View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Research Papers in Economics

IOWA STATE UNIVERSITY

U.S. Financial Transmission Rights: Theory and Practice

Junjie Sun

March 2005
(Revised September 19, 2005)

Working Paper # 05008

Department of Economics Working Papers Series

Ames, Iowa 50011

Iowa State University does not discriminate on the basis of race, color, age, religion, national origin, sexual orientation, gender identity, sex, marital status, disability, or status as a U.S. veteran. Inquiries can be directed to the Director of Equal Opportunity and Diversity, 3680 Beardshear Hall, (515) 294-7612.

U.S. Financial Transmission Rights: Theory and Practice

Junjie Sun *

Department of Economics
Iowa State University

September 19, 2005

Abstract

This paper reviews both theoretical and empirical studies of financial transmission rights (FTRs) in the major U.S. wholesale power markets and provides a simple illustrative two-stage model to study the competitive behaviors of electricity generators (wholesale power sellers) and load serving entities (LSEs)(wholesale power buyers) and the welfare effects of FTRs in the restructuring U.S. wholesale power market framework. The analysis focuses on a competitive two-node electricity network model where there is one generator and one LSE in each node with linear marginal cost and demand function, supervised by an independent system operator (ISO). In the first-stage of modeling, a no-rights benchmark model is developed to solve for the optimal quantity of power production and consumption and derive the locational marginal price for each node, which serve as the building blocks to solve for the optimal FTR hedge positions in the second-stage model. Once a stochastic shock is introduced, the second-stage model shows that the acquisition of optimal FTRs by the risk averse generators and LSEs will increase and in general will strictly increase the social welfare compared with the case where there is no FTRs available. This result presents a counterexample to the somewhat negative views about FTRs held by other economists in the literature and provides some economic explanations to the fact that FTRs are widely adopted as a financial hedge instrument in the major U.S. wholesale power markets.

Keywords: financial transmission rights, locational marginal price, security-constrained economic dispatch, independent system operator, congestion rent

*Please address correspondence to: Junjie Sun, Iowa State University, Department of Economics, 280E Heady Hall, IA, 50011. Tel: (515) 294-7236. Fax: (515) 294-0221. Email: jjsun@iastate.edu. I would like to thank Leigh Tesfatsion, Deddy Koesrindartoto and Sergio Lence for their helpful comments and suggestions.

1 Introduction

As the largest regulated energy industry in the United States, the U.S. electric power industry has undergone a tremendous change to become more competitive (U.S. Department of Energy 2000). One of the central components in the competitive electricity market is to have open access to the transmission system. In the U.S., the major transmission system can be roughly divided into three regions, the East and West Interconnections and the Electricity Reliability Council of Texas (ERCOT) as shown in Figure 1.

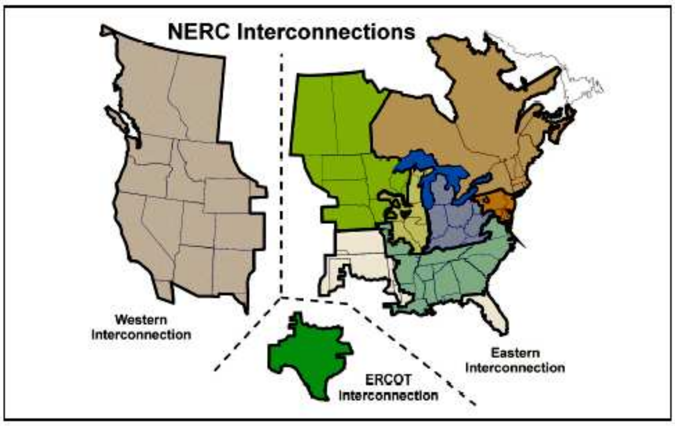

Fig. 1.1 North American Electricity Transmission Systems

The North American electricity transmission system consists of three interconnected systems: the Eastern Interconnection, the Western Interconnection, and most of the state of Texas. Within these interconnections, more than 140 control areas manage electricity operations for local areas and coordinate reliability through 10 regional councils.
Source: NERC 2001.

Figure 1: Three major U.S. interconnected transmission systems

Electricity as an economic good has its unique features. The most distinct one is that its storage cost is enormously high such that almost all the electric power is delivered through transmission lines for immediate consumption once it is produced. As indicated in a recent National Transmission Grid Study (2002), there is now a tendency for U.S. transmission lines to get congested and thus create substantial impact on the locational pricing system and overall reliability of U.S. wholesale power market (see Stoft 2002 and Wilson 2002). The U.S. Federal Energy Regulatory Commission (FERC) responds to this issue by calling a new independent institutional entity to manage and handle transmission assets, i.e., *Independent System Operator* (ISO). By the nature of ISO, it is a non-profit organization whose purpose is to monitor the power flow, collect generator's supply offers and load serving entity (LSE)'s demand bids, and calculate the optimal power dispatch taking into account various network constraints such as energy balancing constraint and thermal limit constraint.

To address the above congestion issue, it is a common practice in the U.S. wholesale power market for ISO to issue *financial transmission rights* (FTRs). According to ISO New England Manual (2003a), an FTR is a financial instrument that entitles the holder to receive compensation for transmission congestion costs that arise when the transmission grid is congested in the day-ahead market. The amount of compensation is based on differences in day-ahead *locational marginal prices* (LMPs) result from the dispatch of generators to relieve the congestion. FTR entitles its holders to a share of the *congestion rents* collected in the day-ahead energy market, thus provides the holder a financial hedge in the day-ahead market for the nodal price difference between a node of receipt (source) to a node of delivery (sink).

In the literature four types of FTRs have been proposed [1], namely, *point-to-point*(PTP) obligation, PTP option, *flowgate* (FG) obligation, and FG option (see Hogan 2002 and 2003). An FTR option entitles its holders to revenue when day-ahead congestion occurs in the desired direction. In contrast,an FTR obligation entitles it holders to a revenue when day-ahead congestion occurs in the desired direction and obligates holders to a payment when day-ahead congestion is in the opposite direction. When using PTP FTRs, market participants can obtain any collection of FTRs corresponding to a feasible power flow in the transmission system. When using FG FTRs, market participants can only obtain FTRs on pre-determined transmission lines (flowgates), which are considered most at risk should the lines get congested.

The definition of a PTP FTR obligation can be more clearly illustrated in the following example. Suppose there are two nodes in the transmission network, node A where power is injected into the transmission network and node B where power is withdrawn from the transmission network. Assuming no transmission losses, the PTP FTR entitles the holder to the difference in day-ahead LMP between node A and B. By its obligation nature, the FTR holder receives a positive payment (LMP(B)-LMP(A)) from the ISO if LMP(B) exceeds LMP(A). On the other hand, the FTR holder is obligated to pay the ISO (LMP(A)-LMP(B)) if LMP(A) exceeds LMP(B). Thus the wholesale power market participants's risks associated with different LMPs are in principle decreased by purchasing FTRs.

To date, FTRs have been widely used to hedge against the potential loss in the transmission congestion in major U.S. wholesale power markets. For example, FTR was introduced in the PJM (Pennsylvania, New Jersey and Maryland) Interconnection since April 1998, in

[1]As recommended in WPMP by FERC (2003), the U.S. electricity industry have favored PTP FTRs due to its simplicity to implement and its successes in the early restructuring markets such as PJM and New York.

New York since September 1999, in California since February 2000, and in New England since March 2003. Note that FTRs have been known under different names in different U.S. power markets. For instance, in PJM FTRs are referred to as *Fixed Transmission Rights*, in New York *Transmission Congestion Contracts* (TCCs), in California *Firm Transmission Rights*, in New England *Financial Transmission Rights*, and in Texas *Transmission Congestion Rights* (TCRs).

In spite of the fact that FTR has been widely used in the major U.S. electricity market, it is still a new market instrument that needs theoretical and empirical evaluations. There are issues remaining questionable such as to what extent, if there is any, can FTRs help facilitate the market to generate orderly, fair, and efficient outcomes despite attempts by market participants to gain individual advantage through strategic behavior? In addition, does the introduction of FTRs create an appropriate incentive for individual firms to invest in the transmission infrastructure?

Although many theoretical models have been proposed and empirical evidences have been discussed in the literature, no attempt has been made to summarize the previous findings about FTRs. The contribution of this paper is to first provide a comprehensive review of various FTR findings from both theoretical and empirical perspectives, and then to better illustrate economic efficiency improvement of introducing FTR in the presence of uncertainty, a simple economic network model is presented and results are discussed. Therefore, this paper is organized as follows. The second section conducts a literature review on both theoretical and empirical studies of FTRs in the U.S. wholesale power market. Section three presents the no-rights benchmark model, which is essentially the competitive equilibrium framework applied to the economic dispatch model in a simple 2-node electricity network. Section four then uses the economic dispatch solution from the benchmark model as the building block to construct a 2-node FTR model where uncertainty is introduced as stochastic shocks to both demand and supply sides. Section five discusses the conclusions and potential extensions of future work.

2 FTRs in Theory and Practice

2.1 Theoretical Studies of FTRs

FTRs and market power

Although FTR advocates argue that tradable FTRs should facilitate electricity trade in the short run through the alleviation of transmission bottlenecks caused by congestion (see Hogan(2003)), in the current economic literature, people hold more negative views toward FTRs.

For example, in a well-known study, Joskow and Tirole (2000) reach a negative conclusion about FTRs. In their two-node network model with cheap cost generators in the north node, expensive cost generators in the south node, and a transmission line linking the North and the South that has a fixed thermal capacity, they argue that the acquisition of financial rights may enhance the market power in the South if the generators in the South are owned by a monopoly firm. In addition, they carry out a welfare comparison and show that the social welfare derived from the absence of transmission rights is at least as high as and in general higher than the social welfare derived from the system with the financial transmission rights. This striking result clearly indicates the negative views about FTRs held by the authors.

Responding to Joskow and Tirole's result, Hogan (2000) provides an example which shows that introducing financial rights enhances monopoly profits but it increases efficiency as well. This is in contrast to Joskow and Tirole's result which implies that the no-rights solution is always the most efficient one. The case in Hogan's comment differed from Joskow and Tirole in that the monopolist controls generation at more than one location and some of its generation is at low cost. The detailed derivation is in Cardel, Hitt and Hogan (1997). This example shows the complex nature of the deregulated U.S. electricity market structure such as having significant different results and policy implications due to different network configurations.

By using a Cournot model of competition in a congested transmission network, Oren (1997) illustrates that even in the absence of market concentration, the expectation of congestion and passive transmission rights can lead to implicit collusion among generators and departure from marginal cost pricing. This invalidates the key premise underlying the indirect implementation of transmission rights trading through optimal dispatch by the ISO. The author concludes that passive transmission rights (in the form of transmission congestion contracts (TCCs)) will be preempted by the active traders who will adjust their prices so as to capture the congestion rents. Price distortions due to congestion and passive transmission ownership can result in short and long term inefficiency.

By re-investigating the issues in Oren (1997), Stoft (1999) demonstrates that financial transmission rights such as TCCs allow their owners to capture at least a portion, and sometimes all, of the congestion rents, and thus is shown to be effective in reducing market power. Moreover, the extent to which TCCs can reduce the market power depends on the extent to which total generation capacity exceeds the capacity of the largest generator. This result is in contrast with Oren's. The author states the reasons why his conclusions differ from Oren's in two perspectives. First, he points out that in Oren's second example, which is intended to be a Cournot model, is mistakenly constructed as a Bertrand model and then mis-analyzed. When the model is re-built along Cournot lines, Oren's conclusion is refuted. Second, in Oren's model, it is assumed that generators could not purchase financial transmission rights while in Stoft's model, this assumption is relaxed.

In another paper, Bushnell (1999) expresses his concern that transmission rights can be manipulated by its owners to reduce transmission capacity made available to the competitive market during hours in which there would otherwise be no congestion. In the short run, such withholding behavior could prove profitable for firms in several ways such as increasing the value of local generators and the value of the transmission rights themselves. The author illustrates his point by using a simple two-node network case with one fixed marginal cost generator at one node and a downward-sloping demand at the other node. Lastly the author argues that due to the concerns about transmission capacity withholding and the inherent network uncertainties, the initial offering of transmission rights in California was to be limited to a level below the full transmission capacity available to the California ISO.

Using human-subject experiment, Kench (2004) conducts an interesting study to test the theoretical results in Joskow and Tirole (2000). Specifically, the author carries out a double-oral auction (DOA) experiment to test the predictions of Joskow and Tirole's theoretical results for a radial electricity market without transmission rights, with financial transmission rights, and with physical rights. The author found that physical rights lead to more "right" market signals, decrease some market power, and remove an uncertainty about electricity transmission congestion better than financial rights or the absence of rights. However, the author also pointed out that one should be very cautious in trying to interpret his experimental results into policy implications because the stylized market setting in his paper does not capture many intricacies (such as the "loop flow effect") of real world electricity markets.

FTRs and auction design issue

Bautista and Quintana (2005) develop a methodology to screen and discriminate FTRs that may exacerbate the market power for some monopoly market participants. The proposed methodology is based upon the use of relative hedging position ratios. These ratios comprise the network configuration, market outcomes, and the participants position in the market, and quantify the relationship between the positions of an FTR bidder in the energy market and in the transmission rights allocation. The authors also point out that since an

FTR scheme has a reduced liquidity, which may be worsened if a discrimination such as in this study is introduced. Due to the potential complexity for carrying out any regulatory intervention on FTRs ownership, the authors suggest to build the FTRs framework upon their allocation to other entities, such as LSEs or traders, rather than generators.

Mendez and Rudnick (2004) propose a new congestion management system under nodal and zonal dispatches with implementation of fixed transmission rights (FTR) and flowgate rights (FGR), respectively. Using a static simulation model, which implements marginal theory where congestion components are introduced in the pricing model, they show that the FTR model is suitable for congestion management in deregulated centralized market structures with nodal dispatch, while the FGR is suitable for decentralized markets. Their application indicates that FGR presents advantages over FTR regarding signals on grid use, but its application is too complicated to make its implementation attractive.

In a related study trying to accommodate both point-to-point and flowgate transmission rights, O'Neill et al (2002) propose a "joint energy and transmission rights auction" (JETRA) to allow transmission users to specify which type of transmission rights, point-to-point or flowgate, they prefer to use and reconfigure them over time. JETRA is able to simultaneously accommodate flowgate and point-to-point options and obligations, along with energy production and consumption futures. Under certain conditions, the authors prove that the auction is revenue adequate for the market operator in the sense that payments to rights holders cannot exceed congestion revenues.

FTRs and transmission investment/expansion

In another set of papers several authors address the issues of transmission investment or expansion in the hope to find the best way to attract investment for the long-term expansion of an electricity transmission network.

Joskow and Tirole (2003) examine the performance of a "merchant transmission" model in which investment in electric transmission capacity rely upon competition and free entry to exploit profitable transmission investment opportunities rather than on regulated monopoly transmission companies. Under strict assumptions, the authors show that the merchant investment model is able to solve the natural monopoly problem traditionally associated with electricity transmission networks. However, when the authors extend their model by introducing assumptions that more accurately reflect the physical and economic attributes of transmission networks, many attractive properties of the merchant model disappear and inefficient transmission investment decisions are made.

In a related study, Kristiansen and Rosellon (2004) propose a merchant mechanism to expand electricity transmission based on long-term FTRs. As the authors argue, the system operator needs a protocol for awarding incremental FTRs that maximize investor's preferences, and preserves certain unallocated FTRs (or proxy awards) so as to maintain revenue adequacy. They define a proxy award as the best use of the current network along the same direction as the incremental awards, and develop a bi-level computational model for allocating long-term FTRs according to this rule and apply it to different network topologies. They find that simultaneous feasibility for a transmission expansion project crucially depends on the investor-preference and the proxy-preference parameters.

In another interesting study, Rudkevich (2004) investigates the investment and bidding strategies for firm transmission rights. The study first addresses the applicability of the Markowitz portfolio theory to investing in firm transmission rights (FTRs) or transmission congestion contracts (TCCs) typical for Northeastern U.S. electricity market. Specifically, the author uses the principal component analysis to select subsets of statistically independent FTRs/TCCs and obtain the necessary and sufficient conditions for arbitrage opportunities. In the second part of paper, the author analyzes the profit-maximizing bidding strategies for large players with significant Auction Revenue Rights (ARRs).

In a survey study on the topic of transmission expansion, Rosellon (2003) studies the three existing approaches to electricity transmission expansion, i.e., transmission expansion through long-term FTRs, through regulatory mechanisms and through strategic behavior of generators (market power). The first approach relies on the auction of long-term FTRs by an independent system operator (also known as the merchant approach). The second approach is to provide a Transco with the incentive to expand transmission by making it confront the social cost of transmission congestion. The last approach defines optimal expansion of the transmission network according to the strategic behavior of generators. After comparing each approach's advantage and disadvantage, the author concludes that there is no single mechanism that guarantees the optimal expansion of the electricity transmission network, and suggest that there may exists the second-best approach which is to combine the merchant and the regulated transmission model.

The vast literature of theoretical FTRs studies is be summarized in Table 1.

2.2 Empirical Studies of FTRs

Siddiqui et al. (2003) analyze the public data from 2000 and 2001, and find out that New York transmission congestion contracts (TCCs) provides market participants with a potentially effective hedge against volatile congestion rents. However, the prices paid for TCCs

Table 1: Overview of Theoretical Studies in FTRs

FTRs and Market Power	FTR Auction Design	Transmission Investment
Joskow and Tirole (2000)	Bautista and Quintana (2005)	Joskow and Tirole (2003)
Hogan (2000)	Mendez and Rudnick (2004)	Kristiansen and Rosellon (2004)
Oren (1997)	O'Neill et al (2002)	Rudkevich (2004)
Stoft (1999)		Rosellon (2003)
Bushnell (1999)		
Kench (2004)		

systematically diviated from the associated congestion rents for distant locations and at high prices. Based on their analyses, the authors suggest that there exist an inﬃcient market for TCCs due to the fact that the price paid for the hedge not being in line with the congestion rents, i.e., unreasonably high risk premiums are being paid. The authors then offer two possible explanations to their empirical finding. One is the low liquidity of TCC markets and the other is the deviation of TCC feasibility requirements from actual energy flows.

In response to Siddiqui et al. (2003) regarding the inefficient pricing of TCCs in New York market, Deng et al. (2004) try to investigate further on the question that whether the price deviations are due to price discovery errors which will eventually vanish or due to inherent inefficiencies in the auction structure. They show that even with perfect foresight of average congestion rents the clearing prices for the FTRs depends on the bid quantity and therefore may not be priced correctly in the FTR auction. The authors conclude that price discovery alone would not remedy the discrepancy between the auction prices and the realized values of the FTRs, and secondary markets or frequent reconfiguration auctions are necessary in order to achieve such convergence.

In a practical study, Lyons et al. (2000) use simple numerical examples to show how the FTRs work in a two-node case network model and give a gentle introduction of various aspects of FTRs such as property rights and transmission expansion, price hedging, and allocation of FTRs. Also the authors conduct a market-wise study and show how various FTRs are handled in PJM, New York and California markets. Their results are summarized and extended in Table 2. Finally, the authors stress that although there is no universally superior model for FTRs, they are still very useful tools in electricity markets with locational pricing.

In another survey study, Kristiansen (2003) investigates how FTRs are acquired and implemented in a range of markets such as PJM, New York, New England, California, Texas, and New Zealand. In each market, the author describe in detail the features of FTRs, some design issues, strength and weakness, and the market performances in different FTR markets. His result along with Lynos et al.(2000) is summarized in Table 2.

