Are Changes in Education Important for the Wage Premium and Unemployment?*

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Abstract

A generalized rise in unemployment rates for both college and high-school graduates, a widening education wage premium, and a sharp increase in college education participation are characteristic features of the transformations observed in the U.S. labor market between 1970 and 1990. This paper investigates the interactions between these changes in the labor market and in educational attainment. First, it develops an equilibrium search and matching model of the labor market where education is endogenously determined. Other important features of the model are a labor market which is segmented by education levels, and the imperfect correlation between skill and education in the labor force. Second, calibrated versions of the model are used to study quantitatively whether either a skill-biased change in technology or a mismatch shock can explain the above facts, and to assess the importance of the links between education and the labor market. The skill-biased shock accounts for a considerable part of the changes but fails to produce the increase in unemployment for the educated labor force. The mismatch shock explains instead much of the change in the four variables, including the wage premium. The endogenous response of education contributes positively to fit better the model's predictions to the data on wages and unemployment, especially so under the mismatch shock.

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1 Introduction

In the U.S., over the period spanning 1970 and 1990, unemployment rates for both high-school graduates and college graduates nearly doubled, the wage differential (or premium) between these two education groups widened considerably, at the same time as the increase in college participation among high-school graduates accelerated markedly.[1] Table 1 reports figures on these four variables for the male population in 1970 and 1990.[2]

Table 1. U.S. labor market 1970-1990

year	college unemployment rate	high-school unemployment rate	wage premium	college participation
1970	1.1	2.4	1.44	0.25
1990	2.1	5.3	1.58	0.33

Is the change experienced by any one variable connected with the changes in the other variables, or is instead the observed association largely accidental? Do these connections have significant implications for the explanation of the facts and the role of labor-market policies and institutions? Although the literature dealing separately with wage inequality, unemployment, and education is significant, the relation between the three has not yet been articulated in an explicit equilibrium framework. This paper takes a first step in this direction.

The first objective is to set up a simple theoretical model where the four key labor-market variables – unemployment rates for college and high-school graduates, wage premium, and college participation rate – are jointly determined. The second objective is to use this model to study two alternative factors as candidate explanations of the U.S. changes documented in Table 1. The two alternative factors considered will be a skill-biased change in technology and a deterioration of the matching conditions between unemployed workers and firms. In previous studies each has been emphasized to analyze a particular subset of the four variables under study. For example, there seems to be compelling evidence —

[1] It is well known, though, that the increase in the wage premium was non-monotonic. In the 70's the premium actually decreased. This paper will not deal with this particular issue.

[2] This choice is not completely arbitrary since the two years correspond to the same phase of the cycle in terms of unemployment. For unemployment and education, the figures are calculated from the Statistical Abstracts of the US, US Census Bureau (1995), Tables 662 and 629, respectively. The unemployment rates refer to male civilian non-institutional population aged 25-64. The college participation of the male labor force aged 25 and over is the percentage with four or more years of college divided by the percentage with four or more years of high school. The wage premium is the average wage of college white male workers over the average wage for high-school white male workers aged 18-64 as reported in Autor et al. (1998).

such as, for example, Krusell *et al.* (2000) — of a skill-biased change in technology which may have driven the rise in wage inequality. The mismatch shock has been considered as an explanation of rising unemployment associated with more turbulent economic conditions. The unobservable heterogeneity in firms' requirements and workers' characteristics may have changed in a way that makes it harder for suitable partners to make contact (see Pissarides (2000,p22-23)).[3] The purpose is, first, to assess the ability of either hypothesis to explain the complete set of facts and, second, establish to which extent changes in education are important to rationalize the observed movements in the wage premium and unemployment rates[4]. A secondary goal is to draw the implications for residual wage inequality as a way of further assessing the merit of the two different explanations.

To pursue these goals, this paper proposes and analyzes an equilibrium matching-search model of a labor market that is segmented in two education levels. Workers are differentiated by innate productive skills and take forward-looking decisions about acquiring education and which labor-market segment (or career) to participate in. Firms decide which education segment to participate in, and which types of workers to hire taking into account the skill composition of the labor force in the different segments. Workers and firms interact in this economy in an otherwise standard framework with search and matching frictions to determine wages and vacancy/unemployment ratios in the two segments.

The analysis has two key features. The first is that it considers a labor market which is segmented. It presupposes that a worker's education is observable ex-ante and that firms post job vacancies that specify the minimal attainment required on prospective candidates to filling the post. More specifically, vacancies in the 'educated' segment require a degree and vacancies in the 'non-educated' segment do not. Segmentation means that both vacant firms and unemployed workers confine their search efforts to the one particular segment they choose, not to both. The second key feature is the assumption that there is an imperfect correlation between skill and education status. That is, the model recognizes that some of

[3]Works that study one or more of these factors to explain one or more, but not all, of the variables of interest are the following. Skill-biased change: Acemoglu (1999), Mortensen and Pissarides (1999), Saint-Paul (1996), Caselli (1999), Albrecht and Vroman (2002), Shi (2002), Wong (2003). Mismatch: Ljungqvist and Sargent (1998), Marimon and Zilibotti (1999).

[4]Some authors, like Acemoglu (1999), consider that key determinants of college enrollment in the early 70's were mainly the baby boom and the Vietnam war. The data in Topel (1997, Figure 4) suggests a strong correspondence between the college/high school wage premium and the enrollment ratio as the fraction of men 20-24 with some college for the period 1963-95. The present paper sets aside the factors emphasized by Acemoglu (1999) and focuses instead on the possibilities of an explanation consistent with Topel (1997)'s findings.

the workers in possession of a degree may have a low skill whereas some workers without a degree may have a high skill. This requires that a person's skill is subject to important early influences other than college education.[5] The imperfect correlation skill-education permits that, by altering the skill composition of the population by education level and the labor force participating in the different segments, endogenous changes in education and career (or segment) choices may have implications for equilibrium unemployment and wages across education groups.

Numerical experiments are conducted to study quantitatively the effects in the model of a skill biased change in technology and a negative shock to the technology of matching. The benchmark parameters are set to match some long-run observations for the US economy and the 1970 values of key endogenous variables. The experiments produce steady-state equilibrium outcomes that are to be compared with the 1990 observations in the data.

The analysis shows that the skill-biased change in technology, while producing a considerable part of the 1970-1990 increase in the wage premium, the non-educated unemployment rate, and college education, fails to bring about the increase in the educated unemployment rate. The direct impact of the skill-biased shock itself explains much of the rise in the wage premium and the non-educated unemployment rate. Nevertheless, the changes in education have a sizable quantitative impact on the amount of variation in these two variables, but not in the educated unemployment rate. On the other hand, the mismatch shock accounts for much of the variation in all the US labor market variables under study, including the wage premium. In this story, the mismatch shock dominates the increase in unemployment rates, whereas the rise in the wage premium is largely the result of a change in the composition of the educated population that improves the relative average skill of the typical graduated worker. The change in education decisions proves crucial, not only quantitatively, but also qualitatively, for the mismatch hypothesis to reproduce the facts. The two theories have also implications for within-group wage inequality. Neither does a good job at predicting rising residual wage inequality. There is one primitive parameter tightly related to the distribution of skills in the population which cannot be calibrated beyond certain boundaries. The main results appear to be fairly, yet not completely, robust to variation in this parameter.

[5] Haveman and Wolf (1995), for example, report that achievement is highly correlated with parents' income and education. Thanks to one of the referees for pointing this out.

A number of papers have also studied the implications of shocks for the distribution of wages and unemployment in a search-matching setup. The literature dealing with the wage premium and unemployment includes Mortensen and Pissarides (1999), Acemoglu (1999), and Saint-Paul (1996). Albrecht and Vroman (2002) add also residual inequality. Hornstein et al. (2002) focus on residual inequality along with general unemployment. Shi (2002) and Wong (2003) restrict their attention to the study of the wage premium and residual inequality. The present paper differs from these works in a number of aspects. First, here education is endogenized and thus the merit of different explanations can be evaluated against their implications for changes in educational attainment. In the rest of papers, education changes are either treated parametrically or simply absent. The second difference is that those works dealing with the wage premium treat education as equivalent to skill. In the current paper, the imperfect correlation between education and skill has an important role and is determined as part of the equilibrium outcomes through the education and career choices of workers with different characteristics. These choices are absent from the papers cited. Third, and related to the previous point, the present model assumes that the labor market is segmented in terms of jobs with different observable education requirements. Technically, there is matching function for each segment. This is also a feature of Saint-Paul (1996). The assumption in Mortensen and Pissarides (1999) is similar but, there, a different segment is associated with each different productivity-skill level, and outcomes in each segment are determined independently of changes that affect other segments such as shifts in the skill composition of the labor force. Shi (2002) has a model of directed search where the endogenous nature of the matching process leads to segmentation. In Acemoglu (1999), Albrecht and Vroman (2002), and Wong (2003) there is no segmentation in the sense of the present paper, so education is not used to sort applicants into job categories through differentiated matching processes.[6] Finally, the present paper uses data on several variables to evaluate the implications of changes in both the technology skill-bias and the matching process. The rest of papers do not analyze the mismatch shock. Changes in

[6] In the equilibria studied in the present paper, all firms within any single segment will be of the same type. This rules out situations with two types of firms catering for workers with different skill level in the same segment, which is precisely the type of situation that corresponds to a *separating* equilibrium in Acemoglu (1999)'s model of a single-segment market. The present paper assumes instead two segments and only considers *pooling* equilibria within each segment. This seems consistent with the focus of the paper on inter-group differences, whereas the approach in Acemoglu (1999) looks more appropriate to study within-group differences. Similar comments carry over to Albrecht and Vroman (2002) and Wong (2003).

the skill supply and/or the general level of technology not studied in this paper have also been considered in the other papers. Hornstein et al. (2002) turns its attention instead to the effects of independently measurable embodied technological change in a model with an endogenous vintage capital structure. It can be viewed as a noteworthy step towards a more fundamental identification of the ultimate source of the changes under study. Only Mortensen and Pissarides (1999), Hornstein et al. (2002), and Wong (2003) share with the present paper a quantitative approach to evaluating the implications of the theory.

