Politically sustainable social insurance.

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1 Introduction

Private information underpins a basic social trade-off between incentives and inequality. Normative models of dynamic incentive provision explore the optimal evolution of this trade-off. Many prescribe relentlessly increasing levels of inequality. Indeed, under a variety of standard preference assumptions, an immiseration result obtains: a privileged measure zero set of agents obtain all resources in the economy, while the rest converge to the minimal possible utility level. The immiseration result makes the political viability of such optima doubtful.[1] Surely, a society could not realistically commit to implementing an allocation that consigns almost all agents to eventual misery? Surely, the immiserated would resist?

This paper confronts these questions. We augment the well known normative model of Atkeson and Lucas

[1] Of course, similar issues occur in Ramsey models in which the government is restricted to using linear tax mechanisms and there is usually a representative agent. The immiseration result gives them extra force in the current setting.

(1992)[2] with political arrangements for revising continuation allocations. To be politically sustainable an allocation must be immune to such revision. Initially, we model these arrangements as a sequence of reduced form continuous "political constraints" on the payoffs delivered to agents by an allocation. We call an allocation politically sustainable if it satisfies these constraints, along with resource and incentive-feasibility conditions. Later, we turn to a class of fully specified political economy games that feature voting over mechanisms, we show that these games deliver reduced form political constraints as equilibrium restrictions. Thus, political sustainability (with respect to an appropriate sequence of political constraints) emerges as a necessary and sufficient condition for an allocation to be an equilibrium one in these games.

To begin with we focus on Pareto optimal politically sustainable allocations. We show that such allocations coincide with those chosen by a "virtual" planner who faces no political constraints, but who uses an endogenously determined discounting scheme. In general, this scheme applies individualized, history dependent discount factors to agents that weakly exceed the agents' true discount factor. The virtual planner's objective is derived from a reorganization of the Lagrangian from the original politically constrained problem; her discount factors strictly exceed those of the agents' whenever the political constraints bind since such binding constraints elevate the shadow value of future agent payouts. In this sense, she exhibits "excessive" social patience. We show that if immiserated allocations (i.e. those that provide the minimal possible utility to almost all agents) violate the political constraints, then these constraints bind infinitely often in the Pareto problem. We give numerical examples in which these constraints bind eventually and in which they bind always. In the latter case, the implied virtual planner uses

[2] The Atkeson-Lucas framework is simple and provides a well known benchmark. Our results can be extended to many other dynamic private information environments.

a constant discount factor in excess of the agent and, hence, the objective assumed in the normative analyses of Farhi and Werning (2006).

The second part of the paper considers a class of political economy games with probabilistic voting. Following, Atkeson and Lucas (1992), in each period t, a government allocates an aggregate quantity of resources R_t amongst a population of long-lived agents. These agents experience privately observed taste shocks that affect their desire to consume and against which they would like to obtain insurance. To provide such insurance, the government must induce them to reveal information about their shocks. It can do this by implementing a sequence of social mechanisms. This sequence rewards agents who reveal a low current desire for consumption with higher future utility and penalizes those who reveal a high current desire with reduced future utility. Thus, future mechanisms are used to elicit information in the present. The mechanism sequence implemented by the government is determined by a process of electoral competition. In each period, two political parties with different characteristics propose mechanisms. Agents then vote over party-mechanism pairs. Following the literature on probabilistic voting, we assume that agents have idiosyncratic and time varying biases towards one party or the other. These biases imply that electoral outcomes are uncertain and that they are not solely determined by economic policy.[3] The repeated holding of elections precludes government commitment; a mechanism will only be adopted if it is in the interests of agents to vote for it in the period of its adoption. As we have noted, future mechanisms play an essential role in the provision of current insurance, but when future elections come such past benefits will be ignored. Agents vote for the party-mechanism pair that maximizes their current expected lifetime payoff. The latter depends not only upon

[3] They also confer a technical advantage: under appropriate assumptions on bias distributions, they ensure that a mechanism's election-winning probability is continuous in the payout of the continuation allocation it induces.

the outcome of the current election, but also on the reporting behavior of agents and the (self-fulfilling) beliefs that agents hold about future policy. This dependence on agent behavior and beliefs can impose discipline on the electoral process. Sequences of mechanisms that reward past information revelation and provide insurance can be sustained by equilibrium reporting behavior that effectively punishes the electorate for choosing an alternative sequence.

In our baseline model, distributions over political biases are uniform - a standard assumption in the literature - and political parties are operated by politicians who are concerned only with winning office in the current period. We show that equilibrium allocations must be resource and incentive-feasible and must satisfy a political constraint that requires they maintain a utilitarian payoff above a constant reservation level. Pareto optimal equilibrium allocations solve virtual planning problems with no political constraints and a common, though potentially time varying discount factor. Our analysis of reduced form Pareto optimal politically sustainable allocations applies - political constraints must bind infinitely often. The virtual planner's problem in this case coincides with that considered by Sleet and Yeltekin (2006). The numerical analysis of that paper suggests that incentive-constraints on agents impart a force for immiseration and a downward drift in the utilitarian payoff, eventually the political constraints bind and arrest this drift. If the initial distribution of Pareto weights is sufficiently dispersed, then political constraints bind in all periods and, as described above, the virtual planner's criterion features a constant discount factor in excess of the agents. In summary, Pareto optimal equilibria feature uncommitted, highly impatient politicians making the same choices as a committed and, from the agents' point of view, excessively patient virtual planner. Despite (indeed because) politicians cannot commit, they behave as if they are more concerned with the long run than agents.

We consider various extensions. Incorporating incumbency advantages or politicians who care about winning future as well as current elections leaves our results intact. Allowing more general (i.e. non-uniform) distributions over biases complicates the analysis, but does not change the basic message. In this case, Pareto optimal equilibrium allocations solve virtual planning problems with individual, history-dependent discount factors. These discount factors capture the endogenously evolving political influence of history-specific sub-populations of agents. Assuming that politicians are rent-seeking rather than office-motivated also leaves the basic message unaltered. Again we recover a political constraint on continuation agent payoffs as an equilibrium restriction and, hence, a patient virtual planner. In this case, the political constraint binds in all periods regardless of the initial distribution of Pareto weights.

Up to this point, the equilibria that we have studied satisfy sequential rationality restrictions: after each history, the continuation strategies of all politicians and agents are optimal. Although, these equilibria avoid the immiserating outcomes that characterize the limits of allocations chosen by committed utilitarian planners, they can, nonetheless, deliver low (continuation) social payoffs. This suggests that a politician may attempt to coordinate the future play of agents and politicians on to a new equilibrium with a higher payoff. Potentially, the politician can enhance her electoral prospects if she can persuade agents that her election will trigger such coordination. In the context of our baseline political economy game, we consider equilibria that are robust to such "political revisions" by introducing an equilibrium refinement in the spirit of Pearce's renegotiation-proofness (Pearce, 1987). We show that outcome allocations induced by such politically revision-proof equilibria are those chosen by a virtual planner with a unit discount factor, i.e. the virtual planner corresponds to the Rawlsian planner of Phelan (2006). Political revision-proofness serves to further tighten equilibrium restrictions on continuation utilitarian payoffs; it translates into equilibria that implement the allocations that would be chosen by a highly patient, committed planner even though, as before, politicians have very short term objectives.

1.1 Literature

The normative literature Atkeson and Lucas (1992) consider the optimal trade-off between incentives and insurance in a normative model that assumes planner commitment and equal private and societal discounting. They provide an immiseration result. Farhi and Werning (2006), Phelan (2006) and Sleet and Yeltekin (2007) drop the equal discounting assumption, but retain planner commitment. They show that when the societal discount factor exceeds the private one, the immiseration result is overturned. In contrast, we dispense with planner commitment and suppose that allocations are determined by an uncommitted political process. By focussing on Pareto optimal and politically provision proof equilibria, we recover a committed "virtual planner" who uses discount factors in excess of the agents.

This link between planner problems and political economy models provides positive micro-political foundations for the former. In the other direction, it indicates the relevance of methods used in solving normative problems with patient planners for political economy games.

Political economy literature The literature on probabilistic voting is large; Banks and Duggan (2003) provide a unifying treatment, Persson and Tabellini give a textbook overview. Recent contributors have extended these models to dynamic macroeconomic settings. Examples include Azzimonti (2004) and Hassler et al (2005, 2006). The focus of these papers is varied, but relative to us they restrict attention to simpler mechanisms and Markov equilibria. On the other hand, they allow for features that are not present here, including capital accumulation, heterogeneous preferences over public goods and aggregate shocks.

Acemoğlu et al (2006) is closest to us in spirit. They provide a model in which resource-allocating mechanisms are implemented by rent-seeking politicians.[4] If a politician attempts to extract more current rents than her strategy prescribes, she is dismissed and her future rents are eliminated. In this model, unlike ours, there is no contemporaneous competition for office amongst politicians. Because political competition is weaker, allocations featuring arbitrarily unequal distributions of utility are sustainable, provided they deliver sufficient rents to the politician. Acemoğlu et al emphasize optimal equilibria in which political constraints bind in the short run, but not in the limit; in contrast, we obtain political constraints that bind infinitely often (i.e. in the limit, but not, generally, in the short run). Our model features an immiserating politically unconstrained optimum and a competitive political system. We conjecture that political constraints that distort intertemporal margins would be a permanent, rather than a transitory phenomenon in other models that share these features

Finally, the current paper builds on Sleet and Yeltekin (2006) who, in the spirit of Ramsey models without commitment, model the government as an uncommitted planner implementing a sequence of mechanisms.

2 An environment with commitment

A continuum of infinitely-lived agents inhabit an economy. The population is initially partitioned into a measure space $(\mathbb{R}, \mathcal{B}, \Psi)$ of types w. For now, we will interpret w simply as device for distinguishing between sub-populations of agents. In each period, agents receive a random taste shock $\theta_t \in \Theta := \{\widehat{\theta}_k\}_{k=1}^{K}$. These shocks are i.i.d. across

[4] Acemoğlu et al do consider a partially benevolent politician. However, they then restrict the shock process to ensure that the provision of incentives does not require too much inequality ex post.

agents and time with distribution π. Let $\theta^t := \{\theta_1, ..., \theta_t\} \in \Theta^t$ denote a t-period history of shocks; let π^t denote the corresponding probability distribution. We assume that for all t and sets $E \subset \Theta^t$, $\pi^t(E)$ gives the fraction of agents with shock history in E.[5]

After each realized history θ^t, an agent receives an allocation of consumption and, hence, utility. In the sequel, it will be convenient to describe allocations directly in terms of the stream of utility they provide rather than stream of resources they use. Define an *individual allocation* to be a sequence of functions $\{\psi_t\}_{t=1}^{\infty}$, with for all t, $\psi_t : \Theta^t \to D \subseteq \mathbb{R}$ and denote an agent's payoff from $\{\psi_t\}_{t=1}^{\infty}$ by

$$U(\{\psi_t\}_{t=1}^{\infty}) = \liminf_T (1-\beta) \sum_{t=1}^{T} \sum_{\Theta^t} \beta^{t-1} \theta_t \psi_t(\theta^t) \pi^t(\theta^t),$$

where $\beta \in (0,1)$ is the agent's discount factor. For now we assume that D is bounded. Similarly, we define a *(population) allocation* $\{\varphi_t\}_{t=1}^{\infty}$ to be a sequence of measurable functions with $\varphi_t : \mathbb{R} \times \Theta^t \to D$. A population allocation specifies the individual allocation obtained by each w-type agent: $\{\varphi_t(w, \cdot)\}_{t=1}^{\infty}$. Let W denote the range of U and A the set of population allocations.

Cost function Let $C(u)$ denote the cost of delivering a utility amount u to an agent. We suppose the function $C : D \to \mathbb{R}_+$ satisfies the following.

Assumption 1 *1) $C : D \to \mathbb{R}_+$ is a strictly increasing and strictly convex function. It is continuously differentiable on the interior of D; 2) $\lim_{u\to\infty} C'(u) = \infty$.*

Of course, C implies a standard utility function over consumption: $v = C^{-1}$.

[5] In making this assumption, we rely on the construction of Sun (2006).

2.1 Feasible Allocations

We require that an allocation be resource and incentive-feasible. Formally, suppose that the economy possesses a finite quantity of resources $R_t \in [0, R]$ that can be allocated to agents in period t. To be resource-feasible an allocation must satisfy:

$$\forall t, \;\; R_t \geq \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi(dw). \tag{1}$$

Since shocks are privately observed by agents, allocations must provide agents with incentives to reveal them. Let $\alpha = \{\alpha_t\}_{t=1}^{\infty}$ denote a reporting strategy for the agents with, for each t, $\alpha_t : \Theta^t \to \Theta$. Let $\alpha^t(\theta^t)$ denote the history of reports induced by α given the shock history θ^t. An allocation provides incentives for truthful reporting if for almost every w and all δ,

$$(1-\beta)\sum_{t=1}^{\infty}\sum_{\Theta^t}\beta^{t-1}\theta_t\varphi_t(w,\theta^t)\pi^t(\theta^t) \geq (1-\beta)\sum_{t=1}^{\infty}\sum_{\Theta^t}\beta^{t-1}\theta_t\varphi_t(w,\alpha^t(\theta^t))\pi^t(\theta^t). \tag{2}$$

Let $\Gamma(\{R_t\}, \Psi)$ denote the set of resource and incentive-feasible allocations $\{\varphi_t\}_{t=1}^{\infty}$ satisfying (1) and (2).

2.2 Planning problem

We may define a family of Pareto planning problems by

$$\sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(R,\Psi)} \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty})\Psi(dw), \tag{3}$$

for some measurable Pareto-Negishi weighting function $\gamma : \mathbb{R} \to \mathbb{R}_+$ with $\int \gamma(w)\Psi(dw) = 1$. This is essentially a primal version of the problem considered by Atkeson and Lucas (1992). For a large class of cost functions C (or utility functions C^{-1}), its solution satisfies an agent immiseration property. Denoting a solution to (3) by $\{\varphi_t^*\}_{t=1}^{\infty}$, this property can be stated as: for Ψ-a.e. w, π^{∞}-a.e. θ^{∞}, $\lim_{t\to\infty} U(\{\varphi_{t+s}^*(w,\theta^t,\cdot)\}_{s=1}^{\infty})$ exists and equals $E[\theta] \inf D$.

When, as here, $\sup D < \infty$, this agent immiseration property implies $\lim_{t\to\infty} \int_{\mathbb{R}} \gamma(w) \sum_{\Theta^t} U(\{\psi^*_{t+s}(\theta^t, \cdot)\}_{s=1}^{\infty})\pi^t(\theta^t) = E[\theta] \inf D$.

3 Political constraints and patient virtual planners

A politically constrained Pareto problem We now augment the Pareto problem (3) with a sequence of political constraints. These constraints capture restrictions on continuation allocations that are necessary to ensure that the allocation is not revised or altered by a benevolent planner or by voters in an election. For the moment, we simply state these constraints and explore their implications. Later we show that they emerge as equilibrium restrictions in a variety of political economy models.

Let $h_t = (w, \theta^{t-1})$ denote an agent's t-period individual history; this history includes the agent's type along with its shocks up to date $t-1$. Let Q_t be the probability measure for t-period histories induced by Ψ and π^t. Given a population allocation $\{\varphi_t\}_{t=1}^{\infty}$, we denote the implied individual continuation allocation after history h_t by $\{\varphi_{t+r-1}|h_t\}_{r=1}^{\infty}$. Define $\mathcal{U}_t = \{u \mid u : \mathbb{R} \times \Theta^{t-1} \to W,\ u \text{ measurable}\}$, a convex subset of $L_\infty(Q_t)$; let Z be a bounded subset of $\mathbb{R}$. We call $Z_t : \mathcal{U}_t \to Z$ and $X : [0, R] \to \mathbb{R}$ *political viability functions* and say that an allocation $\{\varphi_t\}_{t=1}^{\infty}$ is politically sustainable if for all t,

$$Z_t(U(\{\varphi_{t+r-1}|\cdot\}_{r=1}^{\infty})) + X\left(R_t - \int C(\varphi_t(w, \theta^t)\pi^t(\theta^t)\Psi(dw)\right) \geq 0. \tag{4}$$

In the different political economy games we consider below, Z_t is variously, the probability that a continuation allocation and, hence, a political party will win an election and the (negative) of the expected political rents attainable from a political defection. In a model with a benevolent planner, it is the utilitarian payoff and, hence, captures the planner's commitment to the allocation. In some of the games described below X is present, in others not. When it is present it describes the rents extracted by a political party. We assume the following.

Assumption 2 (Z1) *For all t, Z_t is Fréchet differentiable. For $u \in \mathcal{U}_t$ its Fréchet derivative ∂Z_t is of the form:*

$$\partial Z_t(u; f) = \langle \widehat{z}_t(u), f \rangle .$$

where $\langle \widehat{z}_t(u), f \rangle = \int \widehat{z}_t(u)(w, \theta^{t-1}) f(w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw)$, $f \in \mathcal{U}_t$ and the functions $\{|\widehat{z}_t(\cdot)(\cdot)|\}$ are uniformly bounded by some $\overline{z} < \infty$.

Assumption 3 (Z2) *For all t, Z_t is concave.*

Assumption 4 (Z3) *For all t, Z_t is increasing and for all $u \in \mathcal{U}_t$ and almost all w, θ^{t-1}, $\widehat{z}_t(u)(w, \theta^{t-1}) > 0$.*

Assumption 5 (X) *X is non-decreasing, continuous, smooth and concave.*

Assumption 6 (ZX) *There exists an incentive-feasible allocation $\{\varphi_t\}_{t=1}^{\infty}$ such that $\inf_t Z_t(U(\{\varphi_{t+r-1}|\cdot\}_{r=1}^{\infty}) + X(R_t - \int C(\varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw))) > 0$.*

It is often convenient to re-express the sequence of constraints (4) as a single constraint $\{\{\varphi_t\}_{t=1}^{\infty} | \{Z_t \circ U + X\}_{t=1}^{\infty}(\{\varphi_t\}_{t=1}^{\infty}) \geq 0\}$, where $\{Z_t \circ U + X\}_{t=1}^{\infty}(\{\varphi_t\}_{t=1}^{\infty}) = \{Z_t(U(\{\varphi_{t+r-1}|\cdot\}_{r=1}^{\infty})) + X(R_t - \int C(\varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw))\}_{t=1}^{\infty}$. These assumptions are essentially technical, they imply that the set of politically sustainable allocations is convex, has a non empty interior and is described by a smooth function. In applications, (3) is the most problematic, when it does not hold results similar to below continue to hold, but with local Kuhn-Tucker arguments replacing the global optimization arguments used. Assumption 4 captures the intuitive idea that if an allocation is politically sustainable, then one that offers more to everyone is also politically sustainable.

The optimal politically-constrained allocations solve an optimization similar to (3), but augmented with the political constraints (4):

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}} \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty}) \Psi(dw), \tag{5}$$

subject to (1), (2) and (4).

The virtual planner We now show transform (5) into the problem of a virtual planner who uses a perturbed discounting scheme and who faces no political constraints. There are three steps. We first obtain a Lagrangian and show that under our assumptions, an allocation is an optimal politically sustainable one if and only if it attains a saddle point of the Lagrangian. The second step replaces the Lagrangian with a "linearized" Lagrangian, the third reconfigures the linearized Lagrangian to give an objective for a virtual planner.

The Lagrangian is given by:

$$\begin{aligned}(1-\beta)\mathcal{L}(\{\varphi_t\}_{t=1}^{\infty}, \{\mu_t\}_{t=1}^{\infty}, \{q_t\}_{t=1}^{\infty}) \quad = \quad & (1-\beta)\int \gamma(w) \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\Theta^t} \theta_t \varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) \\ & +(1-\beta)\sum_{t=1}^{\infty} \beta^{t-1}\mu_t \left[Z_t \left(\sum_{\Theta^{t+r}} \theta_{t+r}\varphi_{t+r}(\cdot,\cdot,\theta^r)\pi^r(\theta^r) \right) \right. \\ & \left. +X\left(R_t - \int C(\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) \right) \right].\end{aligned} \tag{6}$$

where $\{\mu_t\}_{t=1}^{\infty} \in L = \{\{\mu_t\} \in \mathbb{R}_+^{\infty} | (1-\beta)\sum_{t=1}^{\infty} \beta^{t-1}\mu_t < \infty\}$ denotes a sequence of bounded, summable multipliers on the political constraints. We have the following.

