Microeconomic Inventory Behavior and Aggregate Inventory Dynamics*

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Abstract

This paper provides a unified explanation for two puzzles in the inventory literature: 1. estimates of inventory speeds of adjustment in aggregate data are very small; 2. estimates of inventory speeds of adjustment in firm-level data are significantly higher than in the aggregate. The paper develops a multisector model where inventories are held to avoid stockouts and price markups vary along the business cycle. Countercyclical markups variations introduce a downward bias in estimates of adjustment speeds obtained from partial adjustment models. An aggregation bias obtains when the cyclicality of markups differs across sectors. The paper also shows that these results apply not only to inventories but also to labor demand. Montercarlo simulations of a calibrated version of the model suggest that these biases are quantitatively significant.

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1 Introduction

A distinctive feature of the dynamics of inventory stocks over the business cycle is that they seem to adjust slowly to changes in sales. This fact has been documented in different ways. For example, Ramey and West (1999) compute deviations of inventory stocks from their long-run relationship with respect to sales and show how these deviations are very persistent in quarterly data. Other researchers, starting from Auerbach and Feldstein (1976), have estimated partial adjustment models of inventory stocks and found the estimated speeds of adjustment to be puzzlingly small.[1] Among other reasons, small speeds of adjustment are puzzling because large quarterly variations in inventory stocks amount to only a few days worth of production in the data.

Some recent papers have added a second dimension to this inventory adjustment puzzle. Specifically, Seitz (1993) and Schuh (1996), among others, have shown how inventory speeds of adjustment estimated using firm-level data are significantly larger than their counterparts estimated using aggregate data.[2] For example, Schuh (1996) in a comprehensive study of US manufacturing firms finds that, according to the empirical specification of his model, the weighted average of adjustment speeds estimated using firm-level data is 67 to 105 percent larger than the one obtained using aggregate data constructed from the same panel of firms. Despite this evidence, the effect of aggregation across heterogeneous firms is not well understood yet.[3]

The conceptual framework underlying most of these empirical exercises is represented by Lovell (1961)'s stock-adjustment model and Holt et al. (1960)'s linear-quadratic model. In both

[1]These authors, for example, estimate that firms eliminate on average in a quarter less than 6 percent of the gap between current and desired inventories. Similar results are reported by, among others, Maccini and Rossana (1981), Blinder (1986), and Haltiwanger and Maccini (1989). Using monthly finished-goods inventory data from the Department of Commerce for the period 1967:01-1997:12, I obtain estimated speeds of adjustment equal to approximately 3% and 6% for durable and nondurable goods industries respectively.

[2]Blinder (1986) and Carlson and Dunkelberg (1989) make similar points.

[3]For example, in their survey of the inventory literature, Ramey and West (1999, page 881) ask whether aggregation across heterogeneous firms substantially account for what they call the "persistence of the inventory-sales relationship" (i.e. the small adjustment speed of inventory stocks) and go on to state that they are are not aware of analytical arguments establishing this bias.

models firms face costs of adjusting their inventories either because marginal cost functions are upward sloping or because of other costs of adjusting production, such as recruiting and training costs, etc. Thus, in an expansion inventory stocks adjust slowly to increases in sales resulting in countercyclical and persistent inventory-sales ratios.

This paper provides a unified explanation for these two inventory puzzles that does not rely on the existence of costs of adjusting inventories. It does so by exploring the alternative idea, first advanced by Auerbach and Feldstein (1976), that firms might indeed not *want* to keep a constant inventory-sales ratio over the business cycle even if they did not face any cost of adjusting production or inventories. In particular, this might be true if price markups vary countercyclically over the business cycle, as argued by Bils and Kahn (2000). The paper shows, qualitatively and quantitatively, how failure to account for cyclical changes in inventory targets in empirical partial adjustment models results in a downward bias in estimates of inventory adjustment speeds. Moreover, it shows that this result is not limited to inventories, as cyclical changes in markups tend to produce similar effects on estimates of adjustment speeds for labor.

To make these points I first consider a finished goods inventory model with multiple sectors.[4] Firms in all sectors operate a constant returns to scale production function and do not face any cost of adjusting production or inventories. Inventories contribute to increase firms' sales at a given price by reducing the likelihood of stockouts, as in Bils and Kahn (2000). Then, I ask the following question: suppose that real world data were generated by such model and that these firm-level and aggregate data were used to estimate standard partial adjustment equations; would the estimates obtained from running such regressions be consistent with the results reported by the empirical literature mentioned at the beginning?

[4] In this paper I focus on finished goods inventories rather than input inventories because the evidence that inventory-sales ratios are persistent and countercyclical is stronger for the former. Humprehys, Maccini and Schuh (2001) find that the output inventory ratio is more countercyclical than the input inventory ratio in both the nondurable and durable manufacturing sectors. This evidence is confirmed by Bils and Kahn (2000, footnote 4). They report that the only inventory-sales ratios that are systematically countercyclical outside finished goods inventories are works-in-process for new housing construction.

The answer to this question is affirmative. Countercyclical movements in price markups induce firms to reduce inventory holdings relative to sales in expansions and increase them in recessions. Failure to account for these cyclical variations in price markups leads to an omitted variable problem in partial adjustment regressions and results in downward biased estimates of inventory adjustment speeds. In particular, estimated speeds of adjustment are lower in sectors where price markups are relatively more cyclical. Moreover, sectors where markups are relatively more cyclical tend to affect the aggregate estimates of inventory adjustment speeds more than these sectors' average share of aggregate inventories would suggest. Using time series for sales and inventories simulated from a calibrated version of the model I show how these biases are quantitatively very significant. For example, when sectors displaying the most volatile markup movements account on average for only 10 percent of aggregate inventories, the weighted average of firm-level speeds of adjustment is 135 percent higher than its aggregate counterpart.

The intuition behind these results is as follows. First, in standard partial adjustment models the inventory target is specified as a constant proportion of sales. If markups are countercyclical, however, firms desire to expand their inventory stocks less than proportionally when sales increase. Therefore, partial adjustment models tend to overpredict the increase in inventory targets in expansions, and attribute the discrepancy between the latter and the observed change in inventories to the fact that firms are adjusting slowly.

Second, in standard partial adjustment models the gap between target and current inventories increases in an expansion relatively more for sectors in which markups are more volatile. This is because these sectors experience the same increase in sales as other sectors, but do not expand their inventory stocks as much. Therefore, in an expansion these sectors contribute more to the increase in the aggregate inventory gap, as measured by partial adjustment models, than to the observed increase in inventory stocks. As a result, the adjustment speed estimated from aggregate data tends to reflect disproportionally the behavior of sectors with more cyclical markups.

This paper is related to two literatures. The first one, reviewed by Rotemberg and Woodford (1999), is concerned with measuring the dynamics of price markups over the business cycle. Cyclical changes in the relationship between sales and inventories can potentially provide useful information regarding the behavior of price markups over the cycle, as argued by Bils and Kahn (2000). With respect to the latter this paper makes two novel contributions. First, it shows how failure to account for countercyclical changes in markups in traditional partial adjustment regressions can explain the robust empirical finding of small inventory adjustment speeds at the aggregate level. Second, it shows how adjustment speeds' estimates obtained from aggregate data will tend to be lower than firm-level estimates if sectors differ in the cyclicality of their markups.

