Shareholders Unanimity With Incomplete Markets*

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Abstract

Macroeconomic models with heterogeneous agents and incomplete markets (e.g. Krusell and Smith, 1998) usually assume that consumers, rather than firms, own and accumulate physical capital. This assumption, while convenient, is without loss of generality only if the asset market is complete. When financial markets are incomplete, shareholders will in general disagree on the optimal level of investment to be undertaken by the firm. This paper derives conditions under which shareholders unanimity obtains in equilibrium despite the incompleteness of the asset market. In the general equilibrium economy analyzed here consumers face idiosyncratic labor income risk and trade firms' shares in the stock market. A firm's shareholders decide how much of its earnings to invest in physical capital and how much to distribute as dividends. The return on a firm's capital investment is affected by an aggregate productivity shock. The paper contains two main results. First, if the production function exhibits constant returns to scale and short-sales constraints are not binding, then in a competitive equilibrium a firm's shareholders will unanimously agree on the optimal level of investment. Thus, the allocation of resources in this economy is the same as in an economy where consumers accumulate physical capital directly. Second, when short-sales constraints are binding, instead, the unanimity result breaks down. In this case, constrained shareholders prefer a higher level of investment than unconstrained ones.

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1 Introduction

In recent years macroeconomists have started exploring the implications of dynamic general equilibrium models with heterogeneous agents, incomplete markets and aggregate shocks for a variety of issues, including asset pricing (Krusell and Smith, 1997 and Storesletten, Telmer, and Yaron, 2001), business cycles (Krusell and Smith, 1998), and the distribution of income and wealth (Castaneda, Diaz-Gimenez and Rios-Rull, 1998). In these models consumers face idiosyncratic and aggregate uncertainty and accumulate assets, such as physical capital, in order to smooth consumption over time. The assumption that consumers, rather than firms, accumulate physical capital can either be interpreted literally, or alternatively, as capturing a situation in which each consumer is also an entrepreneur. The latter directly operates a technology by employing his privately accumulated physical capital and by hiring and supplying labor in a competitive labor market.[1]

This paper considers a version of the incomplete markets models cited above in which consumers trade firms' shares rather than physical capital, and firms' shareholders make decisions regarding investment in physical capital. The incompleteness of the asset market implies that, in principle, this setting is *not* equivalent to one in which consumers accumulate physical capital directly. When the asset market is incomplete, a firm's shareholders do not necessarily agree on what the objective of the firm should be, and, in particular, on whether the firm should maximize its stock market value or not (see e.g., Grossman and Hart, 1979). When the asset market is complete, instead, marginal rates of substitution are equalized among shareholders, who, therefore, unanimously agree on the investment decision of the firm. In this context it is irrelevant whether consumers or firms accumulate physical capital.

This paper investigates the conditions under which the allocation of resources in the workhorse incomplete markets model is the same independently of whether firms or consumers are allowed to accumulate physical capital. In doing so it makes three contributions.

First, using a two-period version of the standard incomplete markets model, it shows that if the firms' production function exhibits constant returns to scale in capital and labor, and short-sales constraints are not binding, then in a competitive equilibrium, any firm's shareholders will unanimously agree on the optimal level of investment. As a result, the equilibrium allocation of this economy coincides with the equilibrium allocation of a similar economy in which consumers, rather than firms, accumulate capital. Second, it shows that when short-sales constraints are binding for some initial shareholders, the latter prefer a higher level of investment than unconstrained ones. The paper discusses how this conflict among shareholders might be

[1] The equivalence of these two settings is guaranteed by the standard assumption of constant returns to scale in production and the existence of an economy-wide labor market for labor. These assumptions guarantee that capital-labor ratios will be equalized across production units, independently of whether firms rent capital from consumers or the latter operate their own technology. Angeletos and Calvet (2001) consider a version of this kind of model in which there is no economy-wide labor market. Each entrepreneur employs his own capital and labor in the firm, and, as a result, capital-labor ratios are not equalized across firms.

resolved using majority voting. In this case, provided that constrained shareholders own a minority of the firm's shares, the equilibrium stock price of the firm is always equal to its capital stock and the allocation is again the same as in the version of the economy in which consumers accumulate capital directly. Third, the paper extends the unanimity result to a multiperiod economy.

Our companion paper (Carceles-Poveda and Coen-Pirani, 2004) provides a different, though complementary, approach to the problem of the firm's investment under incomplete markets. In the latter we also compare a setting where firms maximize period-by-period profits (i.e. the standard setting) with a setting where firms make intertemporal investment decisions and households hold their stock. However, instead of directly addressing the question of unanimity among shareholders, we show that there exists a particular objective for the dynamic firm that implies the same equilibrium allocation as in the standard setting. This objective corresponds to discounting the cash flows of the firm with *any* present value price that does not allow for arbitrage opportunities. Differently from the present paper, this result is valid even if portfolio restrictions are binding.

While the macroeconomic literature with incomplete markets has mostly assumed away the problem of joint ownership of the firm, this problem has received plenty of attention in the theory literature, starting from Diamond's (1967) classic paper. The economies considered in the latter literature bear some similarity with the one usually considered by macroeconomists, with some important differences.

First, the theory literature generally consider models in which capital is the only input in production and the production function exhibits decreasing returns to scale (see, e.g., Magill and Quinzii, 1996, page 378). Macroeconomists, instead, typically assume constant returns to scale production technologies that use as inputs capital and labor. The assumption of constant returns to scale is crucial for the unanimity result of this paper.

Second, in the theory literature, borrowing or short-sale constraints are generally assumed not to be binding for shareholders.[2] Binding borrowing constraints, instead, play a more central role in the macroeconomic literature (see e.g. Krusell and Smith, 1998), so it is important to investigate their effect on the investment decision of the firm. This paper shows that the unanimity result derived here breaks down when short-sales constraints are binding, and that in this situation constrained shareholders would like the firm to purchase more capital than unconstrained ones.

Last, the theory literature typically considers two-period models, while the macroeconomic literature considers infinite horizon economies. It turns out that the main intuitions of this paper can be presented in a two period setting. The two-period model also makes it easier to relate the results of this paper with the

[2] This is one of the crucial assumptions made by Grossman and Hart (1979, page 299, footnote 5), for example. With binding short-sale constraints, their approach to the problem of the firm's objectives under incomplete markets based on "competitive price perceptions" would not be applicable.

classic contributions of Diamond (1967), Grossman and Hart (1979), and Ekern and Wilson (1974). This is done in section 3.3. The generalization to multiperiod and infinite horizon economies is introduced in the last section of the paper.

The rest of the paper is organized as follows. Section 2 introduces a standard two-period model economy with incomplete markets and idiosyncratic risk. Section 3 studies the equilibrium of this economy under the assumption that consumers' borrowing constraints are not binding. Section 4 considers the case of binding borrowing constraints. Section 5 analyzes the multiperiod case. Section 6 summarizes the results.

2 Model Economy

In this section we introduce the model economy. Since all relevant intuition can be obtained in a two-period version of the benchmark incomplete markets model used in macroeconomics (see Ayiagari, 1994 and Krusell and Smith, 1998), we will start from this case.[3]

Let time be denoted by $t = 0, 1$. The economy is populated by a continuum of measure 1 of consumers, indexed by $i \in I = [0, 1]$, and a continuum of measure one of firms indexed by $j \in J = [0, 1]$.

Firms produce an homogeneous good that can be either consumed or invested. Each firm operates the production function $y = F(k, l; z)$, where k denotes the physical capital input, l the labor input, and z is a random variable. The production function F is assumed to be twice differentiable with respect to k and l and display constant returns to scale: $F(\mu k, \mu l; z) = \mu F(k, l; z)$, for all $\mu > 0$. The constant returns to scale assumption will play a key role in the analysis.