Proposition 1 *Let Assumptions (3) and (6) hold and let U^* denote the optimal payoff from (5). Then, there is an element $\{\mu_t^*\}_{t=1}^{\infty} \in L$ such that:*

$$U^* = \sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\},\Psi)} (1-\beta)\mathcal{L}(\{\varphi_t\}_{t=1}^{\infty}, \{\mu_t^*\}_{t=1}^{\infty}). \tag{7}$$

Furthermore, if $\{\varphi_t^*\}_{t=1}^\infty$ *attains the supremum in (5), then* $\mathcal{L}$ *has a saddle point at* $\{\varphi_t^*\}_{t=1}^\infty$, $\{\mu_t^*\}_{t=1}^\infty$, *i.e. for all* $\{\mu_t\}_{t=1}^\infty \in L$, $\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\}, \Psi)$,

$$\mathcal{L}(\{\varphi_t\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty) \leq \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty) \leq \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t\}_{t=1}^\infty). \tag{8}$$

Conversely, if $\{\mu_t^*\}_{t=1}^\infty \in L$ *and* $\{\varphi_t^*\}_{t=1}^\infty \in \Gamma(R, \Psi)$ *satisfy (8), then* $\{\varphi_t^*\}_{t=1}^\infty$ *attains the optimum in (5).*

The proof of this and other results is contained in the appendix. It invokes an argument of Rustichini (1996) to establish that the optimal multipliers are summable. Next, we "linearize" the Lagrangian.

Proposition 2 *Let Assumptions (2), (3), (6) hold. Suppose that* $\{\varphi_t^*\}_{t=1}^\infty$ *attains the supremum in (5). Then there is a pair of sequences* $\{\mu_t^*\}_{t=1}^\infty \in L$ *and* $\{\widehat{z}_t^*\}_{t=1}^\infty$, $\widehat{z}_t^* : \mathbb{R} \times \Theta^{t-1} \to \mathbb{R}_+$ *such that* $\{\varphi_t^*\}_{t=1}^\infty$ *solves*

$$\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\}, \Psi)} \mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\}, \{\widehat{z}_t^*\}), \tag{9}$$

where

$$\begin{aligned} \mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\}, \{\widehat{z}_t^*\}) \quad = \quad & \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w, \cdot)\}_{t=1}^\infty) \Psi(dw) \\ & + \sum_{t=1}^\infty \beta^{t-1} \mu_t^* \left[\int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1} | w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw) \right. \\ & \left. + X \left(R_t - \int C(\varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \right) \right] \end{aligned}$$

Conversely, if $\{\varphi_t^*\}_{t=1}^\infty$ *solves (9) for some* $\{\mu_t^*\}_{t=1}^\infty \in L$ *and* $\{\widehat{z}_t^*\}_{t=1}^\infty$, $\widehat{z}_t^* : \mathbb{R} \times \Theta^{t-1} \to \mathbb{R}_+$ *and if*

1. $\partial Z_t(\{\varphi_t^*\}_{t=1}^\infty; \cdot) = \langle \widehat{z}_t^*, \cdot \rangle$;

2. $\{\mu_t^*\} \in \arg\inf_{\{\mu_t\}_{t=1}^\infty \in L} \mathcal{L}^*(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t\})$;

then $\{\varphi_t^*\}_{t=1}^{\infty}$ *solves (5).*

Notice that the "linearized" Lagrangian $\mathcal{L}^*(\{\varphi_t\}_{t=1}^{\infty};\{\mu_t^*\},\{\widehat{z}_t^*\})$ incorporates a history-specific multiplier scheme $\{\mu_t^*\widehat{z}_t^*\}$. Let $\{\mu_t\}_{t=1}^{\infty}\in L$ be a multiplier sequence and $\{\widehat{z}_t\}_{t=1}^{\infty}$ a function sequence with $\widehat{z}_t:\mathbb{R}\times\Theta^{t-1}\to\mathbb{R}_+$.For all w, define $z_1(w,\theta^0)$ by $\mu_1\widehat{z}_1(w,\theta^0)=\mu_1 z_1(w,\theta^0)\gamma(w)$ and for all $t>1$, w, θ^{t-1}, define $z_t(w,\theta^{t-1})$ by $\mu_t\widehat{z}_t(w,\theta^{t-1})$ $=\mu_t z_t(w,\theta^{t-1})\gamma(w)\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}))$. With this renormalization we can rewrite $\mathcal{L}^*(\{\varphi_t\}_{t=1}^{\infty};\{\mu_t\},\{\widehat{z}_t\})$ as

$$\int_{\mathbb{R}}\gamma(w)U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty})\Psi(dw)$$
$$+\sum_{t=1}^{\infty}\beta^{t-1}\int_{\mathbb{R}}\gamma(w)\sum_{\theta^{t-1}}\mu_t z_t(w,\theta^{t-1})\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}))U(\{\varphi_{t+r-1}|w,\theta^{t-1}\})\pi^{t-1}(\theta^{t-1})\Psi(dw).$$

We utilize the following lemma.

Lemma 1 *Assume that* $\{\mu_t\}\in L$ *and* $\sum_{t=1}^{\infty}\int_{\mathbb{R}}\gamma(w)\sum_{\theta^{t-1}}\beta^{t-1}\left(\prod_{r=1}^{t}(1+\mu_r z_r(w,\theta^{r-1}))\right)\pi^{t-1}(\theta^{t-1})\Psi(dw)<\infty$ *then*

$$\begin{aligned}\mathcal{L}^*(\{\varphi_t\}_{t=1}^{\infty};\{\mu_t\},\{\widehat{z}_t\}) &= \int\gamma(w)\sum_{t=1}^{\infty}\sum_{\Theta^t}B_1^t(w,\theta^{t-1})\theta_t\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw)\\ &\quad+\sum_{t=1}^{\infty}\beta^{t-1}\mu_t X\left(R_t-\int C(\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw)\right)\end{aligned}$$

where $B_1^t(w,\theta^{t-1})=\beta^{t-1}\prod_{r=1}^{t}(1+\mu_r z_r(w,\theta^{r-1}))$.

Combining the Lemma 1 and Proposition 2, we obtain the following.

Proposition 3 *Let Assumptions (2)-(6) hold. Suppose that* $\{\varphi_t^*\}_{t=1}^{\infty}$ *attains the supremum in (5). Then there is a pair of sequences* $\{\mu_t^*\}_{t=1}^{\infty}\in L$ *and* $\{z_t^*\}_{t=1}^{\infty}$, $z_t^*:\mathbb{R}\times\Theta^{t-1}\to\mathbb{R}_+$ *with* $\sum_{t=1}^{\infty}\int_{\mathbb{R}}\gamma(w)\sum_{\theta^{t-1}}\beta^{t-1}\left(\prod_{r=1}^{t}(1+\mu_r^* z_r^*(w,\theta^{r-1}))\right)$

$\pi^{t-1}(\theta^{t-1})\ \Psi(dw) < \infty$,such that $\{\varphi_t^*\}_{t=1}^\infty$ solves

$$\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t^*\},\Psi)} \int \gamma(w) \sum_{t=1}^{\infty} \sum_{\Theta^t} B_1^t(w,\theta^{t-1})\theta_t \varphi_t(w,\theta^t)\pi^t(\theta^t) \tag{10}$$

where for each t, $B_1^t(w,\theta^{t-1}) = \beta^{t-1}\prod_{r=1}^t(1+\mu_r^* z_r^*(w,\theta^{r-1})) \geq \beta^{t-1}$ and $R_t^* = R_t - \int C(\varphi_t^*(w,\theta^t)\pi^t(\theta^t)\Psi(dw)$. Conversely, if $\{\varphi_t^*\}_{t=1}^\infty$ solves (10) for some $\{\mu_t^*\}_{t=1}^\infty \in L$, $\{R_t^*\}_{t=1}^\infty$ with $R_t^* = R_t - \int C(\varphi_t^*(w,\theta^t)\pi^t(\theta^t)\Psi(dw)$, $\{z_t^*\}_{t=1}^\infty$, $z_t^* : \mathbb{R}\times\Theta^{t-1} \to \mathbb{R}_+$ with $\sum_{t=1}^\infty \int_{\mathbb{R}} \gamma(w) \sum_{\theta^{t-1}} \beta^{t-1}\left(\prod_{r=1}^t (1+\mu_r^* z_r^*(w,\theta^{r-1}))\right)\pi^{t-1}(\theta^{t-1})\Psi(dw) < \infty$, and if

1. *$\partial Z_t(\{\varphi_t^*\}_{t=1}^\infty;\cdot) = \langle \widehat{z}_t^*,\cdot\rangle$, where $\widehat{z}_t^*(w,\theta^{t-1}) = z_t^*(w,\theta^{t-1})\gamma(w)\prod_{r=1}^{t-1}(1+\mu_r z_r^*(w,\theta^{r-1}))$,*

2. *$\{\mu_t^*\} \in \arg\inf_{\{\mu_t\}_{t=1}^\infty \in L} \mathcal{L}^*(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t\})$,*

then $\{\varphi_t^*\}_{t=1}^\infty$ solves (5).

Notice that the optimization problem (10) features *no* political constraints, they are absorbed into the objective and resource constraints. This objective corresponds to that of a committed planner who uses a history-contingent discounting scheme with discount factors $\beta(1+\mu_t^* z_t^*(w,\theta^{t-1}))$ that weakly exceed those of the agents. If the political constraint binds in period t, then $\mu_t^* > 0$ and the discount factor of the virtual planner strictly exceeds that of the agents. Relative to the agents, the virtual planner exhibits an excessive concern for the long run. In the special case in which for all t, w and θ^{t-1}, $\mu_t^* z_t^*(w,\theta^{r-1}) = (\lambda-\beta)/\beta$, the virtual planner's preferences coincide with those assumed in Farhi and Werning (2006) or Sleet and Yeltekin (2006). More generally, the current model delivers a virtual planner who uses a history-contingent discounting scheme. This scheme may be interpreted as capturing the time varying and history-dependent political influence of agents.

3.1 Dynamic private information and the pattern of binding political constraints

At this point we have not established that the political constraints bind in any period. We now turn to this issue and impose the additional assumption:

Assumption 7 *For all t, $Z_t(\underline{U}_t) + X(0) < 0$, where $\underline{U}_t \in \mathcal{U}$ is such that for Q_t-a.e. (w, θ^{t-1}), $\underline{U}_t(w, \theta^{t-1}) = E[\theta] \inf D$.*

Assumption 7 implies that an allocation that pays out the entire resource bundle to agents and assigns the minimal possible utility to almost all agents at some date t is not politically sustainable. The political constraints that emerge as equilibrium conditions in our later political economy games satisfy this condition. Now, it follows from the existing normative literature (e.g. Atkeson and Lucas, 1992) that if the political constraints did not bind after some date T, then the continuation allocation would converge to an immiserated one (i.e. for Ψ-a.e. w, π^∞-a.e. θ^∞, $\lim_{t\to\infty} U(\{\varphi^*_{t+s}(w, \theta^t, \cdot)\}_{s=1}^\infty) = E[\theta] \inf D$). The continuity of each Z_t and Assumption 7 then imply that this continuation allocation violates the political constraints. Hence, there can be no such date T and the political constraints must bind infinitely often. We have the following result.

Proposition 4 *Let Assumption 7 hold. Then the political constraints in (5) bind infinitely often, i.e. for all T, there is a $t > T$ such that these constraints bind and the corresponding optimal multiplier μ^*_t is greater than zero.*

An immediate implication of the previous propositions is that under their respective assumptions, the discount factor of the virtual planner exceeds that of the agents infinitely often. The result stops short of saying that the political constraints "bind asymptotically", i.e. $\limsup \mu^*_t > 0$. However, we conjecture that this is the case.

Numerical examples suggest that the distribution of utilities becomes more dispersed as the multipliers μ_t^* decline and that if these numbers are persistently small, then the resulting inequality is politically unsustainable.

Remark: Collectively, these results contrast with those in Acemoğlu et al. Their political constraints are of the form $\sum_{t=1}^{\infty} \chi^{t-1} X\left(R_t - \int C(\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw)\right) \geq 0$. Our political constraints set augment theirs with an (increasing) Z_t function and set $\chi = 0$. It is the former difference that is important, since in conjunction with Assumption 7, it ensures that political constraints preclude allocations that deliver low utilities to most agents.

4 A political economy game with office-motivated politicians

4.1 Basic environment

We embed the basic environment of Section 2 into a political economy game. The game incorporates probabilistic voting over political parties. As is typical in the probabilistic voting literature, we assume that agents have heterogeneous and time varying biases towards a particular party. Although stylized, these biases capture the idea that elections are not solely determined by economic policy platforms. They ensure that election outcomes are uncertain and that election probabilities and, hence, the implied political constraints, are continuous in agent payoff functions. We start with the simplest formulation in which 1) political bias distributions are uniform, 2) there are no incumbency advantages and 3) parties are operated by impatient, office-motivated politicians. We then complicate the model by relaxing each of these assumptions.

4.1.1 Players

Politicians There are two political parties $i \in \{A, B\}$. In each period, politicians from the respective parties propose political mechanisms that describe how resources will be allocated in that period. Agents vote over the two proposed mechanisms and the election-winning mechanism is implemented. Politicians are *office-motivated,* i.e. concerned solely with winning elections: the objective of party i's politician is:

$$\sum_{t=1}^{\infty} \chi^{t-1} p_t^i,$$

where p_t^i is the probability that party i wins the election at date t and χ is the politicians discount factor. In much of the paper we assume that $\chi = 0$. The assumption that politicians are completely myopic only makes our results starker. Later we consider the possibility that politicians are concerned with the expected rents that they can extract if elected.

Agents Agents have the same preferences over allocations as before; we augment them with biases for one or the other party. Suppose that the two political parties A and B are distinguished by fixed characteristics that are non-economic in nature and difficult to change. We call these characteristics ideologies. Agent preferences over ideologies are described by two families of random variables $\{\xi\}$ and $\{\delta_t\}$. The first of these represents a permanent and idiosyncratic shock to each agent's relative preference for party B's ideology, it is i.i.d. across agents according to an atomless distribution with c.d.f. F. The second represents a common, time varying shock to the population's preference for party B's ideology, it is i.i.d. across time according to an atomless distribution with c.d.f. G. The lifetime "political" utility of an agent is given by:

$$(1-\beta)E\left[\sum_{t=1}^{\infty}\beta^{t-1}\left[\xi+\delta_t\right]1_t^B\right],$$

where 1_t^B is a random variable that takes the value 1 if party B is in power in period t and 0 otherwise. An agent's total payoff from an allocation $\{\varphi_t\}_{t=1}^{\infty}$ and the probability distribution over electoral outcomes is then:

$$U(\{\varphi_t\}_{t=1}^{\infty})+(1-\beta)E\left[\sum_{t=1}^{\infty}\beta^{t-1}\left[\xi+\delta_t\right]1_t^B\right].$$

For now we make the following assumption on the distributions F and G. It is conventional in applied probabilistic voting models. Later we relax it.

Assumption 8 *1) F is uniform with range $\Xi=\left[-\frac{1}{2\widehat{\xi}},\frac{1}{2\widehat{\xi}}\right]$ and density $\widehat{\xi}$. 2) G is uniform with range $\Delta=\left[-\frac{1}{2\widehat{\delta}},\frac{1}{2\widehat{\delta}}\right]$ and density $\widehat{\delta}$. 3) Let $d=E[\theta](\sup D-\inf D)$. $\widehat{\delta}$ and $\widehat{\xi}$ satisfy $\left[2d+\frac{1}{2\widehat{\delta}}\right]\in\left(0,\frac{1}{2\widehat{\xi}}\right)$ and $2d\in\left(0,\frac{1}{2\widehat{\delta}}\right)$.*

The last part of the assumption rules out inconvenient boundary solutions by ensuring that within each history-contingent sub-population there are some agents who will vote for either party no matter what and at the aggregate level, there is a probability (possibly very small) that a party will win an election no matter what.[6]

4.1.2 The stage game

Mechanisms As before, the economy is endowed with R units of resources in each period. Political parties propose schemes (or "political mechanisms") for allocating these resources amongst agents; they compete in elections with the mechanism of the election-winning party being implemented. At the beginning of period $t\geq 1$,

[6] The second part of the assumption is not necessary for our first model with office-motivated parties. It does simplify our second model with rent-motivated parties.

each agent is publicly identified by an index $w \in \mathbb{R}$ and a history of past messages $m^{t-1} \in M^{t-1}$. A mechanism proposal from party i is a pair $S_t^i = (M_t^i, \varphi_t^i)$, $i \in \{A, B\}$. The first piece of the mechanism is a finite message space that agents use to communicate with the planner, the second is a *utility allocation function* $\varphi_t^i : \mathbb{R} \times M^{t-1} \times M_t^i \to D$ that describes how utility will be awarded to agents contingent on their index and message history inclusive of current message that they send. We suppose that φ_t^i is appropriately measurable.

Remark: We allow for a fairly rich collection of mechanisms, much richer than is typically permitted in the probabilistic voting-political economy literature. In principal, these mechanisms can be used to implement history dependent allocations that provide future rewards and penalties for current behavior and, more specifically, they can be used to implement the optimal allocation of a committed utilitarian planner.

Timing The ensuing stage game consists of three sub-periods. In the first, politicians propose mechanisms. In the second the δ political preference shock is realized and agents vote over mechanisms. In the third sub-period, the election-winning mechanism is executed. Agents receive their taste shocks, they choose a probability distribution over the current message space and draw a message from it. They transmit this message to the government and receive the utility award implied by the election-winning utility allocation function.

4.1.3 Histories and strategies

We place some restrictions on strategies at the outset to rule out especially implausible equilibria. In particular, we assume that no player conditions her behavior on either past election-*loosing* mechanisms or the political identity of past election winners.[7] This assumption precludes strategies that would effectively punish the economy for failing to elect a particular party even if that party makes a very undesirable mechanism proposal. However, it still allows for considerable history dependence of equilibria and is much weaker than a Markov restriction.

Policy strategies We define an *aggregate history* H_t, $t \geq 2$, to be a sequence $\{M_r, \varphi_r\}_{r=1}^{t-1}$ of past election-winning mechanisms such that each $\varphi_r : \mathbb{R} \times \{M_s\}_{s=1}^{r} \to D$. H_1 is the null history. Let $\mathcal{H}^t$, $t \geq 1$, denote the set of t-period aggregate histories and define $\mathcal{S}_t$ to be the set of period t mechanisms. A *policy strategy* for party i is a sequence of functions $\sigma^i = \{\sigma_t^i\}_{t=1}^{\infty}$, where $\sigma_t^i : \mathcal{H}^t \to \mathcal{S}_t$ gives party i's t-period mechanism as a function of the aggregate history. We require that for all i, t and $H_t \in \mathcal{H}^t$, $(H_t, \sigma_t^i(H_t)) \in \mathcal{H}^{t+1}$. Let $\sigma = \{\sigma^A, \sigma^B\}$ denote a profile of policy strategies.

Message strategies A period t *individual history* of an agent $h_t = \left(w, m^{t-1}\right)$ gives the agent's w-type and past message history. Let $\mathcal{J}^1$ denote the null set and for all $t > 1$, $\mathcal{J}^t = \{\{M_r, \varphi_r\}_{r=1}^{t}, w, m^{t-1} : \{M_r, \varphi_r\}_{r=1}^{t} \in \mathcal{H}^{t+1}$, $w \in \mathbb{R}$, $m^{t-1} \in \{M_r\}_{r=1}^{t-1}\}$ and $\mathbb{P}$ the space of finite element probability distributions. The first component of agent behavior is a message strategy, $\lambda = \{\lambda_t\}_{t=1}^{\infty}$, where $\lambda_t : \mathcal{J}^t \times \Theta \to \mathbb{P}$ maps the $t+1$-period aggregate history and the agent's individual history and current shock to a lottery over the election winning message space M_t. Together H_t and λ induce a cross sectional distribution over individual histories h_t. This dispersion may be described by a measure space $(\mathbb{R} \times M^{t-1}, \mathbb{B}^t \otimes \mathbb{M}^{t-1}, Q_t(H_t, \lambda))$, where $Q_t(H_t, \lambda)$ is the probability measure over individual histories induced by H_t and λ. Let $\mathbb{P}(\mathbb{R} \times M^{t-1})$ denote the set of probability measures on $(\mathbb{R} \times M^{t-1}, \mathbb{B} \otimes \mathbb{M}^{t-1})$.

