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A Theory of Advice Based on Information Search Incentives

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A Theory of Advice Based on Information Search Incentives*

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Abstract

This paper investigates whether recourse to a consultant always enhances decision making. Advice given by a consultant changes the manager's belief about his own decision-making ability. This change in belief alters the manager's incentives to make a decision. Taking into account this effect, we characterize the contracts that the firm must offer to the manager when a consultant with a given expertise is hired. Surprisingly, we find that the benefit curve of the firm may decrease as the consultant expertise increases, even if there is no consulting fee. Moreover, we show that the value of advice depends on the "good fit" between the informativeness of the consultant and the manager's incentives to reach the right decision.

Keywords: Advice, value of information, information search, incentives.

JEL Classification: D82 (Asymmetric and Private Information), D83 (Search, Learning, and Information), J30 (Wages, Compensation, and Labor Costs).

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PRN: *What can companies do to become better clients, better able to appreciate and use their advisors*

A.S.: *The first thing is, really good clients are great learners - they have a learning attitude. If you fundamentally get yourself into, "I'm going to learn from this", it's going to open you a lot.*

-Dialog between consultant Andrew Sobel and Public Relation News (Sept. 2000).

1 Introduction

At first sight, the description of the relationship between a consultant and his client appears to be easy to sketch. The consultant helps the client solve a decision-making problem. Thus a simple way to consider professional advice activity is that the client buys an item (information) from a consultant; this relationship can be summarized by a simple transfer of information from a consultant to his client. The greater the expertise of the consultant on the decision problem at stake, the more valuable is the information that he delivers to the decision-maker.

However, managers and professional consultants know that things are not that simple. At least two points can be made to describe the advice process more realistically.

First, in many contexts, decision-makers tend to have an already formed opinion about the decision they are willing to make. For instance, a manager who wants to merge her firm with another company may have some idea about how to proceed. She may even find that, given the corporate culture of her firm, one particular firm may be a superior choice for a successful merger. Of course, she may forget crucial points in her analysis and her opinion may be weak or wrong. Thus, without ruling out the possibility for perfect ignorance, our starting point is that a manager being advised may have an opinion about the decision she has to make.

Second, in any decision-making problem, incentive aspects are never absent from the decision-making process and will greatly influence the quality of the decision made. A decision-maker, uncertain about her level of expertise, can always exert effort to overcome her lack of expertise. "Advised" decision-making processes are not exceptions to the rule and involve incentive problems on the part of the decision-maker (and probably on the part on the consultant). Consultants often deliver general advice that is sometimes out of touch with the firm's specificity. Managers have to understand whether those advice is relevant for their organization; this is not an effort-free exercise. Consultants' specialists agree that the determinants of a successful mission are partly independent of the consultant's expertise and depend crucially on the client's behavior. The success of a mission depends on the client's incentives to be involved in the search for an effective solution to the problem of the firm.

When those two points are taken into account, the decision-making process becomes one in which one expert (the consultant) advises another expert (the manager) who will face the consequences of her decision.

After, receiving the advice, the manager must decide whether or not to enhance her decision. For instance, the advice can take the form of a written report delivered to the manager. The manager can then decide to superficially read the report and follow the advice without criticism, or she can try to improve on that advice and incorporate, for instance, relevant informations she knows about her firm. In the merger example mentioned above, the manager may acquire some culture about merger processes, learn the points that are crucial for successful mergers, and then match those points with what she knows about her firm. There is clearly an incentive question involved here, but the exact nature of the incentive problem can only be addressed by further investigating the effect of the advice on the manager's incentives.

When the manager is uncertain about her exact decision-making talent, the information received from the consultant alters her perception of her own decision-making talent and her incentives to reach a decision.

Indeed, if the advice significantly departs from the manager's prior opinion, then her perception of her expertise tends to be low and she is thus willing to work hard to compensate for her lack of talent; talent and hard work are substitutes in our setting. In this case, advice will boost managerial incentives for the search for information.

Conversely, if the advice turns out to be in accordance with the manager's opinion, we expect her to have few incentives to analyze the consultant's report in depth, because a deep analysis is not likely to alter her final decision. Thus, "consensus" tends to generate few incentives for decision-making.

The fact that the advice endogenizes the manager's incentives for decision-making is a consequence of the two points introduced earlier, and it is a key aspect of the present work. To summarize, *the economic value of advice depends not only on the person who gives it, but also on the one who receives it.*

The purpose of our paper is to integrate the aspects discussed above into a model of decision-making with advice. The model studied here involves a principal (e.g., a firm or a CEO) who decides to hire a consultant to advise an agent (e.g., a CEO or a division manager) in a decision-making problem. The decision-making process is a period of time during which, after the report has been made by the consultant, the manager can deepen her knowledge and understanding of the problem. We assume that the decision to improve her expertise and to forge a new opinion involves some unobservable actions; there is thus a moral hazard component. The firm proposes an incentive contract to the manager to reach the most accurate decision possible. This contractual approach allows us to describe the effect of the informativeness of the advice on managerial incentives. The "private benchmarking" of the manager's talent appears to be crucial because, it is then possible to discriminate between managers who receive "bad news" about their talent and those who receive "good news" from the consultant. The former should be given strong incentives, whereas the latter should be given weak incentives. The help of the consultant is, thus, twofold. It enhances the probability of good decision-making - the traditional role of a consultant - and it may also play a key role in targeting incentives for decision-making.

Several results emerge from the analysis of this setup. We show that if the firm wants the manager to have a say in the decision-making process, and be able to fruitfully criticize an advice given by a consultant, the firm should choose the manager's incentives and the consultant carefully.

More precisely, for the accurateness of a given manager's opinion and her level of incentives for the search of information, there exists a unique level of the consultant's informativeness, such that the firm's profit flow from the decision is maximized. The intuition behind this result can be grasped by first noticing that it is appropriate to incentivize the manager as much as possible when there is a dissent between her and the consultant. This can only happen when the consultant who is as informative as the manager. Indeed, with such a consultant, managerial ignorance will be maximum in the case of dissent, and the manager's incentives to overcome her ignorance are also bound to be the greatest. However, such a consultant will perform poorly on the information side, and the firm also probably wants the consultant to be as much informative as possible. The choice of the optimal consultant is the result of a trade off between these two conflicting objectives[1].

Surprisingly, the value of the information provided by the consultant can decrease with the informativeness of the advice delivered, *even if the advice is free of consulting fees*[2]. The benefit curve of the firm as a function of the consultant's expertise appears to be non-monotonic. Therefore, given a manager with some expertise level and incentives to reach the right decision, there exists a range of consultants' expertise levels that is strictly inefficient (i.e., that are beaten by a consultant with lower expertise).

In the basic model, we assume that the consultant and manager do make mistakes that are statistically independent. In a variant of the model, we study the case in which mistakes in the prediction of the state of nature are correlated. We show that this correlation has a negative effect on incentives since it

[1] Most of the existing economic literature on advice assumes that the decision-maker is ignorant and that the advisor "knows" or is more informed. In our work, we do not assume away the fact that the manager can be smarter than the consultant. The fact that the optimal consultant is more informative than the manager is another result of this paper. However, we also show that the manager can eventually benefit from an advice less informative than her opinion. This finding is close to that of Zabojnik (1998) who finds that a superior may find it worthwhile to delegate to a less-informed subordinate who will implement his own idea.

[2] We assume that there is no consulting fee. Any cost from having a consultant is thus endogenous.

generates two few information searches. Thus, for an equal level of expertise, a consultant with a lower level of correlation is more valuable to the firm. This finding may have an important consequence if we accept the assumption that opinions tend to be more correlated within than across organizations. Indeed, in this case, hiring an external consultant constitutes a credible way to increase managerial incentives for decision-making.

The relationship between the informativeness of the advice and its value for the firm is conducted with a binary level of effort in order to provide explicit solutions to the problem. However, the analysis is also conducted with continuous effort. In this case, we characterize the optimal contract and study the relationship between the level of effort exerted by the manager and the level of information provided by the consultant. We find that an increase in the informativeness of the advice should be followed by an increase in managerial effort. We illustrate our findings in the continuous effort case with a numerical example and we show that the value of information can still be negative if the advice is free. These findings are consistent with the managerial literature which asserts that valuable advice depends not only on the consultant's advice but also on the client's strong involvement in terms of learning effort[3].

The rest of the paper is organized as follows. In section 2 we relate the model to the economic literature. Section 3 presents a simple model of information search with binary level of effort. In section 4 we study the case of consultancy and the associated firm's incentive problem. We derive explicit solutions to the incentive scheme problem, and we also study a variant in which mistakes in opinion are correlated. Section 5 generalizes the model with continuous levels of effort. Section 6 offers future directions for research and concludes.

2 Related literature

To the best of our knowledge, no theoretical discussions have been devoted to the incentive aspects of the manager/consultant relationship. However, this work is related to several streams of the economic literature.

There exist several papers that are mainly concerned with advice. For instance, Calvert (1985) shows that, in a context of political decision-making, a politician who has a prior opinion on a decision he has to make should receive advice from an advisor whose opinion tends to be identical to his. The choice of an advisor biased toward his opinion is justified by the fact that, in case of disagreement (which is a rare event), the contradictory advice is very powerful and valuable to the politician, since it can change his

[3]The managerial literature on this topic is vast. See, for instance, Edvardsson (1989), Fullerton and West (1996), and Stumpf and Longman (2000).

point of view. The fact that the optimal advisor should be biased toward the decision-maker opinion is not a conclusion of our analysis. In fact, the introduction of the incentive aspect in our setting leads to the opposite conclusion: more potential contradiction between opinions generates more incentives on the part of the decision-maker.

The seminal work of Crawford and Sobel (1982) has given rise to several contributions on the economics of advice. These contributions generally focus on the credibility of the expert whose objectives differ from those of the decision-maker. Austin-Smith (1993) explores a situation in which an assembly obtains the advice of two experts who have differing political objectives. The paper analyzes under which conditions the assembly should ask the experts sequentially rather than simultaneously.

Krishna and Morgan (1999) study a setting in which two perfectly informed but biased experts offer their advice sequentially to an uninformed decision-maker. They show that asking two experts with opposing biases may allow the decision-maker to extract all the relevant information.

Another strand of the economic literature is concerned with the study of the transmission of information by agents who have career concerns. Ottaviani and Sorensen (1999) study how experts who want to appear well-informed communicate their information to a decision-maker. Although the experts receive signals of continuously varying intensity and can freely communicate the direction of their signal and its intensity, they show that reputational concerns result in the transmission of very coarse information. Experts are only able, in the most informative equilibrium, to transmit the direction of their signal, but not the intensity[4].

All these models study the strategic transmission of information by the expert, and eventually evaluate the magnitude of these distortions. Our model takes the informativeness of the information delivered as given, and studies the behavior of the decision-maker. Our approach can be seen as complementary to these models.

Moreover and relatedly, these models take for granted that the receiver of advice has no expertise about the decision making problem. This is well-justified in a model of political advice, because the politician is essentially a generalist who needs the advice of a specialist (a scientist, say). We think that in the case of managerial advice, this assumption is not justified in many contexts. Although the manager needs an advice, she has good knowledge of her firm and its problems, and can eventually disagree with

[4] Other contributions dealing with decision-making under reputational concern include, among others, Holmstrom and Ricart y Costa (1986), Scharfstein and Stein (1990), Zweibel (1996), Effinger and Polborn (1998) and Milbourn, Schockley and Thakor (2001). Also related to reputation and consultancy is the paper of Demsky et al. (1999). This paper analyzes the organizational practices of firms that must manage the information of their client (essentially banks, audit firms and consultants). The paper shows that the capacity to protect and to control the client information determines the demand for services that these firms will have.

advice given. The possibility of (non) disagreement then plays a crucial role, since it endogenizes future information searches and analysis.

Our work is also related to the literature that explores the link between the incentives of the agents and the quantity of information generated by the organization. Crémer (1995) explores a model in which a principal may be better off by committing not to acquire information about an agent's productivity, although this information has a strictly positive value. In the present setting, we show that even though the consultant is costless and informative, the principal may optimally refuse this source of information, as it jeopardizes the agent's incentives. Dewatripont and Tirole (1989) present a model of advocates and demonstrate that the creation of advocates for and against issues provides the best incentives to acquire information when there are competing causes. Osband (1989) presents a model of incentives in forecasting. In this work, the agent is a risk-neutral forecaster who can refine his forecast at some private cost. The principal would want to induce the agent to refine his forecast, but neither the cost nor the precision of the forecast can be verified[5]. Optimal incentives schemes for forecasting are derived in this setting.

In our setting, the manager has an external source of information and the informativeness of the information strongly shapes her incentives to acquire further information. The non monotonic shape of the firm's benefit curve as a function of the informativeness of the consultant is explained by the effect of advice on incentives.

3 The model

3.1 State of nature and optimal decision

There are one period and two dates $t = 0$ and $t = 1$.

Suppose that the firm wants a manager to make a decision at date $t = 1$. Let us assume that there are two unobservable possible states of nature A and B and each of those states of nature may arise with equal probability at date 1. We assume that for each state of nature, there exists an appropriate decision. Let us denote by "a" the decision that should be optimally made if the state of nature is A. On the other hand, implementing decision "a" whereas the true state of nature is B is considered a managerial mistake that will be followed by financial losses.

Therefore, let us state that an adequate decision ((A, a) or (B, b)) generates a profit flow of $G > 0$ monetary units, while a mistake ((A, b) or (B, a)) generates a loss of $L > 0$.

[5] Also in a principal-agent setting, Malcomson (1999) presents a more general model in which a risk averse agent must incur an effort and truthfully reveal the outcome of this effort.

Of course, the manager is not a perfect decision-maker, and we now describe how the manager makes-up his mind before making the decision at date 1.

3.2 Type of agent and signal technology

The correctness of the decision of the manager depends on her type as well as on her effort.

We denote by $\theta \in [0,1]$ the unconditional probability that the manager has the correct type for the kind of decision she has to make. From now on, we will call θ "the level of expertise" (or simply "expertise") of the manager. So with probability θ a manager has the good type and receives a signal indicating the true state of nature with probability one; for instance, if the underlying state of nature is A, the manager receives a signal indicating decision a. We slightly abuse notation by directly denoting this signal a.

When the manager is less talented, which occurs with probability $1-\theta$, she just receives a random signal indicating with equal probability decision "a" or "b"[6].

