Risk & Sustainable Management Group

Risk & Uncertainty Program Working Paper: R05_6

Research supported by an Australian Research Council Federation Fellowship
http://www.arc.gov.au/grant_programs/discovery_federation.htm

Outcomes and Strategy Choices in Tullock Contests

Flavio Menezes

Australian Centre of Regulatory Economics,Australian National University

and

John Quiggin
Australian Research Council Federation Fellow, University of Queensland

Schools of Economics and Political Science
University of Queensland
Brisbane, 4072
rsmg@uq.edu.au
http://www.uq.edu.au/economics/rsmg

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Outcomes and Strategy Choices in Tullock Contests

Flavio Menezes
Australian Centre of Regulatory Economics
Australian National University
Canberra, ACT, 0200
Australia
Email: flavio.menezes@anu.edu.au

John Quiggin
School of Economics
University of Queensland
Email: j.quiggin@uq.edu.au

24 March 2005

Abstract

We explore the relationship between the choice of the strategy space and outcomes in Tullock contests. In particular, in a framework where one of the contest's participants moves first, we show that there is an equilibrium where this individual wins the contest with probability one. We also show that not only the nature of the outcome changes (e.g., who wins the contest) with the choice of the strategy space but also that a contest organiser might have preferences over this space. We argue that ultimately the analyst does not have complete freedom to choose the strategy space. Instead, he or she should consider the strategies that are permitted by the organisers of a formal contest, whose interests might lie in maximising returns. That is, the analyst's choice of the strategy space is not neutral.

JEL: C7.

Kewords: Strategy space, Tullock contests.

1 Introduction

There are many strategic interactions where agents spend resources to dispute some rent or prize. There is a large literature, which perhaps can be loosely labelled as the economics of contests, that examines this type of strategic interactions. It is commonly assumed in this literature that agents have only one instrument to influence the outcome's choice by the decision maker – usually this instrument is referred to as effort or payment. [1] However, several authors have pointed out that there are many other instruments available to players in a rent-seeking game or contest. For example, Haan and Schoonbeek (2003) examine a rent-seeking contest where players spend both money and effort to influence decision makers. In the same vein, Konrad (2000) considers the interaction of standard effort and sabotage (effort that reduces particular rivals's performance). Perhaps not surprisingly, these authors obtain results that are quite distinct from those in the literature where agents have only one instrument.

In this note we deal with an even more fundamental problem. The same interaction may be modelled with different descriptions of the instruments available to the players. In an oligopoly game, for example, any given player may be regarded as choosing a price, a quantity or a markup, given the residual demand curve conditional on the choices of the other players. If these different representations of instruments are taken to determine the strategy space available to the players, different equilibrium outcomes arise.[2]

Most of the strategic contests that have been analysed using a game-theoretic framework are 'games without rules.' Unlike, say, poker or chess, or the formalised interactions of an auction, there is no rulebook that specifies the strategies that interest groups disputing a rent might adopt. Hence, any proposed game-theoretic representation is, in essence, a ´conjectural variations' model, in which the conjecture is that other players will choose to hold constant some

[1] For an excellent survey of this literature the reader is refereed to Konrad (2004).

[2] The quantity (Cournot) and price (Bertrand) equilibria are well known. Grant and Quiggin (1994) examine the case of equilibrium in markups.

particular variable, described as their ´strategy.' The fact that a player may be modelled as setting the value of an instrument to influence outcomes does not necessarily imply that the contest is a game with values of the instrument as strategies.

Elsewhere we argue that, in economic analysis, outcomes, and not strategies, are the natural primitives.[3] In this paper we formally show that the mere specification of the rule relating payoffs to contributions tells us very little about the equilibrium outcome(s) of the contest. Indeed, with the appropriate choice of the strategy space, it is possible to obtain distinct and meaningful outcomes.

2 Outcomes and the choice of the strategy space

Consider a Tullock contest, for example an election or an all-pay auction, in which players i make contributions p_i with probability of winning a unit prize given by

$$\pi_i = \frac{p_i}{\sum_j p_j}. \tag{1}$$

The payoff to player i is $u_i(p_i, p_{-i}) = \pi_i - p_i$.[4]

A standard approach to this problem is to model the contest as a game in which the strategy space for player i consists of contribution levels p_i, then to consider possible Nash equilibria of the game. Commonly, the specification of the strategy space is read directly from the contest description given above, with no further discussion of players' beliefs, institutional structures and so on.