The paper is also related to the literature that is exploring the aggregate effects of microeconomic nonlinearities. The effect of aggregation and model misspecification on estimates of partial adjustment models has been recently addressed by Caballero and Engel (2003). They show how failure to account for (S, s)-type of adjustment policies used by firms when estimating partial adjustment models at the firm-level results in an upward bias in the estimates of adjustment speeds. They also show how estimating these models using aggregate data leads to a reduction, but not necessarily to a correction, of this problem. Caballero and Engel focus on the adjustment speed of employment and prices rather than finished goods inventories. While their results are in principle applicable to the question addressed in this paper, it seems reasonable, as traditionally done in the inventory literature, to model finished goods inventories abstracting from microeconomic nonconvexities.[5]

The rest of the paper is organized as follows. Section 2 describes the model economy. Section

[5]Within the inventory literature, Lovell (1993) is close in spirit to the exercise I undertake here. Using a simulation approach he shows how aggregation may bias adjustment speeds' estimates downward. However, he uses a reduced-form model where firms don't optimize and parameters are not calibrated. Moreover, he does not provide an explanation for this result.

3 describes the equilibrium of the model in closed-form. Section 4 discusses, from a qualitative point of view, the effects of cyclical markups on estimates of inventory speeds of adjustment. Section 5 contains the quantitative results of the paper. Section 6 shows how the results for inventories carry over to labor demand as well. Section 7 concludes. The appendix illustrates the derivation of the closed-form solution of the model.

2 The Model Economy

The economy I consider extends Bils and Kahn (2000)'s model of a representative firm to an economy with multiple sectors. Each sector produces a continuum of varieties of a product. In turn, each variety within a sector is produced by a monopolistically competitive firm. Firms within a sector are otherwise homogeneous. The key source of heterogeneity in the model is across sectors. Differences in the properties of demand functions in different sectors lead to heterogeneity in the cyclical properties of markups across sectors.

Sales and inventories

The market structure I consider is the simplest one that captures the following two key elements: a) Firms have some degree of market power, so that it is meaningful to discuss the effects of changes in price markups; b) Firms are ex-ante heterogeneous. Heterogeneity is necessary in order to analyze the effects of aggregation.

The description of the model is as follows. Time is discrete and infinite. The objective of each firm is to maximize the present discounted value of its profits, discounted at the constant rate $\beta \in (0, 1)$. To keep the model as simple as possible I consider a two-sector economy. Each sector, indexed by $k = 1, 2$, produces different varieties of a product. Each variety is produced by one and only one monopolistically competitive firm. Each sector is populated by a continuum of firms of measure one.

The key building block of this model is represented by the relationship between a firm's finished goods inventories and its sales. Following Bils and Kahn (2000), inventories are assumed to contribute to increase a firm's sales at a given price by reducing the likelihood of stockouts.[6] Denoting by a_{kt}^j the sum of firm j's beginning-of-the-period output inventories i_{kt}^j and current production y_{kt}^j, sales for firm j in sector k at time t are given by

$$s_{kt}^j = \gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}}\right)^{-\delta_{kt}} \left(a_{kt}^j\right)^{\phi}, \ 0 < \phi < 1, \ \delta_{kt} > 1. \tag{1}$$

The term $\left(a_{kt}^j\right)^{\phi}$ in this equation captures the revenue-generating role of inventories. The parameter ϕ determines the extent by which a higher stock of goods contributes to generate higher sales at a given price.

In equation (1), a firm's sales in sector k are also assumed to depend on this firm's price relative to a measure P_{kt} of the price level in sector k. The only restriction imposed on P_{kt} is that when all firms in sector k charge the same price p, then $P_{kt} = p$. The elasticity of demand faced by firms operating in sector k is denoted by δ_{kt} and is allowed to change stochastically over time. Cyclical variations in the elasticity of demand give rise to cyclical variations in markups. The parameter μ_k determines the weight of sector k firms in aggregate sales and inventories, and is such that $\mu_1 + \mu_2 = 1$.[7]

Last, sales in all sectors are affected by an aggregate shock γ_t. The latter evolves over time according to the process $\gamma_t = \gamma_{t-1}^{\rho} u_t$, where $0 < \rho < 1$. The random variable u_t is identically and

[6]This approach to modeling the role of inventories acknowledges that in reality firms might stockout even if their observed inventory stocks are not zero, because goods come in different colors, sizes etc. and consumers have tastes over these characteristics. Therefore, having higher inventories decreases the chances of a mismatch between the available stock and the preferences of consumers. Kahn (1987, 1992) develops and tests a structural model of the stockout avoidance motive for holding inventories.

[7]Notice that, since μ_k is a parameter, a sector's sales do not depend on the relative price of goods in the two sectors. The model can be generalized to allow for a dependence of μ_k on the ratio P_{1t}/P_{2t}. Since μ_k is simply a scale factor, this generalization does not affect the inventory-sales ratio in a sector, as will become evident in the next section. The composition of aggregate sales and inventories is, however, affected by μ_k. In footnote (17), I argue that this dependence of μ_k on P_{1t}/P_{2t} would make the results of the paper even stronger.

independently distributed over time according to some distribution function F with positive support. Without loss of generality, I normalize its unconditional mean to one. The timing of the model is such that the demand shock u_t is observed by firms before making their production and pricing decisions.

Production

Technology is assumed to be the same across firms. In order to emphasize the role played by cyclical variations in markups, instead of upward sloping marginal cost curves, in generating low estimates of adjustment speeds, I assume that firms operate a constant returns to scale production function using labor as the only input. Thus, a firm's cost function takes the linear form:

$$c\left(y_{kt}^{j}\right) = c y_{kt}^{j},$$

for some $c > 0$.[8]

A firm j that starts period t with inventories i_{kt}^{j}, produces output y_{kt}^{j} and sells s_{kt}^{j} units of the good, begins period $t+1$ with inventories equal to

$$i_{kt+1}^{j} = a_{kt}^{j} - s_{kt}^{j}, \tag{2}$$

with $a_{kt}^{j} = i_{kt}^{j} + y_{kt}^{j}$. Since inventories cannot be negative it must be the case that $a_{kt}^{j} \geq s_{kt}^{j}$. To simplify the notation in what follows I ignore this non-negativity constraint on inventories. In the simulations presented below this constraint never binds. For completeness though, in the appendix I derive the solution of the model taking this constraint into account.

[8]Many papers in the inventory literature have estimated the slope of the marginal cost function, with different outcomes (see the review by Ramey and West, 1999). While marginal costs curves are usually found to be upward sloping, Ramey (1991) estimates downward sloping marginal cost curves. Bils and Kahn (2000) instead find that for most sectors they consider the marginal cost function is independent of output for given input prices.

Cyclical markups and sectoral heterogeneity

As it will become clear in the next section, a constant elasticity $\delta_{kt} = \delta$ in (1) implies that firms choose a constant markup of price over marginal cost. To generate cyclical variations in markups in a simple way, I allow this elasticity to change over time with the aggregate state of the economy:[9]

$$\delta_{kt} = 1 + (\delta - 1)\,\gamma_t^{\pi_k}, \text{ with } \pi_k < 1,\ \delta > 1. \tag{3}$$

The specification (3) is such that when $\pi_k = 0$ the elasticity of demand is constant (i.e. $\delta_{kt} = \delta$) and price markups are also constant. When $\pi_k \neq 0$ instead, the elasticity becomes time-varying as a function of the exogenous state of demand. In particular, if $\pi_k > 0$, periods when $\gamma_t > 1$ (i.e., "expansions") are also periods when demand is relatively more elastic and markups are lower. Thus, setting $\pi_k > 0$ in (3) gives rise to countercyclical movements in markups.[10]

The key assumption of the model is that the cyclical behavior of markups differs across sectors. In particular, I assume that markups in sectors one and two are both countercyclical ($\pi_1 > 0$ and $\pi_2 > 0$), but that they are more so in sector one than in sector two ($\pi_1 > \pi_2$). The empirical evidence largely supports the assumption that the cyclical properties of markups vary across sectors. Bils (1987) and Rotemberg and Woodford (1991) study two-digit-SIC manufacturing industries and reports that price markups over marginal costs are countercyclical in almost all of these industries. Moreover, their results point to a wide dispersion across industries in the degree of cyclicality of markups (see especially Bils' Table 5 and Rotemberg and Woodford's Table 8).