The rest of the paper is organized as follows. Section 2 presents the model and basic behavior relations. Section 3 characterizes the equilibrium and properties useful for computation purposes. Section 4 describes the calibration procedure. Section 5 reports the results of the comparative statics experiments. Section 6 concludes the paper with an overview of results and final remarks.

2 The Model

Agents in the model consist of workers and firms. Workers can be either acquiring education or actively participating in the labor market. An active worker's type is defined by two characteristics: skill (indexed by j) and education (indexed by i). An active worker can be skilled ($j = s$) or non-skilled ($j = ns$), and educated ($i = e$) or non-educated ($i = ne$). Active workers can be employed or unemployed and searching for a job. Firms can be either posting one job vacancy and searching or producing output with one worker. A firm can be of one of two types $i \in \{e, ne\}$ according to whether it targets educated or non-educated workers. Workers and firms that are searching meet through a matching process. Firms observe the education status of a worker at any time but only observe her skill level after being matched. This implies that the labor market can be segmented by education levels. This will be the case here since the focus will be on equilibria with firms that specify different education requirements when advertising a job. The rest of this section describes in detail the model and the decision problems faced by workers and firms, and characterizes the optimal choices. Attention is restricted to stationary situations with constant decision rules.

2.1 Workers

There is a continuum of workers who are born with an innate type attribute, l, uniformly distributed on the interval $[0, 1]$. The type is distributed so that, ordering agents by increasing l, the measure p_s at the lower end are skilled or high-ability (i.e. if $l < p_s$, then $j = s$). The remaining fraction $1 - p_s$ are non-skilled. Workers may acquire education. Education has no effect on the skill of a worker. There are two components to the cost of acquiring education. There is a time requirement, T_e, which is the length of the period needed to become educated, and a component in terms of disutility, $c_e(l)$, which is increasing in the worker's type l but can take a negative value when education brings positive utility. Thus skilled workers find it less costly (or more rewarding) to get education. Workers face a constant probability of leaving the labor force, ρ. When a worker leaves, another worker of exactly the same type replaces her.

The timing of events for a worker is as follows. First, she decides whether to acquire education. Second, the worker enters the labor force and decides which segment (or career) of the market, $i \in \{e, ne\}$, to participate in. A non-educated worker can only search for jobs in the non-educated segment. This follows because in an equilibrium with segmentation firms posting vacancies in the educated segment only consider contacts with workers holding proof of education. Educated workers can instead search in either segment. Let $\phi_{i|j} \in \{0, 1\}$ represent the decision of an educated worker with skill j whether to participate (value 1) or not in segment i so $\phi_{e|j} = 1 - \phi_{ne|j}$. Figure 1 is introduced to visualize the type of configurations that may arise.

The circle represents the composition of the total labor force in terms of workers' types. Given the exogenous distribution of skills, p_s, the size of the different slices is entirely determined by the education choices. The bottom and top sections on the circle account for the skill composition of the educated and non-educated groups of population respectively. Education and skill are imperfectly correlated when the sections labeled (s, ne) and (ns, e) do exist. The boxes represent the two market segments and the arrows show the possible assignments of the labor force of different types to the market segments. The career decision, $\phi_{i|j}$, determines the direction of this flow for the educated workers in the two bottom sections of the circle. Thus education (the slices in the circle) and career decisions (the arrows) will jointly determine the skill composition of the labor force that participates in each segment,

$p_{j|i}$: the proportion of workers in segment $i \in \{e, ne\}$ with skill $j \in \{s, ns\}$. The skill composition of the educated population (i.e., the relative size of the two bottom slices of the circle) and $p_{j|e}$ will coincide only if $\phi_{e|s} = \phi_{e|ns} = 1$. Figure 1 will be repeatedly used and fully explained throughout the rest of the paper.

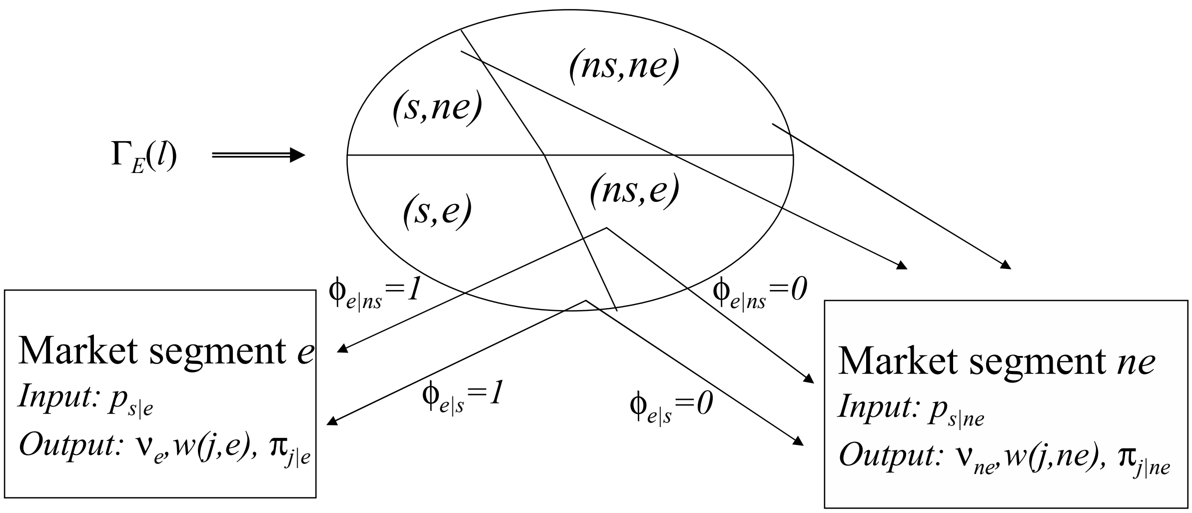

Figure 1. The allocation of workers to segments.

Third, the worker starts searching for a job. Workers and firms are matched randomly. The probability that a worker searching in segment i makes contact with a suitable firm is ν_i. The value of being unemployed and searching to a worker with skill j and seeking employment in segment i is denoted by $U(j, i)$. In a steady-state one can argue that the Bellman values are not indexed by the worker's education type since the worker, even if educated, will never want to exercise the option to switch segment at a later date. Fourth, upon contact with a firm the skill of the worker j is disclosed. The unemployed worker must agree with the vacant firm on whether to create the job or continue searching. The decision of the firm whether to hire the worker is denoted by the indicator $\pi_{j|i} \in \{0, 1\}$, with value 1 if the decision is positive. If the job is created, the wage to the worker is $w(j, i)$ and the value of the match is $W(j, i)$. The job is terminated exogeneously with a Poisson probability λ. In this event, the agent becomes unemployed and searches for a new job. There is a flow value to the unemployed worker that depends on the wage:

$$b(w(j, i)) = b_0 + b_1 w(j, i) \tag{1}$$

The coefficient on the wage can be interpreted as the unemployment benefit replacement rate. The fixed component may include the value of leisure.

The worker seeks to maximize the expected present value of utility minus the utility component of the education cost, c_e, and discounts the future at the constant rate r. The instantaneous utility is given by the value of consumption. Free borrowing and lending is assumed so that in equilibrium the interest rate equals r and hence the worker maximizes the present value of wages plus unemployment compensation. With the notation introduced, the Bellman equation for the value of a job to a worker with skill j and matched with a firm in segment i is

$$(r+\rho)W(j,i) = w(j,i) + \lambda(U(j,i) - W(j,i)). \tag{2}$$

The value of unemployment for a worker with skill j in segment i is

$$(r+\rho)U(j,i) = b(w(j,i)) + \nu_i \pi_{j|i} \max\{W(j,i) - U(j,i), 0\} \tag{3}$$

for $i = e, ne$ and $j = s, ns$. Concerning career (or segment) choices, for a non-educated worker the only segment available is $i = ne$. The choice of segment by an educated worker with skill j can be represented by

$$\phi_{e|j} = 1 - \phi_{ne|j} = \begin{cases} 1 & U(j,e) - U(j,ne) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

A worker of type $l \in [0,1]$ decides to go to college if

$$\Gamma_E(l) > 0 \tag{5a}$$

where, assuming the specification

$$c_e(l) = c_e l - \bar{c}_e, \quad c_e > 0, \tag{5b}$$

$$\Gamma_E(l) \equiv \begin{cases} e^{-(r+\rho)T_e} & [\phi_{e|s}U(s,e) + (1-\phi_{e|s})U(s,ne) - c_e(l)] \\ & -U(s,ne) \qquad\qquad l \in [0, p_s] \\ e^{-(r+\rho)T_e} & [\phi_{e|ns}U(ns,e) + (1-\phi_{e|ns})U(ns,ne) - c_e(l)] \\ & -U(ns,ne) \qquad\qquad l \in (p_s, 1] \end{cases} \tag{5c}$$