Denton and Waterworth (2002) did a comprehensive practical study about how FTRs could be introduced in Australian National Electricity Market (NEM)[2]. The Settlements Residue Auction (SRA) was established shortly after NEM to help market participants manage risks. The authors start their report by stating the rationale for changing the SRA process to create a better environment for implementing FTRs. They compare the FTRs in the U.S. markets such as PJM and New England. Then they introduce a workable FTR solution in line with the modified SRA and discuss how the proposed FTR solution addresses the critical issues in the Australian electricity market.

2.3 The Illustrative Two-stage FTR Study

Although the current literature expresses a mixed feeling about FTRs, it is not unfair to say more negative views are held toward FTRs (Joskow and Tirole 2000, Oren 1997, Bushnell 1999, Siddiqui et al. 2003, Deng et al. 2004, etc). While FTRs are widely adopted as a financial hedging instrument to help market participants to reduce their risks in the major U.S. wholesale power market, it seems not working very well. Why? Is it because of the complicated wholesale power market structure, or because the market participants are still learning how to place the bids and offers more efficiently, or because there is something fundamentally wrong about it?

Some close examination of previous work might give us some clues. For example, in the influential paper by Joskow and Tirole (2000), we found that although the authors demonstrated that introducing FTRs can decrease the overall efficiency, enhance the market power and reduce the welfare, their model seems to be too restrictive in the sense that *there is no uncertainty involved*. Since FTR, by construction, is used as a financial instrument to hedge against *uncertain* profit, if there is no uncertainty, the only conclusion that can be drawn is that FTR at most won't do any good and may in general do worse than the case where there are no FTRs available. In fact, Joskow and Tirole's welfare comparison shows that the social welfare under the absence of FTRs is as high as and in general strictly higher than that with FTRs in the case of no uncertainty.

In this paper, the goal is to illustrate how a simple two-stage FTR model can work to

[2]Although their report mainly focuses on the application in Australian national electricity market, there are indeed many similarities between Australian market and major U.S. markets such as PJM, New York and New England.

Table 2: Comparison of FTRs in Major U.S. Wholesale Power Market (Source: Kristiansen 2003, Lyons et al. 2000, NEPOOL FTR manual 2003b, MISO FTR manual 2005)

	PJM	New York	New England
Name	Fixed Transmission Rights	Transmission Congestion Contracts	Financial Transmission Rights
Contract	Obligations & options , no hedge against losses	Obligations, no hedge against losses	Obligations, no hedge against losses
Duration	Monthly auction, annual network integration service FTRs	6 months and 1, 2 and 5 year auction, monthly reconfiguration	Monthly auction
Acquisition	Network integration service, firm point-to-point service, auction, secondary market	Centralized TCC auction, direct sales, and secondary market	Auction, secondary market, transmission updates, entities paying congestion charges
Auction design	Monthly, single-round, uniform-price auction	Seasonal (multi-round), monthly reconfiguration uniform-price auction	Monthly, single-round, uniform-price auction
Congestion rents	Excess rents distributed to deficiencies in other periods, deficient rents reduce payments proportionally	Excess rents offset transmission system cost, deficit rents covered by the transmission owners	Excess rents distributed to FTR holders, deficit rents reduce payments proportionally
Distribution of revenues	FTR auction revenues are allocated among the regional transmission owners in proportion to their transmission revenue requirements	All revenues received by transmission owners from the sale of TCCs and excess auction revenues, are credited against the transmission owner's cost of service to reduce the transmission service charge	FTR auction revenues are distributed to sellers of FTRs and auction revenue rights recipients
Website	http://www.pjm.com/	http://www.nyiso.com/	http://www.iso-ne.com/

	California	Texas	Midwest
Name	Firm Transmission Rights	Transmission Congestion Rights	Financial Transmission Rights
Contract	Option-like, no hedge against losses	Inter-zonal option	Obligation, phase in option in the future
Duration	Annual auction	Monthly and annual auction	3 months or 1 year auction
Acquisition	Auction, secondary market, hour-ahead market	Auction, secondary market	Auction, secondary market, allocated based on existing transmission rights
Auction design	Annual, multi-round uniform-price auction	Annual, monthly, single-round, 24 simultaneous combinatorial auction	Annual, seasonal(3 months), monthly auction
Congestion rents	Excess rents partly cover the fixed costs of the grid deficient rents reduce payments proportionally	Any rent shortfall is uplifted to load and any surplus is credited against other uplift to load	Excess rents redistributed to FTR holders
Distribution of revenues	The auction proceeds go to the participating transmission owners. Each of them credits its FTR auction proceeds against its access charge	Credited to load entities in proportion to their load ratio share	To be determined
Website	http://www.caiso.com/	http://www.ercot.com/	http://www.midwestiso.org/

improve social welfare should there is any uncertainty. Specifically, we would like to address the following fundamental question: when we introduce uncertainty, does FTR matter now? In addition we want to conduct a welfare comparison in the uncertainty case to see if introducing FTRs is able to improve the social welfare or not. We start in section three (Stage 1) by constructing a benchmark model, which focuses on a two-node electricity network where there is one generator and one LSE in each node with parameterized marginal cost and demand functions, supervised by an independent system operator (ISO). This is essentially the competitive equilibrium (CE) case. By solving this benchmark model as the usual CE case, we obtain a security-constrained economic dispatch solution. Section four (Stage 2) presents the FTR model with stochastic shocks. Using the results from the benchmark model as building blocks, we then solve for the optimal FTR hedge solutions, and show that once uncertainty (even in a very simple form) is introduced, the acquisitions of optimal FTRs by the risk averse generators and LSEs increase and in general strictly increase the social welfare compared with the case where there are no FTRs available. This result thus serves as an counterexample to the somehow negative views of FTRs by other economists in the literature and provides some economic explanations to the fact that FTRs are widely adopted as a financial hedge instrument in the major U.S. wholesale power markets.

3 The No-rights Benchmark Model

The benchmark model consists of a simple two-node electricity network connected by a transmission line with a thermal limit. There is only one good in this model: electricity power, which is supplied by a group of unregulated generating companies (generators for short), wholesale power suppliers, and demanded by a group of Load Serving Entities(LSEs), wholesale power buyers. LSEs can be thought of as the distribution companies that can buy the "bundled" electricity power in the wholesale market and resell it to downstream end-user consumers. There is also an Independent System Operator (ISO) that operates the transmission network and manages the energy market. So there are three types of agents in this model: generators, LSEs and ISO.

Furthermore, each LSE has a price-sensitive and downward sloping demand curve. Each generator supplies the real power with a non-decreasing marginal cost. To obtain a dispatched quantity of power, all generators submit their supply offers and all LSEs submit their demand bids to ISO in the wholesale power market. ISO by its nature is a not-for-profit organization and behaves like a "social planner" to maximize the total net benefit of generators and LSEs based on their submitted offers and bids information by solving the optimal quantities of power supply and demand for each generator and LSE subject to the physical network constraints [3].

3.1 Model Specifications and Assumptions

To make this benchmark model simple, we make the following specifications and assumptions:

- There are only 2 nodes, namely node 1 and node 2, in this electricity network, which implies that power may either flow from node 1 to node 2 or node 2 to node 1 through the transmission line with the maximum power flow equal to the thermal limit capacity T $(T > 0)$. Also assume there is no loss during power transmission.

- For simplicity, suppose there is only one generator at each node, i.e., G_1 at node 1 and G_2 at node 2. Let Q_{G1} and Q_{G2} be the power supply quantities (injections) at node 1 and 2, respectively. The total cost function $TC_i(Q_{Gi})$, variable cost function $VC_i(Q_{Gi})$, and marginal cost function $MC_i(Q_{Gi})$ for generator G_i $(i = 1, 2)$ are specified as follows:

$$TC_1(Q_{G1}) = f_1 + b_1^S Q_{G1} + \frac{1}{2} a_1^S Q_{G1}^2 \tag{1}$$

$$TC_2(Q_{G2}) = f_2 + b_2^S Q_{G2} + \frac{1}{2} a_2^S Q_{G2}^2 \tag{2}$$

$$VC_1(Q_{G1}) = b_1^S Q_{G1} + \frac{1}{2} a_1^S Q_{G1}^2 \tag{3}$$

$$VC_2(Q_{G2}) = b_2^S Q_{G2} + \frac{1}{2} a_2^S Q_{G2}^2 \tag{4}$$

$$MC_1(Q_{G1}) = b_1^S + a_1^S Q_{G1} \tag{5}$$

$$MC_2(Q_{G2}) = b_2^S + a_2^S Q_{G2} \tag{6}$$

  where parameters (a_i^S, b_i^S, f_i) are all positive for $i = 1, 2$.

- For simplicity, suppose there is only one LSE at each node, i.e., LSE_1 at node 1 and LSE_2 at node 2. Let Q_{L1} and Q_{L2} be the power demand quantities (withdrawals) at node 1 and 2, respectively. The demand function $D_j(Q_{Lj})$ and gross consumer surplus $GCS_j(Q_{Lj})$ for LSE_j $(j = 1, 2)$ are specified as follows:

$$D_1(Q_{L1}) = b_1^D - a_1^D Q_{L1} \tag{7}$$

[3] This modelling framework is a simplified version of Standard Market Design(SMD) implemented by ISO New England since March 2003. See ISO New England (2003a) for detailed descriptions.

$$D_2(Q_{L2}) = b_2^D - a_2^D Q_{L2} \tag{8}$$

$$GCS_1(Q_{L1}) = b_1^D Q_{L1} - \frac{1}{2} a_1^D Q_{L1}^2 \tag{9}$$

$$GCS_2(Q_{L2}) = b_2^D Q_{L2} - \frac{1}{2} a_2^D Q_{L2}^2 \tag{10}$$

where parameters (a_j^D, b_j^D) are all positive for $j = 1, 2$. After purchasing Q_{Lj} amount of power in the wholesale market, each LSE_j can then sell the Q_{Lj} amount of power to its local downstream consumers and receive resale revenue equal to R_j [4]

- There are no learning and bidding strategies for either generators or LSEs. Each generator bids his true marginal cost function and each LSE bids his true demand function. The information set consisting of each generator's TC, VC, MC, each LSE's GCS and demand functions, and the structure parameter vector $(a_j^D, b_j^D; a_i^S, b_i^S, f_i; T) > 0$ for $i, j = 1, 2$ is known to public. Moreover, the structure parameter vector is fixed and given in the model. So there is no uncertainty and no private information in this model.

- After collecting the information through generators' supply offers and LSEs' demand bids, ISO solves a *Security-Constrained Economic Dispatch* (SCED)[5] problem by maximizing the total net benefit subject to a set of physical power network constraints in the day-ahead power market[6] to solve for the optimal dispatch quantities for all generators and LSEs and derive the associated *locational marginal prices*[7](LMPs) for each node. Consequently, each generator produces Q_{Gi} amount of power at the ISO's dispatch and is paid by ISO the LMP per unit of its produced power for $i = 1, 2$, while each LSE purchases Q_{Lj} amount of power at the ISO's dispatch and pays ISO the LMP per unit of purchased power for $j = 1, 2$. Recall that since there is a transmission line connecting the two nodes, the total power produced at a local node does not have

[4]The downstream resale revenue for LSE_j could be specified as $R_j(Q_{Lj}) = (\beta_j - \alpha_j Q_{Lj})Q_{Lj}$ for $j = 1, 2$, where β_j and α_j are the parameters of aggregate demand function in the resale market at node 1 ($j = 1$) and node 2 ($j = 2$).

[5]See the following section for a discussion of this SCED problem formulation.

[6]According to ISO New England Standard Market Design (SMD), the real U.S. wholesale power market is a complicated two-settlement system which consists of consists of several submarkets including Day-Ahead, Real-Time, Supply Re-offers, FTR, and bilateral markets to reduce uncertainty for market participants and ensure orderly, fair, and efficient market outcomes. For simplicity, assume the dispatched quantities of powers committed in the Day-Ahead market are exactly carried out in the Real-Time market (Real-Time market is just a duplicate of Day-Ahead market and thus negligible), all generators submit their true marginal cost (so no Supply Re-offer market is needed), and assume bilateral trades are prohibited. Thus in this paper only the Day-Ahead (in this section) and FTR (in the next section) (sub)markets are considered.

[7]Roughly stated, location marginal price at any given node is the minimum incremental cost of providing one additional unit of power at that node.

to match up with its local demand. For example, some low-cost generator may produce more than its local demand and transfer the "overproduced" power through the transmission line to fulfill the residual demand at a high-cost generator node. However the power flow through the transmission line has an upper limit equal to the line thermal capacity T. When the power flow reaches that upper limit T, we call the line is congested. One important consequence of congestion is that the LMPs will no long be the same across all nodes. Assuming no loss during power transmission, the separation of LMPs creates the *congestion rent*(CR) (difference between LMP_1 and LMP_2 multiplied by T), which is accrued to ISO.

- To further simplify the model, assume the minimum production capacities for G_1 and G_2 are both zero implying that it is feasible for generators to stop producing power while bearing the fixed cost. And assume the maximum production capacities for G_1 and G_2 are both infinitely large so that the generators can meet arbitrarily high demands in the power market. Therefore the *locational marginal prices* for node 1 and 2, LMP_1 and LMP_2, are the last unit marginal cost for Generator 1 and 2 or the marginal unit of willingness to pay for LSE 1 and LSE 2 when the thermal constraint T is binding; LMP_1 and LMP_2 become the same and are equal to the market clearing price of the aggregate demand and supply functions when the thermal constraint is not binding.

- The benchmark model can best summarized in Figure 2.

3.2 Model Setup

Based on the above assumptions and specifications, this benchmark model boils down to a *Security-Constrained Economic Dispatch* (SCED) problem [8]. As detailed in Stoft (2002), *dispatch* is the process of determining generator output level for the servicing of LSEs. *Economic Dispatch* means that the dispatch process is efficient. *Security-Constrained Economic Dispatch* (SCED) means that constraints are imposed in the economic dispatch problem to ensure that the power on each node of the transmission line is within the balancing, non-negative and thermal limits.

The objective of this SCED problem is then to maximize the 'total net benefit' (TNB) subject to the balancing, non-negativity, and thermal limit constraints. The balancing constraint should be respected because it represents the physical aspect of the electricity network, which is essentially stated in the Kirchhoff's law: total power injections should be

[8] SCDE is essentially a constrained optimization problem.

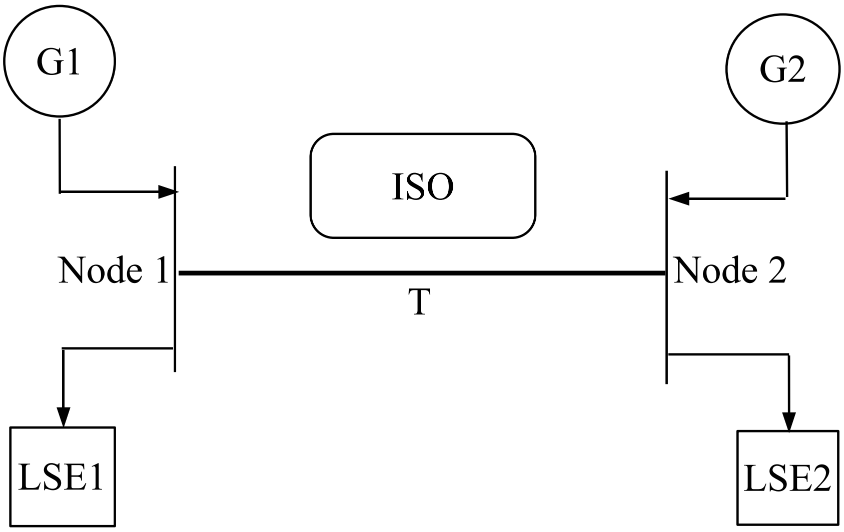

Figure 2: The two-node electricity benchmark model.

equal to total power withdrawals at any time in the electric network. In our benchmark model, this requires that the power supplies by G_1 and G_2 should be equal to power demands by LSE_1 and LSE_2. The non-negativity constraint holds naturally since we only allow the real power production and purchasing in this model [9], and exclude the speculative behaviors such as taking a short position in the day-ahead power market. Lastly the thermal limit constraints have to be respected due to the physical aspect of the transmission line, i.e., the power flow between two nodes simply cannot exceed the thermal capacity limit of the transmission line.

When the thermal constraint becomes binding, it might be necessary to supply a next unit of power by dispatching the relatively expensive local generation *out of merit order*, i.e., in place of the other generation with lower marginal cost. *Locational marginal prices* (LMPs) reflect the cost of this out-of-merit-order dispatch. A separate LMP is calculated for each pricing location (node). Technically, the LMP at any node k is defined to be the change in total system variable costs that would result if one more unit of power were to be serviced at node k. In our simple benchmark model, LMP_k then reduces to the marginal cost of the last unit of power for G_k at node k, for $k = 1, 2$. In the absence of binding thermal limit constraint, and assuming no transmission losses, each node has the same LMP. Otherwise, however, *price separation* can occur, meaning that different nodes can have different LMPs.

[9] This is also the case in the real world wholesale power markets such as New England (ISO New England (2004)).

TNB is defined as the sum of all LSE surpluses and all generator surpluses. In the benchmark model, TNB is just the sum of surpluses from LSE_1, LSE_2, G_1, G_2. Geometrically, TNB represents the summed area under the demand curve less the area under the supply curve (marginal cost curve) over two nodes. Formally the model is set up as follows:

Maximize

$$
\begin{aligned}
TNB &= \int_0^Q [D_1(Q_{L1}) - MC_1(Q_{G1})]dQ + \int_0^Q [D_2(Q_{L2}) - MC_2(Q_{G2})]dQ && (11)\\
&= [GCS_1(Q_{L1}) - VC_1(Q_{G1})] + [GCS_2(Q_{L2}) - VC_2(Q_{G2})] && (12)
\end{aligned}
$$

where

$$GCS_1(Q_{L1}) - VC_1(Q_{G1}) = (b_1^D Q_{L1} - \frac{1}{2}a_1^D Q_{L1}^2) - (b_1^S Q_{G1} + \frac{1}{2}a_1^S Q_{G1}^2) \quad (13)$$

$$GCS_2(Q_{L2}) - VC_2(Q_{G2}) = (b_2^D Q_{L2} - \frac{1}{2}a_2^D Q_{L2}^2) - (b_2^S Q_{G2} + \frac{1}{2}a_2^S Q_{G2}^2) \quad (14)$$

with respect to $Q_{G1}, Q_{G2}, Q_{L1}, Q_{L2}$

subject to:

$$Q_{G1} + Q_{G2} = Q_{L1} + Q_{L2} \text{ (balancing constraint)} \quad (15)$$

$$Q_{G1} \geq 0,\ Q_{G2} \geq 0,\ Q_{L1} \geq 0,\ Q_{L2} \geq 0 \text{ (non-negativity constraint)} \quad (16)$$

$$-T \leq Q_{G1} - Q_{L1} \leq T \text{ (thermal constraint for node 1)} \quad (17)$$

$$-T \leq Q_{G2} - Q_{L2} \leq T \text{ (thermal constraint for node 2)} \quad (18)$$

In this SCED problem, we want to solve for the vector

$$s^* = (Q_{G1}^*, Q_{G2}^*, Q_{L1}^*, Q_{L2}^*)$$

which maximizes (11) or (12) subject to (15) - (18). Based on this solution we can then derive LMP_1 and LMP_2[10]. Note that the SCED solution vector s^* is ISO's dispatch quantities in the day-ahead market, and LMP_1 and LMP_2 are the locational marginal price applied to node 1 and node 2, respectively.

