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Jump Bidding and Overconcentration in Decentralized Simultaneous Ascending Auctions*

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Abstract

A model of English auction that allows jump bidding is proposed. When two objects are sold separately via such English auctions, I construct an equilibrium such that bidders signal via jump bids, thereby forming rational expectations of the prices without relying on any central mediator. This equilibrium eliminates the exposure problem for a bidder whose valuation function is superadditive. Consequently, the auction game overly concentrates the goods to a multi-item bidder and never overly diffuses them to single-item bidders.

JEL Classification: D44, D82

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1 Introduction

In decentralized markets, an economic agent hoping to acquire a bundle of goods often runs into the following dilemma. One of the goods is available at a price above its standalone value. Should he buy it or not? The problem is that the prices of the other items in the bundle are uncertain and may be so high that the total price of the bundle exceeds its total value. Without a "Walras auctioneer" to coordinate across markets, the agent cannot postpone his decision on one good to wait for the realization of the prices for its complements.

This dilemma has been crystallized into the *exposure problem* in auction theory. Milgrom (2000) and Bykowsky et al. (2000) have constructed complete-information examples for this problem. A few authors have analyzed the exposure problem in asymmetric-information models. The typical setup is that two objects are being auctioned off via two separate auctions simultaneously. Some bidders are *local* in the sense that they value only one particular object. The others are *global* in the sense that they value both objects as complements. A global bidder faces the exposure problem. Albano et al. (2001, 2006) analyze two variants of a two-object ascending auction. Krishna and Rosenthal (1996) and Rosenthal and Wang (1996) consider simultaneous sealed-bid auctions for possibly more than two objects. The qualitative predictions are typically that various kinds of inefficiency may occur. Sometimes the objects are *overconcentrated* to a single bidder while efficiency requires that they go to different bidders, and sometimes the goods may be *overdiffused* to separate owners while efficiency requires that a single bidder should own them.[1]

This paper analyzes the exposure problem by proposing a new model of English auctions to capture their open ascending nature. The idea is that the open transparent nature of English auctions allows the bidders to signal to one another across auctions, thereby forming a rational expectation of the prices. A bidder makes his signal credible via jump bidding, i.e., committing to paying for the good at a pledged price (above its current price) if he wins immediately.[2] Based on this model, a continuation equilibrium is constructed that eliminates the exposure problem conditional on any event that the problem may arise (Proposition 2). Given this equilibrium, the exposure problem is eliminated and there is a

[1] The exposure problem may arise in circumstances other than complementarity. For example, a bidder with unit-demand preference may face the exposure problem when he is bidding for multiple homogeneous units simultaneously. The exposure problem may even be driven by a bidder's budget constraint, as in the centralized two-object auction considered by Brusco and Lopomo (2005a).

[2] The literature on jump bidding typically assumes that jump bidding occurs only at an exogenous stage. See for example Avery (1998) and Gunderson and Wang (1998). Recently, Xiong (2007) finds in a single-unit model a strict incentive for bidders to make a jump bid whose magnitude is a priori determined. In my paper, the timing of jump bidding is endogenous, and the magnitude of a jump bid varies with the bidder's type and furthermore almost surely fully reveals the type.

clear-cut qualitative "overconcentration" prediction of the auctions (Proposition 3).[3]

Although sensitive to the two-object assumption, this result conveys the message that, even without central coordination, economic agents may get to form rational expectations of prices through signaling to one another.

2 The Primitives

There are two items for sale, A and B, and several bidders. For each bidder i, the values of winning item A alone, item B alone, and both items are, respectively, $u_i(A)$, $u_i(B)$, and $u_i(AB)$. There are three kinds of bidders: one *global bidder*, named bidder γ, who values items A and B as complements, several *A-local* bidders who value only item A, and several *B-local* bidders who value only item B; a *local* bidder means an A- or B-local bidder. I.e.,

$$\begin{aligned} u_\gamma(AB) &\geq u_\gamma(A) + u_\gamma(B), \\ u_i(AB) - u_i(A) = u_i(B) &= 0 \quad \text{if } i \text{ is A-local}, \\ u_i(AB) - u_i(B) = u_i(A) &= 0 \quad \text{if } i \text{ is B-local}. \end{aligned} \tag{1}$$

For each bidder i, it is commonly known whether i is global, A-local, or B-local, but the *standalone values* $u_i(A)$ and $u_i(B)$, as well as the *synergy* $u_i(AB) - u_i(A) - u_i(B)$ if i is the global bidder γ, are only privately known to i and are independently drawn from distributions $F_{i\alpha}$, $F_{i\beta}$, and (if $i = \gamma$) F_γ. These distributions are commonly known and their supports each have zero as the infimum, with $F_\gamma(0) = 0$.

A bidder's payoff is equal to his value of the package he wins minus his total monetary payment. He is risk neutral in his payoff.

The two items are auctioned off via separate English auctions that start simultaneously. To be eligible for an item, a bidder needs to participate in its auction from the start. Once he drops out from the auction of an item, a bidder cannot raise his bid for it any more. Once sold, the good is not refundable. Bidders' actions are commonly observed.

This setup is *decentralized* in the sense that the auctioneers of the two auctions cannot coordinate with each other on when to close the auctions.

Thus, when the global bidder can acquire an item say A, he may be still uncertain about the price of the other item B. When the price for A is higher than its standalone value, the bidder faces an *exposure problem*: if he drops out from A now, he foregoes the probable opportunity of acquiring both items at a profitable total price; if he buys A now, however, the eventual price for B may turn out to be unprofitably high.

[3] The open transparent nature of English auctions may also facilitate tacit collusion among bidders. For example see Brusco and Lopomo (2002, 2005b) and Garratt et al. (2007).

3 The Exposure Problem under the Clock Model

Let us illustrate the exposure problem when an English auction is modeled by the traditional "clock model": For each item k, the price p_k starts at zero and rises continuously at an exogenous positive speed until all but one bidder have quit bidding for k, at which point item k is immediately sold to the remaining bidder at the current price.[4]

Let p_A and p_B denote the current prices for items A and B, respectively. For any $x \in \mathbb{R}$, let $(x)^+ := \max\{x, 0\}$. Let $\mathbb{E}_X[f((X) \mid g(X) \geq 0]$ denote the expected value of the function $f(X)$ of the random variable X conditional on $g(X) \geq 0$.

An undominated strategy for every local bidder is to continue bidding for his desired item until its price reaches its value. It will be demonstrated that the following strategy for the global bidder γ is a best reply to the undominated strategy.

a. When $p_A = p_B = 0$, participate in both auctions.

b. If neither item has been sold:

   i. If $v_\gamma(A, p_B) > p_A$ and $v_\gamma(B, p_A) > p_B$, where $v_\gamma(A, p_B)$ and $v_\gamma(B, p_A)$ will be defined later, then continue in both auctions.

   ii. If $v_\gamma(A, p_B) \leq p_A$ or $v_\gamma(B, p_A) \leq p_B$:

      I. If $u_\gamma(k) \leq p_k$ for each item $k \in \{A, B\}$, then drop out from both auctions.

      II. If $u_\gamma(k) > p_k$ for some item k, then continue bidding for the item k_* that maximizes $\mathbb{E}_{(u_i(k))_{i\neq\gamma}}[(u_\gamma(k) - \max_{i\neq\gamma} u_i(k))^+ \mid \max_{i\neq\gamma} u_i(k) \geq p_k]$ over $k \in \{A, B\}$ and drop out from the other item.

c. If the bidder has dropped out from an item k, then continue in the auction for the other item, say $-k$, if and only if $p_{-k} < u_\gamma(-k)$.

d. If the bidder has won an item k, then continue in the auction for the item $-k$ if and only if $u_\gamma(AB) - u_\gamma(k) > p_{-k}$.