Returning to Figure 1, one can now outline the interactions that in the complete model will be key to understand the determination of the variables object of this paper: education-specific unemployment rates and wages, and educational attainment. The boxes show that, once the skill composition of the labor force in a particular segment i, $p_{j|i}$, has been determined, the hiring policy of firms $\pi_{j|i}$, the wage structure $w(j,i)$, and the probability of contact for an unemployed worker ν_i will all be determined within the segment. These segment-specific variables will have direct implications for the wages and unemployment rates across education groups, implications which will be mediated by the skill and career composition of these groups (determined by Γ_E and $\phi_{e|j}$ respectively). On the other hand, when deciding whether to educate and which segment to join, a worker compares the returns to the different options which will be largely governed by the segment-specific variables in the different segments. Shifts in education and career choices have in turn two types of implications for education-specific wages and unemployment rates. The first is a *direct* effect: by altering $p_{j|i}$ and thereby the segment-specific outputs as just described. The second is a *composition* effect: changes in the relative proportion of agents with different skill and career within each education group for given segment-specific outputs. The rest of this section lays out the remaining elements of the model that will be used to assess the role of these interacting mechanisms.

2.2 Firms

The timing of events is as follows. First, an inactive firm creates a job vacancy that specifies the education requirement on the worker sought, i (i.e., the market segment). The value of such a vacancy is $V(i)$. A vacancy is posted and there is contact with a suitable job seeker with probability ξ_i. The skill status, j, of the worker met in segment i is not observed by the firm at this stage. The firm holds instead a rational belief about the probability that a matched worker in that segment has skill of type j. This coincides with the equilibrium fraction of workers with skill j within the pool of unemployed workers participating in the market segment i, $us_{j|i}$. Posting a vacancy has a flow recruiting cost c_R.

Second, upon contact, the firm observes the worker's skill, and the firm and the worker agree on whether to create the job. As before, $\pi_{j|i} \in \{0,1\}$ denotes the decision by the firm of type i whether to hire a worker with skill j.

Third, the firm starts operating and the flow of output generated by the firm is $y(j,i)$. The value of the existing job match for the firm is $J(j,i)$. The match terminates as the consequence of exogeneous job failure which occurs with Poisson probability λ. The job can also break down by the worker leaving the labor force which occurs at rate ρ. When the job is terminated, the firm will seek to open a new vacancy type i of the highest value. Like the worker, the firm discounts future values at the constant interest rate r. Formally, the value of a job of type (j,i) obeys the Bellman equation

$$rJ(j,i) = y(j,i) + (\lambda + \rho)\left[\max_{i' \in \{e,ne\}} V(i') - J(j,i)\right] \tag{6}$$

and the value of a vacancy of type i satisfies

$$rV(i) = -c_R + \xi_i \sum_{j=s,ns} u s_{j|i} \pi_{j|i} [J(j,i) - V(i)] \tag{7a}$$

$$\pi_{j|i} = \begin{cases} 0 & J(j,i) - V(i) \leq 0 \\ 1 & J(j,i) - V(i) > 0 \end{cases} \tag{7b}$$

2.3 Technology

The flow of output to a match depends on the worker's productivity. Let η_j denote the productivity of a worker with skill j and assume that $\eta_s > \eta_{ns}$. The output of a match involving a worker with skill j equals the value of the gross income flows

$$\eta_j = w(j,i) + y(j,i). \tag{8}$$

The matching technology is segment-specific. There is a homogenous-of-degree-one matching function that gives the number of matches per period in segment i, $m_i = m(v_i, u_i)$, where v_i is the mass of vacant firms and u_i is the number of unemployed workers in this segment. This matching technology is specified as:

$$m(v,u) = m_0 v^{1-\theta} u^{\theta}, \qquad \theta \in [0,1],$$

and $m_0 > 0$ characterizes the efficiency of the matching process. Then the probabilities of contact that are relevant to firms and workers are

$$\begin{aligned}\xi_i =& m_i/v_i = \xi(v_i/u_i) = m_0 \left(\frac{v_i}{u_i}\right)^{-\theta} \\ \nu_i =& m_i/u_i = \nu(v_i/u_i) = m_0 \left(\frac{v_i}{u_i}\right)^{1-\theta}\end{aligned} \tag{9}$$

so $\xi_i'(.) < 0$ and $\nu_i'(.) > 0$.

2.4 Bargaining, free-entry, and skills of the unemployed

The wage is determined at each instant of time through bargaining over the surplus of a match between the firm and the worker that have agreed to create a job. The solution to the corresponding generalized Nash bargaining problem is

$$w(j,i) = \arg\max\{\beta \log S_W(j,i) + (1-\beta)\log S_F(j,i)\},$$

where S_W and S_F represent the match surplus to the worker and the firm respectively, and β represents the workers' bargaining power. For the worker $S_W(j,i) \equiv W(j,i) - U(j,i)$. For the firm, $S_F(j,i) \equiv J(j,i) - V(i)$. Using Eq.(1)-(3) and (6)-(7), the necessary first-order condition for this problem is:

$$\frac{1-\beta}{\beta(1-b_1)}(W(j,i) - U(j,i)) = J(j,i) - V(i) \tag{10}$$

There is free-entry in vacancies which leads to the exhaustion of pure rents from vacancy creation in both segments $i \in \{e, ne\}$:

$$V(i) = 0 \tag{11}$$

The firms in a segment i take as given the skill composition of the pool of unemployed workers from which matches are drawn, $us_{j|i}$. This depends of the skill-composition of the labor force in this segment, $p_{j|i}$, and the forces determining unemployment, including matching and hiring rates, ν_i and $\pi_{j|i}$. Supposing that workers are always willing to accept the job offers made by firms, then in a steady-state the equalization of the flows in and out of employment will lead to the following expression (see Appendix A):

$$us_{s|i} = 1 - us_{ns|i} = p_{s|i} \left(\frac{\nu_i \pi_{s|i} + \lambda + \rho}{\nu_i \pi_{ns|i} + \lambda + \rho} (1 - p_{s|i}) + p_{s|i} \right)^{-1} \tag{12}$$

3 Equilibrium

An equilibrium is a situation consistent with Eq.'s(1)-(11) above and, then, also Eq.(12).[7] Given the technology described in Eq.(8) and (9), the fundamental endogenous variables in this system are, for $i = e, ne$ and $j = s, ns$, the market tightness v_i/u_i (or, by Eq.(11) the contact probability ν_i), wages $w(j,i)$, hiring decisions $\pi_{j|i}$, labor participation (or career) choices $\phi_{j|i}$, and the distribution of the value of education $\Gamma_E(l)$ on $l \in [0,1]$. These are just the variables displayed in Figure 1. As indicated in that figure, the significance of $\phi_{j|i}$ and $\Gamma_E(l)$ for the outcomes of any segment i will be conveniently summarized by the skill-composition of the labor force $p_{j|i}$ in that segment.

The focus of the analysis will be restricted to equilibrium situations where both the educated and the non-educated labor-market segments are operative[8], and where the proportion of skilled workers is higher among the labor force that participates in the educated segment (i.e., $p_{s|e} > p_{s|ne}$). The latter is intuitive and convenient since it restricts the type of career choices that may arise in equilibrium. In effect, the skilled workers that are educated must decide to participate in the educated segment, or $\phi_{e|s} = 1$. Otherwise, existence of an active educated segment would require the non-skilled educated workers to be the only participants in the educated segment (see again Figure 1). But then non-skilled agents would be most numerous in the educated segment, which is inconsistent with the required condition on the $p_{j|i}$'s. Thus variation in labor-force participation (or career) will occur only through the choice by the educated non-skilled workers, for which the shorter notation $\phi \equiv \phi_{e|ns}$ is introduced.

[7]The decision of a worker whether to accept an offer in Eq.(3) is dominated by the hiring decision of the firm $\pi_{j|i}$ in Eq.(7) since Eq.(10) must hold. Thus unemployed workers accept equilibrium job offers made by firms and Eq.(12) holds.

[8]In such an equilibrium, the two types of vacancies are optimally created and, given free-entry, firms are indifferent between the two types of vacancies given the market tightness and meeting probabilities in each segment. Therefore in this equilibrium there are no incentives for existing vacancies to deviate from their segment choice. It is true nonetheless that, in principle, the model could accommodate situations with a single segment and matching function. For the issues at hand, these situations can be regarded as being of little interest and are therefore set aside.

Exogenous skill composition: This section studies first the equilibrium in each segment when the skill composition is exogenous. When the distributions of skills, $p_{j|i}$, is assumed to be given, then an equilibrium determines the variables v_i/u_i (or, by Eq.(11), ν_i), $w(j,i)$, and $\pi_{j|i}$ for $i = e, ne$ and $j = s, ns$. These are the outputs inside the boxes in Figure 1. They can be characterized by a number of relations. The derivations are in Appendix A.