Ex ante, neither the principal nor the agents (i.e., the manager or the consultant) can know whether the agent is of the right type; that is, whether she always gets the right signal at the first time. We assume that the consultant possesses the same signal technology as the manager, and we denote by λ the unconditional probability that the consultant is smart. Just before $t = 1$, another signal may be obtained by the manager if she exerts a binary effort. The manager may perform either:

- no effort, for which she has no disutility. In this case, the manager obtains exactly the signal she had obtained at the beginning of the first period.
- a strong effort, for which she has a disutility of 1 monetary unit[7]. The signal technology has the following features. If the manager is smart, the second signal is the same as in the beginning of the first period. If the manager is dumb, she obtains a new signal independent from the first, and this signal indicates the true state of nature with probability $\frac{1}{2} < p < 1$.

Therefore, by exerting effort and thus obtaining another signal, a manager can improve on the quality of his decision-making. The existence of this second signal helps us to modelize the fact that talented managers make the right decisions sooner and with no effort. Thus, in this setting the signal that a talented manager gets at date $t = 0$ is the right one, and is identical to the one she will get at date $t = \frac{1}{2}$.

[6] We will sometimes follow the terminology introduced by Scharfstein and Stein (1990) and call the high ability type decision-maker a "smart" manager whereas we will refer to the low ability type decision-maker as the "dumb" manager.

[7] There is no loss of generality in considering one unit rather than $R > 0$. It is just simpler.

Conversely, less talented managers tend to have an immediate right opinion less often and must exert some effort to converge toward the right decision.

We now describe the information sets of the agents, as well as the contracting possibilities of the firm.

3.3 Information, effort and contracts

The signals obtained by the agents do not constitute hard information in the sense developed by Tirole (1986). This means that new signals may be forged costlessly by the manager, if necessary. Therefore, these signals will not be contracted upon by the parties.

The consultant is hired on the consultant's market, and is paid after he reports to the manager. As the consultant does not perform any effort, the payment is a fixed amount $\mu \geq 0$ independent of the realized state of nature. In the rest of the paper, and as we want to focus on the manager incentive problem, we will use $\mu = 0$ to simplify the analysis[8]. Since the outcome is observable and contractible, it is possible for the firm to commit to a payment contingent upon this outcome.

The effort performed by the manager to obtain the second signal is not observable by the firm. However, it can be induced by the design of an incentive contract based on the outcome of the decision.

3.4 Timing of events

The timing of events is as follows:

1. The nature draws a state of nature, $I = \{A, B\}$

2. The manager is proposed a contract that stipulates whether a consultant will be present and the payments to be made contingent upon the outcome.

3. The manager and the consultant (if present) privately receive a signal, $i \in \{a, b\}$ indicating the possibly appropriate decision that fits the underlying state of nature[9,10]. The consultant

[8] A more complete (and realistic) treatment would involve an ex ante payment $\mu(\lambda) > 0$, with $\mu(.)$ being an increasing function of the consultant's ex ante reputation, λ. Any result we get would not be affected if we were to assume that μ was an increasing function of λ.

[9] In fact, it does not matter whether the consultant and the manager receive the signal simultaneously or not.

[10] The assumption that the advice of the consultant is not observable, and hence not contracted upon, could be removed. If this is done, the manager still has private information on his talent but the set of possible contracts is broader. In particular, a contract that stipulates a payment contingent upon the signal announced by the consultant and the decision implemented by the manager would be useful in extracting some of the rent associated with the private information possessed by the manager. An optimal incentive scheme using the information provided by the consultant would take the following form: if the manager successfully implements a decision different from the one advocated by the consultant, then he would receive a greater salary than the one received if the decision implemented is identical to the consultant's advice. We refrain from

(if present) privately reports his signal to the manager and is paid an amount normalized to zero.

4. The manager privately decides to obtain another signal; this decision involves a costly effort.

5. The manager implements her chosen decision, the state of nature is realized and payments are made according to the contract.

The next section concerns with the benchmark case in which the manager acts alone in the decision-making.

3.5 The "no-consultant" case

We now consider the case where the firm must induce the manager to decide alone. We assume that the firm finds it worthwhile to induce the manager to obtain the second signal, and we will provide the formal condition that insures that the manager will effectively expand the costly effort.

In this context, the manager receives a first signal and must decide in the second round whether to produce an effort which will entitle her to obtain a second signal. We denote by w_{nc}^S the wage given to the manager if she makes the right decision. The reward w_{nc}^F is given in case of failure.

The program of the firm is:

$$\underset{(w_{nc}^S, w_{nc}^F)}{Max} \left[\theta + p\left(1-\theta\right)\right]\left[G - w_{nc}^S\right] - \left(1-p\right)\left(1-\theta\right)\left(L + w_{nc}^F\right) \tag{1}$$

subject to the following constraints:

$$\left[\theta + p\left(1-\theta\right)\right] w_{nc}^S + \left(1-p\right)\left(1-\theta\right) w_{nc}^F - 1 \geq \left[\theta + \tfrac{1}{2}\left(1-\theta\right)\right] w_{nc}^S + \tfrac{1}{2}\left(1-\theta\right) w_{nc}^F \tag{2}$$

$$w_{nc}^S \geq 0 \text{ and } w_{nc}^F \geq 0. \qquad \text{(Limited liability constraints)}$$

The objective function states that the manager picks the right decision with probability $\theta + p(1-\theta)$. This is because, with probability θ, she is smart and always receives the right signal. She can also be dumb with probability $1-\theta$, but by exerting effort she is successful with probability $p > \frac{1}{2}$. A right decision generates a profit flow of $G > 0$ and salary w_{nc}^S must be paid to the manager. The manager picks a wrong decision with probability $(1-p)(1-\theta)$ and the firm shows a loss $L > 0$. The salary paid to the manager is w_{nc}^F in this case.

presenting such an analysis because even if the optimal contract is more sophisticated, the results derived have the same flavor as the ones we are going to present; some rent is transferred from the manager to the firm. Moreover, the interpretation of the model changes if the signal is public, since in this case the firm is auditing the manager's talent.

The incentive constraint (2) states that the manager is willing to exert effort.

The solution to this program is

$$w_{nc}^{S*} = \frac{2}{(1-\theta)(2p-1)} \text{ and } w_{nc}^{F*} = 0. \tag{3}$$

By plugging these optimal salaries into the objective function of the firm, we easily obtain the equilibrium profit of the firm when no consultant is hired:

$$B_{nc}^{*} = (\theta + p(1-\theta)) \left[G + L - \tfrac{2}{(1-\theta)(2p-1)}\right] - L \tag{4}$$

and we can now provide a condition which guarantee that the firm is willing to induce the manager to refine his opinion.

Lemma 1 *The firm induces the manager to exert an information search effort when the following condition is verified:*

$$G + L \geq \tfrac{4(p+\theta-p\theta)}{(2p-1)^2(1-\theta)^2}. \tag{5}$$

Proof. The proof consists in the computation of the difference between the benefit to the firm when the incentive scheme (3) is implemented and the benefit when the manager is not induced to do a search effort. This difference is equal to:

$$\Delta B = B_{nc}^{*} - \left(\left[\theta + \frac{1}{2}(1-\theta)\right] G - \frac{1}{2}(1-\theta) L\right).$$

A simple computation shows that ΔB is positive if the following condition on the parameters is satisfied:

$$G + L \geq \frac{4(p+\theta-p\theta)}{(2p-1)^2(1-\theta)^2}.$$

This is condition (5). ■

From now on, we will assume that condition (5) holds; that is, the parameters of the model are always such that the firm wants to induce the manager to obtain another signal when she acts alone. If this condition is not verified, the incentive problem of the firm becomes trivial because payments are independent of the outcome.

We now turn to the case where a consultant is hired to advise the manager.

4 The case of consultancy

This part concerns the determination and analysis of the optimal contracts offered by the firm when a consultant is hired.

The aim of the following subsection is to explain how many signals should be optimally collected by the manager. Moreover, although it is not crucial for the results, the merit of this subsection is to explain and clarify the interaction between the agents. It also explains to what extend each agent participates in the decision-making process because, as mentioned in the introduction, there is often the problem of measuring the real influence of a consultant on the decision-making process.

4.1 Manager and consultant: What does the consultant bring?

In our context, the firm will have to determine the optimal number of signals before offering an incentive contract to the manager. Here, we want to abstract from the problem of inducing information acquisition. Therefore, we focus on the value of an additional signal *if the acquisition of this signal is costless.*

A signal is worth acquiring if the decision that is taken with this additional signal strictly increases the expected profit relative to a situation in which the decision is taken only with two signals. For instance, let us assume that the manager has received i and that the consultant has reported to her signal i. With those two identical signals the optimal decision of the manager is prone to be action i. Acquiring a third signal will enhance decision-making, only it changes the decision made by the manager. Thus, if a signal j is obtained, it must be at least as informative as the two signals ii.

The optimal number of signals that should be obtained by the manager depends on the interplay between Bayesian learning, the ex ante ability of the agents and the productivity of effort. The following proposition characterizes the number of signals that the firm is willing to obtain if those signals are costless.

Proposition 1 *There exist two values of the consultant's expertise*

$$\underline{\lambda} = 2p - 1 \text{ and } \overline{\lambda} = \frac{2p - 1 + 3\theta - 2p\theta}{1 + \theta},$$

such that the following statements are true:

- *whenever* $\lambda < \underline{\lambda}$ *, and whatever the first two signals, a sequence of three signals always strictly benefits the firm.*
- *whenever* $\lambda \in \left[\underline{\lambda}\ ,\ \overline{\lambda}\right]$*, a sequence of three signals is worthwhile if the two first signals differ whereas a sequence of two signals is sufficient if they are similar.*
- *whenever* $\lambda > \overline{\lambda}$ *, a third signal is never worthwhile; the two first signals are always sufficient.*

Proof. The formal proof is given in the appendix. ■

We provide an intuition for proposition 1. If $\lambda = 0$, the signal transmitted by the consultant is non-informative about the true state of nature and acquiring a third signal is always strictly worthwhile. Continuity implies that, this would be true for small values of λ. The third point is easily understood if one assumes that $\lambda = 1$. In such a case, acquiring a third signal is never worthwhile since the one brought by the consultant is perfect (i.e., indicate the true state of nature with probability one). Continuity implies that, this will be true for high values of λ. The last case describes the intermediate case in which the signal brought by the consultant does not totally dominate the information that the manager can generate.

The parameters of the model together determine over which regions an information search is optimal. For instance, an increase in θ increases the length of $\left[\underline{\lambda}, \overline{\lambda}\right]$.

To make the picture complete, we now provide a result stating the optimal decision that the manager makes for a given sequence of signal. This optimal decision depends on the sequence of signal obtained, and on the relative talent of the manager and the consultant.

Corollary 1 *(Optimal Decision Rules)*

- *If the sequence is ij, the optimal decision of the manager is i if $\lambda \leq \theta$, and j otherwise.*
- *If the sequence is ijj, the optimal decision is j.*
- *If the sequence is iji, the optimal decision is i if $\lambda \leq \overline{\lambda}$, and j otherwise.*
- *If the sequence is iij, the optimal decision is j if $\lambda \leq \underline{\lambda}$, and i otherwise.*
- *If the sequence is iii the optimal decision is i.*

Proof. The first part and the last part are obvious; the second is due to the fact that the second signal obtained by the manager is always more accurate. We know from proposition 1 that if $\lambda \leq 2p - 1$, it is always optimal to add a signal after *ii*; thus, if the third signal is *j* it is optimal to announce decision *j*. If $\lambda \leq \overline{\lambda}$, we know from proposition 1 that adding a third signal is always worthwhile; *i* must be implemented. ■

This first section has dealt with signal technology and has assumed that the acquisition of the third signal was costless. However, information acquisition here involves an incentive problem on the part of the manager.

We now turn to the case where the firm must induce the manager to collect information.

4.2 The firm incentive problem

In the next subsection, we describe how the presence of the consultant will affect the manager's incentives.

4.2.1 Advice, learning and incentives

When the firm designs the incentive scheme, it must take into account the fact that the manager learns about her own type after the first two signals. Learning about her type arises because the manager compares her own signal to the one reported to her by the consultant.

If the sequence of signal is *ii*, then using Baye's rule, the manager's private assessment of the probability that she is smart is[11]:

$$p\left(smart \mid ii\right) = \frac{\left(1+\lambda\right)\theta}{1+\lambda\theta} > \theta. \tag{6}$$

We will refer to this kind of manager as *the strong (ability) manager.*

On the other hand, if the signal is *ij*, straightforward Bayesian computation shows:

$$p\left(smart \mid ij\right) = \frac{\left(1-\lambda\right)\theta}{1-\lambda\theta} < \theta.$$

We will refer to this kind manager as *the weak (ability) manager.*

Hence, after she learns the content of the report, the manager has private information about her type, this private information influences the amount of effort the manager is willing to devote to information acquisition.

If there exists a consensus between the manager and the consultant, then the former has little incentive to acquire more information by expanding effort. The reason is that, since she knows that she already has a good information about the likely state of nature, obtaining an additional signal will only slightly improve the quality of her final decision. We expect, in this case, that the firm will find it costly to induce effort from her.

On the other hand, after a dissent, the manager has little information about the likely state of nature. The contradiction with the consultant informs her that the decision she is about to make is rather loose. This creates, in turn, incentives on the part of the manager to collect further information about the right decision. Thus, we can predict that the firm will obtain a high of effort much more easily in this case.

[11] More precisely, the expression (6) is obtained by reducing the following expression:

$$\frac{\theta\lambda + \frac{1}{2}\theta\left(1-\lambda\right)}{\theta\lambda + \frac{1}{2}\theta\left(1-\lambda\right) + \frac{1}{2}\lambda\left(1-\theta\right) + \frac{1}{2}\left(1-\theta\right)\left(1-\lambda\right)}.$$

The denominator of this expression entails the four possible combinations for which two identicals signals can be obtained: a smart manager and a smart consultant always obtain the same (right) signal; a smart manager can be matched with a dumb consultant who obtain (by chance) the right signal; the symmetric case and finally the case in which both agents are dumb. The numerator of this expression entails the cases in which the manager is smart.

Of course, in the no-consultant case, such a learning effect does not exist, and one of the main questions in the paper is the following: Can the firm ever benefit from this learning effect in the consultancy case?

The answer is a question of parameter of the model.

4.2.2 The program of the firm

Compared to the no-consultant case, the profit maximization program of the firm must now incorporate the interaction induced by the consultant. The form of the program of the firm depends on the strength of the incentives it wants to give to the manager. Indeed, we know that, depending on the nature of the information transmitted by the consultant, the manager's incentives to search for information differ. Thus, three types of contracts can be proposed to the manager to incite her to enhance her decision-making.

