Factor Price Equalization in Heckscher-Ohlin Model

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ABSTRACT

This paper investigates the likelihood of factor-price equalization under the simple assumptions of Heckscher-Ohlin Theory. Factor-price equalization is also directly related to whether countries specialize or not in the global market.

A full-equilibrium in the world requires not only the equilibrium in the production side of the economy, but also the suppy-demand equality in the world. However, once we obtain an equilibrium in the production side of the economy, it is always possible to define demand in a way to get supply-demand equality at any production side equilibrium amounts. Therefore, it is not possible to talk about factor-price equalization without specifying demand in the economy.

Using L-P diagrams, the paper demonstrates how both factor-price equalization and non-equalization cases are possible when we look at only the production side of the economy. It is also demonstrated that the equilibrium possibilities will be much larger for factor-price equalization case if the number of commodities is more than the number of factors of production. However, the larger possibilities do not refer to different real equilibria, but only to indeterminacy in production.

When demand is introduced in the economy and supply-demand equality constraints are respected, we see that factor-prices might or might not be equalized depending on factor endowments, production functions and demand. The paper demonstrates this by introducing a model with 2 countries, 2 factors of production, 3 goods and CES utility function. Finally, using comparative statistics on this simple model, the conditions under which the likelihood of factor-price equalization increases are determined.

1 Introduction

In 1933, Bertil Ohlin published the book which was explaining one of the most discussed theories of all economic research history. Even if Ohlin wrote the book alone, Heckscher, who had earlier worked on the problem and who supervised Ohlin in his doctoral thesis, was also credited as a co-developer of the model. Later in 1977, Bertil Ohlin won Nobel Prize for his "path breaking contribution to the theory of international trade and international capital movements"[1]. The interest on his theory was at its peak during the 50s, 60s, and 70s. Over the following decades, this interest calmed down, as academicians like Brander, Dixit, Grosman, Helpman, and Krugman published a vast array of interesting models that focus on economies of scale and strategic interactions.

In order to understand the significance of Heckscher-Ohlin Theory (H-O Theory), it will be useful to look at the Ricardian model of comparative advantage, which was a widely referred model among trade theorists before the introduction of H-O model to the international trade literature. According to Ricardo's model, trade was motivated by different technologies, and therefore different labor productivities between the countries. The model was an essential contribution to trade theory for two reasons: First, it showed that trade can create wealth so that it can benefit both countries involved in it. Second, it estimated the direction of trade of a good: from the country that has comparative advantage to the one that has comparative disadvantage[2].

Even if the argument for technological differences between the countries is acceptable, the necessity of such differences to create mutual benefits to the countries involved in trade was undermining the usefulness of trade. Under such a theory, there will not be any use of trade whenever the technological differences between the countries are eliminated. All countries behave then as if they are autarkies.

In H-O model, there is no need for technological differences for trade to be mutually beneficial. The model is based on identical production technologies throughout the world. The model introduces capital as an additional factor of production along with labor[3], and it implies that trade can create wealth even if all the countries share the same production technologies whenever the countries differ from each other in terms of their factor endowments.

[1] http://nobelprize.org/nobel_prizes/economics/laureates/1977/index.html

[2] Comparative advantage theory explains why trade will be beneficial for both countries involved in it even though one of them can produce every kind of item more efficiently than the other. It states that for trade to be mutully beneficial, what actually matters is the ratio between how easily the two countries can produce different kinds of goods.

[3] We can also have more than two factors of production in H-O model. However, the model was initially defined with two factors of production, namely labor and capital. It has been mostly used in this format afterwards.

Even though Heckscher and Ohlin introduced the idea of factor endowment differences as the foundation of international trade, Ohlin's book was mostly verbose rather than mathematical. Moreover, there were a lot of possible extensions that might be considered using the framework of H-O Theory. As economists analyzed the model further, they come up with strong, and sometimes debatable propositions. Rybcznski[4], Stolper-Samuelson[5], and Factor-Price Equalization[6] theorems were developed using the framework provided by H-O Theory. Especially, the Factor-Price Equalization Theorem was discussed considerably thereafter.

The original H-O model was a 2-2-2[7] model. In this model, given the prices of commodities, the production amount of each good in each country is uniquely determined. Given that the demand functions in the two countries are the same, this also determines who exports what to the neighboring country. So, a new mechanism of comparative advantage was introduced into the international trade literature which was based on the factor intensity ratios of the countries.

Later, academicians looked at 2-3-2 case. J.E. Meade (1950) seems to be the first who recognized the indeterminacy in production[8] in 2-3-2 case. Samuelson (1953) explained the reason for the indeterminacy geometrically by pointing out that there will be flat planes and straight lines on the convex locus, and therefore there will not be a unique tangency when the international trading ratio is exactly equal to one of these ratios. W. P. Travis (1964) showed the indeterminacy diagrammatically and algebraically. Later, Melvin (1968) examined some of the well-known trade propositions under the 2-3-2 case, and he developed a methodology of using Edgeworth Box to depict the case when there are 3 goods.

One other point discussed previously was about how many goods a country will be producing in General Equilibrium (GE). Land (1959) argued that when there

[4] The Rybczynski theorem basically tells that when the endowment of a factor is increased there will be a relative increase in the production of the good using this factor more intensely. As a result, there will be a corresponding decline in that good's relative price.

[5] Stolper-Samuelson theorem tells that — under some economic assumptions (constant returns, perfect competition) — an increase in the relative price of a good results in an increase in the price of the factor which is used more intensively in the production of that good, and a fall in the price of the factor that is used less intensely in the production of that good.

[6] Factor-price-equalization theorem tells that the prices of identical factors of production will be equal as a result of the competition.

[7] 2-2-2 case refers to the situation when there are 2 factors of production, 2 goods and 2 countries. Note that for analogous statements, the first index refers to the number of factors, the second index refers to the number of goods and the third index refers to the number of countries.

[8] I want to clarify what I mean by indeterminacy in production in order to avoid confusion. Given the prices of goods, we talk about indeterminacy in production when the aggregate quantity produced in the world is determinate but the quantites produced in each country are indeterminate at equilibrium. The equilibrium factor-prices are the same for all these equilibria. Since the only thing that is different is the quantity bundles produced in each contry, I consider all these equilibria as the same real equilibrium.

are more goods than the factor endowments, under free trade, the number of goods produced in each country will not be more than the number of factors. However, there is an inherent conflict of this argument. What if the number of goods is more than that of the multiplication of the factors of production and the number of countries in the world? If Land's argument is right, there will not be any positive production for some goods. Assuming that there will be strictly positive demand for any good whatever its price is, we cannot talk of equilibrium if Land is right.

Land is right in proposing that for a random price system given exogenously to the economy, the countries will choose to produce as many goods as the number of factors. This point was also mentioned by Samuelson and Vanek and Bertrand (1971). However, in a GE situation, prices are not random. They are determined endogenously in the economy. Prices adjust in a way to make firms in a country indifferent between producing some of them. This permits the number of goods produced in positive quantity to be more than the number of factors in GE. However, it leads to above-mentioned indeterminacy in production case.

All of the papers that I mentioned above worked on factor-price equalization cases. Also, they worked only on the production side of the economy. Melvin stresses this point and he reminds that the demand conditions are equally important as supply conditions in determining what the final equilibrium will be. He rightly says that it is not possible to conclude that factor-prices are equalized without taking into account the factors on the demand side.

Factor-price equalization is probably the most intensely discussed topic of the H-O Theorem. It is completely interrelated with whether the countries specialize[9] in production or not and whether the aggregate production bundles in countries are indeterminate or not. Whenever we consider only the production part of the economy, it is possible to create equilibrium for both factor-price equalization and non-equalization cases. For both cases, given the aggregate production bundle in the world, it is possible to define the demand side of the economy in such a way so that the consumers demand exactly this aggregate production bundle[10]. This means that it is possible to have a full GE for both factor-price equalization and non-equalization cases. So, a crucial question of academic research arises. When do factor-prices equalize?

[9] Berofe moving forward, I want to clarify what I mean by specialziation. If there are N goods in the world, and if a country produces positive units of only N' goods, where $N' < N$, the country is said to be specialized in the production of these N' goods. That is, I talk about specialization in a country whenever that country is not involved in the production of one or more goods. I call a GE a case of specialization if there is at least one country who specializes in production.

[10] If we assume that all consumer in the world share the same CES utility function, the only thing we need to do to ensure supply-demand equality is to arrange the parameters of the utility function in a way so that the relative demand of goods at the equilibrium prices will be the same as the relative equilibrium production amounts.

Determining whether factor-prices equalize or not, or equivalently whether countries specialize in production of a subset of goods or not, has significance beyond the mechanical test of the H-O trade model. The question is even more important today. As the world gets closer and markets are more globalized, people wonder how the wages in the industrialized western countries will be influenced from the growing competition coming from the workers in the developing countries. Schott (2001) states that if rich and poor countries export the same mix of goods in an open world economy, their workers compete directly and there will be strong ties between the wages of these countries. He claims that the more the countries specialize, the weaker will be the link between the wages of these countries.

The exact factor-price equalization case is not practically so important. We know that factor-prices in the world are not equalized. However, the conditions that make factor-price equalization case more likely are the same conditions that decrease factor-price differentials between the countries. Therefore, when we talk about the conditions that make factor-price equalization case more likely, we are also talking about the conditions which decrease the wage differentials throughout the world.

It is understandable that economists have been keenly interested in determining the conditions under which factor-prices are equalized. Paul A. Samuelson (1948) states the sufficient conditions for factor-price equalization when there are two countries and two goods. His most restricting condition is to assume that both countries produce both goods. With this restriction, he succeeds to save the effort to introduce demand in the economy. Later, McKenzie (1955), Reiter (1961), Kuga (1972), and Helpman and Krugman (1985) weakened Samuelson's assumptions. Blackorby, Schworm and Venables (1992) worked on the conditions in terms of an economy's revenue functions and relaxed some assumptions of H-O model. However, the insistence in evading the introduction of demand in the economy prevailed in the literature.

Given that factor-prices might or might not be equalized, academicians assumed equilibriums of one way or another in their papers. Factor-price equality has been used in trade theory to introduce and test empirically market structure by Helpman and Krugman (1985), uncertainty by Helpman (1988) and endogenous growth by Grossman and Helpman(1991). Schott, on the other hand, criticizes previous tests of H-O theory claiming that they suffer from their focus on the narrower of the model's two potential equilibria which, he says, is the one that assumes that all countries produce all goods. He works on the case where countries with sufficiently disparate endowments specialize in different subsets of goods. With this change, he finds stronger empirical support for H-O specialization unlike previous findings.

Without defining demand, we cannot solve for GE in the world. However, even for the simplistic 2-3-2 case with homogenous CES utility function, there are a lot of equations and economic variables and it is hard to handle the equation system. Moreover, as indicated in this paper, there are 6 different possible equilibria and only one of them is the factor-price equalization case. Whenever we solve the equations for factor-price equalization case, the results we get might not constitute a valid equilibrium. The solution set may imply negative factor allocations in some countries which is not a valid equilibrium. If this happens, it means that the GE of the system is actually one of the remaining specialization cases. Therefore, it is understandable why academicians tried to avoid introduction of demand to the economy. However, the introduction of demand is absolutely necessary to solve for GE. In this paper, factor-price equalization equilibrium is solved for 2-3-2 case and CES utility function. Applying comparative statistics techniques on the numerical example, the paper determines the conditions which makes factor-price equalization in GE more likely.

Before using H-O model's framework to determine the conditions for which factor-prices are more likely to be equalized, I also want to question how realistic the H-O assumptions are. Today, the factors of production are more mobile than how they used to be when Ohlin first developed his theory. We see huge capital movements between countries. This conflicts with model's assumption of immobility of factors and it undermines the prediction power of Ohlin's theory. However, there are still strong restrictions against the movement of labor across the borders. Also, some factors of production are immobile by their nature, e.g. land. Moreover, some other developments in the world economy moved towards H-O assumptions. For instance, technological spillover across the countries are making the production technologies more similar throughout the world. Also, the developments in the transportation systems and the decreasing trade barriers under the guidance of WTO makes common goods-market assumption of the model more acceptable today. I believe that determining the conditions under which factor-prices are equalized in H-O model can contribute to our understanding of changes in factor prices, especially wage, in today's globalizing economy.

Let me give a brief outline of my paper. In the second part, I will introduce Lerner-Pearce Diagram, which I believe is a very powerful tool to depict production side equilibria in a multi-country, multi-good world. I will use the diagram to demonstrate different cases of production side equilibria, namely factor-price equalization and non-equalization. The indeterminacy in production is explained using this diagram. In the third section, I define demand side of the economy by introducing CES utility function, and then I solve for GE for 2-m-n case. Using the equations, I show that the dimensionality of the space of production side equilibria is larger for factor-price equalization case than for non-equalization cases. So, the equilibrium possibilities are larger for factor-price equalization case. However, the larger possibilities do not refer to different real equilibria, but instead to indeterminacy in production. It is demonstrated in this section that the measure of real equilibria is the same for factor-price equalization and non-equalization cases. In the fourth section, I numerically solve the model which I presented in the third section for 2-3-2 case. I show that whether factor-prices are equalized or not in GE depends on the factor endowments, production functions and demand. Using comparative statistic techniques on this numerical example, I determine the conditions under which factor-price equalization case becomes more likely.

2 Lerner-Pearce Diagram and Production side equilibria

2.1 Introduction to Lerner-Pearce Diagram

Lerner-Pearce Diagram (L-P Diagram) is originally developed by Lerner (1952). Pearce (1952) is credited for his discussion with Lerner. Findlay and Grubert (1959) contributed by showing how usefully the diagram can be used. L-P diagram is a powerful tool to depict production side equilibrium in international trade. I recommend Deardorff's paper (2002) for a concise but very instructive resource. He uses the diagram and graphically demonstrates Rybcznski and Stolper-Samuelson theorems on it. The L-P diagram is widely used by trade theorists.

An L-P diagram basically shows the combinations of capital and labor that can be used to produce a dollar's worth of output. In Figure 1, you see an L-P diagram for two goods: machinery and apparel. The curves for machinery and apparel show the combinations of capital and labor amounts that can be used to produce 1$ worth output.

The quantity of good that is worth 1$ depends on the price of the good. Assuming a constant returns to scale production function, this means that as the price of a good changes, the curve of the good shifts inwards or outwards along the rays coming out of the origin. In the figure, the dotted red curve represents how the curve for apparel shifts outwards when the price of apparel decreases. Since apparel is now cheaper, a higher quantity of apparel is needed so that it is worth again 1$. A higher output means using higher factor inputs, so the curve shifts outwards.

