread and the reference rate, with a level of significance of 6%. The estimates of K^{uu} and Σ^{uu} are 1.767 and 1.201, respectively. Thus, even though stationarity is not imposed, the risk-neutral default intensities appear to be stationary under the data-generating measure. Assuming that the reference curve is at its long-run mean, the half-life of shocks to s amounts to less than five months. The actual one-year-ahead probability of the short spread, conditional on the reference rate being equal to its sample average, to be negative is less than 8%. The estimated mean term structure of yield spreads[18] is downward sloping. Figure 3 shows the time series of the implied short spreads of the callable Occidental note. The basic pattern follows that of the yield spreads in Figure 2. Near the first call date, the effective maturity of the callable Occidental bond is basically July 1996. This can be seen in Figure 12 in Appendix B, which displays the time series of the estimated conditional risk-neutral probabilities that optimal exercise occurs at a given date, for each of the seven coupon dates between July 1996 and July 1999.[19] Near the optimal call date, the implied short spread is extremely sensitive to the measured

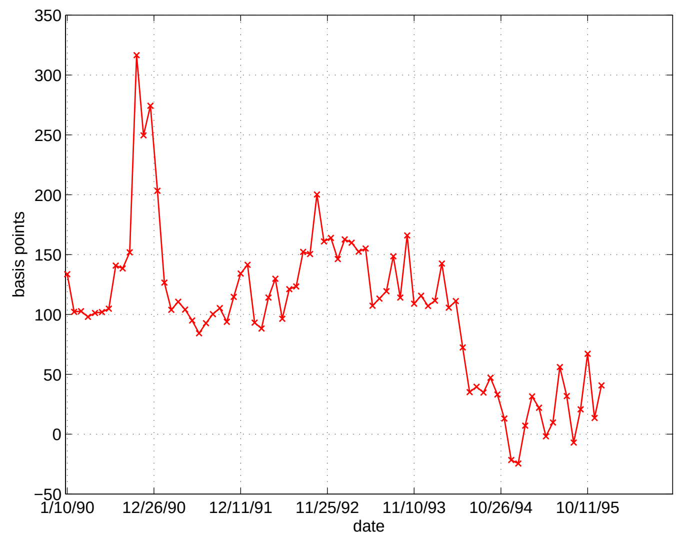

Figure 3: Implied short spreads for the callable Occidental bond.

price.

6.2 Interpreting the Short Spread

The likelihood of default by Occidental is influenced by firm-specific balance-sheet and related macroeconomic variables, on which I now focus. As illustrated in Figure 11 in Appendix B, chemicals (rather than oil and gas) was Occidental's main industry segment throughout the observation period (1/90 to 12/95). Hence, during that time, a boost to the chemicals market would have been expected to lower the probability of default by Occidental, and thereby lower the short spread s (naturally assuming that these are in a monotonic relationship). Further, Occidental's leverage ratio is defined as the book value of its debt divided by the sum of the market value of Occidental's equity plus the book value of its debt. A higher leverage ratio should raise the level of the short spread. As a preliminary examination of the potential influence of such covariates, I regress u, the implied short spread component of the callable bond not explained by the reference rate, onto the Datastream index for U.S chemicals (CHEM) and Occidental's leverage ratio (LEV) using four-weekly data from January 1990 to December 1995.[20] The estimated regression model, in basis points, is

$$\begin{aligned} u_t &= \underset{(5.81)}{14.21} - \underset{(0.11)}{0.71}\,\mathrm{CHEM}_t + \underset{(1.59)}{1.45}\,\mathrm{LEV}_t + \epsilon_t. \end{aligned}$$

I obtain an R^2 of about 69.3%. Newey-West heteroskedasticity-corrected standard errors are reported in parentheses. Both coefficients show the expected sign, although only the estimate for the U.S. chemical index multiplier is significant at conventional levels.[21]

6.3 Decomposing Callable Bond Prices

I am now in a position to value the American option embedded in the callable bond, and thereby disentangle the components of the callable bond prices due callability and due to default and illiquidity risk. Figure 13 in Appendix B shows the disentangled components of Occidental's callable bond prices. It matches Figure 1 in that it shows the market prices of the callable Occidental bond together with the prices at which this bond would trade if it was noncallable and default-free. I am now in a position, however, to show also the prices at which this bond would trade if it was noncallable (but still defaultable). In other words, I can compute the prices of the straight version of the bond, and thereby specify what portion of the total price difference between the callable, defaultable bond and its noncallable, default-free equivalent is the price of the American call option, and what part is due to discounting for default and illiquidity risk.

Figure 4 displays the amounts by which the theoretical price of the noncallable, defaultable bond and the theoretical price of the noncallable, default-free bond exceed the market price of the callable Occidental bond, respectively. The shaded area underneath the graph associated with the noncallable, defaultable bond shows the amounts by which Occidental's bond prices are less than the theoretical values of the underlying straight bond, hence giving the implied values of the American call option embedded in the callable bond. One observes that the call option gains in value throughout time until just prior to the first call date in July 1996. The only exception is the time around the Gulf War, when Occidental's bond prices were so weak that the early redemption option was far out of the money. The noncallable, default-free bond prices exceed the theoretical values of the noncallable, defaultable bond by the amounts highlighted in the shaded area between the corresponding two lines, indicating the reduction in price due to discounting for default and illiquidity risk.

6.4 Relative Pricing of Callable and Straight Debt

In order to investigate the relative pricing of callable and noncallable bonds, I choose the straight Occidental issue as benchmark and treat the callable note as a non-benchmark

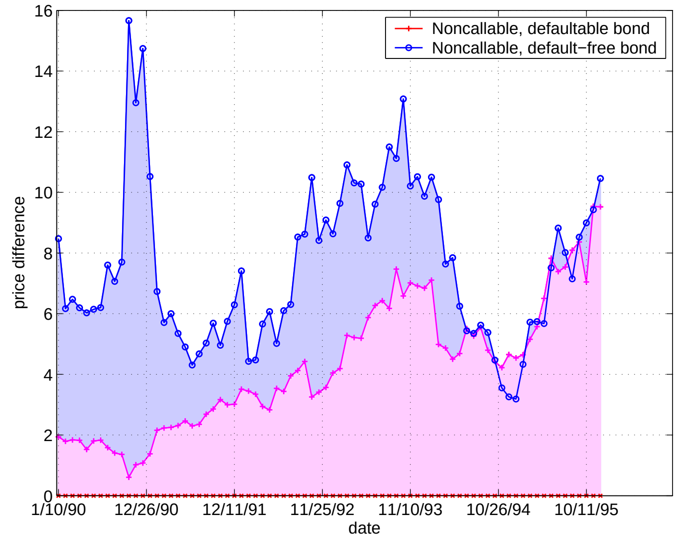

Figure 4: Price differences (relative to callable Occidental bond) of its noncallable, defaultable equivalent (due to callability), and of its noncallable, default-free equivalent (due to discounting for default and callability).