As with policy strategies, agent strategies do not condition on the election-loosing allocations or the identities

[7] This assumption resembles Duggan and Fey's (2006) assumption of outcome stationarity.

of the election winning and loosing parties. In addition, the message strategy does not condition on the (so far unspecified) voting behavior of the agent. This is without loss of generality. In our economy, agents are atomistic and their individual votes do not determine the election winning mechanism. Thus, their current and past votes do not influence the payoff from sending a particular message.

Voting strategies A period t *individual political history* of an agent $h_t^p = \left(\{H_t, S_t^A, h_t\}, \{H_t, S_t^B, h_t\}, \xi, \delta_t\right) \in \mathcal{J}^t \times \mathcal{J}^t \times \Xi \times \Delta$ includes two aggregate-individual histories that coincide up to date $t-1$, but include the current mechanism proposals of, respectively, party A and party B. It also includes the agents current political preference shocks. An agent's *voting strategy* is given by $\zeta = \{\zeta_t\}_{t=1}^{\infty}$, where $\zeta_t : \mathcal{J}^t \times \mathcal{J}^t \times \Xi \times \Delta \to P(\{A, B\})$ maps individual political histories to a probability distribution over $\{A, B\}$. After each h_t^p, the agent draws the name of a party from the distribution $\zeta_t\left(h_t^p\right)$ and then votes for that party. Note that an agent's voting strategy does not condition on her past votes since these are anonymous and do not affect her preferences over the parties.[8]

Resource-feasibility To be resource feasible given λ, a mechanism must consume an amount of resources less than R.

Definition 1 *Given a message strategy λ, a mechanism M is* ***resource-feasible*** *at H_t if:*

$$\int_{\mathbb{R} \times M^t} C(\varphi(h_{t+1})) Q_{t+1}(H_t, S, \lambda)(dh_{t+1}) \leq R.$$

Let $\mathcal{S}_t(\lambda, H_t)$ denote the set of date t resource-feasible mechanisms at H_t given the message strategy λ. Given λ, an aggregate history $H_t = \{S_r\}_{r=1}^{t-1}$ is ***resource-feasible*** *if each $S_r \in \mathcal{S}_r(\lambda, \{S_s\}_{s=1}^{r-1})$. Given λ, let $\mathcal{H}^t(\lambda)$ denote*

[8] As with the policy strategies, voting strategies do not condition on past election loosing policies or the identity of a winning party.

the set of t-period resource-feasible aggregate histories. Finally, given λ, a policy strategy σ^i is ***resource-feasible*** *if after each $H_t \in \mathcal{H}^t(\lambda)$, $\sigma_t^i(H^t) \in \mathcal{S}_t(\lambda, H_t)$.*

Let $\Sigma(\lambda)$ denote the set of resource-feasible policy strategy profiles given λ. We do not allow political parties to make resource-infeasible policy proposals and, henceforth, restrict attention to resource-feasible policy strategies.

Strategy profile outcomes A mechanism is implemented at date t if it either wins more than 50% of the vote or, in the event of a tie, it wins a fair coin toss. The strategy profile (σ, ζ, λ) together with the shock distributions F and G induce a family of conditional probability distributions over current and future mechanisms, incumbent governments and agent utilities.[9] We call this the outcome of the profile and isolate three components. First, let $p_t^i(H_t, S_t^i, S_t^j|\zeta)$ denote the probability that mechanism S_t^i wins an election for party i when the aggregate history is H_t, party j proposes S_t^j and the voting strategy is ζ. Second, any continuation equilibrium induces a continuation outcome allocation. Each of these in turn implies continuation payoffs for agents as functions of their histories. Let $U_t(H_{t+1}, h_t|\sigma, \zeta, \lambda)$ denote the continuation payoff implied by (σ, ζ, λ) after history (H_{t+1}, h_t) and let $W_t(H_t|\sigma, \zeta, \lambda)$ denote the utilitarian payoff at t prior to the determination of the period t election winner.

[9] The fraction of the vote obtained by party A in a competition between mechanisms S_t^A and S_t^B at t is:

$$f_t^A(H_t, S_t^A, S_t^B, \delta_t|\zeta, \lambda) = \int_{\mathbb{R}\times M^t} \zeta_t((H_t, S_t^A, h_t), (H_t, S_t^B, h_t), \xi, \delta_t; A) F(d\xi) Q_t(H_t, \lambda)(dh_t).$$

Since party A does not know the value of δ_t when it makes its policy proposal, it's probability of winning the election given is:

$$p_t^A(H_t, S_t^A, S_t^B|\zeta, \lambda) = \int 1_{\{\delta: f_t^A(H_t, S_t^A, S_t^B, \delta|\zeta,\lambda) > \frac{1}{2}\}} G(d\delta) + \frac{1}{2}\int 1_{\{\delta: f_t^A(H_t, S_t^A, S_t^B, \delta|\zeta,\lambda) = \frac{1}{2}\}} G(d\delta).$$

4.2 Politically credible equilibria

We describe the elements of an equilibrium below and then collect these elements into a formal definition.

Policy strategies Since the politicians operating party i are concerned only with winning the current election, they select σ^i so that for all t, H_t, $j \neq i$,

$$\sigma_t^i(H_t) \in \arg \sup_{S \in \mathcal{S}_t(\lambda, H_t)} p_t^i(H_t, S_t^i, \sigma_t^j(H_t)|\zeta_t). \tag{11}$$

In doing so they ignore the fact that their proposals may reduce the lifetime payoffs obtained by agents in previous or future periods and may, hence, contribute to electoral failure in those periods. In particular, parties cannot commit themselves to future policies.

Message strategies Agents choose message strategies that are optimal after each individual history h_t. For all t, $(H_{t+1}, h_t) \in \mathcal{J}^t$,

$$\forall \widehat{\lambda}, \quad U_t(H_{t+1}, h_t|\sigma, \zeta, \lambda) \geq U_t(H_{t+1}, h_t|\sigma, \zeta, \widehat{\lambda}). \tag{12}$$

Voting strategies Let S_t^i denote the mechanism proposed by party i at time t. The total continuation payoff to an agent, inclusive of ideological payoff, after individual political history $h_t^p = ((H_t, S_t^A, h_t), (H_t, S_t^B, h_t), \xi, \delta_t)$ and a period t election victory for party i is:

$$U_t(H_t, S_t^i, h_t|\sigma, \zeta, \lambda) + (1-\beta)(\xi + \delta_t)1_{\{i=B\}} + (1-\beta)E\left[\sum_{s=1}^{\infty} \beta^s (\xi + \delta_{t+s})1_{t+s}^B | H_t, S_t^i; \sigma, \zeta, \lambda\right].$$

where $1_{\{i=B\}}$ takes the value 1 if i equals B and is 0 otherwise. Let $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda) := U_t(H_t, S_t^A, h_t|\sigma,\zeta,\lambda)$ $- \; U_t(H_t, S_t^B, h_t|\sigma,\zeta,\lambda)$ denote the difference in economic payoffs and

$$\frac{D(H_t, S_t^A, S_t^B \xi, \delta_t|\sigma,\zeta,\lambda)}{1-\beta} := \xi + \delta_t + E\left[\sum_{s=1}^{\infty} \beta^s(\xi + \delta_{t+s}^B)|H_t, S_t^B; \sigma,\zeta,\lambda\right] - E\left[\sum_{s=1}^{\infty} \beta^s(\xi + 1_{t+s}^B)|H_t, S_t^A; \sigma,\zeta,\lambda\right]$$

the (normalized) difference in ideological payoffs across the two proposals to an agent with political history h_t^p. We assume that agents vote as if they were pivotal in the current period and vote for the party that maximizes their current payoff. This corresponds to the requirement that agents eliminate weakly dominated strategies in each stage game (see, for example, Baron and Kalai (1993) or Duggan and Fey (2006)). Thus, their voting strategy satisfies for all t, $h_t^p \in \mathcal{J}^t \times \mathcal{J}^t \times \Xi \times \Delta$,

$$\zeta_t(h_t^p; A) \in \begin{cases} \{1\} & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda) > D(H_t, S_t^A, S_t^B, \xi, \delta_t) \\ [0,1] & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda) = D(H_t, S_t^A, S_t^B, \xi, \delta_t) \\ \{0\} & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda) < D(H_t, S_t^A, S_t^B, \xi, \delta_t). \end{cases} \tag{13}$$

Politically credible equilibria Collecting these components of an equilibrium together we have the following formal definition.

Definition 2 (σ,ζ,λ) *is a politically credible equilibrium (PCE) if* $\sigma \in \Sigma(\lambda)$ *and*

1. *(Party optimality)* $\forall t$, $H_t \in \mathcal{H}^t$, $i \in \{A, B\}$, σ^i *satisfies (11);*

2. *(Agent optimality: messages)* $\forall t$, $(H_{t+1}, h_t) \in \mathcal{J}^t$,$\widehat{\lambda}$, λ *satisfies (12);*

3. *(Agent optimality: voting)* $\forall t$, $h_t^p \in \mathcal{J}^t \times \mathcal{J}^t \times \Xi \times \Delta$, ζ *satisfies (13).*

A symmetric politically credible equilibrium (SPCE) is a PCE that satisfies the symmetry condition:

4. $\sigma^A = \sigma^B$.

These definitions are in the spirit of Chari and Kehoe's sustainable plans equilibrium concept. They require optimality of player strategies after all aggregate and individual histories.

Characterization of equilibria Recall that continuation play of the game depends only on which mechanism wins the election, not which party. If the two parties propose the same mechanism then the continuation outcome path is independent of the electoral outcome and an agent's vote is determined solely by its current political shocks: $\zeta_t(h_t^p; A) = 1$ (resp. 0) if $0 > \xi + \delta_t$ (resp. $0 < \xi + \delta_t$). In this case, party A wins the election with probability $p = G(-F^{-1}(1/2))$ and party B with probability $1 - G(-F^{-1}(1/2))$. Since party A is always able to mimic B and choose the same mechanism, p places a lower bound on its probability of winning. Conversely, party B can always mimic party A and secure a probability of winning of $1 - p$. It follows that in any (continuation) equilibrium party A must win with probability p in each period. We have the following lemma.

Lemma 2 *Party A wins the election with probability $p = G(-F^{-1}(1/2))$ in each period. When F and G satisfy Assumption 8, $p = \frac{1}{2}$.*

We can use Lemma 2 and (13) to substitute an agent's current voting function ζ_t from (11) and, hence, obtain an equilibrium expression for the probability that a party wins an election in terms of current policy proposals, history and future strategies. We have the following.

Proposition 5 *In equilibrium, the probability that party A wins the election given H_t, the current proposals S_t^A*

and S_t^B and the equilibrium strategy profile (σ,ζ,λ) is

$$p_t^{*A}(H_t,S_t^A,S_t^B|\sigma,\zeta,\lambda)=G\left(\delta^*:\int F\left(\frac{\Delta U_t(H_t,S_t^A,S_t^B,h_t|\sigma,\zeta,\lambda)}{1-\beta}-\delta^*\right)Q_t(dh_t)=1/2\right)$$

and $p_t^{*B}(H_t,S_t^A,S_t^B|\sigma,\zeta,\lambda)=1-p_t^{*A}(H_t,S_t^A,S_t^B|\sigma,\zeta,\lambda)$. When F and G satisfy Assumption 8, for $i\in\{A,B\}$, $j\neq i$, $p_t^{*i}(H_t,S_t^i,S_t^j,h_t|\sigma,\zeta,\lambda)=\frac{1}{2}+\frac{\widehat{\delta}}{1-\beta}\int\Delta U_t(H_t,S_t^i,S_t^j,h_t|\sigma,\zeta,\lambda)Q_t(dh_t)$.

Proof: It follows that in any (continuation) equilibrium party A must win with probability p in each period. Thus, *regardless of current play*, $D(H_t,S_t^A,S_t^B,\xi,\delta_t|\sigma,\zeta,\lambda)=(1-\beta)(\xi+\delta_t)$ and the current electoral outcome affects neither the probability distribution over future government incumbents nor expected future political payoffs. Now, the fraction of type-h_t agents who vote for party A given the aggregate shock δ_t is the fraction such that $\Delta U_t(H_t,S_t^A,S_t^B,h_t|\sigma,\zeta,\lambda)>(1-\beta)(\xi+\delta_t)$ or $F\left(\frac{\Delta U_t(H_t,S_t^A,S_t^B,h_t|\sigma,\zeta,\lambda)}{1-\beta}-\delta_t\right)$. The fraction of the population that votes for party A is then $\int F\left(\frac{\Delta U_t(H_t,S_t^A,S_t^B,h_t|\sigma,\zeta,\lambda)}{1-\beta}-\delta_t\right)Q_t(dh_t)$.Finally, the probability that this fraction is above 1/2, resulting in an election victory for A is $G\left(\delta^*:\int F\left(\frac{\Delta U_t(H_t,S_t^A,S_t^B,h_t|\sigma,\zeta,\lambda)}{1-\beta}-\delta^*\right)Q_t(dh_t)=1/2\right)$. It is then immediate that when F and G satisfy Assumption 8, for each $i\in\{A,B\}$, $j\neq i$, $p_t^{*i}(H_t,S_t^i,S_t^j,h_t|\sigma,\zeta,\lambda),Q_t)$ $=\frac{1}{2}+\frac{\widehat{\delta}}{1-\beta}\int\Delta U_t(H_t,S_t^i,S_t^j,h_t|\sigma,\zeta,\lambda)Q_t(dh_t)$. ■

It follows immediately from this proposition that under Assumption 8, the policy optimality condition (11) is equivalent to

$$\sigma_t^i(H_t)\in\arg\sup_{S\in\mathcal{S}_t(\lambda,H_t)}\int U_t(H_t,S,h_t|\sigma,\zeta,\lambda)Q_t(dh_t). \tag{14}$$

Thus, the parties behave as if they are utilitarian planners who take the future message and policy strategies as given.

Remark After any aggregate history, the weighted average ideological payoff to agents is constant across equilibria. Consequently, in the remainder of the paper, we rank equilibria according to the weighted average utilitarian payoff attained by their outcome allocation. ■

4.3 Necessary and sufficient conditions for politically credible outcome paths

In this section, we formally define an outcome path, essentially an event tree of mechanisms coupled with a reporting strategy restricted to aggregate histories on the tree. We give necessary and sufficient conditions for an outcome path to be induced by a PCE. Outcome paths are more complicated than allocations as we have previously defined them, additional randomness is introduced into utility awards by the uncertain nature of elections and by the mixed strategy message strategies employed by agents. In the next section, we specialize PCE so that they induce allocations as defined in Section 2. As a precursor to this analysis we introduce the idea of a no insurance PCE.

No insurance PCE A building block of our subsequent analysis is the no insurance PCE. In this politicians repeatedly propose mechanisms that provide no insurance against taste shocks and agents send messages that maximize their current payoff. Under Assumption 8, reversion to this equilibrium minimizes the probability that a defecting party will win an election and, hence, maximizes the electoral discipline on the parties. If an allocation can be sustained, it can be sustained by reversion to this equilibrium.

A mechanism $S_t = (M_t, \varphi_t)$ offers no insurance against taste shocks if for each h_t, and m_t, m_t' in M_t $\varphi_t(h_t, m_t) = \varphi_t(h_t, m_t')$. Such a mechanism gives agents the same utility regardless of the message that they send and, hence, prevents agents with a larger taste shock from obtaining more resources. We focus on one particular no insurance mechanism in the remainder which we denote $S_t^{NI} = (\Theta, \varphi_t^{NI})$. This mechanism is direct - it uses Θ as its message space - and has an allocation function φ_t^{NI} that satisfies for all (h_t, θ), $\varphi_t^{NI}(h_t, \theta) = u(R)$. We define the *no insurance policy strategy* profile σ^{NI} as for all i, t, H_t, $\sigma_t^i(H_t) = S_t^{NI}$. If the future play of politicians conforms to σ^{NI}, then an agent's continuation payoff is unaffected by her current message and it is optimal for her to send a message that maximizes her current payoff. This motivates our definition of the *no insurance message strategy* λ^{NI}. For all t, H_{t+1}, h_t, θ set $\lambda_t^{NI}(H_t, S_t, h_t, \theta) = 1_\theta$, if $\theta \in \arg\max_{m \in M_t} \varphi_t(h_t, \theta')$ and $\lambda_t^{NI}(H_t, S_t, h_t, \theta) = 1_{m'}$, some $m' \in \arg\max_{m \in M_t} \varphi_t(h_t, \theta)$ otherwise. Thus, λ^{NI} requires that agents maximize their current payoff and, if their current shock achieves this maximum, they truthfully reveal this shock. Set the no insurance voting strategy:

$$\zeta_t^{NI}(h_t^p; A) \in \begin{cases} \{1\} & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t | \sigma^{NI}, \zeta^{NI}, \lambda^{NI}) > D(H_t, S_t^A, S_t^B, \xi, \delta_t) \\ [0,1] & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t | \sigma^{NI}, \zeta^{NI}, \lambda^{NI}) = D(H_t, S_t^A, S_t^B, \xi, \delta_t) \\ \{0\} & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t | \sigma^{NI}, \zeta^{NI}, \lambda^{NI}) < D(H_t, S_t^A, S_t^B, \xi, \delta_t) \end{cases} . \tag{15}$$

Now, by (15), ζ^{NI} satisfies the optimality condition for voting and, as asserted above, λ^{NI} is optimal for an agent given σ^{NI} and ζ^{NI}. The optimality of σ^{NI} follows from the fact that if agents play according to λ^{NI}, then it is not possible to provide them with any insurance, since no agent will send a message resulting in low current consumption. Under Assumption 8, we then have

$$S_t^{NI} \in \arg \sup_{S \in \mathcal{S}_t(\lambda, H_t)} \int U_t(H_t, S, h_t | \sigma^{NI}, \zeta^{NI}, \lambda^{NI}) Q_t(dh_t)$$

and so σ^{NI} is optimal for politicians. It follows immediately that $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ is a PCE. We have the following result.

Lemma 3 *Let Assumption 8 hold. 1) $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ is a PCE. 2) Amongst PCE's $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ delivers the lowest utilitarian payoff.*

Outcome paths The previous proposition identifies the no insurance equilibrium as the one that gives the lowest utilitarian payoff $\underline{W} = E[\theta]u(R)$. We put this to use below and provide necessary and sufficient conditions for a family of mechanisms and message functions to be the outcome of a PCE. In addition to resource-feasibility and message optimality, our requirement is that the family provide a politician with a payoff in excess of the no insurance one at all dates. To state the result, we provide a formal definition of an outcome path.