One type of relation comes from developing the bargaining condition Eq.(10). This is a version of the *job-destruction* curve in Mortensen and Pissarides (1994) which gives the wage as a function of the worker's skill, the segment's tightness and the firm's hiring decisions. It reads as follows:

$$w(j,i) = \omega\left(\eta_j, \Omega\left(\frac{v_i}{u_i}, \pi_{j|i}\right)\right) \equiv \frac{b_0 + \Omega(v_i/u_i, \pi_{j|i})(\eta_j - b_0)}{1 - b_1 + \Omega(v_i/u_i, \pi_{j|i})b_1} \tag{13a}$$

where the effect of tightness on bargaining outcomes is in the term

$$\Omega\left(\frac{v_i}{u_i}, \pi_{j|i}\right) \equiv \frac{\beta(1-b_1)(r+\rho+\lambda+\nu_i(v_i/u_i)\pi_{j|i})}{\beta(1-b_1)(r+\rho+\lambda+\nu_i(v_i/u_i)\pi_{j|i}) + (r+\rho+\lambda)(1-\beta)} \tag{13b}$$

Eq.(13) traces out a positive relation between the wage and the equilibrium market tightness. The interpretation is as follows. A higher probability of meeting a vacancy for the worker, ν_i, means that the outside option of a job is also higher for the worker. Hence the wage has to be also higher to keep the worker into the job,. Since ν_i depends positively of market tightness, the positive relation between v_i/u_i and $w(j,i)$ follows.

The other type of relation comes from developing the free-entry condition Eq.(11), using Eq.(13) to replace the wage terms. This delivers the following:

$$\xi(v_i/u_i) \sum_{j=s,ns} us_{j|i}\pi_{j|i}\Psi\left(\eta_j, v_i/u_i, \pi_{j|i}\right) - c_R = 0 \tag{14a}$$

$$\pi_{j|i} = \begin{cases} 0 & \Psi\left(\eta_j, \frac{v_i}{u_i}, 1\right) < 0 \\ 1 & \text{otherwise} \end{cases} \tag{14b}$$

where the firm's net-profit term is defined as

$$\Psi\left(\eta_j, \frac{v_i}{u_i}, \pi_{j|i}\right) \equiv \frac{1}{r+\lambda+\rho}\left[\eta_j - \omega\left(\eta_j, \Omega\left(\frac{v_i}{u_i}, \pi_{j|i}\right)\right)\right] \tag{14c}$$

with $\omega()$ as defined in Eq.(13) and, using Eq.(12) with (9),

$$us_{s|i} = 1 - us_{ns|i} = p_{s|i}\left(\frac{\nu(v_i/u_i)\pi_{s|i} + \lambda + \rho}{\nu(v_i/u_i)\pi_{ns|i} + \lambda + \rho}(1 - p_{s|i}) + p_{s|i}\right)^{-1} \tag{14d}$$

This expression is associated with the idea of *job creation*. A higher probability of contacting a worker for the firm, ξ_i, increases the expected profits for a given allocation of output between the worker and the firm. Then free-entry would drive the wages (i.e., the terms $\omega(.)$) upwards so as to restore the zero value of creating vacancies. Since ξ_i depends negatively on market tightness, a negative relation between v_i/u_i and wages follows for each $j = s, ns$. However, by Eq.(13), the wages themselves depend on market tightness, and the term $\Psi(...)$ in Eq.(14c) picks up this dependence. For each i, the equilibrium can then be expressed as a market tightness v_i/u_i that satisfies Eq.(14a). It must be consistent with the firm's hiring policy $\pi_{j|i}$ in Eq. (14b) which characterizes the choice of the firm whether to create the job when contacting an unemployed worker with skill j. There is the possibility that a match with a particular skill j is not profitable and the job is not created, but observe that (14a) requires that at least for one j the job is created. Changes in the composition of the labor force $p_{s|i}$ enter this condition through changes in the probabilities that, say, an unemployed worker is skilled in the educated segment, $us_{s|e}$, in Eq.(14d). Since the educated and the non-educated sectors are both operative, any firm must be willing to hire at least the skilled workers so that one can set $\pi_{s|e} = \pi_{s|ne} = 1$. In principle, the non-skilled workers may be hired in either sector, both sectors or none sector. The following proposition makes this characterization more precise.

Proposition. *Assume that $p_{j|i}$ is given. Consider the following conditions:*

$$\Psi(\eta_s, 0, 1) > 0 \tag{C1}$$

$$\Psi(\eta_{ns}, 0, 1) < 0 \tag{C2}$$

$$\xi((v_i/u_i)^*)p_{s|i}\Psi(\eta_s, (v_i/u_i)^*, 1) - c_R < 0 \tag{C3}$$

$$\xi((v_i/u_i)^*)us^*_{s|i}\Psi(\eta_s, (v_i/u_i)^*, 1) - c_R > 0 \tag{C4}$$

where $(v_i/u_i)^*$ is such that $\Psi(\eta_{ns}, (v_i/u_i)^*, 1) = 0$ and $us^*_{s|i}$ is as in Eq.(14d) with $v_i/u_i = (v_i/u_i)^*$, $\pi_{s|i} = 1$, and $\pi_{ns|i} = 0$. The following holds.

(a) *In equilibrium,* $\pi_{s|i} = 1$ *and the pair* $(v_i/u_i, \pi_{ns|i})$ *is unique.*

(b) *Assume* $p_{s|i} \in (0,1)$*. Then C1 is a necessary condition for existence. This condition is also sufficient if* $p_{s|i} = 1$*.*

(c) *Assume* $p_{s|i} \in (0,1)$ *and C1. An equilibrium does not exist if and only if none of conditions C2, C3 and C4 hold. If C2 holds, then the equilibrium has* $\pi_{ns|i} = 0$*. If C3 holds then C4 does not and the equilibrium has* $\pi_{ns|i} = 1$*. If C3 does not hold and C4 does, the equilibrium has* $\pi_{ns|i} = 0$*.*

(d) *Assume* $p_{s|i} = 0$*. That C2 does not hold is a necessary and sufficient condition for existence with* $\pi_{ns|i} = 1$*.*

The meaning and justification of the proposition is best conveyed by using Figure 2. It represents the left-hand side of Eq.(14a) as a decreasing function of v_i/u_i, on account of Eq.(14b)-(14c). Thus an equilibrium must be unique (point a). Also existence requires that the curve lies above the horizontal zero line for low values of the market tightness. Formally, given the limiting properties of the function $\xi(.)$, a necessary condition for existence is that firms be willing to hire at least skilled workers when tightness is very low (point b).

The left-hand side of Eq.(14a) has a discontinuity at the value $(v_i/u_i)^*$ of market tightness where, according to Eq.(14b), $\pi_{ns|i}$ shifts from 1 to 0 or, in other words, the value at which it is no longer profitable for firms in segment i to hire non-skilled workers. At this point, the probability of meeting a skilled worker among the pool of unemployed, $us_{s|i}$ in Eq.(14d), drops because all the non-skilled workers in this segment become unemployed. An equilibrium may not exist due to this discontinuity (first part of point c). When there is a zero of the LHS of Eq.(14a) to the right of $(v_i/u_i)^*$, non-skilled workers are not hired, otherwise both skill types are hired (second part of point c). If all the labor force are non-skilled workers, there is no issue of discontinuity (point d).

This characterization can also be used to derive comparative statics results. For the purpose of the present paper, it is important to retain that v_i/u_i and, through Eq.(13), $w(j,i)$ increase with $p_{s|i}$ as the curve in Figure 2 shifts upwards.

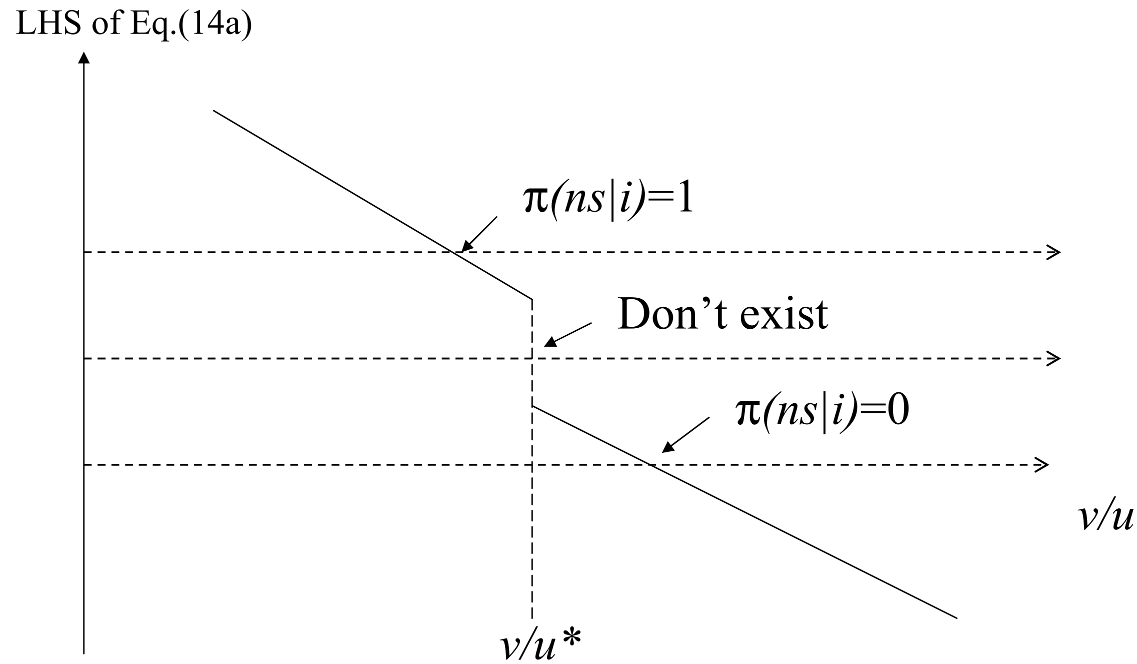

Figure 2. Equilibrium.