[9] As Rotemberg and Woodford (1999, page 1119) observe "the simplest and most familiar model of desired markup variations attributes them to changes in the elasticity of demand faced by the representative firm." Here I assume, for simplicity, that variations in the elasticity of demand are exogenous. Bils (1989) and Gali (1994) show how, when purchasers differ in their elasticity of demand, cyclical changes in the composition of demand can generate endogenous variations in its elasticity.

[10] The specification in (3) guarantees that $\delta_{kt} > 1$ at all times and that in the steady state (i.e. when $\gamma_t = 1$), the elasticity δ_{kt} is the same across sectors: $\delta_{kt} = \delta$.

Evidence of differences across sectors in the cyclical properties of markups can also be found in more highly disaggregated data.[11] For example, Binder (1995) analyzes business cycles across four-digits-SIC manufacturing industries and concludes (at page 27) that in light of his results "findings of a uniform cyclical variation of markups in producer goods manufacturing industries may have to be reconsidered." Domowitz, Hubbard and Petersen (1987) consider 57 four-digits-SIC manufacturing industries from 1958 to 1981 and report a wide dispersion in the yearly standard deviation of markups across industries.[12]

3 Firms' Optimization and Equilibrium

When making its pricing and production choices, a firm j in sector k takes as given the stochastic process for the price index $\{P_{kt}\}$ and for the elasticity $\{\delta_{kt}\}$. Its optimization problem in sequence form is:

$$\max_{\left\{p^j_{kt},a^j_{kt},i^j_{kt+1}\right\}} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(p^j_{kt}s^j_{kt}-c\left(a^j_{kt}-i^j_{kt}\right)\right)\right] \tag{4}$$

$$s.t.$$

$$s^j_{kt}=\gamma_t\mu_k\left(\frac{p^j_{kt}}{P_{kt}}\right)^{-\delta_{kt}}\left(a^j_{kt}\right)^{\phi},$$

$$i^j_{kt+1}=a^j_{kt}-s^j_{kt}, \tag{5}$$

$$\text{given } i_0\geq 0,\ u_0\geq 0,\ (3) \text{ and } \{P_{kt}\}.$$

Letting z_t denote the Lagrange multiplier associated with the constraint (5), the first order

[11] It might be necessary to consider this more disaggregated evidence because Schuh (1996) suggests that industry affiliation, as measured by two-digit-SIC codes, explains a relatively small fraction of the cross-sectional variance of his firm-level estimates of adjustment speeds.

[12] One reason why the cyclical properties of price markups vary across sectors is represented by differences in market power. The latter can be introduced in the model by assuming that the steady state elasticity of demand differs across sectors. This generalization would not affect the results of the paper because differences in average markups, per se, do not explain the inventory puzzles addressed here.

conditions with respect to a^j_{kt}, i^j_{kt+1} and p^j_{kt} are respectively:[13]

$$\phi p^j_{kt}\gamma_t\mu_k\left(\frac{p^j_{kt}}{P_{kt}}\right)^{-\delta_{kt}}\left(a^j_{kt}\right)^{\phi-1}+z_t\left(1-\phi\gamma_t\mu_k\left(\frac{p^j_{kt}}{P_{kt}}\right)^{-\delta_{kt}}\left(a^j_{kt}\right)^{\phi-1}\right)-c=0, \tag{6}$$

$$z_t-\beta c=0, \tag{7}$$

$$-\left(\delta_{kt}-1\right)\gamma_t\mu_k\left(\frac{p^j_{kt}}{P_{kt}}\right)^{-\delta_{kt}}\left(a^j_{kt}\right)^{\phi}+\frac{1}{P_{kt}}\delta_{kt}z_t\gamma_t\mu_k\left(\frac{p^j_{kt}}{P_{kt}}\right)^{-\delta_{kt}-1}\left(a^j_{kt}\right)^{\phi}=0. \tag{8}$$

The first order conditions (6)-(8) have a straightforward interpretation. First, marginally increasing a^j_{kt} entails a production cost c for the firm. An extra unit of the good available for sale at time t results in an increase in current sales and an increase in future inventories. An extra unit of the good held in inventory at the beginning of period $t+1$ allows the firm to save the unit cost of production c in that period. Thus, the current value of an extra unit of inventories i^j_{kt+1} available at $t+1$ must equal βc. Third, a higher price p^j_{kt} causes a loss of current revenue (the first term in equation 8) since the elasticity δ_{kt} is larger than one. For given a^j_{kt}, this reduction in current sales translates into a higher stock of inventories at the beginning of $t+1$, which allows the firm to save on production costs in that period. This marginal benefit of a higher price is represented by the second term in equation (8) and at the margin must exactly compensate the firm for the corresponding loss of current revenue.

In the following I focus on a symmetric equilibrium in which all firms in a given sector make the same production and pricing decisions. Thus, in equilibrium $p^j_{kt} = P_{kt}$ for all firms j in sector k. In what follows I denote by A_{kt}, I_{kt+1}, S_{kt} and Y_{kt} the equilibrium values of a^j_{kt}, i^j_{kt+1}, s^j_{kt} and y^j_{kt}.

Imposing equilibrium ($p^j_{kt} = P_{kt}$) in the first order conditions above and solving for the optimal choices of A_{kt}, I_{kt+1} and P_{kt} one obtains the following closed-form solution to the

[13] See the appendix for a derivation of these conditions and a detailed description of the solution of the model.

model. The closed-form solution is given by:

$$A_{kt} = \left[\frac{\gamma_t \mu_k \phi \beta}{(\delta_{kt} - 1)(1-\beta)}\right]^{\frac{1}{1-\phi}}, \; P_{kt} = \frac{\beta c}{1 - \delta_{kt}^{-1}}, \tag{9}$$

$$S_{kt} = \gamma_t \mu_k A_{kt}^{\phi}, \; I_{kt+1} = A_{kt} - S_{kt}, \; Y_{kt} = A_{kt} - I_{kt}. \tag{10}$$

For this to be the solution of the model at time t the following condition must be satisfied:

$$\delta_{kt} \leq 1 + \frac{\phi\beta}{1-\beta}. \tag{11}$$

As shown in the Appendix, condition (11) guarantees that demand is inelastic enough so that the firm chooses not to stockout at time t. In the Montecarlo simulations I run in section (5) this condition is always verified.

The expressions in (9) and (10) contain the key mechanisms of the model and deserve some comments. The dynamics of the model is driven by exogenous shocks to demand, represented by γ_t. Increases in demand have two opposite effects on the stock of goods A_{kt} that firms make available for sale in a period. On the one hand, for given elasticity of demand, A_{kt} tends to increase in γ_t. However, if the elasticity of demand δ_{kt} is higher in expansions than in recessions, markups of prices over marginal costs are lower in the former than in the latter. Lower markups, in turn, provide incentives to firms to make less goods available for sale for each unit of output they sell. This can be seen by computing the ratio A_{kt}/S_{kt}:

$$\frac{A_{kt}}{S_{kt}} = \frac{\phi\beta}{(\delta_{kt} - 1)(1-\beta)}, \tag{12}$$

and noticing that this ratio is a decreasing function of δ_{kt}. Using this expression and the law of motion for inventories in (10) one obtains that time $t+1$ inventories can be written as:

$$I_{kt+1} = \lambda_{kt} S_{kt}, \tag{13}$$

where

$$\lambda_{kt} = \frac{\phi\beta}{(\delta_{kt} - 1)(1 - \beta)} - 1.$$

Equation (13) says that the inventory target for a sector k firm in period $t+1$ is represented by a time-varying proportion of sales in the previous period. The factor of proportionality λ_{kt} is always positive under condition (11), and changes systematically over the business cycle with the elasticity of demand δ_{kt}. Notice, instead, that standard partial adjustment models specify the inventory target for period $t+1$ as a *constant*, rather than time-varying, proportion of sales. Thus, in the context of this model this specification is correct only when markups are constant, in which case λ_{kt} is also a constant. The next section derives the implications of this observation for the estimates of partial adjustment equations.