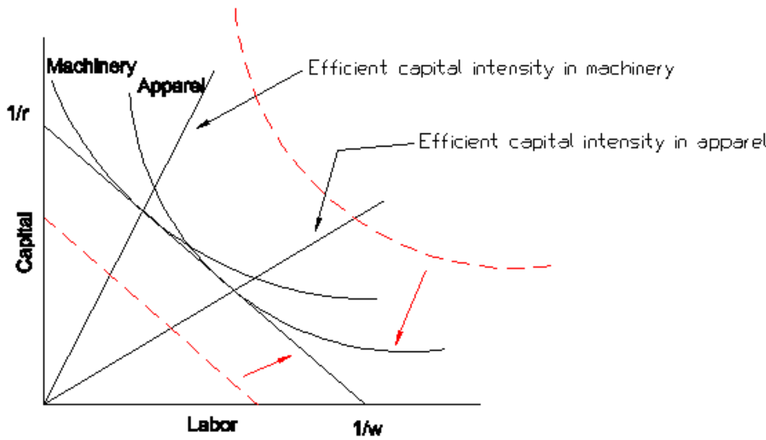

Figure 1: Lerner-Pearce Diagram

The sloped line in the figure, which I call the budget line, represents the combinations of labor and capital that cost 1$. Again, its location depends on the factor prices. The budget line cuts x and y axis at $1/w$ and $1/r$ where w stands for the factor price of labor (wage) and r stands for the factor price of capital (interest rate). The crossing points correspond to capital and labor amounts that is worth 1$ since the usage of the other factor is zero at these points.

Assuming perfect competition among firms, firms pay factor providers exactly the market value of their production. So, factor providers will get 1$ when the output is worth 1$. Also, perfect competition among the firms guarantees that the firms will use the least costly combination of factors in production. These two conditions ensure that the budget line on L-P diagram will be tangent to the curve of a good if that good is produced by the firms in that country.

Whenever a good is produced in a country, and given the price of that good[11], there will be many different lines that are tangent to the same curve. So, factor prices are indeterminate. However, if we know a second good that is produced in that country, we will have a unique line that is tangent to both curves. You see such a line in the figure. Therefore, the prices of factor inputs will be determinate whenever we know the prices of machinery and apparel, and that the country produces both goods. The efficient factor-intensity ratios[12] are also determined. In the figure, I indicate these ratios with rays coming out of the origin.

[11] Note that, given the production function of a good, the exact location of the curve for that good on L-P Diagram is determined once the dollar price of that good is known.

[12] Efficient factor intensity ratios are the ratio of factors of production used in producing a good at equilibrium in the production side of the economy. In the figure, I indicate these ratios with rays coming out of the origin. The rays passes through the origin and the tangency point of the budget line and the curve of the corresponding good. Efficient factor-intensity ratios of machinery and apparel productions are the slopes of the corresponding rays.

Assume that the factor prices are more than w and r. Then, 1$ budget line will be closer to the origin as depicted in Figure 1 by the dotted red line. However, no firms in this country will be willing to hire labor or capital at these factor prices. They lose money if they produce machinery or apparel at these prices. Therefore, the factor providers will decrease what they ask for from the firms, and the budget line will move outwards as depicted in the figure.

2.2 Production side equilibria on L-P Diagram

2.2.1 2-2-2 Case[13]

Non-specialization For 2-2-2 case, if there is no specialization, it means that both countries are producing both goods. I depict such a situation in Figure 2. Notationally, I call the countries by letters K and L, K being the capital-intense and L being the labor-intense country.

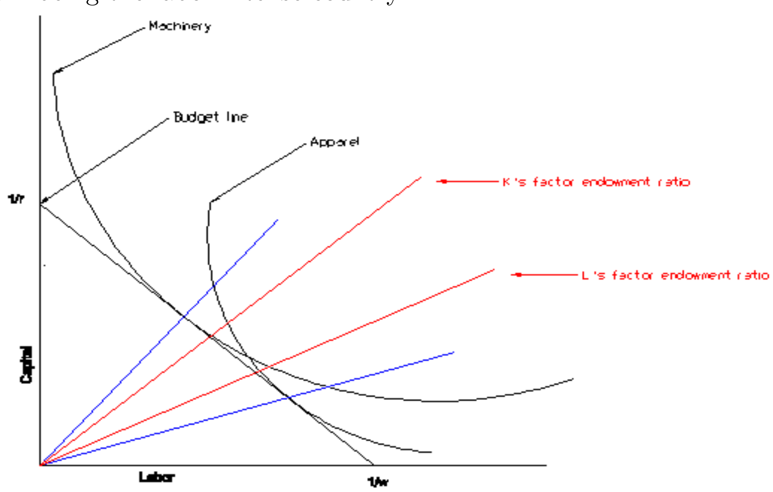

Figure 2: 2-2-2, Non-specialization

As it can be seen in the figure, K and L share the same budget line on the diagram since they produce the same set of goods. So, the factor-prices in the two countries are the same. This situation is true in general. The countries with common factor prices produce the same set of goods. In other words, non-specialization and factor-price equalization are equivalent concepts[14].

[13] In this paper, I will use the terms non-specialization and specialization interchangeably with the terms factor-price equalization and non-equalization respectively.

[14] Of course, the equivalence of these two terms is valid for basic H-O assumptions. For instance, if the technologies in the two countries are not the same, the two terms will not be equivalent any more.

The blue lines in the figure represent the efficient factor-intensity ratios while producing machinery and apparel. They are determined once the prices of goods (or factors) are determined. As it can be seen, capital is more intensely used in the production of machinery than apparel. The red lines in the figure represent the factor endowment ratios in K and L. In each country, the equilibrium allocation of factor resources in production of apparel and machinery happens in a way to ensure that no factor resources are wasted at the end.

Given the prices of goods and factor endowments, the quantities of machinery and apparel produced in K and L are determinate. This can be seen very easily from equations (1), (2), (3) and (4). Assume that the factor endowments in a country are given by $\overline{k}$ and $\overline{l}$. The efficient factor-intensity ratios to produce machinery and apparel are determined as the slopes of blue lines in the figure. I denote these ratios with r_m and r_a. Then, this country should allocate its capital and labor endowments between machinery and apparel by (k_m,k_a) and (l_m,l_a) which satisfies:

$$k_m + k_a = \overline{k} \qquad (1)$$
$$l_m + l_a = \overline{l} \qquad (2)$$
$$k_m / l_m = r_m \qquad (3)$$
$$k_a / l_a = r_a \qquad (4)$$

The above equation system has 4 equations in 4 unknowns, and there will be a unique solution for the unknown set (k_m,k_a,l_m,l_a). Since the factor inputs allocated to each sector are determinate, the production of each good in each country will be determinate. Finally, the aggregate production in the world will also be determinate.

Specialization Now, let us look at the specialization situation for 2-2-2 case. Figure 3 depicts such a situation. The reasoning of this figure is similar to Figure 2. However, two points should be pointed out additionally. First of all, notice that the lines representing the factor endowment ratios of the countries and the efficient factor-intensity ratios coincide. This situation arises, because at equilibrium no factor endowments can be wasted in each country. Since each country specializes in the production of only a single good, this can only be possible if the factor-intensity ratio in a country is the same as the efficient factor-intensity ratio of the good which the country is specialized in.

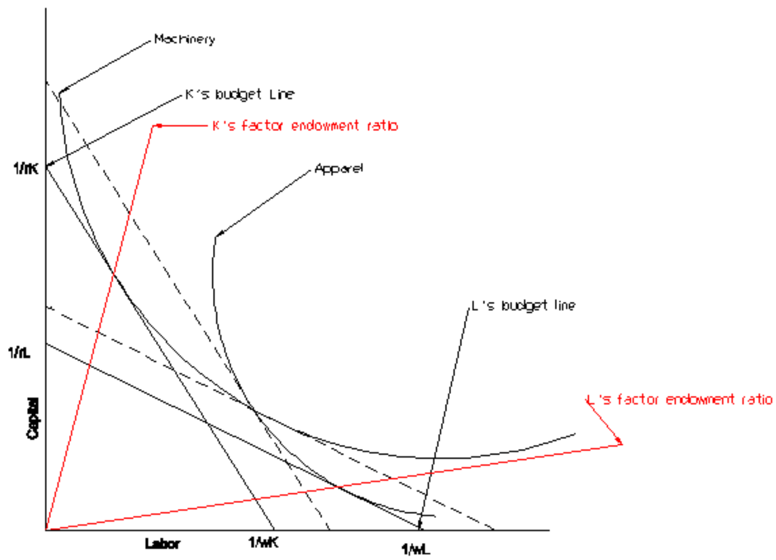

Figure 3: 2-2-2, Non-specialization

The second point I will make is that given the equilibrium prices of goods and factor endowments, at equilibrium, firms in a country should not be willing to produce the good which the other country specializes in. In other words the firms in K should not be willing to produce textile and the firms in L should not be willing to produce machinery. This condition is satisfied if the minimum cost of producing the other good that is worth 1$ is more than 1$ at the prevailing factor prices. The situation depicted in Figure 3 satisfies this condition. This can be seen by the dotted lines on the figure which stand for the minimum cost of producing apparel in K and machinery in L. As it can be seen, the dotted lines are further to the origin. The production of apparel in K and machinery in L both costs more than 1$, and the firms will not be producing them.

It is easy to see that the production of machinery and apparel in K and L, and therefore, in the world are determinate. Both countries allocate all their resources in the production of a single good, and the quantity produced will be determinate given the production functions. The aggregate production amounts of the goods in the world will be the amounts produced in the two countries.

2.2.2 2-3-2 Case

Non-Specialization Figure 4 represents the non-specialization situation in for the 2-3-3 case.

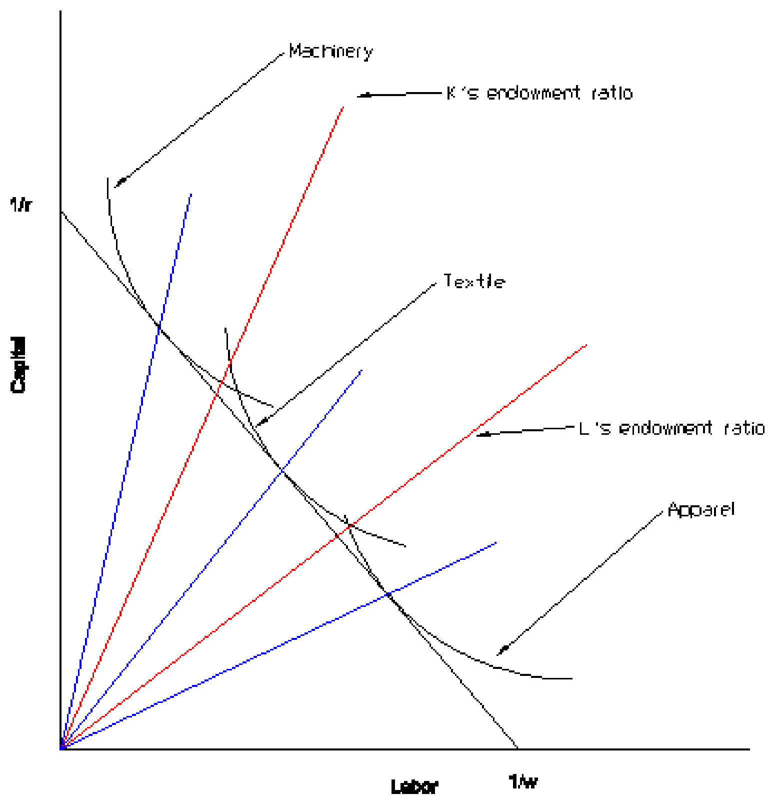

Figure 4: 2-3-2, specialization

All the discussions about 2-2-2 case are also valid for 2-3-2 case. However, there are now 3 goods instead of 2. Once the prices of goods and the factors are determined, each country now solves an equation system with 5 equations in 6 unknowns: (k_m, k_t, k_a) and (l_m, l_t, l_a).

$$k_m + k_t + k_a = \bar{k} \qquad (5)$$
$$l_m + l_t + l_a = \bar{l} \qquad (6)$$
$$k_m/l_m = r_m \qquad (7)$$
$$k_t/l_t = r_t \qquad (8)$$
$$k_a/l_a = r_a \qquad (9)$$

Even if we fix the aggregate production bundle in the world, there will be different ways of achieving this aggregate bundle by allocating the production of the goods among the countries differently. This is the indeterminacy of production mentioned in international trade literature in H-O model. The tables below depict such a situation. Assume that the capital and labor endowments of K and L are (42,26) and (41,75) respectively. Also, let us say that the efficient factor-intensity ratios used to produce 1$ worth output are 3/1, 2/2, and 1/3 for machinery, textile and apparel respectively. Then, both countries use all of their factor endowments if they produce the goods in quantities presented in Table 1 and Table 2. Moreover, the aggregate production bundle in the world is the same for both cases.

		Quantity produced		
Good	Factor usages(capita/labor)	K	L	World
Machinery	3/1	10	3	13
Textile	2/2	5	6	11
Apparel	1/3	2	20	22

Table 1

		Quantity produced		
Good	Factor usages(capita/labor)	K	L	World
Machinery	3/1	9	4	13
Textile	2/2	7	4	11
Apparel	1/3	1	21	22

Table 2

The indeterminacy in production in some country happens when the number goods produced in that country is more than the number of factors of production. It is a theoretical support for strategical industrial policy. It says that a country can keep the industries which it considers as strategically important without violating efficiency. The only two restrictions imposed on a country to be efficient in production are that it should use efficient factor-intensity usage ratios in production of goods, and it should not waste its factor endowments. However, there are many different ways of doing that if the number of goods produced is more than the number of factors. Therefore, a capital-intense country like Japan can keep the labor-intensive rice production sector at home without violating efficiency. Of course, if Japan wants to produce more rice, it needs to shift its production configuration in other sectors to more capital-intensive goods so that it will still be using its endowed factor-intensity ratio on average.

Specialization Figure 5 depicts 2-3-2 specialization case. Note that specialization does not mean that a country produces only one good. Here, each country produces 2 goods. Note also that this is not necessarily the only way of specialization for 2-3-2 case. It might also be the case in equilibrium that one country produces only a single good and the other country produces the remaining two goods. Similarly, a country might be producing all three goods while the other produces only one good.

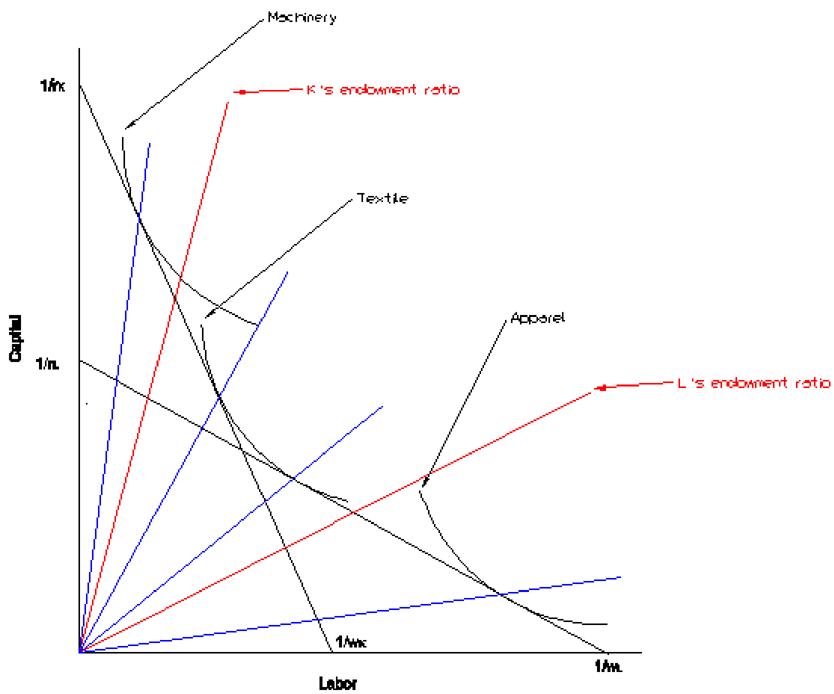

Figure 5: 2-3-2, non-specialization

As for 2-2-2 non-specialization case, a country has a unique way of allocating its capital and labor endowments between two goods without wasting any of its factor resources. An equation system with 4 unknowns in 4 equations will arise here as well. So, given the prices of the goods, the aggregate production bundle in a country, and also in the world will be determinate.