One strong prediction of this model is that there are no Nash equilibria in which only one player contributes, winning with probability 1. The argument is reasonably straightforward. Consider a candidate equilibrium in which $p_1 > 0, p_j = 0$ for $\dot{j} \neq 1$. Then player 1 can benefit by reducing her contribution. Also, if p_1 is small enough, other players can benefit by contributing. More formally,

[3] Menezes and Quiggin (2004).

[4] Under an alternative, somewhat richer formulation, the probability of winning is given by $\pi_i = \frac{\lambda_i p_i}{\sum_j \lambda_j p_j}$, where λ_i is an effectiveness variable.

$\frac{\partial u_1}{\partial p_1} = -1$ at $p_2 = p_3 = ... = p_n = 0$. Similarly, Player 2's best response when $p_1 > 0$ and $p_3 = ... = p_n = 0$ is such that $\frac{\partial u_2}{\partial p_2} = \frac{1}{p_1} - 1 > 0$ at $p_2 = 0$. Thus, Player 2's best reply to $p_1 > 0$ and $p_3 = ... = p_n = 0$ involves a positive effort or contribution.

Indeed, in this game, the unique (symmetric) Nash equilibrium is such that $p_i = \frac{n-1}{2n} = p$ for $i = 1, ..., n$. To see this, note that $\frac{n-1}{2n}$ is the solution to $\frac{\partial u_1}{\partial p_1} |_{p_2=p_3=...=p_n} = \frac{1}{p_1+(n-1)p} - \frac{p_1}{(p_1+(n-1)p)^2} - 1 = 0$.

That is, under a strategy space where players choose a contribution level p_i, the prediction is that all players will make positive and identical contributions. In reality, though, uncontested elections are common. Indeed, we show next that it is possible to obtain this as an equilibrium outcome of asymmetrical games with different specifications of the strategy space, in which the uncontested winner is the first mover.

For example, suppose that player 1's strategy space is given by a probability of winning ${\pi^*}_1$,$0 < \pi_1^* < 1$, with the special interpretation of a minimal contribution δ_1 if all other players choose 0, in which case player 1 receives the prize with probability 1. For $j \neq 1$, the strategy spaces consist of contribution levels p_j, as before, and again we avoid continuity problems by requiring that either $p_j = 0$ or $p_j > \delta_j$ for some $\delta_j > 0$.

That is, having chosen π_1^*, and conditional on the (non-zero) strategies p_j of the other players, player 1 is required to contribute p_1^* such that

$$\pi_1^* = \frac{p_1^*}{\sum_j p_j}$$

It's apparent that the symmetric Nash equilibrium of the standard Tullock game is also a Nash equilibrium of the new game. To check this, note that if player 1 chooses π_1^* consistent with a contribution $p_1 = \frac{n-1}{2n}$, it is a best reply for players $2, ..., n$ to contribute $p = \frac{n-1}{2n}$. Similarly, when players $2, ..., n$ contribute $p = \frac{n-1}{2n}$, player 1's best reply is to choose $\pi_1^* = \frac{1}{n}$, which is consistent with a contribution of $p_1 = \frac{n-1}{2n}$.

However, there is also an additional family of Nash equilibria where player 1 makes the minimal contribution δ and receives the prize with probability 1. To see this, observe that, if $\pi_1 > 1 - \frac{1}{\delta_j}, \forall j \neq 1$, the best-reply strategy for player j is $p_j = 0, \forall j \neq 1$. Conversely, given that all $p_j = 0, \forall j \neq 1$, the choice of π_1 is weakly optimal, since player 1 pays δ_1 and receives the prize regardless of the choice of π_1. Since player 1 moves first, this is subgame perfect. (The family of such equilibria, corresponds to values of π_1^* in the interval $max_j \left\{1 - \frac{1}{\delta_j}, 1\right\}$, but the outcome is the same in each case).