The strategy described above exhausts all possibilities. It will be well-defined if $v_\gamma(A, p_B)$ and $v_\gamma(B, p_A)$ are defined. To define $v_\gamma(A, p_B)$, consider the case where global bidder γ has bought item A at price p_A. Then, given any price $\tilde{p}_B$ for item B, the bidder's payoff will be $u_\gamma(AB) - p_A - \tilde{p}_B$ if he also wins B and $u_\gamma(A) - p_A$ if he loses B. Thus, having bought A, the bidder's optimal action is to continue in the auction for B if and only if $u_\gamma(AB) - u_\gamma(A) > \tilde{p}_B$.

[4] The model considered in this section is similar to one of the models in Albano at al. (2001, 2006). But I do not need the two assumptions in their papers: (i) $u_\gamma(A) = u_\gamma(B)$ and $u_\gamma - u_\gamma(A) - u_\gamma(B)$ is commonly known; (ii) the prices of the two objects rise in the same pace (which they say in Albano at al., 2006, is restrictive).

(This explains provision (d) in the above strategy.) It follows that, if bidder γ buys A at price p_A and if the eventual price for B is $\tilde{p}_B$, then his ex post payoff is equal to

$$(u_\gamma(AB) - u_\gamma(A) - \tilde{p}_B)^+ + u_\gamma(A) - p_A.$$

With item A sold to bidder γ, any other bidder j will continue bidding for B up to the standalone value $u_j(B)$, hence

$$\tilde{p}_B = u_{-\gamma}(B) := \max_{j \neq i} u_j(B).$$

Thus, if bidder γ buys A at price p_A, the expected payoff is equal to

$$\mathbb{E}_{u_{-\gamma}(B)}\left[(u_\gamma(AB) - u_\gamma(A) - u_{-\gamma}(B))^+ \mid u_{-\gamma}(B) \geq p_B\right] + u_\gamma(A) - p_A.$$

Hence bidder γ's expected payoff from buying item A at price p_A is positive if and only if

$$\mathbb{E}_{u_{-\gamma}(B)}\left[(u_\gamma(AB) - u_\gamma(A) - u_{-\gamma}(B))^+ \mid u_{-\gamma}(B) \geq p_B\right] + u_\gamma(A) > p_A.$$

Thus, define

$$v_\gamma(A, p_B) := \mathbb{E}_{u_{-\gamma}(B)}\left[(u_\gamma(AB) - u_\gamma(A) - u_{-\gamma}(B))^+ \mid u_{-\gamma}(B) \geq p_B\right] + u_\gamma(A). \tag{2}$$

Analogously, define

$$v_\gamma(B, p_A) := \mathbb{E}_{u_{-\gamma}(A)}\left[(u_\gamma(AB) - u_\gamma(B) - u_{-\gamma}(A))^+ \mid u_{-\gamma}(A) \geq p_A\right] + u_\gamma(B). \tag{3}$$

Proposition 1 *Assume that the distributions $F_{i\alpha}$ and $F_{i\beta}$ are atomless for all bidders i. The global bidder's strategy (a)–(d), together with the local bidders' undominated strategy of bidding for the valued item up to its value, constitutes a perfect Bayesian equilibrium when each English auction is a clock auction.*

Proof By the atomless assumption in this proposition and Eqs. (2)–(3), $v_\gamma(A, p_B)$ is continuous in p_B, and $v_\gamma(B, p_A)$ is continuous in p_A.

The justification for provisions (a), (c), and (d) in the strategy are obvious. Let us consider the case for provision (b), when neither item has been sold.

First, consider subcase (b.i), where $v_\gamma(A, p_B) > p_A$ and $v_\gamma(B, p_A) > p_B$. By the continuity of $v_\gamma(A, \cdot)$ and $v_\gamma(B, \cdot)$, these strict inequalities will continue to hold for a sufficiently short interval of time. Thus, if the bidder is to continue bidding for A, his expected payoff from staying for item B is positive for at least a while, and the same statement is true when A and B switch roles. This expected payoff is bigger than the expected payoff from staying for only a single item, since $v_\gamma(k, p_{-k}) > u_\gamma(k)$ ($\forall k \in \{A, B\}$) by (2)–(3). Hence it is suboptimal to drop out from one auction now while continuing in the other auction. It is also suboptimal to drop out from both auctions, which yields zero payoff.

Second, consider subcase (b.ii). Without loss, suppose $v_\gamma(A, p_B) \leq p_A$. Since $v_\gamma(A, p_B)$ is weakly decreasing in p_B and the prices are strictly increasing in time, $v_\gamma(A, p'_B) < p'_A$ for any p'_B from now on. Thus, by the construction of $v_\gamma(A, \cdot)$ and $v_\gamma(B, \cdot)$, it is suboptimal to continue bidding for both items. That, however, does not mean the bidder should quit both auctions, because the standalone value of an item may still be above its current price. Hence the justification for provisions b.ii.I and b.ii.II are obvious. ■

The next remark says that the above equilibrium exhibits at least two kinds of inefficiency. One is *overdiffusion*: the two items go to two separate bidders, while $u_\gamma(AB) > \max_j u_j(A) + \max_k u_k(B)$. The second kind is *overconcentration*: for some distinct local bidders i and j, bidder γ wins both items while $u_\gamma(AB) < u_i(A) + u_j(B)$.

Remark 1 *If for all bidders i, distributions $F_{i\alpha}$ and $F_{i\beta}$ have no gap, then at the equilibrium constructed above, overdiffusion and overconcentration are events with strictly positive probability.*

The proof is in Appendix A. The intuition is: As long as both auctions are still going on after the price of an item has reached its standalone value for a global bidder, the bidder will drop out before the total price of the two items reach their combined value. Hence he "underbids" before winning any item. If he has won an item, however, with the payment for that item sunk, the bidder will bid for the other item up to its marginal value, which is greater than the total value minus his payment for the won item. Hence the bidder "overbids" after winning an item. Hence both overdiffusion and overconcentration are probable.

4 The Need for an Alternative Model to Capture the Dynamics of English Auctions

The clock model, albeit widely used in auction theory, has abstracted away most of the dynamic aspects of English auctions. In actual English auctions, bids may be submitted through open outcries, hence a bidder may be able to speed up the rising price through jump-bids and adjust his strategy during the intermission between outcries.

These dynamic aspects of English auctions are important for the presence of the exposure problem. Recall that a bidder faces the exposure problem when he is about to buy an item at its current price without knowing the eventual price for its complement. If prices ascend through open outcries, however, the bidder may be able to partially resolve his uncertainty by making a jump-bid. From the rivals' responses, he could at least partially infer the price of the complement. That information might help the bidder to adjust his actions before he has to commit to buying the first item.

5 An Alternative Model

For each item, the auction is the clock model with the following amendments.

A1. As in the clock model, each *active* bidder, who has not dropped out from the auction, can *continue* bidding by pressing the button for the item.

A2. Besides "continue," an active bidder has the option of *jump bidding*: making a bid higher than the item's current price. This action is done in zero second.

A3. An active bidder can *drop out* from, or briefly *quit*, and auction. That is done by either releasing the button (if the price is ascending through the clock) or crying out "quit" (if the price clock is pausing due to the following amendment).

A4. If a bidder drops out from an auction, the price clock in the auction pauses at the dropout price for a short interval of time, called a *pause*. The maximum duration of the pause is assumed to be exogenously δ seconds.

a1. During the pause, every bidder still active in the auction has three alternative actions: quitting, *staying* in the auction, or *resuming* the auction (by crying out "resume").

a2. If the pause has lasted for δ seconds and there is at least one active bidder in the auction, the price clock resumes at the paused level unless there is only one active bidder, in which case the item is immediately sold to this bidder at the paused price.

a3. During the pause, if an active bidder resumes the auction, then the price clock resumes immediately without finishing the δ seconds, and if this bidder is the only active bidder in the auction, the item is immediately sold to this bidder at the paused price.

a4. If every active bidder drops out during the pause, the pause ends without finishing the δ seconds, and the item is sold according to the tie-breaking rule A6 described below.