4 Mechanisms

In this section I discuss the qualitative implications of cyclical variations in markups for the inventory adjustment speeds estimated using partial adjustment models. It is convenient to consider first an individual sector in isolation. The first subsection shows that if this sector's inventories and sales data were generated by the model above, the speed of adjustment estimated by a standard partial adjustment regression would tend to be less than one.

The second subsection analyzes the case where the volatility of price markups differs across sectors. The result is that in this circumstance the estimates of inventory adjustment speeds obtained using aggregate data tend to be smaller than a weighted average of the sectoral speeds of adjustment, with weights reflecting these sectors' share of aggregate inventories. In other words, in this case an aggregation bias, as defined by Theil (1971), obtains.

4.1 Partial Adjustment Equations and Countercyclical Markups

The standard partial adjustment equation for inventories has the form:

$$I_{kt+1} - I_{kt} = \theta_k \left(I_{kt}^{*} - I_{kt} \right) + \varepsilon_{kt+1}, \tag{14}$$

where I_{kt}^{*} denotes a sector k firm's inventory target at time t and ε_{kt+1} is a mean-zero error term. The adjustment speed parameter θ_k denotes the fraction of the gap between current and desired inventories that sector k 's firms fill in a period. The inventory target is usually specified as a function of sales (see e.g. Ramey and West, 1999, page 894):

$$I_{kt}^{*} = \lambda S_{kt}, \tag{15}$$

where λ denotes the constant fraction of current sales that firms want to hold as inventories at the beginning of period $t+1$. That is, if firms could freely choose how much inventories to hold at the beginning of period $t+1$ they would choose an amount given by (15). In what follows, instead of estimating λ, I use the closed-form solution of the model to specify a value for this coefficient. This simplification allows me to provide a clear intuition for the biases in the estimates of θ_k because in this case the partial adjustment regression (14) is characterized by only one regressor.[14] Specifically, notice that according to the model of the previous section firms do not face any kind of cost to adjusting inventories. For these firms, it is true that actual and desired inventories always coincide, i.e., $I_{kt+1} = I_{kt}^{*}$ at all times. Given this, the inventory

[14] The quantitative results of section 5 are unchanged when λ is estimated rather than specified a-priori.

equation (13) suggests that a reasonable specification for λ is

$$\lambda = \frac{\phi\beta}{(\delta - 1)(1 - \beta)} - 1.$$

This parameter is the long-run, or steady state, ratio I_{kt+1}/S_{kt} implied by the model. This specification of the target equation (15) captures the idea, which characterizes standard partial adjustment models of inventories, that the frictionless inventory target is a *constant* multiple of sales. Replacing equation (15) into (14), gives the empirical specification of a standard partial adjustment model:

$$I_{kt+1} - I_{kt} = \theta_k (\lambda S_{kt} - I_{kt}) + \varepsilon_{kt+1}. \tag{16}$$

In contrast, the actual evolution of inventory investment in sector k implied by the model is given by equation (13). Subtracting I_{kt} from both sides of that equation one obtains:

$$I_{kt+1} - I_{kt} = \lambda_{kt} S_{kt} - I_{kt}. \tag{17}$$

To illustrate the effect of ignoring cyclical variations of λ_{kt} on the estimates of θ_k it is useful to consider the extreme case in which the elasticity of demand in sector two is constant. In this case $\lambda_{2t} = \lambda$ which implies that for firms in sector two the empirical specification (16) is correct. Thus, estimating this equation will always deliver the correct speed of adjustment $\theta_2 = 1$.

For sector one firms, instead, the empirical model (16) is misspecified because it does not take into account the fact that the "true" target parameter λ_{1t} is changing over time due to changes in markups. If an econometrician would try to estimate the parameter θ_1 in (16) she would tend to estimate inventory adjustment speeds less the one. To see this notice that equation (17) for sector $k = 1$ can be rewritten as

$$I_{1t+1} - I_{1t} = \lambda S_{1t} - I_{1t} + (\lambda_{1t} - \lambda) S_{1t},$$

so that estimating equation (16) results in the omission of the relevant variable $(\lambda_{1t} - \lambda) S_{1t}$ from the regression. Applying the omitted variable formula in Greene (1990, page 246) one obtains that the expected value of the ordinary least squares estimator $\widehat{\theta}_1$ of θ_1 is

$$E\left(\widehat{\theta}_1\right) = 1 + \frac{Cov\left(\lambda S_{1t} - I_{1t}, (\lambda_{1t} - \lambda) S_{1t}\right)}{Var\left(\lambda S_{1t} - I_{1t}\right)}. \tag{18}$$

In turn, this is less than one because the covariance term in this equation is negative when markups are countercyclical: as the economy goes into an expansion and S_{1t} increases, markups fall and so does λ_{1t} because firms find it optimal to decrease their inventory-sales ratios. Intuitively, partial adjustment models tend to overpredict the increase in inventory targets in expansions, and attribute the discrepancy between the latter and the observed change in inventories to the fact that firms are adjusting slowly.

Thus, the first empirical implication of countercyclical changes in markups in this setting is that they lead to estimates of inventory adjustment speeds less than one.

4.2 Aggregation Bias

The second kind of evidence that is addressed in this paper is the fact that inventory adjustment speeds estimated using aggregate data tend to be higher than their counterpart estimated using firm-level data. I will now argue that countercyclical changes in markups also tend to give rise to an aggregation bias of this sort.

Consider the aggregate partial adjustment regression

$$I_{t+1} - I_t = \theta\left(\lambda S_t - I_t\right) + \varepsilon_{t+1}, \tag{19}$$

where aggregate inventories I_t and sales S_t are defined as [15]

$$I_t = I_{1t} + I_{2t},\ \ S_t = S_{1t} + S_{2t}.$$

Thus, equation (19) is the aggregate equivalent of the firm-level equations (16). Using equations (17) for $k = 1, 2$, and still assuming for simplicity $\lambda_{2t} = \lambda$, one obtains that the actual evolution of aggregate inventory investment implied by the model can be expressed as:

$$I_{t+1} - I_t = \lambda S_t - I_t + (\lambda_{1t} - \lambda) S_{1t}.$$

Regression (19) suffers from the same omitted variable problem mentioned above, since this regression ignores the term $(\lambda_{1t} - \lambda) S_{1t}$. It follows that

$$E\left(\widehat{\theta}\right) = 1 + \frac{Cov\left(\lambda S_t - I_t, (\lambda_{1t} - \lambda) S_{1t}\right)}{Var\left(\lambda S_t - I_t\right)}, \tag{20}$$

where the covariance term in this equation is again negative due to countercyclical markups.

An aggregation bias (in the sense of Theil, 1971) obtains when

$$E\left(\widehat{\theta}\right) < \omega_I E\left(\widehat{\theta}_1\right) + (1 - \omega_I) E\left(\widehat{\theta}_2\right),$$

with the weight ω_I reflecting the average share of inventories held by sector one firms:

$$\omega_I = E\left(\frac{\sum_t I_{1t}}{\sum_t I_t}\right).$$

[15] Aggregate sales and inventories are constructed as weighted sums of the corresponding firm-level data using as weights sale prices in the model's steady state. Since firms in all sectors charge the same price in the steady state this implies that aggregate inventories and sales can be obtained as simple sums of firm-level variables.