[10] By definition, the locational marginal price (LMP) at node k is the minimum incremental cost of producing one additional unit of power at node k. Recall in this benchmark model, we assume the zero minimum production and infinitely large maximum production capacity, the minimum incremental cost of producing one more unit of power is just the marginal cost at that node. Furthermore, as we will show in the Appendix 1 and 2, LMP is indeed captured by the Lagrangian multiplier associated with the balancing constraint.

3.3 The SCED Solution

To present the solution to this SCED problem in a more orderly fashion, it is proposed in this paper to solve the SCED problem in two steps. In the first step, assume the thermal limit T is so large that the thermal constraints will never get binding (thus the two thermal constraints are ignored), which simplifies the problem at hand to be a standard maximization problem. Then use the solved optimal solution to check if the thermal limit constraints are actually binding or not. If not binding, then we are done; if binding, then proceed to step 2. In step 2, resolve the SCED problem by adding one of the thermal limit constraint as the equality constraint. The formal procedure of solving this model is presented as follows:

3.3.1 Step 1: Thermal constraint T is NOT binding

In this step, suppose the thermal limit T is so large that the thermal constraint will never get binding. According to the model setup section, this is a standard optimization problem with one equality constraint (the balancing constraint) and four inequality constraints (the non-negativity constraints for Q_{G1}, Q_{G2}, Q_{L1} and Q_{L2}). Use μ as the multiplier for equality constraint and λ's as the multipliers for inequality constraints, and formulate the Lagrangian equation:

$$L = (b_1^D Q_{L1} - \frac{1}{2}a_1^D Q_{L1}^2) - (b_1^S Q_{G1} + \frac{1}{2}a_1^S Q_{G1}^2) + (b_2^D Q_{L2} - \frac{1}{2}a_2^D Q_{L2}^2) - (b_2^S Q_{G2} + \frac{1}{2}a_2^S Q_{G2}^2)$$

$$+\mu(Q_{G1} + Q_{G2} - Q_{L1} - Q_{L2}) + \lambda_{G1}Q_{G1} + \lambda_{G2}Q_{G2} + \lambda_{L1}Q_{L1} + \lambda_{L2}Q_{L2} \tag{19}$$

For simplicity, only consider the case where all dispatched quantities are positive, i.e., all non-negativity constraints are not binding [11], we obtain the following non-thermal-constraint solution (denoted with a hat). (The detailed derivation is provided in Appendix 1):

$$\hat{Q_{G1}} = (\mathbb{G}_{\mathbf{1}} + \mathbb{B}_{\mathbf{1}})/\mathbb{A} \tag{20}$$

$$\hat{Q_{G2}} = (\mathbb{G}_{\mathbf{2}} + \mathbb{B}_{\mathbf{2}})/\mathbb{A} \tag{21}$$

$$\hat{Q_{L1}} = (\mathbb{L}_{\mathbf{1}} + \mathbb{C}_{\mathbf{1}})/\mathbb{A} \tag{22}$$

$$\hat{Q_{L2}} = (\mathbb{L}_{\mathbf{2}} + \mathbb{C}_{\mathbf{2}})/\mathbb{A} \tag{23}$$

[11] To be exhaustive, we find 9 other possible solution cases, i.e., (1) $Q_{G1} = 0$; (2) $Q_{G2} = 0$; (3) $Q_{L1} = 0$; (4) $Q_{L2} = 0$; (5) $Q_{G1} = Q_{L1} = 0$; (6) $Q_{G1} = Q_{L2} = 0$; (7) $Q_{G2} = Q_{L1} = 0$; (8) $Q_{G2} = Q_{L2} = 0$; (9) $Q_{G1} = Q_{G2} = Q_{L1} = Q_{L2} = 0$.

where

$$\begin{array}{llll} \mathbb{G}_{\mathbf{1}} = D_2B_1 + a_1^Da_2^SB_2, & \mathbb{B}_{\mathbf{1}} = a_1^DA_2C_1, & \mathbb{L}_{\mathbf{1}} = (D_2 + a_1^SA_2)B_1 - a_1^Sa_2^SB_2, & \mathbb{C}_{\mathbf{1}} = a_1^SA_2C_2; \\ \mathbb{G}_{\mathbf{2}} = D_1B_2 + a_1^Sa_2^DB_1, & \mathbb{B}_{\mathbf{2}} = a_2^DA_1C_2, & \mathbb{L}_{\mathbf{2}} = (D_1 + a_2^SA_1)B_2 - a_1^Sa_2^SB_1, & \mathbb{C}_{\mathbf{2}} = a_2^SA_1C_1; \\ \mathbb{A} \ = D_1A_2 + D_2A_1; & & & \end{array}$$

$$\begin{array}{llll} A_1 = a_1^D + a_1^S, & B_1 = b_1^D - b_1^S, & C_1 = b_2^S - b_1^S, & D_1 = a_1^Da_1^S; \\ A_2 = a_2^D + a_2^S, & B_2 = b_2^D - b_2^S, & C_2 = b_1^S - b_2^S, & D_2 = a_2^Da_2^S. \end{array}$$

Now we solved the non-thermal-constraint SCED problem and need to examine the solution $(\hat{Q_{G1}}, \hat{Q_{G2}}, \hat{Q_{L1}}, \hat{Q_{L2}})$ closely to determine whether the thermal limit constraints are actually binding or not. Before proceeding further, we formally define the term thermal constraint is not binding, binding from 1 to 2 and binding from 2 to 1 as follows:

Definition 1 *In this two-node electricity network model, after solving the non-thermal-constraint SCED problem and obtaining the solution vector* $(\hat{Q_{G1}}, \hat{Q_{G2}}, \hat{Q_{L1}}, \hat{Q_{L2}})$*, regarding the thermal limit* T*, we say,*

- T *is binding from 1 to 2 if* $\hat{Q_{G1}} - \hat{Q_{L1}} > T$ *or* $\hat{Q_{G2}} - \hat{Q_{L2}} < -T$*;*
- T *is binding from 2 to 1 if* $\hat{Q_{G2}} - \hat{Q_{L2}} > T$ *or* $\hat{Q_{G1}} - \hat{Q_{L1}} < -T$*.*
- T *is not binding if* $|\hat{Q_{G1}} - \hat{Q_{L1}}| \leq T$ *or* $|\hat{Q_{G2}} - \hat{Q_{L2}}| \leq T$*;*

Remarks: this definition elaborates the relationship between the optimal SCED solution and network physical condition. Recall in this step we assume the thermal constraint will not be binding and proceed to solve the SCED problem, and its solution is the actual dispatched quantity that each generator will produce and each LSE will purchase. If the SCED solution requires what Generator 1 produces ($\hat{Q_{G1}}$) be greater than what LSE 1 purchases ($\hat{Q_{L1}}$), then the power flow will transport $\hat{Q_{G1}} - \hat{Q_{L1}}$ amount of power from node 1 to node 2 through the transmission line to meet the residual demand, which is equal to $\hat{Q_{L2}} - \hat{Q_{G2}}$ [12], at node 2. However the power flow is not allowed to exceed the upper limit of thermal capacity (T) of the transmission line. So if that does happen, that is, $\hat{Q_{G1}} - \hat{Q_{L1}} > T$ or equivalently, $\hat{Q_{L2}} - \hat{Q_{G2}} > T$, we call the thermal constraint is binding with power flowing from node 1 to node 2, or use the definition, *T is binding from 1 to 2*. In this case, the non-thermal-constraint SCED solution is not appropriate any more, and we will need to continue on to Step 2.

[12] Note that the balancing constraint is observed here, i.e., extra production meets residual demand implying $\hat{Q_{G1}} - \hat{Q_{L1}} = \hat{Q_{L2}} - \hat{Q_{G2}}$, which is equivalent to the balancing constraint $\hat{Q_{G1}} + \hat{Q_{G2}} = \hat{Q_{L1}} + \hat{Q_{L2}}$

If, on the other hand, the SCED solution requires what Generator 2 produces ($\hat{Q_{G2}}$) be greater than what LSE 2 purchases ($\hat{Q_{L2}}$), then the power flow will transport $\hat{Q_{G2}} - \hat{Q_{L2}}$ amount of power from node 2 to node 1 through the transmission line to meet the residual demand, which is equal to $\hat{Q_{L1}} - \hat{Q_{G1}}$ at node 1. By the similar argument, the thermal constraint is binding with power flowing from node 2 to node 1, or use the definition, *T is binding from 2 to 1.* In this case, the non-thermal-constraint SCED solution is not appropriate any more, and we will need to continue on to Step 2.

If the power flow in the above two cases indeed does not exceed thermal limit T, i.e., $|\hat{Q_{G1}} - \hat{Q_{L1}}| \leq T$ or $|\hat{Q_{G2}} - \hat{Q_{L2}}| \leq T$, we call *T is not binding*[13]. In this case, the non-thermal-constraint SCED solution is the right solution we seek, i.e., the solution vector is

$$s^* = (Q^*_{G1}, Q^*_{G2}, Q^*_{L1}, Q^*_{L2})$$

where

$$Q^*_{G1} = \hat{Q_{G1}}$$

$$Q^*_{G2} = \hat{Q_{G2}}$$

$$Q^*_{L1} = \hat{Q_{L1}}$$

$$Q^*_{L2} = \hat{Q_{L2}}$$

$$LMP_1 = LMP_2 = (D_2E_1 + D_1E_2)/\mathbb{A}$$

where $\mathbb{A}, D_1, D_2$ are as previously specified and $E_1 = a_1^D b_1^S + a_1^S b_1^D, E_2 = a_2^D b_2^S + a_2^S b_2^D$

By the nature of this problem, since the thermal constraint is not binding, each generator and LSE are bidding their true marginal cost and demand functions, ISO acts as a "social planner" trying to maximize the total net benefit taking into account of all generator's production cost and all LSE's willingness to pay, there is no strategic behaviors and any other system distortions. From the standard microeconomics point of view, this is both the competitive equilibrium and Pareto optimal outcome and LMPs are the same across two nodes as a result of aggregate market (node) clearing process [14].

[13]In this two-node benchmark model, there is small likelihood that the SCED solution requires what Generator 1 produces happen to be the same as what LSE 1 purchases. Then by the balancing constraint, this implies that what Generator 2 produces has to be the same as what LSE 2 purchases. So there is zero power flow between node 1 and node 2. This case certainly falls into the category of *"T is not binding"*.

[14]It is worth mentioning that when thermal constraint is not binding, the SCED solution can also be obtained through the market clearing point of the aggregate supply (marginal cost) and aggregate demand curves, i.e., finding the aggregate market clearing price (the common LMP) and referring it back to the individual demand and supply curves to obtain the SCED solution.

3.3.2 Step 2: Thermal constraint T is binding

Based on Step 1, if we know T is binding from 1 to 2, i.e., $\hat{Q_{G1}} - \hat{Q_{L1}} > T$, we can set $Q_{G1} - Q_{L1} = T$, the SCED problem does not change from Step 1 other than adding one more constraint $Q_{G1} - Q_{L1} = T$. Denoting μ's as the multipliers for equality constraints and λ's as the multipliers for inequality constraints, and form the Lagrangian equation as follows:

$$L = (b_1^D Q_{L1} - \frac{1}{2}a_1^D Q_{L1}^2) - (b_1^S Q_{G1} + \frac{1}{2}a_1^S Q_{G1}^2) + (b_2^D Q_{L2} - \frac{1}{2}a_2^D Q_{L2}^2) - (b_2^S Q_{G2} + \frac{1}{2}a_2^S Q_{G2}^2)$$

$$+\mu_B(Q_{G1}+Q_{G2}-Q_{L1}-Q_{L2})+\mu_T(T-Q_{G1}+Q_{L1})+\lambda_{G1}Q_{G1}+\lambda_{G2}Q_{G2}+\lambda_{L1}Q_{L1}+\lambda_{L2}Q_{L2} \quad (24)$$

For simplicity, only consider the case where all dispatched quantities are positive, i.e., all non-negativity constraints are not binding [15], we obtain the following thermal-constraint-binding solution $(Q_{G1}^*, Q_{G2}^*, Q_{L1}^*, Q_{L2}^*)$ (The detailed derivation is provided in Appendix 2):

$$Q_{G1}^* = (B_1 + a_1^D T)/A_1 \quad (25)$$

$$Q_{G2}^* = (B_2 - a_2^D T)/A_2 \quad (26)$$

$$Q_{L1}^* = (B_1 - a_1^S T)/A_1 \quad (27)$$

$$Q_{L2}^* = (B_2 + a_2^S T)/A_2 \quad (28)$$

$$LMP_1 = (E_1 + D_1 T)/A_1 \quad (29)$$

$$LMP_2 = (E_2 - D_2 T)/A_2 \quad (30)$$

where

$$\begin{array}{llll} A_1 = a_1^D + a_1^S, & B_1 = b_1^D - b_1^S, & D_1 = a_1^D a_1^S, & E_1 = a_1^D b_1^S + a_1^S b_1^D; \\ A_2 = a_2^D + a_2^S, & B_2 = b_2^D - b_2^S, & D_2 = a_2^D a_2^S, & E_2 = a_2^D b_2^S + a_2^S b_2^D. \end{array}$$

Similarly, if, from Step 1, we know T is binding from 2 to 1, i.e., $\hat{Q_{G2}} - \hat{Q_{L2}} > T$, we can set $Q_{G2} - Q_{L2} = T$, and obtain the following thermal-constraint-binding solution $(Q_{G1}^*, Q_{G2}^*, Q_{L1}^*, Q_{L2}^*)$:

$$Q_{G1}^* = (B_1 - a_1^D T)/A_1 \quad (31)$$

$$Q_{G2}^* = (B_2 + a_2^D T)/A_2 \quad (32)$$

[15] To see the complete solutions, refer to Appendix 2.

$$Q^*_{L1} = (B_1 + a_1^S T)/A_1 \tag{33}$$

$$Q^*_{L2} = (B_2 - a_2^S T)/A_2 \tag{34}$$

$$LMP_1 = (E_1 - D_1 T)/A_1 \tag{35}$$

$$LMP_2 = (E_2 + D_2 T)/A_2 \tag{36}$$

3.4 Solution Discussion

Based on the two-step SCED solution and the associated locational marginal prices LMP_1 and LMP_2, we can obtain the following propositions:

Proposition 1 *In the two-node electricity network, when thermal constraint T is binding, power flows from node 1 to node 2 (or node 2 to node 1) if and only if $LMP_2 > LMP_1$ (or $LMP_1 > LMP_2$)(assuming the dispatched quantities are all positive in the SCED solution). Furthermore,*

$$(*1)\ T \textit{ is binding from 1 to 2} \ \Leftrightarrow\ LMP_2 > LMP_1 \ \Leftrightarrow\ \Omega > T \tag{37}$$

$$(*2)\ T \textit{ is binding from 2 to 1} \ \Leftrightarrow\ LMP_2 < LMP_1 \ \Leftrightarrow\ \Omega < -T \tag{38}$$

$$(*3)\ T \textit{ is NOT binding} \ \Leftrightarrow\ LMP_2 = LMP_1 \ \Leftrightarrow\ -T \leq \Omega \leq T \tag{39}$$

where

$$\begin{aligned}
&\Omega = (A_1E_2 - A_2E_1)/(D_1A_2 + D_2A_1);\\
&A_1 = a_1^D + a_1^S, \quad D_1 = a_1^D a_1^S, \quad E_1 = a_1^D b_1^S + a_1^S b_1^D;\\
&A_2 = a_2^D + a_2^S, \quad D_2 = a_2^D a_2^S, \quad E_2 = a_2^D b_2^S + a_2^S b_2^D.
\end{aligned}$$

Proposition 1 has shown the relationship between the power flow direction and magnitude of LMPs under the condition that the thermal constraint is binding in the two-node electricity network [16]. Recall that if the thermal constraint is not binding, even if there is power flow, the LMPs will be the same across two nodes (see the Step 1 SCED solution). So this proposition basically asserts that whenever the thermal constraint T is binding, the power (which is equal to T) always flows from low LMP node to high LMP node. This result can be derived mathematically from the SCED solution and thermal constraint binding conditions in this benchmark model. The detailed proof of Proposition 1 is provided in Appendix 4.

[16] However, as the counter example in Kirschen and Strbac (2004) shows, the result in this proposition does not generalized to the case where the number of nodes is greater than or equal to three due to the externality brought by the "loop flow" effect.

The economic intuition behind this proposition is that the generator at the high LMP node has a high marginal cost and the generator at the low LMP node has a low marginal cost [17]. So when ISO, acting as a TNB maximizer, dispatches the high cost generator to produce less than its local demand and the low cost generator to produce more than its local demand and transfer the excess supply (which is equal to T) over the transmission line to meet the excess demand (which is equal to T) in the high LMP node, the power is indeed flowing from low LMP node to high LMP node. As we will see in the later section, this proposition serves as the crucial foundation to derive FTR values,

Proposition 2 *In the two-node electricity network, the SCED solution guarantees each generator's profit has a function form as:*

$$\pi_{Gk} = \frac{1}{2} a_k^S Q_{Gk}^2 - f_k \,, \quad \forall \ k = 1, 2 \tag{40}$$

and each LSE's profit has a function form as:

$$\pi_{Lk} = R_k(Q_{Lk}) - LMP_k Q_{Gk} \quad \forall \ k = 1, 2 \tag{41}$$

Proof: As the SCED solution suggests, in the benchmark model, regardless whether thermal constraint is binding or not, each generator submitting its true marginal cost function produces the dispatched quantity Q_{Gk} and receives revenue equal to $LMP_k Q_{Gk}$ while incurring a total cost equal to $f_k + b_k^S Q_{Gk} + \frac{1}{2} a_k^S Q_{Gk}^2$. Also recall that LMP_k is equal to the last unit marginal cost of generator k for $k = 1, 2$. The profit function of generator k is:

$$\begin{aligned} \pi_{Gk} &= LMP_k Q_{Gk} - TC_k(Q_{Gk}) \\ &= (b_k^S + a_k^S Q_{Gk}) Q_{Gk} - (f_k + b_k^S Q_{Gk} + \tfrac{1}{2} a_k^S Q_{Gk}^2) \\ &= \tfrac{1}{2} a_k^S Q_{Gk}^2 - f_k \,, \quad \forall \ k = 1, 2 \end{aligned}$$

Similarly, each LSE submitting its true demand function gets the dispatched quantity Q_{Lk} and receives revenue equal to $R_k(Q_{Lk})$ from downstream consumers while paying a total amount of $LMP_k Q_{Lk}$ for purchasing the power energy. The profit function of LSE k is:

$$\pi_{Lk} = R_k(Q_{Lk}) - LMP_k Q_{Gk} \quad \forall \ k = 1, 2$$

This proposition shows that in the benchmark model, if generator G_k gets dispatched it will produce Q_{Gk} to cover its fixed cost f_k. Note that if a generator does not get any

[17] Recall LMP is defined as the last unit marginal cost of the generator at the local node

dispatch, then G_k must bear the negative profit equal to its fixed cost $-f_k$. Similarly, if LSE_k gets demand dispatch it will purchase Q_{Lk} to meet its downstream consumer demand and acquires profit equal to its resale revenue less its payment.