Recursively define an *outcome path* to be a tuple $\Upsilon = \{\{\mathcal{H}_t^\Upsilon\}, \{\mathcal{J}_t^\Upsilon\}, \{\sigma_t^\Upsilon\}, \{\lambda_t^\Upsilon\}\}$, where $\mathcal{H}_1^\Upsilon = \mathcal{H}_1$, for $i \in \{A, B\}$, $\sigma_1^{\Upsilon,i} : \mathcal{H}_1^\Upsilon \rightarrow \mathcal{S}_1$ and for $t > 1$, $\mathcal{H}_t^\Upsilon = \{\{S_r\}_{r=1}^{t-1} : \{S_r\}_{r=1}^{t-2} \in \mathcal{H}_{t-1}^\Upsilon$ and for some $i \in \{A, B\}$, $S_{t-1} = \sigma_{t-1}^{\Upsilon,i}(\{S_r\}_{r=1}^{t-2})\}$, $\mathcal{J}_t^\Upsilon = \{\{M_r, \varphi_r\}_{r=1}^t, w, \{m_r\}_{r=1}^{t-1} : (\{M_r, \varphi_r\}_{r=1}^{t-1}, w, \{m_r\}_{r=1}^{t-2}) \in \mathcal{J}_t^\Upsilon$, and for some $i \in \{A, B\}$, $(M_t, \varphi_t) = \sigma_t^{\Upsilon,i}(\{S_r\}_{r=1}^{t-1})$ and $\{m_r\}_{r=1}^{t-2} \in \{M_r\}_{r=1}^{t-2}\}$, $\sigma_t^{\Upsilon,i} : \mathcal{H}_t^\Upsilon \rightarrow \mathcal{S}_t$ (with for all i, t, $\{M_r, \varphi_r\}_{r=1}^{t-1} \in \mathcal{H}_t^\Upsilon$, $\sigma_t^{\Upsilon,i}(\{M_r, \varphi_r\}_{r=1}^{t-1})$ equals some (M_t, φ_t) such that $\varphi_t : \mathbb{R} \times \{M_r\}_{r=1}^t \rightarrow D$) and $\lambda_t^\Upsilon : \mathcal{J}_t^\Upsilon \times \Theta \rightarrow \mathbb{P}$ (with for all t, $\{\{M_r, \varphi_r\}_{r=1}^t, w, \{m_r\}_{r=1}^{t-1}, \theta\} \in \mathcal{J}_t^\Upsilon \times \Theta$ $\lambda_t^\Upsilon(\{M_r, \varphi_r\}_{r=1}^t, w, \{m_r\}_{r=1}^{t-1}, \theta)$ placing all probability mass on M_t). We suppose that after each $H_t \in \mathcal{H}_t^\Upsilon$, $\sigma_t^{\Upsilon,A}(H_t)$ is picked with probability p and $\sigma_t^{\Upsilon,B}(H_t)$ with probability $1-p$. An outcome path is simpler than an equilibrium: it does not describe what happens after a history that is not generated by the path and it omits the voting strategy. Moreover, a PCE induces an outcome path. We now provide necessary and sufficient conditions for an outcome path to be an equilibrium one. In the following proposition we use $U_t^\Upsilon(H_{t+1}, h_t|\widehat{\lambda}^\Upsilon)$ to denote the continuation payoff to an agent along the outcome path Υ if she uses the message functions $\widehat{\lambda}^\Upsilon = \{\widehat{\lambda}_t^\Upsilon\}$, $\widehat{\lambda}_t^\Upsilon : \mathcal{J}_{t+1}^\Upsilon \times \Theta \rightarrow \mathbb{P}$. $Q_t^\Upsilon(H_t)$ denotes the cross sectional distribution of individual histories implied by the outcome path at aggregate history H_t.

The next proposition asserts that, under Assumption 8, an outcome path is a PCE outcome path if it satisfies resource and incentive-feasibility condition and a political sustainability condition (18).

Proposition 6 *Let Assumption 8 hold. Let* $\Upsilon = \{\{\mathcal{H}_t^\Upsilon\}, \{\mathcal{J}_t^\Upsilon\}, \{\sigma_t^\Upsilon\}, \{\lambda_t^\Upsilon\}\}$ *be an outcome path and let* $Q_t^\Upsilon(H_t, dh_t)$ *denote the induced distribution over individual histories after* $H_t \in \mathcal{H}_t^\Upsilon$*.* Υ *is an PCE outcome path if and only if:*

1. *For all* t, $H_{t+1} = \{M_r, \varphi_r\}_{r=1}^t \in \mathcal{H}_{t+1}^\Upsilon$,

$$\int_{\mathbb{R}\times M^t} C(\varphi_t(h_{t+1}))Q_{t+1}^\Upsilon(H_{t+1}, dh_{t+1}) \leq R; \tag{16}$$

2. *For all* t, $(H_{t+1}, h_t) \in \mathcal{J}_t^\Upsilon$,

$$\forall \widehat{\lambda}^\Upsilon, \quad U_t^\Upsilon(H_{t+1}, h_t|\sigma^\Upsilon, \lambda^\Upsilon) \geq U_t^\Upsilon(H_{t+1}, h_t|\sigma^\Upsilon, \widehat{\lambda}^\Upsilon); \tag{17}$$

3. *For all* t, $H_t = \{M_r, \varphi_r\}_{r=1}^{t-1} \in \mathcal{H}_t^\Upsilon$,

$$\int_{\mathbb{R}\times M^{t-1}} U_t^\Upsilon(H_t, \sigma_t^{\Upsilon,i}(H_t), h_t|\sigma^\Upsilon, \lambda^\Upsilon)Q_t^\Upsilon(H_t, dh_t) \geq \underline{W}. \tag{18}$$

4.4 Outcome paths to allocations: Two simplifications

A Revelation principle Our definition of PCE does not require that agents truthfully reveal their type. However, any politically credible distribution of payoffs can be implemented by an equilibrium in which 1) political parties always propose direct mechanisms and 2) it is optimal for agents to be truthful along the equilibrium outcome path. More formally, we have the following Revelation principle for our environment.

Definition 3 *A mechanism* $S_t = (M_t, \varphi_t)$ *is direct if* $M_t = \Theta$*. A policy strategy* σ *is direct if for all* i, t, H_t, $\sigma_t^i(H_t)$ *is direct.*

Definition 4 *Let* S_t *be a direct mechanism.* λ *is truthful after individual history* $(H_t, S_t, h_t) \in \mathcal{J}^t$ *if, for all* θ,

$$\lambda_t(H_t, S_t, h_t, \theta) = 1_\theta.$$

Proposition 7 ***(Revelation Principle)*** *Let (σ, ζ, λ) be a PCE. Then there exists another PCE $(\hat{\sigma}, \hat{\zeta}, \hat{\lambda})$ such that 1) $\hat{\sigma}$ is direct, 2) $\hat{\lambda}$ is truthful after all histories $(H_t,\ \sigma_t(H_t), h_t) \in \mathcal{J}^t$ and 3) the equilibrium $(\hat{\sigma}, \hat{\zeta}, \hat{\lambda})$ delivers the same lifetime payoff to each w-type of agent as the original equilibrium (σ, ζ, λ).*

The proof is similar to that given in Sleet and Yeltekin (2006) and is not repeated. It utilizes Proposition 6 above; by this proposition any PCE outcome path can be supported by trigger strategies that revert to the no insurance equilibrium following the electoral victory of a defecting political party. Using the standard proof of the Revelation Principle any equilibrium outcome path can be replaced with one that uses direct mechanisms and delivers the same expected utility to agents if they are truthful as they would have received under the original equilibrium. A political party that makes proposals consistent with this new outcome path generates the same utilitarian payoffs at each date and has the same probability of winning an election as before. A party that deviates triggers play of the no insurance worst equilibrium if it is elected. This lowers the party's probability of winning an election and removes any political incentive to deviate.

It follows from Proposition 7 that, from a welfare perspective, there is no loss of generality in restricting attention to equilibria that use direct mechanisms and induce truthful reporting. Hence, for the remainder of this section, we restrict attention to politically credible equilibria (σ, ζ, λ) satisfying:

Property 1 (Truthful reporting) *σ is direct along its outcome path and λ is truthful along this path.*

Of course, we continue to allow political parties to deviate to non-direct mechanisms.

Symmetry We also simplify in the remainder by focussing on symmetric PCE, i.e. PCE's satisfying:

Property 2 (Symmetry) $\sigma^A = \sigma^B$.

Under these restrictions, a PCE induces an allocation of the form $\{\varphi_t\}_{t=1}^{\infty}$, $\varphi_t : \mathbb{R} \times \Theta^t \to D$. Neither elections nor agent's message strategies introduce any additional uncertainty over an agent's utility award; these awards can be expressed as functions of the agent's w-type and her (truthfully reported) sequence of shocks. We define a *politically credible allocation* to be one induced by a PCE satisfying the above properties.

4.5 Politically credible allocations

The following proposition asserts that an allocation is politically credible if only if it is resource-feasible, incentive-compatible and politically sustainable.

Proposition 8 *Let Assumption 8 hold.* $\{\varphi_t\}_{t=1}^{\infty}$ *is a politically credible allocation if and only if it satisfies (1), (2) and the political constraints*

$$\forall t, \qquad Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty})) = (1-\beta)\int_{\mathbb{R}} \sum_{r=0}^{\infty} \beta^r \sum_{\Theta^{t+r}} \theta_{t+r}\varphi_{t+r}(w, \theta^{t+r})\pi^{t+r}\left(\theta^{t+r}\right)\Psi(dw) - \underline{W} \geq 0. \tag{19}$$

The proof of this proposition is a specialization of that given for Proposition 6 to equilibria that are truthful and symmetric; it is omitted. Given this result, it is clear that we can recover a best politically credible allocation by solving the following problem:

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}} (1-\beta)\int \gamma(w) \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\Theta^t} \theta_t \varphi_t(w, \theta^t)\pi^t(\theta^t)\Psi(dw) \text{ s.t. } (1),\ (2),\ \text{and } (19). \tag{20}$$

Problem (20) is a special case of our reduced form problem (5). The political constraints (19) incorporate relatively simple political viability functions that satisfy Assumptions 2-6. In this case, $X = 0$ and for all t, h_t, $z_r^*(h_t) = 1$. Thus, the following specialization of Proposition 3 holds.

Proposition 9 *Suppose that* $\{\varphi_t^*\}_{t=1}^{\infty}$ *attains the supremum in (20). Then there is a sequence* $\{\mu_t^*\}_{t=1}^{\infty} \in L$ *with* $\sum_{t=1}^{\infty} \int_{\mathbb{R}} \gamma(w) \sum_{\theta^{t-1}} \beta^{t-1} \left(\prod_{r=1}^{t} (1+\mu_r^*)\right) \pi^{t-1}(\theta^{t-1}) \; \Psi(dw) < \infty$*, such that* $\{\varphi_t^*\}_{t=1}^{\infty}$ *solves*

$$\sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(R,\Psi)} \int \gamma(w) \sum_{t=1}^{\infty} \sum_{\Theta^t} B_1^t \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \tag{21}$$

where $B_1^t = \beta^{t-1} \prod_{r=1}^{t}(1+\mu_r^*) \geq \beta^{t-1}$*. Conversely, if* $\{\varphi_t^*\}_{t=1}^{\infty}$ *solves (21) for some* $\{\mu_t^*\}_{t=1}^{\infty} \in L$ *with* $\sum_{t=1}^{\infty} \int_{\mathbb{R}} \gamma(w) \sum_{\theta^{t-1}} \beta^{t-1} \left(\prod_{r=1}^{t}(1+\mu_r^*)\right) \pi^{t-1}(\theta^{t-1}) \Psi(dw) < \infty$*, and if*

1. $\{\mu_t^*\} \in \arg\inf_{\{\mu_t\}_{t=1}^{\infty} \in L} \mathcal{L}^*(\{\varphi_t^*\}_{t=1}^{\infty}, \{\mu_t\})$,

then $\{\varphi_t^*\}_{t=1}^{\infty}$ *solves (20).*

In addition, the functions $\{Z_t\}_{t=1}^{\infty}$ satisfy Assumption 7 and, hence, Proposition 4 applies. We conclude that in an optimal PCE, short-sighted politicians concerned only with winning current elections implement the same allocation as a planner who is more patient than agents. Such politicians cannot commit to severe limiting allocations even if they raise the current utilitarian payoff and, hence, improve a party's current election prospects.

4.6 Numerical examples

TO BE ADDED.

5 Generalizations

5.1 Non -uniform preference distributions [Incomplete]

A general condition for a political strategy to be optimal for impatient office-motivated parties is that:

$$p \geq \min_{(\sigma',\zeta',\lambda')\in E} \max_{S_t^A\in\mathcal{S}_t(H_t,\lambda')} G\left(\delta^* : \int F\left(\frac{U_t(H_t,S_t^A,h_t|\sigma',\zeta',\lambda') - U_t(H_t,\sigma_t^B(H_t),h_t|\sigma,\zeta,\lambda)}{1-\beta} - \delta^*\right) Q_t(dh_t) = 1/2\right) \tag{22}$$

and

$$1-p \geq \min_{(\sigma',\zeta',\lambda')\in E} \max_{S_t^B\in\mathcal{S}_t(H_t,\lambda')} 1-G\left(\delta^* : \int F\left(\frac{U_t(H_t,\sigma_t^A(H_t),h_t|\sigma',\zeta',\lambda') - U_t(H_t,S_t^B,h_t|\sigma,\zeta,\lambda)}{1-\beta} - \delta^*\right) Q_t(dh_t) = 1/2\right), \tag{23}$$

where E denotes the set of PCE's and p is defined as before. If these conditions hold, then each party is better off adhering to the strategy profile and accepting the equilibrium election-winning probabilities p and $1-p$, than defecting to S_t^i and triggering the worst (from the perspective of their electoral opportunities) continuation equilibrium. When F is uniform, then these conditions reduce to the requirement that for all H_t,

$$\int U_t(H_t,\sigma_t^i(H_t),h_t|\sigma,\zeta,\lambda)Q_t(dh_t) \geq \underline{W},$$

which in turn reduces to (18) for politically credible allocations.[10] If G is uniform on Δ, but F not, the analysis is more complicated. To simplify it somewhat suppose that F is symmetric (i.e. $F(x) = -F(-x)$ for all x) and focus on symmetric, truth-telling PCE. As shown in the appendix

$$R(\delta) = \min_{(\sigma',\zeta',\lambda')\in E} \max_{S_t^A\in\mathcal{S}_t(H_t,\lambda')} \int F\left(\frac{U_t(H_t,S_t^A,h_t|\sigma',\zeta',\lambda') - U_t(H_t,\sigma_t^B(H_t),h_t|\sigma,\zeta,\lambda)}{1-\beta} - \delta\right) Q_t(dh_t)$$

[10] In fact the previous uniform assumption on G is not necessary for these results.

is increasing. Thus, neither party will defect from an equilibrium (σ, ζ, λ) if

$$\min_{(\sigma',\zeta',\lambda')\in E} \max_{S_t \in \mathcal{S}_t(H_t,\lambda')} \int F\left(\frac{U_t(H_t, S_t, h_t|\sigma', \zeta', \lambda') - U_t(H_t, \sigma_t(H_t), h_t|\sigma, \zeta, \lambda)}{1-\beta}\right) Q_t(dh_t) \leq \frac{1}{2},$$

where $\sigma_t(H_t)$ is the common policy proposal implied by σ after history H_t. Now any defecting politician will propose a mechanism $(\{m\}, \varphi_t')$ with a message space that consists of a single message and an allocation function that conditions only on past histories h_t. Similarly, the payoff function $\underline{U}_{t+1}$ implied by the defection-triggered continuation equilibrium depends only upon histories h_t. Intuitively, agents will only send a message that is not current utility maximizing if they are compensated with a more generous continuation utility. If eliciting such an message and providing such a continuation utility raises the defecting politician's election-winning probability, the minimizing continuation equilibrium will not provide it. Thus, current message-sending cannot be used to provide insurance and further the electoral prospects of a defecting politician. Indeed, partitioning agents according to their current message may give greater scope for punishing the politician. The implied political constraints on an allocation $\{\varphi_t\}_{t=1}^{\infty}$ are then reduce to:

$$\begin{aligned} 0 &\leq Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty})) \quad (24)\\ &= \frac{1}{2} - \max_{\varphi_t': \int C(\varphi_t'(h_t)Q_t(dh_t) \leq R} \left[\min_{U_{t+1}' \in U_t} \int F\left(\frac{(1-\beta))\, E[\theta]\varphi'(h_t) + \beta U_{t+1}'(h_t) - U(\{\varphi_{t+r}|h_t\}_{r=0}^{\infty})}{1-\beta}\right) Q_t(h_t)\right] \end{aligned}$$

with $X = 0$. Assuming that a pair of optimizing functions φ_t^* and U_{t+1}^* exists for (24), then (see the Appendix) Z_t has a Frechet derivative of the form $\langle \widehat{z}_t, \cdot\rangle$, where $\widehat{z}_t(U_t)(h_t) = \frac{1}{1-\beta} f\left(\frac{(1-\beta))E[\theta]\varphi_t^*(h_t) + \beta U_{t+1}^*(h_t) - U_t(h_t)}{1-\beta}\right)$ and f is the density of F. In general, symmetry of F implies that Z_t is not concave (the exception is the case considered previously, F uniform). By appealing to generalized Kuhn-Tucker arguments, the necessity part of Proposition 3 can still be obtained. We have that if $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}} \int \gamma(w) \sum_{t=1}^{\infty} \beta^t \sum_{\Theta^t} \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \tag{25}$$

subject to (1), (2) and (24), then it also attains the supremum in

$$\sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(R,\Psi)} \int \gamma(w) \sum_{t=1}^{\infty} \sum_{\Theta^t} B_1^t(w, \theta^{t-1}) \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \tag{26}$$

where for each t, $B_1^t(w, \theta^{t-1}) = \beta^{t-1} \prod_{r=1}^{t}(1 + \mu_r^* z_r^*(w, \theta^{r-1})) \geq \beta^{t-1}$ and $\{z_t^*\}_{t=1}^{\infty}$ are recovered from the functions $\widehat{z}_t^*(w, \theta^{t-1}) = \frac{1}{1-\beta} f\left([(1-\beta))\, E[\theta]\varphi_t^*(h_t) \;+\; \beta U_{t+1}^*(h_t) \;-\; \int \gamma(w) \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\Theta^t} \theta_t \varphi_t^*(w, \theta^t) \pi^t(\theta^t) \Psi(dw)]/(1-\beta)\right)$ according to our earlier procedure.

Interpretation The main term in the political constraint gives the probability that party A will win the election at t after making its best possible defection (given continuation play which is designed to inflict the maximum electoral penalty on a defecting party). In our model, the population of agents is partitioned by their past histories. Agents with the same political biases, but different histories will, in general, receive different allocations along both the equilibrium path and the defecting path. Consequently, they will have different attitudes towards a defection. Within each history-specific subpopulation, there will be a swing voter - an agent whose bias value ξ renders him indifferent between the equilibrium path and the defection. The ξ-identity of the swing voter will usually differ across history subpopulations. Political constraints are relaxed at the margin if swing voters are attracted away from the defection. When F is uniform, the number of swing voters within each subpopulation is proportional to the number of agents in the subpopulation, so the political constraint reduces to a bound on a utilitarian payoff. When F is non-uniform, this is no longer the case. The number of swing voters in a sub-population depends on the value of the density f evaluated at that sub-population's gain or loss from the defection. Subpopulations with relatively more swing voters (higher values of f) will receive relatively greater weight in the virtual planner's objective.[11]

5.2 Incumbency advantage

We revert to uniform F and G distributions. Incumbency advantage can be introduced in a simple way by respecifying an agent's political preferences as:

$$(1-\beta)E\left[\sum_{t=1}^{\infty}\beta^{t-1}\left[\xi+\delta_t\right]1_t^B\right]+E\left[\phi_t\right]$$

where $\phi_t = \phi > 0$ if party i is in power at $t-1$ and at t and 0 otherwise, and $\phi \in (0, 1/(2\widehat{\xi}) - 2d - 1/(2\widehat{\delta})]$. All else equal, the introduction of the ϕ_t variable raises the probability of an incumbent party being re-elected. Suppose that party A was in power at $t-1$ and in the period t election, proposed the same mechanism as party B. Since political strategies condition only on election winning outcomes, agents face the same allocation in current and future periods regardless of who they vote for. Thus, agents base their vote on ideological considerations alone. All those agents with $\xi < \phi - \delta_t$ vote for party A and A's probability of winning the election is $G(\phi) > \frac{1}{2}$. As before, mimicking party B need not constitute equilibrium behavior, but it does place a lower bound on A's probability of winning. On the other hand, B could mimic A and win with $1 - G(\phi) < \frac{1}{2}$. This places an upper bound on

[11] In the static probabilistic voting literature, it is usual to assign different uniform F distributions exogenously to different types of agents, thus generating differential weighting in the electoral process. Here, with a common non-uniform shock distribution F this differential weighting arises endogenously. The provision of incentives leads to different continuation allocations for different history-types so creating a dispersed population of swing voters who receive different weights according to f.

the probability with which party A could win. We conclude that if party A is in power at date $t-1$, it wins with probability $G(\phi)$ at date t, while if it was out of power at $t-1$, it wins with probability $1-G(\phi)$. These probabilities hold independently of the specific history of election-winning mechanisms generated by past play.

A very similar argument to that given in the previous section implies that parties seeking to maximize their probability of re-election propose mechanisms that maximize the utilitarian payoff to agents given future play. After specializing to symmetric PCE with truth-telling, we recover the political constraints 18 and Proposition 9. The only change in this setting is to the probability of any given party winning an election conditional on which party was in office in the prior period.