A5. If a bidder jump-bids in an auction, the price clock in the auction jumps to the jump bid instantly and then pauses at the jump bid. The maximum duration of the pause is δ seconds.[5]

[5] The duration of the pause triggered by a jump bid is assumed to be exogenous just for simplicity. Our results can be extended to allow the following case of endogenous duration of a pause. The maximum duration of the pause is equal to the time it takes for the price clock to reach the jump bid level had there not been the jump bid; i.e, say the price of item k jumps by Δ_k and the speed for the price clock is $\dot{p}_k$, then the duration of the pause is equal to $\Delta_k/\dot{p}_k$. The endogeneity of the duration of a pause prevents bidders from slowing down an auction by submitting smaller and smaller "jump" bids, though bidders have no incentive to do that in the equilibrium constructed here.

a1. During the pause, every bidder still active in the auction has three alternative actions: staying, making a higher jump bid, or quitting unless the bidder is the *highest jump-bidder*, who has made a jump bid which is highest among all jump bids.

a2. If all but the highest jump-bidder have quit during the pause, the auction ends without finishing the δ seconds and the highest jump-bidder buys the good at his jump bid.

a3. All jump bids are commonly observed even if they are submitted simultaneously.

a4. If there are multiple active bidders at the end of the pause, the price clock resumes from the highest jump bid.

A6. A *tie* occurs if a bidder drops out from an auction and all the other currently active bidders drop out from the auction either at the same instant or during the pause triggered by the dropout action. The rule to break such ties is:

a1. Each bidder involved in a tie chooses whether to *concede*.

a2. If not all bidders concede, then the object is randomly assigned to those who do not concede with equal probabilities and the selected one buys the good at its current price.

a3. If all bidders concede, then one of them is selected randomly with equal probabilities and the selected one buys the good at its current price.

6 Avoiding Exposure via Jump Bidding

A *decisive event* means: (i) in one of the auctions, all the remaining active local bidders have just dropped out, (ii) at least one local bidder is still active in the other auction, and (iii) the global bidder is still active in both auctions. When a decisive event occurs we will say that the auction is in a *paused phase* if it is the one where all the remaining active local bidders have just dropped out, as its price clock pauses according to amendment A4. If an auction is not in a paused phase and has not ended, we will say it is in an *active phase*.

Proposition 2 *Assume that no local bidder bids for an item above its standalone value. Conditional on any decisive event up to which the current posterior distributions of all bidders' types are commonly known, there exists a continuation equilibrium such that the following event occurs almost surely: the global bidder knows whether he can profitably acquire both items before he buys any item.*

The idea is that the global bidder can, before making a purchase commitment in the paused auction, find out the eventual total price for the two items by making a jump bid in the active auction. Like an equilibrium bid in a first-price auction, the bidder's jump bid reveals his type, i.e., his maximum willingness to pay in the active auction given that he is to win the paused auction at the paused price. Seeing the jump bid, those local bidders whose types are lower than the global bidder's immediately quit, and those with higher types immediately respond with jump bids. They prefer to signal their types through jump bids because the global bidder's maximum willingness to pay in the active auction would jump if he has made a purchase commitment in the paused auction. If all local bidders immediately quit, the global bidder wins at a price equal to his jump bid. Else he learns that the price in the active auction will be too high for him to acquire both items profitably, so he immediately stops bidding for both items. In this case, if he wants, the global bidder can drop out from the paused auction and, during tie-breaking, concede the good to the local bidder whose dropout triggered the pause. Hence the global bidder suffers no exposure problem.

6.1 The Interim Types and Jump Bids during the Pause

Consider a decisive event. Without loss of generality, let the paused auction be the auction of item A (briefly *auction A*), with the price paused at p_A. Let us calculate the global bidder γ's valuation of winning in the other auction, the auction for item B (briefly *auction B*), during the pause of auction A. The proofs of the lemmas in this subsection are in Appendix B.

If $u_\gamma(A) \geq p_A$, bidder γ's decision is straightforward. He would immediately resume auction A, thereby ending the pause and buying item A. Having bought A, bidder γ's valuation of winning B becomes $u_\gamma(AB) - u_\gamma(A)$, which will be his dropout price in auction B.

The case of $u_\gamma(A) < p_A$ is more complicated, which includes the following two subcases: If $u_\gamma(B) \geq u_\gamma(AB) - p_A$, bidder γ's optimal action is to bid for item B alone:

Lemma 1 *If $u_\gamma(B) \geq u_\gamma(AB) - p_A$, then global bidder γ prefers buying item B alone to buying both items or buying A alone.*

If $u_\gamma(B) < u_\gamma(AB) - p_A$, bidder γ wants to acquire both items up to a certain point:

Lemma 2 *If $u_\gamma(B) < u_\gamma(AB) - p_A$ and $u_\gamma(A) < p_A$, then it is dominated for global bidder γ to buy item B alone and, during the pause, it is profitable for him to buy item B if and only if it is profitable for him to buy both items, i.e., if and only if $u_\gamma(AB) - p_A > p_B$.*

Thus, the only nontrivial case for bidder γ during the pause of auction A is "$u_\gamma(B) < u_\gamma(AB) - p_A$ and $u_\gamma(A) < p_A$." In this case, the global bidder's maximum willingness to pay for item B during the pause, by Lemma 2, is equal to

$$t_\gamma := u_\gamma(AB) - p_A. \tag{4}$$

Also denote

$$t_i := u_i(B) \quad \forall i \neq \gamma. \tag{5}$$

Call t_i the *interim type* of bidder i for any i still active during the pause of auction A.[6]

For every bidder i active at the start of the pause, initialize G_i to be the distribution function of t_i, derived from $F_{i\alpha}$, $F_{i\beta}$ and F_γ, conditional on the history of the game up to the start of the pause. Let T_i denote the support of G_i. Let $G_{-i} := (G_j)_{j \neq i}$. Let

$$t_{-i}^{(1)} := \max\{t_j : j \neq i\},$$

and let $T_{-i}^{(1)}$ denote the support of the random variable whose realizations are denoted by $t_{-i}^{(1)}$. For any $t_i \in T_i$, define

$$\beta_{i,G_{-i}}(t_i) := \mathbb{E}_{t_{-i}^{(1)}}\left[t_{-i}^{(1)} \;\middle|\; t_{-i}^{(1)} \leq t_i; G_{-i}\right], \tag{6}$$

i.e., the expected value of the highest rival's interim type conditional on its not exceeding i's interim type, given the distributions G_{-i}. Let

$$\beta_{i,G_{-i}}^{-1}(x_i) := \left\{t_i \in T_i : \beta_{i,G_{-i}}(t_i) = x_i\right\}.$$

Lemma 3 *The function $\beta_{i,G_{-i}}$ is weakly increasing. Furthermore, if $\inf T_i \geq \inf T_{-i}^{(1)}$ and $\beta_{i,G_{-i}}^{-1}(x_i) \neq \varnothing$, then for any $j \neq i$ and almost every t_j (relative to G_j),*

$$t_j \leq \inf \beta_{i,G_{-i}}^{-1}(x_i) \quad \text{or} \quad t_j > \sup \beta_{i,G_{-i}}^{-1}(x_i). \tag{7}$$

6.2 The Proposed Jump-Bidding Equilibrium

Here is a continuation equilibrium starting from the beginning of the pause of auction A at the paused price p_A.

1. If (global) bidder γ drops out from auction A during the pause, the local bidder i whose dropout triggered the pause does not concede item A to γ.