The economy is characterized by both aggregate and individual uncertainty. The former is captured by the random variable z, that is common to all firms. At time zero the value of z is known and equal to z_0. In the second period, instead, z_1 is random and can take a finite number of values in the set $Z = \{z^1, z^2, ..., z^N\}$. Let $s^0 = \{z_0\}$ and $s^1 = \{z_0, z_1\}$ denote the length-0 and length-1 histories of aggregate shocks, and $\pi_z(s^t)$ the unconditional probability of history s^t, with $\pi_z(s^0) = 1$. Also, denote by S^1 the set $Z \times Z$.

The initial capital level of each firm at time zero is denoted by k_0^j. This is exogenously given to each firm and can be different across firms. At time zero the firm decides on the labor input $l_0^j(s^0)$ and on next period's capital stock $k_1^j(s^0)$. At time 1 the firm decides on the labor input $l_1^j(s^1)$ after the realization of the shock z_1.

Individual uncertainty refers to the fact that a consumer's labor endowment at $t = 1$ is random. At $t = 0$ a consumer i is endowed with x_0^i units of labor, where x_0^i is non-random and can differ across consumers.

[3] In Section 5, I extend the model to the multiperiod case.

Let L_0 denote the aggregate labor endowment at time zero:

$$L_0 = \int_0^1 x_0^i di.$$

A consumer's labor endowment at time 1, denoted by x_1^i, is random and can take values in the set $X = \{x^1, x^2, ..., x^L\}$. Let $s^{i0} = \{x_0^i\}$ and $s^{i1} = \{x_0^i, x_1^i\}$ denote the histories of endowment shocks for consumer i. The individual shocks might be correlated with the aggregate one, so that the probability of history s^{i1} might depend on s^1. Denoting by $\pi_x(s^{i1}|s^1)$ this conditional probability, let $\pi(s^{i1}, s^1) = \pi_x(s^{i1}|s^1)\pi_z(s^1)$ indicate the joint probability of s^{i1} and s^1. We assume that the only aggregate source of uncertainty is the productivity shock z_1. Individual uncertainty is assumed to disappear in the aggregate due to a law of large numbers. In particular, the aggregate units of labor available for production at time 1 following aggregate history s^1 are equal to:

$$L_1(s^1) = \int_0^1 \sum_{s^{i1}} x_1^i(s^{i1})\pi_x(s^{i1}|s^1)\,di.$$

Each consumer i is endowed with the following utility function:

$$V^i = U\left(c_0^i\left(s^{i0}, s^0\right)\right) + \beta \sum_{s^1, s^{i1}} \pi\left(s^{i1}, s^1\right) U\left(c_1^i\left(s^{i1}, s^1\right)\right), \tag{1}$$

where c_0 denotes consumption at time zero and c_1 consumption at time one. The momentary utility function U is assumed to be twice differentiable with $U' > 0$ and $U'' < 0$.

The consumer also derives income from trading shares of the firms. At time zero a consumer i is endowed with θ_{0j}^i shares of firm $j \in J$. The timing of the model is such that at the beginning of time zero firms pay dividends $d_0^j(s^0)$, then the stock market opens, and consumers can rebalance their portfolios by buying or selling their initial shares at the (ex-dividend) price $p_0^j(s^0)$. Let $\theta_{1j}^i(s^{i0}, s^0)$ denote the final shares of firm j held by consumer i after trading in the stock market. We assume that there is an exogenously given negative lower bound on holdings of each firm's shares:[4]

$$\theta_{1j}^i\left(s^{i0}, s^0\right) \geq -\underline{\theta}, \tag{2}$$

where $\underline{\theta} > 0$.

At the beginning of period 1 aggregate and individual uncertainties are revealed. Each firm j pays a dividend $d_1^j(s^1)$, and each consumer observes his labor shock and consumes $c_1^i(s^{i1}, s^1)$. A consumer faces

[4]This lower bound must be such that the consumer is always able to pay back his debt in period 1, while enjoying non-negative consumption.

the budget constraints:

$$c_0^i\left(s^{i0},s^0\right)+\int_0^1 p_0^j\left(s^0\right)\theta_{1j}^i\left(s^{i0},s^0\right)dj = \int_0^1 \theta_{0j}^i\left(d_0^j\left(s^0\right)+p_0^j\left(s^0\right)\right)+w_0\left(s^0\right)x_0^i, \tag{3}$$

$$c_1\left(s^{i1},s^1\right) = \int_0^1 \theta_{1j}^i\left(s^{i0},s^0\right)d_1^j\left(s^1\right)dj+w_1\left(s^1\right)x_1^i, \tag{4}$$

where $w_0\left(s^0\right)$ and $w_1\left(s^1\right)$ denote the time 0 and time 1 wages per unit of labor endowment.

The measure of each firm's shares outstanding is normalized to one in both periods:

$$\begin{aligned}\int_0^1 \theta_{0j}^i di &= 1, \text{ for all } j\in J,\\ \int_0^1 \theta_{1j}^i\left(s^{i0},s^0\right)di &= 1, \text{ for all } j\in J.\end{aligned} \tag{5}$$

3 Equilibrium

The key difficulty in solving this model is represented by the fact that each firm is, in general, owned by many consumers-shareholders, and the latter might disagree on its optimal level of investment. Here we assume that initial, rather than final, shareholders make the investment decision.[5] Given this timing, a firm's initial shareholder has to form an expectation on how the firm's stock price is going to change with different levels of investment.

The theoretical literature concerning the objectives of the firm under incomplete markets has mostly proceeded under the assumption of *competitive price perceptions* (from now on CPP), originally introduced by Grossman and Hart in their seminal 1979 paper (see Magill and Quinzii, 1996 for a discussion of this approach). With CPP each shareholder forms expectations about the effects of investment on the firm's stock price using his own state prices. As a result, initial shareholders will unanimously agree to maximize the firm's net value p_0+d_0, while potentially disagreeing on the best way to achieve this result. In turn, this conflict can be either resolved by allowing transfers among shareholders, as suggested by Grossman and Hart (1979) or by some voting procedure (see e.g. DeMarzo, 1993). One problem with the CPP approach is that it is only applicable when there are no binding restrictions on short-sales of a firm's stock.[6] Moreover, the CPP approach implies that in an incomplete markets equilibrium shareholders have different opinions

[5] The results of this section easily apply to the situation in which final shareholders make the investment choice. The assumption that initial shareholders make the investment decision implies that the latter have to form expectations about the effect of investment on the firm's stock price. This would be always the case, independently of whether initial or final shareholders decide on investment, in a model with more than two periods.

[6] See Grossman and Hart (1979, page 299, footnote 5) for a discussion of this point. In case an initial shareholder faces binding short-sale restrictions, his state prices do not contain sufficient information on the change in the firm's stock market price following a change in its investment.

about the sensitivity of stock prices to the level of investment.[7] In this paper, instead, we adopt the standard rational expectations assumption to derive shareholders' expectations on the effects of different investment levels on the firm's stock price. Among other things, this assumption implies that shareholders will always agree on the effect of a change in investment on the firm's stock price, while possibly disagreeing on the optimal level of investment to be undertaken by the firm.