- In the *complete search contract (cs)* , the firm incites the manager to an informational search whatever the information he possesses. This is possible only if $\lambda < \underline{\lambda}$. Indeed, for $\lambda \geq \underline{\lambda}$, a manager who has two identical signals would never want to obtain a third signal, since according to proposition 1, her final decision would be independent of the third signal.
- In the *discriminating contract (dc)*, the firm incites the manager to enhance her decision-making only if she has received two different signals. According to proposition 1, this is possible if $\lambda < \overline{\lambda}$.
- In the *no-incentive contract (nic)*, the manager is never incited to perform an information search, whatever the nature of the information she holds.

Formally, we denote by $P_{I,iii}$ the joint probability of the events "the realized state of nature is I and the sequence of signals collected by the manager is three identical signals[12] i". According to corollary 1, we know that in this case the decision taken by the manager is a success[13]. Of course, the sequence iii can be either aaa or bbb and we have $P_{I,iii} = P_{A,aaa} + P_{B,bbb}$. Moreover, we denote by $P_{I|ii}$ the conditional probability of success of the manager knowing that her first opinion was i and that the advice received from the consultant was also i.

We now present, successively, the programs of the firm.

The Complete Search Contract. In the complete search contract, we denote by w_{cs}^{S} the salary obtained by the manager if her decision is a success and if a consultant has been hired. We denote by

[12] We use small letters for signals and capital letters for states of nature.

[13] We will not write the decisions that are made by the manager, but we know that these decisions follow the optimal decision rules given in corollary 1.

w_{cs}^F the salary in case of failure. The symmetric nature of the signal allows us to simplify the program, letting us denote by Υ the following set:

$$\Gamma = \left\{ \begin{array}{c} (I,J) \in (A,B)\,, \;\; (i,j) \in (a,b)\,,\, i \neq j, \text{ if } I\,(\text{resp } J) = A \text{ then } i\,(\text{resp } j) = a \\ \text{and if } I\,(\text{resp } J) = B \text{ then } i\,(\text{resp } j) = b \end{array} \right\}$$

The program of the firm is thus

$$\underset{(w_{cs}^S, w_{cs}^F)}{Max} \sum_{\Gamma} \left[(p_{I,iii} + p_{J,iij} + p_{I,iji} + p_{J,ijj}) \left(G - w_{cs}^S\right) - (p_{J,iii} + p_{I,iij} + p_{J,iji} + p_{I,ijj}) \left(L + w_{cs}^F\right) \right] \quad (7)$$

subject to the following constraints:

$$p_{iii|ii} \left(p_{I|iii} w_{cs}^S + p_{J|iii} w_{cs}^F\right) + p_{iij|ii} \left(p_{J|iij} w_{cs}^S + p_{I|iij} w_{cs}^F\right) - 1 \;\; \geq \;\; p_{I|ii} w_{cs}^S + p_{J|ii} w_{cs}^F$$

(Effort of the strong manager)

(8)

$$p_{iji|ij} \left(p_{I|iji} w_{cs}^S + p_{J|iji} w_{cs}^F\right) + p_{ijj|ij} \left(p_{J|ijj} w_{cs}^S + p_{I|ijj} w_{cs}^F\right) - 1 \;\; \geq \;\; p_{I|ij} w_{cs}^S + p_{J|ij} w_{cs}^F$$

(Effort of the weak manager when $\theta \geq \lambda$)

(9)

$$p_{iji|ij} \left(p_{I|iji} w_{cs}^S + p_{J|iji} w_{cs}^F\right) + p_{ijj|ij} \left(p_{J|ijj} \; w_{cs}^S + p_{I|ijj} w_{cs}^F\right) - 1 \;\; \geq \;\; p_{J|ij} w_{cs}^S + p_{I|ij} w_{cs}^F$$

(Effort of the weak manager when $\theta < \lambda$)

(10)

$$w_{cs}^S \geq 0 \;\text{ and } w_{cs}^F \geq 0 \quad \text{(Limited liability Constraints)} \quad (11)$$

The first part of the objective function of the program states that, given the optimal decision rules, the manager can make the right decision after each of the four possible sequences of signal, namely iii, iij, iji *and* ijj. In such a case, a positive benefit G accrues to the firm and the manager is paid accordingly. The second part of the objective function states that the manager can fail to pick the right decision, and in such a case the firm shows a loss L, whereas a salary contingent w_{1c}^F is paid to the manager.

The incentive constraint (8) states that the strong manager is induced to obtain an additional signal rather than to content herself with two signals.

The incentive constraint of the weak manager can take two forms, depending on the compared expertise of the manager and the consultant. In the right hand side of the constraint (9), if the manager decides to shirk (i.e., not to acquire an additional signal) she will follow her own first signal rather than the one reported by the consultant. The reason for this is that the manager has a stronger precision in the prediction of the state of nature. On the other hand, if the consultant is stronger than her, then she will use the consultant's signal to make her decision. The rational manager always follows the signal reported by the consultant if the latter has a stronger expertise.

In the next lemma, we give the optimal wage scheme that the firm should offer to the manager to induce a complete search.

Lemma 2 The optimal wage scheme is given by $w_{cs}^{S*} = \frac{2(1+\lambda\theta)}{(1-\theta)(2p-1-\lambda)}$ *and* w_{cs}^{F*} *=0.*

Moreover, for this optimal wage scheme the payoff of the firm is

$$B_{cs} = [\theta + p(1-\theta)] \left(G + L - \frac{2(1+\lambda\theta)}{(1-\theta)(2p-1-\lambda)} \right) - L \tag{12}$$

Proof. One incentive requirement of this problem is $w_{cs}^{S} \geq w_{cs}^{F}$. The problem of the firm is to minimize its wage bill. However, it is constrained by the wealth constraint, which states that $w_{cs}^{F} \geq 0$. Thus the firm should optimally set $w_{cs}^{F*} = 0$ to meet the wealth constraint. The salary w_{cs}^{S} must be adjusted so as to meet the three incentive constraints of the manager. Taking into account the fact that we have $w_{cs}^{F*} = 0$, and using the conditional probabilities computed in the appendix, we can rewrite the three incentive constraints as:

$$w_{cs}^{S} \geq \frac{2(1+\lambda\theta)}{(1-\theta)(2p-1-\lambda)}$$

for the strong manager,

$$w_{cs}^{S} \geq \frac{2(1-\lambda\theta)}{(1-\theta)(2p-1+\lambda)}$$

for the weak manager and $\theta \geq \lambda$ and

$$w_{cs}^{S} \geq \frac{2(1-\lambda\theta)}{2p(1-\theta)+3\theta-\lambda(1+\theta)}$$

for the weak manager and $\lambda < \theta$.

The firm wants to set w_{cs}^{S} as low as possible and it is routine to check that the first binding constraint is (as usual) the incentive constraint of the high type. In expression (12), the probability of success is $[\theta + p(1-\theta)]$ and it is obtained by doing the following reasoning. According to proposition 1, we know that when $\lambda < \underline{\lambda}$, the manager follows the third signal. The probability of success is, thus, the probability of success of the manager of expertise level θ who follow her second signal; that is, with probability θ she is smart and obtains the right signal whereas with probability $1-\theta$, she is dumb and she obtains the good signal with probability p. ■

We now give two results that characterize the complete search contract.

Lemma 3 *At the optimum, the complete search contract B_{cs} is a decreasing function of the consultant's level of expertise, λ, hired by the firm.*

Proof.

$$\frac{\partial B_{cs}^{*}}{\partial \lambda} = -2\frac{1+\theta-2\theta p}{(1-\theta)(2p-1-\lambda)^{2}} < 0. \quad \blacksquare$$

Corollary 2 *The optimal complete search contract is always dominated by a situation in which the firm does not hire any consultant.*

Proof. If we make use of expressions (4) and (12), we obtain

$$\Delta B = B_{nc} - B_{cs} = \frac{2\lambda\left(\theta + p\left(1-\theta\right)\right)\left(1+\theta\left(2p-1\right)\right)}{\left(1-\theta\right)\left(2p-1\right)\left(2p-1-\lambda\right)} > 0$$

which is true for any $\lambda < \underline{\lambda}$. $\blacksquare$

Some points deserve to be stressed, as these comparative statics results may appear to be counterintuitive. These results have two causes.

First, the information brought by the consultant is not utilized by the manager, and from the point of view of the firm is completely useless. Moreover, its effect on the incentive scheme is negative. A manager who observes two identical signals, privately assess that her level of expertise is $\widehat{\theta} = \frac{(1+\lambda)\theta}{1+\lambda\theta} > \theta$. When λ rises, her private assessment increases and the marginal productivity of her search effort diminishes, because the third signal is more likely to be redundant (compared to the first two). Thus, giving incentives becomes more costly to the firm.

INSERT FIGURE 3 HERE

We are aware that this first analysis does not speak in favor of consultancy, as it states that an increase in the consultant's expertise for small values of λ strictly lowers the firm's benefit (net of the consultant fees).

The analysis of the two next programs balances this first (negative) conclusion and shows that consultancy may benefit the firm if the consultant is carefully chosen.

The Discriminating Contract. In this case, the firm does not incite the manager to deepen his decision-making if the first two signals are identical. As a consequence, in this program the incentive constraint of the strong manager (8) is removed compared to the complete search case. We denote by w_{dc}^{S} the wage given to the manager if the decision is a success and by w_{dc}^{F} the wage given if the decision is a failure. In the discriminating case, the firm's program can be written as:

$$\max_{\left(w_{dck}^{S}, w_{dck}^{E}\right)} \sum_{\Gamma} \left[\left(p_{I,ii} + p_{I,iji} + p_{J,ijj}\right)\left(G - w_{dck}^{S}\right) - \left(p_{J,ii} + p_{J,iji} + p_{I,ijj}\right)\left(L + w_{dck}^{F}\right)\right], \; k \in \{1,2\} \qquad (13)$$

subject to constraints (9), (10) and

$$w_{dck}^{S} \geq 0, \qquad w_{dck}^{F} \geq 0\ ,\ \ k \in \{1,2\} \quad \text{(limited liability).} \tag{14}$$

In the firm's objective function, the probability of success is formally written as $p_{I,ii} + p_{I,iji} + p_{J,ijj}$ because the strong manager does not find it worthwhile to perform an information search. The incentive constraint of the manager who has received two different signals remains the same as the one in the complete search program. The solution to this program is given below:

Lemma 4 (Optimal incentives with the discriminating contract) . *Depending on the comparative level of expertise of the manager and the consultant, there exist two optimal wage scheme solutions to the program of the firm:*

- *If* $\theta \geq \lambda$, $w_{dc1}^{S*} = \frac{2(1-\lambda\theta)}{(1-\theta)(2p-1+\lambda)}$ *and* $w_{dc1}^{F*} = 0$. *At this optimum, the benefit to the firm is given by:*

$$B_{dc1} = \tfrac{1}{4}\left[1 + 3\theta + (1-\theta)(2p+\lambda)\right]\left(G + L - \tfrac{1-\lambda\theta}{2(1-\theta)(2p-1+\lambda)}\right) - L.$$

- *If* $\theta < \lambda$, $w_{dc2}^{S*} = \frac{2(1-\lambda\theta)}{2p(1-\theta)+3\theta-1-\lambda(1+\theta)}$ *and* $w_{dc2}^{F*} = 0$. *At this optimum, the benefit to the firm is given by:*

$$B_{dc2} = \tfrac{1}{4}\left[1 + 3\theta + (1-\theta)(2p+\lambda)\right]\left(G + L - \tfrac{1-\lambda\theta}{2[2p(1-\theta)+3\theta-1-\lambda(1+\theta)]}\right) - L. \tag{15}$$

Proof. In the proof of lemma 3, a reduced form of the incentive constraints of the weak manager were already obtained. We know that these constraints are mutually exclusive; that is, if $\theta \geq \lambda$, the firm should only take into account the corresponding constraint and adjust w_{dc1}^{S} so as to make it bind.

The probability of success of the manager who receives two identical signals is computed as follows:

$$P_{I|ii} = \frac{(1+\lambda)(1+\theta)}{2(1+\lambda\theta)}. \tag{16}$$

When the manager receives two different signals, she is induced to obtain a third signal, thus her probability of success is given by

$$b = P_{i|ij} \times P_{I|iji} + P_{j|ij} \times P_{J|ijj} = \frac{\theta + p - p\theta - \lambda\theta}{1 - \lambda\theta}. \tag{17}$$

Using expressions (16) and (17), the ex ante probability of success is given by:

$$P_S = P_{ii} \times P_{I|ii} + P_{ij} \times b = \tfrac{1}{4}\left[1 + 3\theta + (1-\theta)(2p+\lambda)\right]. \ \blacksquare$$

If we denote by $B_{dc}(\lambda)$ the benefit of the firm as a function of λ, this function is defined by

$$B_{dc}(\lambda) = \begin{cases} B_{dc1}(\lambda) & \text{if } \lambda \leq \theta \\ B_{dc2}(\lambda) & \text{if } \lambda > \theta \end{cases}. \tag{18}$$

We must emphasize the results of lemma 4. Let us first focus on $B_{dc}(\lambda)$, when the manager has greater expertise. In this case, we notice that

$$\frac{\partial w_{dc1}^{S*}}{\partial \lambda} = -2\frac{2\theta p - \theta + 1}{(1-\theta)(2p-1+\lambda)^2} < 0 \ .$$

Thus, increasing the expertise level of the consultant allows the firm to decrease the salary offered to the manager. In the discriminating contract, the firm can lower the agency cost by increasing the reputation of the consultant. Thus, the consultant's informativeness and managerial effort in decision-making are complementary[14]. Moreover, hiring a consultant has a simple informational effect. Indeed an inspection of B_{dc} shows that an increase in the consultant's expertise increases the probability of a successful decision; this effect is not present in the complete search contract, where we show that the probability of good decision is independent of consultant expertise. In the discriminating case, the consultant plays his expected role and strictly enhances the manager's decision-making.

The following lemma summarizes these findings.

Lemma 5 *When $\lambda \leq \theta$, the firm's benefit when the discriminating contract is implemented is an increasing function of the expertise of the consultant λ.*

Proof. $\frac{\partial B_{dc1}}{\partial \lambda} = \frac{1}{4}(1-\theta)(G+L) + \frac{8\theta^2 p + (1-\theta)\left(3\theta + \lambda^2\theta + 4\theta p^2 - 2\lambda\theta + 4\lambda\theta p + 2\right)}{8(1-\theta)(2p-1+\lambda)^2} > 0.$ ■

It is important to notice that even if the expertise of the consultant is strictly smaller than the manager's level of expertise, the probability of success is an increasing function of the consultant's expertise. Thus, the manager's learning process of about her type, described before, allows us to modelize a fruitful interaction between two heterogenous agents who confront their points of view, and enhance decision-making with this exchange.