5.3 Patient political parties

We now show that political parties that care about both current and future elections make the same choices as those who care only about the current election. Suppose that the preferences of political parties are given by:

$$(1-\chi)\sum_{t=1}^{\infty}\chi^{t-1}p_t^i,$$

where now $\chi \in (0,1)$ is a discount factor. We assume that there are no incumbency advantages.

As before any triple (σ, ζ, λ) induces an outcome path and, hence, a sequence $\{p_t^i(H_t, \sigma_t(H_t)|\, \zeta)\}$ that gives the conditional probability that party i will win after each history. Let $V^i(\sigma, \zeta, \lambda|H_t)$ denote the continuation payoff to party i from (σ, ζ, λ) after the history H_t; V^i satisfies the recursion:

$$\begin{aligned} V^i(\sigma,\zeta,\lambda|H_t) &= p_t^i(H_t,\sigma_t(H_t)|\zeta)[(1-\chi)+\chi V^i(\sigma,\zeta,\lambda|H_t,\sigma_t^i(H_t))]. \\ &\quad +\chi(1-p_t^i(H_t,\sigma_t(H_t)|\zeta))V^i(\sigma,\zeta,\lambda|H_t,\sigma_t^j(H_t)). \end{aligned}$$

The natural extension of our earlier equilibrium definition is:

Definition 5 (σ, ζ, λ) *is a politically credible equilibrium if* $\sigma \in \Sigma(\lambda)$ *and*

1. *(Party optimality) For all* t, H_{t-1}, $i, j \in \{A, B\}$, $j \neq i$ *and all* $\widehat{\sigma}^i$ *that are resource-feasible given* λ

$$V^i(\sigma^i, \sigma^j, \zeta, \lambda | H_t) \geq V(\widehat{\sigma}^i, \sigma^j, \zeta, \lambda | H_t); \tag{27}$$

2. *(Agent optimality: voting)* $\forall t, H_{t-1}, S_t^A, S_t^B, h_t, \xi, \delta_t$, ζ *satisfies (13);*

3. *(Agent optimality: messages)* $\forall t, H_t$,h_t,$\widehat{\lambda}$, λ *satisfies (12).*

All that has changed here is condition (27), which generalizes our earlier political optimality condition to allow for political discount factors in excess of 0. This criterion initially appears more complicated than before. A policy choice effects both the probability of winning in the current period and, through its affect on the game's history, its probability of winning in subsequent periods. Potentially, a party might trade these probabilities off against each other, sacrificing its chances of winning today, in order to improve its future electoral prospects. In fact this does not happen. Even with these more complicated preferences, each party chooses its current mechanism to maximize the current utilitarian payoff, given its future play and the play of its rival.

Proposition 10 (σ, ζ, λ) *is a PCE of a game with patient parties who have discount factor* $\chi > 0$ *if and only if it is a PCE of a game with impatient parties who have discount factor* $\chi = 0$.

Proof: Suppose that $(\sigma^A, \sigma^B, \zeta, \lambda)$ is a credible equilibrium of a game with patient parties, then it is resource-feasible and satisfies agent optimality. It remains to check that

$$\sigma_t^i(H_t) \in \arg \sup_{S \in \mathcal{S}_t} p_t^i(H_t, S_t^i, \sigma_t^j(H_t) | \zeta). \tag{28}$$

As before, party A can always win elections with probability p (and obtain the payoff p) simply by making the same choices as party B and playing σ^B. Thus, p places a lower bound on party A's payoff. Symmetrically, party B's payoff is also bounded below by $1-p$. But, then

$$p \leq (1-\chi)\sum_{t=1}^{\infty}\chi^{t-1}p_t^A = 1-(1-\chi)\sum_{t=1}^{\infty}\chi^{t-1}p_t^B \leq p.$$

Hence, party A always earn a payoff of p in equilibrium; moreover, by the same argument, a party A's continuation equilibrium payoff after any history is also p. Now consider party A's play after history H_t. The payoff to party A if it defects to S_t^A and then reverts to equilibrium play is:

$$(1-\chi)p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta) + \chi p.$$

Whether party A wins or looses in the present does not affect its equilibrium continuation payoff and, so, it chooses policy simply to maximize its current payoff. It follows that equilibrium strategies must maximize the per period probability of winning. The same logic applies to party B.

Conversely, if $(\sigma^A, \sigma^B, \zeta, \lambda)$ is a PCE of a game with impatient parties ($\chi = 0$), then it is resource-feasible and satisfies the agent optimality conditions. It remains to check that it satisfies the political optimality criteria (27) for $\chi > 0$. Now, in the game with impatient parties, party A wins an election with probability p in each period conditional on the past history of the game. Thus, for all i, t, H_t, $V^A(\sigma^i, \sigma^j, \lambda|H_{t+1}) = p$. It follows that

$$\begin{aligned} V^A(S^A, \sigma^B, \zeta, \lambda|H_t) &= (1-\chi)p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta) \\ &\quad +\chi(1-p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta))V^A(\sigma^A, \sigma^B, \zeta, \lambda|H_t, \sigma_t^B(H_t)) \\ &\quad +\chi p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta)V^A(\sigma^A, \sigma^B, \zeta, \lambda|H_t, S_t^A) \\ &= (1-\chi)p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta) + \chi p. \end{aligned}$$

Since $\sigma_t^A(H_t)$ maximizes $p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta)$, it follows that it also satisfies the "one step no deviation condition"

$$\begin{aligned}\sigma_t^A(H_t) &= \arg\sup_{S_t^A \in \mathcal{S}_t} (1-\chi) p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta) \\ &\quad + \chi(1 - p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta)) V^A(\sigma^A, \sigma^B, \zeta, \lambda | H_t, \sigma_t^B(H_t)) \\ &\quad + \chi p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta) V^A(\sigma^A, \sigma^B, \zeta, \lambda | H_t, S_t^A).\end{aligned}$$

Thus, regardless of the value of χ, party A has no incentive to defect from σ^A for one period and then revert to this strategy. By a standard argument, given the boundedness of payoffs, party A has no incentive to undertake even an infinite series of deviations from σ^A. Thus, σ^A satisfies the party optimality condition (27). By an identical argument σ^B satisfies this condition as well. ■

Thus, the set of PCE's is independent of the politicians' discount factor. This independence extends to symmetric, truth-telling PCE's and, hence, the conditions for a politically credible allocation are unaltered. In particular, with Assumption **??** reinstated, the political constraints (18) continue to hold, as do Propositions 4 and 9. We conclude that in an optimal PCE, politicians implement the same allocation as a planner who is more patient than agents regardless of their discount factor. Such politicians cannot commit to severe limiting allocations even if they raise their electoral prospects. Note that one cannot appeal to a folk theorem-type result to the effect that political constraints are relaxed as politicians become more patient. Punishment for a defection is essentially immediate, but nonetheless the force for immiseration ensures that political constraints (eventually) bind.

6 The model with rents

In this section, we alter the objective of political parties. Previously, we assumed that parties were motivated only by their desire to win office. We now suppose that they care about the rents that they can extract while in government. As before, we assume that parties are impatient so that the objective of party i at date t is: $r_t^i p_t^i$, where r_t^i is the amount of rent extracted by party i if it wins the date t election and p_t^i is the probability that i wins. Parties are now prepared to trade the probability of winning office off against the amount of rent they obtain if they do win.

As before each party proposes a mechanism S_t in successive periods. In contrast to earlier sections, however, any resources not allocated to agents are appropriated by the candidate as rent. We modify our earlier definition of a politically credible equilibrium to accommodate our new political objective.

Definition 6 *(σ, ζ, λ) is a politically credible equilibrium in the model with political rents if $\sigma \in \Sigma(\lambda)$ and*

1. *(Party optimality) t, H_t, $i \in \{A, B\}$, σ^i satisfies*

$$\sigma_t^i(H_t) \in \arg \sup_{S_t^i \in \mathcal{S}_t} r(S_t^i, \lambda | H_t) p_t^i(H_t, S_t^i, \sigma_t^j(H_t) | \zeta). \tag{29}$$

where for $S_t^i = (M_t^i, \varphi_t^i)$,

$$r(S_t^i, \lambda | H_t) := R - \int_w \sum_{M^{t-1} \times M_t^i} C(\varphi_t^i(h_{t+1})) Q_{t+1}(H_t, S_t^i, dh_{t+1}).$$

2. *(Agent optimality: messages) $\forall t, H_{t+1}, h_t, \widehat{\lambda}$, λ satisfies (12);*

3. *(Agent optimality: voting) $\forall t, H_t, S_t^A, S_t^B, h_t, \xi, \delta_t$, ζ satisfies (13).*

Our earlier analysis of PCE's used the fact that in any equilibrium the probability with which a party won an election was independent of the past history aggregate history. This is no longer true in general in the model with political rents. To simplify the analysis, we focus on *symmetric political credible equilibria*. In these both political parties use the same political strategy. Under Assumption **??**, symmetric PCE (without incumbency advantages) imply a probability of electoral victory for either party of $\frac{1}{2}$. Using similar arguments to those in Section 4, the party optimality condition (29) can be rewritten as:

$$\sigma_t^i(H_t) \in \arg\sup_{S_t^i \in \mathcal{S}_t} r(S_t^i, \lambda | H_t) \left[\frac{1}{2} + \widehat{\delta} \int U_t(H_t, S_t^i, h_t | \sigma, \zeta, \lambda) - U_t(H_t, \sigma_t^j(H_t), h_t | \sigma, \lambda) Q_t(H_t, \lambda | dh_t) \right],$$

where the bracketed term gives the probability that party i wins.

Lemma 4 *Let $\underline{W}_t(H_t) = \inf_{\{\sigma,\zeta,\lambda\} \in SE} W_t(\sigma, \zeta, \lambda | H_t)$ denote the worst utilitarian payoff attainable by a continuation SPCE after aggregate history H_t. Let $\underline{W} = \inf_{H_t} \underline{W}_t(H_t)$ and assume that there is some $\widehat{H}_T$ and $(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda})$ such that $W_T(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda} | \widehat{H}_t) = \underline{W}$. Then, there is an equilibrium (σ, ζ, λ) such that $W_1(\sigma, \zeta, \lambda) = \underline{W}$.*

Once again, the period 1 utilitarian payoffs induced by a (symmetric) PCE can be induced by a symmetric PCE that relies on direct mechanisms and induces truth-telling along its equilibrium paths. Again, the basic logic is that concealing information from the government does not lower a party's worst defection payoff and there is no reason to do it. We state this formally in the next lemma, whose proof is omitted.

Proposition 11 ***(Revelation Principle)*** *Let (σ, ζ, λ) be an SPCE equilibrium in the game with political rents. Then there exists another SPCE $(\hat{\sigma}, \hat{\zeta}, \hat{\lambda})$ in this game such that 1) $\hat{\sigma}$ is direct, 2) $\hat{\lambda}$ is truthful after all histories $(H_t,\ \sigma_t(H_t), h_t)$ and 3) the equilibrium $(\hat{\sigma}, \hat{\zeta}, \hat{\lambda})$ delivers the same payoffs to each w-type agent as the original equilibrium (σ, ζ, λ).*

Given this, we can once again restrict attention to equilibria that rely on direct mechanisms and that induce truth-telling along their equilibrium paths. We again refer to such any such allocation induced by an SPCE as a politically credible allocation (PCA).

Proposition 12 $\{\varphi_t\}_{t=1}^{\infty}$ *is a politically credible allocation in the model with political rents if and only if it satisfies (2) and the political constraints*

$$\forall t, \qquad Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty}))) + X\left(R - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Psi(dw)\right) \geq 0, \tag{30}$$

where $Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty}))) = -\sup_{r''\in[0,R]} r''\left\{\frac{1}{2} + \widehat{\delta}\left[E[\theta]u(R-r'') + \beta\underline{W} - W_t(\{\varphi_t\}_{t=1}^{\infty})\right]\right\} \leq 0$ *and* $X(y) = \frac{1}{2}y$.

Proof: Any PCA must be incentive-compatible. It remains to check (**??**). Let (σ, ζ, λ) denote the PCE that induces $\{\varphi_t\}_{t=1}^{\infty}$. A party can always feasibly defect at t to a mechanism $(\Theta, \widetilde{\varphi}_t)$, where for all (w, θ^t), $\widetilde{\varphi}_t(w,\theta^t) = u(R-r)$, $r \in [0,R]$. If the party defects in this way, it induces a continuation equilibrium with payoff $W_{t+1} \geq \underline{W}$. Since it chooses not to make this defection:

$$\begin{aligned}\frac{R - \int_{\mathbb{R}}\sum_{\Theta^t} C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Psi(dw)}{2} &= r\left\{\frac{1}{2} + \widehat{\delta}\left[E[\theta]u(R-r) + \beta W_{t+1} - W_t(\{\varphi_t\}_{t=1}^{\infty})\right]\right\} \\ &\geq r\left\{\frac{1}{2} + \widehat{\delta}\left[E[\theta]u(R-r) + \beta\underline{W} - W_t(\{\varphi_t\}_{t=1}^{\infty})\right]\right\}.\end{aligned}$$

Since r was an arbitrary element of $[0,R]$, we have (**??**).

Suppose we have an allocation satisfying the conditions in the proposition. We need to construct a PCE that supports it. Our procedure is similar to that in Lemma 4. For all i and t, we set $\sigma_t^i(\{\Theta,\varphi_s\}_{s=1}^{t-1}) = (\Theta,\varphi_t)$ and $\lambda_t(\{\Theta,\varphi_s\}_{s=1}^{t}, w, \theta^t) = 1_{\theta_t}$. If a party defects and wins the election in period t with (M_t', φ_t') we set $\lambda_t(\{\Theta,\varphi_s\}_{s=1}^{t-1}, (M_t',\varphi_t'), w, \theta^t) = 1_{m^*}$, where $m^* \in \arg\max_{m\in M_t'} \varphi_t'(w, \theta^{t-1}, m)$. In periods up to and including the first defection, we set the voting strategy to:

$$\zeta_t(h_t^p; A) = \begin{cases} 1 & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) \geq \delta_t + \xi \\ 0 & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) < \delta_t + \xi. \end{cases} \tag{31}$$

Following a defection in period t, we use the equilibrium constructed in Lemma (4) to set continuation strategies from period $t+1$ onwards. This equilibrium induces the utilitarian payoff $\underline{W}$. The strategies (σ, ζ, λ) constructed in this way induce the desired allocation. Since

$$\frac{R - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi(dw)}{2} \geq \sup_{r'' \in [0,R]} r'' \left\{ \frac{1}{2} + \widehat{\delta} \left[E[\theta] u(R - r'') + \beta \underline{W} - W_t(\{\varphi_t\}_{t=1}^{\infty}) \right] \right\} \geq 0, \tag{32}$$

the equilibrium is resource-feasible along its outcome path.

Given that parties adhere to their strategies and no defection wins the election, incentive-compatibility of the allocation ensures that truth-telling message is optimal. Following defection, subsequent play reverts to the worst equilibrium with utilitarian payoff $\underline{W}$; this continuation equilibrium makes no use of previously revealed information and so it is optimal for agents to send the message that maximizes their current payoff in the period of the defection. Given the constructed message strategy, a party's optimal defection at date t is to a mechanism $(\Theta, \widetilde{\varphi}_t)$, where for all (w, θ^t), $\widetilde{\varphi}_t(w, \theta^t) = u(R - r_t^*)$, where r_t^* solves:

$$W_t^{def} = \sup_{r \in [0,R]} r \left\{ \frac{1}{2} + \widehat{\delta} \left[E[\theta] u(R - r) + \beta \underline{W} - W_t(\{\varphi_t\}_{t=1}^{\infty}) \right] \right\}$$

But, (**??**) no party has an incentive to undertake such a defection (or any sequence of such defections). Agents voting strategies are optimal in the periods up to and including a defection. Finally, the use of the equilibrium with payoff $\underline{W}$ to construct continuation strategies following a defection ensures the optimality of player decisions in the periods following a defection. Hence, (σ, ζ, λ) is a (symmetric) PCE and the allocation is a PCA as desired.

■

Remark In this case, given Assumption 8, $Z_t(U) < 0$ for all $U \in \mathcal{U}_t$ and the political constraints bind in all periods. Moreover, for all t, the political constraints require $X\left(R - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi(dw)\right) > 0$ and so the resource constraints are non-binding. ■

We now check that the political constraints in the rent-seeking model satisfy our earlier Assumptions 3-4.

Lemma 5 *The functions* $\{Z_t\}_{t=1}^{\infty}$ *satisfy Assumptions 2, 3 and 4. X satisfies 5.* $\{Z_t\}_{t=1}^{\infty}$ *and X satisfy Assumption* **??**.

Proof: Define:

$$Z(B) = -\max_{\widehat{r} \in [0,R]} \widehat{r}\left[\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R-\widehat{r}) + \beta \underline{W} - B\}\right]$$

has derivative: $Z'(B) = r^*(B)$, where $r^*(B) = \arg\max_{\widehat{r} \in [0,R]} \widehat{r}\left[\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R-\widehat{r}) + \beta \underline{W} - B\}\right]$. But the first order condition for $r^*(B)$ is:

$$\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R - r^*(B)) + \beta \underline{W} - B\} - \widehat{\delta} r^*(B)(1-\beta)E[\theta]u'(R - r^*(B)) = 0.$$

Hence,

$$\frac{\partial r^*}{\partial B}(B) = \frac{1}{-2(1-\beta)E[\theta]u'(R - r^*(B)) + r^*(B)(1-\beta)E[\theta]u''(R - r^*(B))} < 0.$$

Thus, $Z''(B) = \frac{\partial r^*}{\partial A}(B) < 0$. It follows that $Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty}))) = Z(\int_w \sum_{r=0}^{\infty} \sum_{\Theta^t} \beta^{r-1}\theta_{t+r}\varphi_{t+r}(w, \theta^{t+r})\pi(\theta^{t+r})\Psi(dw))$ is concave, verifying Assumption 3. It is easy to show that the Fréchet derivative of Z_t is given by $\langle \widehat{z}_t(u), \cdot \rangle$, where for all (w,θ^{t-1}), $\widehat{z}_t(w,\theta^{t-1}) = \widehat{\delta} r_t^*(\int_w \sum_{r=0}^{\infty} \sum_{\Theta^t} \beta^{r-1}\theta_{t+r}\varphi_{t+r}(w,\theta^{t+r})\pi(\theta^{t+r})\Psi(dw)) \geq 0$ and

$$r_t^* = \arg\max_{\widehat{r}\in[0,R]} \widehat{r}\left[\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R-\widehat{r}) + \beta\underline{W} - \int_w \sum_{r=0}^{\infty}\sum_{\Theta^t} \beta^{r-1}\theta_{t+r}\varphi_{t+r}(w,\theta^{t+r})\pi(\theta^{t+r})\Psi(dw))\}\right]$$

This verifies Assumptions 2 and 4. That X satisfies 5 is immediate; 6 is readily verified. ■

It follows from Proposition 12 and the above remark that a Pareto-optimal, politically credible allocation in the model with political rents solves

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}} \int_w \sum_{t=1}^{\infty}\sum_{\Theta^t} \beta^{t-1}\theta_t\varphi_t(w,\theta^t)\pi(\theta^t)\Psi(dw) \tag{33}$$

subject to (1), (2) and (30). Given Lemma 5, we can apply Proposition 3 to obtain the following.