There are many other specifications of strategies we might consider, consistent with the outcome description. For example, players might specify demand curves for the good, . To work this out a bit further, suppose each player nominates a value v_i for the good, indicating willingness to pay $p_i = \sqrt{\pi} v_i$ for probability π of receiving the good. Then an equilibrium is a set of payments $p_i = \sqrt{\pi_i} v_i$ such that $\sum_i \pi_i = 1$ and

$$\pi_i = \frac{\sqrt{\pi_i} v_i}{\sum_j \sqrt{\pi_j} v_j}$$

Observe that, in this context, if one player reduces their offer, the others increase theirs.

To illustrate, consider the case where $n = 2$ where we can write

$$\pi_1 = \frac{\sqrt{\pi_1} v_1}{\sqrt{\pi_1} v_1 + \sqrt{(1 - \pi_1)} v_2}$$

and observe that this is solved by

$$\pi_1 = \frac{v_1^2}{v_1^2 + v_2^2}$$

so the expected return to player 1 is given by

$$\begin{aligned} \pi_1 - \sqrt{\pi_1} v_1 &= \frac{v_1^2}{v_1^2 + v_2^2} - \frac{v_1^2}{\sqrt{v_1^2 + v_2^2}} \\ &= v_1^2 \left(\frac{1}{v_1^2 + v_2^2} - \frac{1}{\sqrt{v_1^2 + v_2^2}} \right). \end{aligned}$$

To find a symmetric equilibrium we set $v_2 = y$ and find Player 1's best reply:

$$\frac{\partial u_1}{\partial v_1} = \frac{-2v_1^3}{(v_1^2+y^2)^2} + \frac{v_1^3}{(v_1^2+y^2)^{\frac{3}{2}}} + \frac{2v_1}{v_1^2+y^2} - \frac{2v_1}{\sqrt{v_1^2+y^2}}$$

In the symmetric equilibrium, $\frac{\partial u_1}{\partial v_1} \mid_{v_1=y} = 0$. This yields

$$v_1 = v_2 = \frac{\sqrt{2}}{3}$$

and $\pi_1 = \pi_2 = 0.5, p_1 = p_2 = \frac{1}{3}$. That is, a Nash equilibrium in this setup will not normally be a Nash equilibrium of the original game. Moreover, as the total amount paid will be higher in this case, organisers of an all-pay auction might prefer this rule.

3 Conclusion

The main point, then, is that the mere specification of the rule relating payoffs to contributions tells us very little about the equilibrium outcome(s) of the contest. Only if we have information about the strategy space can we apply tools of game theory such as Nash equilibrium. In the case of a formal contest, such as an all-pay auction, such information might be directly observable. Alternatively, we might be able to collect behavioral information on the set of players, indicating what kinds of descriptions of patterns of play correspond to the game-theoretic notion of strategies.

The fact that equation 1 provides a simple and compact rule for determining the winning probabilities does not necessarily mean that it is relevant in determining the strategy space. Most obviously, in the case of a formal contest, the strategies are those permitted by the organisers, whose interests lie in maximising returns rather than in simple and compact rules. Even in an informal game, the fact that some description of the outcomes is simple and compact does not mean that participants will employ this representation of their interactions with other players.

References

[1] Haan, M. and L. Schoonbeek, 2003, ´Rent seeking with efforts and bids,' *Journal of economics 79(3)*, 215-235.

[2] Grant, S. and Quiggin, J. (1994), 'Nash equilibrium with markup-pricing oligopolists', *Economics Letters* 45, 245–51.

[3] Konrad, K. A., 2004, ´Strategy in tournaments and contests,' Mimeo, available at www.wz-berlin.de/mp/mps/teaching/script-november-04.pdf.

[4] Konrad, K. A., 2000, ´Sabotage in rent-seeking contests,' *Journal of Law, Economics and Organization 16(1)*, 155-165.

[5] Menezes, F. M. and J. Quiggin, ´Games without rules,' Risk and Uncertainty Program Working Paper 7/R04, available at http://www.uq.edu.au/economics/rsmg/WP/WP7R04.pdf.