Proposition 3 *In this two-node electricity model, the social welfare can be measured by total net benefit (TNB), and TNB increases as the thermal limit T increases, provided that the thermal constraint is still binding. The comparative statics are shown as follows:*

$$\frac{\partial TNB}{\partial T} = \begin{cases} \frac{E_2}{A_2} - \frac{E_1}{A_1} - \left(\frac{D_1}{A_1} + \frac{D_2}{A_2}\right) T > 0 & \textit{iff } T \textit{ is binding from 1 to 2;} \\ \frac{E_1}{A_1} - \frac{E_2}{A_2} - \left(\frac{D_1}{A_1} + \frac{D_2}{A_2}\right) T > 0 & \textit{iff } T \textit{ is binding from 2 to 1.} \end{cases} \tag{42}$$

where

$$A_1 = a_1^D + a_1^S, \quad D_1 = a_1^D a_1^S, \quad E_1 = a_1^D b_1^S + a_1^S b_1^D;$$
$$A_2 = a_2^D + a_2^S, \quad D_2 = a_2^D a_2^S, \quad E_2 = a_2^D b_2^S + a_2^S b_2^D.$$

The proof is provided in Appendix 4. This proposition has a rich economic meaning and important policy implications. It states that if the thermal limit of transmission line T can be increased it will increase TNB [18] and thus lead to a more efficient production and a higher social welfare, provided that T is still binding. (Once the thermal constraint T becomes non-binding, according to the SCED solution, we've already obtained the first-best outcome in the sense that it's both competitive and Pareto optimal solution. Further investment in the transmission line will thus be a waste of resources, provided that there is no uncertainty.) However, expanding the capacity of transmission line (so as to increase the thermal limit T) involves issues such as 'free ride' due to its public good feature. So how to create incentives for market participants to make transmission investment remains an important and yet challenging concern to ISO.

To finish the benchmark model and proceed to the next section, we define a useful definition of congestion rent.

Definition 2 *In the two-node electricity network, when thermal constraint is binding, i.e, the transmission line is congested, ISO acquires the congestion rent (CR) as its revenue,*

[18] Recall that total net benefit (TNB) consists of two components, consumer surplus (CS) and producer surplus (PS). This proposition only shows that TNB increases when thermal limit T increases. It does not indicate the individual effect of CS and PS. As a matter of fact, in one of their examples, Kirschen and Strbac (2004) shows that when the thermal limit T increases, in some circumstances, CS will decrease and then increase while PS is monotonically increasing. So the policy implication is that to promote the idea of transmission investment may improve the payoffs of generators at the cost of worsening the payoff of LSEs (for some range of thermal capacity T) although the total net effect is Pareto improvement.

which is equal to the difference in LMPs multiplied by T, that is,

$$CR = |LMP_2 - LMP_1|T \tag{43}$$

Remarks: (a) When the thermal constraint T is binding from 1 to 2, i.e., T amount of power flowing from node 1 to node 2, the nature of the SCED problem must lead to ISO to dispatch Q_{G1} for G1, Q_{L1} for LSE1 and Q_{G2} for G2, Q_{L2} for LSE2. At node 1, G1 receives revenue $LMP_1 * Q_{G1}$ from ISO and LSE1 makes payment $LMP_1 * Q_{L1}$ to ISO. Since T is binding from 1 to 2, $Q_{G1} - Q_{L1} = T$. Therefore ISO has a revenue deficit equal to $-LMP_1 * T$. Conversely, in node 2, G2 receives revenue $LMP_2 * Q_{G2}$ from ISO and LSE2 makes payment $LMP_2 * Q_{L2}$ to ISO. Since T is binding from 1 to 2, $Q_{L2} - Q_{G2} = T$. Therefore ISO has a revenue surplus equal to $LMP_2 * T$. The ISO's clearing process can be expressed as:

$$\begin{aligned}
\text{ISO's revenue} &= -LMP_1 Q_{G1} + LMP_1 Q_{L1} - LMP_2 Q_{G2} + LMP_2 Q_{L2} \\
&= -LMP_1 T + LMP_2 T \\
&= (LMP_2 - LMP_1)T
\end{aligned}$$

So the ISO's revenue (congestion rent) is equal to $(LMP_2 - LMP_1)T$. This congestion rent is positive since $LMP_2 > LMP_1$ when T is binding from 1 to 2 by Proposition 1.

(b) On the other hand, when the thermal constraint T is binding from 2 to 1, i.e., T amounts of power flowing from node 2 to node 1, by the similar argument, the congestion rent that is accrued to ISO is equal to $(LMP_1 - LMP_2)T$. This congestion rent is positive since $LMP_1 > LMP_2$ when $T$is binding from 2 to 1 by Proposition 1.

Hence by (a) and (b) we conclude that when thermal constraint is binding the congestion rent accrued to ISO is equal to $|LMP_2 - LMP_1|T$. This is a natural consequence of having a binding thermal constraint. In other words, the fact that thermal constraint is binding implies that the more expensive generation has been dispatched locally which could otherwise be serviced by the less expensive generation had the thermal constraint were not binding.

Note that the congestion rent can be related to the tariff issue in the international trade literature with the difference that tariff is imposed by government to purposely protect domestic producers while congestion rent is the natural outcome of having a congested transmission line. Just like in international trade, decreasing the tariff would increase total social welfare, decreasing the congestion rent, thus increasing thermal limit T, would also enhance total net benefit (TNB) in this two-node electricity network (see Proposition 3).

4 The FTR Model with Stochastic Shocks

Since the benchmark model depends exclusively on the structural parameters ($a_j^D, b_j^D; a_i^S, b_i^S, f_i$; $T\ \forall\, i, j = 1, 2$) that are fixed and known to public, there is no uncertainty and no private information, which implies that the SCED solution derived from the benchmark model is already the competitive equilibrium (first-best) outcome. Therefore there is no incentive for agents to purchase FTRs, and introducing FTRs can at best do no good to the model economy. Indeed as Joskow and Tirole (2000) indicate in their model, the existence of FTRs in the absence of uncertainty will only decrease social welfare compared with the case there is no FTRs available. However, the benchmark model is very important because the SCED solution and corresponding propositions serve as the building blocks to solve for the FTR model in this section.

Since the absence of uncertainty dooms the fate of FTRs, we are now interested to see whether introducing uncertainty into this model would create an incentive for agents to purchase FTRs, and if yes, to what extent could FTRs possibly help enhance the social welfare.

Based on the benchmark case, we will introduce a simple source of uncertainty into the model: the parameter values that characterize the cost attributes of generators and demand attributes of LSEs are now under stochastic shocks so that the direction of power flow and the magnitude of LMPs are no longer known in advance. This should create an incentive for both generators and LSEs to hedge against their uncertain profit streams through purchasing FTRs. We will develop a formal model to investigate this hypothesis and analyze the associated welfare effects.

Recall that a *financial transmission right* (FTR) is a financial hedging instrument that entitles the holder to receive compensation for transmission congestion costs that arise when a transmission line is congested. The *Wholesale Power Market Platform* (WPMP) proposed by FERC (2003) recommends that transmission congestion be managed by the ISO through the issuance of *point-to-point* (PTP) FTRs obligation in the day-ahead power market. Holders of PTP FTR obligations would be charged or credited based on the congestion components in day-ahead market LMPs.

Two issues here need to be clarified before we proceed further on. First, recall that there are four types of FTRs (PTP obligation, PTP option, FG [19] obligation, and FG option) currently available in the U.S. wholesale power markets. For simplicity, this model investigates only the first one, namely, the PTP FTR obligation. Hereafter if not stated

[19]FG stands for *flowgate*, which is mainly implemented by ERCOT in Texas and partly implemented by CAISO in California.

explicitly, FTR means PTP FTR obligation. The second issue is concerned with the time horizon of the model. Recall that in this model, generators and LSEs can purchase FTRs to hedge against their future profit in the day-ahead power market. So the FTR market works like a forward market (denoted with time t = 0), and day-ahead power market works like a spot market [20] (denoted with time t = 1). Hence terms such as FTR forward market or day-ahead spot market should not cause any confusions.

4.1 Model Specifications and Assumptions

- This model consists of two markets, one is FTR forward market and the other is day-ahead power market. The basic day-ahead power market structure remains the same as in the benchmark model, i.e., the two-node electricity network with one generator and one LSE at each node and an ISO in the middle to manage the transmission network and collect the congestion rent if the line is congested. All the cost and demand function forms also remain the same as those in the benchmark model. Also for simplicity, assume the dispatched quantities are all positive from the SCED problem in the benchmark model.

- To make the case of FTR interesting, assume the thermal limit constraint T is so small that it is always binding. The justification is that if the thermal constraint is not binding, there will be no price separation, i.e., $LMP_1 = LMP_2$, which directly implies that the value of FTR based on the difference of LMPs becomes zero for sure regardless whether is uncertainty or not. Therefore to preclude this trivial case, T is assumed to be binding all the time.

- Introduce a stochastic shock to the two-node electricity network with a binding thermal limit constraint T: in the FTR forward market (t=0) all agents know they will be in one of the two states, *state 1* or *state 2*, in the day-ahead power spot market (t=1) such that if in *state 1*, T is binding from node 1 to node 2 with probability *prob*; if in *state 2*, T is binding from node 2 to node 1 with probability *1 - prob*. Then according

[20] The actual flow of activities in the U.S. wholesale power markets shows that the FTR market is usually operated once a month and the day-ahead market is operated once a day. In fact, day-ahead market is operated one day ahead of real-time market. But since in this paper we have assumed that there is no changes of power supplies and demands between day-ahead market and real-time market, day-ahead market and real-time market collapse to be one market. See ISO New England (2004) for details.

to Proposition 1, we have:

$$\begin{cases} \textit{state 1}: T \text{ is binding from 1 to 2} \Leftrightarrow LMP_2 > LMP_1 \Leftrightarrow \Omega > T & \text{with } \textit{prob}; \\ \textit{state 2}: T \text{ is binding from 2 to 1} \Leftrightarrow LMP_2 < LMP_1 \Leftrightarrow \Omega < -T & \text{with } \textit{1 - prob}. \end{cases}$$

To differentiate the notations in two states, denote the realized values of parameters in state 1 with a $\prime$ and state 2 with a $\prime\prime$, that is, all the structure parameters except thermal limit T are random variables denoted by a ˜such that

$$\begin{aligned} \tilde{a}_k^Z &= \begin{cases} a_k^{Z\prime} & \text{with } \textit{prob}; \\ a_k^{Z\prime\prime} & \text{with } \textit{1 - prob}, \quad \forall\, k = 1, 2; Z = D, S. \end{cases} \\ \tilde{b}_k^Z &= \begin{cases} b_k^{Z\prime} & \text{with } \textit{prob}; \\ b_k^{Z\prime\prime} & \text{with } \textit{1 - prob}, \quad \forall\, k = 1, 2; Z = D, S. \end{cases} \\ \tilde{f}_k &= \begin{cases} f_k' & \text{with } \textit{prob}; \\ f_k'' & \text{with } \textit{1 - prob}, \quad \forall\, k = 1, 2. \end{cases} \end{aligned}$$

$$\begin{cases} \textit{state 1}: T \text{ is binding from 1 to 2} \Leftrightarrow LMP_2' > LMP_1' \Leftrightarrow \Omega' > T & \text{with } \textit{prob}; \\ \textit{state 2}: T \text{ is binding from 2 to 1} \Leftrightarrow LMP_2'' < LMP_1'' \Leftrightarrow \Omega'' < -T & \text{with } \textit{1 - prob}. \end{cases}$$

In reality, the shocks may come from various sources. For example, the changing weather may suddenly increase/decrease LES's demand attributes. Or the changing price of raw material for producing electricity such as coal or oil may suddenly increase/decrease generator's cost attributes. Since these kinds of changes are really out of control of any market participants, it may be reasonable to introduce these random shocks into the model to create a simple form of uncertainty.

- Introduce two types of FTRs in this model, FTR_{12} and FTR_{21}. (a) Define FTR_{12} as the PTP FTR obligation that obligates the owner to get paid if thermal constraint T is binding from 1 to 2 or get charged if thermal constraint T is binding from 2 to 1. The total amount of payments or charges are equal to the number of FTR contracts times $LMP_2 - LMP_1$. (b) Similarly, define FTR_{21} as the PTP FTR obligation that obligates the owner to get paid if thermal constraint T is binding from 2 to 1 or get charged if thermal constraint T is binding from 1 to 2. The total amount of payments or charges are equal to the number of FTR contracts times $LMP_1 - LMP_2$.

- In this model, it is ISO who has the authority to issue these two types of FTRs at pre-announced prices. Generators and LSEs can choose to buy these FTR contracts from ISO by paying the corresponding FTR prices and benefit from its payoffs. On the other hand, ISO receives the FTR sales revenue while paying for its associated payoffs to generators or LSEs. Recall that ISO still receive some amount of congestion rent (CR) (what differs from the benchmark model is that now ISO does not know exactly how much CR it will obtain in at time t = 0, but it can use the expected CR as an approximation). So the ISO's revenue adequacy condition is respected in expectation. Lastly, ISO also gets to set the maximum amounts of FTRs for sale.

- Relying on the literature of corporate risk management, which argues that firms could benefit from hedging market risks (Smith and Stulz (1985), Stulz (1990), Bessembinder (1991), Froot et al. (1993)), it is argued in this study that firms (generators and LSEs) in the electric power market are risk averse and are likely to benefit from reducing the risk of their profits. Therefore, we assume generators and LSEs are risk averse with a *constant relative risk aversion* (CRRA) utility function. Furthermore, to make the calculation simpler, assume all generators and LSEs possess a logarithmic utility function.

- Finally in this model, assume generators and LSEs can only buy FTRs (can take long positions) but they cannot sell them (cannot take short positions), i.e., the FTR secondary market is not available in this model. Furthermore, assume G_1 and LSE_2 can only buy FTR_{12} and G_2 and LSE_1 can only buy FTR_{21}. The reason is that since the agents are all assumed to be risk averse, they will not be willing to purchase a financial instrument that will increase the risk of their profits even higher. For example, if G_1 can buy FTR_{21}, it will only make its profit stream more volatile, that is, when G_1 buys FTR_{21}, if $LMP_1 > LMP_2$, FTR_{12} can bring $LMP_1 - LMP_2$ amount of per contract profit to G_1, but G_1 is already enjoying the high LMP_1; similarly, if $LMP_1 < LMP_2$, G_1 will incur $LMP_1 - LMP_2$ amount of per contract loss for buying FTR_{21} , but G_1 is already suffering the low LMP_1. So purchasing the FTR_{12} will only make the G_1's profit even riskier. Similar arguments apply to G_2, L_1 and L_2 too.

4.2 Model Setup

Generator's and LSE's total profits come from two parts: profit from power supply (or demand) and profit from purchasing FTRs.

Denote G_k's profit from power production as the random variable $\tilde{\pi}_{Gk}$. Then by Proposition 2, we have:

$$\tilde{\pi}_{Gk} = \begin{cases} \pi'_{Gk} = LMP'_k Q'_{Gk} - TC_k(Q'_{Gk}) = \frac{1}{2} a_k^{S\prime} Q'^2_{Gk} - f'_k & \text{with } prob; \\ \pi''_{Gk} = LMP''_k Q''_{Gk} - TC_k(Q''_{Gk}) = \frac{1}{2} a_k^{S\prime\prime} Q''^2_{Gk} - f''_k & \text{with } 1 \text{ - } prob, \quad \forall\, k = 1, 2. \end{cases} \tag{44}$$

Denote LSE_k's profit from purchasing wholesale power from generator and reselling it to downstream consumers as the random variable $\tilde{\pi}_{Lk}$. Then by Proposition 2, we have:

$$\tilde{\pi}_{Lk} = \begin{cases} \pi'_{Lk} = R_k(Q'_{Lk}) - LMP'_k Q'_{Lk} & \text{with } prob; \\ \pi''_{Lk} = R_k(Q''_{Lk}) - LMP''_k Q''_{Lk} & \text{with } 1 \text{ - } prob, \quad \forall\, k = 1, 2. \end{cases} \tag{45}$$

where Q'_{Gk} and Q'_{Lk} are the Step 2 thermal-constraint-binding SCED solution (binding from 1 to 2) in the benchmark model, and Q''_{Gk} and Q''_{Lk} are the Step 2 thermal-constraint-binding SCED solution (binding from 2 to 1) in the benchmark model. Furthermore, to make the case interesting, it is reasonable to assume $\frac{1}{2} a_k^{S\prime} Q'^2_{Gk} > f'_k$ and $\frac{1}{2} a_k^{S\prime\prime} Q''^2_{Gk} > f''_k$ so that $\tilde{\pi}_{Gk} > 0$. This means in either state, the generator has a positive production profit thus does not go to bankrupt. For the similar reason, assume $R_k(Q'_{Lk}) > LMP'_k Q'_{Lk}$ and $R_k(Q''_{Lk}) > LMP''_k Q''_{Lk}$ so that $\tilde{\pi}_{Lk} > 0$.

Denote FTR_{12}'s per contract payoff function as the random variable $\tilde{H}_{12}$. By the definition of an FTR and LMP solutions from Step 2 benchmark model, we have:

$$\tilde{H}_{12} = \begin{cases} H'_{12} = LMP'_2 - LMP'_1 = \frac{(A'_1 E'_2 - A'_2 E'_1) - (D'_2 A'_1 + D'_1 A'_2) T}{A'_1 A'_2} > 0 & \text{with } prob; \\ H''_{12} = LMP''_2 - LMP''_1 = \frac{(A''_1 E''_2 - A''_2 E''_1) + (D''_2 A''_1 + D''_1 A''_2) T}{A''_1 A''_2} < 0 & \text{with } 1 \text{ - } prob. \end{cases} \tag{46}$$

Denote FTR_{21}'s per contract payoff function as the random variable $\tilde{H}_{21}$. By the definition of an FTR, we have $\tilde{H}_{21} = -\tilde{H}_{12}$.

At this point, it may be of some interest to know the relationship between the FTR payoff spread and the thermal limit T. Define the FTR payoff spread (FTR^{SP}) as the net difference in the realized FTR payoffs in two states ($|H' - H''|$). The following proposition will show that the increasing thermal limit T will always decrease the FTR payoff spread FTR^{SP}.

Proposition 4 *Define the FTR payoff spread as the net difference in the realized FTR payoffs in two states, that is,* $FTR^{SP}_{12} = H'_{12} - H''_{12}$ *and* $FTR^{SP}_{21} = H''_{21} - H'_{21}$. *Then we have the following:*

$$\frac{\partial FTR^{SP}_{12}}{\partial T} < 0, \frac{\partial FTR^{SP}_{21}}{\partial T} < 0, \tag{47}$$

Proof: From the definition of FTR payoff function and FTR payoff spread, we have:

$$\begin{aligned}\frac{\partial FTR_{12}^{SP}}{\partial T} &= \frac{\partial H'_{12}}{\partial T} - \frac{\partial H''_{12}}{\partial T} \\ &= -\frac{D'_2A'_1 + D'_1A'_2}{A'_1A'_2} - \frac{D''_2A''_1 + D''_1A''_2}{A''_1A''_2} \\ &< 0 \quad \text{(since all parameters are positive)}\end{aligned}$$

$$\begin{aligned}\frac{\partial FTR_{21}^{SP}}{\partial T} &= \frac{\partial H''_{21}}{\partial T} - \frac{\partial H'_{21}}{\partial T} \\ &= -\frac{D''_2A''_1 + D''_1A''_2}{A''_1A''_2} - \frac{D'_2A'_1 + D'_1A'_2}{A'_1A'_2} \\ &< 0 \quad \text{(since all parameters are positive)}\end{aligned}$$

Q.E.D.

Now let's look at ISO's revenue components. Similar to generators and LSEs, ISO's total revenue comes from two parts too: one part from collecting the congestion rent and the other part from selling FTRs.