In what follows, the plan is to first define an exchange equilibrium for the economy described above by taking the production and investment plans of each firm as given. This exchange equilibrium determines the relationship between firm j's stock price p_0^j and its capital stock in period 1, k_1^j, for given capital chosen by the other firms in the economy. The rational expectations approach to solving the investment decision problem of a firm consists of deriving the sensitivity of a firm's stock price to variations in k_1^j from this relationship, rather than from the agent's individual state prices. Given this pricing function we can then turn to the issue of determining the optimal investment level for firm j. It turns out that in this economy, under constant returns to scale in production, the set of unconstrained initial shareholders of firm j will unanimously agree on its optimal investment k_1^j.[8]

3.1 Exchange Equilibrium

Let a firm's dividends paid in periods 0 and 1 be defined as:

$$\begin{aligned} d_0^j\left(s^0\right) &= F\left(k_0^j, l_0^j\left(s^0\right); z_0\right) - w_0\left(s^0\right) l_0^j\left(s^0\right) - \left(k_1^j\left(s^0\right) - (1-\delta) k_0^j\right), \qquad (6) \\ d_1^j\left(s^1\right) &= F\left(k_1^j\left(s^0\right), l_1^j\left(s^1\right); z_1\right) - w_1\left(s^1\right) l_1^j\left(s^1\right) + (1-\delta) k_1^j\left(s^0\right). \qquad (7) \end{aligned}$$

Before defining an exchange equilibrium for this economy, it is important to point out that the assumption of constant returns to scale in production implies that

$$d_1^j\left(s^1\right) = k_1^j\left(s^0\right) q\left(w_1\left(s^1\right), z_1\right), \qquad (8)$$

where $q\left(w_1\left(s^1\right), z_1\right)$ is common among all firms:

$$q\left(w_1\left(s^1\right), z_1\right) \equiv F\left(1, g\left(w_1\left(s^1\right), z_1\right); z_1\right) - w_1\left(s^1\right) g\left(w_1\left(s^1\right), z_1\right) + 1 - \delta.$$

To see why this is the case, it is convenient to consider a firm's labor demand. Since this is a static choice, its shareholders will always agree about hiring labor up to the point where its marginal product is

[7] See section 3.3 for further discussion on the CPP approach.

[8] Final shareholders will agree as well, but they are not the ones that are assumed to make the investment decision.

equal to the wage:

$$\begin{aligned} w_0\left(s^0\right) &= F_L\left(k_0^j, l_0^j\left(s^0\right); z_0\right), \quad (9) \\ w_1\left(s^1\right) &= F_L\left(k_1^j\left(s^0\right), l_1^j\left(s^1\right); z_1\right). \quad (10) \end{aligned}$$

Therefore, since under constant returns to scale $F_L(k,l;z)$ is homogeneous of degree zero in k and l, all firms will have the same capital-labor ratio. In particular,

$$l_1^j\left(s^1\right) = k_1^j\left(s^0\right) g\left(w_1\left(s^1\right), z_1\right), \quad (11)$$

where g is a function of $w_1\left(s^1\right)$ and z_1 only. Replacing this expression into (7) and using again the constant returns to scale assumption to collect $k_1^j\left(s^0\right)$ yields equation (8). An important implication of this equation is that, since the capital stock $k_1^j\left(s^0\right)$ is determined in period 0, the vectors of dividends $(d_1^j\left(s^1\right))$ paid by any two firms in period 1 are always linearly dependent.

An exchange equilibrium for this economy is defined as follows.

Definition (Exchange Equilibrium). Given the dividends $((d_t^j(s^t))_{t=0,1})_{j\in J}$ paid by firms and the initial distribution of shares $((\theta_{0j}^i)_{j\in J})_{i\in I}$, an exchange equilibrium is a represented by a collection of stock prices $(p_0^j\left(s^0\right))_{j\in J}$, a consumption allocation $((c_t^i\left(s^{it}, s^t\right))_{t=0,1})_{i\in I}$ and portfolio choices $((\theta_{1j}^i\left(s^{i0}, s^0\right))_{j\in J})_{i\in I}$, such that:

(i) Given $(p_0^j\left(s^0\right))_{j\in J}$, $(c_t^i\left(s^{it}, s^t\right))_{t=0,1}$ and $(\theta_{1j}^i\left(s^{i0}, s^0\right))_{j\in J}$ are optimal for consumer $i \in I$, i.e., they maximize (1) subject to (3), (4) and (2).

(ii) $(p_0^j\left(s^0\right))_{j\in J}$ is such that the market for the shares of each firm clears, i.e., (5) holds.

A key property of an exchange equilibrium of this economy is summarized in the following proposition:

Proposition 1. In an exchange equilibrium, the rates of return on the stocks of all firms are equalized for all possible realizations of the aggregate shock z_1 in period 1. Formally:

$$\frac{d_1^{j'}\left(s^1\right)}{p_0^{j'}\left(s^0\right)} = \frac{d_1^j\left(s^1\right)}{p_0^j\left(s^0\right)} \text{ for all } j, j', \text{ and } s^1. \quad (12)$$

To see this, suppose that there existed a couple of firms j and j' and a state of the world z_1 such that equation (12) did not hold. Then, using the expression (8) for a firm's dividend, it must be the case that, say:

$$\frac{k_1^j\left(s^0\right) q\left(w_1\left(s^1\right), z_1\right)}{p_0^j\left(s^0\right)} > \frac{k_1^{j'}\left(s^0\right) q\left(w_1\left(s^1\right), z_1\right)}{p_0^{j'}\left(s^0\right)}, \quad (13)$$

so that the equality condition is in fact violated for all possible histories s^1. Thus, equation (13) implies that the rate of return on firm j's stock is always higher than the rate of return on firm j''s stock, for all possible realizations of z_1. Then, clearly, there would be an incentive for a consumer holding firm j''s stock to sell it and buy instead firm j's stock. This in turn would imply that the economy is not at an exchange equilibrium. Therefore, equation (12) must hold.

This condition is important because it provides us with a way to compute the effects of a variation in firm j's investment on its stock price. In particular, rearranging (12) and using (8) to replace $d_1^j\left(s^1\right)$ yields:

$$p_0^j\left(s^0\right)=k_1^j\left(s^0\right)\frac{p_0^{j'}\left(s^0\right)}{k_1^{j'}\left(s^0\right)}\text{ for all } j'. \tag{14}$$

When deciding on $k_1^j\left(s^0\right)$, firm j's shareholders take as given the stock price and the investment decision made by all other firms. Thus, equation (14) provides them with information about the effect of different investment levels on their firm's stock price. Using this equation we obtain:

$$\frac{\partial p_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}=\frac{p_0^{j'}\left(s^0\right)}{k_1^{j'}\left(s^0\right)}\text{ for all } j'. \tag{15}$$

Notice that with an arbitrary production function, changes in the level of capital $k_1^j\left(s^0\right)$ would not give rise to proportional variations in the dividend vector $\left(d_1^j\left(s^1\right)\right)$, as in equation (8). The period 1 dividends paid out by firms characterized by different levels of capital would not be linearly dependent, and thus it would not be possible to infer the effect of variations in a firm's investment on its stock price from the stock prices of other firms.

We conclude this section by stating the first order condition associated with a consumer's optimal portfolio choice in an exchange equilibrium:

$$p_0^j\left(s^0\right)\geq\sum_{s^1,s^{i1}}m_0^i\left(s^{i1},s^1\right)d_1^j\left(s^1\right),\text{ for all } j\in J, \tag{16}$$

where

$$m_0^i\left(s^{i1},s^1\right)\equiv\beta\pi\left(s^{i1},s^1\right)\frac{U'\left(c_1^i\left(s^{i1},s^1\right)\right)}{U'\left(c_0^i\left(s^{i0},s^0\right)\right)}. \tag{17}$$

Equation (16) holds with equality if for consumer i: $\theta_{1j}^i\left(s^{i0},s^0\right)>-\underline{\theta}$. Notice that, because of equation (12), if condition (16) holds with equality for firm j, then it must hold with equality for all other firms $j'\neq j$.