Endogenous skill composition: The skill composition $p_{s|i}$ has been taken as given so far. In equilibrium, as indicated by the arrows in Figure 1, it will depend on career and education decisions of workers. The career choice in equilibrium is determined by Eq.(4), with the equilibrium values of searching in alternative segments determined by

$$(r+\rho)U(j,i) = b(w(j,i)) + \nu(v_i/u_i)\pi_{j|i}\frac{w(j,i) - b(w(j,i))}{r+\rho+\lambda+\nu(v_i/u_i)\pi_{j|i}}, \tag{15}$$

where the equality follows from Eq.(2) and (3). Note that the equilibrium condition that the career choice of skilled educated workers must be to participate in the educated segment (i.e., $\phi_{e|s} = 1$) requires that $U(s,e) > U(s,ne)$. In general, one has to account for the possibility that two equilibria exist for given education (i.e., $\Gamma_E(l)$), one with $\phi = \phi_{e|ns} = 0$ and the other with $\phi = 1$.

Endogenizing education choices will complete the characterization of the equilibrium. An equilibrium requires that the education decisions satisfy Eq.(5). The assumption in (5b) implies that the curve $\Gamma_E(l)$ is piece-wise decreasing on the regions of values for l below and above p_s. Therefore the distribution of education over the labor force can be characterized by two numbers, $p_{e1} \in [0, p_s]$ and $p_{e2} \in [p_s, 1]$, such that $l \in [0, p_{e1}] \cup [p_s, p_{e2}]$

are educated. Figure 3 depicts a typical configuration.

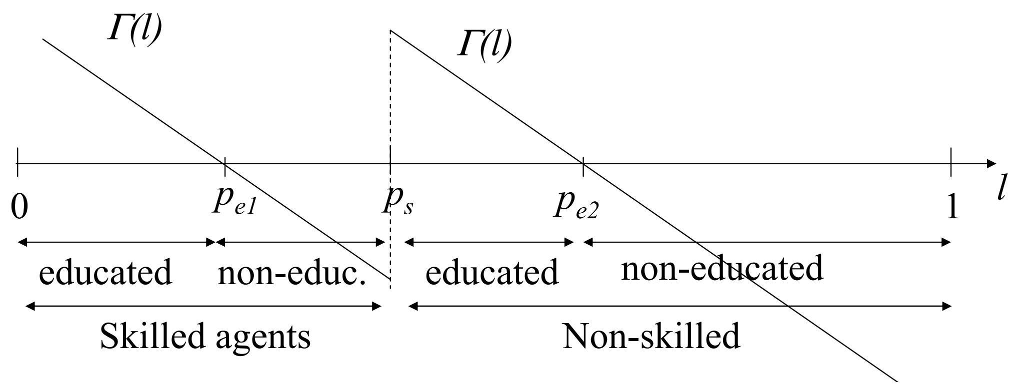

Figure 3. Distribution of types.

Provided that in an equilibrium skilled workers that are educated will decide to participate in the educated segment, the distribution of skills within the labor force in each segment can be written explicitly as follows:

$$p_{s|e} = \frac{p_{e1}}{p_{e1} + \phi(p_{e2} - p_s)} \tag{16a}$$

$$p_{s|ne} = \frac{p_s - p_{e1}}{p_s - p_{e1} + (1 - p_{e2}) + (1 - \phi)(p_{e2} - p_s)} \tag{16b}$$

An equilibrium is a fixed point in the education outcome which can be visualized using Figure 1 again. A particular Γ_E — or, equivalently, (p_{e1}, p_{e2}) — yields equilibrium values for ϕ, and through $p_{s|i}$'s in Eq.(16), the $\{\pi_{j|i}, w(j, i), \nu_i\}_{i\in\{e,ne\},j\in\{s,ns\}}$. These values do in turn affect Γ_E and (p_{e1}, p_{e2}). The equilibrium (p_{e1}, p_{e2}) must be consistent with the equilibrium it generates. By aggregating appropriately the segment-specific variables, such an equilibrium delivers implications for educational attainment and education-specific unemployment rates and wages. The *direct* and *composition* effects discussed at the end of section 2.1 will be crucial to interpret these implications. Appendices B and C contain details on computation.

4 Calibration

One model's period is assumed to correspond to one quarter. The number of parameters to be determined is 15: b_1, r, T_e, ρ, λ, θ, p_s, β, η_s, η_{ns}, b_0, c_R, m_0 and either p_{e1} and p_{e2}, if education is exogenous, or $\bar{c}_e$ and c_e, if education is endogenous. The procedure adopted here is divided in two steps. First, the parameters are chosen assuming that education is exogenous. In the second step, the parameters of the education technology, $\bar{c}_e$ and c_e, are calibrated so that the equilibrium outcomes with endogenous education are consistent with the values of p_{e1} and p_{e2} chosen in the first step.

Then 7 of these parameters can be set directly. The choice of b_1 is consistent with an unemployment benefit (UI) replacement rate of 20% from OECD (1997). US Bureau of Labor Statistics (2002) finds on March CPS data an average separations rate in 1970 of 25%. This includes employment-to-unemployment and employment-to-employment transitions. This is approximately 6% per quarter, the number used for λ in the calibration. A 5% annual interest rate in Cooley and Prescott (1995) implies a value for r. A life-expectancy in the labor market of 45 years implies $\rho = 1/(45 \times 4)$. The matching elasticity θ is set following the estimate in Blanchard and Diamond (1990). Four full years required for a college degree in Autor et al. (1998) determine T_e. Since the calibration procedure will target figures for unemployment rates, one can argue that the choice of either m_0 or c_R is a normalization and can fix m_0.

To determine the remaining parameters with exogenous education, outcomes are restricted to be consistent with targets for the key endogenous variables corresponding to the year 1970. There are two blocks to this task. One block of the calibration procedure consists of matching the four targets for the wage premium, the unemployment rates of educated and non-educated workers, and the value of hiring cost as a proportion of wage income by choice of the four parameters η_s, b_0, β, and c_R. It is assumed that $\eta_s + \eta_{ns} = 3$, so η_s determines η_{ns} directly and changes in η_s can be regarded as skill-biased changes in technology. The targets for the wage premium and unemployment are taken from the 1970 data of Table 1 in Section 1. Concerning hiring costs, Hamermesh (1993) estimated that average hiring costs represent about 2 per cent of the wage bill.

In the other block, the parameters p_{e1}, p_{e2}, and p_s are calibrated to match the targets of educational attainment and measures of inequality within defined occupational categories that can be associated with the two segments in the model. The target for education is the 1970 figure for the proportion of college participation in Table 1. Concerning inequality within job categories, Gould (2002, Fig. 1b) reports the variance of the OLS log-wage residuals from uniform March CPS data for white males, after controlling for years of schooling, experience, region of residence, marital status, and living in a standard metropolitan statistical area. These measures are provided for various years within three different occupation groups: professional sector, service sector, and blue collar sector.[9] In the calibration, it is assumed that the educated segment and the non-educated segment correspond to the professional sector and the blue-collar sector, respectively. For 1970 the residual variance of log wages for the professional and blue-collar sectors are 0.18 and 0.12 respectively. The target to match will be the differential residual variance of educated over non-educated jobs in 1970 which is thus 0.06.[10] Since only the differential in residual variance is targeted, rather than the two absolute values, this block would leave one degree of freedom in the choice of p_s. To deal with this, the model has been calibrated for alternative values of p_s around the target for education 0.25. With a large value such as $p_s = 0.30$ it was found that the calibration procedure reaches a point where existence fails before meeting all the targets. With $p_s = 0.25$ the model can be calibrated but then for small increases in η_s an equilibrium fails to exist which precludes conducting some of the experiments of interest. For p_s below 0.17 the difference in residual variance across groups is too big to match the corresponding target. The benchmark calibration features $p_s = 0.20$.[11] This choice will be subject to sensitivity analysis.

Table 2 displays the benchmark calibration with exogenous education and summarizes the procedure. All targets set out are matched exactly, with the educated and non-educated residual variances being 0.07 and 0.01.

[9] The professional sector includes all workers in the professional, technical, managerial, and academic occupations; the service sector, all service workers as well as clerical and sales workers; the blue collar sector, all construction workers, craftsmen, machinists, operatives, and laborers.

[10] Hence there must exist positive residual inequality in the educated segment so the calibrated equilibrium must necessarily imply that $p_{e2} > p_s$ and $\phi = 1$. In the model one can calculate the variance within a segment i as $(\log w_{s,i} - \log w_{ns,i})^2 p_{s|i}(1 - p_{s|i})$ where $p_{s|i}$ is the share of skilled in the segment.

[11] Notice that, since this is smaller than the education target 0.25, then the number of skilled not educated is less than the number of non-skilled that are educated, or $p_s - p_{e1} < p_{e2} - p_s$.