Denote the congestion rent that accrues to ISO as the random variable $\widetilde{CR}$. Then by Definition 2, we have:

$$\widetilde{CR} = \begin{cases} CR' = (LMP'_2 - LMP'_1)T & \text{with } prob; \\ CR'' = (LMP''_1 - LMP''_2)T & \text{with } 1 \text{ - } prob. \end{cases} \tag{48}$$

So the total profits for G_1, G_2, LSE_1, and LSE_2 and the total revenue for ISO can be expressed as random variables $\tilde{\Pi}_{G1}$, $\tilde{\Pi}_{G2}$, $\tilde{\Pi}_{L1}$, $\tilde{\Pi}_{L2}$, and $\tilde{\Pi}_{ISO}$, respectively, that is,

$$\tilde{\Pi}_{G1} = \tilde{\pi}_{G1} + (\tilde{H}_{12} - \eta_{12})FTR_{12(G1)} \tag{49}$$

$$\tilde{\Pi}_{G2} = \tilde{\pi}_{G2} + (\tilde{H}_{21} - \eta_{21})FTR_{21(G2)} \tag{50}$$

$$\tilde{\Pi}_{L1} = \tilde{\pi}_{L1} + (\tilde{H}_{21} - \eta_{21})FTR_{21(L1)} \tag{51}$$

$$\tilde{\Pi}_{L2} = \tilde{\pi}_{L2} + (\tilde{H}_{12} - \eta_{12})FTR_{12(L2)} \tag{52}$$

$$\tilde{\Pi}_{ISO} = \tilde{CR} + (\eta_{12} - \tilde{H}_{12})\overline{FTR}_{12} + (\eta_{21} - \tilde{H}_{21})\overline{FTR}_{21} \tag{53}$$

where η_{12} and η_{21} are the ISO pre-announced prices of FTR_{12} and FTR_{21} at beginning of FTR market, i.e, at time t = 0. To make the case interesting, assume $H'_{12} > \eta_{12}$ and $H''_{21} > \eta_{21}$, that is, ISO sets the FTR price below its positive payoff so that generators and

LSEs know that if they buy FTRs they are not losing money for sure. To see this, suppose $H'_{12} < \eta_{12}$and take generator G_1 for example. If G_1 is in *state 1*, FTR's total payoff is $(\tilde{H}_{12} - \eta_{12})FTR_{12(G1)} = (H'_{12} - \eta_{12})FTR_{12(G1)} < 0$; if G_1 is in *state 2*, FTR's total payoff is $(\tilde{H}_{12} - \eta_{12})FTR_{12(G1)} = (H''_{12} - \eta_{12})FTR_{12(G1)} = [(LMP''_2 - LMP''_1) - \eta_{12}]FTR_{12(G1)} < 0$. Since there is no private information, G_1 knows for sure that he will lose money if he purchases $FTR_{12(G1)}$. Similar argument applies to $H''_{21} > \eta_{21}$. $\overline{FTR}_{12}$ and $\overline{FTR}_{21}$ are the maximum amounts of FTR_{12} and FTR_{21} that are available to sell.

Recall in the benchmark model, the main problem is for ISO to maximize the TNB subject to a set of constraints in the day-ahead spot market while generators and LSEs have no control at all. In this model, however, the main problem is for generators and LSEs in the FTR forward market to choose their optimal numbers of FTR contracts to hedge against the profit risks in the day-ahead spot market in order to maximize their expected utility of profit[21]. The total number of FTRs must satisfy ISO's revenue adequacy constraint (RAC), which in turn will ensure ISO passes the simultaneous feasibility test (SFT) (see Hogan (2002)).

Finally, to illustrate the point that FTRs really serve as hedging instruments in the sense that FTRs can shrink the total profit spread of agents in two states and thus risk averse agents are willing to pay some amount of premium to buy FTRs, let's look at the case for G_1.

With probability *prob*, G_1's total profit becomes Π'_{G1} such that

$$\begin{aligned} \Pi'_{G1} &= \pi'_{G1} + (H'_{12} - \eta_{12})FTR_{12(G1)} \\ &= LMP'_1 Q'_{G1} - TC_{G1}(Q'_{G1}) + (LMP'_2 - LMP'_1 - \eta_{12})FTR_{12(G1)} \end{aligned}$$

We see that with probability *prob*, G_1 is in *state 1* where LMP'_1 is less than LMP'_2 which makes FTR_{12} bring positive profit to G_1, but G_1 was suffering from receiving the low LMP'_1 (relative to LMP'_2), which directly decreases its production profit π'_{G1}. Thus in this case, the FTR compensates G_1 for being in its unfavorable state by paying G_1 a positive amount of profit.

On the other hand, with probability *1 - prob*, G_1's total profit becomes Π''_{G1} such that

$$\begin{aligned} \Pi''_{G1} &= \pi''_{G1} + (H''_{12} - \eta_{12})FTR_{12(G1)} \\ &= LMP''_1 Q''_{G1} - TC_{G1}(Q''_{G1}) + (LMP''_2 - LMP''_1 - \eta_{12})FTR_{12(G1)} \end{aligned}$$

[21] After generators and LSEs make their FTR purchasing decision in the forward market, they wait until the states get revealed. At that time they will be in the day-ahead spot market and everything follows the results derived from benchmark model.

We see that with probability *1 - prob*, G_1 is in *state 2* where LMP_2'' is less than LMP_1'' which makes FTR_{12} bring negative profit to G_1, but G_1 was enjoying in receiving the high LMP_1'' (relative to LMP_2''), which directly increases its production profit π_{G1}''. Thus in this case, the FTR penalizes G_1 for being in its favorable state by taking away part of G_1's production profit.

So whether in *state 1* or *state 2*, FTR_{12}'s profit stream will always be in the opposite direction of G_1's production profit to fulfill its hedging purpose. Similar argument applies to $G2$, LSE_1 and LSE_2. Therefore, FTRs are indeed hedging instruments for generators and LSEs to reduce their systematic profit risks.

4.3 FTR Solutions

Assume all generators and LSEs possess logarithmic utilities and maximize their expected utility of total profit [22] by choosing the optimal FTR contracts, i.e., optimal hedge positions subject to the ISO's revenue adequacy constraint (RAC). Then G_1, G_2, LSE_1 and LSE_2's problem can be expressed as follows:

$$G_1: \quad \text{Max } E[U(\tilde{\Pi}_{G1})] = prob\,\log(\Pi'_{G1}) + (1 - prob)\log(\Pi''_{G1}) \quad \text{w.r.t. } FTR_{12(G1)} \tag{54}$$

$$G_2: \quad \text{Max } E[U(\tilde{\Pi}_{G2})] = prob\,\log(\Pi'_{G2}) + (1 - prob)\log(\Pi''_{G2}) \quad \text{w.r.t. } FTR_{21(G2)} \tag{55}$$

$$LSE_1: \quad \text{Max } E[U(\tilde{\Pi}_{L1})] = prob\,\log(\Pi'_{L1}) + (1 - prob)\log(\Pi''_{L1}) \quad \text{w.r.t. } FTR_{21(L1)} \tag{56}$$

$$LSE_2: \quad \text{Max } E[U(\tilde{\Pi}_{L2})] = prob\,\log(\Pi'_{L2}) + (1 - prob)\log(\Pi''_{L2}) \quad \text{w.r.t. } FTR_{12(L2)} \tag{57}$$

subject to:

$$FTR_{12(G1)} + FTR_{12(L2)} \leq \overline{FTR}_{12}$$

$$FTR_{21(G2)} + FTR_{21(L1)} \leq \overline{FTR}_{21}$$

$$E(\tilde{\Pi}_{ISO}) = E(\tilde{CR}) + (\eta_{12} - E(\tilde{H}_{12}))\overline{FTR}_{12} + (\eta_{21} - E(\tilde{H}_{21}))\overline{FTR}_{21} \geq 0 \quad \text{(RAC)}$$

Solving for the first order conditions (FOCs), we obtain the following FTR optimal hedge solutions (OHSs):

$$\text{(OHS1)} \quad FTR^*_{12(G1)} = \frac{prob(H'_{12} - \eta_{12})\pi''_{G1} + (1 - prob)(H''_{12} - \eta_{12})\pi'_{G1}}{(H'_{12} - \eta_{12})(\eta_{12} - H''_{12})} \tag{58}$$

[22] Since the underlying parameters are not normally distributed, the expected utility is not linear in expected profit and the variance of profit. Thus the usual mean-variance analysis does not work well here.

$$\text{(OHS2)} \quad FTR^{*}_{21(G2)} = \frac{prob(H'_{21} - \eta_{21})\pi''_{G2} + (1 - prob)(H''_{21} - \eta_{21})\pi'_{G2}}{(H''_{21} - \eta_{21})(\eta_{21} - H'_{21})} \tag{59}$$

$$\text{(OHS3)} \quad FTR^{*}_{21(L1)} = \frac{prob(H'_{21} - \eta_{21})\pi''_{L1} + (1 - prob)(H''_{21} - \eta_{21})\pi'_{L1}}{(H''_{21} - \eta_{21})(\eta_{21} - H'_{21})} \tag{60}$$

$$\text{(OHS4)} \quad FTR^{*}_{12(L2)} = \frac{prob(H'_{12} - \eta_{12})\pi''_{L2} + (1 - prob)(H''_{12} - \eta_{12})\pi'_{L2}}{(H'_{12} - \eta_{12})(\eta_{12} - H''_{12})} \tag{61}$$

First we derive the following important proposition:

Proposition 5 *In a two-node electricity network model facing the uncertain parameter shocks, all risk averse agents, i.e., generators and LSEs (assuming log utilities), will hold a positive amount of FTRs if and only if the shock probability satisfies the following regularity condition (RC):*

$$\max\{\overline{prob}_{G1}, \overline{prob}_{L2}\} < prob < \min\{\overline{prob}_{G2}, \overline{prob}_{L1}\} \qquad \textit{(RC)} \tag{62}$$

where

$$\overline{prob}_{G1} = \frac{(\eta_{12} - H''_{12})\pi'_{G1}}{(H'_{12} - \eta_{12})\pi''_{G1} + (\eta_{12} - H''_{12})\pi'_{G1}}$$

$$\overline{prob}_{G2} = \frac{(\eta_{21} - H''_{21})\pi'_{G2}}{(H'_{21} - \eta_{21})\pi''_{G2} + (\eta_{21} - H''_{21})\pi'_{G2}}$$

$$\overline{prob}_{L1} = \frac{(\eta_{21} - H''_{21})\pi'_{L1}}{(H'_{21} - \eta_{21})\pi''_{L1} + (\eta_{21} - H''_{21})\pi'_{L1}}$$

$$\overline{prob}_{L2} = \frac{(\eta_{12} - H''_{12})\pi'_{L2}}{(H'_{12} - \eta_{12})\pi''_{L2} + (\eta_{12} - H''_{12})\pi'_{L2}}$$

This proposition states that when there is a stochastic shock in this two-node electricity network, the risk-averse generators and LSEs will hold a positive amount of FTRs to hedge against the uncertain profit in the energy spot market, provided that the shock probability satisfies the regularity condition.

Proof:

Recall that $\tilde{\pi}_{Gk} > 0$ and $\tilde{\pi}_{Lk} > 0$ implies $\pi'_{Gk} > 0$, $\pi''_{Gk} > 0$, $\pi'_{Lk} > 0$ and $\pi''_{Lk} > 0$, for $k = 1, 2$.

Then from (OHS1)—(OHS4) we have the following:

$$
\begin{aligned}
& FTR^*_{12(G1)} = \frac{prob(H'_{12} - \eta_{12})\pi''_{G1} + (1 - prob)(H''_{12} - \eta_{12})\pi'_{G1}}{(H'_{12} - \eta_{12})(\eta_{12} - H''_{12})} > 0 \\
\iff \quad & prob(H'_{12} - \eta_{12})\pi''_{G1} + (1 - prob)(H''_{12} - \eta_{12})\pi'_{G1} > 0 \quad (\because H'_{12} - \eta_{12} > 0, H''_{12} - \eta_{12} < 0) \\
\iff \quad & prob > \frac{(\eta_{12} - H''_{12})\pi'_{G1}}{(H'_{12} - \eta_{12})\pi''_{G1} + (\eta_{12} - H''_{12})\pi'_{G1}} \in (0,1) \quad (\because \pi'_{G1} > 0, \pi''_{G1} > 0) \\
& \qquad = \overline{prob}_{G1}
\end{aligned}
$$

$$
\begin{aligned}
& FTR^*_{21(G2)} = \frac{prob(H'_{21} - \eta_{21})\pi''_{G2} + (1 - prob)(H''_{21} - \eta_{21})\pi'_{G2}}{(H''_{21} - \eta_{21})(\eta_{21} - H'_{21})} > 0 \\
\iff \quad & prob(H'_{21} - \eta_{21})\pi''_{G2} + (1 - prob)(H''_{21} - \eta_{21})\pi'_{G2} > 0 \quad (\because H'_{21} - \eta_{21} < 0, H''_{21} - \eta_{21} > 0) \\
\iff \quad & prob < \frac{(\eta_{21} - H''_{21})\pi'_{G2}}{(H'_{21} - \eta_{21})\pi''_{G2} + (\eta_{21} - H''_{21})\pi'_{G2}} \in (0,1) \quad (\because \pi'_{G2} > 0, \pi''_{G2} > 0) \\
& \qquad = \overline{prob}_{G2}
\end{aligned}
$$

$$
\begin{aligned}
& FTR^*_{21(L1)} = \frac{prob(H'_{21} - \eta_{21})\pi''_{L1} + (1 - prob)(H''_{21} - \eta_{21})\pi'_{L1}}{(H''_{21} - \eta_{21})(\eta_{21} - H'_{21})} > 0 \\
\iff \quad & prob(H'_{21} - \eta_{21})\pi''_{L1} + (1 - prob)(H''_{21} - \eta_{21})\pi'_{L1} > 0 \quad (\because H'_{21} - \eta_{21} < 0, H''_{21} - \eta_{21} > 0) \\
\iff \quad & prob < \frac{(\eta_{21} - H''_{21})\pi'_{L1}}{(H'_{21} - \eta_{21})\pi''_{L1} + (\eta_{21} - H''_{21})\pi'_{L1}} \in (0,1) \quad (\because \pi'_{L1} > 0, \pi''_{L1} > 0) \\
& \qquad = \overline{prob}_{L1}
\end{aligned}
$$

$$
\begin{aligned}
& FTR^*_{12(L2)} = \frac{prob(H'_{12} - \eta_{12})\pi''_{L2} + (1 - prob)(H''_{12} - \eta_{12})\pi'_{L2}}{(H'_{12} - \eta_{12})(\eta_{12} - H''_{12})} > 0 \\
\iff \quad & prob(H'_{12} - \eta_{12})\pi''_{L2} + (1 - prob)(H''_{12} - \eta_{12})\pi'_{L2} > 0 \quad (\because H'_{12} - \eta_{12} > 0, H''_{12} - \eta_{12} < 0) \\
\iff \quad & prob > \frac{(\eta_{12} - H''_{12})\pi'_{L2}}{(H'_{12} - \eta_{12})\pi''_{L2} + (\eta_{12} - H''_{12})\pi'_{L2}} \in (0,1) \quad (\because \pi'_{L2} > 0, \pi''_{L2} > 0) \\
& \qquad = \overline{prob}_{L2}
\end{aligned}
$$

Therefore, to ensure that $FTR^*_{12(G1)} > 0$, $FTR^*_{21(G2)} > 0$, $FTR^*_{21(L1)} > 0$, and $FTR^*_{12(L2)} > 0$, we need to have $prob > \overline{prob}_{G1}$, $prob > \overline{prob}_{L2}$, $prob < \overline{prob}_{G2}$, and $prob < \overline{prob}_{L1}$, which is equivalent to $\max\{\overline{prob}_{G1}, \overline{prob}_{L2}\} < prob < \min\{\overline{prob}_{G2}, \overline{prob}_{L1}\}$. Q.E.D.

Furthermore, when we investigate how the optimal FTR hedge positions change with the change of shock probability $prob$, we have the following proposition:

Proposition 6 *In a two-node electricity network model facing uncertain parameter shocks,* *the optimal FTR_{12} increases with increasing prob while the optimal FTR_{21} decreases with increasing prob, provided that prob satisfies the regularity condition. The comparative statics are shown as follows:*

$$\frac{\partial FTR_{12(G1)}}{\partial prob} > 0, \ \frac{\partial FTR_{12(L2)}}{\partial prob} > 0, \ \frac{\partial FTR_{21(G2)}}{\partial prob} < 0, \ \textit{and} \ \frac{\partial FTR_{21(L1)}}{\partial prob} < 0. \tag{63}$$

The economic intuition behind this proposition is straightforward. Recall that *prob* is the probability that the transmission line is congested from node 1 to node 2. Increasing *prob* thus implies that the transmission line is more likely to get congested from node 1 to node 2. Since congestion from node 1 to node 2 makes FTR_{12} bring positive profit to its owner but makes FTR_{21} bring negative profit to its owner, the agents who own FTR_{12} (G_1 and L_2) will tend to buy more of FTR_{12} while the agents who own FTR_{21} (G_2 and L_1) will tend to buy less of FTR_{21}. The formal proof is provided below.

Proof:

Recall that $H'_{12} - \eta_{12} > 0$, $H''_{12} - \eta_{12} < 0$ and $H'_{21} - \eta_{21} < 0$ $H''_{21} - \eta_{21} > 0$.

$$\begin{aligned}
\frac{\partial FTR_{12(G1)}}{\partial prob} &= [(H'_{12} - \eta_{12})\pi''_{G1} + (\eta_{12} - H''_{12})\pi'_{G1}]/[(H'_{12} - \eta_{12})(\eta_{12} - H''_{12})] > 0; \\
\frac{\partial FTR_{12(L2)}}{\partial prob} &= [(H'_{12} - \eta_{12})\pi''_{L2} + (\eta_{12} - H''_{12})\pi'_{L2}]/[(H'_{12} - \eta_{12})(\eta_{12} - H''_{12})] > 0; \\
\frac{\partial FTR_{21(G2)}}{\partial prob} &= [(H'_{21} - \eta_{21})\pi''_{G2} + (\eta_{21} - H''_{21})\pi'_{G2}]/[(H'_{21} - \eta_{21})(\eta_{21} - H''_{21})] < 0; \\
\frac{\partial FTR_{21(L1)}}{\partial prob} &= [(H'_{21} - \eta_{21})\pi''_{L1} + (\eta_{21} - H''_{21})\pi'_{L1}]/[(H'_{21} - \eta_{21})(\eta_{21} - H''_{21})] < 0; \quad \text{Q.E.D.}
\end{aligned}$$

Now we are in a position to establish the most important proposition in this paper, that is, to show that the existence of FTRs actually increases the social welfare in this two-node electricity network model under stochastic parameter shocks.

Proposition 7 *In a two-node electricity network model facing uncertain parameter shocks, the acquisition of optimal FTRs by the risk averse generators and LSEs increases and in general strictly increases the social welfare compared with the case where there is no FTRs. Social welfare function W can be measured by generators and LSEs' total expected utilities. Denote the welfare under optimal FTRs as W_F and the welfare without FTRs as W_0. Then,*

$$W_F \geq W_0 \tag{64}$$

The economic intuition behind this proposition is that since there is uncertainty in this model, generators and LSEs are not sure about their future profits: they may enjoy high profits in one state or suffer low profit in the other state. However they know ISO is issuing a financial instrument, namely FTR, which can be used to hedge against their risky profit by reducing the profit spread between the two states. The risk averse generators and LSEs are thus willing to pay some premium to buy FTRs in order to maximize their expected utilities of future profits. If all generators and LSEs maximize their expected utilities by purchasing FTRs, then we can say FTRs increase the social welfare which is measured by total expected utilities. The formal proof of the proposition is provided in Appendix 5.