In what follows we will separately consider the case in which the constraint (2) is never binding for any agent from the case in which there is a positive measure of agents for which it is binding. The distinction is important because in the former case all initial shareholders of a firm will unanimously agree on the amount of capital to be invested. In the latter case, instead, there will be disagreement among initial shareholders, and it will be necessary to specify a mechanism to aggregate these heterogeneous preferences into one investment decision.

3.2 A Firm's Investment Decision When Short-Sales Constraints are Not Binding

In this section we consider the case where the short-sales constraint (2) is never binding for any agent i.[9] Under this assumption let's now turn to the decision problem faced by a firm in this economy. A firm in this model makes three decisions. It chooses the labor input at time 0 and at time 1 in each aggregate state of the world. It also chooses a level of physical capital $k_1^j(s^0)$ to be used as an input in production in period 1. The capital stock depreciates at the rate δ after production.

The static nature of the labor input decision and its timing imply that all shareholders will agree on the optimal quantity of labor to hire in a given period. The dynamic nature of the investment decision in this incomplete markets setting, instead, suggests that there might be scope for disagreement among initial shareholders on the optimal level of investment. The main result of this section is that, under constant returns to scale in production and no binding short-sales constraints, there will be unanimity among shareholders.

Consider the investment decision of firm j. Investment is assumed to be financed entirely out of the firm's retained earnings at time zero.[10] An initial shareholder i of the firm would choose $k_1^j\left(s^0\right)$ in order to maximize the utility function V^i subject to (6) and (7), taking as given the price of labor and the investment decisions of other firms.[11] The first order condition for his problem is:

$$\theta_{0j}^i\left(\frac{\partial p_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}+\frac{\partial d_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}\right)-\theta_{1j}^i\left(s^{i0},s^0\right)\left(\frac{\partial p_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}-\sum_{s^1,s^{i1}}m_0^i\left(s^{i1},s^1\right)\frac{\partial d_1^j\left(s^1\right)}{\partial k_1^j\left(s^0\right)}\right)=0, \tag{18}$$

where

$$\frac{\partial d_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}=-1.$$

Equation (18) is the key to understanding the effect of incomplete markets on shareholders' investment

[9] Sufficient conditions for this to be the case are: i) $\underline{\theta}=0$; ii) the minimum possible labor endowment realization in period 1 is zero, i.e., $x^1=0$; iii) the marginal utility of consumption at zero is infinite, i.e. $U'(0)=+\infty$. These assumptions imply that a consumer will never choose $\theta_{1j}^i\left(s^{i0},s^0\right)=0$ for any j. Suppose, in fact, that for some firm j, $\theta_{1j}^i\left(s^{i0},s^0\right)=0$ and that equation (16) holds as an inequality. Then, equation (12) would imply that the first order condition (16) holds as an inequality for all firms $j\in J$. Thus, $\theta_{1j}^i\left(s^{i0},s^0\right)=0$ for all $j\in J$. In this case, since $x^1=0$, a consumer will experience zero consumption with positive probability. The assumption that $U'(0)=+\infty$ is sufficient to rule this case out. Thus, the short-sale constraint will never bind.

[10] It is straightforward to add bonds to this economy without altering the unanimity result of the paper. As long as all initial shareholders are unconstrained in their portfolio choice, the Modigliani-Miller theorem holds in this setup and the financial structure of the firm is indeterminate.

[11] By assumption, agents for whom $\theta_{0j}^i<0$ do not participate in the investment decision of the firm. Their goal would be to minimize rather than maximize the firm's value.

decisions. As initial owners of the firm, shareholders care about the way investment in physical capital affects the net stock market value of the firm. This effect is captured by the term multiplying θ^i_{0j} in (18). As final owners of the firm, shareholders care about the way the firm's investment affects what Magill and Quinzii (1996, page 380) refer to as the "*spanning services* of the firm's equity contract": its stock price in period 0 and the dividends it pays out in period 1. This effect is captured by the term multiplying $\theta^i_{1j}\left(s^{i0}, s^0\right)$ in (18). Under complete financial markets, the individual state prices $m^i_0\left(s^{i1}, s^1\right)$ are equalized among shareholders. Thus, the second term in (18) is always zero.[12]

When the asset market is incomplete, instead, state prices are not necessarily equal among shareholders. In this circumstance, the assumption of constant returns to scale in production guarantees that trading in the stock market is sufficient to make shareholders agree on the expected discounted value of an extra unit of investment. To see this, recall that from equation (8)

$$\frac{\partial d^j_1\left(s^1\right)}{\partial k^j_1\left(s^0\right)} = \frac{d^j_1\left(s^1\right)}{k^j_1\left(s^0\right)},$$

and from equation (15):

$$\frac{\partial p^j_0\left(s^0\right)}{\partial k^j_1\left(s^0\right)} = \frac{p^j_0\left(s^0\right)}{k^j_1\left(s^0\right)}. \tag{19}$$

Replacing these two equations into (18), this first order condition for investment becomes

$$\theta^i_{0j}\left(\frac{p^j_0\left(s^0\right)}{k^j_1\left(s^0\right)} - 1\right) - \theta^i_{1j}\left(s^{i0}, s^0\right)\left(\frac{p^j_0\left(s^0\right)}{k^j_1\left(s^0\right)} - \sum_{s^1, s^{i1}} m^i_0\left(s^{i1}, s^1\right)\frac{d^j_1\left(s^1\right)}{k^j_1\left(s^0\right)}\right) = 0. \tag{20}$$

The expression in parenthesis multiplying $\theta^i_{1j}\left(s^{i0}, s^0\right)$ is equal to zero because stock market trading equalizes final shareholders' valuations of future dividends, i.e. equation (16) holds with equality. This observation leads us to the following proposition:

Proposition 2. The unconstrained initial shareholders of a firm j will unanimously agree in setting its level

[12] With complete markets the first order condition for holding stock is:

$$p^j_0\left(s^0\right) = \sum_{s^1} m_0\left(s^1\right) d^j_1\left(s^1\right),$$

where $m_0\left(s^1\right)$ denotes the price at time zero of a security that pays one unit of consumption in state s^1 and zero in all other states. Given that a firm takes these prices as given, this equation can be used to obtain:

$$\frac{\partial p^j_0\left(s^0\right)}{\partial k^j_1\left(s^0\right)} = \sum_{s^1} m_0\left(s^1\right)\frac{\partial d^j_1\left(s^1\right)}{\partial k^j_1\left(s^0\right)}.$$

This condition implies that the second term in (18) is equal to zero for all shareholders, independently of whether the production function displays constant returns to scale or not.

of capital for period 1 according to the condition

$$\frac{\partial p_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}=1,$$

where the derivative on the left hand side of this equation is defined in equation (19).

It follows that for all initial shareholders the optimal investment strategy for each firm $j \in J$ is such that $k_1^j\left(s^0\right)=p_0^j\left(s^0\right)$. Notice that all firms make the same decisions regarding period 1 capital stock, $k_1^j\left(s^0\right)$, independently of their initial capital stock k_0^j. In fact, the decision about $k_1^j\left(s^0\right)$ depends only on the cost of investment, which is equal to one, and the present discounted value of period 1 dividends, which is independent of a firm's initial capital.