bond. This selection is based on the observation that the straight note is of higher volume, and that it appears to be more liquid across the observation period (1/10-12/95). The "recursive" short-spread model described in (8) through (10) allows estimation of the parameters of the short spread of the straight bond, s^s, in a first stage, followed by the estimation of the parameters of the idiosyncratic factor of the callable Occidental bond, η^c, in a second step. Estimates of the parameters in (8) through (10) for the straight Occidental bond are displayed in Table 3,[22] together with their MC distribution. I then treat the estimates for α, β^r, K^{uu}, and Σ^{uu} as the true parameters, and estimate the parameters of the idiosyncratic spread-risk component in (11) and (12) for the callable issue.

For the benchmark bond, I impose over-identifying restrictions analogous to those described in Section 6.1. In particular, I enforce $\beta^v = 0$ and $\tilde{K}^{uu} = 0$. The coefficient β^r is again estimated to be negative, but is no longer significant at conventional levels. In order to improve the interpretability of the parameter estimates associated with the short spread of the non-benchmark callable issue, s^c, facilitating the comparison of the implied values for s^s and s^c, I impose that s^c is a one-dimensional Ornstein-Uhlenbeck intensity process under Q, by taking $\tilde{\kappa}^c = \tilde{K}^{uu}$. Here, $\tilde{\kappa}^c$ denotes the risk-neutral mean-reversion parameter of η^c. From (11) and (12) we have $\tilde{\kappa}^c = \kappa^c + \lambda^{c,\eta}\sigma^c = 0$, which translates to

Table 3: Parameter estimates for the benchmark straight Occidental bond and for the idiosyncratic factor of the non-benchmark callable issue, together with their MC distribution.

Parameter	Estimate	MC distribution mean	std. dev.
straight bond			
α	1.202	1.024	0.642
β^v	0.000	–	–
β^r	−0.046	−0.021	0.083
K^{uu}	2.198	2.947	1.093
Σ^{uu}	1.020	1.029	0.088
λ_0^u	0.000	–	–
λ_1^u	−2.155	–	–
callable bond			
ϑ^c	−0.355	−0.348	0.042
κ^c	14.752	17.503	6.668
σ^c	1.205	1.240	0.195
λ_0^c	0.000	–	–
$\lambda_1^{c,s}$	0.000	–	–
$\lambda_1^{c,\eta}$	−12.242	–	–

$\lambda^{c,\eta} = -\frac{\kappa^c}{\sigma^c}$. I also require that the market price of risk component $\lambda_1^{c,s}$ of η^c is equal to zero. Since $\tilde{\kappa}^c = 0$, I can set $\lambda_0^c = 0$ without loss of generality. Estimates of the parameters in (11) and (12) for the idiosyncratic factor of the non-benchmark callable Occidental bond are displayed in Table 3, together with their MC distribution.[23]

The results suggest that the callable bond short (or yield) spreads were indeed significantly more volatile. Moreover, the estimate of ϑ^c of -36 basis points, more than eight times its standard error, suggests that the callable bond traded "rich" relative to the noncallable. This might be due to an assumption by investors of "suboptimal" calling behavior by Occidental. Figure 5 displays the time series of the implied short spreads of the benchmark straight Occidental note and of the non-benchmark callable Occidental bond. Again, the basic patterns for both issues follow those of the associated yield spreads in Figure 2.

As a diagnostic check, I examine the behavior of the standardized innovations ϵ^c of

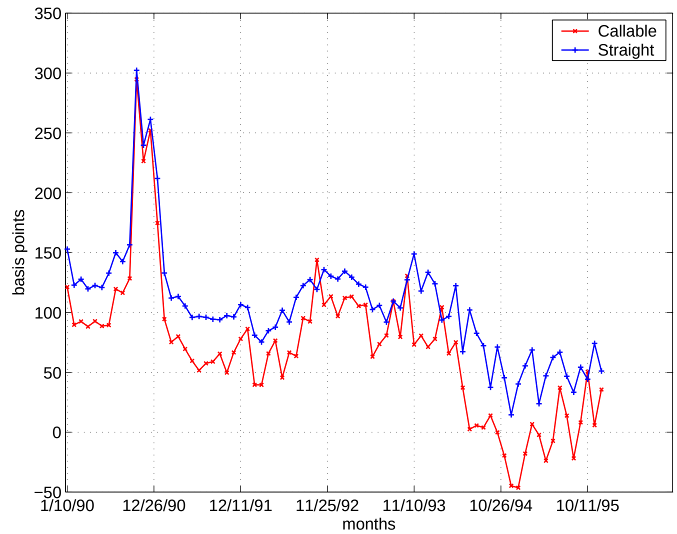

Figure 5: Implied short spreads for the benchmark straight Occidental bond, and for the non-benchmark callable Occidental bond.

η^c, defined by

$$\eta^c_{t+h} \quad = \quad e^{-\kappa^c h}\eta^c_t + \sigma^c \sqrt{\frac{1 - e^{-2\kappa^c h}}{2\kappa^c}}\, \epsilon^c_{t+h},$$

for some time step h. Under the specified model (11), with respect to the actual probability measure, these innovations are independent standard normals. The sample mean and the sample standard deviation of the fitted versions of these standardized innovations are -0.0066 and 1.0516, respectively. Figure 6 shows the associated histogram.

In further applications, I will compare the yield spread of the straight version of the callable bond that is theoretically implied by the short spread of the callable bond (the "call-corrected" yield spread) with the yield spread that is implied by the short spread of the straight bond. Additionally, given a simple model of expected recovery at default and for the mean fractional liquidity cost rate, I am able to estimate the implied risk-neutral probabilities of default. The actual probabilities of default can be estimated on the basis of estimated risk premia. Finally, using the parameter estimates and implied short spreads from Tables 2 and 3 and Figures 3 and 5, I can compute model-implied prices for out-of-sample bonds which were not included in the estimation procedure, but were issued by Occidental and had similar contractual features.