2. The strategy of bidder γ is:

   a. if $u_\gamma(A) \geq p_A$,

      i. immediately resume auction A, thereby buying A and ending the pause,

      ii. and bid for item B if and only if the price p_B of B is below $u_\gamma(AB) - u_\gamma(A)$;

[6] Although global bidder γ's private information has three dimensions, $u_\gamma(A)$, $u_\gamma(B)$, and $u_\gamma(AB)$, his behavior is tractable because, given one auction pausing and the other auction expected to finish within the pause, his private information is reduced to only one dimension, $u_\gamma(AB) - p_A$.

b. if $u_\gamma(B) \geq u_\gamma(AB) - p_A$ and $u_\gamma(A) < p_A$,

i. drop out from auction A immediately,

ii. concede A to the local bidder(s) whose dropout triggered the pause,

iii. continue in auction B as long as $u_\gamma(B) > p_B$;

c. if $u_\gamma(B) < u_\gamma(AB) - p_A$ and $u_\gamma(A) < p_A$,

i. immediately submit a jump bid equal to $\beta_{\gamma,G_{-\gamma}}(t_\gamma)$ for item B,

ii. if bidder γ has submitted a jump bid x_γ, then during the pause of auction B triggered by the jump bid,

A. if a local bidder i drops out or does not respond with a jump bid x_i such that $\sup \beta^{-1}_{i,G_{-i}}(x_i) > \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$, the posterior distribution G_i of t_i is updated by $t_i \leq \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$,

B. if every local bidder i either drops out or does not respond with a jump bid x_i such that $\sup \beta^{-1}_{i,G_{-i}}(x_i) > \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$, and if $x_\gamma = \beta_{\gamma,G_{-\gamma}}(t_\gamma)$, then bidder γ immediately takes actions 2.a.i and 2.a.ii,

C. if a local bidder i submits a jump bid x_i such that $\inf \beta^{-1}_{i,G_{-i}}(x_i) \geq t_\gamma$, then bidder γ immediately drops out of auction B and takes actions 2.b.i and 2.b.ii,

D. if some local bidder i jump-bids but there is no jump bid x_i from any local bidder i such that $t_\gamma \leq \inf \beta^{-1}_{i,G_{-i}}(x_i)$, then bidder γ plays strategy b in the equilibrium constructed in §3.

3. The strategy of any active local bidder i ($i \neq \gamma$) is:

a. unless bidder γ has made a jump bid, stay in auction B without jump-bidding and quit at $p_B = u_i(B)$,

b. if bidder γ has made a jump bid x_γ during the pause of auction A,

i. if $t_i \leq \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$, drop out immediately,

ii. else immediately make a jump bid equal to $\beta_{i,G_{-i}}(t_i)$, with G_{-i} being the posterior distributions updated by bidder γ's jump bid,

A. if $t_i \leq \inf \beta^{-1}_{j,G_{-j}}(x_j)$ given the jump bid x_j from some local bidder $j \neq i$, drop out immediately unless $t_i = \beta^{-1}_{j,G_{-j}}(x_j) = \max_k \beta^{-1}_{k,G_{-k}}(x_k)$, in which case i drops out if and only if $i < j$ (so that not all local bidders quit),

B. else stay in auction B without jump-bidding and quit at $p_B = u_i(B)$.

6.3 The Proof of Proposition 2

On the path of the proposed equilibrium, global bider γ almost surely resolves his price uncertainty. To show that, recall Lemma 3. It implies that, from bidder γ's jump bid $\beta_{\gamma,G_{-\gamma}}(t_\gamma)$, every local bidder i almost surely can tell (i) $t_\gamma \geq t_i$ apart from (ii) $t_\gamma < t_i$. In case (i), bidder i immediately drops out. In case (ii), bidder i immediately makes a jump bid $\beta_{i,G_{-i}}(t_i)$ based on the updated posteriors G_{-i}.

If all local bidders belong to case (i), bidder γ wins item B at the known price $\beta_{\gamma,G_{-\gamma}}(t_\gamma)$ before the pause of auction A ends. Since $\beta_{\gamma,G_{-\gamma}}(t_\gamma) \leq t_\gamma$ by definition of the jump bid, the bidder knows, by definition of t_γ, Eq. (4), that it is profitable for him to buy both items.

If some local bidder i belongs to case (ii), upon seeing i's jump bid, bidder γ knows that almost surely $t_\gamma < t_i \leq t_{-\gamma}^{(1)}$ and hence the price for B will be greater than t_γ if bidder γ does not drop out (contingent plan 3.b.ii.B). I.e., bidder γ knows that it is almost surely unprofitable for him to buy both items. Again he learns that during the pause of auction A. (Aside: The contingency in plan 2.c.ii.D is a zero-probability event if bidder γ's jump bid was equal to the proposed $\beta_{\gamma,G_{-\gamma}}(t_\gamma)$.)

We still need to verify every bidder's incentive to follow the proposed equilibrium.

6.3.1 The Incentive for Contingent Plans 1, 2.a, and 2.b

Plan 1 follows from the fact that each local bidder's profit from buying item A is nonnegative, as his dropout price does not exceed his standalone value of the item, according to the assumption of the proposition.

Contingent plan 2.a has been justified by the second paragraph of §6.1.

Contingent plan 2.b follows from Lemma 1, which implies that bidder γ would take actions 2.b.i and 2.b.ii, quitting from auction A and conceding A to the local bidders. Since the local bidders do not concede (plan 1), bidder γ frees himself from any obligation of buying A. With only item B to consider, the optimality of plan 2.b.iii is obvious.

6.3.2 The Global Bidder's Incentive for Contingent Plan 2.c

Under the contingency of plan 2.c, $u_\gamma(B) \geq u_\gamma(AB) - p_A$ and $u_\gamma(A) < p_A$, so Lemma 2 applies. Then by the definition of t_γ, Eq. (4), bidder γ buys both items if he knows that the price p_B of item B is less than or equal to his interim type t_γ, and he buys neither item if he knows that p_B is greater than t_γ.

Contingent plan 2.c.ii.B is optimal, because the updating rule 2.c.ii.A coupled with bidder γ's obedience to the jump bid function ($x_\gamma = \beta_{\gamma,G_{-\gamma}}(t_\gamma)$) implies that $t_\gamma \geq t_{-\gamma}^{(1)} \geq p_B$. Hence bidder γ buys both items by taking actions 2.a.i and 2.a.ii according to plan 2.c.ii.B.

Contingent plan 2.c.ii.C is optimal because the contingency $\inf \beta^{-1}_{i,G_{-i}}(x_i) \geq t_\gamma$ under this plan, coupled with bidder γ's expectation that other players abide to the jump bidding function, implies that $t_\gamma \leq t^{(1)}_{-\gamma}$. Then p_B will be greater than or equal to t_γ if bidder γ does not drop out (contingent plan 3.b.ii.B). Hence it is optimal for bidder γ to drop out of both auctions by following plan 2.c.ii.C.

Claim: Under the contingency of plan 2.c, bidder γ prefers making a jump bid to not doing so. If he does not jump bid, the continuation play is the equilibrium presented in §3, as the local bidders will stay without jump bidding until the price reaches their values (plan 3.a). Then bidder γ will follow strategy b in that equilibrium. His expected payment upon winning in this continuation equilibrium is the same as the one if he jump-bids, because in both cases, p_A has been fixed, and the price for item B will be equal to $t^{(1)}_{-i}$ in expectation. The events in which bidder γ wins are different in the two cases. If bidder γ makes a jump bid equal to $\beta_{\gamma,G_{-\gamma}}(t_\gamma)$, the event where he wins is exactly the event where his profit is positive conditional on winning. In contrast, if the global bidder does not jump-bid and hence follows the equilibrium in §3, the event where he wins is not aligned with the event where his profit is positive conditional on winning, because overdiffusion and overconcentration are both probable. Thus, bidder γ would rather jump-bid according to the proposed equilibrium.

Lemma 4 *If bidder γ is to make a jump bid, his optimal jump bid is equal to $\beta_{\gamma,G_{-\gamma}}(t_\gamma)$.*

Proof First observe, by definition of interim types, $t_\gamma - p_B = u_\gamma(AB) - p_A - p_B$ is equal to bidder γ's payoff if he wins in auction B at price p_B during the pause of auction A.