A competitive production-exchange equilibrium for this economy is therefore defined as follows:

Production-Exchange Equilibrium. A production-exchange equilibrium for this economy is represented by a stock price $p_0\left(s^0\right))$, capital stock $k_1\left(s^0\right)$, labor demands $((l_t^j\left(s^t\right))_{t=0,1})_{j\in J}$, wages $(w_t\left(s^t\right))_{t=0,1}$, dividends $(d_t\left(s^t\right))_{t=0,1}$, a consumption allocation $((c_t^i\left(s^{it},s^t\right))_{t=0,1})_{i\in I}$ and portfolio choices $((\theta_{1j}^i\left(s^{i0},s^0\right))_{i\in I,j\in J}$ such that:

i) $p_0\left(s^0\right)$, $((\theta_{1j}^i\left(s^{i0},s^0\right))_{i\in I,j\in J}$, and $((c_t^i\left(s^{it},s^t\right))_{t=0,1})_{i\in I}$ constitute an exchange equilibrium for given dividends $(d_t\left(s^t\right))_{t=0,1}$.

ii) Shareholders of all firms unanimously agree to set $k_1\left(s^0\right)=p_0\left(s^0\right)$.

iii) The labor market clears: if wages $(w_t\left(s^t\right))_{t=0,1}$ are given by (9) and (10), aggregate labor demand equals aggregate labor supply in $t=0,1$.

iv) Dividends $(d_t\left(s^t\right))_{t=0,1}$ are given by equations (6) and (7).

As a corollary to Proposition 2, we have the following result:

Corollary 1. Unanimity among initial shareholders implies that the equilibrium allocation of consumption and capital in this incomplete markets economy is the same as in an economy where consumers, instead of firms, accumulate physical capital directly.

To show this point formally, consider a version of the economy where consumers, instead of firms, accumulate physical capital. In this version the budget constraint of consumer i reads:

$$\begin{aligned} c_0^i\left(s^{i0},s^0\right)+k_1^i\left(s^{i0},s^0\right) &= \left(r_0\left(s^0\right)+1-\delta\right)k_0^i+w_0\left(s^0\right)x_0^i, &(21)\\ c_1\left(s^{i1},s^1\right) &= \left(r_1\left(s^1\right)+1-\delta\right)k_1^i\left(s^{i0},s^0\right)+w_1\left(s^1\right)x_1^i, &(22)\end{aligned}$$

where $r_t\left(s^t\right)$ is the rental rate of the stock of capital owned by consumer i in period t. Letting aggregate capital be defined as:

$$k_0 = \int_0^1 k_0^i di, \text{ and } k_1\left(s^0\right) = \int_0^1 k_1^i\left(s^{i0}, s^0\right) di,$$

the rental rates are equal to the marginal products of capital in the two periods:

$$r_0\left(s^0\right) = F_K\left(k_0, L_0; z_0\right) \text{ and } r_1\left(s^1\right) = F_K\left(k_1\left(s^0\right), L_1\left(s^1\right); z_1\right).$$

Further, each consumer accumulates physical capital according to the marginal condition

$$1 = \sum_{s^1, s^{i1}} m_0^i\left(s^{i1}, s^1\right)\left(r_1\left(s^1\right) + 1 - \delta\right). \tag{23}$$

It is easy to see that the equations that characterize the equilibrium of this economy are the same as the conditions that characterize a production-exchange equilibrium. First, the first order condition (23) is the same as (16) because

$$r_1\left(s^1\right) + 1 - \delta = \frac{d_1\left(s^1\right)}{k_1\left(s^0\right)},$$

and $k_1\left(s^0\right) = p_0\left(s^0\right)$. Second, the budget constraints (21) and (22) are equivalent to the ones in the economy of section 2 (equations 3 and 4), after setting[13]

$$k_0^i = \theta_0^i k_0, \; k_1^i\left(s^{i0}, s^0\right) = \theta_1^i\left(s^{i0}, s^0\right) p_0\left(s^0\right),$$

and noticing that

$$r_0\left(s^0\right) = \frac{d_0\left(s^0\right)}{k_0} - (1 - \delta).$$

3.3 Discussion and Relationship with the Literature

The key assumptions for the unanimity result obtained above are that the production function F displays constant returns to scale and that short-sales constraints are not binding. These assumptions jointly guarantee that the term in parenthesis multiplying $\theta_{1j}^i\left(s^{i0}, s^0\right)$ in equation (18) is equal to zero: 1) The constant returns to scale assumption guarantees that the latter term can be written as in equation (20); 2) The fact that the portfolio decision is interior guarantees that the first order condition for agent i's portfolio choice can be used to set this term to zero.

It is easy to show that this unanimity result is robust to extending the model to consider: 1) preferences

[13] I am abstracting from the firm sub-index j here because in a production-exchange equilibrium there is symmetry across all firms.

characterized by non-expected utility; 2) preference heterogeneity among consumers; 3) different "opinions" among consumers about the likelihood of a given state of the world; 4) adjustment costs in the installation of new capital, as long as the adjustment cost function is homogeneous of degree one in current capital and investment (as in Hayashi, 1982).[14]

In what follows we discuss how the unanimity result obtained here relates to some of the main contributions to the theoretical literature on the objectives of the firm under incomplete markets. Consider, first, Diamond (1967)'s classic paper. Diamond focused his attention on the case where: 1) uncertainty faced by firms takes a multiplicative form; 2) capital is the only input used in production; 3) production occurs under decreasing returns to scale according to a production function of the form $zG(k)$, where $G' > 0$ and $G'' < 0$; and 4) capital fully depreciates within a period, i.e., $\delta = 1$. Under these assumptions a firm's period 1 dividend is $d_1^j(s^1) = z_1 G(k_1^j(s^0))$. Given that aggregate uncertainty affects dividends multiplicatively, an argument analogous to the one developed in section 3.1 implies the equalization of ex-post rates of return among firms (equation 12). Thus the equivalent of equation (14) is

$$p_0^j(s^0) = p_0^{j'}(s^0) \frac{G(k_1^j(s^0))}{G(k_1^{j'}(s^0))} \text{ for all } j, j'.$$

It follows that

$$\frac{\partial p_0^j(s^0)}{\partial k_1^j(s^0)} = G'(k_1^j(s^0)) \frac{p_0^j(s^0)}{G(k_1^j(s^0))}. \tag{24}$$

Replacing (24) and the interior version of the first order condition (16) in (18) yields the equation that determines the optimal choice of capital for shareholder i:

$$\theta_{0j}^i \left(G'(k_1^j(s^0)) \frac{p_0^j(s^0)}{G(k_1^j(s^0))} - 1 \right) = 0.$$

Therefore shareholders will be unanimous in their choice of $k_1^j(s^0)$. Now, consider the relationship between this result and the one obtained in the previous section. To do this, consider a possible extension of Diamond's result to an economy where production occurs using capital and labor while assumptions 1) and 3)-4) are preserved. Denote the production function by $zG(k,l)$, where G does not need to exhibit constant returns to scale. The dividend in period 1 is

$$d_1^j(s^1) = z_1 G(k_1^j(s^0), l_1^j(s^1)) - w_1(s^1) l_1^j(s^1). \tag{25}$$

[14] This homogeneity guarantees that period 1 dividends can still be written as $k_1^j(s^0)$ times a term that depends only on aggregate variables.