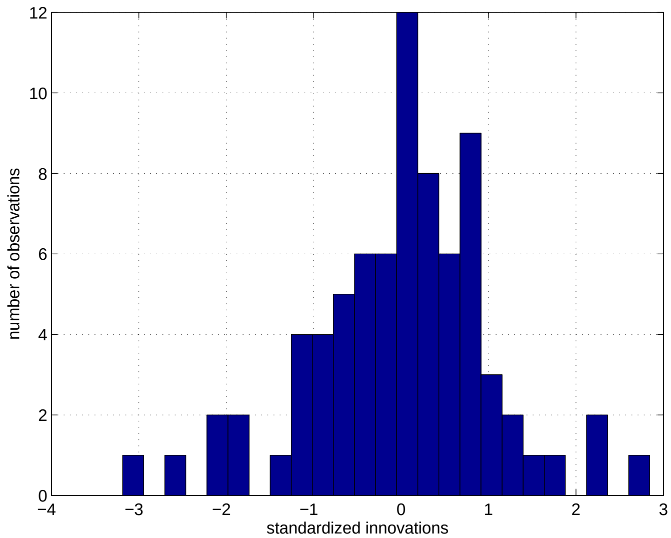

Figure 6: Histogram of the standardized innovations ϵ^c for the callable Occidental bond.

6.5 Call-Corrected Yield Spreads

For a callable bond, the call-corrected yield spread is defined as the theoretical yield spread, relative to the reference curve, of the underlying straight bond (see Figure 4). Figure 7 shows the call-corrected yield spreads of the callable Occidental bond, using the stand-alone parameter estimates and model-implied short spreads reported in Table 2 and Figure 3. Unlike the quoted yield spreads of a callable bond (see Note 15), these spreads are computed based on a fixed maturity. Call-corrected yield spreads may be of particular use when communicating credit pricing information among traders and investors, for example when comparing them to the yield spreads of other, callable or straight, bonds of the same issuer, or other issuers. These yield spreads offer a uniform cross-market measure of the credit risk and illiquidity priced into corporate bonds.

I now turn to a comparison of the noncallable yield spreads of Occidental's callable bond with those based on a common market practice for pricing callable corporate debt. Practitioners often value callable defaultable bonds based on a term-structure model calibrated to (or estimated from) the prices of straight bonds of the same credit rating. As outlined in the introduction to this paper, this does not allow for idiosyncratic risk factors specific to the callable bond under investigation.[24] I implement this approach by using the parameter estimates (Table 3) and corresponding short spreads (Figure 5) inferred from the straight bond prices to value the straight bond underlying the callable

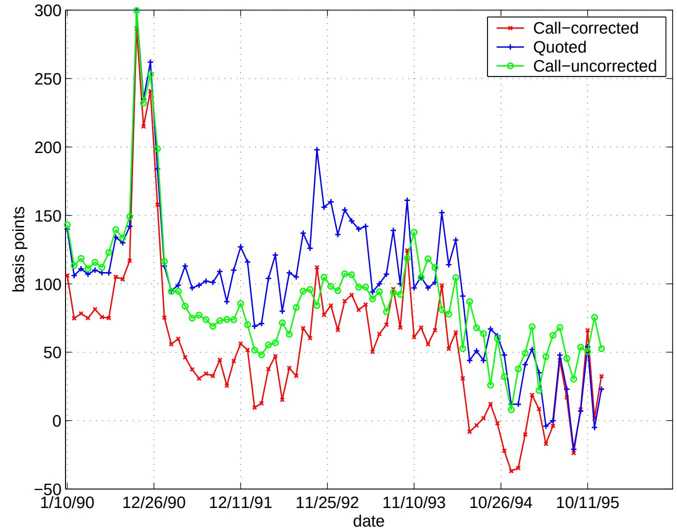

Figure 7: Noncallable yield spreads (relative to Dollar swap curve) of the callable Occidental bond using the short spread estimated from callable bond prices (call-corrected) in Section 6.1, as quoted (see Figure 2), and estimated from straight bond prices (call-uncorrected).

Occidental issue. Figure 7 shows the associated call-uncorrected yield spreads, together with the call-corrected yield spreads computed from the parameter estimates and model-implied short spreads for the callable bond as reported in Section 6.1. For example, the call-uncorrected yield spread exceeds its call-corrected counterpart by 46 basis points in June 1991, and on average underprices the callable issue by 34 basis points. Consequently, in the case of Occidental, using call-uncorrected yield spreads as a substitute for call-corrected yield spreads produces misleading results.

6.6 Estimating Implied Default Probabilities

Given a simple model of recovery at default and for the mean fractional cost rate capturing the illiquidity risk, I am now in a position to estimate the implied risk-neutral probabilities of default. For $t < \bar{t}$, let $\tilde{p}(t, \bar{t})$ denote the risk-neutral probability of default before time $\bar{t}$, given that default did not occur by t. Under technical conditions discussed, for example, in Duffie (2001),

$$\tilde{p}(t, \bar{t}) \;=\; 1 - E_t^Q \left(e^{-\int_t^{\bar{t}} h_u \, du} \right). \tag{14}$$

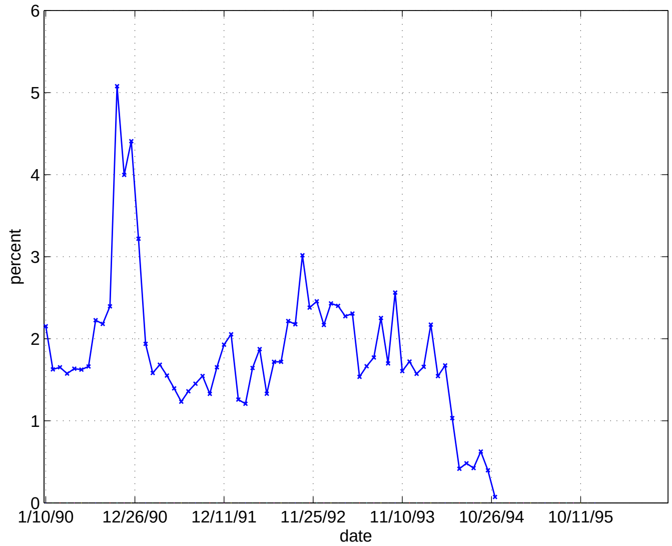

Figure 8: Implied risk-neutral one-year default probabilities for the callable Occidental bond in Section 6.1.