When $\theta < \lambda$, we have

$$\frac{\partial w_{dc2}^{S*}}{\partial \lambda} = 2\frac{(1-\theta)(1+3\theta-2\theta p)}{(-2p+2\theta p-3\theta+1+\lambda+\lambda\theta)^2} > 0 \ ,$$

and in this case the positive incentive effect of the consultant has disappeared. Now, if the firm wants to increase the level of expertise of the consultant, it must also increase the salary offered to the manager if it still wants to incite her to exert high effort. However, the informational effect remains; that is,

[14] This incentive effect is not due to the signal technology that assumed in figure 1. We could develop a variant of this model in which the signals are continuous and the true state of nature is a real unkown number μ. The manager would receive a signal drawn from a continuous (e.g. normal) distribution with mean μ but with a variance that would be a function of her type; a smart manager would obtain a signal from a distribution with precision strictly greater than a dumb one. The euclidian distance between the signal of the manager and the signal of the consultant would provide some information on the manager's type and would affect her incentive for decison-making. In such a variant, the firm's problem would be to determine the distance above which it would incite the manager to search for information.

increasing the consultant's expertise increases the probability of a right decision. Hence, when $\lambda > \theta$, increasing λ has an ambiguous effect on the firm's profit.

According to proposition 1, we know that it is not possible to induce an information search when $\lambda > \overline{\lambda} = \frac{2p-1+3\theta-2p\theta}{1+\theta} > \theta$. If we consider $w_{dc2}^{S\,*}$ it is obvious that when λ approaches $\overline{\lambda}$ this salary becomes infinite and the firm's benefit will eventually become (infinitely) negative. Thus, starting from a level $\lambda = \overline{\lambda}$, slightly decreasing λ would increase the firm's benefit. However, when $\lambda = \theta$, decreasing λ is never profitable, according to the previous lemma. What we just said suggests the following result:

Proposition 2 (Optimal informativeness of advice) . *If one denotes by B_{dc} the optimal benefit curve in the discriminating contract, then there exists a unique level of consultant expertise λ^* such that the firm benefit is maximized. λ^* is strictly greater than θ and it is given explicitly by*

$$\lambda^* = \overline{\lambda} - \frac{\sqrt{8(1+3\theta-2\theta p)(2\theta+p-\theta p)}}{(1+\theta)\sqrt{((G+L)(1+\theta)-2\theta)}} \ .$$

Proof. The proof can be found in the appendix. ■

This result, about the optimality of the team formed by the manager and the consultant, is that given the managerial's prior expertise and the fact that the firm wants her to participate in the decision process, the level of expertise of the consultant chosen to advise the manager should be equal to λ^*[15]. The optimal level of informativeness of the consultant is shown to be strictly greater than the level of the manager. This level λ^* trades off the need for incentives provision and the informativeness of the advice. When $G + L$ (a rough measure of the variance) increases, the optimal value λ^* increases and get closer from $\overline{\lambda}$; that is accepting project at the first round (i.e., after a sequence *ii*) becomes more costly and the firm is ready to pay higher salaries to the manager. Indeed when λ^* is increased, the manager is less prone to search for new information and is willing to content herself with the information delivered by the consultant. The exact relationship between the level of effort of the manager and the informativeness of the consultant is derived in the last section where we introduce continuous level of effort (see lemma 8).

[15] In an old version of this paper (Marcoul (1999)), we investigate the robustness of the conclusions reached in the case of symmetric payoffs in each state of nature in a situation in which the manager can either implement a risky decision for the firm or implement the status quo that has no financial consequence for the firm. We show that the optimal incentive scheme is identical to the one derived by Snyder and Levitt (1997). Indeed, when the manager receives "bad news" about the decision, the firm would like her to stop the implementation, and provides her with incentives to report the bad news. It turns out that the properties about the value of advice derived in the case of symmetric state of nature payoffs remain true when states of nature no longer have symmetric payoffs. In particular, the shape of the curve is preserved. These results are available upon request.

It is important to note that the consultant that would be chosen by the firm is not the one that the manager would pick. Indeed if we denote the manager's (equilibrium) utility by U_m^*, it can be written as

$$U_m^* = P_S w_{dc}^{S*} - 1.$$

When $\lambda \geq \theta$, the derivative of U_m^* with respect to the consultant's reputation λ is

$$\frac{\partial U_m^*}{\partial \lambda} = \frac{1}{8} \frac{(1-\theta)\left(9\theta^2(1-p)+\lambda^2\theta^2+4p^2\theta^2+4p((1-\theta)(1-\lambda\theta)+\theta(1-p))+2\lambda\theta+3\theta(1-p\theta)+6\theta(1-\lambda\theta)+\lambda^2\theta\right)}{(2p-2p\theta+3\theta-1-\lambda-\lambda\theta)^2} > 0.$$

The manager would thus choose a stronger consultant than the one the firm would like to give him. Interestingly, the manager would not choose a consultant whose level is greater than $\overline{\lambda}$ because in this case decision-making is completely externalized and the manager plays no role in it and he would loose any rent associated with information gathering. The feature of the model may help us to understand why some corporations have set up a severe control on consultant's choice and expenses[16]. Without anticipating the results of proposition 4, the manager would like to choose the consultant in the set Υ. This set is one in which an increase in the advisor's expertise leads to a lower profit for the firm.

Figure 3 represents the benefit curve of the firm as a function of λ in the discriminating case. B_{dc} is a continuous curve because $B_{dc1}(.)$ and $B_{dc2}(.)$ do cross for $\lambda = \theta$. The curve is not defined for values of λ that are greater than $\overline{\lambda}$, this is the result of proposition 1. We denote by B_{nc} the case where no consultant is hired, figure 3 shows clearly that for some values of λ (including λ^*) the firm should hire a consultant. This result may be surprising because the firm does not find it profitable to increase the consultant's expertise after the level λ^*.

INSERT FIGURE 4 HERE

The no-incentive contract. We now proceed to the analysis of the last case. This case is more simple than the previous ones, since neither kind of manager is induced to collect a third signal[17]. The manager must then be given, in any state of nature, her reservation wage normalized to 0. When $\theta < \lambda$, the manager just transmits the signal reported to her by the consultant. Otherwise when $\theta \geq \lambda$, the manager just transmits her own signal, as it is stronger than the consultant's. Therefore, the benefit curve of the firm is simply given by

$$B_{nic} = \begin{cases} \left(\lambda + \frac{1}{2}(1-\lambda)\right) G - \frac{1}{2}(1-\lambda) L \text{ if } \theta < \lambda \\ \left(\theta + \frac{1}{2}(1-\theta)\right) G - \frac{1}{2}(1-\theta) L \text{ if } \theta \geq \lambda \end{cases}.$$

[16] The reader can refer to the book of O'Shea and Madigan (1997) to find striking examples of consultant's overuse by managers (and the response of the corporation).

[17] This case would be equivalent to the case in which the manager is totally ignorant.

In figure 4, we have represented the benefit function of the firm in the case where the manager is not induced to obtain a third signal. Therefore, with two signals the manager follows her own signal when her ability is stronger than the consultant's (i.e., $\theta \geq \lambda$) and the consultant piece of advice otherwise. Thus, the benefit curve depends on λ only for $\theta < \lambda$.

INSERT FIGURE 5 HERE

4.3 Optimality of the consultant

After the analysis of the different programs of the firm, we now look for the properties of the benefit function. We want to determine the optimal level of the consultant's expertise, knowing that the consultant has an impact on the manager's incentives. We restrain the analysis to the cases in which the manager is induced to exert a positive effort ex ante.

Proposition 3 *In the consultancy framework, when the firm wants the manager to participate in decision-making (i.e., search for information with a positive probability), the optimal consultant has always a level of expertise equal to* λ^*.

Proof. The proof can be found in the appendix. ■

The case where the consultant has an ability equal to 0 has a straightforward interpretation, because such a consultant reports a perfectly random signal independent of the real state of nature. This signal is not taken into account by the manager; this case is perfectly equivalent to a situation in which there is no consultant. The proof shows that the benefit derived in (4) is always strictly smaller than the benefit derived with λ^* whenever the condition in lemma 1 is satisfied. There is thus room for a positive consulting fee[18]. Therefore, given that the manager brings in her knowledge into the decision-making process, there exists a unique level of "external information" that maximizes the firm's benefit. This level of external information takes into account the incentive scheme of the manager. The next result presents a simple condition under which (free) advice will decrease the firm's profit.

Proposition 4 (Negative value of increased infomativeness of advice) *In the consultancy framework, if the following sufficient condition is realized:*

$$c = G + L > \bar{c},$$

then for $\lambda = \lambda^*$, *the firm derives strictly more profit when it induces the manager to exert effort than when the consultant makes the decision alone. Moreover, there exists a non-empty set of the consultants'*

[18] It is straightforward to assume that the fee is an increasing function of the level of λ (with possibly infinite fees when it becomes perfect, i.e. $\lambda = 1$). It would still give a unique level $0 \leq \lambda^* < 1$ such that the firm's profit is maximized.

informativeness $\Upsilon =]\lambda^*, \lambda_{\sup}[$ *such that every consultant picked in* Υ *will bring a strictly lower profit to the firm than* $B_{dc}(\lambda^*)$. *This set is described by* $\Upsilon =]\lambda^*, \lambda_{\sup}[$ *where* $\lambda_{\sup}$ *is defined as*

$$B_{dc2}(\lambda^*) = \frac{1}{2}(1 + \lambda_{\sup})\, c.$$

This result completes the picture described in proposition 3. For projects with a magnitude c greater than $\overline{c}$, when $\lambda = \lambda^*$, the firm prefers to involve the manager into the decision-making process rather than to let the consultant decide alone. Interestingly, increasing the informativeness of the consultant above λ^* will strictly decrease the firm's profit whenever $c = G + L > \overline{c}$. Thus, the decision-making process can be made strictly less efficient with stronger advice. Having the consultant decide alone is only worthwhile if his level is increased above $\lambda_{\sup}$.

It seems worth noticing that the set Υ belongs to the set of consultant that are more informative than the manager ex ante (because $\lambda^* > \theta$). It emphasizes the potential harm of choosing a consultant level of informativeness that does not fit well with the manager effort, and talent. We believe that this result sheds light on why it is practically a problem to manage "external information" and managerial incentives[19].

After the analysis of each of the firm's programs , we wonder what is the exact shape of the benefit curve of the firm as a function of the consultant ability λ. This curve which we denote B_c is defined by

$$B_c = \sup(B_{cs}, B_{dc}, B_{nic}).$$

Figure 6 represents B_c in the case where hiring a consultant is profitable; that is, hiring a consultant whose ability is near λ^* generates a greater gross benefit than without a consultant. To know whether a consultant actually will be hired one must take into account the compensation that is given to the consultant. This question depends on the consultancy market conditions, and is out of the scope of this paper.

INSERT FIGURE 6 HERE

We now develop a variant of this model in which we introduce some correlation between the signals obtained by weak agents. We analyze the consequences for the discriminatory contract.

4.4 Correlated signals: Is "external information" superior ?

Up until now, we have described a situation in which the signals received by the consultant and the manager are either perfectly correlated - when agents are both smart, they both receive the right signal -

[19] The recent managerial literature tends to emphasize the client's involvment in the mission. On the client role, see for instance Hislop (2002).

or completely independent - when at least one of the agent is mediocre, the signals received are completely independent. This last assumption may be restrictive.

Indeed, in most organizations the necessity to rationalize production processes leads to a standardization of the practices used for dealing with problems. For instance, when time is of essence, common language between employees helps the organization to react more rapidly to a change in the competitive environment. This common way of thinking is often referred to as *corporate culture* by social scientists, and the economic aspects of corporate culture have been studied by several economists[20]. Most of the time, corporate culture must lead to a common way of analyzing problems within an organization; employees analyze the problem in the same manner simply because they are used to doing so[21]. As a result the solution that are adopted tend to be similar and mistakes also tend to be correlated.

The purpose of this section is to address the importance of correlated mistakes in the decision-making process of the manager. To modelize correlation, we make the following assumption.

Assumption: *When the manager and the consultant are both mediocre, the signals that they receive are identical, with a probability* $\mu \in \left(\frac{1}{2}, 1\right)$.

Since we know that the complete search contract is dominated by hiring a consultant of level λ^*, we focus on the discriminatory contract. The objective function of the firm is identical to (13). The constraints are (9), (10) and the limited liability constraints. Using, the probabilities of success (when $\mu \geq \frac{1}{2}$) computed in the table 1, we easily obtain the optimal salaries offered to the manager in the discriminating contract:

$$w_{dc1}^{S*} = \frac{2-\theta-\lambda-2\mu(1-\theta)(1-\lambda)}{(1-\theta)(2\mu\lambda p-1+\lambda+2p-p\lambda-2p\mu+\mu-\mu\lambda)} \text{ and } w_{dc1}^{F*} = 0 \text{ if } \theta \geq \lambda,$$

and

$$w_{dc2}^{S} = \frac{2-\theta-\lambda-2\mu(1-\lambda)(1-\theta)}{\theta+(\mu\lambda+1-\mu)(2p-1)(1-\theta)-p\lambda-\theta\lambda(1-p)} \text{and } w_{dc2}^{F*} = 0 \text{ if } \lambda > \theta.$$

The probability of success can be computed as

$$P_S = \frac{1}{2}(1-\theta)\left((2p+\lambda)(1-\mu) + 2\mu\lambda p + \mu - \lambda p\right) + \theta.$$

If $\theta \geq \lambda$, the benefit of the firm is given by

[20] Crémer (1993) emphasizes the fact that organizations do develop among their employees a shared knowledge of the norms of behavior. For a survey of the literature on the economics of corporate culture, see Hermalin (2000).

[21] The economic literature on strategic communication has emphasized the prevalence of herding in communications made by experts. Here, the fact that people have the same point of view is just the result of their standardized way of thinking (the consultant may not observe the manager's opinion, for instance). In the conclusion, we discuss for possibility of strategic communication by the consultant.

$$B^*_{dc1} = P_S\left(G + L - \frac{(2-\theta-\lambda-2\mu(1-\lambda)(1-\theta))}{(1-\theta)(\lambda(1-p)+(2p-1)(1-\mu(1-\lambda)))}\right) - L$$

or, otherwise, by

$$B^*_{dc2} = P_S\left(G + L - \frac{2-\theta-\lambda-2\mu(1-\lambda)(1-\theta)}{\theta+(\mu\lambda+1-\mu)(2p-1)(1-\theta)-p\lambda-\theta\lambda(1-p)}\right) - L.$$

The next result analyze the effect of changes in μ on the equilibrium profit.