In order to extend Diamond's result to such an economy one has to show that, once the optimal $l_1^j\left(s^1\right)$ has been replaced into $d_1^j\left(s^1\right)$, the latter can be written as:

$$d_1^j\left(s^1\right) = h(k_1^j\left(s^0\right))\widetilde{q}\left(z_1, w_1\left(s^1\right)\right), \tag{26}$$

where h and $\widetilde{q}$ are two functions. If this were the case then we could apply Diamond's argument summarized above. The problem with this argument is that, even if uncertainty enters multiplicative in the *original* production function, there is no guarantee that the decomposition (26) applies. Consider for example the decreasing returns to scale production function:

$$zG(k,l) = z\log\left(1 + k + l^{\frac{1}{2}}\right).$$

Solving the labor demand problem and plugging the optimal $l_1^j\left(s^1\right)$ back into (25) yields:

$$d_1^j\left(s^1\right) = z_1 \log\frac{1}{2}\left\{1 + k_1^j\left(s^0\right) + \left[(1 + k_1^j\left(s^0\right))^2 + \frac{2z_1}{w_1\left(s^1\right)}\right]^{\frac{1}{2}}\right\},$$

which clearly does not take the form (26).[15] In summary, in the model considered here, Diamond's assumption of multiplicative uncertainty does not, in general, give rise to unanimity among shareholders if the production function exhibits decreasing returns to scale in capital and labor. The assumption of constant returns to scale in production, instead, is sufficient to obtain unanimity.[16] Of course, as the example of footnote (15) indicates, the assumption of constant returns to scale in production is not *necessary* in order to obtain the unanimity result.

It is interesting to contrast the approach taken here in solving the model, with the one pursued by Grossman and Hart in their seminal (1979) paper. They do not assume that F is constant returns to scale. Instead, they postulate that consumers have CPP, i.e., that they use their own state prices $m_0^i\left(s^{i1}, s^1\right)$ to evaluate the effect of a change in $k_1^j\left(s^0\right)$ on the firm's stock price:

$$\left.\frac{\partial p_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}\right|_i = \sum_{s^1, s^{i1}} m_0^i\left(s^{i1}, s^1\right)\frac{\partial d_1^j\left(s^1\right)}{\partial k_1^j\left(s^0\right)}, \tag{27}$$

[15] Notice, however, that when the production function takes the commonly used Cobb-Douglas form this problem does not arise. For example, if $G\left(k,l\right) = k^\alpha l^\sigma$, with $\alpha + \sigma < 1$, then:

$$d_1^j\left(s^1\right) = \left(k_1^j\left(s^0\right)\right)^{\frac{\alpha}{1-\sigma}}\left(\frac{\sigma z_1}{w_1\left(z_1\right)}\right)^{\frac{\sigma}{1-\sigma}} z_1\left(1-\sigma\right),$$

which satisfies (26).

[16] Notice that if the production function displays constant returns to scale, the aggregate shock z must affect this function multiplicatively.

where the subindex i on the left-hand side indicates that the perception of this derivative varies across shareholders. This amounts to setting the second term in equation (18) equal to zero *by assumption*. Thus, while shareholders will be unanimous in their desire to maximize the firm's net stock market value, by setting

$$\left.\frac{\partial p_0^j\left(s^0\right)}{\partial k_1^j\left(s^0\right)}\right|_i = 1,$$

they will, in general, disagree on the choice of $k_1^j\left(s^0\right)$. Grossman and Hart resolve this conflict by allowing for income transfers among a firm j's initial shareholders at time zero. In their approach, an optimal investment plan for the firm is such that there is no other investment plan and set of income transfers among shareholders such that all shareholders are better off.

In the model considered here, even if shareholders use their own state prices to evaluate $\partial p_0^j\left(s^0\right)/\partial k_1^j\left(s^0\right)$, they still agree on the magnitude of this derivative. This is because trading in the stock market and constant returns to scale in production guarantee that the right-hand side of equation (27) is the same among all shareholders. An advantage of *not* postulating CPP is that we can extend our approach to the case where short-sales constraints are binding (see section 4). In this case the CPP approach is not applicable because the right-hand side of (27) does not measure the marginal benefit of higher investment by the firm for a constrained shareholder.

The unanimity result obtained here can also be interpreted as a special case of the "spanning" result derived by Ekern and Wilson (1974). They showed that if the asset market is incomplete shareholders will be unanimous in approving investment plans generating vectors of dividends that are spanned by the payoffs of existing securities. In this model this condition is satisfied. Each firm j's period 1 dividend vector $(d_1^j\left(s^1\right))$ is just a multiple of the dividend vector of any other firm j' because $d_1^j\left(s^1\right) = (k_1^j\left(s^0\right)/k_1^{j'}\left(s^0\right))d_1^{j'}\left(s^1\right)$ for every s^1. Different levels of a firm's investment do not alter the spanning properties of its dividends vector. Thus, the effects of different levels of investment on a firm's stock price can be inferred from the information contained in other firms' stock prices. Notice that the reason why Ekern and Wilson's spanning condition holds here is non-trivial, in the sense that it is not an assumption, but rather an implication of constant returns to scale in production. The key to spanning lies in the fact that from the point of view of period 0, the production function in period 1 effectively displays constant returns to scale in physical capital only. This is because the labor input in period 1 is chosen after the capital stock is already in place, and, by constant returns to scale, the optimal labor input in period 1 is a linear function of capital (see equation 11).

4 Binding Short-Sales Constraints

In this section we analyze the case where the short-sales constraint (2) binds for some initial shareholders of a firm. In this case, for these initial shareholders the term multiplying $\theta^i_{1j}\left(s^{i0}, s^0\right)$ in equation (18) is not equal to zero. Except for the case in which the binding short-sales constraint is $\underline{\theta}= 0$, in which case the second term in (18) disappears, this situation introduces the possibility of disagreement among initial shareholders about the optimal size of investment to be undertaken by the firm.

Consider an initial shareholder i for whom the short-sales constraint (2) binds. As noticed above, if this constraint binds for one firm j, then it must also bind for all firms. The introduction of a short-sale constraint of the kind (2) implies that this shareholder's first order condition with respect to $\theta^i_{1j}\left(s^{i0}, s^0\right)$ (equation 16) holds as an inequality. Thus, there exists a $\xi^i_0\left(s^0\right) \in (0, 1)$ such that

$$p^j_0\left(s^0\right)\xi^i_0\left(s^0\right) = \sum_{s^1,s^{i1}} m^i_0\left(s^{i1}, s^1\right) d^j_1\left(s^1\right), \text{ for all } j \in J.$$

Replacing this into the first order condition for the shareholder i's preferred investment level (equation 18), taking into account equation (19) and the constraint $\theta^i_{1j}\left(s^{i0}, s^0\right) = -\underline{\theta}$, yields:

$$\theta^i_{0j}\left(\frac{p^j_0\left(s^0\right)}{k^j_1\left(s^0\right)} - 1\right) + \underline{\theta}\frac{p^j_0\left(s^0\right)}{k^j_1\left(s^0\right)}\left(1 - \xi^i_0\left(s^0\right)\right) = 0.$$

It follows that:

Proposition 4. Constrained initial shareholders of firm j prefer a higher level of period 1 capital than unconstrained shareholders. In particular, constrained shareholder i's preferred investment is given by:

$$k^j_1\left(s^0\right) = p^j_0\left(s^0\right)\left[1 + \frac{\underline{\theta}}{\theta^i_{0j}}\left(1 - \xi^i_0\left(s^0\right)\right)\right], \tag{28}$$

while all unconstrained shareholders would want to choose $k^j_1\left(s^0\right) = p^j_0\left(s^0\right)$.

A constrained initial shareholder would like to set the firm's investment to the point where the derivative $\partial p^j_0\left(s^0\right)/\partial k^j_1\left(s^0\right)$ is smaller than one, i.e., for each dollar of additional investment in the firm, the firm's stock market price increases by less than a dollar. Despite the fact that the increase in the firm's price is smaller than the cost of increasing the firm's capital, a constrained shareholder benefits from this choice. In fact, the higher stock price translates into greater proceedings from short-selling the firm's stock. Given that the shareholder is constrained, each extra dollar obtained from short-selling can be consumed today, while the discounted marginal cost of repaying this debt back tomorrow is only $\xi^i_{0j}\left(s^0\right) < 1$.