Second, observe that making a jump bid is equivalent to the action of picking a point $\hat{t}_\gamma \in T_\gamma$ and announcing:

> My interim type is equal to $\hat{t}_\gamma$ such that $\hat{t}_\gamma = \inf\{t'_\gamma \in T_\gamma : \beta_{\gamma,G_{-\gamma}}(t'_\gamma) = \beta_{\gamma,G_{-\gamma}}(\hat{t}_\gamma)\}$, hence my payment is equal to $\beta_{i,G_{-i}}(\hat{t}_\gamma)$ if I win immediately.

The lemma is proved if it is optimal for bidder γ to pick $\hat{t}_\gamma = t_\gamma$.

Suppose $\hat{t}_\gamma < t_\gamma$. If $t^{(1)}_{-\gamma} \leq \hat{t}_\gamma$, bidder γ wins immediately (contingent plan 3.b.i); if $t^{(1)}_{-\gamma} > \hat{t}_\gamma$, bidder γ cannot win immediately, and given local bidders' response 3.b.ii.B if γ does not quit, the best he can hope for is that he wins if and only if $t^{(1)}_{-\gamma} \leq t_\gamma$ and he pays $t^{(1)}_{-\gamma}$ upon winning. (He cannot do better than that if the price uncertainty is not resolved within the pause of auction A.) Thus, in picking $\hat{t}_\gamma < t_\gamma$, the expected payoff for bidder γ is less than or equal to

$$\begin{aligned}
& \text{Prob}\left\{t_{-\gamma}^{(1)} \le \hat{t}_\gamma\right\}\left(t_\gamma - \beta_{\gamma,G_{-\gamma}}(\hat{t}_\gamma)\right) + \mathbb{E}_{t_{-\gamma}^{(1)}}\left[\left(t_\gamma - t_{-\gamma}^{(1)}\right)\mathbf{1}_{\hat{t}_\gamma < t_{-\gamma}^{(1)} \le t_\gamma}\right] \\
= \; & \text{Prob}\left\{t_{-\gamma}^{(1)} \le \hat{t}_\gamma\right\}\left(t_\gamma - \mathbb{E}_{t_{-\gamma}^{(1)}}\left[t_{-\gamma}^{(1)} \mid t_{-\gamma}^{(1)} \le \hat{t}_\gamma\right]\right) + \mathbb{E}_{t_{-\gamma}^{(1)}}\left[\left(t_\gamma - t_{-\gamma}^{(1)}\right)\mathbf{1}_{\hat{t}_\gamma < t_{-\gamma}^{(1)} \le t_\gamma}\right] \\
= \; & t_\gamma \text{Prob}\left\{t_{-\gamma}^{(1)} \le t_\gamma\right\} - \mathbb{E}_{t_{-\gamma}^{(1)}}\left[t_{-\gamma}^{(1)}\left(\mathbf{1}_{t_{-\gamma}^{(1)} \le \hat{t}_\gamma} + \mathbf{1}_{\hat{t}_\gamma < t_{-\gamma}^{(1)} \le t_\gamma}\right)\right] \\
= \; & \text{Prob}\left\{t_{-\gamma}^{(1)} \le t_\gamma\right\}\left(t_\gamma - \beta_{\gamma,G_{-\gamma}}(t_\gamma)\right),
\end{aligned}$$

which is equal to bidder γ's expected payoff from picking $\hat{t}_\gamma = t_\gamma$.

Suppose $\hat{t}_\gamma \ge t_\gamma$. Then bidder γ wins if and only if $t_{-\gamma}^{(1)} \le \hat{t}_\gamma$. That is because if $t_{-\gamma}^{(1)} \le \hat{t}_\gamma$, he wins immediately (contingent plan 3.b.i); if $t_{-\gamma}^{(1)} > \hat{t}_\gamma$, even if bidder γ does not drop out, the price for item B will be $t_{-\gamma}^{(1)}$ (contingent plan 3.b.ii.B), which is greater than $\hat{t}_\gamma \ge t_\gamma$. Since $\beta_{\gamma,G_{-\gamma}}$ is weakly increasing, bidder γ's winning probability

$$q_\gamma(\hat{t}_\gamma) := \text{Prob}\left\{t_{-\gamma}^{(1)} : \hat{t}_\gamma \ge t_{-\gamma}^{(1)}\right\}$$

is weakly increasing in his announced type $\hat{t}_\gamma$. Furthermore, when he announces $\hat{t}_\gamma$, bidder γ's expected payment is equal to his winning probability times his jump bid, i.e.,

$$\begin{aligned}
q_\gamma(\hat{t}_\gamma)\beta_{i,G_{-\gamma}}(\hat{t}_\gamma) &= q_\gamma(\hat{t}_\gamma)\mathbb{E}_{t_{-\gamma}^{(1)}}[t_{-\gamma}^{(1)} \mid t_{-\gamma}^{(1)} \le \hat{t}_\gamma] \qquad \text{by Eq. (6)} \\
&= \mathbb{E}_{t_{-\gamma}^{(1)}}\left[t_{-\gamma}^{(1)}\mathbf{1}_{t_{-\gamma}^{(1)} \le \hat{t}_\gamma}\right] \qquad \text{by definition of } q_\gamma \\
&= \int_0^{\hat{t}_\gamma} z\,dq_\gamma(z) \\
&= \hat{t}_\gamma q_\gamma(\hat{t}_\gamma) - \int_0^{\hat{t}_\gamma} q_\gamma(s)ds \qquad \text{integration by parts.}
\end{aligned}$$

Thus, the payment scheme induced by the jump-bidding strategy satisfies the envelope formula for all alleged interim types $\hat{t}_\gamma \ge t_\gamma$.

It follows that it is optimal for bidder 1 to reveal his interim type t_γ truthfully by submit the jump bid $\beta_{\gamma,G_{-\gamma}}(t_\gamma)$. ■

Thus, we have shown bidder γ's incentive to follow strategy 2.c.

6.3.3 A Local Bidder's Incentive for Contingent Plan 3

The optimality of plan 3.a is obvious, since the global bidder does not jump bid. Action 3.b.ii.B is dominant. Action 3.b.ii.A is a best reply because bidder i has learned that some other local bidder $j \neq i$ has a higher value than he does. With local bidder j following contingent plan 3.b.ii.B, the price for item B will be higher than the value for i.

Let us verify the incentive to follow the jump-bidding strategy 3.b.ii. Consider the contingency under 3.b.ii, i.e., bidder γ has made a jump bid x_γ.

Claim 1: Conditional on not dropping out, a local bidder i prefers making a jump bid that signals $t_i > \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$ to not doing so. Expecting the global bidder to buy item A immediately if no local bidder responds with a jump bid that signals the above event (contingent plan 2.c.ii.B), if bidder i does not make a jump bid in this manner, there is a positive probability with which the global bidder buys A immediately, thereby raising the global bidder's valuation of item B from $u_\gamma(AB) - p_A$ to $u_\gamma(AB) - u_\gamma(A)$ (since $u_\gamma(A) < p_A$) and hence reducing bidder i's winning probability, which by the envelope formula, reduces i's expected payoff. Thus, bidder i prefers to submit a jump bid that signals $t_i > \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$.

Claim 2: If bidder γ's jump bid reveals to local bidder i that $t_i > \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$, then local bidder i does not want to drop out. That is because there is a positive probability with which i wins with positive profits.

Claim 3: If a local bidder i will respond with a jump bid in the contingency under plan 3.b.ii, it is optimal for i to represent his type t_i truthfully in the jump bid, i.e., to make a jump bid equal to $\beta_{i,G_{-i}}(t_i)$ given the updated posterior G_{-i}. The proof is the same as that for Lemma 4, with the substitutions $\hat{t}_\gamma \to \hat{t}_i$, $t^{(1)}_{-\gamma} \to t^{(1)}_{-i}$, $q_\gamma \to q_i$, and $G_{-\gamma} \to G_{-i}$.