Proposition 13 *Let Assumptions (2)-(6) hold. Suppose that $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in (33). Then there is a sequence $\{\mu_t^*\}_{t=1}^{\infty} \in L$ such that $\{\varphi_t^*\}_{t=1}^{\infty}$ solves*

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}\in\Gamma(\{R_t^*\},\Psi)} \int \gamma(w) \sum_{t=1}^{\infty} B_1^t(w) \sum_{\Theta^t} \theta_t\varphi_t(w,\theta^t)\pi^t(\theta^t) \tag{34}$$

where for each t, w, $B_1^t(w) = \beta^{t-1}\prod_{r=1}^{t}(1+\mu_r^* z_r^*(w)) \geq \beta^{t-1}$, $\mu_1\widehat{\delta} r_1^* = \mu_1 z_1^*(w)\gamma(w)$, for all $t>1$, $\mu_t^*\widehat{\delta} r_t^* = \mu_t z_t^*(w)\gamma(w)\prod_{r=1}^{t-1}(1+\mu_r^* z_r^*(w))$,

$$r_t^* = \arg\max_{\widehat{r}\in[0,R]} \widehat{r}\left[\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R-\widehat{r}) + \beta\underline{W} - \int_w \sum_{r=0}^{\infty}\sum_{\Theta^t} \beta^{r-1}\theta_{t+r}\varphi_{t+r}^*(w,\theta^{t+r})\pi(\theta^{t+r})\Psi(dw))\}\right] \tag{35}$$

and $R_t^* = R_t - \int C(\varphi_t^*(w,\theta^t)\pi^t(\theta^t)\Psi(dw)$. Conversely, if $\{\varphi_t^*\}_{t=1}^{\infty}$ solves (34) for some $\{\mu_t^*\}_{t=1}^{\infty} \in L$, $\{R_t^*\}_{t=1}^{\infty}$ with $R_t^* = R_t - \int C(\varphi_t^*(w,\theta^t)\pi^t(\theta^t)\Psi(dw)$, $\{z_t^*\}_{t=1}^{\infty}$, $z_t^*: \mathbb{R}\times\Theta^{t-1} \to \mathbb{R}_+$ with $\sum_{t=1}^{\infty}\beta^{t-1}\prod_{r=1}^{t}(1+\mu_r^* z_r^*) < \infty$, and if

1. *$\mu_1\widehat{\delta} r_1^* = \mu_1 z_1^*(w)\gamma(w)$, for all $t>1$, $\mu_t^*\widehat{\delta} r_t^* = \mu_t z_t^*(w)\gamma(w)\prod_{r=1}^{t-1}(1+\mu_r^* z_r^*(w))$, where r_t^* satisfies (35),*

2. $\{\mu_t^*\} \in \arg\inf_{\{\mu_t\}_{t=1}^{\infty} \in L} \mathcal{L}^*(\{\varphi_t^*\}_{t=1}^{\infty}, \{\mu_t\})$,

then $\{\varphi_t^*\}_{t=1}^{\infty}$ *solves (33).*

This is clearly in the class of problems considered by Farhi and Werning (2006) or Sleet and Yeltekin (2005). Thus, the optimal Pareto problems from the model with political rents can be reformulated as ones in which committed planners use discount factors that exceed those of the agents and must implement particular distributions of utility promises. It follows that political credibility considerations provide microfoundations for the sorts of problems considered by FW and SY and, in particular, for the high planner discount factors assumed in both of their papers.

Conversely, given an optimal planning problem of the sort considered by FW and SY, we can parameterize a model with political rents so that it induces the solution of the planning problem as a (constrained) Pareto optimal SPCA. The FW-SY papers focus on planning problems of the form:

$$\sup_{\{\psi_t\}_{t=1}^{\infty}} \int_w \sum_{t=1}^{\infty} b^{t-1} \sum_{\Theta^t} \theta_t \psi_t(w, \theta^t) \pi^t(\theta^t) \Phi_0(dw) \tag{36}$$

subject to (2),

$$R - \int_w \sum_{\theta^t \in \Theta^t} C(\psi_t(v, \theta^t)) \pi(\theta^t) \Phi_0(dv) \geq 0 \tag{37}$$

and, for all v

$$v = (1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\Theta^t} \psi_t(v, \theta^t) \pi^t(\theta^t)$$

They establish that there are values for $(R^*,\ \Phi_0^*)$ such that the solution to this problem induces a Markov process over promises of utility with Φ_0^* as its invariant distribution. At $(R^*,\ \Phi_0^*)$, the solution to (36) has a sequence of Lagrange multipliers on the resource constraints (37) of the form $\{q^* b^{t-1}\}_{t=1}^{\infty}$. Given these values and this solution, we can recover a Lagrangian of the form (**??**) by setting $B_1^t = b^{t-1}$ and $q_t = q^* b^{t-1}$ and by using Φ_0 and the Lagrange multipliers on the promise-keeping constraints to recover a distribution over Pareto weights Ψ_0. Time invariant values for ϕ and r^* can be obtained using the formulas: $b = (1 + \delta\phi r^*)$ and $q^* = \frac{\phi}{1+\hat{\delta}\phi r^*}$. Finally, we can set the parameters of the sustainability constraint (**??**), R and $\underline{W}'$ to ensure that this constraint holds. If $\underline{W}'$ is the utilitarian payoff attainable from some SPCE, then we have constructed an SPCA that is Pareto optimal (for the distribution Ψ) given the use of the punishment equilibrium with utilitarian payoff $\underline{W}'$.

7 Political revision-proofness

7.1 Politically revision proof equilibria

We now revert to our baseline political economy game, with impatient office motivated parties and political bias distributions satisfying Assumption 8. As we have seen PCE in this game can be reasonably severe; the worst from a utilitarian perspective offers no insurance against shocks at all. This suggests that a political party may seek to coordinate agents onto a new equilibrium with a higher utilitarian payoff in order to improve its current electoral prospects. To this end, a party may propose a mechanism and a continuation equilibrium that describes how the game should be played if the party is elected. How should voters evaluate such a proposal? We will assume that proposals for future play are only accepted if voters believe that they are immune from further revision. If they are not, voters disregard them and assume that future play will adhere to the original equilibrium. This leads us to define an equilibrium as politically revision-proof if all possible proposed revisions to play are themselves vulnerable to further revisions. To formalize this idea, we introduce a renegotiation-proof equilibrium refinement in the spirit of Pearce (1987).

To begin with, we generalize somewhat our notion of a mechanism and a political strategy. Previously, we fixed an initial distribution of types Ψ. An agent's type w identified the subpopulation to which the agent belonged. Agents of different types might receive different weights in social criteria; they could also be treated differently by a mechanism. We now allow political parties to assign types (or "names") to agents in the initial period in lotteries. Thus, the initial type or name distribution is endogenous rather than a parameter. For completeness, we allow parties to rename agents in subsequent periods.[12]

Suppose that at the beginning of each period t, agents are publicly distinguished by a history of names and messages (w^{t-1}, m^{t-1}). Each party i proposes a mechanism, which now consists of a triple $\widehat{S}_t^i = (\rho_t^i, M_t^i, \varphi_t^i)$. Here ρ_t^i maps an an agent's history to a lottery over current names. As before, M_t^i is a message set and φ_t^i is an allocation function (that now depends on histories of the form (w^t, m^t) that include past and current lottery outcomes). The lottery ρ_t^i allows an incumbent government to name or rename agents. We revise all of earlier definitions to accommodate these augmented mechanisms in the obvious way. For example, political strategies map past histories of election-winning mechanisms to an augmented mechanism; voting and message strategies now depend on past histories of these mechanisms, past (and in the case of message strategies current) lottery outcomes as well as the relevant message histories and political preference parameters. Our previous definition and analysis of PCE go through with only minor modification.

[12] This latter ability, although it is not used on the equilibrium path, ensures that equilibria are recursive and that there is no difference between what a party can do in period 1 relative to later periods.

Political revisions and political revision-proofness We formalize the idea of a political revision. Suppose that when confronted with a continuation equilibrium $(\sigma, \zeta, \lambda | H_t)$, party i has the option of conforming to the equilibrium or defecting and proposing a current mechanism S_1^i *and* a new continuation equilibrium $(\sigma', \zeta', \lambda' | S_1^i)$ that describes how everyone will play if party i wins the election.[13] The defecting party's manifesto might say: "If you vote for us, we will forget the past and implement S_1^i. Furthermore, a vote for us will be a vote to coordinate play on the continuation equilibrium $(\sigma', \zeta', \lambda' | S_1^i)$." To persuade agents to coordinate in this way and, hence, vote for the party in the first place, party i must at a minimum persuade them that there will not be further revisions to this equilibrium. This motivates the following definition.

Definition 7 *A triple (σ, ζ, λ) is a* ***politically revision-proof equilibria*** *(PRPE) if 1) (σ, ζ, λ) is a PCE and 2) there exists no date t, history $H_t = \{S_s\}_{s=1}^{t-1}$ and alternative equilibrium $(\sigma', \zeta', \lambda')$ such that for all s and H_s*

$$W_s(\sigma', \zeta', \lambda' | H_s) \geq y > W_t(\sigma, \zeta, \lambda | H_t).$$

An allocation is politically revision-proof if it is the outcome of a politically revision-proof equilibrium.

The intuition behind this refinement is as follows. For agents to accept a political revision after some history, they must be persuaded that there will be no attempt to revert to the original equilibrium at some point in the future. To persuade agents of this, the revision must deliver continuation utilitarian payoffs that exceed the payoff from the original continuation equilibrium. In this case, any subsequent proposal to revert to the old continuation equilibrium will strictly lower a party's election probability. Agents may reasonably suppose that no such proposal

[13]Without loss of generality, the new mechanism and continuation equilibrium does not condition on past messages and lottery outcomes.

will be made, making the revision politically viable. An equilibrium is politically revision-proof if it admits no such politically viable revisions. Our next lemma characterizes and provides an alternative interpretation of the refinement.

Lemma 6 *Let E denote the set of politically credible equilibria. $(\sigma, \zeta, \lambda) \in E$ is politically revision-proof if and only if it solves:*

$$\widehat{W} = \sup_{(\sigma',\zeta',\lambda')\in E} \inf_{H_t} W_t(\sigma', \zeta', \lambda'|H_t). \tag{38}$$

Proof: Suppose that $(\sigma, \zeta, \lambda) \in E$ satisfies the condition in the lemma. Consider a planner whose defection proposal at some H_t consists of S_t and the continuation equilibrium $(\sigma', \zeta', \lambda'|H_t, S_t)$. The pair $((H_t, S_t), (\sigma', \zeta', \lambda'|H_t, S_t))$ induces a family of continuation utilitarian payoffs. It is easy to check that this family can also be induced by some $(\sigma'', \zeta'', \lambda'') \in E$. It follows that either $W_t(\sigma', \zeta', \lambda'|H_t, S_t) \leq \widehat{W} \leq W_t(\sigma, \zeta, \lambda|H_t)$ or for some $H^{s-1} \succeq (H^{t-1}, S_t)$, $W_s(\sigma', \zeta', \lambda'|H_s) \leq \widehat{W} \leq W_t(\sigma, \zeta, \lambda|H_t)$. Thus, (σ, ζ, λ) is politically revision-proof.

For the converse, suppose that (σ, ζ, λ) is politically revision-proof, but does not solve (38). Then there is some history H_t and some $\varepsilon > 0$ such that $W_t(\sigma, \zeta, \lambda|H_t) < \widehat{W} - \varepsilon$. But then, we can find an alternative equilibrium $(\sigma', \zeta', \lambda')$ such that for all s, $W_s(\sigma', \zeta', \lambda'|H_s) \geq \widehat{W} - \varepsilon > W_t(\sigma, \zeta, \lambda|H_t)$. This contradicts the political revision-proofness of (σ, ζ, λ). ■

It follows from this lemma that PRPE maximize the worst continuation utilitarian payoff that might conceivably occur. In this sense, they rely on "mild punishments" for a political defection. Our next lemma shows us that given any politically revision-proof equilibrium, we can construct a simpler equilibrium that uses direct mechanisms, lotteries over names only in the initial period, keeps promises and induces truth-telling. Moreover, this equilibrium induces the same continuation utilitarian payoffs along the outcome path as the original equilibrium.

Lemma 7 *(Simplifying politically revision-proof equilibria) Let* (σ, ζ, λ) *denote a PRPE. There exists a PRPE* $(\sigma', \zeta', \lambda')$ *satisfying:*

1. σ' *is symmetric (i.e.* $\sigma^{i\prime} = \sigma^{j\prime}$);

2. σ' *is direct along the outcome path of the equilibrium;*

3. λ' *is truthful along the outcome path of the equilibrium;*

4. *The allocation functions induced by* σ' *along the outcome path of the equilibrium condition only upon the lottery outcome in the initial period;*

5. $(\sigma', \lambda', \zeta')$ *"keeps promises" in the sense that if an agent is assigned the name* w *in the initial lottery* ρ_1' *then the continuation equilibrium delivers her an expected lifetime utility of* w;

6. $(\sigma', \lambda', \zeta')$ *induces the same continuation utilitarian payoffs along its outcome path as* (σ, ζ, λ).

The proof is straightforward, but long and we omit it. Equilibria satisfying conditions (1)-(5) of the above lemma are significantly more tractable than general PRPE. In particular, such PRPEs induce an initial name distribution-allocation pair $(\rho_1, \{\varphi_t\}_{t=1}^{\infty})$ along their equilibrium paths. We call such a pair a politically revision-proof allocation or PRPA.

Our goal is to show that PRPA solves a virtual planning problem with a patient planner. In fact, as we argue below, subject to a technical qualification, *all* PRPA solve the problem of a Rawlsian planner who has a unit discount factor[14]. Thus, by refining our notion of political equilibrium and, implicitly, tightening the political restrictions on allocations, we reinforce our earlier result that political systems populated with short-lived politicians behave like (excessively) patient planners.

Relative to previous sections our method of proof is quite different. Rather than obtaining explicit political constraints as equilibrium restrictions and then reformulating the implied Lagrangian, we proceed by directly comparing PRPA with allocations that are optimal for the Rawlsian planner.

7.2 Rawlsian planning problems

The sequence Rawlsian problem More formally, the Rawlsian planner solves:

$$W^{\text{rawls}} = \sup_{\{\Psi_1, \{\varphi_t\}_{t=1}^{\infty}\}} \liminf_{T \to \infty} \left\{ \frac{1}{T} \sum_{t=1}^{T} \int_W \sum_{s=0}^{\infty} \sum_{\Theta^{t+s}} \beta^s \theta_{t+s} \varphi_{t+s}(w, \theta^{t+s}) \pi(\theta^{t+s}) \Psi_1(dw) \right\} \tag{39}$$

subject to the resource, incentive-compatibility and promise-keeping constraint, for all $w \in W$,

$$w = (1-\beta) \sum_{s=0}^{\infty} \beta^s \theta_{t+s} \varphi_{t+s}(w, \theta^{t+s}) \pi(\theta^{t+s}). \tag{40}$$

Stationary Rawlsian problem Closely, related to this Rawlsian problem is the following auxiliary stationary cost problem. In this a planner chooses an invariant measure over agent utility promises Ψ and a pair of measurable policy functions $\varphi : W \times \Theta \to D$ and $w' : W \times \Theta \to W$. The first of the policy functions gives an agent's current utility as a function of its current promise and shock, the second gives an agent's continuation promise as a function of these variables. The policy functions are chosen to satisfy recursive versions of the promise keeping

[14] The label "Rawlsian planner" stems from viewing an infinitely lived agent as dynasty of single period lived, partially altruistic generations. The Rawlsian planner uses intergenerational weights. See Phelan (2006).

and incentive compatibility constraints. The function w' and the probability distribution over shocks π imply a Markov process for utility promises. Ψ is required to be an invariant measure for this process. Finally, the mean utility implied by Ψ is required to exceed an exogenously given utility amount W. Formally, the stationary cost problem at W is given by:

$$\textbf{Stationary Cost Problem} \qquad \inf_{\{\Psi,\varphi,w'\}} \int_{W\times\Theta} C(\varphi(w,\theta))\Psi(dw)\pi(\theta) \tag{41}$$

subject to the recursive promise-keeping condition:

$$\forall w \in W, \qquad w = \sum_{\Theta}[(1-\beta)\theta\varphi(w,\theta) + \beta w'(w,\theta)]\pi(\theta), \tag{42}$$

the recursive incentive-compatibility condition:

$$\forall w \in W, k, j \in \{1,\cdots,K\}, \qquad (1-\beta)\hat{\theta}_k\varphi(w,\hat{\theta}_k) + \beta w'(w,\hat{\theta}_k) \geq (1-\beta)\hat{\theta}_k\varphi(w,\hat{\theta}_j) + \beta w'(w,\hat{\theta}_j), \tag{43}$$

the steady state condition:

$$\forall B \in \mathcal{B}(W), \qquad \Psi(W) = \int_{W\times\Theta} 1_{\{w'(w,\theta)\in B\}}\Psi(dw)\pi(\theta), \tag{44}$$

and the recursive aggregate utility condition:

$$W \leq \int_{W} w\Psi(dw). \tag{45}$$

Any triple (Ψ, φ, w') induces an allocation. It is routine to check that this allocation satisfies the (non-recursive) incentive-compatibility constraints. Proposition 14 below makes use of this stationary Rawlsian problem.

7.3 Political revision-proofness and the Rawlsian planner

We now turn to the main result of this section and show, subject to a technical qualification, that all PRPA solve the problem of a Rawlsian planner. Conversely, any solution to this problem that delivers a constant payoff at all dates (i.e. is payoff-stationary) is a PRPA. The proof works by showing that all PRPA are feasible for the Rawlsian planner and, hence, any solution to the Rawlsian planner's problem must deliver a weakly higher (Rawlsian) payoff. Working in the reverse direction, a name distribution and a politically credible allocation (i.e. an allocation induced by a PCE) is obtained that attains the Rawlsian payoff and gives a constant payoff after all aggregate histories. By (38), any PRPA must give a weakly higher payoff after all histories than this allocation and so its Rawlsian payoff must weakly exceed that attained by the Rawlsian planner. Combining the arguments, gives the desired result.

Proposition 14 *Suppose the stationary cost solution has a solution at W^{rawls}, then*

1. *Any PRPE (σ, ζ, λ) induces an allocation that solves the Rawlsian planner's problem*

2. *Any payoff stationary solution to the Rawlsian planner's problem is a politically revision-proof allocation.*

Proof: Any politically revision-proof equilibrium (σ, ζ, λ) induces an name distribution-allocation pair that is resource-feasible. In light of Lemma 7, we may, without loss of generality, assume that this pair can be represented by $(\Psi_1, \{\varphi_t\}_{t=1}^{\infty})$, where each $\varphi_t : W \times \Theta \to D$. Using Lemma 7, we may also assume, again without loss of generality, that the allocation is incentive-compatible and satisfies promise-keeping. Hence, it is feasible for the Rawlsian problem and $W^{\text{rawls}} \geq W^r(\Psi_1, \{\varphi_t\}_{t=1}^{\infty})$, where $W^r(\Psi_1, \{\varphi_t'\}_{t=1}^{\infty})$ is the Rawlsian payoff implied by the pair $(\Psi_1, \{\varphi_t\}_{t=1}^{\infty})$.

We now seek to show the reverse inequality. To do so, we use two arguments in Phelan (2006). First, by Lemma 1 of Phelan (2006), if $(\Psi_1, \{\varphi_t\}_{t=1}^{\infty})$ solves the Rawlsian problem, then it also solves the following cost problem at aggregate utility amount W^{rawls} and does so with cost objective R:

$$\textbf{Cost Problem} \qquad J^{\text{dual}}(W) = \inf_{\{\Psi_1, \{\psi_t\}_{t=1}^{\infty}\}} \left[\limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \int_W \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi_1(dw) \right] \tag{46}$$

subject to the incentive-compatibility (2) and promise-keeping (40) constraints, and the aggregate utility constraint:

$$W \le \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \int_W \sum_{s=0}^{\infty} \beta^s \sum_{\Theta^{t+s}} \theta_{t+s}\varphi_{t+s}(w, \theta^{t+s})\pi^{t+s}(\theta^{t+s})\Psi_1(dw). \tag{47}$$

Second, we can use the construction in Lemma 2, Phelan (2006), to obtain a stationary distribution-allocation pair from $(\Psi_1, \{\varphi_t\}_{t=1}^{\infty})$ that 1) attains a per period aggregate cost less than $R + \varepsilon$ for arbitrary $\varepsilon > 0$, 2) has invariant measure Ψ_1 and 3) delivers a payoff of at least W^{rawls} to the Rawlsian planner. It follows that if a solution $(\Psi_1^*, \varphi^*, w'^*)$ exists to the stationary cost problem at W^{rawls}, then it has a cost of less than or equal to R. Hence, the pair induced by this solution is resource-feasible. It is also incentive-compatible, keeps promises and attains a constant planner payoff of at least W^{rawls}. Denote this pair by $(\Psi_1^*, \{\varphi_t^*\}_{t=1}^{\infty})$.