Table 2. Calibration with exogenous education

parameter	value	target to match	source
b_1	0.2	UI replacement 20%	OECD (1997)
r	0.013	annual interest 5%	Cooley et al. (1995)
T_e	16	time in college 4 years	Autor et al. (1998)
ρ	0.0055	working life 45 years	
λ	0.06	annual separation rate 25%	Bureau Labor Stat. (2002)
θ	0.5	matching elast. 0.5	Blanchard et al. (1990)
m_0	1	normalize to unity	
p_s	0.20	college partic. 25%	Census Bureau (1995)
p_{e1}	0.165	residual ineq. diff. 0.06	Gould (2002)
p_{e2}	0.285		
$\eta_s = 3 - \eta_{ns}$	1.915	wage premium 1.44	Autor et al. (1998)
b_0	0.775	unemp. educ. 1.1%	Census Bureau (1995)
β	0.15	unemp. non-educ. 2.4%	Census Bureau (1995)
c_R	0.10	hiring costs 2%	Hamermesh (1993)

The next step is to calibrate the parameters of the cost of education so that the specific values for p_{e1} and p_{e2} obtain as the outcome of endogenous education choices. The outcomes are in Table 3.

Table 3. Calibration with endogenous education

parameter	value	target to match	source
$\overline{c}_e$	42.1	p_{e1}	Table 2
c_e	79.698	p_{e2}	Table 2

5 Numerical Experiments

This section studies separately the equilibrium effects of skill-biased shocks to technology and shocks to the matching function. The implications of the corresponding shock for unemployment rates, the wage premium, and educational attainment will be reported and analyzed. The purpose is to use the implications to assess these sources of shocks as alternative hypotheses about the causes of US labor market changes between 1970 and 1990. The detailed analysis of the mechanisms at work will highlight the nature and quantitative importance of the interaction between the key endogenous variables.

5.1 The skill-biased change hypothesis

The skill-biased hypothesis can be represented by a rise in the productivity of the skilled workers η_s and a corresponding decline in the productivity of the non-skilled workers $\eta_{ns} = 3 - \eta_s$ between 1970 and 1990.[12] The rest of this section studies the effect of the skill-biased shock on the calibrated benchmark equilibrium. The first column of figures in Table 4,'Benchmark', shows the value of the parameter η_s and the endogenous variables in this benchmark equilibrium. The top portion of the column corresponds to the key observable variables of interest which take on the values of the 1970 data from Table 1. The second column reproduces the 1990 data on these same variables for the purpose of comparison with the implications of the theoretical experiments.

5.1.1 Results for the skill-biased change

In the first experiment the education decisions are left to respond endogenously to the shock. The shift in η_s from 1.915 to 1.965 has been calibrated so that, after the changes, the model reproduces a wage premium comparable with the value 1.58 observed in the 1990 data. The third column of Table 4, 'Endog. Edu.', shows this new value for the parameter as well as the corresponding values for the endogenous variables. One can see that the skill-biased shock is able to produce rising unemployment for the non-educated and an increase in the wage premium. Quantitatively, the non-educated unemployment rate after the shock 0.037 falls short of the U.S. 1990 figure of 0.053, and accounts for near 50 per cent of the observed increase on the 0.024 rate in 1970. On the other hand, the unemployment rate for the educated does not undergo any increase, thus failing to reproduce the rise recorded in the data. Regarding education, the rate of college participation shows a rise from 0.25 to 0.297 which, short of the 0.33 figure in the 1990 data, accounts for nearly 60 per cent of the observed increase on the 0.25 rate in the 1970 benchmark.

In order to understand the consequences just reported of the skill-biased shock, one has to study the associated changes undergone by variables of the model that feature in the relations depicted in Figure 1. Values for these variables are reported in the bottom portion of Table 4. The unemployment rate for non-educated workers increases as a consequence of the decline in a worker's matching rate in this segment, ν_{ne}, which follows from the fall in

[12] Equivalently, one might postulate a widening gap with positive rises in both productivities while the economy wide parameters increase at an average rate that must exceed the rise in non-skilled productivity.

tightness in the non-educated segment, v_{ne}/u_{ne}, where all non-educated workers participate. On its part, it is the increased tightness in the educated segment, v_e/u_e, that tends to reduce unemployment for the educated skilled workers, although the quantitative effect is hardly visible as it stays very close to the benchmark 0.011. Regarding the wage premium, the rise in the wage of the educated workers that are skilled, $w(s, e)$, tends to increase the average wage of the educated labor force. There is also a small composition effect at work driven by the increase in the fraction of high-wage skilled within this group, $p_{s|e}$, at the expense of the low-wage non-skilled. On its part, the reduction in the wage of the non-skilled non-educated workers, $w(ns, ne)$, drives the reduction in the wage of the non-educated group, effect which is reinforced by the decline in the proportion of high-wage skilled within the non-educated labor force, $p_{s|ne}$. Regarding the college decision, a larger number of non-skilled workers attend college, which is the optimal response to the equilibrium shifts in the value of the relevant options. As a result of the shock, the value of participating in education, $(r + \rho)U(ns, e)$, and the value of not participating, $(r + \rho)U(ns, ne)$, both fall. Because the change in the value of education is discounted ($\exp(-(r + \rho)T_e) \approx 0.74$), the optimal decisions imply the rise in education participation by the non-skilled, p_{e2}. This is an *opportunity cost* mechanism governed by the deterioration of conditions in the labor market for the non-skilled. For the skilled, the rising return to education, $(r + \rho)U(s, e)$, along with the decline in the return to not taking education, $(r + \rho)U(s, ne)$, dictates that college attendance, p_{e1}, goes up.

5.1.2 The role of education under the skill-biased change

The second experiment calculates the equilibrium outcomes for the η_s calibrated in the first experiment while holding the benchmark 1970 education choices constant. The fourth column, 'Exog. educ.', of Table 4 shows the result. Both the unemployment rate for the non-educated and the wage premium still increase, but by smaller amounts than in the experiment with endogenous education and, therefore, they do not meet the 1990 data as closely. With exogenous education, the market tightness in the non-educated segment, v_{ne}/u_{ne}, after the shock is visibly higher than with endogenous education. Thus the *direct* effect accounts for much of the contribution of education to the change in the non-educated unemployment rate. Since wages, $w(j, i)$'s, are similar whether education is exogenous or endogenous, education must have had a predominantly *composition* effect on the wage premium. Regarding the educated unemployment rate, with exogenous education it stays nearly constant and, therefore, provides about as poor a match to the 1990 data as in the experiment with endogenous education.

Therefore, under the skill-biased hypotheses, the behavior of education contributes to the quantitative response of the wage premium and non-educated unemployment, but not to that of unemployment for the educated workers.

Table 4. Skill-biased technical change

	'70 Benchmark	'90 Data	Endog. educ.	Exog. educ.
Exog. parameter η_s	1.915	–	1.965	1.965
unemp. educated	0.011	0.021	0.011	0.011
unemp. non-educ.	0.024	0.053	0.037	0.028
wage premium	1.44	1.58	1.58	1.51
college partic.	0.25	0.33	0.30	0.25
ϕ	1		1	1
p_{e1}	0.165		0.200	0.165
p_{e2}	0.285		0.297	0.285
$p_{s\|e}$	0.66		0.67	0.66
$p_{s\|ne}$	0.05		0.00	0.05
v_e/u_e	34.55		36.10	35.42
v_{ne}/u_{ne}	7.36		2.87	5.04
$w(s,e)$	1.85		1.90	1.90
$w(s,ne)$	1.78		1.76	1.80
$w(ns,e)$	1.08		1.03	1.03
$w(ns,ne)$	1.07		1.02	1.02
$(r+\rho)U(s,e)$	1.84		1.89	
$(r+\rho)U(s,ne)$	1.77		1.74	
$(r+\rho)U(ns,e)$	1.08		1.03	
$(r+\rho)U(ns,ne)$	1.07		1.02	
variance edu.	0.066		0.082	0.083
variance non-edu.	0.011		0.000	0.014

5.2 The mismatch shock hypothesis

The mismatch hypothesis will be represented by a decline in the matching parameter m_0 between 1970 and 1990. This section studies the effect of this mismatch shock on the calibrated model. The first column in Table 5, 'Benchmark', shows the value of the parameter m_0 and the endogenous variables in the calibrated benchmark equilibrium. The top portion of the column corresponds to the key observable variables of interest which take on the values of the 1970 data from Table 1. The second column reproduces the 1990 data against which the theoretical experiments will be assessed.

5.2.1 Results for the mismatch shock

In the first experiment, just as in subsection 5.1.1, education responds endogenously. The shift in m_0 from 1.0 to 0.55 has been calibrated so that the model reproduces an unemployment rate for the non-educated comparable with the 1990 figure of 0.053. The third column of Table 5, 'Endog. Edu.', shows this new value for the parameter as well as the corresponding values for the endogenous variables. One can see that the mismatch shock is able to cause rising unemployment for both the educated and the non-educated labor force, and an increase in the wage premium. Quantitatively, the educated unemployment rate after the shock, 0.0204, closely matches the U.S. 1990 figure of 0.021. The wage premium reaches 1.483, which falls short of the U.S. 1990 value of 1.58, and accounts for about 30 per cent of the observed increase on the 1.44 premium in 1970. Regarding education, the rate of college participation experiences a rise from 0.25 to 0.297 which, short of the 0.33 figure in 1990 data, accounts for nearly 60 per cent of the observed increase on the 0.25 rate in the 1970 benchmark.