2.3 Dimensionality of the set of production side equilibria

2.3.1 2-2-2 Case

Specialization If there will be an equilibrium of specialization case, the capital intense country (K) should specialize in capital-intense commodity, i.e. machinery; and the labor-intense country should specialize in labor intense commodity, i.e. apparel. Figure 6 demonstrates how we can determine the dimensionality of the set of production side equilibria for specialization case. The picture shows how many variables we can choose freely so that the equilibrium of the system will be identified. The number of variables that we choose freely is the degrees of freedom (DOF) we have. So, the set of equilibria will be a correspondence to a bounded space with dimensionality equal to DOF.

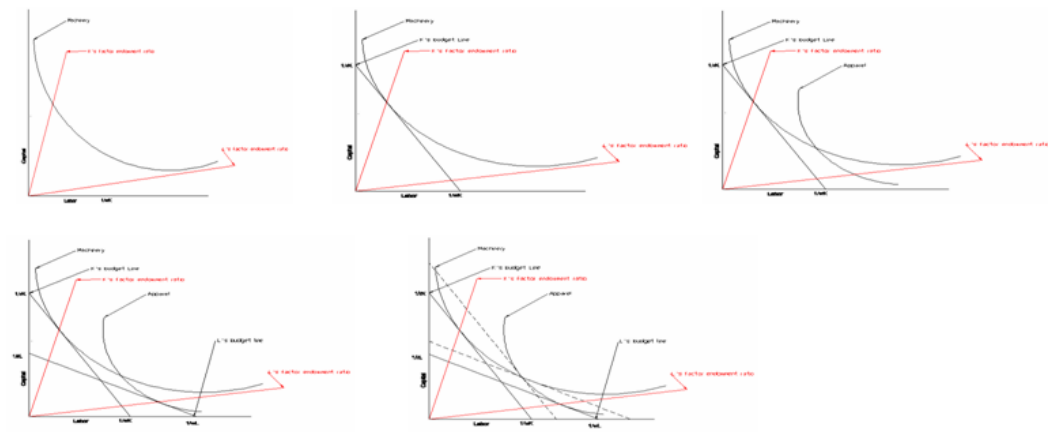

Figure 6: DOF in 2-2-2, specialization case

First of all, the dollar means nothing if we do not define its value in terms of a real commodity. In other words, we need a numeraire good to fix the value of 1$. That means fixing the dollar price of a commodity so that dollar will have a real value in terms of that commodity. Let us assume that we fix the price of machinery. This will fix the location of the curve for machinery. Also, the lines that represent the factor endowment ratios of K and L are exogenously given. These lines along with the curve for machinery can be seen in the first L-P diagram in Figure 6.

We also know that at the equilibrium no factors of production will be wasted due to the competition in factor suppliers. So, the tangency of the budget line for K should happen at the point where the curve for machinery production and the line defining the factor endowment ratio of K intersects. There is only a unique line satisfying this property. As a result, the location of the line and therefore, the factor prices in K are determined. This budget line can be seen in the second L-P diagram in Figure 6.

The same arguments will be valid for L and the production of apparel as it is drawn in the third and fourth L-P diagrams in Figure 6. The only difference is that, now we can change the price of apparel freely. In an L-P diagram, it means that we can determine the location of the curve for apparel on the diagram freely[15]. We will have an equilibrium point for each price that we choose.

To sum up, we can arrange an equilibrium in the production side of the economy by changing the relative price of apparel with respect to machinery. The number of equilibrium points is a correspondence with a line segment, each point

[15] However, keep in mind that the price level should be adjusted in an interval which makes the production of apparel in K and machinery in L more costly than 1$. This condition is satisfied for the above picture, which can be seen from the dashed lines in the last L-P diagram.

corresponding to a different price ratio between machinery and apparel. For all these equilibrium points, the quantity of apparel and machinery produced in each country is the same. Each country is allocating all its resources for the production of the same single good.

Non-specialization Figure 7 depicts how we can determine the dimensionality of production side equilibria for 2-2-2 non-specialization case.

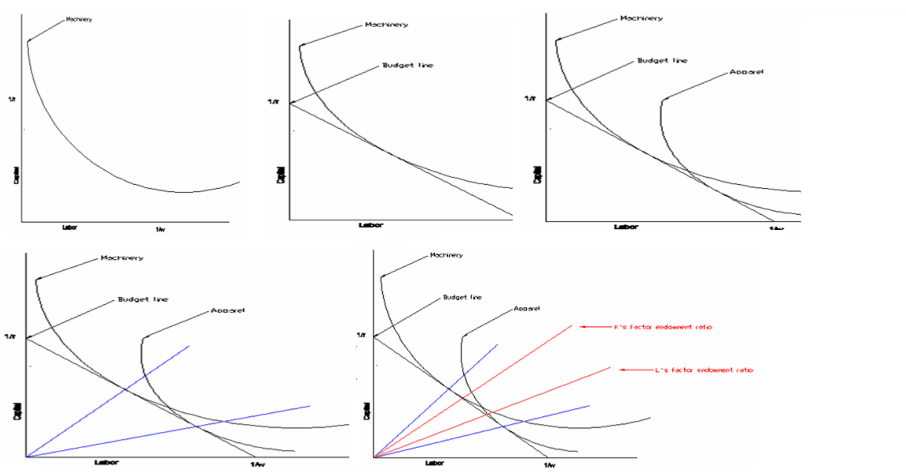

Figure 7: DOF in 2-2-2, non-specialization case

We again fix the position of the curve for machinery to fix its price in terms of dollars. This curve is drawn in the first L-P diagram in Figure 7.

The budget line defined by factor prices should be tangent to this curve. However, this time there is no need for the tangency point to be on the point where the curve and K's endowment ratio line intersects. Since K produces both machinery and apparel, K can allocate its endowments between the production of apparel and machinery in a way so that no factor resources are wasted in the end. This gives us the freedom to choose where the budget line intersects the curve. We can choose r (or w) to determine the position of this line uniquely in the second L-P diagram. After drawing the budget line, the factor prices are determined.

There is only one point where the curve for apparel intersects the budget line. So, the price of apparel, and therefore the location of the apparel curve on L-P diagram will be uniquely determined. You can see this in the third L-P diagram in Figure 7.

Once the intersection points are determined, both countries will have only one way of allocating their factor endowments between the production of apparel and machinery so that they do not waste any of their factor resources at the end. I indicated this point above with the equation system defined by (1), (2), (3), and (4). As a result, the aggregate production of apparel and machinery in the world will also be uniquely determined.

Again, we see that the solution set is a correspondence with a line segment. We only have the freedom to choose the price of one factor. Then, the equilibrium in the production side of the economy will be uniquely determined. However, unlike the specialization case, this time each of these equilibria means different aggregate production bundles of machinery and apparel in each country and also in the world.

2.3.2 2-3-2 Case

Specialization Figure 8 shows how 2-3-2 specialization case[16] arises. In the figure, K specializes in textile and machinery, and L specializes in textile and apparel.

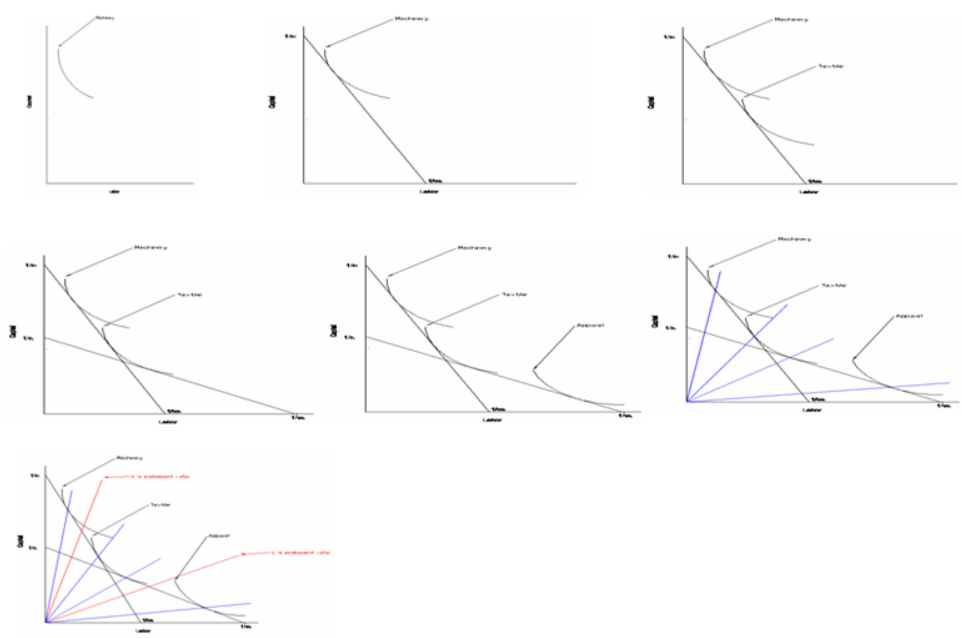

Figure 8: DOF in 2-3-2, specialization case

Again, we first fix the dollar price of machinery. So, the position of the curve for machinery is fixed. Then, we choose r in K freely. The tangency budget line will

[16]Remember that this is not the only specialization case for 2-3-2 setting.

be uniquely determined afterwards. Also, the curve's position for textile will be uniquely determined. You can see this in the third L-P diagram in Figure 8.

We also choose r in L freely. The budget line tangent to the textile curve will be uniquely determined afterwards. This will fix the apparel curve's location as in the fifth L-P diagram.

After the graph is drawn, each country will have a unique way of allocating its factor endowments between the production of two goods so that no resources are wasted. To sum up, the degrees of freedom that we have is 2, which means that the number of production side equilibria will be a correspondence to a two dimensional bounded area.

Non-specialization Figure 9 depicts the non-specialization, or equivalently the factor-price equalization case.

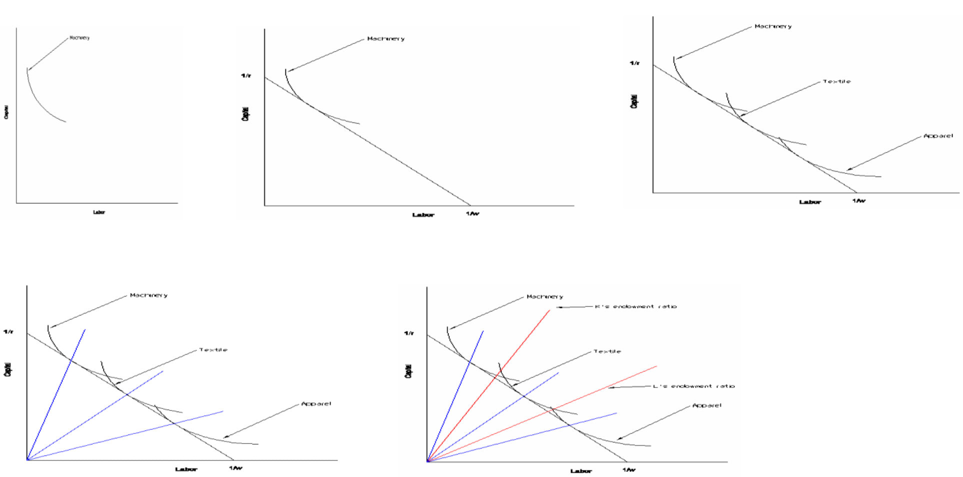

Figure 9: DOF in 2-3-2, non-specialization case

Again, we fix the position of the curve for machinery as in the first graph. Then, we can choose one factor price, say r, freely. We will have a unique tangency line to the curve as in the second L-P diagram in Figure 9. The factor prices in both countries will be determined. After the budget line is drawn, the positions of the curves for textile and apparel will also be uniquely determined as well as their prices.

After the prices of factors and goods and the efficient factor-intensity usage ratios are determined, each country will have an equation system of 6 variables in 5 equations as in (5), (6), (7), (8) and (9). So, we have 5 equations for 6 unknowns, which enables us to choose one of the unknowns freely in each country. This means that we have two more degrees of freedom. In sum, we will have 3 degrees of freedom.

So, do we actually have more equilibria for 2-3-2 case if factor prices are equalized? The answer is yes if we accept production indeterminacy cases as different equilibria. However, the aggregate production in the world and all other real economic variables are the same for all these equilibria. Therefore, I do not accept different instances of indeterminacy in production as different real equilibria. As I will demonstrate in section 3 when we assume that they are the same real equilibrium, the number of equilibria is the same for factor-price equalization and non-equalization cases.

3 General Equilibrium in 2-m-n case

As it is indicated in Section 2, both factor-price equalization and non-equalization cases might arise as the production side equilibrium. However, in order to attain GE in the economy, we also need to check whether consumers demand exactly what is produced at relevant prices. Using CES utility function, I will show that it is always possible to define demand in a way so that we will also have supply-demand equality for a production side equilibrium. Then, I will solve for GE of the model for 2-m-n case.

3.1 DEMAND

Assume CES utility function of the form:

$$u(c_1, c_{2,}c_3, ..., c_n) = [\theta_1 c_1^p + \theta_2 c_2^p + \theta_3 c_3^p + ... + \theta_n c_n^p]^{1/p}$$

Then, a consumer who has wealth W will solve the following utility maximization problem given the prices of the goods by $(P_1, P_2, P_3, ..., P_n)$:

$$max_{\{c_1,c_2,c_3,..,c_n\}}[\theta_1 c_1^p + \theta_2 c_2^p + \theta_3 c_3^p + ... + \theta_n c_n^p]^{1/p}$$

$$s.t.$$

$$P_1 * c_1 + P_2 * c_2 + P_3 * c_3 + ... + P_n * c_n \leq W$$

$\mathcal{L} = [\theta_1 c_1^p + \theta_2 c_2^p + \theta_3 c_3^p + ... + \theta_n c_n^p]^{1/p} - \lambda(P_1 * c_1 + P_2 * c_2 + P_3 * c_3 + ... + P_n * c_n - W)$

Let us define: $\nu = [\theta_1 c_1^{*p} + \theta_2 c_2^{*p} + \theta_3 c_3^{*p} + ... + \theta_n c_n^{*p}]$ where $(c_1^*, c_2^*, c_3^*, .., c_n^*)$ is the optimal consumption series. Then:

$$\frac{\partial \mathcal{L}}{\partial c_i} = \frac{1}{p} * \nu^{(\frac{1}{p}-1)} * \theta_i * p * c_i^{*p-1} = \lambda * P_i \quad i = 1, 2, ..., n$$

$\Rightarrow$:

$$\frac{\frac{\partial \mathcal{L}}{\partial c_i}}{\frac{\partial \mathcal{L}}{\partial c_1}} : \frac{\theta_i}{\theta_1}\left(\frac{c_i^*}{c_1^*}\right)^{p-1} = \frac{P_i}{P_1}$$

$$\Rightarrow \frac{c_i^*}{c_1^*} = \left(\frac{P_i}{P_1} * \frac{\theta_1}{\theta_i}\right)^{\frac{1}{p-1}} \quad i = 2, 3, ..., n \qquad (10)$$

From (10) we see that the relative consumption ratio of two goods depends only on the prices of the goods and the parameters of the utility function. It is independent of the wealth of the household. Since this ratio is the same for all households independent of their wealths, it will also be the case for the aggregate consumption amounts in the world. In other words, if W denotes the total income of households in the world, the solution set of the above maximization problem will give us the aggregate consumption choices of households in the world.