This proposition has important economic implications. First, it shows that in this simple two-node electricity network, once we introduce uncertainty (even in a very simple form), the acquisition of FTRs by risk averse agents can increase total social welfare. Moreover, as the proof shows, this result is strong and robust in the sense that regardless whether agents take long or short positions [23], the social welfare with FTRs is higher and in general strictly higher than that without FTRs. This result thus refutes the far more negative views of FTRs by other economists such as Joskow and Tirole (2000), and provides an economic explanation to the fact that FTRs are widely used in the major U.S. wholesale power markets.

Finally, in an attempt to endogenize the prices of FTRs, η_{12} and η_{21}, consider an ISO's problem. Since all information is public, that is, ISO knows that generators and LSEs will purchase FTRs to hedge against their risky profit in the energy spot market. Then ISO can solve generators and LSEs' problems to get the optimal FTR hedge solutions and substitute them into ISO's revenue adequacy constraint (RAC):

$$E(\tilde{\Pi}_{ISO}) = E(\tilde{CR}) + (\eta_{12} - E(\tilde{H}_{12}))\overline{FTR}_{12} + (\eta_{21} - E(\tilde{H}_{21}))\overline{FTR}_{21} \geq 0$$

where

$$\overline{FTR}_{12} = FTR^*_{12(G1)} + FTR^*_{12(L2)} = \frac{prob(H'_{12} - \eta_{12})(\pi''_{G1} + \pi''_{L2}) + (1 - prob)(H''_{12} - \eta_{12})(\pi'_{G1} + \pi'_{L2})}{(H'_{12} - \eta_{12})(\eta_{12} - H''_{12})}$$

$$\overline{FTR}_{21} = FTR^*_{21(G2)} + FTR^*_{21(L1)} = \frac{prob(H'_{21} - \eta_{21})(\pi''_{G2} + \pi''_{L1}) + (1 - prob)(H''_{21} - \eta_{21})(\pi'_{G2} + \pi'_{L1})}{(H''_{21} - \eta_{21})(\eta_{21} - H'_{21})}$$

$$E(\tilde{CR}) = prob\, CR' + (1 - prob)CR'' = prob(LMP'_2 - LMP'_1)T + (1 - prob)(LMP''_1 - LMP''_2)T$$

$$E(\tilde{H}_{12}) = prob\, H'_{12} + (1 - prob)H''_{12} = prob(LMP'_2 - LMP'_1) + (1 - prob)(LMP''_2 - LMP''_1)$$

$$E(\tilde{H}_{21}) = prob\, H'_{21} + (1 - prob)H''_{21} = prob(LMP'_1 - LMP'_2) + (1 - prob)(LMP''_1 - LMP''_2)$$

[23] Even if taking short position of FTRs is not allowed in this model.

Now ISO has several options to proceed. (a) The simplest option is to adjust η_{12} and η_{21} so that the RAC becomes binding. Then the relationship between η_{12} and η_{21} can be obtained as an implicit function denoted as $g_1()$ such that $g_1(\eta_{12}, \eta_{21}) = 0$. (b) The more complicated option that ISO can adopt is to adjust η_{12} and η_{21} so that it can extract a maximum amount of residual congestion rent (RCR). Then ISO invests this RCR to expand the transmission line, i.e., increase thermal limit T, which will reduce the uncertain profit spread, enhance efficiency, and increase social welfare. In this option, ISO can also get another set of relationship between η_{12} and η_{21} in an implicit function denoted as $g_1()$ such that $g_1(\eta_{12}, \eta_{21}) = 0$.

To possibly obtain a unique solution for η_{12} and η_{21}, we need to turn around and look at the problem from generators and LSEs' point of views. Since all generators and LSEs are assumed to be risk averse, they must be willing to pay certain amount of premiums to reduce the profit risks. Then in equilibrium the risk premiums are equivalent to the price of FTRs multiplied by the corresponding FTR contracts, that is, we have,

$$U[E(\tilde{\pi}_{G1} + \tilde{H}_{12}FTR_{12(G1)}) - \eta_{12}FTR_{12(G1)}] = E[U(\tilde{\pi}_{G1} + \tilde{H}_{12}FTR_{12(G1)})] \tag{65}$$

$$U[E(\tilde{\pi}_{G2} + \tilde{H}_{21}FTR_{21(G2)}) - \eta_{21}FTR_{21(G2)}] = E[U(\tilde{\pi}_{G2} + \tilde{H}_{21}FTR_{21(G2)})] \tag{66}$$

$$U[E(\tilde{\pi}_{L1} + \tilde{H}_{21}FTR_{21(L1)}) - \eta_{21}FTR_{21(L1)}] = E[U(\tilde{\pi}_{L1} + \tilde{H}_{21}FTR_{21(L1)})] \tag{67}$$

$$U[E(\tilde{\pi}_{L2} + \tilde{H}_{12}FTR_{12(L2)}) - \eta_{12}FTR_{12(L2)}] = E[U(\tilde{\pi}_{L2} + \tilde{H}_{12}FTR_{12(L2)})] \tag{68}$$

With the logarithmic utilities, in principle we can solve for $FTR^{**}_{12(G1)}$, $FTR^{**}_{21(G2)}$, $FTR^{**}_{21(L1)}$, and $FTR^{**}_{12(L2)}$ such that:

$$E(\tilde{\pi}_{G1}) + (E(\tilde{H}_{12}) - \eta_{12})FTR^{**}_{12(G1)} = (\pi'_{G1} + H'_{12}FTR^{**}_{12(G1)})^{prob}(\pi''_{G1} + H''_{12}FTR^{**}_{12(G1)})^{1-prob}$$

$$E(\tilde{\pi}_{G2}) + (E(\tilde{H}_{21}) - \eta_{21})FTR^{**}_{21(G2)} = (\pi'_{G2} + H'_{21}FTR^{**}_{21(G2)})^{prob}(\pi''_{G2} + H''_{21}FTR^{**}_{21(G2)})^{1-prob}$$

$$E(\tilde{\pi}_{L1}) + (E(\tilde{H}_{21}) - \eta_{21})FTR^{**}_{21(L1)} = (\pi'_{L1} + H'_{21}FTR^{**}_{21(L1)})^{prob}(\pi''_{L1} + H''_{21}FTR^{**}_{21(L1)})^{1-prob}$$

$$E(\tilde{\pi}_{L2}) + (E(\tilde{H}_{12}) - \eta_{12})FTR^{**}_{12(L2)} = (\pi'_{L2} + H'_{12}FTR^{**}_{12(L2)})^{prob}(\pi''_{L2} + H''_{12}FTR^{**}_{12(L2)})^{1-prob}$$

where $E(\tilde{H}_{12})$ and $E(\tilde{H}_{21})$ are as defined as above and $E(\tilde{\pi})$'s are defined as follows:

$$E(\tilde{\pi}_{G1}) = prob\,\pi'_{G1} + (1 - prob)\pi''_{G1};$$

$$E(\tilde{\pi}_{G2}) = prob\,\pi'_{G2} + (1 - prob)\pi''_{G2};$$

$$E(\tilde{\pi}_{L1}) = prob\, \pi'_{L1} + (1 - prob)\pi''_{L1};$$

$$E(\tilde{\pi}_{L2}) = prob\, \pi'_{L2} + (1 - prob)\pi''_{L2}.$$

After substituting the FTR^{**} solutions into the ISO's RAC and let ISO adjust η_{12} and η_{21}. Regardless whether ISO chooses option(a) or option(b), we can, in principle, derive another relationship between $\eta_{12}$and η_{21} in an implicit function denoted as $g_2()$ such that $g_2(\eta_{12}, \eta_{21}) = 0$.

Hence by solving the system of equation for η_{12} and η_{21},

$$\begin{cases} g_1(\eta_{12}, \eta_{21}) = 0; \\ g_2(\eta_{12}, \eta_{21}) = 0. \end{cases}$$

in principle we can solve for the equilibrium FTR price vector $\eta^* = (\eta^*_{12}, \eta^*_{21})$ such that

$$\begin{cases} g_1(\eta^*_{12}, \eta^*_{21}) = 0; \\ g_2(\eta^*_{12}, \eta^*_{21}) = 0. \end{cases}$$

5 Conclusions and Extensions

In this paper, we've studied the competitive behaviors of electricity generators and LSEs, and analyzed welfare effects of financial transmission rights (FTRs) in a restructured U.S. wholesale power market model. The analysis focuses on a competitive two-node electricity network model where there is one generator and one LSE in each node with parameterized marginal cost and demand function, supervised by an independent system operator (ISO). In the first part of the paper, a no-rights benchmark model is developed to solve for the optimal quantity of power production and consumption (the SCED solutions) and derive the locational marginal prices for each node, which serve as the building blocks to solve for the optimal FTR hedge positions in the second model. Then in the second model, we introduce a stochastic parameter shock into the two-node electricity network model, and manage to show that in the absence of market power the acquisition of optimal FTRs by the risk averse generators and LSEs increases and in general strictly increases the social welfare compared with the case where there is no FTRs available. This result refutes the somehow negative views of FTRs by other economists in the literature and provides the economic explanations to the fact that FTRs are widely adopted as a financial hedge instrument in the major U.S. wholesale power markets.

This study can be extended in several ways. First, we can extend the model to have an arbitrary number of generators and LSEs in each node. Admittedly, this extension adds the burden of calculations, but it does not change the essence of the solution. Mainly what we should be concerned about is to obtain an aggregate marginal cost (supply) function $AS_k(G_k)$ and an aggregate demand function $AD_k(L_k)$ for each node $k = 1, 2$. Then proceed to solve the model as if there were one 'representative' generator and one 'representative' LSE. After the aggregate solution is acquired, the solution quantities can be referred back through LMPs to get the individual dispatched quantities. Although the process of solving the problem is more tedious, the essence of the solution algorithm in this paper remains. We expect that including multiple generators and LSEs at each node will not have dramatic effects on the solution outcomes.

Second, we can extend our two-node electricity network model to three nodes or more. Then we will be introducing an important feature of real world electricity network, the "loop flow effect", which considerably increases the modeling complications. Basically, the "loop flow effect" is associated with the fact that electrons follow the path of least resistance. In an electric network with a transmission grid consisting of multiple connection lines, the patterns of electricity flows follow the famous Kirchhoff's laws in physics. For example, in a three-node network, if there is a power injection Q at one node, say node 1 and an equal amount of withdrawal at another node, say node 2, then depending on the reactance of line 1-2, line 1-3 and line 2-3, a proportion amount of power, say αQ flows from node 1 to node 2 while the rest $(1 - \alpha)Q$ flows from node 1 to node 3 then to node 2. For instance, if the line reactance is the same for all three lines, then $\alpha = 2/3$. In this case, we need to add one more variable, the phase angle (ϕ), in order to control the power flows between transmission lines [24]. Although there is significant amount of work involved when we model the three-node case, the result is expected to be closer to reality than the two-node case.

Third, in our two-node model, we assume generators and LSEs always submit their true marginal cost and true demand function to ISO, so we always get the competitive solution which is also Pareto optimal [25]. But what if we relax this assumption so that generators and LSEs can strategically submit their marginal cost and demand functions in the hope that they can gain individual advantages through strategic behaviors.

Fourth, how about extending the static two-node model into a dynamic model with multiple periods, where in each period, generators and LSEs submit their strategic bids and offers in a double auction framework in both FTR and day-ahead power markets. They could be endowed with an initial wealth, and if they don't make enough profits within several periods, then they are forced to bankrupt. Moreover, these generators and LSEs can

[24] Technically, we need to model the 3-node case using a Direct Current (DC) power flow formulation.

[25] In the 3-node case, it becomes unclear that the outcome will still be Pareto optimal because of externality brought by "loop flow" effect.

'learn' what is the best strategies for them over time. The learning methods may include reinforcement learning and anticipatory learning, etc.

With these complicated extensions, it seems almost impossible to proceed with analytical tools. A natural candidate that may fit very well for this purpose is the agent-based computational approach. For a comprehensive introduction of Agent-based Computational Economics (ACE), see the ACE survey by Tesfatsion (2003). The next stage of this study is to extend the static and competitive two-node electricity model into a dynamic multi-node electricity model with learning agents bidding through double auction markets in a sequel paper using agent-based computational approach.

Appendix 1

The non-thermal-constraint SCED solution in Step 1 is derived as follows:

In step 1, when the thermal limit T never binds, the SCED problem is to maximize the 'total net benefit'(TNB) subject to the balancing and non-negativity constraints. This is just a standard optimization problem with one equality constraint (the balancing constraint) and four inequality constraints (the non-negativity constraints for Q_{G1}, Q_{G2}, Q_{L1} and Q_{L2}). Using μ's as the multipliers for equality constraint and λ's as the multipliers for inequality constraints, and formulate the Lagrangian equation:

$$L = (b_1^D Q_{L1} - \frac{1}{2}a_1^D Q_{L1}^2) - (b_1^S Q_{G1} + \frac{1}{2}a_1^S Q_{G1}^2) + (b_2^D Q_{L2} - \frac{1}{2}a_2^D Q_{L2}^2) - (b_2^S Q_{G2} + \frac{1}{2}a_2^S Q_{G2}^2)$$

$$+\mu(Q_{G1} + Q_{G2} - Q_{L1} - Q_{L2}) + \lambda_{G1}Q_{G1} + \lambda_{G2}Q_{G2} + \lambda_{L1}Q_{L1} + \lambda_{L2}Q_{L2}$$

Derive the first order conditions (FOCs):

$$\frac{\partial L}{\partial Q_{L1}} = b_1^D - a_1^D Q_{L1} - \mu + \lambda_{L1} = 0$$

$$\frac{\partial L}{\partial Q_{G1}} = -b_1^S - a_1^S Q_{G1} + \mu + \lambda_{G1} = 0$$

$$\frac{\partial L}{\partial Q_{L2}} = b_2^D - a_2^D Q_{L2} - \mu + \lambda_{L2} = 0$$

$$\frac{\partial L}{\partial Q_{G2}} = -b_2^S - a_2^S Q_{G2} + \mu + \lambda_{G2} = 0$$

$$\frac{\partial L}{\partial \mu} = Q_{G1} + Q_{G2} - Q_{L1} - Q_{L2} = 0$$

$$Q_{G1} \geq 0,\ \lambda_{G1} \geq 0,\ \lambda_{G1}Q_{G1} = 0$$

$$Q_{G2} \geq 0,\ \lambda_{G2} \geq 0,\ \lambda_{G2}Q_{G2} = 0$$

$$Q_{L1} \geq 0,\ \lambda_{L1} \geq 0,\ \lambda_{L1}Q_{L1} = 0$$

$$Q_{L2} \geq 0,\ \lambda_{L2} \geq 0,\ \lambda_{L2}Q_{L2} = 0$$

For simplicity, only consider the case where all dispatched quantities are positive, i.e., all non-negativity constraints are not binding ($\lambda_{G1} = \lambda_{G2} = \lambda_{L1} = \lambda_{L2} = 0$)[26]. Thus from

[26] To be exhaustive, we find 9 other possible solution cases, i.e., (1) $Q_{G1} = 0$; (2) $Q_{G2} = 0$; (3) $Q_{L1} = 0$; (4) $Q_{L2} = 0$; (5) $Q_{G1} = Q_{L1} = 0$; (6) $Q_{G1} = Q_{L2} = 0$; (7) $Q_{G2} = Q_{L1} = 0$; (8) $Q_{G2} = Q_{L2} = 0$; (9) $Q_{G1} = Q_{G2} = Q_{L1} = Q_{L2} = 0$.

FOCs we have:

$$\begin{cases} a_1^S Q_{G1} + a_1^D Q_{L1} = b_1^D - b_1^S\,; \\ a_1^S Q_{G1} + a_2^D Q_{L2} = b_2^D - b_1^S\,; \\ a_2^S Q_{G2} + a_2^D Q_{L2} = b_2^D - b_2^S\,; \\ Q_{G1} + Q_{G2} = Q_{L1} + Q_{L2}\,; \\ \mu = b_1^S + a_1^S Q_{G1}\,. \end{cases}$$

Notice the last equation shows that the Lagrangian multiplier associated with balancing constraint is equal to the marginal cost, which by the nature of this SCED problem is also the LMP. Solving 5 unknown variables for 5 equations, we obtain the Step 1 non-thermal-constraint SCED solution:

$$\hat{Q_{G1}} = \frac{a_2^D a_2^S(b_1^D - b_1^S) + a_1^D a_2^S(b_2^D - b_2^S) + a_1^D(a_2^D + a_2^S)(b_2^S - b_1^S)}{a_1^D a_1^S(a_2^D + a_2^S) + a_2^D a_2^S(a_1^D + a_1^S)}$$

$$\hat{Q_{G2}} = \frac{a_1^D a_1^S(b_2^D - b_2^S) + a_1^S a_2^D(b_1^D - b_1^S) - a_2^D(a_1^D + a_1^S)(b_2^S - b_1^S)}{a_1^D a_1^S(a_2^D + a_2^S) + a_2^D a_2^S(a_1^D + a_1^S)}$$

$$\hat{Q_{L1}} = \frac{(a_2^D a_2^S + a_1^S a_2^D + a_1^S a_2^S)(b_1^D - b_1^S) - a_1^S a_2^S(b_2^D - b_2^S) - a_1^S(a_2^D + a_2^S)(b_2^S - b_1^S)}{a_1^D a_1^S(a_2^D + a_2^S) + a_2^D a_2^S(a_1^D + a_1^S)}$$

$$\hat{Q_{L2}} = \frac{(a_1^D a_1^S + a_1^D a_2^S + a_1^S a_2^S)(b_2^D - b_2^S) - a_1^S a_2^S(b_1^D - b_1^S) + a_2^S(a_1^D + a_1^S)(b_2^S - b_1^S)}{a_1^D a_1^S(a_2^D + a_2^S) + a_2^D a_2^S(a_1^D + a_1^S)}$$

$$LMP_1 = LMP_2 = \hat{\mu} = b_1^S + a_1^S\hat{Q_{G1}} = \frac{a_2^D a_2^S(a_1^D b_1^S + a_1^S b_1^D) + a_1^D a_1^S(a_2^D b_2^S + a_2^S b_2^D)}{a_1^D a_1^S(a_2^D + a_2^S) + a_2^D a_2^S(a_1^D + a_1^S)}$$

which can be expressed as:

$$\hat{Q_{G1}} = (\mathbb{G}_{\mathbf{1}} + \mathbb{B}_{\mathbf{1}})/\mathbb{A}$$

$$\hat{Q_{G2}} = (\mathbb{G}_{\mathbf{2}} + \mathbb{B}_{\mathbf{2}})/\mathbb{A}$$

$$\hat{Q_{L1}} = (\mathbb{L}_{\mathbf{1}} + \mathbb{C}_{\mathbf{1}})/\mathbb{A}$$

$$\hat{Q_{L2}} = (\mathbb{L}_{\mathbf{2}} + \mathbb{C}_{\mathbf{2}})/\mathbb{A}$$

$$LMP_1 = LMP_2 = \hat{\mu} = b_1^S + a_1^S\hat{Q_{G1}}$$

where

$$\begin{aligned} &\mathbb{G}_{\mathbf{1}} = D_2B_1 + a_1^D a_2^S B_2, \quad \mathbb{B}_{\mathbf{1}} = a_1^D A_2 C_1, \quad \mathbb{L}_{\mathbf{1}} = (D_2 + a_1^S A_2)B_1 - a_1^S a_2^S B_2, \quad \mathbb{C}_{\mathbf{1}} = a_1^S A_2 C_2; \\ &\mathbb{G}_{\mathbf{2}} = D_1B_2 + a_1^S a_2^D B_1, \quad \mathbb{B}_{\mathbf{2}} = a_2^D A_1 C_2, \quad \mathbb{L}_{\mathbf{2}} = (D_1 + a_2^S A_1)B_2 - a_1^S a_2^S B_1, \quad \mathbb{C}_{\mathbf{2}} = a_2^S A_1 C_1; \\ &\mathbb{A} \;= D_1A_2 + D_2A_1; \end{aligned}$$

$$A_1 = a_1^D + a_1^S, \quad B_1 = b_1^D - b_1^S, \quad C_1 = b_2^S - b_1^S, \quad D_1 = a_1^D a_1^S;$$
$$A_2 = a_2^D + a_2^S, \quad B_2 = b_2^D - b_2^S, \quad C_2 = b_1^S - b_2^S, \quad D_2 = a_2^D a_2^S.$$

Appendix 2

The binding-thermal-constraint SCED solution in Step 2 is derived as follows:

(a) Based on Step 1 non-thermal-constraint SCED solution, if we know T is binding from 1 to 2, i.e., $\hat{Q_{G1}} - \hat{Q_{L1}} > T$ or $\hat{Q_{L2}} - \hat{Q_{G2}} > T$. We can set either $Q_{G1} - Q_{L1} = T$ or $Q_{L2} - Q_{G2} = T$. But one of them is redundant due to the fact that the balancing constraint ($Q_{G1} + Q_{G2} = Q_{L1} + Q_{L2}$) always holds in this two-node electricity network. So without loss of generality, let $Q_{G1} - Q_{L1} = T$.