These results draw on the underlying changes operated in other endogenous variables of the model. These are reported in the bottom portion of Table 5. The upward shift in the unemployment rate in the two education groups is driven largely by the drop in matching rates, ν_i, that follow from both the direct impact of lower m_0 and the downward shift in market tightness, v_i/u_i. Given that the wages of non-educated workers, $w(j, ne)$, do not appear to fall proportionately more than the wages for the educated, $w(j, e)$, much of the rise in the wage premium must have been caused by the shifts in the composition of the labor force. The changes in education imply that a larger proportion of educated workers become high-wage skilled, $p_{s|e}$, at the same time as the proportion of the non-educated that are high-wage skilled workers, $p_{s|ne}$, falls. Regarding the college decisions, a larger number of non-skilled workers attend college. As a result of the shock, the value of participating in education, $(r + \rho)U(ns, e)$, and the value of not participating, $(r + \rho)U(ns, ne)$, both fall. Because the value of education is discounted, the *opportunity cost* effect leads to the rise in education participation by the non-skilled, p_{e2}. For the skilled there is also a decline in both, the return to education, $(r + \rho)U(s, e)$, and the return to not taking education, $(r+\rho)U(s, ne)$. Again, discounting means that the fall in the opportunity cost of education dictates that college attendance by the skilled, p_{e1}, goes up.

5.2.2 The role of education under the mismatch shock

The second experiment calculates the equilibrium outcomes for the m_0 calibrated in the first experiment while holding the benchmark 1970 education choices constant. The fourth column, 'Exog. educ.', of Table 5 shows the result. The unemployment rate for the non-educated still increases, but by a smaller amount than in the experiment with endogenous education and, therefore, does do not meet the 1990 data as closely. With exogenous education, market tightness in the non-educated segment, v_{ne}/u_{ne}, after the shock is visibly higher than with endogenous education. Thus the *direct* effect accounts for much of the contribution of education to the change in the non-educated unemployment rate. Regarding the wage premium, with education held constant it declines relative to the 1970 benchmark and, unlike in the experiment with endogenous education, fails to meet even the qualitative features of the change in the data to 1990. Since wages, $w(j, i)$'s, are similar whether education is exogenous or endogenous, education has had to have a predominantly *composition* effect on the wage premium. Concerning the educated unemployment rate, the change with exogenous education nearly accounts for the shift observed in the data and, therefore, provides as good a match to the 1990 data as in the experiment with endogenous education.

Therefore, under the mismatch hypotheses, the behavior of education contributes not only to the quantitative response of the wage premium and non-educated unemployment, but also decisively to the qualitative sign of the former.

Table 5. Mismatch shock

	'70 Benchmark	'90 Data	Endog. educ.	Exog. educ.
Exog. parameter m_0	1.00	–	0.55	0.55
unemp. educated	0.011	0.021	0.020	0.021
unemp. non-educ.	0.024	0.053	0.053	0.045
wage premium	1.44	1.58	1.48	1.43
college partic.	0.25	0.33	0.30	0.25
ϕ	1		1	1
p_{e1}	0.165		0.200	0.165
p_{e2}	0.285		0.296	0.285
$p_{s\|e}$	0.66		0.68	0.66
$p_{s\|ne}$	0.05		0.00	0.05
v_e/u_e	34.55		32.90	32.23
v_{ne}/u_{ne}	7.36		4.55	6.38
$w(s,e)$	1.85		1.80	1.80
$w(s,ne)$	1.78		1.67	1.69
$w(ns,e)$	1.08		1.07	1.07
$w(ns,ne)$	1.07		1.05	1.06
$(r+\rho)U(s,e)$	1.84		1.78	
$(r+\rho)U(s,ne)$	1.77		1.63	
$(r+\rho)U(ns,e)$	1.08		1.07	
$(r+\rho)U(ns,ne)$	1.07		1.05	
variance edu.	0.066		0.059	0.060
variance non-edu.	0.011		0.000	0.010

5.3 Residual inequality

Along with a rising wage premium, there has been a substantial widening of wage inequality within specified education groups in the U.S. (see Juhn et al.(1993)). This is typically attributed to variation in unobservable skill attributes. The variance of the log of wages is a common way of measuring this residual inequality. For the numerical experiments here, this measure has been calculated within the two education groups and is displayed in the two final rows of Tables 4 and 5.[13]

In the model, residual inequality within an education group exists when there are both skilled and unskilled individuals who have the same education attribute. It is useful to break down changes in wage inequality into two components: the spread in the wage between skilled and non-skilled wages within the group, and the relative weight of the two skill levels

[13]See also footnote 10.

in the group. As expected, the skill-biased shock widens the spread in the two education groups and leads invariably to increased residual inequality in the educated group. In the non-educated group, wage inequality declines to zero as only non-skilled workers remain in this group after the shock. Under the mismatch shock, the spread of wages within the educated group decreases slightly but the typical shift in composition towards the skilled worker tends to balance out the effect on inequality. As a result there is a mild decline in variance within the educated group. On the other hand, the shift in composition of the non-educated group towards non-skilled workers drives the sharp drop in inequality within this group as well.

Thus, none of the explanations appears to be apt to account for the documented rising trend in residual wage inequality in the U.S., especially among the non-educated workers.

5.4 Sensitivity analysis

The experiments reported above correspond to a version of the model calibrated for a particular $p_s = 0.20$. As pointed in Section 4, this choice has been made so that the equilibrium meets certain requirements of existence under changes in parameters. But this is not the only possible choice for which these conditions hold. This section repeats the above experiments for the calibrations corresponding to two more extreme choices of p_s, 0.17 and 0.23, around the previous baseline choice of 0.20. For each of the two, the top row of Table 6 shows the calibrated values for the fixed parameters b_0, β, c_R, $\overline{c}_e$, and c_e. The main body of Table 6 contains the values for the parameters η_s and m_0 set in the benchmark (BM) 1970 calibration and the two experiments.[14] The rest of entries show the equilibrium values of endogenous variables across the different settings.

With $p_s = 0.23$, the mismatch shock does at least as good a job as in the leading experiment reported in Table 5. In contrast with that, however, now the implied change in the wage premium is quantitatively comparable with the data, the match of the observed rise in the educated unemployment rate being more modest though. The increase in educational attainment is closer to the data in this case too. With a smaller $p_s = 0.17$, however, the mismatch shock fails to produce the increase in the wage premium as well as in education. The poor results in these two fronts are related. For such a low p_s, the targets on residual

[14]The remaining parameters are as in Table 2.

variance and education require a calibration where most of the skilled workers are educated in 1970 (i.e., p_{e1} must be 0.16, very close to $p_s = 0.17$). Thus there is little room for a positive response of education by this group and, therefore, for a quantitatively sizable decline in the skill contents within the non-educated labor force as it is already a small 0.013 in the 1970 benchmark. Hence the composition effect that drives the rise in the wage premium in the previous experiments becomes very thin in this case.

The predicted effects of a skill-biased shock under the calibrations for p_s equal to 0.17 and 0.23 are similar to those obtained in the leading experiment of Table 4, including the failure to raise the unemployment rate of the educated labor force. The only apparent difference is the quantitative response of college participation, which is larger in the case with $p_s = 0.23$ and smaller when $p_s = 0.17$.

Further calibrations associated with other intermediate values of the parameter p_s have been considered as well. One can conclude that the results obtained earlier in Section 5.1-5.3 on the benchmark calibration prove largely robust across these different settings. In particular, the mismatch shock continues to fit the facts well as long as p_s is not too small.

Table 6. Sensitivity Analysis

	Calibration with $p_s = 0.23$ $b_0 = .78, \beta = .15, c_R = .10$ $\overline{c}_e = 34.8, c_e = 65.50$			Calibration with $p_s = 0.17$ $b_0 = .77, \beta = .15, c_R = .10$ $\overline{c}_e = 43.3, c_e = 91.62$		
	'70 BM	mismatch	skill-bias	'70 BM	mismatch	skill-bias
η_s	1.965	1.965	1.978	1.900	1.900	1.995
m_0	1.00	0.76	1.00	1.00	0.50	1.00
unemp. educated	0.011	0.014	0.011	0.011	0.023	0.011
unemp. non-educ.	0.024	0.053	0.045	0.024	0.053	0.045
wage premium	1.44	1.59	1.63	1.44	1.43	1.58
college partic.	0.25	0.33	0.33	0.25	0.27	0.28
ϕ	1	1	1	1	1	1
p_{e1}	0.16	0.23	0.23	0.16	0.17	0.17
p_{e2}	0.32	0.33	0.33	0.26	0.27	0.28
$p_{s\|e}$	0.64	0.70	0.70	0.64	0.63	0.62
$p_{s\|ne}$	0.09	0.00	0.00	0.01	0.00	0.00

6 Conclusion and final remarks

This paper extends a standard search-matching model by introducing education choices in a segmented labor market where education and skill are not equivalent attributes of a worker. The endogeneity of education is one major contribution of this paper to a growing body of literature which studies wage inequality in related models. Equilibrium properties of the model are characterized. Then a calibrated setting is used to study a skill-biased change in technology and a mismatch shock as causes of the changes in education-specific unemployment rates, the wage premium, and college education attainment in the U.S. economy between 1970 and 1990.