In order to have supply-demand equality in the world, aggregate consumption and production amounts should be the same, i.e.:

$$c_1 = q_1,\ c_2 = q_2, ...,\ c_n = q_n \qquad (11)$$

All capital and labor endowments are owned by households. And households are fully paid for the value of their production. Therefore, the value of the total production in the world will be equal to the total income of households in the world. Then, if $(c_1, c_{2,}c_3, ..., c_n)$ and $(q_1, q_{2,}q_3, ..., q_n)$ are the aggregate consumption and production amounts, we have:

$$W = P_1 * c_1 + P_2 * c_2 + P_3 * c_3 + ... + P_n * c_n = P_1 * q_1 + P_2 * q_2 + P_3 * q_3 + ... + P_n * q_n \qquad (12)$$

Proposition: (11) holds if and only if $\frac{c_i}{c_1} = \frac{q_i}{q_1}$ for $i = 2, ..., n$ and (12) holds.

Proof: It is obvious that (11) implies (12) and $\frac{c_i}{c_1} = \frac{q_i}{q_1}$ for $i = 2, ..., n$. I will show how $\frac{c_i}{c_1} = \frac{q_i}{q_1}$ for $i = 2, ..., n$ and (12) implies (4):

From (12), $W = P_1 * c_1 + P_2 * c_2 + P_3 * c_3 + ... + P_n * c_n$ and $W = P_1 * q_1 + P_2 * q_2 + P_3 * q_3 + ... + P_n * q_n$. If I divide these equations by c_1 and q_1 respectively:

$$\frac{W}{c_1} = P_1 + P_2 * \frac{c_2}{c_1} + P_3 * \frac{c_3}{c_1} + ... + P_n * \frac{c_n}{c_1}$$

$\frac{W}{q_1} = P_1 + P_2 * \frac{q_2}{q_1} + P_3 * \frac{q_3}{q_1} + ... + P_n * \frac{q_n}{q_1}$

The right hand sides of the equations are the same since $\frac{c_i}{c_1} = \frac{q_i}{q_1}$ for $i = 2, ..., n$.. Then, we have:

$\frac{W}{c_1} = \frac{W}{q_1}$ and $c_1 = q_1$. Using this equality in $\frac{c_i}{c_1} = \frac{q_i}{q_1}$ for $i = 2, ..., n$, we can also show that $c_i = q_i$ for $i = 2, 3, .., n$.

To sum up, since we know (12) will hold in general equilibrium, supply-demand equality will be satisfied for any production set $(q_1, q_{2,} q_3, ..., q_n)$ that satisfies:

$$\frac{q_i}{q_1} = \left(\frac{P_i}{P_1} * \frac{\theta_1}{\theta_i}\right)^{\frac{1}{p-1}} \quad i = 2, 3, .., n \qquad (13)$$

Proposition: It is always possible to define a utility function of the CES form so that supply-demand equality will be satisfied for some aggregate production bundle in the world.

Proof: Given $(q_1, q_{2,} q_3, ..., q_n)$, prices and p, we can solve for θ_i using (13):

$$\theta_i = \theta_1 \left(\frac{q_i}{q_1}\right)^{1-p} * \frac{P_i}{P_1} \quad i = 2, 3, .., n$$

For some fixed value of θ_1, when we define θ_i as above we will get supply-demand equality. Therefore, it is always possible to define demand in the economy in CES form so that we can attain GE for any production side equilibrium.

3.2 PRODUCTION

- Production functions:

$$q_i = \gamma_i * k^{\alpha_i} * l^{(1-\alpha_i)} \; i = 1, 2, .., n$$

Assume i is produced in j with $i = 1, 2, .., n$ denoting the index of good and $j = 1, 2, .., m$ denoting the index of the country. Also, let us say that a firm in country j wants to produce q' units of good i. At equilibrium, the firm solves the following cost minimization problem:

$$min_{\{k,l\}} \quad r_j * k + w_j * l$$
$$s.t.$$
$$\gamma_i * k^{\alpha_i} * l^{(1-\alpha_i)} \geq q'$$

$$\mathcal{L} = r_j k + w_j l + \lambda_{ij}(q' - \gamma_i * k^{\alpha_i} * l^{(1-\alpha_i)})$$

Then:

$\frac{\partial \mathcal{L}}{\partial k} : r_j - \lambda_{ij} * \alpha_i * \gamma_i * k^{\alpha_i - 1} * l^{(1-\alpha_i)} = 0$

$\Rightarrow r_j = \lambda_{ij} * \alpha_i * \gamma_i * k^{\alpha_i - 1} * l^{(1-\alpha_i)}$

$\frac{\partial \mathcal{L}}{\partial k} : w_j - \lambda_{ij} * (1-\alpha_i) * \gamma_i * k^{\alpha_i} * l^{-\alpha_i}$

$\Rightarrow w_j = \lambda_{ij} * (1-\alpha_i) * \gamma_i * k^{\alpha_i} * l^{-\alpha_i}$

When we divide these two equations, we find the efficient factor-intensity ratio (capital/labor) used in producing i^{th} good in j^{th} country, which I denote by e_{ij}:

$\frac{r_j}{w_j} = \frac{\alpha_i}{(1-\alpha_i)} \frac{l}{k}$

Then:

$$e_{ij} = \frac{k}{l} = \frac{w_j}{r_j} * \frac{\alpha_i}{1-\alpha_i} \qquad (14)$$

- Also, the cost of production should be exactly equal to the revenue raised by the firm:

$$r_j k + w_j l = P_i * \gamma_i * k^{\alpha_i} * l^{(1-\alpha_i)}$$

Divide both sides by l:

$r_j \left(\frac{k}{l}\right) + w_j = P_i * \gamma_i * \left(\frac{k}{l}\right)^{\alpha_i}$

$\Rightarrow r_j * e_{ij} + w_j = P_i * \gamma_i * (e_{ij})^{\alpha_i}$

$\Rightarrow P_i = \frac{r_j * e_{ij} + w_j}{\gamma_i * (e_{ij})^{\alpha_i}}$

Replace e_{ij} using (14) :

$$P_i = \frac{r_j * \frac{w_j}{r_j} * \frac{\alpha_i}{1-\alpha_i} + w_j}{\gamma_i * \left(\frac{w_j}{r_j} * \frac{\alpha_i}{(1-\alpha_i)}\right)^{\alpha_i}} = \frac{w_j * \left(\frac{\alpha_i}{1-\alpha_i} + 1\right)}{\gamma_i * \left(\frac{w_j}{r_j} * \frac{\alpha_i}{(1-\alpha_i)}\right)^{\alpha_i}} = (1-\alpha_i)^{-1+\alpha_i} \alpha_i^{-\alpha_i} \gamma_i^{-1} w_j^{1-\alpha_i} r_j^{\alpha_i}$$

$$P_i = (1-\alpha_i)^{-1+\alpha_i} \alpha_i^{-\alpha_i} \gamma_i^{-1} w_j^{1-\alpha_i} r_j^{\alpha_i}$$

Define $\varsigma_i = (1-\alpha_i)^{-1+\alpha_i} \alpha_i^{-\alpha_i} \gamma_i^{-1}$. Then:

$$P_i = \varsigma_i w_j^{1-\alpha_i} r_j^{\alpha_i} \qquad (15)$$

To sum up, if i^{th} good is produced in j^{th} country, the capital labor ratios used in the production and the price of that good will be as in (14) and (15):

$$e_{ij} = \frac{k}{l} = \frac{w_j}{r_j} * \frac{\alpha_i}{1-\alpha_i} \qquad (14)$$
$$P_i = \varsigma_i w_j^{1-\alpha_i} r_j^{\alpha_i} \qquad (15)$$

- Denote factor prices in countries $j = 1, 2, .., m$ by $(w_1, r_1), (w_2, r_2), ..., (w_m, r_m)$. Note that the factor prices in two countries can be the same or different. Let us group the countries who share the same factor prices and index these groups by $f = 1, 2, ..., F$. Since the sets are disjoint and each set has at least one element, $F \leq m$. Let us denote the factor prices in each set by:

$$(w_1, r_1), (w_2, r_2), .., (w_f, r_f), .., (w_F, r_F)$$

Also, let us change the indices so that $\frac{w_f}{r_f} > \frac{w_{f'}}{r_{f'}}$ for $f < f'$. I will continue with some propositions.

1) At equilibrium, each country will be producing at least one good.

Proof:

Without loss of generality, assume the first country ($j = 1$) does not produce anything. Then, no firms in that country is producing anything and all factor resources are unemployed. Factor providers will be ready to sell their endowments for any strictly positive price.

Assume i is produced in $j \neq 1$. Then:

$$r_j k + w_j l = P_i * \gamma_i * k^{\alpha_i} * l^{(1-\alpha_i)}$$

But then, a firm can buy k and l in the first country by paying (w_1, r_1) such that $0 < r_1 < r_j$ and $0 < w_1 < w_j$ and earn strictly positive profit which is:

$$P_i * \gamma_i * k^{\alpha_i} * l^{(1-\alpha_i)} - r_1 k + w_1 l > P_i * \gamma_i * k^{\alpha_i} * l^{(1-\alpha_i)} - r_j k + w_j l = 0$$

Since the firms will take advantage of any profitable opportunity, this situation cannot be an equilibrium. Each country produces at least one good at equilibrium.

2) Two countries with factor prices (w_f, r_f) and $(w_{f'}, r_{f'})$ with $f \neq f'$ can produce at most one good in common.

Proof:

Each time a good is produced in a country, we have an equation of the form (15). Given that two goods are produced in that country, we can find the factor prices for that country using the two equations of the form (15) for any given prices of the goods. But then, if the same two goods are produced between two countries, the factor prices that we find for these two countries will be the same. This contradicts with our initial assumption. So, at equilibrium two countries with different factor prices can produce at most one good in common.

3) $w_f > w_{f'}$ if and only if $r_f < r_{f'}$.

Proof:

Assume $w_f > w_{f'}$ and $r_f \geq r_{f'}$ By 1, a good will be produced in a country who has factor prices (w_f, r_f). But then, a firm in a country with factor prices $(w_{f'}, r_{f'})$ can produce the same good by using the same factor intensity ratio as in the other country at a lower cost. Then, this firm makes profit, which is not possible. So, between two countries with different factor prices, labor will be cheaper in one of them, and capital will be cheaper in the other. The proof from the other direction is similar.

4) For $w_1 > w_2 > w_3$, there cannot be any good that is produced by 1 and 3 but not by 2.

Proof:

By 3, $r_1 < r_2 < r_3$. So, $\frac{w_1}{r_1} > \frac{w_2}{r_2} > \frac{w_3}{r3}$. Assume that there is a good i which is produced by 1 and 3 but not by 2. Using (15):

$$P_i = \varsigma_i w_1^{1-\alpha_i} r_1^{\alpha_i} = \varsigma_i w_3^{1-\alpha_i} r_3^{\alpha_i}$$

Since i is not produced in 2, it means that the minimum cost of producing this good in the second country is more than P_i[17]:

$$\Rightarrow P_i < \varsigma_i w_2^{1-\alpha_i} r_2^{\alpha_i}$$

$$\Rightarrow \varsigma_i w_1^{1-\alpha_i} r_1^{\alpha_i} = \varsigma_i w_3^{1-\alpha_i} r_3^{\alpha_i} < \varsigma_i w_2^{1-\alpha_i} r_2^{\alpha_i}$$

$$\Rightarrow \frac{w_1}{w_2} < \left(\frac{r_2}{w_2}\frac{w_1}{r_1}\right)^{\alpha_i} \text{ and } \frac{w_3}{w_2} < \left(\frac{r_2}{w_2}\frac{w_3}{r_3}\right)^{\alpha_i} \qquad (16).$$

By 1, the second country will be producing some good. I index this good by ii. Since this good is produced in the second country, the first and the third countries cannot produce it less costly than the second country. So, the implied price of the good using (15) in the second country should be less than or equal to the implied prices in the other two countries:

$$\varsigma_i w_1^{1-\alpha_{ii}} r_1^{\alpha_{ii}} \geq \varsigma_i w_2^{1-\alpha_{ii}} r_2^{\alpha_{ii}}$$
$$\varsigma_i w_3^{1-\alpha_{ii}} r_1^{\alpha_{ii}} \geq \varsigma_i w_3^{1-\alpha_{ii}} r_2^{\alpha_{ii}}$$
$$\Rightarrow \frac{w_1}{w_2} \geq \left(\frac{r_2}{w_2}\frac{w_1}{r_1}\right)^{\alpha_{ii}} \text{ and } \frac{w_3}{w_2} \geq \left(\frac{r_2}{w_2}\frac{w_3}{r_3}\right)^{\alpha_{ii}} \qquad (17)$$

(16) and (17) implies:

$$\left(\frac{r_2}{w_2}\frac{w_1}{r_1}\right)^{\alpha_i} > \frac{w_1}{w_2} \geq \left(\frac{r_2}{w_2}\frac{w_1}{r_1}\right)^{\alpha_{ii}} \text{ and } \left(\frac{r_2}{w_2}\frac{w_3}{r_3}\right)^{\alpha_i} > \frac{w_3}{w_2} \geq \left(\frac{r_2}{w_2}\frac{w_3}{r_3}\right)^{\alpha_{ii}}$$

[17] Remember that Pi is both the price and cost of one unit of i^{th} good.

Since $\frac{w_1}{r_1} > \frac{w_2}{r_2} > \frac{w_3}{r3}$, we have $\frac{r_2}{w_2}\frac{w_1}{r_1} > 1$ and $\frac{r_2}{w_2}\frac{w_3}{r_3} < 1$. Using $\frac{r_2}{w_2}\frac{w_1}{r_1} > 1$ above, we have $\alpha_i > \alpha_{ii}$. However, using $\frac{r_2}{w_2}\frac{w_3}{r_3} < 1$ above, we have $\alpha_i < \alpha_{ii}$. This is a contradiction.

5) 2 and 4 implies that for the ordered factor prices set $\frac{w_1}{r_1} > \frac{w_2}{r_2} > ... > \frac{w_f}{r_f} > .. > \frac{w_F}{r_F}$, we can only have a common good produced for the factor price sets of $f = 1, 2$, $f = 2, 3$ etc. If there are b goods that are produced by countries with different factor prices, this means that $b < F \leq m$.

6) Each country will be in one factor price set. Let us denote the number of countries who has factor prices (w_f, r_f) by m_f, $f = 1, 2, .., F$. Since each country is only in one set, $\sum_{f=1}^{F} m_f = m$.

7) The cost of producing a good is the same in countries that share the same factor prices. Therefore, the firms in these countries can produce the same set of goods without making loss. So, I can index the number of goods produced by f^{th} set by n_f. When there are b goods that are produced in two sets, we have $\sum_{f=1}^{F} n_f = n + b$.