To establish a relationship between $E\left(\widehat{\theta}\right)$, $E\left(\widehat{\theta}_1\right)$ and $E\left(\widehat{\theta}_2\right)$, notice that

$$Cov\left(\lambda S_{2t} - I_{2t}, \left(\lambda_{1t} - \lambda\right) S_{1t}\right) \approx 0$$

because inventory-sales ratios in sector two tend to be constant over time while they exhibit persistent cyclical changes in sector one. In this case, one can easily show that

$$E\left(\widehat{\theta}\right) \approx \alpha E\left(\widehat{\theta}_1\right) + (1-\alpha) E\left(\widehat{\theta}_2\right),$$

where

$$\alpha = \frac{Var\left(\lambda S_{1t} - I_{1t}\right)}{Var\left(\lambda S_t - I_t\right)}. \tag{21}$$

In words, the inventory speed of adjustment estimated from aggregate data is approximately equal to a weighted average of the adjustment speeds estimated from firm-level data, with weights defined as in (21).[16] An aggregation bias occurs when $\alpha > \omega_I$. The kind of heterogeneity across sectors considered in this paper provides a reason to expect that this condition is satisfied in this model. The intuition for this claim is that when markups are countercyclical in sector one, this sector's inventory stock I_{1t} exhibits persistent deviations from its long-run target λS_{1t}. Sector's two inventory stock I_{2t}, instead, tends to stay close to its target λS_{2t} because markups in this sector are constant. It follows that most of the variance in the denominator of (21) is accounted for by changes in the inventory-sales relationship in sector one. This argument can be extended to the case where firms in sector two display countercyclical markups that are less variable than in sector one.[17]

[16] Notice that α is not restricted to be in the interval $(0,1)$, so in principle $E\left(\widehat{\theta}\right)$ could be smaller than $E\left(\widehat{\theta}_1\right)$.

[17] The argument developed in this section implies that the aggregation bias would also obtain in a version of the model where μ_1 is allowed to depend negatively on the relative price of the two goods (i.e. $\mu_1\left(P_{1t}/P_{2t}\right)$). In this case, as the economy expands and prices in sector one fall relative to prices in sector two, the share of aggregate sales and inventories accounted for by sector one would increase. Since the parameter λ does not

Thus, the second empirical implication of countercyclical changes in markups in this setting is that they lead to an aggregation bias. Of course, it is important to evaluate these biases not only qualitatively but also quantitatively. This task is undertaken in the next section.

5 Quantitative Results

The following two subsections respectively describe the calibration of the model of section 2 and verify that it can account for the aggregate moments of sales, production and inventories that have been emphasized in the inventory literature (see for example Blinder and Maccini, 1991).

The third subsection reports results on the Montecarlo experiment where artificial data on inventories and sales are generated from the calibrated version of the model and then used to estimate the speed of adjustment parameters θ_k and θ in equations (16) and (19).

5.1 Calibration

Calibration of the model requires choosing values for the following parameters: β, σ, ρ, ϕ, δ, c, μ_1, π_1, π_2. The model is calibrated at a monthly frequency. The discount factor β is set equal to two percent per month. This includes not only the real interest rate, but also storage and goods' depreciation costs for the firm. The distribution function F is taken to be lognormal with mean one and standard deviation σ. The autocorrelation parameter ρ and the standard deviation σ are set by estimating the following autoregressive process for the logarithm of linearly detrended sales in the manufacturing sector:

$$\ln S_{t+1} = \rho \ln S_t + \varepsilon_{t+1}, \ \mathrm{std}\left(\varepsilon_{t+1}\right) = \sigma.$$

depend on μ_1, the inventory gap $\lambda S_{1t} - I_{1t}$ would increase even more than when μ_1 is constant. Consequently, the variance ratio in equation (21) would tend to be higher when μ_1 is allowed to depend on P_{1t}/P_{2t}. The size of the aggregation bias would then increase.

Estimating this equation for the period 1967:01-1997:12 with US manufacturing data yields point estimates of $\rho = 0.94$ for nondurable sectors and $\rho = 0.96$ for durables. I thus set $\rho = 0.95$. The estimate of the monthly standard deviation of the shock is approximately 0.02 for durables and 0.01 for nondurables. I therefore choose a value $\sigma = 0.015$.

In order to calibrate the parameters δ and ϕ it is convenient to use a steady-state version of the policy functions in (9) by setting the demand shock γ_t equal to its unconditional mean of one. In this case, the policy functions are the same in the two sectors, since what distinguishes them is only the cyclical dynamics of markups. The price equation in (9) can be used to obtain an expression for the average markup in the two sectors:

$$\frac{P_k}{c} = \frac{\beta}{1 - \delta^{-1}}, \text{ for } k = 1, 2.$$

The steady state markup is set equal to 1.2, so that given β the corresponding price elasticity of demand is $\delta = 5.45$. This choice for the average markup is consistent with the available empirical estimates of price markups (see e.g. Morrison, 1992) and lies within the range of values commonly used in calibrated macroeconomic models with monopolistic competition (Rotemberg and Woodford, 1995).

To calibrate the elasticity of sales with respect to goods available for sale, solve expression (12) for ϕ:

$$\phi = \frac{A}{S} \frac{(\delta - 1)(1 - \beta)}{\beta}.$$

The value for δ derived above, together with a ratio $A/S = 1.5$, imply that $\phi = 0.13$. The choice of A/S is consistent with the data from the manufacturing sector where the average ratio A/S is equal to 1.46 for durables and to 1.50 for nondurables in the period 1967:01-1997:12.

The parameter c is just a scale parameter that determines the price level. It is set equal to the inverse of the average markup to normalize the steady state price charged by firms to one.

The parameter μ_1 is also scale parameter that determines the average share of sector one's firms in aggregate sales and inventories. As can be noticed from the decision rules above, this parameter affects a firms' sales, inventories and production in the same proportion. This property implies that a firm's inventory-sales ratio and the estimate of its inventory adjustment speed is independent of μ_1. The only role played by μ_1 is to determine the weight of the two sectors when computing aggregate inventories and sales. This in turn has an effect on the magnitude of the aggregation bias. In what follows I present the results on aggregation bias for different values of μ_1 in $(0, 1)$. To establish a benchmark, the parameter μ_1 is chosen jointly with π_1 and π_2, so that $\mu_1 = 0.13$, $\pi_1 = 0.90$ and $\pi_2 = 0.12$. This value for μ_1 implies that firms in sector one account on average for ten percent of aggregate inventories. The values for π_k imply that the average speeds of adjustment for sector one and sector two firms are respectively 0.015 and 0.45. These figures are consistent with what reported by Schuh (1996, Table 3). He finds that for ten percent of the divisions in his sample the estimated inventory speeds of adjustment were less than 0.13, while for the next eighty percent of firms they were between 0.13 and 1.03, with a median value of 0.40.

To verify that these choices of π_1 and π_2 give rise to cyclical markups variations that are empirically plausible, notice that the price equation in (9) and the price elasticity equation (3) imply that

$$\left(\widehat{\frac{P_k}{c}}\right)_t = -\pi_k \left(\frac{P_k}{c} - \beta\right) \widehat{\gamma}_t,$$

where a hat denotes the percentage deviation of a variable from its value in the steady state. According to this calibration when sector one sales are one percent above their steady state, the markup for sector one firms is about 0.2 percent below its steady state P_k/c. For comparison Rotemberg and Woodford (1999, page 1065) present estimates of this elasticity as high as 0.4.

Table 1 summarizes the benchmark values of the parameters.

Table 1 - Benchmark calibration

Parameter	β	σ	ρ	ϕ	δ	c	μ_1	π_1	π_2
Value	0.98	0.015	0.95	0.13	5.45	0.83	0.13	0.90	0.12

The Montecarlo experiment proposed here is useful only if the calibrated version of the model is successful in replicating the business cycle dynamics of aggregate inventories, sales and output. The next section analyzes the cyclical implications of the model for these variables.