Proposition 5 (Benefit from less correlation in mistakes) *When $\lambda \geq \theta$, an increase in the mistake correlation factor μ always leads to a smaller benefit in the discriminating contract if $\lambda \in [\theta, \lambda_0]$ and $c\,(= G + L) \geq c_0$, or if $\lambda \in \left[\lambda_0, \overline{\lambda}\,(\mu)\right]$. This level of c_0 is given by*

$$\frac{1}{P_S} \cdot \frac{\partial P_S}{\partial \mu}\left(c_0 - w^S\right) = \frac{\partial w^S}{\partial \mu}.$$

Proof. The proof can be found in the appendix. ■

This result is due to the nature of the discriminating contract. When μ is very high, dumb decision-makers tend to share the same point of view and a lot of wrong decisions are made. Basically, a decrease in μ leads to an increase in the probability of dissent between the manager and the consultant. Those decisions are worth being reevaluated by the manager.

The economic literature on experts relies essentially on a one-shot transmission of information, in which the value of advice is summarized by the ability of the expert; namely, his expertise.

This result is interesting because it introduces, besides pure expertise (i.e., the probability of being right when advising), another component of what makes advice valuable. Indeed, the consultant is valuable not only because of his level of expertise, θ, but also because of the way he is mistaken.

If we accept the fact that mistakes tend to be more correlated within than across organizations, then for equal ability, an "external consultant" is expected to be of greater value for the firm than an internal one[22]. The external consultant has more value because his confrontation with the manager generates more incentives on the part of the manager; the firm can then use the consultant as an incentive device.

We see that, to the extend to which hiring an external consultant is observable by a third party (e.g., an investor), the firm may use this action to credibly commit to devoting more emphasis (than usual) on a given project. This result can help us to understand why, to some extent, external consulting works because the firm can always replicate the advisory process internally.

[22] For instance, we could assume that an internal consultant has μ_i, whereas an external one is characterized by μ_{ext} with $\mu_i > \mu_{ext}$.

5 Continuous effort levels

We now develop a version of the present model with continuous effort levels. The aim of this section is to test the robustness of some of the results found previously. In particular, the fact that the value of information may strictly decrease when the advice is more accurate seems more questionable with continuous effort levels. We found, however, that it is still true with continuous effort levels. Moreover, allowing for continuous effort makes it possible to describe how the manager's incentives should be adjusted when the informativeness of the advice is changed.

5.1 The model with continuous effort levels

We assume that the model remains the same up to the point where the manager has to perform effort to obtain a third signal. To modelize continuous effort, we slightly modify the information research of the manager.

Let $p \in (0, 1)$ be the probability that the manager obtains the right signal. We assume that with the complementary probability, she obtains a signal identical to her first signal she had. This assumption implies that with any strictly positive level of effort, the manager is able to generate a new signal (or new opinion) that "beats" her original signal.

The manager can generate p only by expanding effort. Let $C(p)$ represent the monetary cost supported by the manager if she wants to generate p. We assume that this cost function verifies the following usual properties:

Assumption 1: $C(.)$ is positive and its first two derivatives are positive. Moreover, it verifies $C(0) = 0$, $C(1) = +\infty$, $C'(0) = 0$ and $C'(1) = +\infty$.

This set of conditions is imposed to guarantee an interior solution to the problem of the choice of effort level of the manager.

Although effort is a continuous variable, the question remains whether the manager should acquire a third signal. From corollary 2, we know that inducing her to obtain a signal when she already possesses two identical signals is not optimal for the firm. Thus, we concentrate only on the discriminating contract; i.e., the manager is induced to a search only if she has been contradicted. The firm will propose a pair of salaries (w^S, w^F) contingent upon the outcome.

The program Ω of the firm can be stated as:

$$\underset{\{p, w^S, w^F\}}{Max} \; P_s \left[G - w^S\right] - (1 - P_s)\left[L + w^F\right] \qquad (\Omega)$$

subject to

$$p \in \arg\max_{\widetilde{p}\in[0,1]} P_{S/ij,\widetilde{p}} w^S + \left(1 - P_{S/ij,\widetilde{p}}\right) w^F - C\left(\widetilde{p}\right) \tag{IC1}$$

$$P_{S/ij,p} w^S + \left(1 - P_{S/ij,p}\right) w^F - C\left(p\right) \geq P_{S/ij} w^S + P_{F/ij} w^F \tag{IA1}$$

$$w^S \geq 0 \text{ and } w^F \geq 0 \tag{LL}$$

The incentive constraint $(IC1)$ ensures that it is optimal for a manager to choose $\widetilde{p} = p$ voluntarily. The information acquisition constraint $(IA1)$ states that the decision-maker must be induced to search for information when she has been contradicted by the consultant. The quality of the third signal acquired by the decision-maker makes her better-off than if she was to use her prior opinion and the information provided by the consultant only. As before, two cases should be considered, depending on the relative strength of the consultant and the manager.

If $\theta \leq \lambda$, then when only two signals are used, the probability of success is the probability that the right decision is the one proned by the consultant. $P_{S/ij}$ (or equivalently $P_{J/ij}$) can be written as

$$P_{S/ij} = \frac{(1-\theta)(1+\lambda)}{2(1-\theta\lambda)};$$

it is an increasing function of the consultant's ability and a decreasing function of the manager's ability.

Conversely, if $\theta > \lambda$ then when only two signals are used, the probability of success is the probability that the right decision is the one defended by the manager. $P_{S/ij}$ (or equivalently $P_{I/ij}$) can be written as

$$P_{S/ij} = \frac{(1+\theta)(1-\lambda)}{2(1-\theta\lambda)}, \tag{19}$$

a decreasing function of the consultant's ability and an increasing function of the manager's ability.

$P_{S/ij,p}$ is the probability of right decision when the manager follows her own signal after exerting an effort p. Some computations give an analytical expression of $P_{S/ij,p}$. We obtain:

$$P_{S/ij,p} = \frac{(1+\theta)(1-\lambda) + (1-\theta)(1+\lambda)\,p}{2(1-\theta\lambda)}. \tag{20}$$

This expression is identical to (19) when $p = 0$.

We know that any solution to the principal's problem must be such that[23] $w^{S*} > w^{F*} = 0$. Given the assumptions made about the cost of effort, the decision-maker's payoff is strictly concave in p. Her effort choice problem has, thus, an interior solution, and we can replace (IC 1) by the first order condition:

$$w^S = \frac{2(1-\theta\lambda)}{(1-\theta)(1+\lambda)} C'\left(p\right). \tag{IC'1}$$

[23] This assertion will not be proven in the paper, since this is a standard result of contract theory. However, a proof is available upon request.

That is, the firm must offer a wage equal to $\frac{2(1-\theta\lambda)}{(1-\theta)(1+\lambda)}C^{'}(p)$ if it wants to obtain a level of effort p from a manager who has received a different signal from the consultant. By placing $(IC^{'}1)$ into $(IA1)$, we obtain the modified information acquisition constraint in the two cases. For $\theta \leq \lambda$, we obtain

$$\frac{(1-\lambda)(1+\theta)-(1-\theta)(1+\lambda)(1-p)}{(1-\theta)(1+\lambda)}C^{'}(p) \geq C(p), \qquad (IA^{'}1)$$

whereas for $\theta \geq \lambda$, we have

$$pC^{'}(p) \geq C(p). \qquad (IA^{'}2)$$

We notice that $(IA^{'}2)$ is never binding, since $C(0)=0$ and $C^{''}(.)>0$. The intuition behind this is: a manager exerting an arbitrarily small effort would always find it optimal to use the new signal generated by this effort. Indeed, this signal would strictly dominate her own prior signal. When $\lambda > \theta$, this is no longer true, since the strength of the new signal acquired by the manager should be compared to the one brought by the consultant. Thus, a minimal effort is required of the manager if she wants to "outperform" the consultant's signal.

Using $(IA^{'}1)$ and computing P_s, we can rewrite the simplified program $\Omega^{'}$ equivalent to Ω as

$$\underset{p}{Max}\ \tfrac{1}{4}\left(2(1+\theta)+(1-\theta)(1+\lambda)p\right)\left(G+L-\tfrac{2(1-\theta\lambda)}{(1-\theta)(1+\lambda)}C^{'}(p)\right)-L \qquad (\Omega^{'})$$

subject to

$$\tfrac{(1-\lambda)(1+\theta)-(1-\theta)(1+\lambda)(1-p)}{(1-\theta)(1+\lambda)}C^{'}(p) \geq C(p). \qquad (IA^{'}1)$$

For expositional convenience, we make the following assumption:

Assumption 2:

$$C^{'''}(p) > -\tfrac{2(1-\theta)(1+\lambda)}{2(1+\theta)+(1-\theta)(1+\lambda)p}C^{''}(p).$$

The assumption made above guarantees that the firm's optimization problem is globally concave and has a unique solution[24]. Before going further, we give the following lemma that will help us to state the solution to $\Omega^{'}$.

Lemma 6 *When $\lambda \geq \theta$ there exists a unique a level of consultant's ability λ_b such that the information acquisition constraint $(IA^{'}1)$ binds when $\lambda \geq \lambda_b$ and may not bind when $\lambda < \lambda_b$.*

Proof. The proof can be found in the appendix. ■

The next proposition gives a solution to the simplified program $\Omega^{'}$ for all possible values of θ and λ.

[24]Note that the assumption is implied by $C^{'''}(p) \geq 0$.

Proposition 6 (Optimal incentives with continuous effort) *In the case $\theta > \lambda$, the unique optimal contract solving Ω' implements $p_{opt} = p^*$. This level of effort p^* is characterized by*

$$(1-\theta)(G+L) = \tfrac{2(1-\theta\lambda)}{1+\lambda}\left[C'(p) + \left(p + \tfrac{2(1+\theta)}{(1-\theta)(1+\lambda)}\right)C''(p)\right]. \quad (21)$$

In the case $\theta < \lambda_b \leq \lambda$, the unique optimal contract solving Ω' implements $p_{opt} = \overline{p}$. This level of effort is characterized by

$$\tfrac{(1-\lambda)(1+\theta)-(1-\theta)(1+\lambda)(1-\overline{p})}{(1-\theta)(1+\lambda)}C'(\overline{p}) = C(\overline{p}). \quad (22)$$

In the case $\theta \leq \lambda < \lambda_b < 1$, the optimal contract solving Ω' implements $p_{opt} = \max\{p^*, \overline{p}\}$. The optimal salaries contingent upon the outcome are $w^{S*} = \frac{2(1-\theta\lambda)}{(1-\theta)(1+\lambda)}C'(p_{opt})$ and $w^{F*} = 0$.

Proof. The proof is immediate. When the acquisition constraint is not binding, the first order condition of (Ω') gives (21). Assumption 2 makes (21) sufficient. When the acquisition constraint is binding for sure, (22) gives the effort level $\overline{p}$. When the acquisition constraint may be binding, the optimal will be given by the maximum of these two levels, since we want effort to be exerted. ■

Corollary 3 *The equilibrium profit function generated by the optimal contract characterized above admits a unique, optimal level λ^* of consultant informativeness.*

Proof. The proof can be found in the appendix. ■

This corollary extends the result of the two-effort-case. For any decision-maker who must be given incentives to search for information, there exists an optimal level of informativeness of the advice she can receive.

A way to interpret this result is that the consultant will achieve two goals: first, he will bring some information to the decision-making process, and second, he will enhance the incentives for decision-making of the manager when needed. The first objective will be fulfilled with a very strong consultant. However, to fulfill the second, only a consultant with talent comparable to the manager's should be hired. The optimal level λ^* balances these two objectives.

This result is somewhat general and continues to hold in the case of n signals. One important question about the discriminating contract concerns its optimality and, more particularly, whether it can dominate,for any λ, the case in which the manager performs the information search and decision-making alone.

It can be shown that this will be the case for any $(\theta, \lambda) \in [0,1]^2$ when the number of states of nature (and possible decisions) becomes great. The intuition behind this result is straightforward: when n is great, the probability that the manager and the consultant obtain the same signal when at least one of them is not talented is low. This is because an untalented agent has the probability $\frac{1}{n}$ to obtain a signal identical to the one obtained by a talented one. Thus, the probability of a wrong decision with two identical signals becomes small as n increases, and the discriminating contract tends to perfectly sort decisions that have to be rethought and decisions that do not.

Once the consultant has been hired, the question remains whether he should be utilized alone or if the manager should be induced to make some effort after information has been exchanged. Of course, since giving incentives is costly, any consultant close to $\lambda = 1$ should be used alone. However, we can show the following result:

Proposition 7 *There always exists a non-empty set of a consultant's levels of informativeness, strictly greater than the manager's talent θ, such that the discriminating contract strictly dominates the case in which the manager is not involved in decision-making.*

Proof. The proof can be found in the appendix. ■

This result states that even a consultant strictly stronger than the manager should not be left alone; the manager, when contradicted, should be given incentives to build on that contradiction and exerts efforts toward decision-making. This result shows formally the necessary involvement of the manager in the consultant's mission; this involvement has been emphasized very much in the business literature[25]. The next result clarifies the relationship between the effort exerted by the manager and the consultant's informativeness.

Lemma 7 (Informativeness and managerial effort level) *The optimal level of effort $p_{opt} = \max\{p^*, \overline{p}\}$ increases whenever the informativeness of the consultant λ is increased.*

Proof. The proof can be found in the appendix. ■

Again, this lemma shows that the effort of the manager and the consultant's informativeness should not be decided separately. As emphasized in the business literature, if the consultant delivers more informative signals, the manager should be even more involved in the decision-making process in case of dissent. Loose intuitions suggest that the information delivered by the consultant and the manager's effort are substitute; that is, the stronger the signal of the consultant is, the smaller should be the manager's effort.

The conclusion of our model turns out to be more subtle. Either the effort exerted by the manager should be 0 in the case of consent, or in the case of dissent, the manager should be prepared to deliver

[25] It is interesting to note that once you allow for strictly positive consulting fees, $F(.)$, increasing in λ and for $F(\lambda)$ being high when $\lambda = 1$, then, our simulations show that the optimal contract is one in which the manager is involved.

a "high performance" in term of the accurateness of the signal she obtains. Managerial effort and the consultant's informativeness are complementary in the latter case. The next paragraph makes the results obtained in this section more palatable.