Based on the stochastic model of the short spread s, and given estimates for the parameter vector Θ, one can use the relationship $h = \frac{s-l}{L}$ from (1) to calculate the default probability $\tilde{p}(t, \bar{t})$ using (14). The actual sample mean of loss given default, as a fraction of face value, during our sample period was reported by Altman, Brady, Resti, and Sironi (2003) to be approximately 59%. Using 59% as a rough estimate for L for Occidental's senior unsecured bonds, and ignoring the component due to illiquidity,[25] Figure 8 shows the associated risk-neutral one-year probabilities of default of the callable Occidental bond. These values are computed using the parameter estimates and implied short spreads of the callable Occidental bond from Table 2 and Figure 5.

The data-generating (actual) probabilities of default can be estimated on the basis of estimated risk premia (see, for example, Driessen (2003) and the literature cited therein). Under the simplifying assumption that the default timing risk has no risk premium, the actual intensity of default is also h. (See Jarrow, Lando, and Yu (2001) for a discussion of conditions under which this property holds.) I emphasize that this conclusion does not imply that actual and risk-neutral default probabilities are the same, for while the respective default intensity processes are assumed to have the same outcomes in each state, their probability distributions under $\mathcal{P}$ and $\mathcal{Q}$ may differ. If the actual intensity of default is also h, for $t < \bar{t}$, the actual probability of default before time $\bar{t}$, provided that

Table 4: Contractual features of the out-of-sample bond issued by Occidental. Source: FISD.

Security	Callable 10yr note
Issue date	5/1/88
Maturity	5/1/98
Amount issued (MM)	$200
Coupon	10.75%
Credit rating (Moody's)	Baa3
Seniority	Senior/Unsecured
First call date	5/1/95
Redemption	5/1/95 at par

default did not occur by t, is

$$p(t, \bar{t}) \;=\; 1 - E_t\left(e^{-\int_t^{\bar{t}} h_u\,du}\right),$$

which can be estimated using the estimate for the data-generating mean-reversion parameter K^{uu}. For Occidental, the actual one-year default probabilities are, one average across our sample period, 69 basis points lower than their risk-neutral counterparts.

6.7 Pricing Out-of-Sample Bonds

From Datastream, I obtain market price information for an additional (out-of-sample) U.S.-dollar denominated Occidental debenture, whose contractual characteristics are summarized in Table 4. This callable ten-year note was issued on 5/1/88, with an initial principal of $200 million and a semi-annual coupon at an annual rate of 10.75%. The issue was rated Baa3 by Moody's on 3/17/94, and neither downgraded nor upgraded by that rating agency thereafter. This debenture was redeemable at any time on or after 5/1/95, at Occidental's option, at a redemption price of 100% of the principal amount. The entire issue was called in at par effective 5/1/95, its first call date.

Figure 9 shows the market prices and the model-implied prices of this out-of-sample callable Occidental bond. The model-implied price labeled as "model-implied (callable)" is defined as the market value of the out-of-sample bond when priced using the parameter estimates and implied short spreads of the callable Occidental bond, from Table 2 and Figure 3 respectively. Figure 9 also shows the model-implied prices of the out-of-sample debenture that are computed based on the parameter estimates and the implied short

Figure 9: Empirical and model-implied prices of the out-of-sample Occidental bond using the short spread estimated from callable bond prices (callable) in Section 6.1, and estimated from straight bond prices (straight).

spreads that are associated with the straight Occidental bond. The two models fit the out-of-sample debenture prices reasonably well. In particular, the model-implied prices fluctuate around the actual observed market prices, and do not consistently over- or underestimate them. The average absolute relative pricing error amounts to 1.3% and 1.7% for the model-implied prices relative to the callable and the straight bond, respectively. It is noteworthy, however, that during the two years prior to the first call date (May 1995) of the out-of-sample bond, its market prices are comparably high and "jumpy," while the lower model-implied prices smoothly approach the strike price.

7 Summary and Conclusion

This paper develops a method for estimating the term structure of callable corporate bond yield spreads, using as data the prices of callable debt. The approach allows these term structures to be estimated in the absence of a noncallable equivalent, and in contrast to existing methods, dispenses with the stringent assumption that both callable and noncallable bonds of the same issue have equal credit spreads. A further contribution of this work is the first empirical analysis, in the setting of standard arbitrage-free term-structure models, of the time-series behavior of callable corporate bond yield spreads, explicitly incorporating valuations of the American call options. By effectively disentangling the components of callable corporate bond prices associated with discounting at market interest rates, discounting for default and illiquidity risk, and callability, it became possible to value the American option embedded in the callable bond.

Some interesting observations in the case of Occidental include the very low yield spreads and implied credit spreads, even occasionally trading through LIBOR, for the callable bond. One possible explanation for the observed overpricing when approaching the first call date is that investors had assigned a (significant) positive probability to the event that Occidental would not redeem this issue at the first possible call date. For Occidental, the time-series behavior of callable bond yield spreads significantly differs from that of straight bond yield spreads. Using the latter as a substitute for call-corrected credit spreads would produce misleading results. My findings suggest that the callable bond short (or yield) spreads were significantly more volatile, and that the callable bond traded "rich" relative to the noncallable. This might be due, at least partly, to issue-specific illiquidity risks caused by different clientele trading patterns, or asymmetrically informed traders, or data noise, and to an assumption by investors of "suboptimal" calling behavior by Occidental.

A Proofs

Lemma 1 (Identification of u_t given Θ). *Let us fix some time t_0 before maturity T. Take as given the parameter vector Θ, as well as the initial values for v_{t_0}, r_{t_0}, and u_{t_0}. If $\mathcal{Q}(\tau^* > t_0 | v_{t_0}, r_{t_0}, u_{t_0}; \Theta) > 0$, where τ^* denotes an optimal stopping time, then*

$$V_{t_0}((v_{t_0}, r_{t_0}, u_{t_0}); \Theta) \quad > \quad V_{t_0}((v_{t_0}, r_{t_0}, u_{t_0} + x); \Theta), \quad \textit{for all } x > 0.$$