Notice that while the investment level preferred by constrained shareholders in equation (28) depends on the agent-specific multiplier $\xi_0^i\left(s^0\right)$, unconstrained shareholders agree among themselves on the level of investment to be undertaken by the firm. This property suggests that, as long as unconstrained shareholders initially hold more than fifty percent of the firm's shares, their preferred level of investment would prevail if the latter was determined by majority voting. In this case, the price of one share of firm j coincides with the price of a unit of physical capital. Thus, as in section 3.2, the equilibrium allocation of this economy would be the same as in an economy where consumers, rather than firms, accumulate capital.

When initial constrained shareholders own more than fifty percent of the firm's shares the analysis becomes more complex. In a majority voting equilibrium, provided it exists, period 1 capital stock tends to be higher than a firm's stock price: $k_1^j\left(s^0\right) > p_0^j\left(s^0\right)$, as preferred by constrained shareholders. While interesting, this case seems unlikely to occur in the class of incomplete markets models analyzed in the macroeconomic literature. In the latter, in fact the measure of constrained agents is usually negligible and their share of the aggregate capital stock (the equivalent of the initial shares θ_{0j}^i in an economy where consumers accumulate capital) even smaller.

5 Extension to Multiple Periods

In this section we show how the unanimity result derived in section 3.2 extends to a multiperiod, possibly infinite horizon, economy. The additional feature of the model in this context is the fact that stock returns depend not only on future dividends but also on future stock prices.

Consider first the case where T is finite. Denote by s^{it} the length-t history of individual shocks for agent i and by s^t the history of aggregate shocks. Each consumer maximizes the following utility function

$$V^i = \sum_{t=0}^{T} \sum_{s^t, s^{it}} \pi\left(s^{it}, s^t\right) U\left(c_t^i\left(s^{it}, s^t\right)\right),$$

where $\pi\left(s^{it}, s^t\right)$ denotes the probability, as of time zero, of histories $\left(s^{it}, s^t\right)$. As a consumer, agent i faces the sequence of budget constraints:

$$c_t^i\left(s^{it}, s^t\right) + \int_0^1 p_t^j\left(s^t\right) \theta_{t+1j}^i\left(s^{it}, s^t\right) dj = \int_0^1 \theta_{tj}^i\left(s^{it-1}, s^{t-1}\right)\left(d_t^j\left(s^t\right) + p_t^j\left(s^t\right)\right) + w_t\left(s^t\right) x_t^i\left(s^{it}, s^t\right),$$

where the notation is analogous to the one of section 3. For simplicity, in this section we rule out short-selling:[17]

$$\theta^i_{t+1j}\left(s^{it},s^t\right)\geq 0. \tag{29}$$

The first order condition for holding stocks of firm j is given by

$$p^j_t\left(s^t\right)\geq \sum_{s^{t+1}|s^t,s^{it+1}|s^{it}} m^i_t\left(s^{it+1},s^{t+1}\right)\left(d^j_{t+1}\left(s^{t+1}\right)+p^j_{t+1}\left(s^{t+1}\right)\right),\ \text{for all } j\in J, \tag{30}$$

where

$$m^i_t\left(s^{it+1},s^{t+1}\right)\equiv\beta\frac{\pi\left(s^{it+1},s^{t+1}\right)}{\pi\left(s^{it},s^t\right)}\frac{U\left(c^i_{t+1}\left(s^{it+1},s^{t+1}\right)\right)}{U\left(c^i_t\left(s^{it},s^t\right)\right)},$$

and the notation $s^{t+1}|s^t$ denotes the length-$t+1$ history s^{t+1} that follows history s^t.

A firm's dividends are defined as

$$d^j_t\left(s^t\right)=F\left(k^j_t,l^j_t\left(s^t\right);z_t\right)-w_t\left(s^t\right)l^j_t\left(s^t\right)+(1-\delta)\,k^j_t\left(s^{t-1}\right)-k^j_{t+1}\left(s^t\right). \tag{31}$$

Since shareholders will unanimously choose $k^j_{T+1}\left(s^T\right)=0$, by constant returns to scale in production, $d^j_T\left(s^T\right)$ can be written as

$$d^j_T\left(s^T\right)=k^j_T\left(s^{T-1}\right)q\left(w_T\left(s^T\right),z_T\right). \tag{32}$$

Thus, period $T-1$ is exactly the same as period 0 in the two-period economy of section 3. Initial shareholders in $T-1$ will unanimously agree to maximize the net stock market value of the firm, leading to the condition:[18]

$$k^j_T\left(s^{T-1}\right)=p^j_{T-1}\left(s^{T-1}\right).$$

Using the previous equation together with (31), one obtains

$$\begin{aligned} d^j_{T-1}\left(s^{T-1}\right)+p^j_{T-1}\left(s^{T-1}\right) &= F\left(k^j_{T-1}\left(s^{T-2}\right),l^j_{T-1}\left(s^{T-1}\right);z_{T-1}\right)-w_{T-1}\left(s^{T-1}\right)l^j_{T-1}\left(s^{T-1}\right)+(1-\delta)\,k^j_{T-1}\left(s^{T-2}\right)\\ &= k^j_{T-1}\left(s^{T-2}\right)q\left(w_{T-1}\left(s^{T-1}\right),z_{T-1}\right), \end{aligned} \tag{33}$$

where the second equality follows from the constant returns to scale assumption. At the beginning of period $T-2$, a shareholder of firm j would want to choose $k^j_{T-1}\left(s^{T-2}\right)$ in order to maximize his utility function.

[17] This assumption rules out disagreement among constrained and unconstrained initial shareholders. Initial shareholders that are constrained in the stock market choose to hold zero shares of the firm, and therefore only care about maximizing its net stock market value.

[18] Notice that some initial shareholders will be unconstrained, while others will be constrained. However, assumption (29) ruling out short-sales, implies that also constrained shareholder will want to maximize the net market value of the firm. This can be easily seen by setting $\theta^i_{T-1j}\left(s^{iT-1},s^{T-1}\right)=0$ in the first order condition of a constrained shareholder (the equivalent of equation 18).

This gives rise to the first order condition:

$$
\begin{aligned}
&\theta^{i}_{T-2j}\left(\frac{\partial p^{j}_{T-2}\left(s^{T-2}\right)}{\partial k^{j}_{T-1}\left(s^{T-2}\right)}+\frac{\partial d^{j}_{T-2}\left(s^{T-2}\right)}{\partial k^{j}_{T-1}\left(s^{T-2}\right)}\right) \qquad (34)\\
&-\theta^{i}_{T-1j}\left(s^{iT-2},s^{T-2}\right)\left(\frac{\partial p^{j}_{T-2}\left(s^{T-2}\right)}{\partial k^{j}_{T-1}\left(s^{T-2}\right)}-\sum_{\substack{s^{T-1}|s^{T-2}\\ s^{iT-1}|s^{iT-2}}} m^{i}_{T-2}\left(s^{iT-1},s^{T-1}\right)\frac{\partial d^{j}_{T-1}\left(s^{T-1}\right)+\partial p^{j}_{T-1}\left(s^{T-1}\right)}{\partial k^{j}_{T-1}\left(s^{T-2}\right)}\right)\\
&-\sum_{\substack{s^{T-1}|s^{T-2}\\ s^{iT-1}|s^{iT-2}}}\theta^{i}_{Tj}\left(s^{iT-1},s^{T-1}\right)\frac{\partial k^{j}_{T}\left(s^{T-1}\right)}{\partial k^{j}_{T-1}\left(s^{T-2}\right)}\left(\frac{\partial p^{j}_{T-1}\left(s^{T-1}\right)}{\partial k^{j}_{T}\left(s^{T-1}\right)}-\sum_{\substack{s^{T}|s^{T-1}\\ s^{iT}|s^{iT-1}}} m^{i}_{T-1}\left(s^{iT},s^{T}\right)\frac{\partial d^{j}_{T}\left(s^{T}\right)}{\partial k^{j}_{T}\left(s^{T-1}\right)}\right)=0.
\end{aligned}
$$