8) A country with factor prices (w_f, r_f) will allocate its factor endowments between n_f goods. Let $(k_{ii}^{j})_f$ and $(l_{ii}^{jj})_f$ denote the capital and labor resources allocated for the production of ii^{th} good by jj^{th} country in f^{th} set where $ii = 1, 2, .., n_f$, $jj = 1, 2, .., m_f$, and $f = 1, .., F$. Similarly, let $(\mathbf{k}^{jj})_f$ and $(\mathbf{l}^{jj})_f$ be the factor endowments of this country.

Notice that I introduced a new way of indexing goods and countries. Now each country will be identified by its (f, jj) indices. There is a one-to-one relationship between j and (f, jj): $j \longleftrightarrow (f, jj)$.

Similarly, a good is identified by (f, ii). However, a good can be produced by more than one group of countries with the same factor prices (actually at most two as we proved in 2). So, the relationship between i to (f, ii) is one-to-many, and (f, ii) to i is many-to-one.

I define the following indicator function for future use:

$I\{(f, ii), i\}$:1 if the good indexed by (f, ii) and i are the same good, 0 otherwise.

Given this new way of indexation, we should have the following factor-availability constraints:

$$\sum_{ii=1}^{n_f} (k_{ii}^{jj})_f = (\mathbf{k}^{jj})_f \qquad (18)$$

$$\sum_{ii=1}^{n_f} (l_{ii}^{jj})_f = (\mathbf{l}^{jj})_f \qquad (19)$$

The factor intensities used should be equal to efficient capital-labor ratios which we have calculated in (14):

$$\frac{(k_i^{jj})_f}{(l_{ii}^j)_f} = \frac{w_f}{r_f} * \frac{(\alpha_{ii})_f}{1-(\alpha_{ii})_f} \qquad (20)$$

$(\alpha_{ii})_f$ stands for the capital share in the production of the ii^{th} good in f^{th} set.

3.3 GENERAL EQUILIBRIUM

1) Supply-Demand Equality Constraints

$$\frac{c_i}{c_1} = \frac{q_i}{q_1} = \left(\frac{P_i}{P_1} * \frac{\theta_1}{\theta_i}\right)^{\frac{1}{p-1}} \;\; i = 2, 3, .., n \qquad (GE1)$$

2) Production side equilibrium

- Factor-availability constraints:

$$\sum_{ii=1}^{n_f} (k_{ii}^{jj})_f = (\mathbf{k}^{jj})_f \qquad jj = 1, 2, .., m_f,\; f = 1, .., F \qquad (GE2)$$

$$\sum_{ii=1}^{n_f} (l_{ii}^{jj})_f = (\mathbf{l}^{jj})_f \qquad jj = 1, 2, .., m_f,\; f = 1, .., F \qquad (GE3)$$

- Efficient factor-intensity usage constraints

$$\frac{(k_i^{jj})_f}{(l_{ii}^j)_f} = \frac{w_f}{r_f} * \frac{(\alpha_{ii})_f}{1-(\alpha_{ii})_f} \qquad ii = 1, 2, .., n_f;\; jj = 1, 2, .., m_f;\; f = 1, .., F \qquad (GE4)$$

- From firm's problem

$$P_i = \varsigma_i w_f^{1-\alpha_i} r_f^{\alpha_i} \qquad \forall i, f \text{ s.t. } \exists ii \text{ with } I\{(f, ii), i\} = 1 \qquad (GE5)$$

- Aggregate quantities produced

$q_i = \sum_{f=1}^{F} \sum_{ii=1}^{n_f} \sum_{jj}^{m_f} I\{(f,ii),i\} * \gamma_i * ((k_{ii}^{jj})_f)^{\alpha_i} * ((l_i^{jj})_f)^{(1-\alpha_i)}$ $i = 1,2,3,..,n$ $(GE6)$

Remark: The equality constraints are necessary but not sufficient conditions for GE. To ensure GE, any solution to above equations should also be checked for the two conditions below, which I call the feasibility conditions:

F.1: $(k_{ii}^{jj})_f \geq 0$, $(l_{ii}^{jj})_f \geq 0$ for $ii = 1,2,...n_f$, $jj = 1,2,..,m_f$, $f = 1,..,F$[18]

F.2: If i is produced by countries in sets $f = a$ and $f = a+1$, i should be the good with the lowest capital intensity in a^{th} set and highest capital-intensity in b^{th} set.

The second condition is required in order to eliminate anomalous solutions of the equations like the case depicted with an L-P diagram below. The case in figure satisfies all production side equality constraints. If it satisfies the equality constraint ensuring supply-demand equality, we may mistakenly conclude that the solution is GE even if it is not. For the case depicted below, the firms in the labor-intense country can produce the second good profitably.

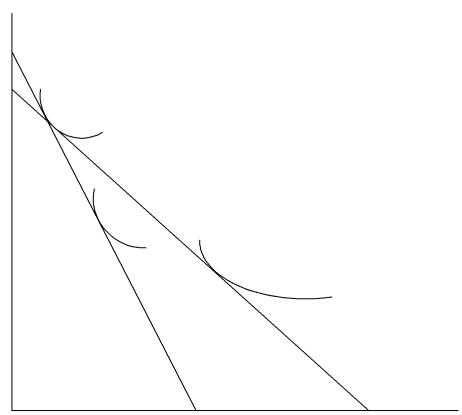

Figure 10: Anomalous solution that might arise if solution is not checked for the second feasibility constraint.

3.4 How many solutions are there?

of unknowns:

capital allocations: $\sum_{f=1}^{F} n_f * m_f$

[18] We cannot have negative factor allocations at equilibrium.

labor allocations: $\sum_{f=1}^{F} n_f * m_f$
good prices: n
factor prices: $2 * F$
quantities produced: n

# of equations

$(GE1) : n - 1$
$(GE2) : \sum_{f=1}^{F} m_f = m$
$(GE3) : \sum_{f=1}^{F} m_f = m$
$(GE4) : \sum_{f=1}^{F} n_f * m_f$
$(GE5) : n + b$
$(GE6) : n$
1 equation to define the numeraire good: e.g.$r_1 = 1\$$

$\#of\ unknowns - \#of\ equations = \Big(2 * \sum_{f=1}^{F} n_f * m_f + 2n + 2F\Big) - \Big(n - 1 + 2m + \sum_{f=1}^{F} n_f * m_f + n + b + n + 1\Big)$

$$= \sum_{f=1}^{F} n_f * m_f + (2F - n - 2m - b) \qquad (21)$$

DOF in production side[19]:

$$= \Big(2 * \sum_{f=1}^{F} n_f * m_f + 2n + 2F\Big) - \Big(2m + \sum_{f=1}^{F} n_f * m_f + n + b + n + 1\Big)$$

$$= \sum_{f=1}^{F} n_f * m_f + (2F - 2m - b - 1) \qquad (22)$$

Note that the dimensionality of equilibriums in the production side of the economy which we determined using L-P diagram in Section 3 for 2-2-2 and 2-3-2 cases are instances of the above equation[20].

- DOF in production side when factor-prices equalize:

$F = 1,\ b = 0,\ n_1 = n,\ m_1 = m :$
$DOF = n * m + 2 - 2m - b - 1$
$\Rightarrow DOF = (n - 2) * m + 1 \qquad (23)$

[19] Without n-1 supply-demand equality constraints.
[20] e.g. 2-3-2 non-specialization case: $F = 1, b = 0, n_1 = 3, m_1 = 2$
$\Rightarrow DOF = 6 + 2 - 4 - 1 = 3$
2-3-2 specialization case with $F = 2, b = 1, n_1 = 2, n_2 = 2, m_1 = 1, m_2 = 1 :$
$\Rightarrow DOF = 2 * 1 + 2 * 1 + 4 - 4 - 1 - 1 = 2$

- Maximum DOF that can be attained in production side when factor-prices do not equalize, i.e., $F > 1$:

$$DOF = \left(2 * \sum_{f=1}^{F} n_f * m_f + 2n + 2F\right) - \left(2m + \sum_{f=1}^{F} n_f * m_f + n + b + n + 1\right)$$
$$\Rightarrow DOF = \sum_{f=1}^{F} n_f * m_f + 2F - 2m - b - 1 \qquad (24)$$

Given F and b, the above term is maximized when $\sum_{f=1}^{F} n_f * m_f$ is maximized. Since $\sum_{f=1}^{F} n_f = n + b$ and $\sum_{f=1}^{F} m_f = m$, the value of $\sum_{f=1}^{F} n_f * m_f$ will be maximized when we choose (n_f, m_f) as $(1, 1)$ for all but one pair. Let us say this is the pair for $f = 1$. Then:

$$\sum_{f=1}^{F} n_f * m_f = (n + b - (F - 1)) + (m - (F - 1)) + F - 1$$
$$= F^2 - 2F + 1 - (F - 1)(m + n + b) + m(n + b) + F - 1$$

Maximized DOF for fixed F and b and for $n > 2$ becomes:

$$= 2F - 2m - b - 1 + (n + b - (F - 1)) + (m - (F - 1)) + F - 1$$
$$= F^2 - 2F + 1 - (F - 1)(m + n + b) + m(n + b) + F - 1$$
$$\Rightarrow DOF = (n-2)*m+1+(\underbrace{(m - F)}_{\geq 0} * \underbrace{(-F + 1 + b)}_{\leq 0} + \underbrace{(F - 1)}_{> 0} * \underbrace{(-n + 2)}_{< 0}) \qquad (25)$$
$$< (n - 2) * m + 1$$

Therefore, the degrees of freedom is more for the factor-price equalization case if $n > 2$, i.e. when the number of factors is more than the number of goods. The number of equilibrium points for factor-price equalization case is larger than the number of equilibrium points when factor-prices do not equalize.

Nevertheless, when we consider only the real equilibria, we see that the number of equilibria in the production side of the economy is the same.[21] The countries who share the same factor prices behave as if they are the same country. The only thing that matters is how this joint country allocates its aggregate factor endowments. How this allocation is allocated between the constituent countries does not matter. Different equilibria as a result of indeterminacy in production do not differ from each other in terms of real economic variables.

In order to calculate the number of real equilibria, we replace m_f by 1 and m by F in the above equations. This way, we do not count indeterminacy of production as different equilibria. We only look at how the aggregate factor endowments of the countries who share common factor prices are allocated. Then, DOF in production side becomes:

[21] What is meant by real equiliria is the equilibria with different aggregate production bundles and factor prices. In this sense, different instances of indeterminacy in production are the same real equilibrium.

$\Big(2 * \sum_{f=1}^{F} n_f * m_f + 2n + 2F\Big) - \Big(2m + \sum_{f=1}^{F} n_f * m_f + n + b + n + 1\Big)$
$= 2 * (n + b) + 2n + 2F - 2F - (n + b) - 2n - b - 1$
$\Rightarrow DOF = n - 1 \qquad (26)$

What we found by (26) is very important. The dimensionality of the set of production side *real* equilibria is the same whether factor prices are equalized or some countries are specialized in production. The dimensionality of the set is the same for different specialization cases too[22]. So, the measures of all these sets are the same and the factor-price equalization is a real possibility[23]. It happens depending on the parameters and factor allocations in the countries.

Note that when we assume additional $n - 1$ restrictions from supply-demand equality constraints, the number of variables and equations is exactly the same. So, we expect a single solution of the above equation systems for any F and b values that we choose. Of course, for some solution to be a valid GE, feasibility constraints should be satisfied too.

4 A Numerical Example, 2-3-2 Case

Note that there are 6 different possibilities of GE for 2-3-2 case, which are:

1.$F = 1, b = 0, n_1 = 3, n_2 = 3$
2.$F = 2, b = 1, n_1 = 3, n_2 = 1$
3.$F = 2, b = 1, n_1 = 2, n_2 = 2$
4.$F = 2, b = 1, n_1 = 1, n_2 = 3$
5.$F = 2, b = 0, n_1 = 2, n_2 = 1$
6.$F = 2, b = 0, n_1 = 1, n_2 = 2$

Figure 11 below depicts these possibilities:

[22] In terms of the notation I introduced above, different specialization instances are different (F, b) bundles where $0 \leq b < F < m$ and $F \neq 1$. Remember that $F = 1$ means factor-price equalization, i.e. non-specialization.

[23] What I mean by real possibility is that it is not a corner-solution which happens with zero probability.

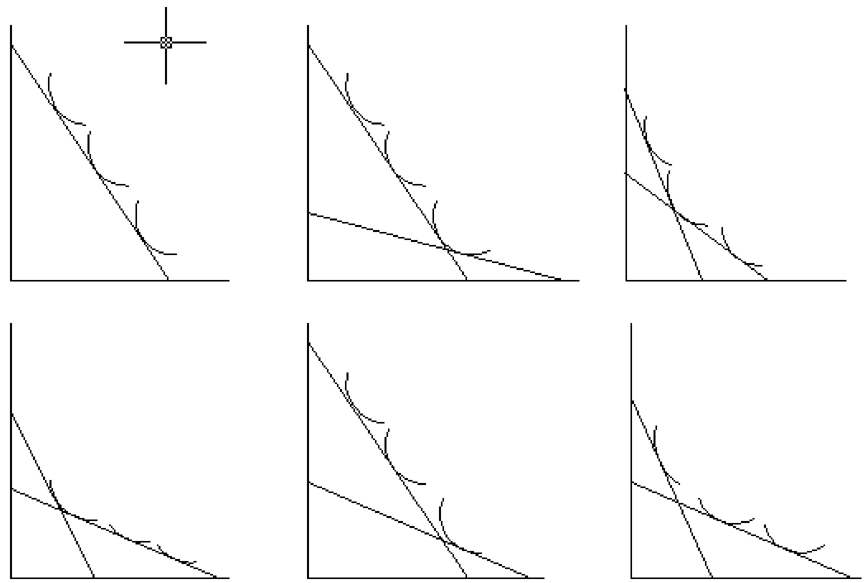

Figure 11: Different production side equilibrium possibilities for 2-3-2 case on L-P diagram.

We are only interested in the first case which is factor-price equalization. I will solve for GE for factor-price equalization using the equation system defined in 3.3:

4.1 GE Equation System

Let us use the cost of capital as numeraire good and fix its dollar price to 1$, i.e. $r = 1$.