5.2 An example of decreasing value of information

The aim of this paragraph is to develop a numerical example to show that the conclusions reached in the two-effort-levels case will still hold when the effort of the manager is treated as a continuous variable. Indeed, failing to coordinate the two agents will result in a loss of profit if the consultant brings a more informative signal.

Let us consider the following cost function:

$$C(p) = \frac{p^2}{(1-p)^{\frac{1}{8}}}.$$

This function verifies the usual properties given in assumption 1, since

$$C'(p) = \tfrac{1}{8}p\frac{16-15p}{(1-p)^{\frac{9}{8}}} > 0,\ C''(p) = \tfrac{1}{64}\frac{105p^2-224p+128}{(1-p)^{\frac{17}{8}}} > 0,\ C(0) = C'(0) \text{ and } C(1) = \infty.$$

Moreover, we have

$$C'''(p) = \tfrac{3}{512}\frac{112(1-p)+14+35p^2}{(1-p)^3\sqrt[8]{(1-p)}} > 0,$$

and by assumption 2 we know that the program of firm is globally concave. Solving equation (22) in p, we obtain

$$\overline{p} = \frac{83\lambda-37-\sqrt{(4649\lambda^2-7038\lambda+2713)}}{2(7\lambda+7)}\ . \tag{23}$$

It is easy to show that $\frac{\partial\overline{p}}{\partial\lambda}$ is strictly positive; an increase in the advice informativeness should be followed by an increase in managerial effort. Figure 1 illustrates all the results we established in the continuous effort version of the model. The benefit to the firm when it uses a consultant corresponds to the plain line curve and is denoted B_c.

We have represented the level λ^*, the value of λ^* ($\simeq 0.889$) depends on the initial talent of the manager θ ($= 0.6$), the benefit in case of success and failure ($G = L = 200$), and the cost function described above.

The set Υ is shown to exist in this example; that is, for any value of λ lying between $\Upsilon \simeq [0.889, 0.92]$, the benefit from advice will be strictly lower for the firm. However, we can also exhibit cost function for which Υ will be an empty set[26].

A consultant can be used alone with no involvement on the part of the manager. In this case, the profit of the firm is a straight increasing line. The form of the profit as a function of λ can be explained

[26] There are cases in which, the benefit curve is always increasing. However, all of our simulations show that there is a region of expertise level λ (strictly greater than θ) for which the benefit curve is almost non-increasing.

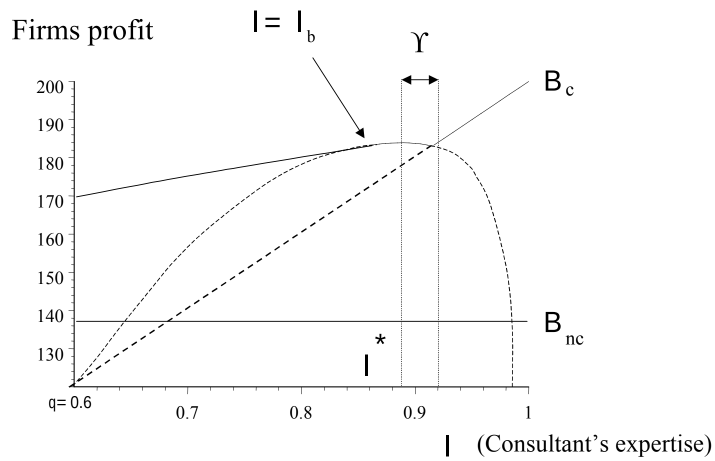

Figure 1: The value of advice when effort is a continuous variable.

as follows. Starting from $\lambda = \theta$, an increase in the value of λ always increases the benefit to the firm. At some point for $\lambda = \lambda_b$, the information acquisition constraint becomes binding and p_{opt} is determined according to (23). For $\lambda > \lambda_b$, further increases in the value of λ are possible only if the value of $p_{opt} = \overline{p}$ is increased even more. For high values of λ, it becomes very expensive to reward the manager for gathering information, and the profit associated with the discriminating contract becomes smaller than the profit to the firm with no involvement[27].

It should be pointed out that managerial involvement in the discriminating contract is only partial. For instance, in the example where the hired consultant has level $\lambda^* \simeq 0.889$, the probability that a manager of initial talent $\theta = 0.6$ must exert effort is 0.23 (i.e. $\frac{1-\theta\lambda^*}{2}$).

This example illustrates the fact that, even if the firm optimizes the manager's effort, the benefit curve can still be shown to be decreasing in λ. This result is not an artifact of the binary effort model.

6 Conclusion

In this paper, we have analyzed the relationship between a consultant and his client. We assumed that a manager's opinion about a given problem could be, at some private cost, enhanced. We first studied the simple case in which a firm wants to induce such an improvement decision-making. We then introduce an external source of information for the manager. In this setting, the analysis shows that the presence of a consultant generates several contradictory effects on a manager's incentives, and these effects's direction

[27] This is not very surprising since we have assumed away consulting fees. A perfect decision-maker ($\lambda = 1$) is trivially dominant in our setting. This would certainly not be true if the fees for the perfect adviser were high enough.

is, *a priori*, ambiguous. As shown by the analysis of benefit curve, an increase in the ability of the consultant is not always followed by an increase of the firm's benefit. Thus, when the manager has, ex ante, a positive probability of exerting effort, we characterize an optimal level of the consultant's expertise. We characterize the form of the benefit curve when the manager is an active information searcher.

We believe that the assumption made to generate the incentive effect of consultancy is not a narrow one, and that in most of real situations, the manager has the (costly) opportunity to obtain more information. Hence, a consequence is that the benefit of consultancy should not be assessed only on the value of the information transmitted by the consultant but also on the incentives effect it has on the part of the manager.

However, from a practical viewpoint, the value of a consultant's intervention remains difficult to measure. The recent management literature does indeed focus largely on the relationship between the consultant and his client, and argues that caring about client involvement during the mission is a key aspect for valuing advice.

An interesting by-product of the paper is that it gives a "visual" representation of the value of the consultant for the firm. Figure 5 shows that, the value of advice is a rather convex function of its informativeness. In a model in which the decision-maker is ignorant (thus, without any interaction with the client) the curve would be linearly increasing with λ. In our setting, the payoff for high reputation (above $\overline{\lambda}$) seems to be high compared to those arising from an average reputation. This helps us to understand why, it is (overly) important for consultants to build a high reputation.

In order to simplify the analysis, we have made the assumption that the external source of information is an exogenous parameter. The goal was to study the effect of external information on the manager's incentives, we therefore treated the consultant as an honest communicator. However, the fact that the value of information can be negative suggests that if we allow the consultant to have more degrees of freedom in his communication, he may indeed behave strategically.

Consultants usually try to extract managerial resources by distorting their communication. For instance, in our setting, any profit maximizing consultant, with a level of expertise belonging to Υ, would like to transmit less information to the manager and stick to a level λ^* of expertise. Interestingly here, the consultant would diminish the quality of the information transmitted to the decision-maker and, nevertheless increase the firm's profit[28]!

[28] There remains the question of how the consultant credibly lowers the quality of the information transmitted. However, this findings is reminiscent of Ottaviani and Sorensen's (1999) result but arises for different reasons; the consultant wants to "extract" effort from the manager.

It should be interesting also to investigate the case in which two agents (or more) devote efforts to making a decision, and exchange information. Our model suggests that agents will have to decide how much information to communicate under two countervailing forces. When they communicate, the two agents will have to balance the need to generate incentives for information searches and the need to be informative enough to help the other side reach a satisfactory opinion. We believe that the design of committee with endogenous information is an important topic. This design should entail how the compensation of the members will induce them to interact fruitfully[29]. Moreover, the problems studied by those committee often relate to several areas of expertise[30], there is also a need to study the interaction of experts within such committees. We hope that the study of endogenous incentives for decision-making created by the relationship between the manager and the consultant is a step in that direction.

[29] There exist contributions that study how committee members are induced to search information through different voting mechanisms. See, for instance Persico (1999) and Bose (2001).

[30] For instance, the evaluation of risks is made by several types of specialists. The relevant decisions concerning a risk of volcano erruption (e.g., the need for population evacuation) would be made with the help of a volcanologist, a security specialist, and perhaps a psychologist (to determine the best way to communicate those decisions to the population).

7 Appendix

The next table presents each possible probability that is used in the optimization program of the firm. Because the states of nature are equally likely, we can restrict to signal i as the first signal received by the manager. For instance, in the first row we compute the ex ante unconditional probability of receiving signal sequence ii. If an effort is performed, then the probability that a signal j occurs, given that ii has been received. is in turn given. To finish, given the three signal sequence, we compute the probability of success if the manager picks a given decision. The probabilities of failure are easily found, as the probability of success of an event is one minus the probability of failure.

The conditional probabilities are computed using Baye's rule, for instance

$$p_{j|ii} = \frac{(1-\theta)\left[\lambda(1-p)+\frac{1}{2}(1-\lambda)(1-p)+\frac{1}{2}(1-\lambda)p\right]}{2\left(\lambda\theta+\lambda\frac{1}{2}(1-\theta)+\theta\frac{1}{2}(1-\lambda)+\frac{1}{2}(1-\theta)(1-\lambda)\right)} = \frac{1}{2}\frac{(1-\theta)(1+\lambda-2\lambda p)}{1+\lambda\theta}.$$

The numerator presents all the events in which sequence iij can arise. To obtain a signal j whereas she previously obtained i, the manager must be dumb with probability one; therefore, this happens with probability $1-\theta$. Then, either she faces a smart consultant (i.e., who always gets the right signal) and she is unlucky and gets the opposite signal with probability $1-p$; or she faces a dumb consultant. The latter is then lucky with probability $\frac{1}{2}(1-\lambda)$, and the manager is unlucky and contradicts him with probability $(1-p)$. The consultant is now unlucky again with probability $\frac{1}{2}(1-\lambda)$ and the manager is lucky with probability p.

The denominator presents all the events in which sequence ii arises. The manager and the consultant are both smart and receive the same signal; this arises with probability $\lambda\theta$. The consultant is smart and faces a lucky manager, the reverse holds. A dumb consultant faces a dumb manager who receives, by chance, the same signal he does. All these events do arise for sequence "*aa*" and sequence "*bb*". Using the same Bayesian logic, one can compute all the conditional probabilities of the game. Those probabilities are inserted in figure 2.

Proof of proposition 1. We first show that whenever $\lambda < \underline{\lambda}$, obtaining a third signal is always worthwhile for the firm.

Let us suppose that two identical signals have been obtained, and denote these signals by ii. It is worth adding a signal to the previous two only if doing so can change the final (optimal) decision. One would only want to change the decision if the third signal contradicts the previous two; that is, if the manager obtains signal j.

If the sequence is iij, the manager knows that she is dumb. Therefore, the probability that her signal is correct is p, whereas the probability that the correct signal is i is $\lambda+\frac{1}{2}(1-\lambda)$. The best decision is j

Event	ii	$I \mid ii$	$J \mid ii$	□
Probability	$\frac{1+\lambda\theta}{2}$	$\frac{(1+\lambda)(1+\theta)}{2(1+\lambda\theta)}$	$\frac{(1-\lambda)(1-\theta)}{2(1+\lambda\theta)}$	□
Event	ii	$j \mid ii$	$J \mid iij$	$I \mid iij$
Probability	$\frac{1+\lambda\theta}{2}$	$\frac{1}{2}\frac{(1-\theta)(1+\lambda-2\lambda p)}{1+\lambda\theta}$	$\frac{(1-\lambda)p}{1+\lambda-2\lambda p}$	$\frac{(1-p)(1+\lambda)}{1+\lambda-2\lambda p}$
Event	ii	$i \mid ii$	$J \mid iii$	$I \mid iii$
Probability	$\frac{1+\lambda\theta}{2}$	$\frac{3\theta\lambda+\theta+2p\lambda-2p\lambda\theta+1-\lambda}{2(1+\lambda\theta)}$	$\frac{(1-\theta)(1-\lambda)(1-p)}{3\lambda\theta+\theta+2p\lambda-2p\lambda\theta+1-\lambda}$	$\frac{(1+\lambda)(p+2\theta-p\theta)}{3\theta\lambda+\theta+2p\lambda-2p\lambda\theta+1-\lambda}$
Event	ij	$i \mid ij$	$J \mid iji$	$I \mid iji$
Probability	$\frac{1-\lambda\theta}{2}$	$\frac{1+\lambda+\theta-3\theta\lambda-2\lambda p+2p\lambda\theta}{2(1-\lambda\theta)}$	$\frac{(1-p)(1-\theta)(1+\lambda)}{1+\lambda+\theta-3\theta\lambda-2\lambda p+2p\lambda\theta}$	$\frac{(1-\lambda)(2\theta+p-p\theta)}{1+\lambda+\theta-3\theta\lambda-2\lambda p+2p\lambda\theta}$
Event	ij	$j \mid ij$	$J \mid ijj$	$I \mid ijj$
Probability	$\frac{1-\lambda\theta}{2}$	$\frac{(1-\theta)(1-\lambda+2p\lambda)}{2(1-\lambda\theta)}$	$\frac{(1+\lambda)p}{1-\lambda+2\lambda p}$	$\frac{(1-\lambda)(1-p)}{1-\lambda+2\lambda p}$
P(success $\mid$ 2 signals), $\mu > \frac{1}{2}$		P(success $\mid$ 2 signals), $\mu > \frac{1}{2}$	P(success $\mid$ 3 signals)	P(success $\mid$ 3 signals), $\mu > \frac{1}{2}$
$\frac{(1-\theta)(\mu\lambda+1-\mu)}{2-2\mu-\theta-\lambda-2\mu\theta\lambda+2\mu\theta+2\mu\lambda}$ if $\lambda \geq \theta$		$\frac{(1-\lambda)(\mu\theta+1-\mu)}{2-2\mu-\theta-\lambda-2\mu\theta\lambda+2\mu\theta+2\mu\lambda}$ if $\theta \geq \lambda$	$\frac{p-p\theta+\theta-\theta\lambda}{1-\theta\lambda}$	$\frac{2p(1-\theta)(1-\mu(1-\lambda))-p\lambda(1-\theta)+\theta(1-\lambda)}{2-\theta-\lambda-2\mu+2\mu\lambda+2\mu\theta-2\mu\theta\lambda}$

Figure 2: Relevant events and their probability of occurence.

if $p > \frac{1}{2}(1+\lambda)$ that is if $\lambda < \underline{\lambda} = 2p - 1$.