This is a standard optimization problem subject to two equality constraints (balancing and thermal constraint) and four inequality constraints (the non-negativity constraints for Q_{G1}, Q_{G2}, Q_{L1} and Q_{L2}). Using μ's as the multipliers for equality constraints and λ's as the multipliers for inequality constraints, and formulate the Lagrangian equation:

$$L = (b_1^D Q_{L1} - \frac{1}{2} a_1^D Q_{L1}^2) - (b_1^S Q_{G1} + \frac{1}{2} a_1^S Q_{G1}^2) + (b_2^D Q_{L2} - \frac{1}{2} a_2^D Q_{L2}^2) - (b_2^S Q_{G2} + \frac{1}{2} a_2^S Q_{G2}^2)$$

$$+\mu_B (Q_{G1} + Q_{G2} - Q_{L1} - Q_{L2}) + \mu_T (T - Q_{G1} + Q_{L1}) + \lambda_{G1} Q_{G1} + \lambda_{G2} Q_{G2} + \lambda_{L1} Q_{L1} + \lambda_{L2} Q_{L2}$$

Recall all the parameters are positive, i.e., $T > 0$, $a_j^D > 0, b_j^D > 0,$, and $a_i^S > 0, b_i^S > 0$ for $i, j = 1, 2$. Derive the FOCs:

$$\frac{\partial L}{\partial Q_{L1}} = b_1^D - a_1^D Q_{L1} - \mu_B + \mu_T + \lambda_{L1} = 0$$
$$\frac{\partial L}{\partial Q_{G1}} = -b_1^S - a_1^S Q_{G1} + \mu_B - \mu_T + \lambda_{G1} = 0$$
$$\frac{\partial L}{\partial Q_{L2}} = b_2^D - a_2^D Q_{L2} - \mu_B + \lambda_{L2} = 0$$
$$\frac{\partial L}{\partial Q_{G2}} = -b_2^S - a_2^S Q_{G2} + \mu_B + \lambda_{G2} = 0$$
$$\frac{\partial L}{\partial \mu_B} = Q_{G1} + Q_{G2} - Q_{L1} - Q_{L2} = 0$$
$$\frac{\partial L}{\partial \mu_T} = T - Q_{G1} + Q_{L1} = 0$$
$$Q_{G1} \geq 0, \ \lambda_{G1} \geq 0, \ \lambda_{G1} Q_{G1} = 0$$

$$Q_{G2} \geq 0,\ \lambda_{G2} \geq 0,\ \lambda_{G2} Q_{G2} = 0$$

$$Q_{L1} \geq 0,\ \lambda_{L1} \geq 0,\ \lambda_{L1} Q_{L1} = 0$$

$$Q_{L2} \geq 0,\ \lambda_{L2} \geq 0,\ \lambda_{L2} Q_{L2} = 0$$

Rearranging the FOCs w.r.t Q_{L1} and Q_{G1}, the FOCs w.r.t Q_{L2} and Q_{G2}, and the FOCs w.r.t μ_B and μ_T, we have:

$$\begin{cases} a_1^S Q_{G1} + a_1^D Q_{L1} = b_1^D - b_1^S + \lambda_{L1} + \lambda_{G1}\,; \\ a_2^S Q_{G2} + a_2^D Q_{L2} = b_2^D - b_2^S + \lambda_{L2} + \lambda_{G2}\,; \\ Q_{G1} - Q_{L1} = T\,; \\ Q_{L2} - Q_{G2} = T\,. \end{cases}$$

Solving four unknown variables for four equations , we have:

$$Q_{G1} = \frac{b_1^D - b_1^S + a_1^D T + \lambda_{L1} + \lambda_{G1}}{a_1^D + a_1^S}$$

$$Q_{G2} = \frac{b_2^D - b_2^S - a_2^D T + \lambda_{L2} + \lambda_{G2}}{a_2^D + a_2^S}$$

$$Q_{L1} = \frac{b_1^D - b_1^S - a_1^S T + \lambda_{L1} + \lambda_{G1}}{a_1^D + a_1^S}$$

$$Q_{L2} = \frac{b_2^D - b_2^S + a_2^S T + \lambda_{L2} + \lambda_{G2}}{a_2^D + a_2^S}$$

Now to tackle the corner solution, first let the solutions be all positive, i.e., $Q_{G1} > 0, Q_{G2} > 0, Q_{L1} > 0, Q_{L2} > 0$, so $\lambda_{G1} = \lambda_{G2} = \lambda_{L1} = \lambda_{L2} = 0$. We then can get the following:

$$Q_{G1} = \frac{b_1^D - b_1^S + a_1^D T}{a_1^D + a_1^S}$$

$$Q_{G2} = \frac{b_2^D - b_2^S - a_2^D T}{a_2^D + a_2^S}$$

$$Q_{L1} = \frac{b_1^D - b_1^S - a_1^S T}{a_1^D + a_1^S}$$

$$Q_{L2} = \frac{b_2^D - b_2^S + a_2^S T}{a_2^D + a_2^S}$$

For the solutions indeed to be all positive, the parameters ($b_j^D, a_j^D, b_i^S, a_i^S, T$, for $i, j = 1, 2$) must satisfy the following conditions (in other words, any violations to the following conditions will lead to corner solutions):

$$(*1) \qquad b_1^D - b_1^S + a_1^D T > 0 \qquad \text{or} \qquad Q_{G1} > 0$$

$$(*2) \qquad b_2^D - b_2^S - a_2^D T > 0 \qquad \text{or} \qquad Q_{G2} > 0$$

$$(*3) \qquad b_1^D - b_1^S - a_1^S T > 0 \qquad \text{or} \qquad Q_{L1} > 0$$

$$(*4) \qquad b_2^D - b_2^S + a_2^S T > 0 \qquad \text{or} \qquad Q_{L2} > 0$$

Close examination on the above conditions indicates that Condition (*1) and (*4) will not be violated here because in this case the thermal constraint is binding from node 1 to node 2, i.e., node 1 as the net export node (NEN) and node 2 as the net import node (NIN). Recall in the simplifying assumptions we assume that there is only one generator and one LSE in each node. So node 1 as the NEN and node 2 as the NIN would imply that G_1 has to supply a positive amount of power over the transmission line and LSE_2 has to demand a positive amount of power sent from G_1 in this two-node electricity network. Hence the total power supply by G_1, Q_{G1}, and the total power demand by LSE_2, Q_{L2}, must be greater than zero, which in turn proves that (*1) and (*4) will always hold in this case.

Then the Complementary Slackness Conditions (CSCs) for Q_{G1} and Q_{L2} will give us $\lambda_{G1} = 0$ and $\lambda_{L2} = 0$.

In summary, the general solution (GS) would look like:

$$(GS1) \qquad Q_{G1} = \frac{b_1^D - b_1^S + a_1^D T + \lambda_{L1}}{a_1^D + a_1^S}$$

$$(GS2) \qquad Q_{G2} = \frac{b_2^D - b_2^S - a_2^D T + \lambda_{G2}}{a_2^D + a_2^S}$$

$$(GS3) \qquad Q_{L1} = \frac{b_1^D - b_1^S - a_1^S T + \lambda_{L1}}{a_1^D + a_1^S}$$

$$(GS4) \qquad Q_{L2} = \frac{b_2^D - b_2^S + a_2^S T + \lambda_{G2}}{a_2^D + a_2^S}$$

Since Condition (*1) and (*4) will always hold, we only need to examine Condition (*2) and (*3) to get the SCED solutions. There are four cases to consider, i.e., [i] both (*2) and (*3) hold; [ii] (*2) holds while (*3) is violated; [iii] (*3) holds while (*2) is violated; [iv] both (*2) and (*3) are violated.

Case I: Both (*2) and (*3) hold (interior solution)

When (*2) and (*3) hold, i.e., $Q_{G2} > 0$ and $Q_{L1} > 0$, the CSCs for Q_{G2} and Q_{L1} will give us: $\lambda_{G2} = 0$ and $\lambda_{L1} = 0$.

Then the Step 2 SCED solution vector is simply $s^* = (Q^*_{G1}, Q^*_{G2}, Q^*_{L1}, Q^*_{L2})$, where

$$Q^*_{G1} = \frac{b_1^D - b_1^S + a_1^D T}{a_1^D + a_1^S}$$

$$Q^*_{G2} = \frac{b_2^D - b_2^S - a_2^D T}{a_2^D + a_2^S}$$

$$Q^*_{L1} = \frac{b_1^D - b_1^S - a_1^S T}{a_1^D + a_1^S}$$

$$Q^*_{L2} = \frac{b_2^D - b_2^S + a_2^S T}{a_2^D + a_2^S}$$

$$LMP_1 = b_1^S + a_1^S Q^*_{G1} = \frac{a_1^D b_1^S + a_1^S b_1^D + a_1^D a_1^S T}{a_1^D + a_1^S}$$

$$LMP_2 = b_2^S + a_2^S Q^*_{G2} = \frac{a_2^D b_2^S + a_2^S b_2^D - a_2^D a_2^S T}{a_2^D + a_2^S}$$

Case II: (*2) holds while (*3) is violated

When (*2) holds, i.e., $Q_{G2} > 0$, then the CSC for Q_{G2} will give us $\lambda_{G2} = 0$. Then from the general solution (GS2) and (GS4), we know that Q^*_{G2} and Q^*_{L2} are the same as in Case I.

When (*3) is violated, i.e., $Q_{L1} < 0$, then by the non-negativity constraint for Q_{L1} we have $Q^*_{L1} = 0$. From (GS3) we have $Q^*_{L1} = \frac{b_1^D - b_1^S - a_1^S T + \lambda^*_{L1}}{a_1^D + a_1^S} = 0$. Solving for $\lambda^*_{L1} = b_1^S - b_1^D + a_1^S T$ and substituting it into (GS1), we have $Q^*_{G1} = T$

So the Step 2 SCED solution vector is $s^* = (Q^*_{G1}, Q^*_{G2}, Q^*_{L1}, Q^*_{L2})$, where

$$\begin{aligned}
Q^*_{G1} &= T \\
Q^*_{G2} &= \tfrac{b_2^D - b_2^S - a_2^D T}{a_2^D + a_2^S} \\
Q^*_{L1} &= 0 \\
Q^*_{L2} &= \tfrac{b_2^D - b_2^S + a_2^S T}{a_2^D + a_2^S} \\
LMP_1 &= b_1^S + a_1^S Q^*_{G1} = b_1^S + a_1^S T \\
LMP_2 &= b_2^S + a_2^S Q^*_{G2} = \tfrac{a_2^D b_2^S + a_2^S b_2^D - a_2^D a_2^S T}{a_2^D + a_2^S}
\end{aligned}$$

Case III: (*3) holds while (*2) is violated

When (*3) holds, i.e., $Q_{L1} > 0$, then the CSC for Q_{L1} will give us $\lambda_{L1} = 0$. Then from the general solution (GS1) and (GS3), we know that Q^*_{G1} and Q^*_{L1} are the same as in Case I.

When (*2) is violated, i.e., $Q_{G2} < 0$, then by the non-negativity constraint for Q_{G2}

we have $Q^*_{G2} = 0$. From (GS2) we have $Q^*_{G2} = \frac{b_2^D - b_2^S - a_2^D T + \lambda^*_{G2}}{a_2^D + a_2^S} = 0$. Solving for $\lambda^*_{G2} = b_2^S - b_2^D + a_2^D T$ and substituting it into (GS4), we have $Q^*_{L2} = T$.

So the Step 2 SCED solution vector is $s^* = (Q^*_{G1}, Q^*_{G2}, Q^*_{L1}, Q^*_{L2})$, where

$$\begin{aligned}
Q^*_{G1} &= \tfrac{b_1^D - b_1^S + a_1^D T}{a_1^D + a_1^S} \\
Q^*_{G2} &= 0 \\
Q^*_{L1} &= \tfrac{b_1^D - b_1^S - a_1^S T}{a_1^D + a_1^S} \\
Q^*_{L2} &= T \\
LMP_1 &= b_1^S + a_1^S Q^*_{G1} = \tfrac{a_1^D b_1^S + a_1^S b_1^D + a_1^D a_1^S T}{a_1^D + a_1^S} \\
LMP_2 &= b_2^D - a_2^D Q^*_{L2} = b_2^D - a_2^D T
\end{aligned}$$

Case IV: Both (*2) and (*3) are violated

When (*2) is violated, i.e., $Q_{G2} < 0$, then by the non-negativity constraint for Q_{G2} we have $Q^*_{G2} = 0$. Q_{L2} is the same as in Case III, i.e., $Q^*_{L2} = T$. When (*3) is violated, i.e., $Q_{L1} < 0$, then by the non-negativity constraint for Q_{L1} we have $Q^*_{L1} = 0$. Q_{G1} is the same as in Case II, i.e., $Q^*_{G1} = T$

So the Step 2 SCED solution vector is $s^* = (Q^*_{G1}, Q^*_{G2}, Q^*_{L1}, Q^*_{L2})$, where

$$\begin{aligned}
Q^*_{G1} &= T \\
Q^*_{G2} &= 0 \\
Q^*_{L1} &= 0 \\
Q^*_{L2} &= T \\
LMP_1 &= b_1^S + a_1^S Q^*_{G1} = b_1^S + a_1^S T \\
LMP_2 &= b_2^D - a_2^D Q^*_{L2} = b_2^D - a_2^D T
\end{aligned}$$

So the Step 2 SCED solutions can be summarized as follows:

Step 2 SCED Solution (T is binding from 1 to 2)

Case I	Case II	Case III	Case IV
$Q^*_{G1} = \frac{B_1 + a_1^D T}{A_1}$	$Q^*_{G1} = T$	$Q^*_{G1} = \frac{B_1 + a_1^D T}{A_1}$	$Q^*_{G1} = T$
$Q^*_{G2} = \frac{B_2 - a_2^D T}{A_2}$	$Q^*_{G2} = \frac{B_2 - a_2^D T}{A_2}$	$Q^*_{G2} = 0$	$Q^*_{G2} = 0$
$Q^*_{L1} = \frac{B_1 - a_1^S T}{A_1}$	$Q^*_{L1} = 0$	$Q^*_{L1} = \frac{B_1 - a_1^S T}{A_1}$	$Q^*_{L1} = 0$
$Q^*_{L2} = \frac{B_2 + a_2^S T}{A_2}$	$Q^*_{L2} = \frac{B_2 + a_2^S T}{A_2}$	$Q^*_{L2} = T$	$Q^*_{L2} = T$
$LMP_1 = \frac{E_1 + D_1 T}{A_1}$	$LMP_1 = b_1^S + a_1^S T$	$LMP_1 = \frac{E_1 + D_1 T}{A_1}$	$LMP_1 = b_1^S + a_1^S T$
$LMP_2 = \frac{E_2 - D_2 T}{A_2}$	$LMP_2 = \frac{E_2 - D_2 T}{A_2}$	$LMP_2 = b_2^D - a_2^D T$	$LMP_2 = b_2^D - a_2^D T$

where

$$A_1 = a_1^D + a_1^S, \quad B_1 = b_1^D - b_1^S, \quad D_1 = a_1^D a_1^S, \quad E_1 = a_1^D b_1^S + a_1^S b_1^D;$$
$$A_2 = a_2^D + a_2^S, \quad B_2 = b_2^D - b_2^S, \quad D_2 = a_2^D a_2^S, \quad E_2 = a_2^D b_2^S + a_2^S b_2^D.$$

(b) If, on the other hand, we know T is binding from 2 to 1 based on Step 1 non-thermal-constraint SCED solution, i.e., $\hat{Q_{G2}} - \hat{Q_{L2}} > T$ or $\hat{Q_{L1}} - \hat{Q_{G1}} > T$. We can set either $Q_{G2} - Q_{L2} = T$ or $Q_{L1} - Q_{G1} = T$. But one of them is redundant due to the fact that the balancing constraint ($Q_{G1} + Q_{G2} = Q_{L1} + Q_{L2}$) always holds in this two-node electricity network. So without loss of generality, let $Q_{G2} - Q_{L2} = T$.