Then:

1) Supply-Demand Equality Constraints

$$\frac{q_2}{q_1} = \left(\frac{P_2}{P_1} * \frac{\theta_1}{\theta_2}\right)^{\frac{1}{p-1}} \quad (27)$$

$$\frac{q_3}{q_1} = \left(\frac{P_3}{P_1} * \frac{\theta_1}{\theta_3}\right)^{\frac{1}{p-1}} \quad (28)$$

2) Production side equilibrium

- Factor-availability constraints:

$k_1^1 + k_2^1 + k_3^1 = \mathbf{k}^1 \qquad (29)$
$k_1^2 + k_2^2 + k_3^2 = \mathbf{k}^2 \qquad (30)$
$l_1^1 + l_2^1 + l_3^1 = \mathbf{l}^1 \qquad (31)$
$l_1^2 + l_2^2 + l_3^2 = \mathbf{l}^2 \qquad (32)$

- Efficient factor-intensity usage constraints

$\frac{k_1^1}{l_1^1} = \frac{w}{r} * \frac{\alpha_1}{1-\alpha_1} \qquad (33)$
$\frac{k_2^1}{l_2^1} = \frac{w}{r} * \frac{\alpha_2}{1-\alpha_2} \qquad (34)$
$\frac{k_3^1}{l_3^1} = \frac{w}{r} * \frac{\alpha_3}{1-\alpha_3} \qquad (35)$
$\frac{k_1^2}{l_1^2} = \frac{w}{r} * \frac{\alpha_1}{1-\alpha_1} \qquad (36)$
$\frac{k_2^2}{l_2^2} = \frac{w}{r} * \frac{\alpha_2}{1-\alpha_2} \qquad (37)$
$\frac{k_3^2}{l_3^2} = \frac{w}{r} * \frac{\alpha_3}{1-\alpha_3} \qquad (38)$

- From firm's problem

$P_1 = \varsigma_1 w^{1-\alpha_1} r^{\alpha_1} \qquad (39)$
$P_2 = \varsigma_2 w^{1-\alpha_2} r^{\alpha_2} \qquad (40)$
$P_3 = \varsigma_3 w^{1-\alpha_3} r^{\alpha_3} \qquad (41)$

- Aggregate quantities produced

$q_1 = \gamma_1 * (k_1^1)^{\alpha_1} * (l_1^1)^{(1-\alpha_1)} + \gamma_1 * (k_1^2)^{\alpha_1} * (l_1^2)^{(1-\alpha_1)} \qquad (42)$
$q_2 = \gamma_2 * (k_2^1)^{\alpha_2} * (l_2^1)^{(1-\alpha_2)} + \gamma_2 * (k_2^2)^{\alpha_1} * (l_2^2)^{(1-\alpha_2)} \qquad (43)$
$q_3 = \gamma_3 * (k_3^1)^{\alpha_3} * (l_3^1)^{(1-\alpha_3)} + \gamma_3 * (k_3^2)^{\alpha_3} * (l_3^2)^{(1-\alpha_3)} \qquad (44)$

4.2 SOLUTION OF THE SYSTEM

1) Given w, we can solve for P_1,P_2, and P_3 using (39), (40) and (41):

$P_1 = \varsigma_1 w^{1-\alpha_1} \qquad (45)$
$P_2 = \varsigma_2 w^{1-\alpha_2} \qquad (46)$
$P_3 = \varsigma_3 w^{1-\alpha_3} \qquad (47)$

2) We can find $q2overq1sd = \frac{q_2}{q_1}$ and $q3overq1sd = \frac{q_3}{q_1}$ using P_1,P_2, and P_3 in (27) and (28):

$q2overq1sd = \frac{q_2}{q_1} = \left(\frac{\varsigma_2 w^{1-\alpha_2}}{\varsigma_1 w^{1-\alpha_1}} * \frac{\theta_1}{\theta_2}\right)^{\frac{1}{p-1}} = w^{\frac{\alpha_1-\alpha_2}{p-1}} \left(\frac{\theta_1 \varsigma_2}{\theta_2 \varsigma_1}\right)^{\frac{1}{p-1}} = w^{\frac{\alpha_1-\alpha_2}{p-1}} * \Psi_{21} \qquad (48)$

$q3overq1sd = \frac{q_3}{q_1} = \left(\frac{\varsigma_3 w^{1-\alpha_3}}{\varsigma_1 w^{1-\alpha_1}} * \frac{\theta_1}{\theta_3}\right)^{\frac{1}{p-1}} = w^{\frac{\alpha_1-\alpha_3}{p-1}} \left(\frac{\theta_1 \varsigma_3}{\theta_3 \varsigma_1}\right)^{\frac{1}{p-1}} = w^{\frac{\alpha_1-\alpha_3}{p-1}} * \Psi_{31}$ (49)

where $\Psi_{21} = \left(\frac{\theta_1 \varsigma_2}{\theta_2 \varsigma_1}\right)^{\frac{1}{p-1}}$, $\Psi_{31} = \left(\frac{\theta_1 \varsigma_3}{\theta_3 \varsigma_1}\right)^{\frac{1}{p-1}}$

3) Using (33), (34), (35), (36), (37), and (38), we can find the capital allocations in each country in terms of labor allocations:

$k_1^1 = w * \frac{\alpha_1}{1-\alpha_1} * l_1^1$ (50)
$k_2^1 = w * \frac{\alpha_2}{1-\alpha_2} * l_2^1$ (51)
$k_3^1 = w * \frac{\alpha_3}{1-\alpha_3} * l_3^1$ (52)
$k_1^2 = w * \frac{\alpha_1}{1-\alpha_1} * l_1^2$ (53)
$k_2^2 = w * \frac{\alpha_2}{1-\alpha_2} * l_2^2$ (54)
$k_3^2 = w * \frac{\alpha_3}{1-\alpha_3} * l_3^2$ (55)

Also, efficiency ratios are as:

$e_1^1, e_1^2 = w * \frac{\alpha_1}{1-\alpha_1}$ (56)
$e_2^1, e_2^2 = w * \frac{\alpha_2}{1-\alpha_2}$ (57)
$e_3^1, e_3^2 = w * \frac{\alpha_3}{1-\alpha_3}$ (58)

4) Using (29) & (31), and (30) & (32), we can solve for l_2^1 & l_3^1 and l_2^2 & l_3^2 in terms of l_1^1 and l_1^2 :

$l_1^1 + l_2^1 + l_3^1 = \mathbf{l}^1$
$\Rightarrow l_3^1 = \mathbf{l}^1 - l_1^1 - l_2^1$

Use (50), (51), and (52) in (29) and replace l_3^1 with what we found above:

$$w * \frac{\alpha_1}{1-\alpha_1} * l_1^1 + w * \frac{\alpha_2}{1-\alpha_2} * l_2^1 + w * \frac{\alpha_3}{1-\alpha_3} * (\mathbf{l}^1 - l_1^1 - l_2^1) = \mathbf{k}^1$$

$$\Rightarrow l_2^1 = -\frac{1}{w} \frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} k - \frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} \frac{\alpha_3}{\alpha_3-1} l - l_1^1 \frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} \left(-\frac{\alpha_3}{\alpha_3-1} + \frac{\alpha_1}{\alpha_1-1}\right)$$

Define:

$$\mu_1 = -\frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} \qquad \mu_2 = -\frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} \frac{\alpha_3}{\alpha_3-1} \qquad \mu_3 = -\frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} \left(-\frac{\alpha_3}{\alpha_3-1} + \frac{\alpha_1}{\alpha_1-1}\right)$$

$\Rightarrow$

$l_2^1 = \frac{1}{w} * \mu_1 * \mathbf{k}^1 + \mu_2 * \mathbf{l}^1 + \mu_3 * l_1^1$
$l_3^1 = \mathbf{l}^1 - l_1^1 - \left(\frac{1}{w} * \mu_1 * \mathbf{k}^1 + \mu_2 * \mathbf{l}^1 + \mu_3 * l_1^1\right)$

$l_3^1 = \mathbf{l}^1 - (\mu_3 + 1)l_1^1 - \left(\frac{1}{w} * \mu_1 * \mathbf{k}^1 + \mu_2 * \mathbf{l}^1\right)$
$l_3^1 = (1 - \mu_2)\,\mathbf{l}^1 - (\mu_3 + 1)l_1^1 - \frac{1}{w} * \mu_1 * \mathbf{k}^1$

The calculations will be similar for the second country.

Summary

$l_1^1 = l_1^1$
$l_2^1 = \frac{1}{w} * \mu_1 * \mathbf{k}^1 + \mu_2 * \mathbf{l}^1 + \mu_3 * l_1^1 \qquad (59)$
$l_3^1 = (1 - \mu_2)\,\mathbf{l}^1 - (\mu_3 + 1)l_1^1 - \frac{1}{w} * \mu_1 * \mathbf{k}^1 \qquad (60)$
$l_1^2 = l_1^2$
$l_2^2 = \frac{1}{w} * \mu_1 * \mathbf{k}^2 + \mu_2 * \mathbf{l}^2 + \mu_3 * l_1^2 \qquad (61)$
$l_3^2 = (1 - \mu_2)\,\mathbf{l}^2 - (\mu_3 + 1)l_1^2 - \frac{1}{w} * \mu_1 * \mathbf{k}^2 \qquad (62)$

where:
$\mu_1 = -\frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} \qquad \mu_2 = -\frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}}\frac{\alpha_3}{\alpha_3-1} \qquad \mu_3 = -\frac{1}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}}\left(-\frac{\alpha_3}{\alpha_3-1} + \frac{\alpha_1}{\alpha_1-1}\right)$

5) Quantities produced are:

$q_{11} = \gamma_1 * (w * \frac{\alpha_1}{1-\alpha_1} * l_1^1)^{\alpha_1} * (l_1^1)^{(1-\alpha_1)} = w^{\alpha_1} * \gamma_1(\frac{\alpha_1}{1-\alpha_1})^{\alpha_1} * l_1^1$
$\Rightarrow q_{11} = \Phi_1 * w^{\alpha_1} * l_1^1 \qquad (63)$

$q_{12} = \gamma_1 * (w * \frac{\alpha_1}{1-\alpha_1} * l_1^2)^{\alpha_1} * (l_1^2)^{(1-\alpha_1)} = w^{\alpha_1} * \gamma_1(\frac{\alpha_1}{1-\alpha_1})^{\alpha_1} * l_1^2$
$\Rightarrow q_{12} = \Phi_1 * w^{\alpha_1} * l_1^2 \qquad (64)$

$q_{21} = \gamma_2 * (w * \frac{\alpha_2}{1-\alpha_2} * l_2^1)^{\alpha_2} * (l_2^1)^{(1-\alpha_2)} = w^{\alpha_2} * \gamma_2(\frac{\alpha_2}{1-\alpha_2})^{\alpha_2} * l_2^1$
$\Rightarrow q_{21} = \Phi_2 * w^{\alpha_2} * l_2^1 \qquad (65)$

$q_{22} = \gamma_2 * (w * \frac{\alpha_2}{1-\alpha_2} * l_2^2)^{\alpha_1} * (l_2^2)^{(1-\alpha_2)} = w^{\alpha_2} * \gamma_2(\frac{\alpha_2}{1-\alpha_2})^{\alpha_2} * l_2^2$
$\Rightarrow q_{22} = \Phi_2 * w^{\alpha_2} * l_2^2 \qquad (66)$

$q_{31} = \gamma_3 * (w * \frac{\alpha_3}{1-\alpha_3} * l_3^1)^{\alpha_3} * (l_3^1)^{(1-\alpha_3)} = w^{\alpha_3} * \gamma_3(\frac{\alpha_3}{1-\alpha_3})^{\alpha_3} * l_3^1$
$\Rightarrow q_{31} = \Phi_3 * w^{\alpha_3} * l_3^1 \qquad (67)$

$q_{32} = \gamma_3 * (w * \frac{\alpha_3}{1-\alpha_3} * l_3^2)^{\alpha_3} * (l_3^2)^{(1-\alpha_3)} = w^{\alpha_3} * \gamma_3(\frac{\alpha_3}{1-\alpha_3})^{\alpha_3} * l_3^2$
$\Rightarrow q_{32} = \Phi_3 * w^{\alpha_3} * l_3^2 \qquad (68)$

where $\Phi_1 = \gamma_1(\frac{\alpha_1}{1-\alpha_1})^{\alpha_1}, \Phi_2 = \gamma_2(\frac{\alpha_2}{1-\alpha_2})^{\alpha_2}, \Phi_3 = \gamma_3(\frac{\alpha_3}{1-\alpha_3})^{\alpha_3}$

7) Find the ratios of productions, equate to what you found in 2):

$$\frac{q_{21}+q_{22}}{q_{11}+q_{12}} = \frac{\Phi_2 * w^{\alpha_2}}{\Phi_1 * w^{\alpha_1}} * \frac{l_2^1 + l_2^2}{l_1^1 + l_1^2} = \frac{\Phi_2 * w^{\alpha_2}}{\Phi_1 * w^{\alpha_1}} * \frac{\left(\frac{1}{w} * \mu_1 * \mathbf{k}^1 + \mu_2 * \mathbf{l}^1 + \mu_3 * l_1^1\right) + \left(\frac{1}{w} * \mu_1 * \mathbf{k}^2 + \mu_2 * \mathbf{l}^2 + \mu_3 * l_1^2\right)}{l_1^1 + l_1^2}$$

$$= \frac{\Phi_2 * w^{\alpha_2}}{\Phi_1 * w^{\alpha_1}} * \left(\frac{1}{w}\mu_1 \frac{\mathbf{k}^1+\mathbf{k}^2}{l_1^1+l_1^2} + \mu_2 \frac{\mathbf{l}^1+\mathbf{l}^2}{l_1^1+l_1^2} + \mu_3\right)$$

$$\Rightarrow w^{\frac{\alpha_1-\alpha_2}{p-1}} * \Psi_{21} = \frac{\Phi_2 * w^{\alpha_2}}{\Phi_1 * w^{\alpha_1}} * \left(\frac{1}{w}\mu_1 \frac{\mathbf{k}^1+\mathbf{k}^2}{l_1^1+l_1^2} + \mu_2 \frac{\mathbf{l}^1+\mathbf{l}^2}{l_1^1+l_1^2} + \mu_3\right)$$

$$\left(\frac{w^{\frac{\alpha_1-\alpha_2}{p-1}} * \Psi_{21}}{\frac{\Phi_2 * w^{\alpha_2}}{\Phi_1 * w^{\alpha_1}}} - \mu_3\right) * \left(l_1^1 + l_1^2\right) = \frac{1}{w}\mu_1 \left(\mathbf{k}^1 + \mathbf{k}^2\right) + \mu_2 \left(\mathbf{l}^1 + \mathbf{l}^2\right)$$

$$\Rightarrow$$

$$l_1^1 + l_1^2 = \frac{\frac{1}{w}\mu_1\left(\mathbf{k}^1+\mathbf{k}^2\right)+\mu_2\left(\mathbf{l}^1+\mathbf{l}^2\right)}{\frac{w^{\frac{\alpha_1-\alpha_2}{p-1}} * \Psi_{21}}{\frac{\Phi_2 * w^{\alpha_2}}{\Phi_1 * w^{\alpha_1}}} - \mu_3}$$

$$\frac{q_3}{q_1} = \frac{q_{31}+q_{32}}{q_{11}+q_{12}} = \frac{\Phi_3 * w^{\alpha_3} * l_3^1 + \Phi_3 * w^{\alpha_3} * l_3^2}{\Phi_1 * w^{\alpha_1} * l_1^1 + \Phi_1 * w^{\alpha_1} * l_1^2}$$

$$= \frac{\Phi_3 * w^{\alpha_3}}{\Phi_1 * w^{\alpha_1}} * \frac{l_3^1+l_3^2}{l_1^1+l_1^2} = \frac{\Phi_3 * w^{\alpha_3}}{\Phi_1 * w^{\alpha_1}} * \frac{\left((1-\mu_2)\mathbf{l}^1-(\mu_3+1)l_1^1-\frac{1}{w}*\mu_1*\mathbf{k}^1\right)+\left((1-\mu_2)\mathbf{l}^2-(\mu_3+1)l_1^2-\frac{1}{w}*\mu_1*\mathbf{k}^2\right)}{l_1^1+l_1^2}$$

$$\Rightarrow l_1^1 + l_1^2 = \frac{-\frac{1}{w}\mu_1\left(\mathbf{k}^1+\mathbf{k}^2\right)+(1-\mu_2)\left(\mathbf{l}^1+\mathbf{l}^2\right)}{\frac{w^{\frac{\alpha_1-\alpha_3}{p-1}} * \Psi_{31}}{\frac{\Phi_3 * w^{\alpha_3}}{\Phi_1 * w^{\alpha_1}}} + (\mu_3+1)}$$