The other case is the one in which the first signals are ij. In this case the optimal decision is looser than the one taken in the ii case. The opportunity of adding a third signal to ij is, thus, obvious.

We now turn to the case in which obtaining two signals is sufficient. Assume again that ij has been received and that the consultant has a stronger expertise than the manager. Then the optimal decision with two signals is to pick decision j. In such a case, it is useless to obtain an additional signal if it does not change the optimal decision with respect to the two-signals case. A third signal can be j. Thus, the optimal decision is still j, and obviously the third signal does not alter the decision with respect to the two-signals' case.

A third signal can be i, and in such a case the manager has received two identical signals ii. The consultant's report is j and we wonder whether the third signal is useless or not. This is the case if:

$$\frac{1-\lambda}{2}\theta + \frac{1-\lambda}{2}\left(\frac{1}{2}p(1-\theta)\right) < (1-\theta)\frac{1}{2}(1-p)\left(\lambda + \frac{1}{2}(1-\lambda)\right).$$

The left hand side of the inequality states that after the sequence iji a success can be obtained either by a smart manager and a dumb consultant who contradicts him by chance, or by a dumb manager who obtains two identical signals and is contradicted by a dumb consultant.

The right hand side states that after a sequence iji a success can be obtained if a dumb manager who obtains two identical (wrong) signals faces a consultant who reports the right signal. This inequality is equivalent to $\lambda > \overline{\lambda} = \frac{2p-1+3\theta-2p\theta}{1+\theta}$. When the manager has more expertise, it is obvious that a third signal is optimal. But, this does not contradict the conclusion reached before, as inequality $\overline{\lambda} > \theta > \lambda$ holds in this case.

We now turn to the case of intermediate values of λ.If the first signals are ij, one knows from the previous explanation that if $\lambda \leq \overline{\lambda}$, obtaining a third signal is strictly optimal. Moreover, from the first case we know that there is no point in adding a signal if the sequence is ii and $\lambda \geq \underline{\lambda}$. ■

Proof of proposition 2. We have

$$\frac{\partial B_{dc2}}{\partial \lambda} = \frac{1}{4}(1-\theta)c - \frac{(1-\theta)\left(9\theta^2+\lambda^2\theta^2-6\lambda\theta^2+4\lambda\theta^2 p-12\theta^2 p+4\theta^2 p^2-4\lambda\theta p+2\lambda\theta-4\theta p^2+9\theta+\lambda^2\theta+4p\right)}{2(2\theta p-2p-3\theta+1+\lambda+\lambda\theta)^2}$$

where $c = G + L$.

The local maximum must verify the following first-order condition $\frac{\partial B_{3c}}{\partial \lambda} = 0$. We know that λ, if it exists, must be such that $0 < \lambda < \overline{\lambda}$. Therefore the only root that satisfies this condition is

$$\lambda^* = \frac{2p-2\theta p+3\theta-1}{1+\theta} - \frac{\sqrt{8(1-\theta)^2(1+3\theta-2\theta p)(2\theta+p-\theta p)}}{(1-\theta)(1+\theta)\sqrt{c\theta-2\theta+c}}.$$

The second order condition is

$$\frac{\partial^2 B_{dc2}}{\partial \lambda^2} = \frac{(1-\theta)(1+3\theta-2\theta p)(2\theta+p-\theta p)}{(-2p+2\theta p-3\theta+1+\lambda+\lambda\theta)^3} < 0,$$

which is true because $\lambda < \overline{\lambda} = \frac{2p(1-\theta)+3\theta-1}{1+\theta}$. Hence, we have a local maximum of B_{dc2} for λ^*. To show that $\lambda^* > \theta$, we compute the difference between those two terms. It can be easily shown that it is equal to

$$\lambda^* - \theta = \frac{(1-\theta)(2p-1+\theta)\sqrt{c\theta-2\theta+c}-2\sqrt{2}\sqrt{(1+3\theta-2\theta p)(2\theta+p-\theta p)}}{(1+\theta)\sqrt{(c\theta-2\theta+c)}}.$$

The previous expression is positive if the numerator N is positive.

Let us compute the first derivative of N with respect to c. We obtain:

$$\frac{\partial N}{\partial c} = \frac{(1-\theta)(2p-1+\theta)(1+\theta)}{2\sqrt{(c\theta-2\theta+c)}} > 0.$$

Thus, the minimum of N is obtained for the smallest value of c equal to $\underline{c} = \frac{4(p+\theta-p\theta)}{(1-\theta)^2(2p-1)^2}$ according to lemma 1. Plugging $\underline{c}$ into N, we obtain

$$\mathrm{N}(\underline{c}) = \frac{2(\theta+2p-1)A+4(2p-1)\sqrt{(1+3\theta-2\theta p)(p+2\theta-\theta p)A}}{(2p-1)^2(1-\theta)} > 0,$$

where $A = 4\theta(1-p)\left(p-\theta p+\theta^2 p+\theta\right)+(2p+\theta)\left(1-\theta^2\right)+4\theta^2 p^2$. Hence $\lambda^* > \theta$. As we have $B_{dc1}(\theta) = B_{dc2}(\theta)$ and $\frac{\partial B_{dc1}}{\partial \lambda} > 0$, λ^* is a maximum of B_{dc1}, and B_{dc2} for $0 \leq \lambda < \overline{\lambda}$. ■

Proof of proposition 3. A maximum of B_{dc} is reached for $\lambda = \lambda^*$ on the interval $\left[0, \overline{\lambda}\right]$. The function B_{cs} is defined over the interval $[0, \underline{\lambda}]$ and it is strictly decreasing on this interval, its maximum is for $\lambda = 0$. Thus, when a consultant is hired if $B_{dc}(\lambda^*) > B_{cs}(0)$ then we have $\lambda_{opt} = \lambda^*$.

We compute the difference between $B_{dc}(\lambda^*)$ and $B_{cs}(0)\,(= B_{nc})$. Using (5) and (15), we can write this difference as

$$\Delta B = B_{dc}(\lambda^*) - B_{cs}(0) = \Delta p.c + \left((\theta + p(1-\theta))\, w_{nc}^{S*} - \tfrac{1}{4}(1 + 3\theta + (1-\theta)(2p + \lambda^*))\, w_{dc2}^{S*}\right)$$

We show successively that two terms of this equation are positive. The first term is the difference between the probabilities of success, it is

$$\Delta p.c = \tfrac{1}{4}(1-\theta)(1 + \lambda^* - 2p)\, c.$$

After replacing λ^* by its value (see proposition 2), we find that

$$\Delta p.c = \tfrac{1}{2}(1-\theta)\left(\frac{2\theta(1-p)}{(1+\theta)} + \frac{\sqrt{2}\sqrt{(1+3\theta-2\theta p)(2\theta+p-\theta p)}}{(1+\theta)\sqrt{(c\theta-2\theta+c)}}\right) c > 0.$$

The difference between the expected salaries can be written as

$$\left(\frac{2(\theta+p(1-\theta))}{(1-\theta)(2p-1)}\right) - \frac{(1+3\theta+(1-\theta)(2p+\lambda^*))(1-\lambda^*\theta)}{8(2p(1-\theta)+3\theta-1-\lambda^*(1+\theta))}$$

That is

$$\Delta w = \frac{4\left(\theta^3-3-5\theta^2+7\theta\right)\theta p^2+\left(22\theta+18-58\theta^2-8\theta^4+26\theta^3\right)p+3\theta^4-1+46\theta^2+16\theta}{8(1-\theta)(2p-1)(1+\theta)^2} - \frac{\sqrt{2}A\left(4p\theta(1-\theta)^2(1-p)+p+\theta^2(1-p)+\theta^2(2-\theta)\right)}{2(1+\theta)^2(2p-1)B}$$

when $\lambda = \lambda^*$ and estimated at $c = \underline{c}$, and where $A = \sqrt{((1 + 3\theta - 2\theta p)(2\theta + p - \theta p))}$ and

$$B = \sqrt{\left(8\theta p + 8\theta^2 - 20\theta^2 p - 8\theta p^2 + 2\theta + 16\theta^2 p^2 - 8\theta^3 p^2 + 8\theta^3 p - 2\theta^3 + 4p\right)}.$$

A is increasing with θ, since

$$\frac{\partial A}{\partial \theta} = \frac{6\theta(1-p)+p(1-\theta)(1-p)+\theta p^2+1}{\sqrt{(1+3\theta-2\theta p)(2\theta+p-\theta p)}} > 0.$$

It is thus strictly smaller than $2\sqrt{2}$. Similarly, B is increasing with θ since

$$\frac{\partial B}{\partial \theta} = \frac{4(1-p)\left(p(1-\theta)^2+\theta^2 p+\theta(1-p)\right)+3\theta\left(1-p+p^2\right)(1-\theta)+\theta+(1-p\theta(1-\theta)(1-p))}{\sqrt{\left(8\theta p+8\theta^2-20\theta^2 p-8\theta p^2+2\theta+16\theta^2 p^2-8\theta^3 p^2+8\theta^3 p-2\theta^3+4p\right)}} > 0.$$

It is thus strictly greater than $2p$. We thus have a major value for the second term, u_2, of Δw. It is

$$U_2 = \frac{\left(4\theta p^2-8\theta^2 p^2+4\theta^3 p^2-4\theta^3 p+9\theta^2 p-4\theta p-p+\theta^3-3\theta^2\right)}{(1+\theta)^2(2p-1)p} > u_2\ .$$

We can show that the first term of Δw, u_1, is strictly greater than U_2. We find that

$$\Delta w > u_1 - U_2 = \frac{14\theta^2\left(p^2(1-\theta)+\theta(2p-1)\right)+6\theta^2(1-\theta)(1-p)(4\theta p+3)+3p+24\theta^2p^2}{8(1+\theta)^2(2p-1)(1-\theta)p} + \frac{8p^2(1-p\theta)+4p\left(1-\theta p^2\right)+\theta^2\left(1-\theta^2 p\right)\left(4\theta^4p^3+1\right)+10p(1-\theta)\left(2\theta^2p^2+4\theta+p\right)}{8(1+\theta)^2(2p-1)(1-\theta)p} + \frac{+\theta^2+32\theta^3p(1-p)+2\theta^3p+8\theta^4+8\theta^2p^3}{8(1+\theta)^2(2p-1)(1-\theta)p} + \theta^2\frac{\theta^2p^3-\theta^4p^3+1+\theta^6p^4-\theta^2p}{2(1+\theta)^2(2p-1)(1-\theta)p} > 0$$

Δw is positive for $c = \underline{c}$. To complete the proof, we now have to show that $\frac{\partial \Delta B}{\partial c} > 0$. We have $\frac{\partial \Delta B}{\partial c} = \frac{\partial}{\partial c}\left(\frac{1}{2}(1-\theta)\left(\frac{2\theta(1-p)}{(1+\theta)} + \frac{\sqrt{2}A}{(1+\theta)\sqrt{(c\theta-2\theta+c)}}\right)c - \frac{(1+3\theta+(1-\theta)(2p+\lambda^*))(1-\lambda^*\theta)}{8(2p(1-\theta)+3\theta-1-\lambda^*(1+\theta))}\right)$ ■

$\frac{\partial \Delta B}{\partial c} = (1-p)\,\theta\frac{1-\theta}{1+\theta} + \frac{1}{16}A\sqrt{2}\,(1-\theta)\,\frac{3c+3c\theta-16\theta}{(c\theta-2\theta+c)^{\frac{3}{2}}(1+\theta)} > 0$ if $c \geq \underline{c}$.

Proof of proposition 4. We compute the difference between B_{dc2} and B_{nic}. ΔB, computed for $\lambda = \lambda^*$ must be positive. The function ΔB is easily computed:

$$\Delta B = B_{dc2} - B_{nic} = \tfrac{1}{4}(2p\,(1-\theta) + 3\theta - 1 - \lambda\,(1+\theta))c - \tfrac{1}{2}\tfrac{(1-\lambda\theta)(1+3\theta+(1-\theta)(2p+\lambda))}{(2p(1-\theta)+3\theta-1-\lambda(1+\theta))}$$

by plugging in λ^* we get the following expression:

$$\frac{\sqrt{2}[\theta(1+3\theta-2\theta p)(2\theta+p-\theta p)(c(1+\theta)+2(1-\theta))-\sqrt{(1+3\theta-2\theta p)(2\theta+p-\theta p)(c\theta-2\theta+c)}\left(2\theta^3p-8\theta^2p-\theta+13\theta^2-3\theta^3-1+6\theta p\right)]}{(1+\theta)^2\sqrt{(1+3\theta-2\theta p)(2\theta+p-\theta p)(c\theta-2\theta+c)}}.$$

This expression is positive if the numerator is positive. Thus, there exists a level $\overline{c}$ such that any c strictly greater will make $\Delta B\,(\lambda = \lambda^*)$ strictly positive. This is a sufficient condition, in many case $c > \overline{c}$ is not required to obtain $\Delta B > 0$.