We can now construct a PCE that implements $(\Psi_1^*, \{\varphi_t^*\}_{t=1}^{\infty})$. In this the equilibrium political strategies are such that both parties sequentially propose $(\rho_t^*, \Theta, \varphi_t^*)$, where $\rho_1^* = \Psi_1^*$ and ρ_t^*, $t > 1$, is arbitrary, i.e. $\sigma_t^i(\{\rho_s^*, \Theta, \varphi_s^*\}_{s=1}^{t-1}) = (\rho_t^*, \Theta, \varphi_t^*)$. For other histories, recursively construct σ_t^i as follows. Given a history $\{\rho_s, M_s, \varphi_s\}_{s=1}^{t-1}$, let r denote the date of the last defection from σ. Set $\sigma_t^i(\{\rho_s, M_s, \varphi_s\}_{s=1}^{t}) = (\rho_{t-r}^*, \Theta, \varphi_{t-r}^*)$. Set the message strategies of agents as follows. If r denotes the date of the last election-winning defection and $r < t$, $\lambda_t(\{\rho_s, M_s, \varphi_s\}_{s=1}^{t}, w, m^{t-1}) = 1_\theta$; if $r = t$, set $\lambda_t(\{\rho_s, M_s, \varphi_s\}_{s=1}^{t}, w, m^{t-1}) = 1_{m^*}$, $m^* \in \arg\max_{m\in M_t} \varphi_t(w, m^{t-1}, m)$.

Set the voting strategy according to:

$$\zeta_t(h_t^p; A) = \begin{cases} 1 & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, w, m^{t-1}|\sigma, \zeta, \lambda) \geq \delta_t + \xi \\ 0 & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, w, m^{t-1}|\sigma, \zeta, \lambda) < \delta_t + \xi \end{cases} \tag{48}$$

This strategy is clearly optimal given the message and policy strategies. Moreover, it ensures that choosing a current mechanism to maximize the probability of winning is equivalent to choosing such a mechanism to maximize the continuation utilitarian payoff of agents. Along the outcome path of this equilibrium, the pair $(\Psi_1^*, \{\varphi_t^*\}_{t=1}^\infty)$ with payoff $W^{\text{rawls}} > \underline{W}$ is induced. If a party defects and wins at t, the policy strategies call for the parties to restart this pair by successively proposing $(\rho_s^*, \Theta, \varphi_s^*)$ in periods $t+s$. Agents are truthful provided a party has not defected and won the election in the current period, in which case, they give the message that maximizes their current payoff. Given this behavior of agents, the continuation utilitarian payoff and, hence, the election probability of a defecting party, is maximized by a defection to a no insurance mechanism. But since $W^{\text{rawls}} > \underline{W}$, even this defection lowers the party's election winning probability. The party will not wish to make it. Since the stationary solution to the Rawlsian problem is incentive-compatible, it is optimal for agents to be truthful during its implementation. Since the political strategy described above calls for parties to ignore past messages and begin reimplementing the solution to the Rawlsian problem following the election victory of a defecting party, it is optimal for agents to send messages that maximize their current payoff in the aftermath of a defection. Thus, these strategies constitute a PCE. Denote it $(\sigma^r, \zeta^r, \lambda^r)$.

Recall that any PRPE (σ, ζ, λ) solves $\sup_{(\sigma', \zeta', \lambda') \in E} \inf_{H_t} W_t(\sigma', \zeta', \lambda'|H_t)$. Now, the equilibrium derived above, $(\sigma^r, \zeta^r, \lambda^r)$, delivers the payoff W^{rawls} after all histories. Hence, any PRPE (σ, ζ, λ) must deliver a continuation utilitarian payoff weakly in excess of W^{rawls}, and so its induced pair $(\Psi_1, \{\varphi_t\}_{t=1}^\infty)$ has a Rawlsian payoff weakly in excess of W^{rawls}, i.e. $W^r(\Psi_1, \{\varphi_t\}_{t=1}^{\infty}) \geq W^{\text{rawls}}$. Combining this inequality with that derived earlier, we conclude that if (σ, ζ, λ) is politically revision-proof then its induced name distribution-allocation pair satisfies $W^r(\Psi_1, \{\varphi_t\}_{t=1}^{\infty}) = W^{\text{rawls}}$ and this pair solves the Rawlsian planner's problem.

Suppose that $(\Psi_1, \{\varphi_t\}_{t=1}^{\infty})$ is a payoff stationary pair that solves the Rawlsian problem. Then by an argument similar to that given above, we can construct a PCE (σ, ζ, λ) that induces this pair. This continuation equilibrium attains the Rawlsian payoff W^{rawls} after each history. If this continuation equilibrium is not politically revision proof, then there exists another pair $(\sigma', \zeta', \lambda')$ satisfying $\min_{H_t} W_t(\sigma', \zeta', \lambda' | H_t) > W^{\text{rawls}} + \varepsilon$, some $\varepsilon > 0$. But then, using Lemma 7, the distribution-allocation pair induced by this continuation equilibrium is feasible for and solves the Rawlsian problem with a Rawlsian payoff strictly greater than W^{rawls}. This is a contradiction. ■

8 Appendix: Proofs

Proof of Proposition 1 By assumption, the constraint functions $\{Z_t \circ U + X\}$ map from $\Gamma(\{R_t\}, \Psi)$ to a subset of ℓ_∞ with an interior point. Since each Z_t is concave and X is concave, $\{\{\varphi_t\}_{t=1}^\infty | \{Z_t \circ U + X\}(\{\varphi_t\}_{t=1}^\infty) \geq 0\}$ is convex. $\Gamma(\{R_t\}, \Psi)$ is convex and $\int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w, \cdot)\}_{t=1}^\infty) \Psi(dw)$ is concave. Consequently, by Luenberger Theorem 1, p.217, there is an element $\mu^* \in \ell'_\infty$ such that

$$\begin{aligned} U^* \quad = \quad & \sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(R,\Psi)} (1-\beta) \int \gamma(w) \sum_{t=1}^\infty \beta^{t-1} \sum_{\Theta^t} \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \\ & + \left\langle \mu^*, \left\{ Z_t \left(\sum_{\Theta^{t+r}} \theta_{t+r} \varphi_{t+r}(\cdot, \cdot, \theta^r) \pi^r(\theta^r) \right) + X \left(R_t - \int C(\varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \right) \right\}_{t=1}^\infty \right\rangle \end{aligned} \tag{49}$$

If $\{\varphi_t^*\}_{t=1}^\infty$ attains the supremum in (5), then the Lagrangian on the right hand side of (49) has a saddle point at μ^*, $\{\varphi_t^*\}_{t=1}^\infty$. That μ^* can be represented by an element in M follows from Rustichini Corollary 5.6. Thus, the saddle point condition (8) holds.

The converse follows from Luenberger Theorem 2, p. 221 and Rustichini Corollary, 5.6. ■

Proof of Proposition 2 For the first part, note that if $\{\varphi_t^*\}_{t=1}^\infty$ attains the supremum in (5), then by the previous proposition, there is a sequence $\{\mu_t^*\}_{t=1}^\infty \in L$ such that $\{\varphi_t^*\}_{t=1}^\infty \in \arg\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)} \mathcal{L}(\{\varphi_t\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty)$. Let $\{\varphi'_t\}_{t=1}^\infty$ be an element of $\Gamma(\{R_t\}, \Psi)$, then $\mathcal{L}(\{\varphi'_t\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty) \leq \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty)$. Let $\{\widehat{z}_t^*\}_{t=1}^\infty$ be such that $\partial Z_t(\{\varphi_t^*\}_{t=1}^\infty; \cdot) = \langle \widehat{z}_t^*, \cdot \rangle$. By Assumption 2, $\mathcal{L}^*(\cdot; \{\mu_t^*\}_{t=1}^\infty, \{\widehat{z}_t^*\})$ is well defined. Suppose $\mathcal{L}^*(\{\varphi'_t\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{\widehat{z}_t^*\})$

$> \mathcal{L}^*(\{\varphi_t^*\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{\widehat{z}_t^*\})$. Then combining the inequalities, we have

$$\begin{aligned} & \sum_{t=1}^\infty \beta^{t-1}\mu_t^* \left[Z_t\left(U(\{\varphi_{t+r-1}^*|\cdot,\cdot)\right) - Z_t\left(U(\{\varphi'_{t+r-1}|\cdot,\cdot)\right)\right] \\ > & \sum_{t=1}^\infty \beta^{t-1}\mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w,\theta^{t-1})[U(\{\varphi_{t+r-1}^*|w,\theta^{t-1}) - U(\{\varphi'_{t+r-1}|w,\theta^{t-1})]\pi^t(\theta^t)\Psi(dw). \end{aligned}$$

But by concavity of Z_t, each $\int \widehat{z}_t^*(w,\theta^{t-1})[U(\{\varphi_{t+r-1}^*|w,\theta^{t-1}) - U(\{\varphi'_{t+r-1}|w,\theta^{t-1})]\pi^t(\theta^t)\Psi(dw) \geq Z_t\left(U(\{\varphi_{t+r-1}^*|\cdot,\cdot)\right)$ $- Z_t\left(U(\{\varphi'_{t+r-1}|\cdot,\cdot)\right)$, a contradiction.

For the converse, suppose $\{\varphi_t^*\}_{t=1}^\infty \in \sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)} \mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\}, \{\widehat{z}_t^*\})$, for some $\{\mu_t^*\} \in L$ and for $\{\widehat{z}_t^*\}$ satisfying (1) in the proposition. If for some $\{\varphi'_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)$, then $\mathcal{L}(\{\varphi'_t\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty) > \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty)$ and the concavity of each Z_t again implies a contradiction. Hence, $\{\varphi_t^*\}_{t=1}^\infty \in \sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)} \mathcal{L}(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\})$. It then follows from (2) in the proposition that $\{\varphi_t^*\}_{t=1}^\infty$ and $\{\mu_t^*\}$ satisfy the saddle point condition (8) and, hence, $\{\varphi_t^*\}_{t=1}^\infty$ is optimal in (5). ■

Proof of Lemma 1 Let $\{\varphi_t\}_{t=1}^\infty$ be an allocation in $\Gamma(\{R_t\},\Psi)$. Fix $T \in \{1,2,\ldots\}$ and for each history θ^T and $t \in \{1,\ldots,T\}$, set $\varphi'_t(w,\theta^T) := \varphi_t(w,\theta^t(\theta^T))$, where $\theta^t(\theta^T)$ is a t-period sub-history of θ^T. We then have

$$\begin{aligned} \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^\infty)\Psi(dw) &= \int_{\mathbb{R}} \gamma(w) \sum_{t=1}^\infty \beta^{t-1} \sum_{\Theta^t} \theta_t \varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) \\ &= \int_{\mathbb{R}} \gamma(w) \sum_{\Theta^T} \left[\sum_{t=1}^T \beta^{t-1}\theta_t(\theta^T)\varphi'_t(w,\theta^T) + \beta^T U(w,\theta^T)\right]\pi^T(\theta^T)\Psi(dw) \end{aligned}$$

Also,

$$\sum_{t=1}^{T}\beta^{t-1}\mu_t\int_{\mathbb{R}}\gamma(w)\sum_{\theta^{t-1}} z_t(w,\theta^{t-1})\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}))U(\{\varphi_{t+r-1}|w,\theta^{t-1})\pi^{t-1}(\theta^{t-1})\Psi(dw).$$

$$\begin{aligned}
= \quad & \int_{\mathbb{R}}\gamma(w)\sum_{\Theta^T}\sum_{t=1}^{T}\mu_t z_t(w,\theta^{t-1}(\theta^T))\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T))) \\
& \qquad \times\left\{\sum_{r=1}^{T-t+1}\beta^{t+r-1}\theta_{t+r-1}(\theta^T)\varphi'_{t+r-1}(w,\theta^T)+\beta^T U(w,\theta^T)\right\}\pi^T(\theta^T)\Psi(dw).
\end{aligned}$$

Combining these terms (using the linearity of integrals) gives:

$$\begin{aligned}
& \int_{\mathbb{R}}\gamma(w)U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty})\Psi(dw) \\
& +\sum_{t=1}^{T}\beta^{t-1}\mu_t\int_{\mathbb{R}}\gamma(w)\sum_{\theta^{t-1}} z_t(w,\theta^{t-1})\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}))U(\{\varphi_{t+r-1}|w,\theta^{t-1}))\pi^{t-1}(\theta^{t-1})\Psi(dw) \\
= \quad & \int_{\mathbb{R}}\gamma(w)\sum_{\Theta^T}\left\{\sum_{t=1}^{T}\beta^{t-1}\theta_t(\theta^T)\varphi'_t(w,\theta^T)+\beta^T U(w,\theta^T)\right. \\
& +\sum_{t=1}^{T}\left[\mu_t z_t(w,\theta^{t-1}(\theta^T))\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\right. \\
& \left.\left.\times\left\{\sum_{r=1}^{T-t+1}\beta^{t+r-1}\theta_{t+r-1}(\theta^T)\varphi'_{t+r-1}(w,\theta^T)+\beta^T U(w,\theta^T)\right\}\right]\right\}\pi^T(\theta^T)\Psi(dw).
\end{aligned}$$

Now, recall Abel's Lemma (see, for example, Theorem 3.41 p.70, Rudin (1976)):

$$\sum_{t=1}^{T}a_t b_t=\sum_{t=1}^{T-1}\left(\sum_{r=1}^{t}a_s\right)(b_t-b_{t+1})+\left(\sum_{s=1}^{T}a_r\right)b_T. \tag{50}$$

Fix (w,θ^T) and let $b_t=\sum_{r=1}^{T-t+1}\beta^{t+r-1}\theta_{t+r-1}(\theta^T)\varphi'_{t+r-1}(w,\theta^T)+\beta^T U(w,\theta^T)$ and $a_1=\gamma(w)+\mu_1 z_1(w)$ and $a_t=\mu_t z_t(w,\theta^{t-1}(\theta^T))\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))$, then Abel's Lemma implies

$$
\begin{aligned}
& \gamma(w)\sum_{t=1}^{T}\beta^{t-1}\theta_t(\theta^T)\varphi'_t(w,\theta^T)+\beta^T U(w,\theta^T) \\
& +\sum_{t=1}^{T}\mu_t z_t(w,\theta^{t-1}(\theta^T))\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\left\{\sum_{r=1}^{T-t+1}\beta^{t+r-1}\theta_{t+r-1}(\theta^T)\varphi'_{t+r-1}(w,\theta^T)+\beta^T U(w,\theta^T)\right\} \\
= \; & \gamma(w)\sum_{t=1}^{T}\beta^t\left(\prod_{r=1}^{t}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\right)\theta_t(\theta^T)\varphi'_t(w,\theta^T)+\beta^T\left(\prod_{r=1}^{T}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\right)U(w,\theta^T).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& \int_{\mathbb{R}}\gamma(w)U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty})\Psi(dw) \\
& +\sum_{t=1}^{T}\beta^{t-1}\int_{\mathbb{R}}\gamma(w)\sum_{\theta^{t-1}}\mu_t z_t(w,\theta^{t-1})\prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1}))U(\{\varphi_{t+r-1}|w,\theta^{t-1})\pi^{t-1}(\theta^{t-1})\Psi(dw) \\
= \; & \int_{\mathbb{R}}\gamma(w)\sum_{\Theta^T}\sum_{t=1}^{T}\left\{\beta^t\left(\prod_{r=1}^{t}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\right)\theta_t(\theta^T)\varphi'_t(w,\theta^T)\right. \\
& \left.+\beta^T\left(\prod_{r=1}^{T}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\right)U(w,\theta^T)\right\}\pi^T(\theta^T)\Psi(dw) \\
= \; & \int_{\mathbb{R}}\gamma(w)\sum_{t=1}^{T}\beta^t\sum_{\Theta^t}\beta^t\left(\prod_{r=1}^{t}(1+\mu_r z_r(w,\theta^{r-1}(\theta^t)))\right)\theta_t\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) \\
& +\int_{\mathbb{R}}\gamma(w)\sum_{\Theta^T}\beta^T\left(\prod_{r=1}^{T}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\right)U(w,\theta^T)\pi^T(\theta^T)\Psi(dw)
\end{aligned}
$$

Now, given that $\sum_{t=1}^{\infty}\int_{\mathbb{R}}\gamma(w)\sum_{\theta^{t-1}}\beta^{t-1}\left(\prod_{r=1}^{t}(1+\mu_r z_r(w,\theta^{r-1}))\right)\pi^{t-1}(\theta^{t-1})\Psi(dw)<\infty$, $\lim_{T\to\infty}\int_{\mathbb{R}}\gamma(w)\sum_{\Theta^T}\beta^T\left(\prod_{r=1}^{T}(1+\mu_r z_r(w,\theta^{r-1}(\theta^T)))\right)\pi^T(\theta^T)\,\Psi(dw)=0$, using the non-negativity of each z_r and the boundedness of the allocation functions $\{\varphi_t\}_{t=1}^{\infty}$ and the dominated convergence theorem, we have

$$\begin{aligned}
&\int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty})\Psi(dw) \\
&+\sum_{t=1}^{\infty} \beta^{t-1}\mu_t \int_{\mathbb{R}} \gamma(w) \sum_{\theta^{t-1}} z_t(w,\theta^{t-1}) \prod_{r=1}^{t-1}(1+\mu_r z_r(w,\theta^{r-1})) U(\{\varphi_{t+r-1}|w,\theta^{t-1})\pi^{t-1}(\theta^{t-1})\Psi(dw) \\
= \;& \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty})\Psi(dw) + \lim_{T\to\infty} \sum_{t=1}^{\infty} \beta^{t-1}\mu_t \int_{\mathbb{R}} \sum_{\theta^{t-1}} z_t(w,\theta^{t-1}) U(\{\varphi_{t+r-1}|w,\theta^{t-1})\pi^{t-1}(\theta^{t-1})\Psi(dw) \\
= \;& \lim_{T\to\infty} \int_{\mathbb{R}} \gamma(w) \sum_{t=1}^{T} \beta^t \sum_{\Theta^t} \beta^t \left(\prod_{r=1}^{t} (1+\mu_r z_r(w,\theta^{r-1}(\theta^t))) \right) \theta_t \varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) \\
&+ \lim_{T\to\infty} \int_{\mathbb{R}} \gamma(w) \sum_{\Theta^T} \beta^T \left(\prod_{r=1}^{T} (1+\mu_r z_r(w,\theta^{r-1}(\theta^T))) \right) U(w,\theta^T)\pi^T(\theta^T)\Psi(dw) \\
= \;& \int_{\mathbb{R}} \gamma(w) \sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^T \left(\prod_{r=1}^{T} (1+\mu_r z_r(w,\theta^{r-1}(\theta^t))) \right) \beta^t \theta_t \varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw).
\end{aligned}$$

Since $[0,R]$ is compact, X continuous and $\{\mu_t\} \in M$, $\sum_{t=1}^{\infty} \beta^{t-1}\mu_t X(R_t - \int C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Psi(dw)) < \infty$, adding this term to both sides of the previous equality gives the desired result. ■

Proof of Proposition ?? From the Proposition 2, if $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in (5), then there is a pair of sequences $\{\mu_t^*\}_{t=1}^{\infty} \in L$ and $\{\widehat{z}_t^*\}_{t=1}^{\infty}$, $\widehat{z}_t^* : \mathbb{R} \times \Theta^{t-1} \to \mathbb{R}_+$ such that $\{\varphi_t^*\}_{t=1}^{\infty}$ solves:

$$\sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\},\Psi)} \mathcal{L}^*(\{\varphi_t\}_{t=1}^{\infty}; \{\mu_t^*\}, \{\widehat{z}_t^*\}), \tag{51}$$

where

$$\begin{aligned}
\mathcal{L}^*(\{\varphi_t\}_{t=1}^{\infty}; \{\mu_t^*\}, \{\widehat{z}_t^*\}) \;=\; & \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty})\Psi(dw) \\
&+ \sum_{t=1}^{\infty} \beta^{t-1}\mu_t^* \Bigg[\int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w,\theta^{t-1}) U(\{\varphi_{t+r-1}|w,\theta^{t-1})\pi^{t-1}(\theta^{t-1})\Psi(dw) \\
&\qquad + X\left(R_t - \int C(\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw)) \right) \Bigg].
\end{aligned}$$

By Assumption 2 and $\{\mu_t^*\}_{t=1}^\infty \in L$, for all (w, θ^{t-1}), $\sum_{t=1}^\infty \beta^t \mu_t^* \widehat{z}_t^*(w, \theta^{t-1}) < \overline{z} \sum_{t=1}^\infty \beta^t \mu_t^*$. Thus, defining $\{z_t^*\}$ as in the main text, $\sum_{t=1}^\infty \int_{\mathbb{R}} \gamma(w) \sum_{\theta^{t-1}} \beta^{t-1} \left(\prod_{r=1}^t (1 + \mu_r^* z_r^*(w, \theta^{r-1}))\right) \pi^{t-1}(\theta^{t-1}) \Psi(dw) < \infty$. Set $R_t^* = \int C(\varphi_t^*(w, \theta^t)\pi^t(\theta^t)\Psi(dw)$. Consider the problem

$$\begin{aligned}
\overline{V} \quad = \quad & \sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t^*\}, \Psi)} \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w, \cdot)\}_{t=1}^\infty) \Psi(dw) \\
& + \sum_{t=1}^\infty \beta^{t-1} \mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1}|w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw)
\end{aligned} \tag{52}$$

$\{\varphi_t^*\}_{t=1}^\infty$ is feasible for this problem and so $\overline{V} \geq \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t^*(w, \cdot)\}_{t=1}^\infty) \Psi(dw) + \sum_{t=1}^\infty \beta^{t-1} \mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r}^*$ If $\overline{V} > \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t^*(w, \cdot)\}_{t=1}^\infty) \Psi(dw) + \sum_{t=1}^\infty \beta^{t-1} \mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1}^*|w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw)$, then there exists some allocation $\{\varphi_t'\}_{t=1}^\infty \in \Gamma(\{R_t^*\}, \Psi)$ and some small $\varepsilon > 0$ such that

$$\begin{aligned}
& \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t'(w, \cdot)\}_{t=1}^\infty) \Psi(dw) \\
& + \sum_{t=1}^\infty \beta^{t-1} \mu_t^* \left[\int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1}^*|w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw) \right. \\
& \qquad \left. + X\left(R_t - \int C(\varphi_t'(w, \theta^t)\pi^t(\theta^t)\Psi(dw)\right) \right] \\
\geq \; & \overline{V} - \varepsilon + \sum_{t=1}^\infty \beta^{t-1} \mu_t^* X \left(R_t - \int C(\varphi_t'(w, \theta^t)\pi^t(\theta^t)\Psi(dw)\right) \\
> \; & \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t^*(w, \cdot)\}_{t=1}^\infty) \Psi(dw) \\
& + \sum_{t=1}^\infty \beta^{t-1} \mu_t^* \left[\int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1}^*|w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw) \right. \\
& \qquad \left. + X\left(R_t - \int C(\varphi_t^*(w, \theta^t)\pi^t(\theta^t)\Psi(dw)\right) \right].
\end{aligned}$$

Since $\{\varphi_t'\}_{t=1}^\infty \in \Gamma(\{R_t^*\}, \Psi) \subset \Gamma(\{R_t\}, \Psi)$, this contradicts the optimality of $\{\varphi_t^*\}_{t=1}^\infty$ in (**??**). Hence, $\{\varphi_t^*\}_{t=1}^\infty$ is optimal in (52). Applying Lemma 1 allows us to rearrange the objective in (52) to obtain the first part of the result.