The first line of this equation captures the effect of a higher $k^{j}_{T-1}\left(s^{T-2}\right)$ on the net stock market value of the firm for an initial shareholder i at time $T-2$. The second line reflects the effect of a higher $k^{j}_{T-1}\left(s^{T-2}\right)$ on the expected discounted value, as perceived by agent i, of the firm's dividend and stock value at $T-1$. The third line captures the effect of a higher $k^{j}_{T-1}\left(s^{T-2}\right)$ on the expected market value of the firm at $T-1$ and dividends at T. While the first two terms are familiar from the analysis of section 3.2, the last one is new. In a setting with more than two periods, in fact, a shareholder has to consider the effect of his investment choice on all the future values of the firm's stock and dividends. In this case, the time $T-2$ shareholder has to consider the effect of his choices not only on time $T-1$ variables, but also on time T variables.

Using the arguments already developed above, it is possible to simplify equation (34) considerably. First, notice that by equation (32):

$$\frac{\partial d^{j}_{T}\left(s^{T}\right)}{\partial k^{j}_{T}\left(s^{T-1}\right)}=\frac{d^{j}_{T}\left(s^{T}\right)}{k^{j}_{T}\left(s^{T-1}\right)}$$

and, as in equation (12):

$$\frac{\partial p^{j}_{T-1}\left(s^{T-1}\right)}{\partial k^{j}_{T}\left(s^{T-1}\right)}=\frac{p^{j}_{T-1}\left(s^{T-1}\right)}{k^{j}_{T}\left(s^{T-1}\right)}.$$

Thus, the first order condition (30) implies that:

$$\frac{\partial p^{j}_{T-1}\left(s^{T-1}\right)}{\partial k^{j}_{T}\left(s^{T-1}\right)}-\sum_{\substack{s^{T}|s^{T-1}\\ s^{iT}|s^{iT-1}}} m^{i}_{T-1}\left(s^{iT},s^{T}\right)\frac{\partial d^{j}_{T}\left(s^{T}\right)}{\partial k^{j}_{T}\left(s^{T-1}\right)}=0. \qquad (35)$$

Using (35) and (33) into (34) then yields

$$\theta_{T-2j}^{i}\left(\frac{\partial p_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}+\frac{\partial d_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}\right) \tag{36}$$

$$-\theta_{T-1j}^{i}\left(s^{iT-2},s^{T-2}\right)\left(\frac{\partial p_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}-\sum_{\substack{s^{T-1}|s^{T-2}\\ s^{iT-1}|s^{iT-2}}} m_{T-2}^{i}\left(s^{iT-1},s^{T-1}\right)\frac{d_{T-1}^{j}\left(s^{T-1}\right)+p_{T-1}^{j}\left(s^{T-1}\right)}{k_{T-1}^{j}\left(s^{T-2}\right)}\right).$$

Notice, further, that (33) implies the no-arbitrage condition

$$\frac{d_{T-1}^{j}\left(s^{T-1}\right)+p_{T-1}^{j}\left(s^{T-1}\right)}{p_{T-2}^{j}\left(s^{T-2}\right)}=\frac{d_{T-1}^{j'}\left(s^{T-1}\right)+p_{T-1}^{j'}\left(s^{T-1}\right)}{p_{T-2}^{j'}\left(s^{T-2}\right)} \text{ for all } j,j',s^{T-2},s^{T-1}.$$

The argument for why this relationship must hold is the same as in section 3.1. It follows that:

$$\frac{\partial p_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}=\frac{p_{T-2}^{j}\left(s^{T-2}\right)}{k_{T-1}^{j}\left(s^{T-2}\right)}.$$

Replacing this into (36) and using the first order condition (30) for holding stock yields:

$$\frac{\partial p_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}-\sum_{\substack{s^{T-1}|s^{T-2}\\ s^{iT-1}|s^{iT-2}}} m_{T-2}^{i}\left(s^{iT-1},s^{T-1}\right)\frac{d_{T-1}^{j}\left(s^{T-1}\right)+p_{T-1}^{j}\left(s^{T-1}\right)}{k_{T-1}^{j}\left(s^{T-2}\right)}=0.$$

Therefore, the first order condition for investment by shareholder i of firm j in period $T-2$ simplifies to

$$\frac{\partial p_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}=1,$$

or $k_{T-1}^{j}\left(s^{T-2}\right)=p_{T-2}^{j}\left(s^{T-2}\right)$. All initial shareholders of firm j in period $T-2$ will unanimously agree to set the level of capital to be used in production in $T-1$ equal to the stock price of the firm in $T-2$. Using the same logic, this unanimity result can be generalized to any period $t<T$ until the initial period $t=0$.

It follows that:

Proposition 5. In the multiperiod economy with no short-sales of stocks, all initial shareholders of a firm j agree in each period $t<T$ to set its capital stock in $t+1$, denoted by $k_{t+1}^{j}\left(s^{t}\right)$, according to the condition:

$$\frac{\partial p_{t}^{j}\left(s^{t}\right)}{\partial k_{t+1}^{j}\left(s^{t}\right)}=1.$$

When $T\rightarrow\infty$ there is no final period that can be used as a starting point for the backward induction type of argument illustrated above. The definition of dividends and the assumption of constant returns to scale in production imply that

$$d_t^j\left(s^t\right)+k_{t+1}^j\left(s^t\right)=k_t^j\left(s^{t-1}\right)q\left(w_t\left(s^t\right),z_t\right),$$

where the lack of a terminal point prevents us from directly replacing $p_t^j\left(s^t\right)$ instead of $k_{t+1}^j\left(s^t\right)$ on the left hand side of this equation (as in equation 33). The analysis in this case must proceed by first guessing that in fact $p_t^j\left(s^t\right)=k_{t+1}^j\left(s^t\right)$, and then verifying that this guess is valid. Showing that the guess is valid amounts to showing that there is unanimity among initial shareholders regarding the investment decision of every firm. The latter point can be proved exactly as above for the finite horizon economy.

6 Summary

In this paper we have studied versions of the standard incomplete markets economy introduced by Aiyagari (1994) and Krusell and Smith (1998) under the assumption that firms, rather than consumers, accumulate physical capital. If borrowing constraints are not binding and production occurs under constant returns to scale, the equilibrium allocation of this economy is shown to coincide with the one that characterizes the standard model. In particular, a firm's shareholders will unanimously agree on the firm's optimal level of investment.

In the case in which borrowing constraints are binding, instead, the unanimity result breaks down. Constrained initial shareholders would like the firm to invest more than unconstrained ones. Given that in a typical macroeconomic model with incomplete market (Krusell and Smith, 1998), the measure of agents for whom short-sale constraints are binding is negligible, it is unlikely that taking this disagreement explicitly into account would have a significant effect on its quantitative properties. Therefore, taken as a whole, this paper suggests that, in practice, there might not be any significant loss of generality in focusing on incomplete markets models a la Aiyagari-Krusell-Smith where consumers, rather than firms, undertake the capital accumulation decision.

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