Proof: Fix some $x > 0$, and let u_t^x denote the process solving the stochastic differential equation (9) with the initial condition $u_{t_0}^x = u_{t_0} + x$. Let $\tilde{K}^{uu}$ denote the mean-reversion parameter of the process u under the risk-neutral measure. Then, $u_t^x = e^{-\tilde{K}^{uu}(t-t_0)} x + u_t$, for all $t_0 \leq t < \infty$, holds $\mathcal{Q}$-almost surely. Since $\mathcal{Q}(\tau^* > t_0 | v_{t_0}, r_{t_0}, u_{t_0}; \Theta) > 0$, there exists some $\underline{t} > t_0$ such that $\mathcal{Q}(\tau^* \geq \underline{t} | v_{t_0}, r_{t_0}, u_{t_0}; \Theta) > 0$. Therefore, suppressing v_{t_0}, r_{t_0} and Θ, $V_{t_0}(u_{t_0})$ exceeds the value of the callable bond given $u_{t_0}^x = u_{t_0} + x$ and assuming exercise according to strategy τ^*. Consequently, $V_{t_0}(u_{t_0})$ dominates the market value of the callable bond given $u_{t_0}^x = u_{t_0} + x$ and assuming that the bond is called so as to minimize its market value, $V_{t_0}(u_{t_0} + x)$. □

Theorem 2. *Assume that X follows a N-factor uncorrelated Gaussian model, in that*

$$dX_t \quad = \quad K(\theta - X_t) + \Sigma \, dW_t,$$

where W is a N-dimensional standard Brownian motion under $\mathcal{P}$, θ is a N-dimensional vector, and K and Σ are $N \times N$ diagonal matrices. For some $N \times N$ matrix $A = (a_{ij})_{i,j=1}^N$, set $Z_t = AX_t$. Then $\xi = \{\xi_t, \mathcal{F}_t; 0 \leq t < \infty\}$ with

$$\xi_t \quad = \quad e^{\sum_{i=1}^N \int_0^t Z_s^{(i)} \, dW_s^{(i)} - \frac{1}{2} \int_0^t |Z_s|^2 \, ds}, \quad \textit{for all } t \geq 0,$$

is a $\mathcal{P}$-martingale.

Proof: Let $|x|$ denote the Euclidean norm of x, and refer to $C[0, \infty)^N$ as the set of all continuous functions from $[0, \infty)$ to $\mathbb{R}^N$. Define a vector $\mu = (\mu^{(1)}, \ldots, \mu^{(N)})$, $\mu^{(i)} : [0, \infty) \times C[0, \infty) \to \mathbb{R}$, of progressively measurable functionals on $C[0, \infty)^N$ via

$$\mu(t, z(\cdot)) \quad = \quad A \, [\theta + e^{-Kt}(x - \theta) + \Sigma(z(t) + K e^{-Kt} \int_0^t e^{Ks} z(s) \, ds)].$$

Define $z^*(t) = \max_{0 \leq s \leq t} |z(s)|$ and $a^* = \max_{i=1,\ldots,N} |a_{i,\cdot}|$, as well as $K^* = |\text{diag}(K)|$ and

$\Sigma^* = |\mathrm{diag}(\Sigma)|$. Then, for each $T \geq 0$ and any $t \in [0, T]$,

$$\begin{aligned} |\mu(t, z(\cdot))| &\leq N\, a^* \left[|\theta|(1 + e^{K^* t}) + e^{K^* t}|x| + \Sigma^*(2 + e^{K^* t}) z^*(t) \right] \\ &\leq K_T \,(1 + z^*(t)), \end{aligned} \tag{A.1}$$

where $K_T = N\, a^* \left[|\theta|(1 + e^{K^* T}) + e^{K^* T}|x| + \Sigma^*(2 + e^{K^* T}) \right]$. On the other hand, we can rewrite X_t as

$$\begin{aligned} X_t &= \theta + e^{-Kt}(x - \theta) + \Sigma e^{-Kt} \int_0^t e^{Ks}\, dW_s \\ &= \theta + e^{-Kt}(x - \theta) + \Sigma \left(W_t - K e^{-Kt} \int_0^t e^{Ks} W_s\, ds \right), \end{aligned}$$

where the last equation uses Itô's formula. Consequently, $Z_t = \mu(t, W_.)$. From this last equation, together with (A.1), it follows that ξ is a $\mathcal{F}_t$-martingale (see, for example, Karatzas and Shreve (1997), pg. 199-200). $\square$

B Additional Background Figures

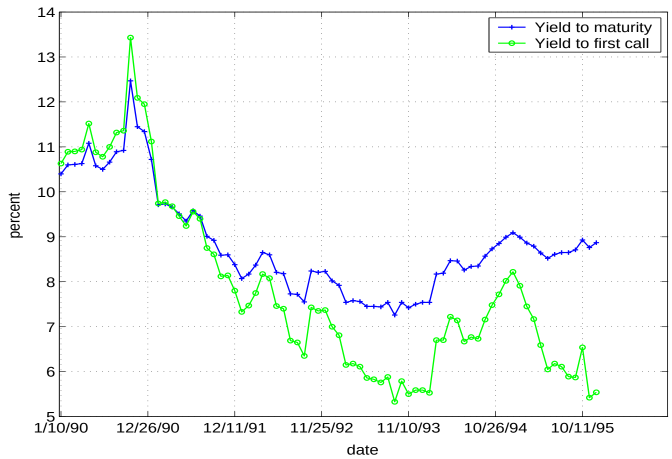

Figure 10: Yield to maturity and yield to first call of the callable Occidental bond. Source: Datastream.

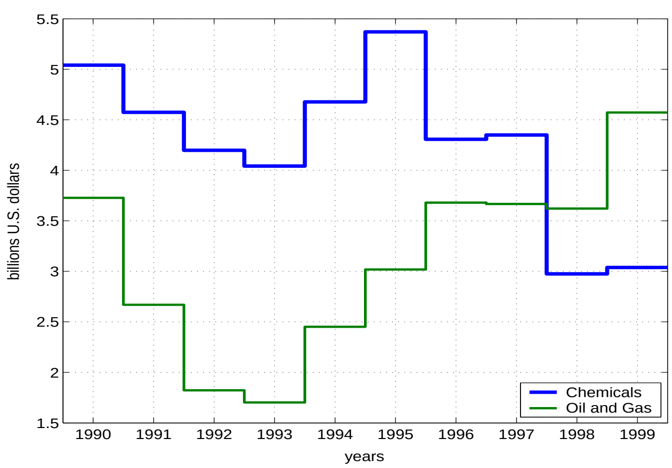

Figure 11: Net sales and operating revenues from Occidental's oil and gas operations and from its chemicals operations. Source: Global Access.