We now show that the jump-bidding strategy 3.b.ii is a best reply for any local bidder i. Under the contingency of 3.b.ii, $t_i > \inf \beta^{(1)}_{\gamma,G_{-\gamma}}(x_\gamma)$, so Claim 2 says that bidder i does not drop out immediately, Claim 1 implies that he would make a jump bid to signal the event "$t_i > \inf \beta^{(1)}_{\gamma,G_{-\gamma}}(x_\gamma)$," and Claim 3 implies that his jump bid signals his type truthfully.

Finally, contingent plan 3.b.i is a best reply for any local bidder i, because under the contingency of 3.b.i, bidder γ's jump bid x_γ has revealed to i that $t_i \leq \inf \beta^{-1}_{\gamma,G_{-\gamma}}(x_\gamma)$. If bidder i does not drop out immediately, Claim 1 implies that bidder i would make a jump bid to signal "$t_i > \inf \beta^{-1}_{-1,G_{-\gamma}}(x_\gamma)$." But this signal is not truthful, so it is suboptimal for the bidder according to Claim 3. It follows that it is optimal for bidder i to drop out immediately as recommended by 3.b.i.

7 Overconcentration

If the continuation equilibrium in Proposition 2 is expected, the global bidder in the simultaneous auctions can bid for both items without the risk of negative profits, and he has no price uncertainty when buying any item. That effectively allows the global bidder to bid for the package $\{A, B\}$. Analogous to the case in a single-object English auction with private values, the next lemma follows. Its proof is in Appendix C.

Lemma 5 *If the continuation equilibrium constructed in Proposition 2 is played once any decisive event occurs, then in any perfect Bayesian equilibrium of the simultaneous-auction* *game and before any decisive event occurs,*

i. if the global bidder γ loses both auctions simultaneously then almost surely

$$u_\gamma(AB) \leq \max_{i \neq \gamma, j \neq \gamma} \{u_i(A) + u_j(B)\}, \tag{8}$$

ii. if γ loses the auction for item $k \in \{A, B\}$ while he is continuing for the other item $-k$, then almost surely

$$u_\gamma(-k) \geq u_\gamma(AB) - \max_{i \neq \gamma} u_i(k). \tag{9}$$

This lemma, coupled with Proposition 2, implies that the global bidder does not give up bidding for an item until he knows for sure that his valuation of the item cannot exceed those of his rivals. The local bidders, in contrast, may choose to stop bidding for an item, thereby risking a positive probability of losing the item, even when there is a positive probability with which his valuation is higher than the global bidder's. For instance, when an A-local bidder becomes the only one competing with the global bidder for item A while the auction for item B is still going on, if the local bidder drops out at a price p_A, he may still get to buy item A at price p_A because the B-local bidders may outbid the global bidder, who would then concede item A to this A-local bidder. Hence a local bidder may free ride the others.[7] While the local bidders cannot overcome the incentive constraint for them to cooperate fully in their competition against the global bidder, the global bidder can overcome the exposure problem due to the jump-bidding signals. That implies the overcentration prediction.

Proposition 3 *Assume that the distributions of the bidders' valuations are atomless and that (8) does not almost surely hold according to the prior distributions. If the continuation equilibrium constructed in Proposition 2 is played once any decisive event occurs, then in any perfect Bayesian equilibrium of the simultaneous-auction game, overconcentration occurs with a positive probability and overdiffusion occurs with zero probability.*

Proof Consider any perfect Bayesian equilibrium specified by the hypothesis of the proposition. Claim (i) of Lemma 5, coupled with Proposition 2, implies that overdiffusion occurs with zero probability at this equilibrium.

To establish the overconcentration claim, consider the following event, denoted by E_A:

> At some current prices, global bidder γ is active for both items, some bidder α is the only active bidder for item A (the other A-local bidders either have dropped out a while ago or have just dropped out with bidder α deciding whether to drop out immediately), and for some $m = 1, 2, \ldots$, bidders $i_1, \ldots, i_m$ are the B-local bidders active for item B.

[7] This is the *threshold problem* typically attached to package auctions.

Analogously, let E_B denote the event such that items A and B switch roles (hence the A-local bidder α is replaced by a B-local bidder β).

The event $E_A \cup E_B$ occurs with strictly positive probability. Otherwise, the global bidder would have zero probability of winning both items. Then Claim (i) of Lemma 5 implies that (8) holds almost surely according to prior distributions, which is impossible by the hypothesis of the proposition.

Therefore, to prove the overconcentration claim, it suffices to prove that, conditional on event E_A (resp., E_B), there is a positive probability with which bidder α (resp., β) drops out at a price below $u_\alpha(A)$ (resp., $u_\beta(B)$). Let us see why that suffices. By symmetry, consider only the case of E_A. If bidder α drops out at a price $p_A < u_\alpha(A)$, then a decisive event occurs and by the continuation equilibrium in Proposition 2, the two items are allocated efficiently except that the dropout price p_A is taken as bidder α's valuation of A, hence overconcentration occurs with strictly positive probability.

Thus, consider event E_A and let $u_\alpha(A)$ be the value of item A for the remaining A-local bidder α. By the previous paragraph, the proof is complete if at the equilibrium there is a positive measure of prices $p_A < u_\alpha(A)$ such that bidder α quits when the price of item A reaches p_A under event E_A. Hence suppose that bidder α's equilibrium strategy requires him to stay active until the price reaches $u_\alpha(A)$ as long as the auction of item B is still going on. We shall prove a contradiction by showing that the bidder strictly prefers deviating from such equilibrium strategy when the current price of item A is sufficiently close to $u_\alpha(A)$.

Suppose bidder α drops out at the current price p_A, then a decisive event occurs and in its continuation equilibrium, global bidder γ eventually loses item A if and only if one of the following two events occurs:

$$p_A > u_\gamma(A) \quad \text{and} \quad p_A + \max\nolimits_{k=1,\dots,m} u_{i_k}(B) > u_\gamma(AB) \quad \text{and} \quad u_\gamma(B) < u_\gamma(AB) - p_A, \tag{10}$$
$$\text{or} \quad p_A > u_\gamma(A) \quad \text{and} \quad u_\gamma(B) > u_\gamma(AB) - p_A. \tag{11}$$

(Condition (10) corresponds to the event that bidder γ loses both items in the jump-bidding equilibrium, and (11) the event that γ loses item A because winning both items is worse-off than winning only B.) Upon winning item A, local bidder α pays p_A. Hence the expected payoff for bidder α to drop out now is equal to

$$(u_\alpha(A) - p_A)\text{Prob}\{A \to \alpha \mid (p_A, p_B)\},$$

where $\text{Prob}\{A \to \alpha \mid (p_A, p_B)\}$ is equal to a positive fraction (which is equal to one if no other bidder has just dropped out from item A at the current price p_A) of the probability of

$$\max\{u_\gamma(A), u_\gamma(AB) - \max\nolimits_{k=1,\dots,m} u_{i_k}(B)\} < p_A < u_\gamma(AB) - u_\gamma(B)$$
$$\text{or} \quad \max\{u_\gamma(A), u_\gamma(AB) - u_\gamma(B)\} < p_A$$

conditional on the current history. There is a positive-probability set of valuation functions such that the conditional probability $\text{Prob}\{A \to \alpha \mid (u_\alpha(A), p_B)\}$ is a positive number. Thus, for any $p_A < u_\alpha(A)$ sufficiently close to $u_\alpha(A)$, $\text{Prob}\{A \to \alpha \mid (p_A, p_B)\}$ is bounded from below by some $M > 0$, so the expected payoff for bidder α to drop out at the current price p_A is bounded from below by

$$(u_\alpha(A) - p_A)M = O(u_\alpha(A) - p_A).$$

Compare this to the expected payoff for bidder α if α follows the equilibrium strategy of not dropping until the price of A reaches $u_\alpha(A)$. For any $t_\alpha \in [p_A, u_\alpha(A)]$, consider the event in which bidder α wins item A with a nonnegative payoff if his true type is t_α instead of $u_\alpha(A)$ and if he follows the equilibrium strategy. Bidder α achieves such an outcome only if the global bidder γ drops out from the auction for A during the interval when its price rises from the current p_A to bidder α's valuation t_α. If bidder γ does so, γ quits either (i) before or (ii) after a decisive event occurs.