For the converse, apply Lemma 1 to obtain the objective in (52). Let

$$\overline{W} = \sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\},\Psi)} \mathcal{L}^*(\{\varphi_t\}_{t=1}^{\infty}; \{\mu_t^*\}, \{\widehat{z}_t^*\}). \tag{53}$$

$\{\varphi_t^*\}_{t=1}^{\infty}$ is feasible for this problem and so $\overline{W} \geq \mathcal{L}^*(\{\varphi_t^*\}_{t=1}^{\infty}; \{\mu_t^*\}, \{\widehat{z}_t^*\})$. If this inequality is strict, then there exists some allocation $\{\varphi_t'\}_{t=1}^{\infty} \in \Gamma(\{R_t^*\}, \Psi)$ and some small $\varepsilon > 0$ such that

$$\begin{aligned}
& \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t'(w,\cdot)\}_{t=1}^{\infty}) \Psi(dw) \\
& + \sum_{t=1}^{\infty} \beta^{t-1} \mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1}' | w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw) \\
\geq \; & \overline{W} - \varepsilon - \sum_{t=1}^{\infty} \beta^{t-1} \mu_t^* X \left(R_t - \int C(\varphi_t'(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \right) \\
> \; & \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t^*(w,\cdot)\}_{t=1}^{\infty}) \Psi(dw) \\
& + \sum_{t=1}^{\infty} \beta^{t-1} \mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1}^* | w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw) \\
& + \sum_{t=1}^{\infty} \beta^{t-1} \mu_t^* X \left(R_t - \int C(\varphi_t^*(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \right) \\
& - \sum_{t=1}^{\infty} \beta^{t-1} \mu_t^* X \left(R_t - \int C(\varphi_t'(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \right) \\
> \; & \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t^*(w,\cdot)\}_{t=1}^{\infty}) \Psi(dw) \\
& + \sum_{t=1}^{\infty} \beta^{t-1} \mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w, \theta^{t-1}) U(\{\varphi_{t+r-1}^* | w, \theta^{t-1}) \pi^{t-1}(\theta^{t-1}) \Psi(dw).
\end{aligned}$$

Since $\{\varphi_t'\}_{t=1}^{\infty} \in \Gamma(\{R_t^*\}, \Psi)$, this contradicts the optimality of $\{\varphi_t^*\}_{t=1}^{\infty}$ in (52). Hence, $\{\varphi_t^*\}_{t=1}^{\infty}$ is optimal in (**??**). Now, apply Proposition 2. ■

Proof of Lemma 3 That $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ forms a PCE follows from the argument preceding Lemma 3. It attains the utilitarian payoff $W_1(\sigma, \zeta, \lambda) = E[\theta]u(R)$.

We show now that it is a worst PCE. Let (σ, ζ, λ) denote an arbitrary PCE. Let its payoff be given by $W_1(\sigma, \zeta, \lambda)$. Consider a political party that proposes a no insurance allocation S^{NI} at date 1. This party, if it won the election ,would deliver the utilitarian payoff of: $(1-\beta)E[\theta]u(R) + \beta W_2(S^{NI}|\sigma, \zeta, \lambda)$. Given Assumption 8, σ satisfies (14). Since S^{NI} need not be the election winning choice prescribed by σ, we have: $W_1(\sigma, \zeta, \lambda) \geq (1-\beta)E[\theta]u(R) + \beta W_2(S^{NI}|\sigma, \zeta, \lambda)$. Now consider a candidate who proposes S^{NI} in period 2 after the history S^{NI}. Once more this proposal need not be that prescribed by σ and so, $W_2(S^{NI}|\sigma, \zeta, \lambda) \geq (1-\beta)E[\theta]u(R) + \beta W_3(S^{NI}, S^{NI}|\sigma, \zeta, \lambda)$. Similarly, for any period t, we obtain: $W_t(\{S^{NI}\}_{t=1}^{t-1}|\sigma, \zeta, \lambda) \geq (1-\beta)E[\theta]u(R) + \beta W_{t+1}(\{S^{NI}\}_{t=1}^{t}|\sigma, \zeta, \lambda)$. Combining these inequalities, gives $W_1(\sigma, \zeta, \lambda) \geq (1-\beta)\sum_{t=1}^{T} \beta^{t-1} E[\theta]u(R) + \beta^{T+1} W_{T+1}(\{S^{NI}\}_{t=1}^{T}|\sigma, \zeta, \lambda)$. Taking the limit in T and using the fact that payoffs are bounded: $W_1(\sigma, \zeta, \lambda) \geq E[\theta]u(R)$.Thus, $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ delivers the lowest utilitarian payoff amongst PCE. ■

Proof of Proposition 6 (Necessity) Let (σ, ζ, λ) be a PCE and $\Upsilon(\sigma, \zeta, \lambda)$ it's induced outcome path. Since $\sigma \in \Sigma(\lambda)$, $\Upsilon(\sigma, \zeta, \lambda)$ must satisfy (1) above. Since λ satisfies (12), $\Upsilon(\sigma, \zeta, \lambda)$ must satisfy (2) above. We also have the following.

$$\begin{aligned}
\int U_t(H_t, \sigma_t^i(H_t), h_t|\sigma, \zeta, \lambda)Q_t(dh_t) &\geq \int U_t(H_t, S_t^{NI}, h_t|\sigma, \zeta, \lambda)Q_t(dh_t) \\
&= (1-\beta)E[\theta]u(R) + \beta W_{t+1}(H_t, S_t^{NI}, h_t|\sigma, \zeta, \lambda) \\
&\geq (1-\beta)E[\theta]u(R) + \beta W_{t+1}(H_t, S_t^{NI}, h_t|\sigma^{NI}, \zeta^{NI}, \lambda^{NI}) = \underline{W}.
\end{aligned}$$

Here the first inequality stems from the fact that $\sigma_t^i(H_t)$ solves $\max_{\mathcal{S}_t(H_t,\lambda)} \int U_t(H_t, S_t, h_t|\sigma,\zeta,\lambda)Q_t(dh_t)$ and that $S_t^{NI} \in \mathcal{S}_t(H_t,\lambda)$, the equality follows from the definition of S_t^{NI}, the second inequality follows from the fact that $(\sigma^{NI},\zeta^{NI},\lambda^{NI}|H_t,S_t^{NI})$ delivers the lowest utilitarian payoff amongst continuation PCE's. Consequently, $\Upsilon(\sigma,\zeta,\lambda)$ satisfies (3) above.

(Sufficiency) For the converse, we construct a strategy profile that induces $\Upsilon = \{\{\mathcal{H}_t^\Upsilon\}, \{\mathcal{J}_t^\Upsilon\}, \{\sigma_t^\Upsilon\}, \{\lambda_t^\Upsilon\}\}$ and then verify that if Υ satisfies (1)-(3) above, then it is an equilibrium strategy profile. For $H_t \in \mathcal{H}_t^\Upsilon$, set $\sigma_t(H_t) = \sigma_t^\Upsilon(H_t)$, for $(H_t, m^{t-1}) \in \mathcal{J}_t^\Upsilon$, $\lambda_t(H_t, m^{t-1}, w, \theta) = \lambda_t^\Upsilon(H_t, m^{t-1}, w, \theta)$. For $H_t \notin \mathcal{H}_t^\Upsilon$, set $\sigma_t(H_t) = \sigma_t^{NI}(H_t)$, for $(H_t, m^{t-1}) \notin \mathcal{J}_t^\Upsilon$, $\lambda_t(H_t, m^{t-1}, w, \theta) = \lambda_t^{NI}(H_t, m^{t-1}, w, \theta)$. Finally, set ζ so that for all t, h_t^p $\zeta_t(h_t^p; A) = \{1\}$ if $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda) > D(H_t, S_t^A, S_t^B, \xi, \delta_t)$, $\zeta_t(h_t^p; A) = [0,1]$ if $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda)$ $= D(H_t, S_t^A, S_t^B, \xi, \delta_t)$ and $\zeta_t(h_t^p; A) = \{0\}$ if $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda) < D(H_t, S_t^A, S_t^B, \xi, \delta_t)$. It is straightforward to verify that these strategies are resource-feasible and optimal for agents. To verify the optimality of the political strategies note that by (3), for $H_t \in \mathcal{H}_t^\Upsilon$ and all $S_t \in \mathcal{S}_t(H_t, \lambda^{NI})$,

$$\begin{aligned}
\int U_t(H_t, \sigma_t^i(H_t), h_t|\sigma,\zeta,\lambda)Q_t(H_t, dh_t) &= \int U_t^\Upsilon(H_t, \sigma_t^{\Upsilon,i}(H_t), h_t|\sigma^\Upsilon,\lambda^\Upsilon)Q_t^\Upsilon(H_t, dh_t) \\
&\geq \underline{W} = (1-\beta)E[\theta]u(R) + \beta W_{t+1}(H_t, S_t^{NI}|\sigma^{NI},\zeta^{NI},\lambda^{NI}) \\
&\geq \int U_t(H_t, S_t, h_t|\sigma,\zeta,\lambda)Q_t(H_t, dh_t).
\end{aligned}$$

where the maximality of S_t^{NI} in $\mathcal{S}_t(H_t,\lambda^{NI})$ delivers the final inequality. ■

Proof of Lemma 4 **Proof:** Let $\{S_{T-1+t}\}_{t=1}^\infty = \{M_{T-1+t}, \varphi_{T-1+t}\}_{t=1}^\infty$ denote the outcome sequence of mechanisms induced by the continuation equilibrium $(\widehat{\sigma},\widehat{\zeta},\widehat{\lambda}|\widehat{H}_T)$. Let $\{\widehat{\lambda}_{T-1+t}(\widehat{H}_T, \{S_{T-1+r}\}_{r=1}^t, h_T, \theta^t; m^{T-1+t})\}_{t=1}^\infty$ denote the equilibrium sequence of distributions over message histories in periods $T-1+t$ conditional on the aggregate history of the game, the agent's individual history up to T and realized sequence of shocks after T. We proceed in two steps. First, we replace the original sequence of mechanisms with a new direct sequence: $\{S'_{T-1+t}\}_{t=1}^{\infty} = \{\Theta, \varphi'_{T-1+t}\}_{t=1}^{\infty}$, where for each (h_T, θ^t), $\varphi'_{T-1+t}(h_T, \theta^t) = \sum_{M^{T-1+t}} \varphi_{T-1+t}(h_{T+t})$ $\widehat{\lambda}_{T-1+t}(\widehat{H}_T, \{S_{T-1+r}\}_{r=1}^{t}, h_T, \theta^s; m^{T-1+t}))$. Next, we use this sequence to obtain a new mechanism sequence $\{S''_t\}_{t=1}^{\infty} = \{\Theta, \varphi''_t\}_{s=1}^{\infty}$ by integrating out individual histories prior to period T, for all (w, θ^t)

$$\varphi''_t(w, \theta^t) = \int \varphi'_{T-1+t}(h_T, \theta^t) Q_t(\widehat{H}_t, \widehat{\lambda} | dh_t).$$

We now set the policy strategies recursively as follows. For each history H_t, let $n < t$ denote the date of the last election-winning defection, where $n = 0$ if no defection has occurred. Set for each i and t, $\sigma_t^i(H_t) = S''_{t-n}$. Thus, the strategy implements the mechanism sequence $\{S''_t\}_{t=1}^{\infty}$ and if there has ever been a defection, it begins implementing this sequence from the beginning. Set $\lambda_t(H_{t+1}, h_t, \theta_t) = 1_{\theta_t}$ if there was no defection in period t; otherwise, set $\lambda_t(H_{t+1}, h_t, \theta_t) = 1_{m^*}$, where $m^* \in \arg\sup_{M_t} \varphi_t(h_t, m)$. The voting strategy is set so that:

$$\zeta_t(h_t^p; A) = \begin{cases} 1 & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t | \sigma, \zeta, \lambda) \geq \delta_t + \xi \\ 0 & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t | \sigma, \zeta, \lambda) < \delta_t + \xi \end{cases} \tag{54}$$

Given our restriction to symmetric PCE, this is optimal for agents and, as usual, ties the probability of winning an election to the utilitarian continuation payoff.

Next, we check that the message strategy of agents is optimal. This strategy implies truthtelling if the parties are adhering to the sequence $\{S'_t\}_{t=1}^{\infty}$. Recall that this mechanism sequence is constructed in two steps. In the first, a sequence of continuation mechanisms $\{\Theta, \varphi'_{T-1+t}\}_{t=1}^{\infty}$ is obtained such that if each agent is truthful she obtains the same expected payoff as she received in the original continuation equilibrium (given her past history (w, m^{T-1})). Moreover, any alternative message strategy gives the agent a payoff she could have obtained by deviating from

$\widehat{\lambda}$ in the original continuation equilibrium. Since agents chose not to deviate in this way truth-telling is optimal for agents when confronted with $\{\Theta, \varphi'_{T-1+t}\}_{t=1}^{\infty}$. In the second step, individual histories (w, m^{T-1}) are integrated out. Since it is optimal for agents to report truthfully regardless of their past history (w, m^{T-1}) after the first adjustment, it remains optimal for them to do so after this second adjustment. Thus truth-telling is optimal for agents confronted with the mechanism sequence $\{S''_t\}_{t=1}^{\infty}$. If a party defects and wins the election at t, the political strategy σ calls for parties to ignore in future periods any messages sent in the present and to resume implementing $\{S''_t\}_{t=1}^{\infty}$. Knowing this it is optimal for agents to send a message that maximizes their current payoff in the period of a defection. Thus, the message strategy λ is optimal for the agents.

It remains to check that the policy strategy is optimal for the political parties. First, note that along the equilibrium path (σ, ζ, λ) delivers the same sequence of continuation utilitarian payoffs as the original continuation equilibrium $(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_T)$ did along its equilibrium path. Moreover, (σ, ζ, λ) uses weakly less resources along its outcome path than does $(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_T)$. Let $\{r_t\}_{t=1}^{\infty}$ denote the sequence of political rents induced by (σ, ζ, λ) along its outcome path and let $\{\widehat{r}_{T-1+t}\}_{t=1}^{\infty}$ the sequence induced by $(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_t)$ along its outcome path. It follows that $r_t \geq \widehat{r}_{T-1+t}$, each t and the allocation induced by the outcome path of (σ, ζ, λ) is resource-feasible.

Now, consider one step defections by political parties from the outcome path implied by (σ, ζ, λ). Given that 1) agents report their best current message in the period of a defection and 2) defections result in a resumption of the mechanism sequence $\{S''_t\}_{t=1}^{\infty}$ and of truth-telling in the period after a defection, the best one step defection that a party can make in period t is to a mechanism of the form $\widetilde{S}_t = (\Theta, \widetilde{\varphi}_t)$, where for all (w, θ^t), $\widetilde{\varphi}_t(w, \theta^t) = u(R - r_t^*)$ and r_t^* solves:

$$W_t^{def} = \sup_{r \in [0,R]} r \left\{ \frac{1}{2} + \widehat{\delta} \left[E[\theta] u(R-r) + \beta \underline{W} - W_t(\sigma, \zeta, \lambda | \{S''_s\}_{s=1}^{t-1}) \right] \right\}$$

Let $\widehat{r}_{T-1+t}$ denote the rents obtained by each party at date $T-1+t$ under $(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_t)$. Since, at each date $T-1+t$ along the continuation path both parties were better off implementing the mechanism S_{T-1+t} and collecting the associated rents if elected, than undertaking a (feasible) deviation to a mechanism of the form $\widetilde{S}_{T-1+t} = (\Theta, \widetilde{\varphi}_{T-1+t})$, where for all (w, m^{T-1+t}), $\widetilde{\varphi}_{T-1+t}(w, m^{T-1+t}) = u(R-r)$, $r \in [0, R]$, we have

$$\begin{aligned} \frac{r_t}{2} &\geq \frac{\widehat{r}_{T-1+t}}{2} \geq \sup_{r\in[0,R]} r \left\{ \frac{1}{2} + \widehat{\delta} \left[E[\theta]u(R-r) + \beta W_{T+t}(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_T, \{S_{T-1+s}\}_{s=1}^{t-1}, \widetilde{S}_{T-1+t}) - W_{T+t}(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_T, \{S_{T-1+s}\} \right. \right. \\ &\geq \sup_{r\in[0,R]} r \left\{ \frac{1}{2} + \widehat{\delta} \left[E[\theta]u(R-r) + \beta \underline{W} - W_{T-1+t}(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_T, \{S_{T-1+s}\}_{s=1}^{t-1}) \right] \right\} \\ &= W_t^{def}. \end{aligned}$$

Where we use the fact that 1) $\underline{W} \leq W_{T+t}(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_T, \{S_{T-1+s}\}_{s=1}^{t-1}, \widetilde{S}_{T-1+t})$ and $W_{T-1+t}(\widehat{\sigma}, \widehat{\zeta}, \widehat{\lambda}|\widehat{H}_T, \{S_{T-1+s}\}_{s=1}^{t-1})$ $= W_t(\sigma, \zeta, \lambda|\{S''_s\}_{s=1}^{t-1})$. Thus, no party has an incentive to make even this best one step deviation. Note that any sequence of future defections will not affect W_t^{def}, they will merely lower the party's payoff in future periods. Thus, the party has no incentive to defect from $\{S''_s\}_{s=1}^{\infty}$. Clearly, the same analysis can be applied to any history that incorporates a defection (and a resumption of the sequence $\{S''_s\}_{s=1}^{\infty}$). We conclude that the strategy profile is a PCE that delivers $\underline{W}$. ■

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