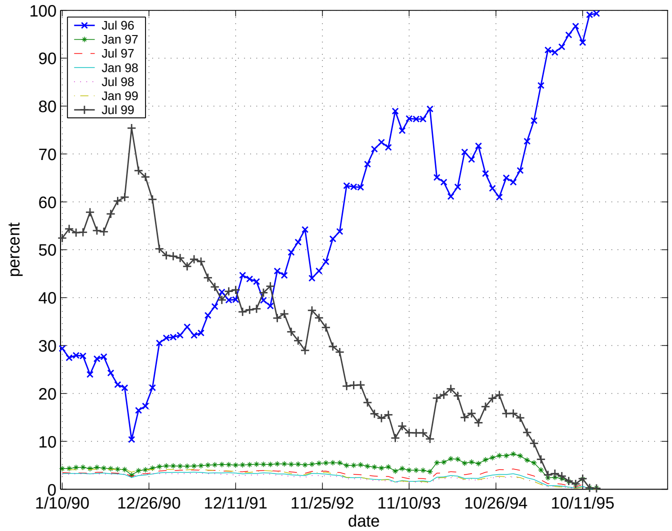

Figure 12: Implied conditional risk-neutral distribution of the optimal exercise date for the callable Occidental bond.

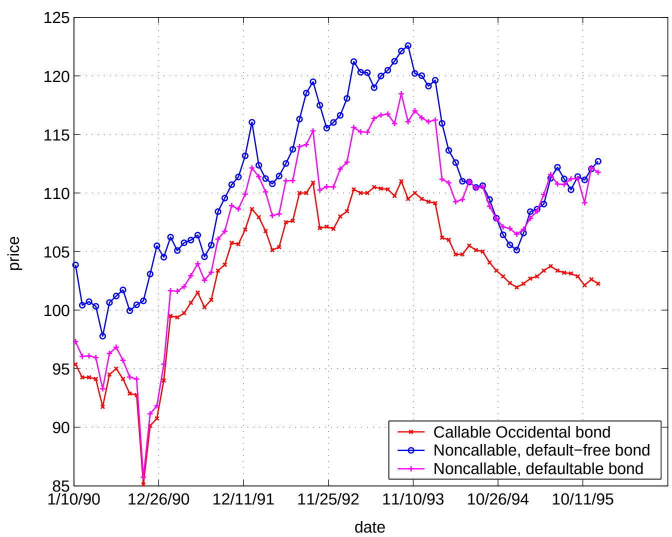

Figure 13: Prices of the callable Occidental bond, of its noncallable, default-free equivalent, and of its noncallable, defaultable equivalent.

References

Acharya, V. V., and J. N. Carpenter, 2002, "Corporate Bonds: Valuation, Hedging, and Optimal Call and Default Policies," *Review of Financial Studies*, 15, 1355-83.

Altman, E., B. Brady, A. Resti, and A. Sironi, 2003, "The Link between Default and Recovery Rates: Theory, Empirical Evidence and Implications," working paper, Stern School of Business New York University NY.

Andersen, L., 2000, "A Simple Approach to the Pricing of Bermudan Swaptions in the Multi-Factor Libor Market Model," *Journal of Computational Finance*, 2, 5-32.

Andersen, L., and M. Broadie, 2004, "Primal-Dual Simulation Algorithm for Pricing Multidimensional American Options," *Management Science*, 50, 1222-1234.

Barraquand, J., and D. Martineau, 1995, "Numerical Valuation of High-Dimensional Multivariate American Securities," *Journal of Financial and Quantitative Analysis*, 30, 383-405.

Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637-654.

Bossaerts, P., 1989, "Simulation Estimators of Optimal Early Exercise," Working Paper, Carnegie-Mellon University.

Boyle, P., M. Broadie, and P. Glasserman, 1997, "Monte Carlo Methods for Security Pricing," *Journal of Economic Dynamics and Control*, 21, 1267-1321.

Boyle, P. P., 1977, "Options: A Monte Carlo Approach," *Journal of Financial Economics*, 4, 1-38.

Brennan, M., and E. Schwartz, 1980, "Analyzing Convertible Bonds," *Journal of Financial and Quantitative Analysis*, 15, 907-29.

Broadie, M., and P. Glasserman, 1997, "Pricing American-Style Securities Using Simulation," *Journal of Economic Dynamics and Control*, 21, 1323-52.

——— , 2004, "A Stochastic Mesh Method for Pricing High-Dimensional American Options," *Journal of Computational Finance*, 7.

Carr, P., and G. Yang, 2001, "Simulating Bermudan Interest Rate Derivatives," in *Quantitative Analysis in Financial Markets*, ed. by M. Avellaneda. World Scientific Publishing, vol. II, pp. 295-316.

Carrière, J., 1996, "Valuation of Early-Exercise Price of Options Using Simulations and Nonparametric Regression," *Insurance: Mathematics and Economics*, 19, 19-30.

Clément, E., D. Lamberton, and P. Protter, 2002, "An Analysis of a Least Squares Regression Method for American Option Pricing," *Finance and Stochastics*, 6, 449-71.

Clewlow, L., and C. Strickland, 1998, "Pricing Interest Rate Exotics by Monte Carlo Simulation," in *Monte Carlo: Methodologies and Applications for Pricing and Risk Management*. Risk Books, London, U.K., chap. 17.

Collin-Dufresne, P., and R. S. Goldstein, 2001, "Do Credit Spreads Reflect Stationary Leverage Ratios?," *Journal of Finance*, 56, 1929-57.

——— , 2002, "Pricing Swaptions within an Affine Framework," *Journal of Derivatives*, 10, 1-18.

Collin-Dufresne, P., R. S. Goldstein, and J. Hugonnier, 2004, "A General Formula for Valuing Defaultable Securities," *Econometrica*, 72, 1377-1408.

Das, S., and P. Tufano, 1995, "Pricing Credit Sensitive Debt When Interest Rates, Credit Ratings, and Credit Spreads are Stochastic," *Journal of Financial Engineering*, 5, 161-98.

Delbaen, F., and W. Schachermayer, 1999, "A General Version of the Fundamental Theorem of Asset Pricing," *Mathematische Annalen*, 300, 463-520.

Driessen, J., 2003, "Is Default Event Risk Priced in Corporate Bonds?," forthcoming, Review of Financial Studies.

Duan, J.-C., 2002, "Nonparametric Option Pricing by Transformation," Working Paper, University of Toronto.