In case (i), Lemma 5 implies that

$$u_\gamma(AB) \le t_\alpha + \max_{k=1,\dots,m} u_{i_k}(B) \quad \text{or} \quad u_\gamma(B) \ge u_\gamma(AB) - t_\alpha,$$

which implies that

$$\begin{aligned} t_\alpha &\ge \min\left\{u_\gamma(AB) - \max_{k=1,\dots,m} u_{i_k}(B), u_\gamma(AB) - u_\gamma(B)\right\} \\ &= u_\gamma(AB) - \max\left\{\max_{k=1,\dots,m} u_{i_k}(B), u_\gamma(B)\right\}. \end{aligned}$$

In case (ii), since bidder α abides to the equilibrium strategy of not dropping out until the price of item A reaches t_α as long as auction B is still going on, the decisive event must be triggered by the B-local bidders at some price p'_B of item B. Then by Proposition 2, with the roles of A and B switched, bidder α's winning event is

$$\begin{gathered} u_\gamma(B) \ge p'_B, \quad u_\gamma(AB) - u_\gamma(B) \le t_\alpha, \quad \text{or} \\ u_\gamma(B) < p'_B, \quad u_\gamma(A) \ge u_\gamma(AB) - p'_B, \quad u_\gamma(A) \le t_\alpha, \quad \text{or} \\ u_\gamma(B) < p'_B, \quad u_\gamma(A) < u_\gamma(AB) - p'_B, \quad u_\gamma(AB) \le p'_B + t_\alpha. \end{gathered}$$

The first line implies that $t_\alpha \ge u_\gamma(AB) - u_\gamma(B)$; the second and third lines each imply that $t_\alpha \ge u_\gamma(AB) - p'_B$. This, coupled with the fact that $p'_B \le \max_{k=1,\dots,m} u_{i_k}(B)$, implies that

$$\begin{aligned} t_\alpha &\ge \min\{u_\gamma(AB) - p'_B, u_\gamma(AB) - u_\gamma(B)\} \\ &\ge \min\left\{u_\gamma(AB) - \max_{k=1,\dots,m} u_{i_k}(B), u_\gamma(AB) - u_\gamma(B)\right\} \\ &= u_\gamma(AB) - \max\left\{\max_{k=1,\dots,m} u_{i_k}(B), u_\gamma(B)\right\}. \end{aligned}$$

Thus, if bidder α with type t_α follows the equilibrium strategy, then he wins A only if

$$u_\gamma(AB) - \max\left\{u_\gamma(B), \max_{k=1,\ldots,m} u_{i_k}(B)\right\} \le t_\alpha.$$

Let

$$q_\alpha(t_\alpha \mid p_A) := \text{Prob}\left\{u_\gamma(AB) - \max\left\{u_\gamma(B), \max_{k=1,\ldots,m} u_{i_k}(B)\right\} \le t_\alpha \;\middle|\; H(p_A, p_B)\right\}, \quad (12)$$

where $H(p_A, p_B)$ denotes the event that both auctions have been continuing up to the current prices (p_A, p_B), with global bidder γ active in both auctions and local bidders $i_1, \ldots, i_m$ active in auction B. In particular, $H(p_A, p_B)$ implies that

$$u_\gamma(AB) - \max\left\{u_\gamma(B), \max_{k=1,\ldots,m} u_{i_k}(B)\right\} \ge p_A, \quad (13)$$

otherwise bidder γ would have quit from auction A by now.

By the above calculation, $q_\alpha(t_\alpha \mid p_A)$ is the upper bound of the probability with which bidder α wins item A by following the equilibrium, given $u_\alpha(A) = t_\alpha$, conditional on the current history $H(p_A, p_B)$. Since the random variable

$$u_\gamma(AB) - \max\left\{u_\gamma(B), \max_{k=1,\ldots,m} u_{i_k}(B)\right\}$$

is independent of the equilibrium under consideration, with the prior distributions atomless, the probability for this random variable to be bounded between p_A and t_α is in the order of $t_\alpha - p_A$. Thus,

$$q_\alpha(t_\alpha \mid p_A) = O(t_\alpha - p_A). \quad (14)$$

By the Milgrom-Segal envelope theorem, bidder α's equilibrium expected payoff is less than or equal to

$$\int_{p_A}^{u_\alpha(A)} q_\alpha(t_\alpha \mid p_A) dt_\alpha = (u_\alpha(A) - p_A) q_\alpha(\tau_\alpha \mid p_A) \le (u_\alpha(A) - p_A) O(u_\alpha(A) - p_A) = o(u_\alpha(A) - p_A).$$

for some τ between t_α and p_A. Thus, for any $p_A < u_\alpha(A)$ sufficiently close to $u_\alpha(A)$, bidder α's expected payoff from following the equilibrium is less than his expected payoff from dropping out at the current price p_A. Thus, local bidder α is better-off quitting slightly before the price reaches his value. This contradiction proves the desired assertion. ■

8 Conclusion

Due to the open transparent nature of an English auction, it should not be surprising that the simultaneous auctions analyzed above may admit multiple equilibria. An open question is whether there exists an equilibrium where the exposure problem is not eliminated or partially mitigated. While addressing this question is beyond the scope of this paper, it can be noted that the answer may vary with the fine details of the model. For example, when the last A-local bidder drops out, the global bidder may somehow expect that the remaining B-local bidders would never jump bid, hence he would buy good A immediately without waiting for their signals. In that case, the exposure problem is not mitigated at all. However, if we slightly modify the model so that no one can buy the item in a paused auction until the exogenous δ-second pause expires, then one can show that this exposure-prone equilibrium cannot survive, because a remaining B-local bidder would then have time to signal through jump bids.

The main point of this paper is not that the exposure problem can be mitigated if the decentralized simultaneous auctions are modified to allow jump bidding. Rather, the message is that if we build models that capture the dynamic details of English auctions, we can construct new self-enforcing arrangements in which economic agents signal and forecast prices without relying on any central coordination.

A The Proof of Remark 1

Without loss of generality, suppose there are only three distinct bidders, an A-local bidder α, a B-local bidder β, and the global bidder γ. Suppose that

$$u_\alpha(A) > u_\gamma(A) > 0, \qquad u_\beta(B) > u_\gamma(B) > 0,$$

which is an event with positive probability because F_β and F_α have no gap. In this event, the eventual prices for A and B for bidder γ are respectively $u_\alpha(A)$ and $u_\beta(B)$ if γ does not quit.

We claim that, within this event, overconcentration and overdiffusion are each possible with strictly positive probability.