5.2 Business Cycle Implications of the Model

In order to better understand the working of the model it is useful to analyze the dynamics of the artificial data it generates in a typical "recession". Figure 1 presents the impulse responses of production, sales and inventories in the benchmark version of the model following a negative sales shock. Specifically, I consider the following process for the sales shock γ_t:

$$\begin{aligned} \gamma_0 &= 1 - \sigma, \\ \gamma_t &= \gamma_{t-1}^{\rho} \text{ for } t \geq 1. \end{aligned} \tag{22}$$

All the variables in this figure are represented as percentage deviations from their steady state value. A period after the sales shock occurs the aggregate inventory stock falls. However, the inventory-sales ratio tends to rise because countercyclical changes in markups induce firms not to reduce their inventory stocks proportionally to sales. Production falls on impact in the same period by more than aggregate sales.

The next table presents the moments for these aggregate variables implied by the benchmark model and the corresponding moments for the durables and nondurables industries in the US manufacturing sector.

Table 2* - Aggregate Statistics

	$\frac{var(Y)}{var(S)}$	$c(\Delta I, S)$	$c(S/A, Y)$	$c\left(S/A, (S/A)_{-1}\right)$	$std\,(S/A)$
Model Economy	1.03	0.17*	0.99*	0.94*	0.009
	(0.01)	(0.03)	(0.00)	(0.02)	(0.001)
U.S. Manufacturing					
Durables	1.02	0.18*	0.86*	0.96*	0.019
Nondurables	0.99	-0.04	0.67*	0.95*	0.011

*Statistics for durables and nondurables have been computed using constant dollars data on finished-goods inventories and sales from the Department of Commerce for the period 1967:01-1997:12. The Y and S series in the data have been linearly detrended. An asterisk indicates that the correlation is significant at the one percent level. Model statistics have been obtained by simulating the economy for 360 periods for 1,000 times and then computing averages. The standard deviation across simulations is reported in parenthesis. std=standard deviation, c=correlation, var=variance.

As the table shows the benchmark version of the model is consistent with the main features of aggregate inventories, production and sales data. Notice, in particular, that the model correctly predicts a procyclical, persistent and volatile ratio of sales to goods available for sale. It tends to underpredict the volatility of this ratio. For the sake of comparison, if markups were constant, the equilibrium ratio S/A would also be constant, as implied by equation (12).[18]

The next section uses the data generated by this calibrated version of the model to investigate the extent to which aggregation tends to bias estimates of inventory adjustment speeds.

[18]If markups were procyclical, instead, the ratio S/A would be negatively correlated with output over the cycle.

5.3 Inventory Adjustment Speeds: Results from Simulated Data

This section presents the estimates of inventory adjustment speeds obtained using artificial firm-level and aggregate data generated by the calibrated version of the model. Specifically, the model is simulated $1,000$ times and for each set of data equations (16) and (19) are estimated by ordinary least squares. The length of the data series in each simulation is 100 periods, which approximately corresponds to the one in Schuh (1996). Table 3 reports the aggregate speeds of adjustment estimated using aggregate and firm-level data generated by the calibrated version of the model for different values of μ_1.

Table 3 - Estimates of aggregate adjustment speeds for inventories*

	Aggregate data		*Firm-level data*		*Aggregation Bias*
	$E\left(\widehat{\theta}\right)$	$E\left(\widehat{\theta}_1\right)$	$E\left(\widehat{\theta}_2\right)$	$\omega_I E\left(\widehat{\theta}_1\right) + (1-\omega_I) E\left(\widehat{\theta}_2\right)$	
$\mu_1 = 0.13$	0.17	0.015	0.45	0.40	135%
$[\omega_I = 0.10]$	(0.07)	(0.012)	(0.10)	(0.09)	
$\mu_1 = 0.33$	0.0023	0.015	0.45	0.31	13,300%
$[\omega_I = 0.30]$	(0.0015)	(0.012)	(0.10)	(0.07)	
$\mu_1 = 0.59$	-0.0083	0.015	0.45	0.18	2,068%**
$[\omega_I = 0.60]$	(0.0074)	(0.012)	(0.10)	(0.05)	
$\mu_1 = 0.87$	0.0093	0.015	0.45	0.06	545%
$[\omega_I = 0.90]$	(0.01)	(0.012)	(0.10)	(0.02)	

*The standard deviation of the estimates across simulations is reported in parenthesis. For other details about the construction of this table see the main text. **This bias is computed using the absolute value of the aggregate speed of adjustment which is slightly negative.

The second column of Table 3 shows the average (across simulations) estimate of θ obtained using aggregate data. The third and fourth columns show the average speed of adjustment estimated for sector one and sector two firms. The fifth column represents the weighted (by ω_I) sum of firm-level estimates of adjustment speeds. Finally, the last column shows the percentage amount by which the aggregate adjustment speed computed using firm-level data (column 5) exceeds the one computed using aggregate data (column 2). The rows of the table report in square brackets the value of ω_I corresponding to each value of μ_1.

Table 3 shows several interesting results. First, it confirms the qualitative predictions developed in section 4: countercyclical changes in markups tend to bias the inventory adjustment speeds estimated from partial adjustment equations downward. The bias gets larger as markups become more cyclical, as can be inferred from comparing columns three and four. Moreover, countercyclical changes in markups result in an aggregation bias, in the sense that the adjustment speeds estimated from aggregate data are significantly smaller than the weighted average of firm-level speeds of adjustment. In interpreting these results it is useful to keep in mind that if markups were constant in both sectors the estimated speeds of adjustment, at both the firm and aggregate levels, would always be equal to one. In this case there would not be any aggregation bias.

Second, the quantitative effects of time-varying markups on inventory adjustment speeds are quite large. For example, the adjustment speed for sector one firms is on average statistically indistinguishable from zero. The aggregation bias is large for a wide range of values of μ_1 and is highest when sector one firms account on average for 30 percent of aggregate inventories.

Third, notice that the results of Table 3 are broadly consistent with the ones reported by Schuh (1996). In particular, the benchmark calibration of the model (i.e. $\mu_1 = 0.13$) implies that the weighted average of firm-level adjustment speeds is 0.40, while Schuh (1996, Table 5) reports a value of 0.45 for divisions in the M3 Longitudinal Research Database. He also estimates an adjustment speed of 0.27 based on aggregate data, which is close to the figure of

0.17 obtained here.

6 Countercyclical Markups, Inventories and Labor Demand

This model focuses on the adjustment speed of finished goods inventories. It is interesting to ask, though. whether cyclical changes in markups will also generate an aggregation bias of the kind described in this paper in the estimated adjustment speeds of other variables of interest to macroeconomists. Extending the analysis to variables other than inventories also represents a further consistency check for the mechanism emphasized in this paper.[19] In fact, for many macroeconomic variables, such as aggregate employment and prices, speeds of adjustment estimated using aggregate data tend to be small (see e.g. Topel, 1982 and Eichenbaum, 1984). It also appears that considering more disaggregated units, such as firm and sectors, leads to higher estimates of adjustment speeds for these variables than what is obtained with aggregate data (see Caballero and Engel, 2003).