So, we have:

$$\Rightarrow l_1 = l_1^1 + l_1^2 = \frac{-\frac{1}{w}\mu_1\left(\mathbf{k}^1+\mathbf{k}^2\right)+(1-\mu_2)\left(\mathbf{l}^1+\mathbf{l}^2\right)}{\frac{w^{\frac{\alpha_1-\alpha_3}{p-1}} * \Psi_{31}}{\frac{\Phi_2 * w^{\alpha_3}}{\Phi_1 * w^{\alpha_1}}} + (\mu_3+1)} = \frac{\frac{1}{w}\mu_1\left(\mathbf{k}^1+\mathbf{k}^2\right)+\mu_2\left(\mathbf{l}^1+\mathbf{l}^2\right)}{\frac{w^{\frac{\alpha_1-\alpha_2}{p-1}} * \Psi_{21}}{\frac{\Phi_2 * w^{\alpha_2}}{\Phi_1 * w^{\alpha_1}}} - \mu_3} \tag{69}$$

The only unknown in this equation is w. Unfortunately, we cannot solve this equation explicitly for w. We need to use numerical methods to calculate it. Once we solve for w, we determine $l_1 = l_1^1 + l_1^2$. Notice that we cannot solve for l_1^1 and l_1^2, but only for their sum. This is the so-called indeterminacy of production. What matters is what the sums of factors allocated for the production of each good. It does not matter who allocates how much. As you will see below, once we solve for l_1, we can also solve for $l_2 = l_2^1 + l_2^2$ and $l_3 = l_3^1 + l_3^2$ but not for l_2^1, l_2^2, l_3^1 and l_3^2:

$$l_2^1 = \frac{1}{w} * \mu_1 * \mathbf{k}^1 + \mu_2 * \mathbf{l}^1 + \mu_3 * l_1^1$$
$$l_2^2 = \frac{1}{w} * \mu_1 * \mathbf{k}^2 + \mu_2 * \mathbf{l}^2 + \mu_3 * l_1^2$$
$$\Rightarrow l_2 = l_2^1 + l_2^2 = \frac{1}{w} * \mu_1 \left(\mathbf{k}^1 + \mathbf{k}^2\right) + \mu_2 \left(\mathbf{l}^1 + \mathbf{l}^2\right) + \mu_3 \left(l_1^1 + l_1^2\right) \tag{70}$$

$$l_3^1 = (1 - \mu_2)\,\mathbf{l}^1 - (\mu_3 + 1)l_1^1 - \frac{1}{w} * \mu_1 * \mathbf{k}^1$$
$$l_3^2 = (1 - \mu_2)\,\mathbf{l}^2 - (\mu_3 + 1)l_1^2 - \frac{1}{w} * \mu_1 * \mathbf{k}^2$$
$$\Rightarrow l_3 = l_3^1 + l_3^2 = (1-\mu_2)\left(\mathbf{l}^1 + \mathbf{l}^2\right) - \frac{1}{w}\mu_1(\mathbf{k}^1+\mathbf{k}^2) - (\mu_3+1)\left(l_1^1 + l_1^2\right) \tag{71}$$

4.3 ALGORITHM TO SOLVE FOR GE WITH FACTOR-PRICE EQUALIZATION

Given all the parameters and factor endowments of the countries, we can solve for the factor-price equalization w using numerical procedure in (69) as above. Then, we also determine the aggregate factor allocations to each good, first (l_1, l_2, l_3) by (69), (70), and (71) and then (k_1, k_2, k_3) using (50)+(53), (51)+(54) and (52) + (55). However, we need to fix one factor allocation in some country, e.g. l_1^1, to determine the remaining factor allocations, i.e. l_2^1, l_3^1, l_1^2, l_2^2, l_3^2, k_1^1, k_2^1, k_3^1, k_1^2, k_2^2 and k_3^2.

Nonetheless, the solution set that we find might not constitute a valid equilibrium. We should also check for the feasibility constraints which are stated by *F.1* and *F.2* in 3.3. *F.2* is not relevant to factor-price equalization case. However, we should check for non-negativity of factor allocations. At GE, $(l_1^1, l_1^2, l_2^1, l_2^2, l_3^1, l_3^2)$ should be all non-negative[24].

In order to write an algorithm that checks whether there is a GE where factor-prices are equalized, we should first determine the relationships between factor allocations. A sign analysis of μ_3 indicates[25]:

$$\mu_3 = -\frac{\frac{\alpha_1}{\alpha_1-1} - \frac{\alpha_3}{\alpha_3-1}}{\frac{\alpha_2}{\alpha_2-1} - \frac{\alpha_3}{\alpha_3-1}} = -\frac{\frac{\alpha_1*\alpha_3-\alpha_1-\alpha_1*\alpha_3+\alpha_3}{(\alpha_1-1)*(\alpha_3-1)}}{\frac{\alpha_2*\alpha_3-\alpha_2-\alpha_2*\alpha_3+\alpha_3}{(\alpha_2-1)(\alpha_3-1)}} = -\frac{\frac{-\alpha_1+\alpha_3}{(\alpha_1-1)}}{\frac{-\alpha_2+\alpha_3}{(\alpha_2-1)}}$$

$$= -\frac{(\alpha_2-1)}{(\alpha_1-1)}\frac{-\alpha_1+\alpha_3}{-\alpha_2+\alpha_3} < -1$$

So, both μ_3 and $\mu_3 + 1$ are negative. Using this information in the equations above, we determine the following relationships between factor allocations:

$$l_2^1 \downarrow, l_3^1 \uparrow, l_1^1 \uparrow$$
$$l_2^2 \downarrow, l_3^2 \uparrow, l_1^2 \uparrow$$

Since we can solve for $l_2 = l_2^1 + l_2^2$ given the factor allocations and parameters, we have the following relationship between l_2^1 and other factor allocations:

$$l_2^1 \downarrow \Rightarrow l_3^1 \uparrow, l_1^1 \uparrow, l_2^2 \uparrow, l_3^2 \downarrow, l_1^2 \downarrow$$

Using this relationship, we can use the following algorithm to check whether factor-prices equalize or not given factor endowments and parameters:

[24] We do not need to check additionally for non-negativity of $(k_1^1, k_1^2, k_2^1, k_2^2, k_3^1, k_3^2)$. This will automatically be satisfied when $(l_1^1, l_1^2, l_2^1, l_2^2, l_3^1, l_3^2)$ is non-negative since efficient factor-intensity ratios are positive.

[25] Remember that $\alpha_1 > \alpha_2 > \alpha_3$ by definition.

ALGORITHM

1) Solve for w in (69) using numerical methods. Then calculate l_1, l_2, l_3 using (69), (70), and (71) :

2) Check whether $l_1 > 0$, $l_2 > 0$, and $l_3 > 0$. If not, then there is no factor-price equalization If yes, go step 3).

3) Assign $l_2^1 = \mathbf{l}^1$ and solve for .l_1^2 and l_3^2[26]:

$l_2^2 = l_2 - l_2^1 = l_2 - \mathbf{l}^1$

Use (62), (31) and l_2^2 to find l_1^2 and l_3^2.

$$\Rightarrow l_1^2 = \frac{l_2 - \mathbf{l}^1 - \frac{1}{w} * \mu_1 * \mathbf{k}^2 - \mu_2 * \mathbf{l}^2}{\mu_3}$$
$$\Rightarrow l_3^2 = \mathbf{l}^2 - l_2 + \mathbf{l}^1 - \frac{l_2 - \mathbf{l}^1 - \frac{1}{w} * \mu_1 * \mathbf{k}^2 - \mu_2 * \mathbf{l}^2}{\mu_3}$$

Check $l_1^2 > 0$ and $l_3^2 > 0$. If not, then there is no factor-price equalization equilibrium. If yes, go step 4).

4) Solve for $l_2^1 = Boundary1$ for which $l_1^2 = 0$. Also, solve for $l_2^1 = Boundary2$ for which $l_3^2 = 0$. Calculate $Boundary = \max(Boundary1, Boundary2, 0)$[27]:

From (62):

$$l_2^2 = \frac{1}{w} * \mu_1 * \mathbf{k}^2 + \mu_2 * \mathbf{l}^2 + \mu_3 * l_1^2 = \frac{1}{w} * \mu_1 * \mathbf{k}^2 + \mu_2 * \mathbf{l}^2$$

Then:

$$Boundary1 = l_2^1 = l_2 - l_2^2 = l_2 - \frac{1}{w} * \mu_1 * \mathbf{k}^2 - \mu_2 * \mathbf{l}^2$$

From (63):

$$l_3^2 = 0 = (1 - \mu_2)\, \mathbf{l}^2 - (\mu_3 + 1) l_1^2 - \frac{1}{w} * \mu_1 * \mathbf{k}^2$$

$$\Rightarrow (\mu_3 + 1) l_1^2 = (1 - \mu_2)\, \mathbf{l}^2 - \frac{1}{w} * \mu_1 * \mathbf{k}^2$$

$$l_1^2 = \frac{(1 - \mu_2)}{(\mu_3 + 1)} \mathbf{l}^2 - \frac{\frac{1}{w} * \mu_1}{(\mu_3 + 1)} * \mathbf{k}^2$$

$$l_1^1 = l_1 - l_1^2 = l_1 - \frac{(1 - \mu_2)}{(\mu_3 + 1)} \mathbf{l}^2 + \frac{\frac{1}{w} * \mu_1}{(\mu_3 + 1)} * \mathbf{k}^2$$

[26] As l_2^1 increases, l_1^2 and l_3^2 increases. So, given w, l_1^2 and l_3^2 attains their maximum values for the largest value that l_2^1 can take, which is $\mathbf{l}^1$. If the values of l_1^2 and l_3^2 are negative for $l_2^1 = \mathbf{l}^1$, then there is no way to make them positive for w.

[27] As $l_2^1 \downarrow \Rightarrow l_1^2 \downarrow$ and $l_3^2 \downarrow$. So, *Boundary* is the smallest value that we can assign l_2^1 for which l_2^1, l_1^2 and l_3^2 are all non-negative.

When we use l_1^1 in (60):

$$l_2^1 = \frac{1}{w}*\mu_1*\mathbf{k}^1+\mu_2*\mathbf{l}^1+\mu_3*l_1^1 = \frac{1}{w}*\mu_1*\mathbf{k}^1+\mu_2*\mathbf{l}^1+\mu_3*\left(l_1 - \frac{(1-\mu_2)}{(\mu_3+1)}\mathbf{l}^2 + \frac{\frac{1}{w}*\mu_1}{(\mu_3+1)} * \mathbf{k}^2\right)$$
$$\Rightarrow Boundary2 = \frac{1}{w} * \mu_1 * \mathbf{k}^1 + \mu_2 * \mathbf{l}^1 + \mu_3 * \left(l_1 - \frac{(1-\mu_2)}{(\mu_3+1)}\mathbf{l}^2 + \frac{\frac{1}{w}*\mu_1}{(\mu_3+1)} * \mathbf{k}^2\right)$$

6) For $l_2^1 = Boundary$, calculate l_3^1, l_1^1, l_2^2. Check $l_3^1 > 0, l_1^1 > 0, l_2^2 > 0$[28] :

Use $l_2^1 = Boundary$ in (60):

$$\Rightarrow l_1^1 = \frac{l_2^1 - \frac{1}{w}*\mu_1*\mathbf{k}^1 - \mu_2*\mathbf{l}^1}{\mu_3} = \frac{Boundary - \frac{1}{w}*\mu_1*\mathbf{k}^1 - \mu_2*\mathbf{l}^1}{\mu_3}$$

Then:

$$l_1^2 = l_1 - l_1^1 = l_1 - \frac{l_2^1 - \frac{1}{w}*\mu_1*\mathbf{k}^1 - \mu_2*\mathbf{l}^1}{\mu_3}$$

From (61):

$$l_3^1 = (1-\mu_2)\,\mathbf{l}^1 - \frac{1}{w} * \mu_1 * \mathbf{k}^1 - (\mu_3+1)l_1^1$$
$$\Rightarrow l_3^1 = (1-\mu_2)\,\mathbf{l}^1 - \frac{1}{w} * \mu_1 * \mathbf{k}^1 - (\mu_3+1)\frac{Boundary - \frac{1}{w}*\mu_1*\mathbf{k}^1 - \mu_2*\mathbf{l}^1}{\mu_3}$$

From (62):

$$\Rightarrow l_2^2 = \frac{1}{w} * \mu_1 * \mathbf{k}^2 + \mu_2 * \mathbf{l}^2 + \mu_3 * l_1^2 = l_2 - l_2^1$$

Check whether $l_3^1 > 0, l_1^1 > 0$, and $l_2^2 > 0$. If not, then there is no factor-price equalization equilibrium. If yes, there is factor price equalization.

4.4 COMPARATIVE STATISTICS

In an autarky, there is a single goods market and a single factors market. In Heckscher-Ohlin model, while the goods market is common, there are different factors markets in each country. What factor-price equalization theorem says is that the common goods market assumption might ensure the equality of factor prices in different countries even if the factors markets are not the same.

The common goods market always works in a way to reduce the differences between factor prices in different countries. However, whether its influence will be strong enough to ensure factor-price equality depends on endowments,

[28] As l_2^1 decreases, l_3^1, l_1^1 and l_2^2 increases. Since $Boundary$ is the smallest value we can assign for l_2^1, l_3^1, l_1^1 and l_2^2 should be all non-negative for $l_2^1 = Boundary$. If not, it means that there is no GE for w.

demand and production functions. When can factor prices be equalized even if the factors markets are separate? This will happen when the mobility of goods removes the necessity for factors to move, i.e., when factor-immobility is no longer a restriction on the system. In that case, we can first solve for GE in the autarkic world[29] and then allocate the aggregate production bundle in the world among the constituent countries in a way so that each country uses exactly its factor endowments to produce the bundle which it is allocated.

Then, the question we should answer becomes: When is the mobility of goods good enough to ensure factor-price equality? In other words, what should be the characteristics of endowments, demand and production functions so that factor prices in different countries are equalized?

The question is answered for endowments. Factor prices in two countries will be equalized if the factor endowment ratios in the two countries are close enough. In order to understand the intuition behind this proposition, let us think about the extreme instance of it which is the equality of factor endowment ratios between all countries. Notationally, if (K_i, L_i) and $(\overline{K}, \overline{L})$ are the factor endowments in i^{th} country and world respectively, equality of factor endowments in each country corresponds to $\frac{K_i}{L_i} = \frac{\overline{K}}{\overline{L}} = \pi$[30] for all i. Then, if the autarky production amounts in the world are $(q_1, q_2, ..., q_n)$ and $\frac{K_i}{\overline{K}} = \frac{L_i}{\overline{L}} = \kappa_i$, we can assign i^{th} country $(\kappa_i q_1, \kappa_i q_2, ..., \kappa_i q_n)$. i will produce exactly this bundle at factor prices equal to world autarkic factor prices. Therefore, factors do not need to move when the factor endowment ratios in each country is the same, and we will always have factor-price equalization.