To prove that, pick $(u_\gamma(AB), u_\gamma(A), u_\gamma(B))$ such that $u_\gamma(AB) - u_\gamma(A)$ and $u_\gamma(AB) - u_\gamma(B)$ are interior points of the supports of F_β and F_α, respectively. There is a positive probability of such $(u_\gamma(AB), u_\gamma(A), u_\gamma(B))$. By the choice of $(u_\gamma(AB), u_\gamma(A), u_\gamma(B))$ and the assumption that F_α and F_β have no gap, we know that if $0 \le p_B < u_\gamma(AB) - u_\gamma(A)$ and $0 \le p_A < u_\gamma(AB) - u_\gamma(B)$,

$$0 < \mathbb{E}_{u_\beta(B)}\left[(u_\gamma(AB) - u_\gamma(A) - u_\beta(B))^+ \mid u_\beta(B) \ge p_B\right] < u_\gamma(AB) - u_\gamma(A) - p_B \quad (15)$$

$$0 < \mathbb{E}_{u_\alpha(A)}\left[(u_\gamma(AB) - u_\gamma(B) - u_\alpha(A))^+ \mid u_\alpha(A) \ge p_A\right] < u_\gamma(AB) - u_\gamma(B) - p_A \quad (16)$$

Overdiffusion: By Eqs. (2)–(3) and the second inequality of (15) and that of (16), $v_\gamma(A, p_B) < u_\gamma(AB) - p_B$ and $v_\gamma(B, p_A) < u_\gamma(AB) - p_A$ if $0 \le p_B < u_\gamma(AB) - u_\gamma(A)$ and

$0 \le p_A < u_\gamma(AB) - u_\gamma(B)$. It then follows from provision (b) of the equilibrium strategy that, conditional on both auctions still going on, bidder γ will drop out from at least one auction when the price trajectory reaches some (p_A^*, p_B^*), with $u_\gamma(AB) < p_A^* + p_B^*$. Thus, it is probable to have $(u_\gamma(AB), u_\alpha(A), u_\beta(B))$ such that $u_\gamma(AB) > u_\alpha(A) + u_\beta(B)$ and "$v_\gamma(A, u_\beta(B)) < u_\alpha(A)$ or $v_\gamma(B, u_\alpha(A)) < u_\beta(B)$," i.e., overdiffusion.

Overconcentration: By Eq. (2) and the first inequality of (15), $v_\gamma(A, p_B) > u_\gamma(A)$ if $0 \le p_B < u_\gamma(AB) - u_\gamma(A)$. Thus, $p_A^* > u_\gamma(A)$ for the aforementioned dropout price for bidder i. By the no-gap assumption of F_α, there is a positive probability with which $p_A^* > u_\alpha(A) > u_\gamma(A)$. Because $p_A^* > u_\alpha(A)$, bidder γ wins item A. Consequently, he bids for item B up to $u_\gamma(AB) - u_\gamma(A)$. With a positive probability, $u_\beta(B) < u_\gamma(AB) - u_\gamma(A)$ (hence bidder γ also wins B) and $u_\beta(B) > u_\gamma(AB) - u_\alpha(A)$ (so it is more efficient to award the items separately to bidders α and β). Hence this is an overconcentration event.

B The Proofs of Lemmas 1–3

Lemma 1 Let p_B denote the eventual price for item B. By $u_\gamma(B) \ge u_\gamma(AB) - p_A$,

$$u_\gamma(B) - p_B \ge u_\gamma(AB) - p_A - p_B. \tag{17}$$

Then bidder γ would rather bid only for item B than bid for both items. Furthermore, coupled with (1), Ineq. (17) implies that $u_\gamma(B) - p_B \ge u_\gamma(A) + u_\gamma(B) - p_A - p_B$, hence $0 \ge u_\gamma(A) - p_A$, so bidder γ does not want to buy item A alone.

Lemma 2 By the hypothesis $u_\gamma(B) < u_\gamma(AB) - p_A$,

$$u_\gamma(B) - p_B < u_\gamma(AB) - p_A - p_B,$$

hence the payoff from buying both items is greater than the payoff from buying item B alone. Also, it is unprofitable to buy A alone since $u_\gamma(A) < p_A$. Thus, during the pause, bidder γ chooses between two goals: (i) to buy both items, which if realized would yield a payoff equal to $u_\gamma(AB) - p_A - p_B$; (ii) to buy none, which if realized would yield zero payoff. Thus, during the pause, the valuation of winning item B is equal to the valuation of buying both items, i.e., $u_\gamma(AB) - p_A - p_B$, as asserted.

Lemma 3 By definition (6), the function $\beta_{i,G_{-i}}$ is weakly increasing. Suppose it is not strictly increasing, then for some $t_i, t_i' \in T_i$ with $t_i < t_i'$, $\beta_{i,G_{-i}}(t_i) = \beta_{i,G_{-i}}(t_i')$. Then (6) implies that $t_{-i}^{(1)}$ has zero mass in $(t_i, t_i']$, i.e.,

$$\prod_{j \ne i} G_j(t_i') = \prod_{j \ne i} G_j(t_i). \tag{18}$$

Note $t_i \geq \inf T_i \geq \inf T_{-i}^{(1)}$. For any $x > t_i$, $\prod_{j \neq i} G_j(x) > 0$. Thus,

$$\forall x > t_i \, \forall j \neq i \;\; G_j(x) > 0. \tag{19}$$

Pick any $k \neq i$. If $t_k \in (t_i, t_i']$, then (19), applied to $x = t_k$, implies that there is a positive probability with which t_k is the realized $t_{-i}^{(1)}$. Then (18) implies $\mathrm{Prob}\{t_i < t_k \leq t_i'\} = 0$, hence (7).

C The Proof of Lemma 5

First, in any (perfect Bayesian) equilibrium of any single-unit English auction game with private values, the allocation is ex post efficient. That is because, when the current price has exceeded a bidder's valuation of the good, the unique best response for this bidder is to drop out. Thus, if a bidder is to win the English auction, the price he pays does not exceed the highest value of his rivals. Therefore, a bidder would avoid any action that results in losing with a positive probability if his posterior belief still assigns a positive probability to the event that his value is higher than his rivals'. With all actions commonly observed, his posterior belief is the same as the (commonly known) equilibrium posterior belief. Thus, the equilibrium probability for inefficient allocation is zero.

Second, in our simultaneous auctions game, once the global bidder γ has lost at least one of the two auctions, the allocation of any continuation equilibrium is ex post efficient in the sense that each item goes to a bidder whose standalone value of the good is the highest among all the bidders remaining active for that item. This follows from the first assertion because, once the global bidder is no longer active in both auctions, each auction becomes an independent single-unit English auction, with each active bidder's private value being the bidder's standalone value.

Third, for any local bidder i who values only item $k \in \{A, B\}$, if the total current price of all the items that he is bidding for has exceeded $u_i(k)$, then the unique best response for i is to drop out from all the items that he is bidding for. (We allow the possibility for a local bidder to bid for the item that he does not value.) Then, if the global bidder γ is to win both items, the total price of the two items does not exceed the right-hand side of (8). Thus, as in the first assertion, as long as (8) does not almost surely hold, γ would not take any action that with strictly positive probability results in losing both items simultaneously. Therefore, claim (i) of the lemma holds.

To prove claim (ii), consider bidder γ's options before any decisive event occurs. If he loses in auction $k \in \{A, B\}$ while continuing in the other auction $-k$, then it follows from the second assertion that his expected payoff is $\mathbb{E}[(u_\gamma(k) - \max_{i \neq \gamma} u_i(k))^+]$. If he continues for both items until either (8) holds or a decisive event occurs, then by Proposition 2 and individual rationality of the local bidders, his expected payoff is greater than or equal to

$$\mathbb{E}\left[\max\left\{0, u_\gamma(A) - \max_{i\neq\gamma} u_i(A), u_\gamma(B) - \max_{i\neq\gamma} u_i(B), u_\gamma(AB) - \max_{i\neq\gamma} u_i(A) - \max_{i\neq\gamma} u_i(B)\right\}\right]. \tag{20}$$

By Jensen's inequality and the fact that $\max\{x, y, z\}$ is a convex function of (x, y, z), (20) is greater than or equal to $\mathbb{E}[(u_\gamma(k) - \max_{i\neq\gamma} u_i(k))^+]$ for each $k \in \{A, B\}$, and the inequality is strict unless bidder γ knows that almost surely

$$u_\gamma(k) - \max_{i\neq\gamma} u_i(k) \geq u_\gamma(AB) - \max_{i\neq\gamma} u_i(k) - \max_{i\neq\gamma} u_i(-k),$$

i.e., (9) holds almost surely, as asserted by claim (ii).

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