If $B_{dc2}\,(\lambda^*) > \frac{1}{2}\,(1+\lambda^*)\,(B+L)$, then providing incentives to the manager to search for information while advised by a consultant of reputation λ^* is strictly worth. The inequality will still hold for any consultant with reputation $\lambda^*+\varepsilon$ where ε is arbitrarily small but positive. But, we know from proposition 2 that λ^* is the unique maximum of $B_{dc}\,(.)$. Thus for $\lambda^*+\varepsilon$ the discriminating contract still dominates the no effort contract (by continuity) but it yields a strictly lower profit to the firm. If λ is increased significantly, the benefit of the no effort contract will increased to the point where it is equal to $B_{dc2}\,(\lambda^*)$. ■

Proof of proposition 5. When $\mu > \frac{1}{2}$, the upper bound on the consultant's level is

$$\overline{\lambda}\,(\mu) = \frac{2\theta + 2p - 2p\theta - 1 - 2\mu p + 2\mu p\theta + \mu - \mu\theta}{\theta - \mu\theta - 2\mu p + 2\mu p\theta + \mu + p - p\theta}.$$

Since we have $\frac{\partial\overline{\lambda}(\mu)}{\partial\mu} = -\frac{(1-p)(2p-1)(1-\theta)^2}{(\theta-\mu\theta-2\mu p+2\mu p\theta+\mu+p-p\theta)^2} < 0$. $\overline{\lambda}\,(\mu)$ is minimum for $\mu = \frac{1}{2}$ (independent signals). It is routine to show that $\theta < \lambda_0 = \theta\frac{3-2p}{1+2\theta-2p\theta} < \overline{\lambda}$. The derivative of the benefit with respect to μ can be written as

$$\frac{\partial B}{\partial\mu} = \underbrace{\frac{\partial P_S}{\partial\mu}\left(c - w^S\right)}_{-} - \underbrace{P_S\frac{\partial w^S}{\partial\mu}}_{+/-}. \tag{24}$$

where

$$\frac{\partial w^S}{\partial \mu} = -\frac{(1-\lambda)(1-\theta)(3\theta-\lambda-2\theta\lambda-2p\theta+2\theta\lambda p)}{(1-2\theta-2\mu\lambda p+2\mu\lambda p\theta+\mu\lambda-\mu\lambda\theta-2p+2p\theta+2\mu p-2\mu p\theta-\mu+\mu\theta+p\lambda+\theta\lambda-\theta\lambda p)^2}$$

is negative as long as $\lambda \leq \lambda_0 = \theta\frac{3-2p}{1+2\theta-2p\theta}$. Therefore, if $c \geq c_0$, $\frac{\partial B}{\partial \mu}$ is negative. If $\lambda > \lambda_0$ then $\frac{\partial B}{\partial \mu}$ is negative. ■

Proof of lemma 7. When $\lambda = \theta$, the first order condition of the relaxed problem can be written as

$$G + L = 2\left[C'(p) + \left(p + \frac{2}{(1-\theta)}\right) C''(p)\right]. \tag{25}$$

Let us denote by $p(\theta)$ the level of effort that solves (25). When $\lambda = 1$, the first order condition can be written as

$$G + L = \left[C'(p) + \left(p + \frac{2(1+\theta)}{1-\theta}\right) C''(p)\right].$$

Considering the two conditions and the convexity of the cost function, we must have $1 > p(1) > p(\theta) > 0$. The constraint $(IA'1)$ reduces to $pC'(p) \geq C(p)$ when $\lambda = \theta$. Thus, any $p \in (0,1)$ would satisfy the constraint. In particular, for $p = p(\theta)$, the constraint would be slack. When $\lambda \to 1$, the constraint $(IA'1)$ will continue to hold only if the denominator remains positive. The denominator remains positive only if p also tends toward 1. The existence of λ_b follows. ■

Proof of corollary 3. The first step consists of showing that the objective function of the maximization program Ω' is an increasing function of λ when the information acquisition constraint is not binding.

When $\theta \geq \lambda$, we know that the information acquisition constraint does not bind since $(IA'2)$ holds. By applying the envelope theorem, we can obtain the effect of an increase in λ on the objective function of program Ω'.

$$\frac{\partial \Pi(\lambda)}{\partial \lambda} = \frac{1}{4}(1-\theta)p(G+L) + \frac{p(1+\lambda)(1-\theta)(2+\theta-\theta\lambda)+2(1+\theta)^2}{2(1-\theta)(1+\lambda)^2} C'(p) > 0.$$

The profit of the firm is, thus, an increasing function of λ whenever the acquisition constraint does not bind.

For $1 > \lambda \geq \lambda_b$, p is equal to $\overline{p}$ and its value is given by (22). When λ increases from λ_b, then p must increase accordingly, to keep equality (22) satisfied. When λ is close to 1, $\overline{p}$ also approaches 1 and the salary $w^S = \frac{2(1-\theta\lambda)}{(1-\theta)(1+\lambda)} C'(\overline{p})$ should approach infinity. Eventually, the profit must then become (infinitely) negative. This shows the existence of λ^*. Unicity is guaranteed by the continuity of the equilibrium profit function. ■

Proof of proposition 7. We want to show that there exist some values of λ verifying $\lambda > \theta$, such that the profit of the firm is greater when a manager participates in decision-making (i.e., is offered an incentive contract). We first compute the value of B_{dc} for $\lambda = \theta$, which is equal to

$$B_{dc} = \frac{1}{4}\left[2\left(1+\theta\right) + \left(1-\theta\right)\left(1+\theta\right)p\right]\left(G - 2C'\left(p\right)\right) - \frac{1}{4}\left(2 - p\left(1+\theta\right)\right)\left(1-\theta\right)L.$$

When the consultant is used alone, the benefit to the firm is

$$B_{nic} = \left(\theta + \frac{1}{2}\left(1-\theta\right)\right)G - \frac{1}{2}\left(1-\theta\right)L \text{ when } \lambda = \theta.$$

The difference between the two expressions above can be written as

$$\Delta B_{nic} = \frac{1}{4}\left(1+\theta\right)\left(1-\theta\right)p\left(G+L\right) - \frac{1}{2}\left(2\left(1+\theta\right) + \left(1-\theta\right)\left(1+\theta\right)p\right)C'\left(p\right)$$

and is positive only if

$$\left(1+\theta\right)\left(1-\theta\right)p\left(G+L\right) \geq 2\left(2\left(1+\theta\right) + \left(1-\theta\right)\left(1+\theta\right)p\right)C'\left(p\right) \tag{26}$$

is verified. When $\lambda = \theta$, we know that since the information acquisition constraint is not binding, the optimal level of effort $p^*\left(\theta\right) > 0$ is given by the first order condition. From (21) we have

$$\left(1-\theta\right)\left(G+L\right) = 2\left(1-\theta\right)C'\left(p\right) + 2\left(1-\theta\right)pC''\left(p\right) + 4C''\left(p\right).$$

By replacing $G+L$ in (26) by its expression from the equation above, we obtain

$$\frac{1}{2}\left(1-\theta\right)\left(p^*\left(\theta\right)\right)^2 C''\left(p^*\left(\theta\right)\right) + p^*\left(\theta\right)C''\left(p^*\left(\theta\right)\right) > C'\left(p^*\left(\theta\right)\right).$$

This is true for any $p > 0$ if, and only if, the function $g\left(p\right) = \frac{1}{2}\left(1-\theta\right)p^2 C''\left(p\right) + pC''\left(p\right)$ is strictly greater than $f\left(p\right) = C'\left(p\right)$. For $p = 0$, those two functions are equal. The computation of the first derivative shows that $g'\left(p\right) > f'\left(p\right)$ if, and only if,

$$C'''\left(p\right) > -\frac{2(1-\theta)}{2+(1-\theta)p}C''\left(p\right).$$

By assumption 2, this is true when $\lambda = \theta$.

Proof of lemma 8. Implicitly differentiating expression (22) with respect to λ yields:

$$\frac{d\overline{p}}{d\lambda} = \frac{2(1+\theta)}{(1+\lambda)[(1-\lambda)(1+\theta)-(1-\theta)(1+\lambda)(1-p)]} \cdot \frac{C'(p)}{C''(p)} > 0 \ .$$

This expression is always positive because the denominator is always positive over the range of λ, for which the information acquisition constraint holds. Similarly, the implicit differentiation of expression (21) yields:

$$\frac{dp^*}{d\lambda} = \frac{\frac{2(1+\theta)}{1+\lambda}\left(C'(p)+pC''(p)+\frac{2+\theta-\theta\lambda}{(1-\theta)(1+\lambda)}C''(p)\right)}{(1-\theta\lambda)\left(2C''(p)+\left(p+\frac{2(1+\theta)}{(1+\lambda)(1-\theta)}\right)C'''(p)\right)} > 0 \ .$$

By assumption 2, the denominator must be positive. ■

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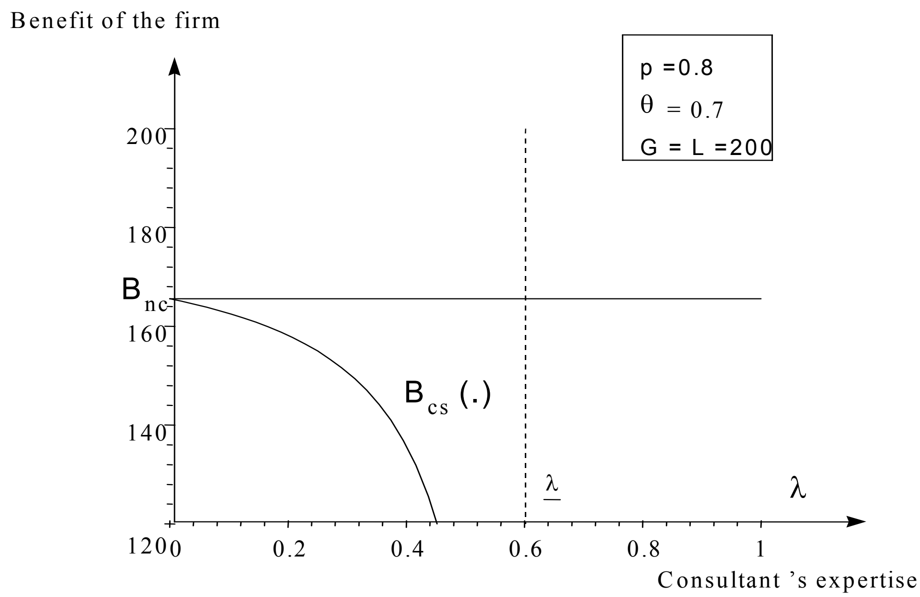

Figure 3: The benefit of the firm when the complete search contract is offered.

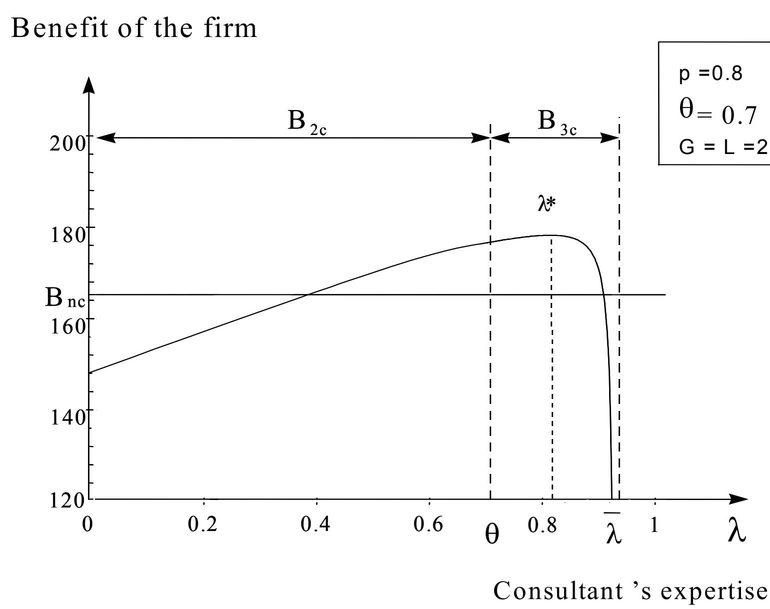

Figure 4: Discriminating Contract.

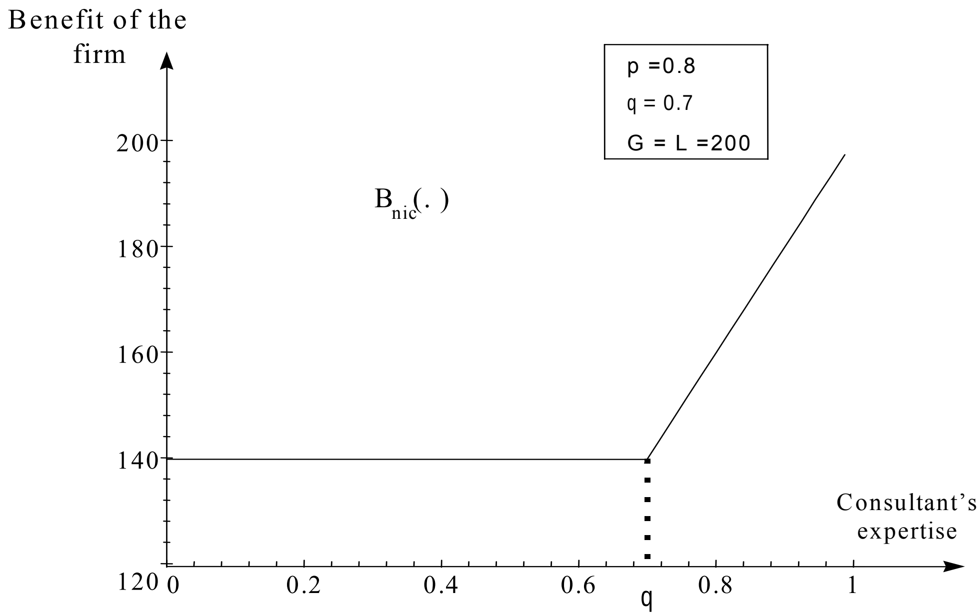

Figure 5: The manager is given no incentive.

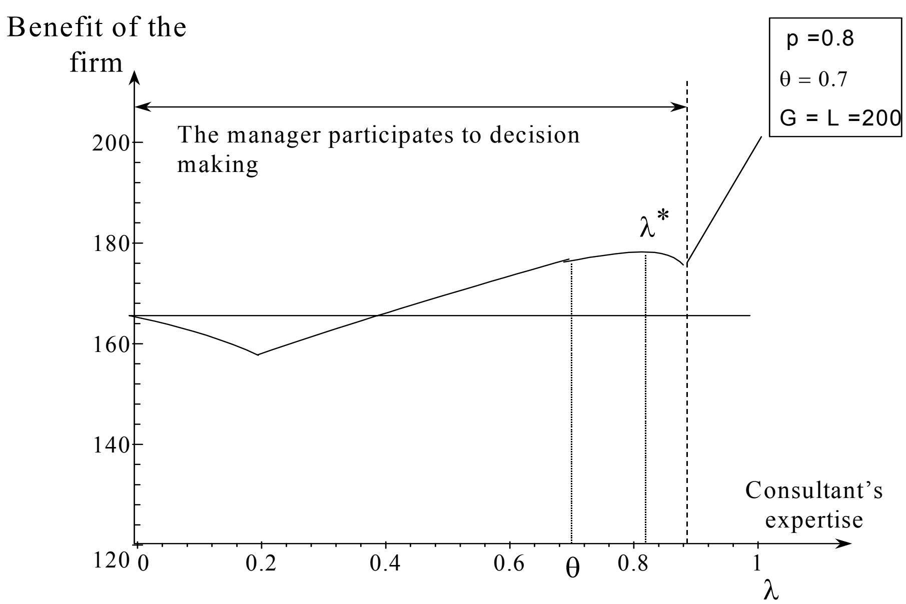

Figure 6: Hiring a consultant can be profitable, λ^* is the optimal level of the consultant's expertise.