Duffee, G. R., 1999, "Estimating the Price of Default Risk," *Review of Financial Studies*, 12, 197-226.

Duffie, D., 2001, *Dynamic Asset Pricing Theory*. Princeton University Press, Princeton, NJ, third edn.

Duffie, D., D. Filipovic, and W. Schachermayer, 2003, "Affine Processes and Applications in Finance," *Annals of Applied Probability*, 13.

Duffie, D., L. H. Pedersen, and K. J. Singleton, 2003, "Modeling Sovereign Yield Spreads: A Case Study of Russian Debt," *Journal of Finance*, 58, 119-60.

Duffie, D., and K. J. Singleton, 1999, "Modeling Term Structures of Defaultable Bonds," *Review of Financial Studies*, 12, 687-720.

Elton, E. J., M. J. Gruber, D. Agrawal, and C. Mann, 2001, "Explaining the Rate Spread on Corporate Bonds," *Journal of Finance*, 56, 247-77.

Fan, R., J. G. Haubrich, R. Ritchken, and J. B. Thomson, 2003, "Getting the Most Out of a Mandatory Subordinated Debt Requirement," *Journal of Financial Services Research*, 24, 149-179.

FISD, 2002, "The Fixed Investment Securities Database," LJS Global Investment Services, Inc.

Fu, M. C., S. B. Laprise, D. B. Madan, Y. Su, and R. Wu, 2001, "Pricing American Options: A Comparison of Monte Carlo Simulation Approaches," *Journal of Computational Finance*, 4.

Gallant, A. R., and G. Tauchen, 1996, "Which Moments to Match?," *Econometric Theory*, 12, 657-81.

Garcia, D., 2002, "A Monte Carlo Method for Pricing American Options," forthcoming, Journal of Economic Dynamics and Control.

Guntay, L., 2002, "Pricing the Risks of Callable Defaultable Coupon Bonds," Working Paper, University of Maryland.

Hansen, L. P., 1982, "Large Sample Properties of Generalized Methods of Moments Estimators," *Econometrica*, 50, 1029-64.

Harrison, M., and D. Kreps, 1979, "Martingales and Arbitrage in Multiperiod Securities Markets," *Journal of Economic Theory*, 20, 381-408.

Haugh, M. B., and L. Kogan, 2004, "Pricing American Options: A Duality Approach," *Operations Research*, 52, 258-270.

Huang, J.-Z., and M. Huang, 2003, "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk," Working Paper, Penn State University.

Jarrow, R. A., D. Lando, and S. M. Turnbull, 1997, "A Markov Model for the Term Structure of Credit Risk Spreads," *Review of Financial Studies*, 10, 481-523.

Jarrow, R. A., D. Lando, and F. Yu, 2001, "Default Risk and Diversification: Theory and Empirical Implications," forthcoming, Mathematical Finance.

Jarrow, R. A., H. Li, S. Liu, and C. Wu, 2003, "Valuing Callable Corporate Bonds in a Reduced Form Model Using a Call Intensity Process," Working Paper, Cornell University.

Jarrow, R. A., and S. M. Turnbull, 1995, "Pricing Derivatives on Financial Securities Subject to Credit Risk," *Journal of Finance*, 50, 53-86.

Karatzas, I., and S. E. Shreve, 1997, *Brownian Motion and Stochastic Calculus*. Springer, New York, NY, second edn.

Kim, I. J., K. Ramaswamy, and S. Sundareasan, 1993, "Does the Default Risk in Coupons Affect the Valuation of Corporate Bonds ?," *Financial Management*, 22, 117-31.

Kusuoka, S., 2003, "Monte Carlo Method for Pricing of Bermuda Type Derivatives," *Advances in Mathematical Economics*, 5, 153-66.

Lai, T. L., and S. P.-S. Wong, 2003, "Valuation of American Options via Basis Functions," *IEEE Transactions on Automatic Control*, 49, To appear.

Li, B., and G. Zhang, 1996, "Hurdles Removed," Working Paper, Merrill Lynch, New York.

Longstaff, F. A., and E. S. Schwartz, 1995, "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt," *Journal of Finance*, 50, 789-819.

——— , 2001, "Valuing American Options by Simulation: A Simple Least-Squares Approach," *Review of Financial Studies*, 14, 113-47.

Madan, D. B., and H. Unal, 1998, "Pricing the Risks of Default," *Review of Derivatives Research*, 2, 121-60.

Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449-70.

Moreno, M., and J. F. Nava, 2003, "On the Robustness of Least-Squares Monte Carlo (LSM) for Pricing American Derivatives," *Review of Derivatives Research*, 6, 107-28.

Nielsen, L. T., J. Saá-Requejo, and P. Santa-Clara, 1993, "Default Risk and Interest Rate Risk: The Term Structure of Default Risk," Working Paper, INSEAD, France.

Peterson, S., and R. C. Stapleton, 2003, "The Pricing of Options on Credit-Sensitive Bonds," Working Paper, Strathclyde University, U.K.

Picazo, J. A., 2000, "American Option Pricing: A Classification-Monte Carlo (CMC) Approach," in *Monte Carlo and Quasi-Monte Carlo Methods 2000*.

Protter, P., 1990, *Stochastic Integration and Differential Equations*. Springer-Verlag, New York, NY.

Pye, G., 1974, "Gauging the Default Premium," *Financial Analyst's Journal*, 30, 623-36.

Rogers, C., 2002, "Monte Carlo Valuation of American Options," .

Sarkar, S., 2000, "Probability of Call and Likelihood of the Call Feature in a Corporate Bond," *Journal of Banking and Finance*, 25, 505-33.

Singleton, K. J., and L. A. Umantsev, 2002, "Pricing Coupon-Bond Options and Swaptions in Affine Term Structure Models," *Mathematical Finance*, 12, 427-46.

Tilley, J., 1993, "Valuing American Options in a Path Simulation," *Transactions of the Society of Actuaries*, 45, 83-104.

Tsitsiklis, J., and B. V. Roy, 1999, "Optimal Stopping of Markov Processes: Hilbert Space Theory, Approximation Algorithms, and an Application to Pricing High-Dimensional Financial Derivatives," *IEEE Transactions on Automatic Control*, 44, 1840-51.

——— , 2001, "Regression Methods for Pricing Complex American-Style Options," *IEEE Transactions on Neural Networks*, 12, 694-703.

Uglum, J., 2001, "When is Exercise Bad for You? An Analysis of the Cancellable Swap Market," Working Paper, Financial Research Group, Morgan Stanley Dean Witter.