Using the same procedure as in (a), we can derive another set of Step 2 SCED solutions:

Step 3 SCED Solution (T is binding from 2 to 1)

Case I	Case II	Case III	Case IV
$Q^*_{G1} = \frac{B_1 - a_1^D T}{A_1}$	$Q^*_{G1} = \frac{B_1 - a_1^D T}{A_1}$	$Q^*_{G1} = 0$	$Q^*_{G1} = 0$
$Q^*_{G2} = \frac{B_2 + a_2^D T}{A_2}$	$Q^*_{G2} = T$	$Q^*_{G2} = \frac{B_2 + a_2^D T}{A_2}$	$Q^*_{G2} = T$
$Q^*_{L1} = \frac{B_1 + a_1^S T}{A_1}$	$Q^*_{L1} = \frac{B_1 + a_1^S T}{A_1}$	$Q^*_{L1} = T$	$Q^*_{L1} = T$
$Q^*_{L2} = \frac{B_2 - a_2^S T}{A_2}$	$Q^*_{L2} = 0$	$Q^*_{L2} = \frac{B_2 - a_2^S T}{A_2}$	$Q^*_{L2} = 0$
$LMP_1 = \frac{E_1 - D_1 T}{A_1}$	$LMP_1 = \frac{E_1 - D_1 T}{A_1}$	$LMP_1 = b_1^D - a_1^D T$	$LMP_1 = b_1^D - a_1^D T$
$LMP_2 = \frac{E_2 + D_2 T}{A_2}$	$LMP_2 = b_2^S + a_2^S T$	$LMP_2 = \frac{E_2 + D_2 T}{A_2}$	$LMP_2 = b_2^S + a_2^S T$

Appendix 3

Proof of Proposition 1:

First we want to show $LMP_2 > LMP_1 \;\Leftrightarrow\; \Omega = (A_1E_2 - A_2E_1)/(D_1A_2 + D_2A_1) > T$. Recall in Step 2 SCED solution, we know $LMP_1 = \frac{E_1 + D_1 T}{A_1}$ and $LMP_2 = \frac{E_2 - D_2 T}{A_2}$. Then

$$\begin{aligned} LMP_2 > LMP_1 \;&\Leftrightarrow\; \frac{E_2 - D_2T}{A_2} - \frac{E_1 + D_1T}{A_1} > 0 \\ &\Leftrightarrow\; \frac{A_1E_2 - A_2E_1 - (D_1A_2 + D_2A_1)T}{A_1A_2} > 0 \\ &\Leftrightarrow\; \frac{A_1E_2 - A_2E_1}{D_1A_2 + D_2A_1} > T \quad (\text{since } A_1, A_2, D_1, D_2 > 0) \end{aligned}$$

Next, let $B_1 = b_1^D - b_1^S$, $B_2 = b_2^D - b_2^S$ and $C = b_2^S - b_1^S$ and we want to show T is binding from 1 to 2 $\Leftrightarrow$

$\frac{A_1E_2-A_2E_1}{D_1A_2+D_2A_1} > T$. Recall in the Step 1 SCED solution and Definition 1,

$$T \text{ is binding from 1 to 2} \Leftrightarrow \hat{Q_{G1}} - \hat{Q_{L1}} > T, \text{ where}$$

$$\hat{Q_{G1}} = \frac{D_2B_1 + a_1^Da_2^SB_2 + a_1^DA_2C}{D_1A_2 + D_2A_1}$$

$$\hat{Q_{L1}} = \frac{(D_2 + a_1^Sa_2^D + a_1^Sa_2^S)B_1 - a_1^Sa_2^SB_2 - a_1^SA_2C}{D_1A_2 + D_2A_1}$$

$$\begin{aligned}
\hat{Q_{G1}} - \hat{Q_{L1}} > T \quad &\Leftrightarrow \quad \frac{a_2^SB_2A_1 - a_1^SB_1A_2 + A_1A_2C}{D_1A_2 + D_2A_1} > T \\
&\Leftrightarrow \quad \frac{a_2^S(b_2^D - b_2^S)A_1 - a_1^S(b_1^D - b_1^S)A_2 + A_1A_2(b_2^S - b_1^S) > T}{D_1A_2 + D_2A_1} > T \\
&\Leftrightarrow \quad \frac{A_1(a_2^S(b_2^D - b_2^S) + b_2^S(a_2^D + a_2^S)) - A_2(a_1^S(b_1^D - b_1^S) + b_1^S(a_1^D + a_1^S)) > T}{D_1A_2 + D_2A_1} > T \\
&\Leftrightarrow \quad \frac{A_1(a_2^Sb_2^D + a_2^Db_2^S) - A_2(a_1^Sb_1^D + a_1^Db_1^S)}{D_1A_2 + D_2A_1} > T \\
&\Leftrightarrow \quad \frac{A_1E_2 - A_2E_1}{D_1A_2 + D_2A_1} > T \\
&\Leftrightarrow \quad \Omega > T
\end{aligned}$$

Since we showed T is binding from 1 to 2 $\Leftrightarrow \Omega > T$ and $LMP_2 > LMP_1 \;\Leftrightarrow\; \Omega > T$, we've proved (*1) in Proposition 1. Similar procedures can easily be applied to prove (*2) and (*3), and thus is omitted here. Q.E.D.

Appendix 4

Proof of Proposition 3:
(i) T is binding from node 1 to node 2
Recall in the benchmark model total net benefit (TNB) is defined as the total net surplus for all generators and LSEs, that is,

$$TNB = (b_1^DQ_{L1} - \frac{1}{2}a_1^DQ_{L1}^2) - (b_1^SQ_{G1} + \frac{1}{2}a_1^SQ_{G1}^2) + (b_2^DQ_{L2} - \frac{1}{2}a_2^DQ_{L2}^2) - (b_2^SQ_{G2} + \frac{1}{2}a_2^SQ_{G2}^2)$$

$$
\begin{aligned}
\frac{\partial TNB}{\partial T} &= (b_1^D\frac{\partial Q_{L1}}{\partial T} - a_1^D Q_{L1}\frac{\partial Q_{L1}}{\partial T}) - (b_1^S\frac{\partial Q_{G1}}{\partial T} + a_1^S Q_{G1}\frac{\partial Q_{G1}}{\partial T}) \\
&+ (b_2^D\frac{\partial Q_{L2}}{\partial T} - a_2^D Q_{L2}\frac{\partial Q_{L2}}{\partial T}) - (b_2^S\frac{\partial Q_{G2}}{\partial T} + a_2^S Q_{G2}\frac{\partial Q_{G2}}{\partial T}) \\
&= -\frac{a_1^S b_1^D}{A_1} + \frac{a_1^D a_1^S Q_{L1}}{A_1} - \frac{a_1^D b_1^S}{A_1} - \frac{a_1^D a_1^S Q_{G1}}{A_1} + \frac{a_2^S b_2^D}{A_2} - \frac{a_2^D a_2^S Q_{L2}}{A_2} - \frac{a_2^D b_2^S}{A_2} - \frac{a_2^D a_2^S Q_{G2}}{A_2} \\
&= \frac{a_2^S b_2^D + a_2^D b_2^S}{A_2} - \frac{a_1^S b_1^D + a_1^D b_1^S}{A_1} - \frac{a_1^D a_1^S (Q_{G1} - Q_{L1})}{A_1} - \frac{a_2^D a_2^S (Q_{L2} - Q_{G2})}{A_2} \\
&= \frac{E_2}{A_2} - \frac{E_1}{A_1} - (\frac{D_1}{A_1} + \frac{D_2}{A_2})T
\end{aligned}
$$

$$
\begin{aligned}
\Longrightarrow \frac{\partial TNB}{\partial T} > 0 \ &\Leftrightarrow \ \frac{E_2}{A_2} - \frac{E_1}{A_1} - (\frac{D_1}{A_1} + \frac{D_2}{A_2})T > 0 \\
&\Leftrightarrow \ \frac{A_1E_2 - A_2E_1}{A_1A_2} > \frac{(D_1A_2 + D_2A_1)T}{A_1A_2} \\
&\Leftrightarrow \ \frac{A_1E_2 - A_2E_1}{D_1A_2 + D_2A_1} > T \quad (\text{since } A_1, A_2, D_1, D_2 > 0)
\end{aligned}
$$

From Proposition 1 we know that

$$
T \text{ is binding from 1 to 2} \Leftrightarrow \frac{A_1E_2 - A_2E_1}{D_1A_2 + D_2A_1} > T
$$

Therefore

$$
\frac{\partial TNB}{\partial T} = \frac{E_2}{A_2} - \frac{E_1}{A_1} - \left(\frac{D_1}{A_1} + \frac{D_2}{A_2}\right) T > 0 \Leftrightarrow \text{ T is binding from 1 to 2;}
$$

(ii) T is binding from node 2 to node 1

Very similar, we can show

$$
\frac{\partial TNB}{\partial T} = \frac{E_1}{A_1} - \frac{E_2}{A_2} - \left(\frac{D_1}{A_1} + \frac{D_2}{A_2}\right) T > 0 \Leftrightarrow \text{T is binding from 2 to 1.}
$$

Q.E.D.

Appendix 5

Proof of Proposition 7:

By a definition of social welfare, we have $W_F = E[U(\tilde{\Pi}_{G1})] + E[U(\tilde{\Pi}_{G2})] + E[U(\tilde{\Pi}_{L1})] + E[U(\tilde{\Pi}_{L2})]$ and $W_0 = E[U(\tilde{\pi}_{G1})] + E[U(\tilde{\pi}_{G2})] + E[U(\tilde{\pi}_{L1})] + E[U(\tilde{\pi}_{L2})]$.

We can prove $W_F - W_0 = \Delta_{G1} + \Delta_{G2} + \Delta_{L1} + \Delta_{L2} \geq 0$ if we can show (i)–(iv) are satisfied.

$$\begin{aligned}
(i) &\quad \Delta_{G1} \geq 0; \\
(ii) &\quad \Delta_{G2} \geq 0; \\
(iii) &\quad \Delta_{L1} \geq 0; \\
(iv) &\quad \Delta_{L2} \geq 0.
\end{aligned}$$

where

$$\begin{aligned}
\Delta_{G1} &= E[U(\tilde{\Pi}_{G1})] - E[U(\tilde{\pi}_{G1})]; \\
\Delta_{G2} &= E[U(\tilde{\Pi}_{G2})] - E[U(\tilde{\pi}_{G2})]; \\
\Delta_{L1} &= E[U(\tilde{\Pi}_{L1})] - E[U(\tilde{\pi}_{L1})]; \\
\Delta_{L2} &= E[U(\tilde{\Pi}_{L2})] - E[U(\tilde{\pi}_{L2})].
\end{aligned}$$

We'll prove (i)–(iv) one by one as follows:
Part (i), denote $p \equiv prob$, then,

$$\begin{aligned}
\Delta_{G1} &= E[U(\tilde{\Pi}_{G1})] - E[U(\tilde{\pi}_{G1})] \\
&= pU(\pi'_{G1} + (H'_{12} - \eta_{12})FTR_{12(G1)}) + (1-p)U(\pi''_{G1} + (H''_{12} - \eta_{12})FTR_{12(G1)}) \\
&\quad - pU(\pi'_{G1}) - (1-p)U(\pi''_{G1}) \\
&= p\,\log\left(1 + \frac{(H'_{12} - \eta_{12})FTR_{12(G1)}}{\pi'_{G1}}\right) + (1-p)\log\left(1 + \frac{(H''_{12} - \eta_{12})FTR_{12(G1)}}{\pi''_{G1}}\right) \\
&= p\log\left(1 + \frac{p(H'_{12} - \eta_{12})\pi''_{G1} + (1-p)(H''_{12} - \eta_{12})\pi'_{G1}}{(\eta_{12} - H''_{12})\pi'_{G1}}\right) \\
&\quad + (1-p)\log\left(1 + \frac{p(H'_{12} - \eta_{12})\pi''_{G1} + (1-p)(H''_{12} - \eta_{12})\pi'_{G1}}{(\eta_{12} - H'_{12})\pi''_{G1}}\right) \\
&= \log\left(p^p(1-p)^{1-p}\left[1 + \frac{(H'_{12} - \eta_{12})\pi''_{G1}}{(\eta_{12} - H''_{12})\pi'_{G1}}\right]^p\left[1 + \frac{(\eta_{12} - H''_{12})\pi'_{G1}}{(H'_{12} - \eta_{12})\pi''_{G1}}\right]^{1-p}\right) \\
&= \log\left(p^p(1-p)^{1-p}\left[1 + X_{G1}\right]^p\left[1 + \frac{1}{X_{G1}}\right]^{1-p}\right)
\end{aligned}$$

where

$$X_{G1} \equiv \frac{(H'_{12} - \eta_{12})\pi''_{G1}}{(\eta_{12} - H''_{12})\pi'_{G1}} > 0$$

Then in order to show $\Delta_{G1} \geq 0$, we need to show

$$p^p(1-p)^{1-p}\left[1+X_{G1}\right]^p\left[1+\frac{1}{X_{G1}}\right]^{1-p} \geq 1$$

For notation simplicity, let $x \equiv X_{G1} > 0$, and $A \equiv p^p(1-p)^{1-p} > 0$, and define a function $f(\cdot)$ such that

$$f(x) = A(1+x)^p(1+\frac{1}{x})^{1-p}$$

Notice that when $x = \frac{1}{p} - 1$, $f(\frac{1}{p} - 1) = 1$. So to prove $f(x) \geq 1$ is equivalent to prove the function $f(x)$ is monotonically decreasing over the domain $(0, \frac{1}{p} - 1)$ and monotonically increasing over the domain $(\frac{1}{p} - 1, +\infty)$. Rewrite $f(x)$ as follows:

$$f(x) = A(1+x)x^{p-1}$$

$$\begin{aligned} f'(x) &= Ax^{p-1} + A(1+x)(p-1)x^{p-2} \\ &= Ax^{p-2}[p(1+x)-1] \\ &= Ax^{p-2}p[x - (\frac{1}{p} - 1)] \end{aligned}$$

$$\Rightarrow f'(x) \begin{cases} < 0 & \text{if } x < \frac{1}{p} - 1; \\ > 0 & \text{if } x > \frac{1}{p} - 1; \\ = 0 & \text{if } x = \frac{1}{p} - 1. \end{cases}$$

That is, f(x) has a global minimum at $x = \frac{1}{p} - 1$. The minimum is:

$$f(\frac{1}{p} - 1) = 1$$

Hence, $f(x) \geq 1$, $\forall\, x \in (0, \infty)$, or $f(p) \geq 1$, $\forall\, p \in (0, 1)$. Notice if the regularity condition is satisfied we have

$$1 > prob > \overline{prob}_{G1} = \frac{(\eta_{12} - H''_{12})\pi'_{G1}}{(H'_{12} - \eta_{12})\pi''_{G1} + (\eta_{12} - H''_{12})\pi'_{G1}} = \frac{1}{\frac{(H'_{12} - \eta_{12})\pi''_{G1}}{(\eta_{12} - H''_{12})\pi'_{G1}} + 1} = \frac{1}{X_{G1} + 1}$$

$$\Leftrightarrow \quad p \in (\frac{1}{1+x}, 1) \subseteq (0, 1) \qquad \text{(Recall } p \equiv prob,\ x \equiv X_{G1})$$

So when the regularity condition is satisfied, i.e., G_1 is taking long positions in the FTR market, we certainly have $f(p) > 1$, which directly implies $\Delta_{G1} > 0$. If, on the other hand, the regularity condition is not satisfied, it can be easily shown that it corresponds to the case where $p \in (0, \frac{1}{1+x})$ and G_1 is taking short positions (although it is not allowed in this model) in the FTR market, which also implies $\Delta_{G1} > 0$. Finally, in the degenerate case where p happens to be $\frac{1}{1+x}$, then G_1 takes zero position in the FTR market and $\Delta_{G1} = 0$. Therefore regardless whether G_1 takes long, short or zero position in the FTR market, $\Delta_{G1} \geq 0$.

It is straightforward to verify that (ii), (iii) and (iv) are true using the exactly same procedures as in (i). Since we have showed that (i)–(iv) are all satisfied, we've proved the proposition result, $W_F \geq W_0$. Q.E.D.

References

Bautista, G., and V. Quintana. (2005). "Screening and Mitigation of Exacerbated Market Power Due to Financial Transmission Rights," IEEE Transactions on Power Systems, 20(1): 213-222.

Bessembinder, H. (1991), "Forward Contracts and Firm Value: Investment Incentive and Contracting Effects," Journal of Financial and Quantitative Analysis 26: 519-532.

Bushnell, J. (1999), "Transmission Rights and Market Power," The Electricity Journal, 12(8):77-85.

Cardell, J., Hitt, C., and Hogan, W. (1997). "Market Power and Strategic Interaction in Electricity Networks," Resource and Energy Economics 18(4): 107-141.

Deng, S., S. Oren, and S. Meliopoulos (2004). "The Inherent Inefficiency of Simultaneously Feasible Financial Transmission Rights Auctions," IEEE Proceedings of the 37th Hawaii International Conference on System Sciences.

FERC (2003), Notice of White Paper (U.S. Federal Energy Regulatory Commission, Issued April 28).

Froot, K., D. Scharfstein, and J. Stein (1993), "Risk Management: Coordinating corporate Investment and Financing Policies," Journal of Finance 48: 1629-1658.

Hogan, W. (2000). "Market Power in Theory and Practice, Comments on Joskow and Tirole (2000) and Johnsen, Verma and Wolfram (1999)," POWER Conference, University of California, Berkeley, Berkeley, CA, March 17.

Hogan, W. (2002). "Financial Transmission Right Formulations," Working Paper, Center for Business and Government, John F. Kennedy School of Government, Harvard University, Cambridge, MA, March 15.

Hogan, W. (2003). "Transmission Market Design," Paper prepared for the conference "Electricity Deregulation: Where From Here?" at the Bush Presidential Conference Center, Texas A & M University, April 4.

ISO New England (2003a). Market Rule 1 - NEPOOL Standard Market Design, Revision 2, September 9. Available at http://www.iso-ne.com/smd/

ISO New England (2003b). NEPOOL Manual for Financial Transmission Rights, Manual M-06, Revision 2, April 4. Available at http://www.iso-ne.com/smd/

ISO New England (2004). NEPOOL Manual for Market Operations, Manual M-11, Revision 8, May 10. Available at http://www.iso-ne.com/smd/

Joskow, P., and J. Tirole, (2000). "Transmission Rights and Market Power on Electric Power Networks," RAND Journal of Economics 31(3): 450-487.

Joskow, P., and J. Tirole, (2003). "Merchant Transmission Investment," NBER working paper, No. 9534. Kamat, R., and S. Oren, (2004). "Two-settlement Systems for Electricity Markets under Network Uncertainty and Market Power," Journal of Regulatory Economics 25(1): 5-37.

Kench, B., (2004). "Let's Get Physical! Or Financial? A Study of Electricity Transmission Rights," Journal of Regulatory Economics 25(2): 187-214.

Kirschen, D., and G. Strbac, (2004). Fundamentals of Power System Economics (John Wiley & Sons, Ltd., Hoboken, NJ). Kristiansen, T. (2003). "Markets for Financial Transmission Rights," Working Paper, Norwegian University of Science and Technology, Department of Electrical Power Engineering.

Lyons, K, H. Fraser, and H. Parmesano, (2000). "An Introduction to Financial Transmission Rights," Electricity Journal 13(10): 31-37.

Mendez, R., and H. Rudnick, (2004). "Congestion Management and Transmission Rights in Centralized Electric Markets," IEEE Transactions on Power Systems, 19(2): 889-896.

Midwest ISO, 2005, MISO Financial Transmission Rights Market Participant User's Manual. Available at http://www.midwestiso.com/

O'Neill, R, U. Helman, and B. Hobbs, (2002). "A Joint Energy and Transmission Rights Auction: Proposal and Properties," IEEE Transactions on Power Systems, 17(4): 1058-1067.

Oren, S., (1997). "Economic Inefficiency of Passive Transmission Rights in Congested Electricity Systems with Competitive Generation," The Energy Journal, 18(1): 63-83.

Rosellon, J. (2003). "Different Approaches Towards Electricity Transmission Expansion," Review of Network Economics, 2(3): 238-269.

Rudkevich, A. (2004). "Investment and Bidding Strategies in Markets for Firm Transmission Rights," IEEE Proceedings of the 37th Hawaii International Conference on System Sciences.

Siddiqui, A., Bartholomew, E., Marnay, C., and Oren, S. (2003). "On the Efficiency of the

New York Independent System Operator Market for Transmission Congestion Contracts," Working Paper, Environmental Energy Technologies Division, Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, CA, April 30.

Smith, C., and R. Stulz (1985), Determinants of Firm's Hedging Policies, Journal of Financial and Quantitative Analysis 20: 391-405.

Stoft, S. (1999). "Financial Transmission Rights Meet Cournot: How TCCs Curb Market Power,", The Energy Journal, 20(1): 1-4.

Stoft, S. (2002). Power System Economics: Designing Markets for Electricity, IEEE Press, Wiley-Interscience, New York, NY.

Stulz, R. (1990), Managerial Discretion and Optimal Financing Policies, Journal of Financial Economics 26: 3-28.

Tesfatsion, L, (2003). "Agent-Based Computational Economics," Department of Economics, Iowa State University, Ames, Iowa 50011-1070, ISU Economics Working Paper No. 1, Revised August 24.

U.S. Department of Energy (2000) "The Changing Structure of the Electric Power Industry 2000: An Update," Energy Information Administration, Washington DC.

U.S. Department of Energy (2002) "National Transmission Grid Study," May. http://www.ntgs.doe.gov/

Wilson, R. (2002). "Architecture of Power Markets," Econometrica, 70(4): 1299-1340.