If we slightly shift from the exact equality case, we can still obtain factor-price equality by changing production bundles allocated to each country. For instance, if factor-intensity ratio in a country is slightly more than π, it should be assigned slightly more from a good which uses capital more intensely. So, we will still be able to obtain factor-price equality. However, if the factor endowment ratios in each country becomes sufficiently different, we will no longer be able to make this adjustment feasibly.

I will use the above property to make comparative statistics analysis in my 2-3-2 example. We know that factor-prices will always equalize if the factor endowments between the two countries are close enough. However, how close they should be depends on demand and production functions. In this section, I fix the capital and labor endowments in the first country (K_1 and L_1), which is defined to be the capital-intense country. I also fix the labor endowment in the second country (L_2), which is defined to be the labor-intense country. Then, for the remaining parameter set, I solve for the minimum capital endowment in the

[29] One country with factor endowments being the sums of factor endowments in each country, and demand being the aggregated demand in the world.

[30] π is the capital/labor factor endowment ratio in the world.

second country (K_2) for which factor prices are equalized. I name this critical value *Critical* K_2. Then, there will be factor-price equalization for K_2 values in [*Critical* $K_2, \frac{K_1}{L_1}L_2$] interval.

In order to find *Critical* K_2, I define a grid for K_2 which is dense enough. Then, I calculate whether factor prices are equalized at each point on the grid using the algorithm presented in Section 3.4. Then, I assign *Critical* K_2 as the minimum K_2 on the grid where factor prices are equalized.

The plots presented in this section show the relationship between *Critical* K_2 and some parameters. All relevant algorithm is written and calculations are done by MATLAB. For each plot, I fix the remaining parameters and change only the value of the relevant parameter. If *Critical* K_2 value increases as one parameter value changes, it means that the factor endowment ratios should be closer for factor-price equalization. So, the change makes factor-price equalization more unlikely[31].

4.4.1 Changes in α

Figure 12 shows how *Critical* K_2 changes as α_1, α_2, and α_3 changes. As it can be seen from the figures, factor-price equalization case becomes more likely when α_1 and α_3 gets closer to their limits, which are 1 and 0 respectively. The relationship between α_2 and *Critical* K_2 is not strict. Factor-price equalization becomes more unlikely as α_2 gets closer to some intermediate value, and it becomes more likely as α_2 goes away from this intermediate value. The result is clear. As capital's share in the production functions of different goods become more dissimilar, the probability of getting factor-price equalization increases.

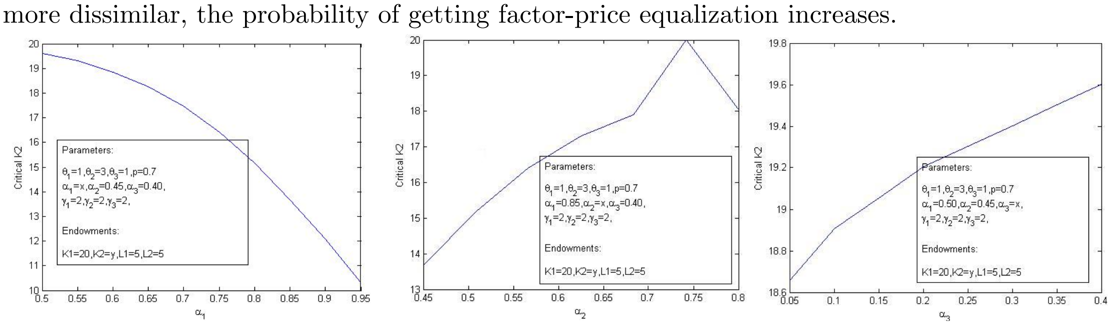

Figure 12: α and *Critical* K_2

[31] In this section, I frequently use the terms "likely" and "unlikely". Actually, given factor endowments and the parameter set, there is no sense in talking about the"likelihood" of factor-price equalization or not. Factor prices are either equalized or not. However, when the capital allocation in the second country is unknown, we can talk about the likelihood of factor-price equalization. As one parameter changes, *Critical* K_2 value increases or decreases. Therefore, for random K_2, the likelihood of factor-price equalization changes as one parameter value changes.

This becomes more understandable if we think of the extreme case, which is having goods with only capital's shares of 1 or 0 in the production functions. Then, the production functions are $f(K, L) = \gamma_i K$ and $f(K, L) = \gamma_j L$, where $\alpha_i = 1$ and $\alpha_j = 0$ If the prices of the goods are P_i and P_j, we will have $r = P_i \gamma_i$ and $w = P_j \gamma_j$ in both countries, whatever the factor endowments are. So, factor-price equalization is guaranteed.

Figure 13 shows more clearly how the probability of getting factor-price equalization increases as the capital shares of the goods become more dissimilar. In this figure, α_2 is fixed and α_1 and α_3 are assigned $\alpha_2 + x$ and $\alpha_2 - x$ respectively. As it is clear from the figure, *Critical* K_2 decreases as α values become more dissimilar. For very similar α values[32], *Critical* K_2 becomes very close to 20 and we need almost factor endowment ratios to be equal in order to get factor-price equalization.

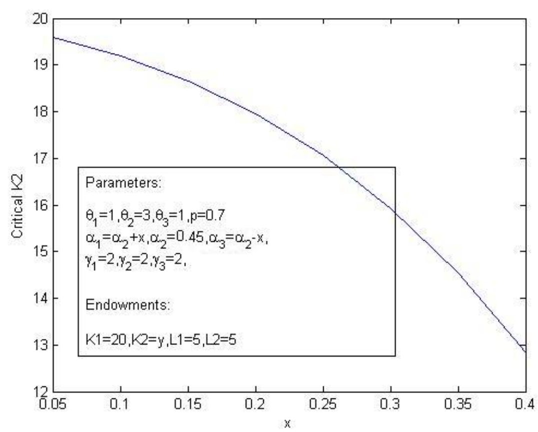

Figure 13: α similarities and *Critical* K_2

4.4.2 Changes in p

We know from Economic Theory that $\frac{1}{1-p}$ is the elasticity of substitution for CES utility function. So, an increase in p means an increase in elasticity of substitution. Figure 14 shows the relationship between p and *Critical* K_2. As p goes 1, this means that the substitutability of the goods increases. Figure 14 shows that as the goods become more substitutable, factor-price equalization case becomes more unlikely.

[32] When x is close to zero.

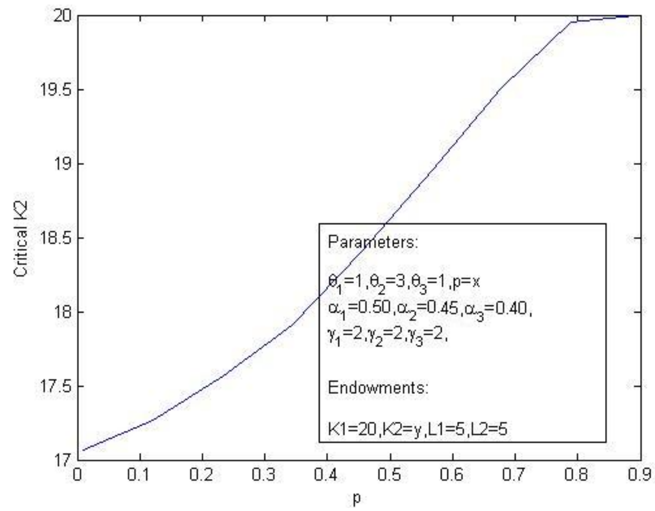

Figure 14: p and *Critical* K_2

This result is intuitive. As the goods become more substitutable, the willingness of countries to trade goods will decrease. The goods missing in a country can be more easily substituted by the ones it produces in excess. Therefore, less goods will be traded and the equilibrium will resemble more to the case where each country is an autarky. For large values of p, factor prices are unlikely to be equalized unless the factor endowments in the two countries are very similar.

4.4.3 Changes in γ

Figure 15 shows how changes in total factor productivities (TFPs) change *Critical* K_2. While increases in total factor productivities of the first and the third good decrease *Critical* K_2 and work on behalf of factor-price equalization, the case is on contrary for the second good.

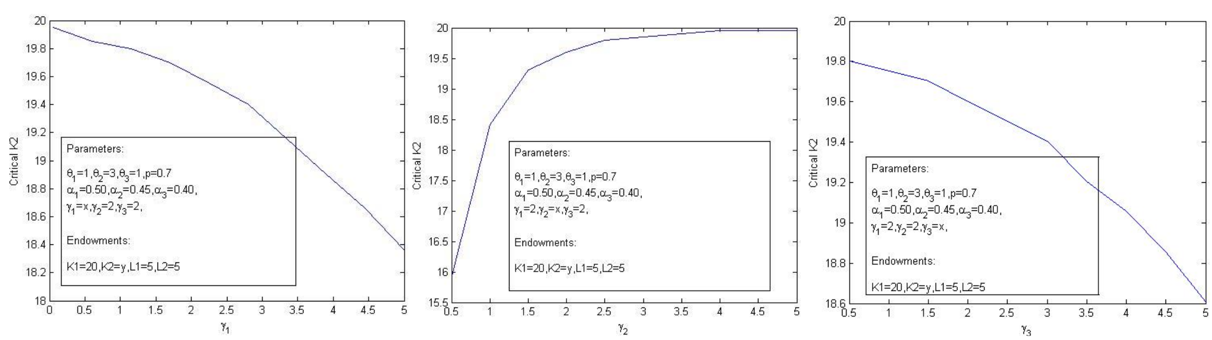

Figure 15: γ and *Critical* K_2

The improvements in technologies (TFPs) to produce extreme goods increase the likelihood of factor-price equalization. However, the technological improvements to produce the intermediary goods make factor-price equalization more unlikely.

4.4.4 Changes in θ

Figure 16 shows the relationships between θ values and *Critical* K_2. These relationships are the most difficult to interpret. As it can be seen, while the increase in θ_2 decreases the probability of attaining factor-price equalization, the relationships for θ_1 and θ_3 are not strict.

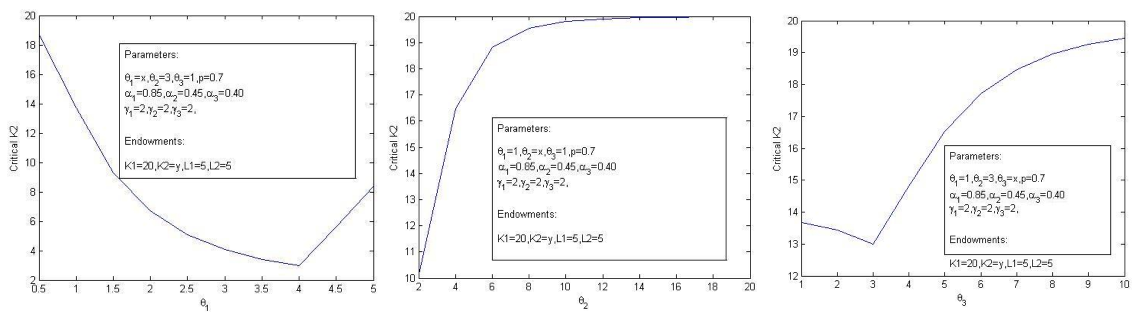

Figure 16: θ and *Critical* K_2

As it is clear from the utility function, an increase in θ means stronger demand for that good. The second plot in Figure 16 shows that stronger demand for intermediate goods makes factor-price equalization unlikely. The first and third plots show that an increase in demand for the extreme goods might or might not increase the probability of getting factor-price equalization.

If θ_1 is large enough to force the first country to completely specialize in producing the first good, the probability of getting factor-price equalization decreases if θ_1 increases. A rise in θ_1 only encourages the first country further to specialize in the first good. So, *Critical* K_2 increases. However, if θ_1 is not large enough to force the first country to produce just the first good, the increase in θ_1 will increase r and decrease w in the first country since the demand for good 1 stronger and good 1 is capital intensive. This will decrease the factor price differentials between the two countries, and finally lead factor-price equalization. The argument is the similar for the third good.

The relationship is strict for the second good. If the demand for the second good decreases, its price will decrease. The second good is relatively labor intensive for the first country and capital intensive for the second country. As its price decreases, the first country shifts its production to the first good and the second country shifts its production to the third good. Therefore, the capital will be more valuable in the first country and labor will be more valuable in the second country. The factor price differentials will decrease and ultimately lead to factor-price equalization[33].

5 Conclusion

The exact factor-price equalization case is not practically so important. We know that factor-prices in the world are not equalized. Most of the assumptions of the naive Heckscher-Ohlin model which I analyzed above are not right. The production technologies are neither exactly constant returns to scale, nor the same between different countries. Moreover, there are many trade imperfections in the real world like transportation costs, trade barriers etc.

However, the conditions that make factor-price equalization case more likely are the same conditions that decrease factor-price differentials between the countries. Therefore, when we talk about the conditions that make factor-price equalization case more likely, we are also talking about the conditions which decrease the wage differentials throughout the world.

This paper investigates the likelihood of factor-price equalization under the simple assumptions of Heckscher-Ohlin Theory. Factor-price equalization is also directly related to whether countries specialize or not in the global market.

In the second section of the paper, L-P diagram is introduced and it is showed that it can be used effectively to demonstrate production side equilibria. It is also showed that there are different possibilities of equilibria, namely factor-price equalization and specialization, when we look at only the production side of an economy.

In the third section, GE problem is solved for 2-m-n case for naive Heckscher-Ohlin model and introducing homogenous CES utility function to define demand in the economy. It is demonstrated that the set of equilibrium possibilities for factor-price equalization case is much larger if the number of commodities is more than the number of factors of production. However, the larger possibilities do not refer to different real equilibria, but only to indeterminacy in production. If we exclude indeterminacy of production from our analysis, we see that the measure of GE sets for factor-price equalization and non-equalization cases are the same, which is actually zero. Therefore, there is only a unique solution of the equation system defining GE for factor price equalization case and different specialization cases. However, being a solution to the equation system does not

[33] Remember that for factor-price non-equalization cases, capital is cheaper and labor is more expensive in the first country since it is capital abundant and labor deficient by definition. So, a change which inreases capital's value in the first country and labor's value in the second country will decrease the factor price differentials between the two countries.

guarantee that the solution is a valid GE. The feasibility constraints should also be satisfied, which are presented in this section.

In the fourth section, 2-3-2 case is solved numerically. The comparative statistics analysis of this example implied the following results about the likelihood of factor-price equalization:

- The probability of getting factor-price equalization increases if the factor endowment ratios of the countries become closer.
- The probability of getting factor-price equalization increases if capital's share in the production functions of different goods become more dissimilar.
- The probability of getting factor-price equalization decreases if the goods become more substitutable.
- The improvements in technologies (TFPs) to produce extreme goods increase the likelihood of factor-price equalization. However, the technological improvements to produce the intermediary goods decrease the likelihood of factor-price equalization..
- The probability of getting factor-price equalization decreases if demand for the intermediary goods increases. The relationship for the extreme goods changes depending on the parameters and endowments.

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