Umantsev, L. A., 2001, "Econometric Analysis of European LIBOR-based Options within Affine Term-Structure Models," Ph.D. thesis, Stanford University.

Zhou, C., 1997, "A Jump-Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities," Working Paper, Federal Reserve Board.

Notes

[1]The idea of using regression methods previously appeared in Carrière (1996), and is also discussed in Tsitsiklis and Roy (1999, 2001).

[2]Including Brennan and Schwartz (1980), Kim, Ramaswamy, and Sundareasan (1993), Nielsen, Saá-Requejo, and Santa-Clara (1993), Longstaff and Schwartz (1995), Zhou (1997), and Collin-Dufresne and Goldstein (2001).

[3]In recent contributions to the pricing of European optionality within the affine framework, numerically accurate and computationally efficient approximations to swaption prices have been developed. For example, Singleton and Umantsev (2002) provide theoretical results based on an affine approximation of the decision boundary and Fourier inversion methods, whereas Collin-Dufresne and Goldstein (2002) use an Edgeworth-expansion technique to estimate the probability distribution of the future asset price. Although suitable for the European swaption, these papers do not treat the usual American case of callable corporate debt.

[4]This is a filtration satisfying the usual conditions. See Protter (1990) for a complete definition.

[5]The short-rate process r is progressively measurable with respect to $\{\mathcal{F}_t : t \geq 0\}$, with $\int_0^t |r_s|\, ds < \infty$ $\mathcal{Q}$-almost surely and $E^{\mathcal{Q}}\left[e^{-\int_0^t r_s\, ds}\right] < \infty$, for all t. See Protter (1990) for details.

[6]Merton (1974) proposed this approach of value minimization over stopping times in a structural model for pricing callable defaultable bonds.

[7]The LSM algorithm, as proposed by Longstaff and Schwartz (2001), can be applied to value American-style options embedded in callable, defaultable bonds, after replacing the default-free discount rate r by the default- and liquidity-adjusted short rate R.

[8]In practice, one often estimates the conditional expected value of the bond at T_i, if not exercised, as a function in polynomials of the intrinsic value $V_{T_i}(T)$ and of all unmatured discount bond prices $B(T_i, T_j)$, with maturity dates T_j up to and including the final maturity date of the bond.

[9]Additionally, Clément, Lamberton, and Protter (2002) show that the normalized error of the LSM procedure, after replacing the conditional expected values of continuation by projections on a finite set of basis functions, is asymptotically Gaussian.

[10]Kusuoka (2003), however, adapts the LSM procedure in a more expensive manner than initially suggested by Longstaff and Schwartz (2001) in that he proposes to simulate,

at each exercise date t_k, a new set of sample paths in order to approximate the value function, while using the approximate stopping rules previously determined at exercise times $t_{k+1}, \ldots, t_K = T$.

[11]Lai and Wong (2003) provide some guideline to the choice of basis functions and their underlying theory. These authors apply neuro-dynamic programming. That is, they use neural networks and regression splines to approximate the regression functions.

[12]For example, a variation of the Generalized Methods of Moments of Hansen (1982) or the Efficient Methods of Moments of Gallant and Tauchen (1996) could be used to match the model-implied and actual moments of the observable variables.

[13]Here, for a given choice of parameters Θ, I first calculate $G_{t_n}(\cdot;\Theta)$ by MC simulation for a sufficiently wide range of outcomes of s_{t_n}. Next, I use Chebyshev polynomials to approximate $G_{t_n}(\cdot;\Theta)$ and $DG_{t_n}(\cdot;\Theta)$. Finally, I invert $G_{t_n}(\cdot;\Theta)$ (approximated) in V_{t_n} to retrieve an approximation of $s^{\Theta}_{t_n}$, at which I evaluate $DG_{t_n}(\cdot;\Theta)$.

[14]On 12/20/99, Occidental announced the completion of a tender offer of this issue. The amount tendered and repurchased totaled \$240.286 million, leaving an amount of \$89.714 million outstanding at that time. Evaluation was based on the yield to maturity of the 5.875% U.S. Treasury note due 11/30/01 at the time of the tender plus 37.5 basis points, and plus interest accrued.

[15]The "yield" on the callable bond is recorded, in practice, as the minimum of the yield to maturity and the yield to first call.

[16]The values for α and Σ^{uu} are shown in percent.

[17]That is, I treat the parameter estimates in Table 2 as the true parameters, generate an independent sample of the time series of observed callable Occidental bond prices, and re-estimate the parameters using MLE. I repeat this procedure 100 times in order to retrieve the MC distribution of the estimated parameters. For the callable Occidental bond, I adopt a grid search strategy to re-estimate the parameters.

[18]That is, the term structure of zero-coupon yield spreads evaluated at the implied sample means of the reference curve and the process u.

[19]At any given time point, using the parameter estimates and implied noncallable spreads from Table 2 and Figure 3, I obtain the sample distribution, with respect to the risk-neutral measure, of the optimal exercise date. In December 1992, for example, the conditional risk-neutral probabilities that optimal exercise occurs on the first (July 1996) and the final (July 1999) call date are estimated as 47% and 34%, respectively.

[20]Both covariates were centered around their respective sample means.

[21]I repeat the regression analysis for the time period that excludes the Gulf War, that

is, April 1991 to December 1995. The resulting estimated regression model is

$$u_t = \underset{(3.13)}{-11.20} - \underset{(0.05)}{0.39}\,\mathrm{CHEM}_t + \underset{(1.03)}{2.96}\,\mathrm{LEV}_t + \epsilon_t,$$

with standard error estimates, corrected for heteroskedasticity using the Newey-West method, shown in parentheses. As expected, without the evident macroeconomic effects of the Gulf War, I now find it harder to explain Occidental's short spread. This difficulty is reflected by the lower R^2 of 62.5%. Both coefficients, however, show the expected sign and are significant at conventional levels.

[22]The values for α and Σ^{uu} are shown in percent.

[23]The values for ϑ^c and σ^c are shown in percent.

[24]Fan, Haubrich, Ritchken, and Thomson (2003) price American-style puttable fixed-rate bonds relative to credit spreads that are calibrated to the prices of noncallable bank loans.

[25]Data limitations make it difficult to determine a breakdown of Occidental's total short spreads into the risk-neutral mean fractional loss rate due to credit risk, hL, and the risk-neutral mean fractional cost rate due to illiquidity, l.