The variable I focus on in this section is the labor input. In this simple model with constant returns to scale in production labor demand in sector k, denoted by L_{kt}, is linear in output:

$$L_{kt} = \alpha Y_{kt},$$

for some $\alpha > 0$. In the following I estimate a partial adjustment model for labor of the following form:

$$L_{kt+1} - L_{kt} = \vartheta \left(L^*_{kt} - L_{kt} \right), \tag{23}$$

where L^*_{kt} denotes the target amount of labor for period $t+1$, and ϑ denotes the speed of

[19] I thank a referee for making this point and suggesting this extension of my analysis.

adjustment of labor towards this target. Several papers in the inventory literature have made the point that firms will adjust to a cyclical decline in demand by choosing some combination of lower labor input and higher inventories.[20] Many of these papers then estimate versions of the partial adjustment model (23) where the labor target L^*_{kt} is assumed to be a function not only of some measure of demand, but also on the available stock of inventories. I capture these ideas by specifying the target L^*_{kt} as a function of time $t+1$ sales and of the amount of inventories held by the firm at the beginning of period $t+1$:

$$L^*_{kt} = \frac{(1+\lambda)}{\alpha} S_{kt+1} - \frac{1}{\alpha} I_{kt+1}. \tag{24}$$

Intuitively, this equation specifies the target level for labor demand as increasing in sales and decreasing in inventories. Notice that, instead of estimating the parameters of this relationship, for simplicity I specify them in advance using the knowledge of the decision rules of the model. To derive (24) I use the definition of labor demand at time $t+1$:

$$L_{kt+1} = \frac{A_{kt+1} - I_{kt+1}}{\alpha},$$

and replace A_{kt+1} by $(1+\lambda)\, S_{kt+1}$, as suggested by (12). Notice that, as in the case of the inventory target I^*_{kt}, I am assuming, consistently with the empirical literature, that the target for labor demand is a linear function of sales and inventories. The time-varying coefficient λ_{kt} is therefore replaced by its steady state value λ. This specification has the added advantage of establishing a useful benchmark, because when markups are constant, $\lambda_{kt} = \lambda$, and the empirical model (23)-(24) is correctly specified. In this case estimating ϑ_k always delivers the "true" speed of adjustment of labor, i.e., $\vartheta_k = 1$. It follows that in this context any estimate of

[20]For example Topel (1982, page 769) writes that "both excess capacity (idle resources) and inventories are devices by which firms might economize on the costs of rapid adjustments when faced with unstable market conditions." Other references include Eichenbaum (1983, 1984), Maccini and Rossana (1984), Ramey (1989) and Haltiwanger and Maccini (1989).

ϑ_k different from 1 must be ascribed to countercyclical changes in markups.

Estimating this partial adjustment model for labor demand leads to the same kind of biases discussed in section 4 in relation to inventories. First, countercyclical changes in markups tend to generate estimates of adjustment speeds for labor demand lower than one. To see this notice consider an increase in sales S_{kt+1}. According to (24) this translates into a proportional increase in the labor demand target L^*_{kt} and the labor demand gap $L^*_{kt} - L_{kt}$. However, in reality, since markups are lower in an expansion firms do not expand labor demand by the amount $L^*_{kt} - L_{kt}$. Instead, the true change in labor demand is given by

$$L_{kt+1} - L_{kt} = L^*_{kt} - L_{kt} + \left(\frac{\lambda_{kt} - \lambda}{\alpha}\right) S_{kt+1}.$$

Since $\lambda_{kt} - \lambda < 0$ the actual change in labor demand will be less than the increase in the labor demand gap. The estimation rationalizes this fact by selecting a value for the adjustment speed parameter ϑ_k less than one.

Second, if markups in sector one are more volatile than in sector two, the estimate of the adjustment speed for aggregate labor obtained from aggregate data - denoted by $\widehat{\vartheta}$ - will suffer from an aggregation bias. That is, it will reflect the behavior of sector one firms more than what would be predicted on the basis of their average labor share:[21]

$$\omega_L = E\left(\frac{\sum_t L_{1t}}{\sum_t L_t}\right).$$

[21] The argument developed in section also applies in this case. Assuming for simplicity that markups in sector two are constant, so that $\widehat{\vartheta}_2$ always equals one, yields

$$E\left(\widehat{\vartheta}\right) \approx \eta E\left(\widehat{\vartheta}_1\right) + (1-\eta) E\left(\widehat{\vartheta}_2\right),$$

where

$$\eta = \frac{Var\left(L^*_{1t} - L_{1t}\right)}{Var\left(L^*_t - L_t\right)}.$$

An aggregation bias in the sense of Theil (1971) obtains when $\eta > \omega_L$. This condition is more likely to be verified when sector one markups are more volatile than sector two markups.

Table 4 presents the results obtained by estimating the partial adjustment equations (23)-(24) with the artificial data generated by the benchmark calibration of the model for different values of μ_1. The description of the different columns of this table matches exactly the one for Table 3 so it is omitted.

Table 4 - Estimates of aggregate adjustment speeds for labor*

	Aggregate data	*Firm-level data*			*Aggregation Bias*
	$E\left(\widehat{\vartheta}\right)$	$E\left(\widehat{\vartheta}_1\right)$	$E\left(\widehat{\vartheta}_2\right)$	$\omega_L E\left(\widehat{\vartheta}_1\right) + (1-\omega_L) E\left(\widehat{\vartheta}_2\right)$	
$\mu_1 = 0.13$	0.74	0.11	0.89	0.81	9%
$[\omega_L = 0.10]$	(0.08)	(0.05)	(0.04)	(0.04)	
$\mu_1 = 0.33$	0.43	0.11	0.89	0.65	51%
$[\omega_L = 0.30]$	(0.10)	(0.05)	(0.04)	(0.04)	
$\mu_1 = 0.59$	0.18	0.11	0.89	0.42	133%
$[\omega_L = 0.60]$	(0.06)	(0.05)	(0.04)	(0.04)	
$\mu_1 = 0.87$	0.12	0.11	0.89	0.19	58%
$[\omega_L = 0.90]$	(0.05)	(0.05)	(0.04)	(0.04)	

*The standard deviation of the estimates across simulations is reported in parenthesis. For other details about the construction of this table see the main text.

As the table shows, countercyclical changes in markups tend to generate relatively small estimates of adjustment speeds for labor demand (column 3). Moreover, the adjustment speeds estimated from aggregate data (column 2) are lower than the ones obtained as the weighted average of firm-level speeds (column 5). As a reference for comparison, if markups were constant in both sectors, the estimated speeds of adjustment in all columns of Table 4 would be equal to one and the aggregation bias would disappear.

From a quantitative point of view the results concerning the aggregation bias for labor demand are significant but not as large as those for inventories. Notice, however, that countercyclical variations in markups generate a large downward bias in estimates of adjustment speeds for sector one firms.

7 Conclusions

The main contribution of this paper is to provide a coherent explanation for the following facts: 1. estimated speeds of adjustment of aggregate inventory stocks are relatively small; 2. speeds of adjustment estimated using firm-level data tend to be higher than the ones estimated using aggregate data; 3. similar patterns also characterize other macroeconomic variables such as, for example, aggregate employment. The paper shows how countercyclical variations in markups can account for these facts, even when firms can freely adjust production and inventories.

The Montecarlo approach taken here tries to answer the following question: suppose that real world data were generated by an inventory model characterized by countercyclical price markups; suppose also that we were to use these data to estimate a standard partial adjustment model; would the estimates obtained by running such regressions be in line with the corresponding estimates obtained using real world data?

The answer to this question is affirmative. Countercyclical changes in price markups, that are not controlled for in partial adjustment models, contribute to bias downward the estimates of inventory speeds of adjustment. Moreover, if different sectors of the economy display different degrees of variations in markups over the cycle, the adjustment speeds estimated from aggregate data will tend to disproportionally reflect the behavior of the sectors characterized by more variable markups. These effects are quantitatively significant.

It is worth noticing that the result of faster speeds of adjustment at the level of the firm than at the aggregate level is not to be interpreted from a structural point of view. In the model firms adjust inventories and labor demand quickly to their target, because there are no costs of adjusting inventories or the labor input. However, cyclical changes in markups cause the target itself to adjust slowly over time. This induces a specification error in partial adjustment equations and results in small estimates of adjustment speeds.

Methodologically, this paper complements Bils and Kahn (2000) in two ways. First, they attempt to measure markups directly and show how their cyclical properties help account for the dynamics of inventories over the business cycle. This paper, instead, takes the countercyclical variations of markups as a starting point and shows how these variations can shed light on some puzzling results in the inventory literature. Second, while Bils and Kahn consider a representative firm model, this paper analyzes the cross-sectional implications of cyclical markups and the related aggregation issues. An interesting avenue for future research is to use disaggregated data to test directly the key cross-sectional implication of the model that sectors where markups are relatively more cyclical *appear* to adjust inventories and labor more slowly than sectors where markups are relatively constant over the business cycle.

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A The First Order Conditions and Decision Rules of the Model

In this section I show how to derive the first order conditions (6)-(8) and the closed-form solution of the model. In doing so, I explicitly consider the non-negativity constraint on inventories

$$a_{kt}^{j} \geq s_{kt}^{j}. \tag{25}$$

Letting q_t denote the multiplier associated with this constraint, the Lagrangian for the optimization problem (4) is:

$$\begin{aligned}
L \;=\; & E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \left(p_{kt}^j \gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}} \right)^{-\delta_{kt}} \left(a_{kt}^j \right)^{\phi} - c \left(a_{kt}^j - i_{kt}^j \right) \right) \right. \\
& -\beta^t z_t \left(i_{kt+1}^j - a_{kt}^j + \gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}} \right)^{-\delta_{kt}} \left(a_{kt}^j \right)^{\phi} \right) \\
& \left. -\beta^t q_t \left(\gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}} \right)^{-\delta_{kt}} \left(a_{kt}^j \right)^{\phi} - a_{kt}^j \right) \right\}.
\end{aligned}$$

The first order conditions are:

$$\frac{\partial L}{\partial a_{kt}^j} = \phi p_{kt}^j \gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}} \right)^{-\delta_{kt}} \left(a_{kt}^j \right)^{\phi-1} - c - \left(q_t + z_t \right) \left(\phi \gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}} \right)^{-\delta_{kt}} \left(a_{kt}^j \right)^{\phi-1} - 1 \right) = 0,$$

$$\frac{\partial L}{\partial i_{kt+1}^j} = -z_t + \beta c = 0,$$

$$\frac{\partial L}{\partial p_{kt}^j} = -\left(\delta_{kt} - 1 \right) \gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}} \right)^{-\delta_{kt}} \left(a_{kt}^j \right)^{\phi} + \frac{1}{P_{kt}} \delta_{kt} \left(q_t + z_t \right) \gamma_t \mu_k \left(\frac{p_{kt}^j}{P_{kt}} \right)^{-\delta_{kt}-1} \left(a_{kt}^j \right)^{\phi} = 0.$$

The associated complementarity-slackness conditions are

$$z_t \geq 0,\ z_t \left(i^j_{kt+1} - a^j_{kt} + \gamma_t \mu_k \left(\frac{p^j_{kt}}{P_{kt}} \right)^{-\delta_{kt}} \left(a^j_{kt} \right)^{\phi} \right) = 0,$$

$$q_t \geq 0,\ q_t \left(\gamma_t \mu_k \left(\frac{p^j_{kt}}{P_{kt}} \right)^{-\delta_{kt}} \left(a^j_{kt} \right)^{\phi} - a^j_{kt} \right) = 0.$$

In a symmetric equilibrium we have $p^j_{kt} = p^{j'}_{kt}$ and $a^j_{kt} = a^{j'}_{kt}$ for all j, j'. Therefore, $p^j_{kt} = P_{kt}$ and $a^j_{kt} = A_{kt}$ for all j. Taking into account the first order condition for i^j_{kt+1}, the ones for a^j_{kt} and p^j_{kt} simplify to:

$$\begin{aligned} \phi P_{kt} \gamma_t \mu_k A^{\phi-1}_{kt} - c - (q_t + \beta c) \left(\phi \gamma_t \mu_k A^{\phi-1}_{kt} - 1 \right) &= 0, \\ (1 - \delta_{kt}) + \frac{1}{P_{kt}} \delta_{kt} (\beta c + q_t) &= 0. \end{aligned} \tag{26}$$

Use the second equation to solve for the equilibrium price in sector k:

$$P_{kt} = \frac{\beta c + q_t}{1 - \delta^{-1}_{kt}}. \tag{27}$$

Replace this expression in equation (26):

$$\phi \frac{\beta c + q_t}{1 - \delta^{-1}_{kt}} \gamma_t \mu_k A^{\phi-1}_{kt} - c - (q_t + \beta c) \left(\phi \gamma_t \mu_k A^{\phi-1}_{kt} - 1 \right) = 0.$$

Collect the terms in $A^{\phi-1}_{kt}$:

$$A^{\phi-1}_{kt} \phi \gamma_t \mu_k (q_t + \beta c) \left(\frac{1}{1 - \delta^{-1}_{kt}} - 1 \right) = c - (q_t + \beta c).$$

Solve for A_{kt}:

$$A_{kt} = \left[\frac{\phi \gamma_t \mu_k (\beta c + q_t)}{(\delta_{kt} - 1)(c(1 - \beta) - q_t)} \right]^{\frac{1}{1-\phi}}. \tag{28}$$

When the constraint (25) is not binding and $q_t = 0$, the solution is

$$\begin{aligned} A_{kt} &= \left[\frac{\gamma_t \mu_k \phi \beta}{(\delta_{kt} - 1)(1 - \beta)}\right]^{\frac{1}{1-\phi}}, \\ P_{kt} &= \frac{\beta c}{1 - \delta_{kt}^{-1}}. \end{aligned}$$

When the constraint is binding, instead, the solution for A_{kt} is:

$$A_{kt} = \gamma_t \mu_k A_{kt}^{\phi} \rightarrow A_{kt} = (\gamma_t \mu_k)^{\frac{1}{1-\phi}}. \tag{29}$$

To find the equilibrium price in this case we need to solve for the multiplier q_t. The latter is obtained by setting the expressions for A_{kt} in equations (29) and (28) equal and solving for q_t. This yields:

$$q_t = \frac{(\delta_{kt} - 1) c (1 - \beta) - \phi \beta c}{\phi + \delta_{kt} - 1}. \tag{30}$$

Notice that the constraint binds when $q_t > 0$, that is, when

$$\delta_{kt} > 1 + \frac{\phi \beta}{1 - \beta}.$$

That is, firms choose to stockout when the elasticity of demand is relatively high, i.e., markups are relatively low. To find P_{kt} when the non-negativity constraint on inventories binds, replace the expression for q_t (equation 30) in equation (27) and simplify to obtain:

$$P_{kt} = \frac{\delta_{kt} c}{\phi + \delta_{kt} - 1}.$$

To summarize, the equilibrium price and stock available for sale in sector k are given by:

$$A_{kt} = \left[\frac{\gamma_t \mu_k \phi \beta}{(\delta_{kt} - 1)(1 - \beta)}\right]^{\frac{1}{1-\phi}} \text{ and } P_{kt} = \frac{\beta c}{1 - \delta_{kt}^{-1}}, \text{ for } \delta_{kt} \leq 1 + \frac{\phi \beta}{1 - \beta}$$

and by

$$A_{kt} = (\gamma_t \mu_k)^{\frac{1}{1-\phi}} \text{ and } P_{kt} = \frac{\delta_{kt} c}{\phi + \delta_{kt} - 1}, \text{ for } \delta_{kt} > 1 + \frac{\phi\beta}{1-\beta}.$$

The other variables can easily be found using the definition of Y_{kt}, S_{kt}, and $I_{kt+1} - I_{kt}$.

Figure 1 - Impulse Responses of Aggregate Variables to a Negative Sales Shock