A skill-biased change in technology goes part of the way towards explaining the changes in the wage premium, the college participation rate, and the unemployment rate of non-educated labor force. However the rise in the unemployment rate of the educated cannot be accounted for by this shock. The endogenous changes in education decisions make a positive, albeit limited, contribution to the quantitative effects of this shock. On the other hand, a mismatch shock explains a great deal of the changes in the wage premium, the unemployment rates for both the educated and non-educated labor force, and the college participation rate. The mismatch hypothesis can thus account for the changes in these four labor market variables. In this case, the change in education is essential to understand the transformations in the labor market which, in turn, govern the response of education. Rises in education occur mostly among the already skilled workers, and this brings about the shifts in the skill composition of the two education groups which are key to explain the increase in the wage premium. Given the recent emphasis placed on the skill-biased interpretation of U.S. labor-market transformations, it is remarkable that this paper shows that a mismatch explanation might work at least as well.

This paper is based on a highly stylized model though. This means that the analysis leaves as many questions open as it answers. For one thing, the model is not consistent with the evidence of rising residual inequality under either shock. This not being the focus of the present paper, points in the direction of necessary further research. In this respect, there are at least two features of the present analysis that deserve consideration. First, the quantitative exercises always imply equilibria where educated workers, regardless of their skills, invariably choose to participate in the segment of jobs that require a degree. This precludes the existence of overeducation and its role in wage inequality. Second, by assumption all jobs have exactly the same technology irrespective of the segment they belong to. Decisions on equipment investment on the part of the firms would cause heterogeneity across job types with consequences for wage disparities. Future work will also extend the approach of this paper in order to explore the ability of different hypotheses to explain how the institutions and policies in European countries may have led to labor-market outcomes that differ from the US experience. This type of analysis may shed new light on the possible differential role of education policies across countries.

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APPENDICES

A Equations (12), (13), and (14)

Denote by $U_{j|i}$ the mass of unemployed workers with skill j in segment i. By definition, the mass of employed workers with these characteristics is $p_{j|i} - U_{j|i}$. The flow equation is $\dot{U}_{j|i} = (p_{j|i} - U_{j|i})(\lambda + \rho) - \nu_i \pi_{j|i} U_{j|i}$. In a steady-state $p_{j|i}(\lambda + \rho) = (\nu_i \pi_{j|i} + \lambda + \rho)U_{j|i}$. Then the definition of $us_{s|i} \equiv U_{s|i}/(U_{s|i} + U_{ns|i})$ leads to Eq.(12).

The decision of a worker whether to accept an offer in Eq.(3) is dominated by the hiring decision of the firm $\pi_{j|i}$ in Eq.(7) since Eq.(10) must hold. This permits to remove the max term in Eq. (3) and simplify the algebra in what follows. Then Eq.(13) results from developing the surplus-sharing bargaining Eq.(10) using the optimal behavior of firms (6)-(7) and workers (2)-(3), and the technology relations (8)-(9), with Eq.(11) to pin down the value of a vacancy in equilibrium. On the other hand, Eq.(14) comes from the free-entry condition Eq.(11) upon using the equilibrium firm's Bellman equations (6)-(7), the technology relations (8)-(9), and Eq.(13) to substitute the wage terms $w(j,i)$.

B Computation of Equilibrium

An outline of the main steps involved in the computation follows.

1. Set a pair $(p_{e1}, p_{e2}) \in [0, p_s] \times [p_s, 1]$.

2. Use Eq.(16) to find $p_{j|i}$'s. This must be done for both $\phi \in \{0, 1\}$ if the region (ns, e) in Figure 1 exists, that is if $p_{e2} > p_s$.

3. Use the Proposition to establish existence and the $\pi_{ns|i}$'s within each segment. Do so for each candidate ϕ, if pertinent.

4. Use Eq.(4) to verify that $\phi_{e|s} = 1$ and, if needed, determine existence and value(s) of ϕ. This requires computing the equilibrium for given p_{e1}, p_{e2}, and $\pi_{ns|i}$'s for all the relevant alternative values of ϕ.

5. Compute equilibrium outcomes for all surviving candidate ϕ's. Update the pair (p_{e1}, p_{e2}) using Eq.(5) and go back to step 2 until convergence.

In step 3, if $p_{s|i} = 1$, by the Proposition, $\pi_{ns|i}$ remains indeterminate and must be found through Eq.(14b) in equilibrium. This is not a problem as the equilibrium can be computed independently of $\pi_{ns|i}$ in this case. In steps 4 and 5 the equilibrium is computed by solving Eq.(14) using, for each segment, a Newton-Rapson iteration on market tightness.

C Unemployment rates and wage premium

The calibration and the derivation of results requires computing education-specific unemployment rates and wages.

C.1 Unemployment rates

Let U_e denote the mass of educated workers that are unemployed, and L_e the size of the educated labor force. Also let $U_{j|e}$ denote the mass of workers with skill j within the unemployed pool of educated workers. The flow equation is

$$\begin{aligned}\dot{U}_e &= (\lambda+\rho)(L_e-U_e)-U_{s|e}\nu_e\pi_{s|e} \\ &\quad -U_{ns|e}\left[\phi\nu_e\pi_{ns|e}+(1-\phi)\nu_{ne}\pi_{ns|ne}\right].\end{aligned}$$

In the steady-state, the unemployment rate is

$$\left(\frac{U_e}{L_e}\right)=\frac{\lambda+\rho}{\lambda+\rho+\left(\frac{U_{s|e}}{U_e}\right)\nu_e\pi_{s|e}+\left(\frac{U_{ns|e}}{U_e}\right)\left[\phi\nu_e\pi_{ns|e}+(1-\phi)\nu_{ne}\pi_{ns|ne}\right]}.$$

It remains to calculate the proportions $U_{j|e}/U_e$ involved. Use the equation $\dot{U}_{s|e}=(\lambda+\rho)(p_{e1}-U_{s|e})-U_{s|e}\nu_e\pi_{s|e}$ and $\dot{U}_{ns|e}=(\lambda+\rho)((p_{e2}-p_s)-U_{ns|e})-U_{ns|e}\left[\phi\nu_e\pi_{ns|e}+(1-\phi)\nu_{ne}\pi_{ns|ne}\right]$. In a steady-state $\dot{U}_{s|e}=\dot{U}_{ns|e}=0$ so

$$U_{s|e}=\frac{(\lambda+\rho)(p_{e1})}{\nu_e\pi_{s|e}+\lambda+\rho},$$

$$U_{ns|e}=\frac{(\lambda+\rho)(p_{e2}-p_s)}{\phi\nu_e\pi_{ns|e}+(1-\phi)\nu_{ne}\pi_{ns|ne}+\lambda+\rho}.$$

Since $U_e=U_{s|e}+U_{ns|e}$,

$$\begin{aligned}\left(\frac{U_{s|e}}{U_e}\right) &= \left[(p_{e1})(\phi\nu_e\pi_{ns|e}+(1-\phi)\nu_{ne}\pi_{ns|ne}+\lambda+\rho)\right]\times \\ &\quad \left[(p_{e1})(\phi\nu_e\pi_{ns|e}+(1-\phi)\nu_{ne}\pi_{ns|ne}+\lambda+\rho)+\right. \\ &\quad \left.(p_{e2}-p_s)(\nu_e\pi_{s|e}+\lambda+\rho)\right]^{-1}.\end{aligned}$$

Similarly, for non-educated workers the unemployment rate is

$$\left(\frac{U_{ne}}{L_{ne}}\right)=\frac{\lambda+\rho}{\lambda+\rho+\left(\frac{U_{s|ne}}{U_{ne}}\right)\nu_{ne}\pi_{s|ne}+\left(\frac{U_{ns|ne}}{U_{ne}}\right)\nu_{ne}\pi_{ns|ne}},$$

with

$$\left(\frac{U_{s|ne}}{U_{ne}}\right)=\frac{(p_s-p_{e1})[\nu_{ne}\pi_{ns|ne}+\lambda+\rho]}{(p_s-p_{e1})[\nu_{ne}\pi_{ns|ne}+\lambda+\rho]+(1-p_{e2})(\nu_{ne}\pi_{s|ne}+\lambda+\rho)}.$$

C.2 Wages

For the educated workers, the total wage bill is

$$\begin{aligned}\tilde{w}_e &= w(s,e)\frac{p_{e1}}{p_{e1}+(p_{e2}-p_s)}\left(1-\frac{\lambda+\rho}{\lambda+\rho+\pi_{s|e}\nu_e}\right)+ \\ &\quad (\phi w(ns,e)+(1-\phi)w(ns,ne))\frac{(p_{e2}-p_s)}{p_{e1}+(p_{e2}-p_s)} \\ &\quad \left(1-\frac{\lambda+\rho}{\lambda+\rho+\phi\pi_{ns|e}\nu_e+(1-\phi)\pi_{ns|ne}\nu_{ne}}\right).\end{aligned}$$

and then the average wage for this group is

$$\begin{aligned}w_e &= \tilde{w}_e\left[\frac{p_{e1}}{p_{e1}+(p_{e2}-p_s)}\left(1-\frac{\lambda+\rho}{\lambda+\rho+\pi_{s|e}\nu_e}\right)+\right. \\ &\quad \left.\frac{(p_{e2}-p_s)}{p_{e1}+(p_{e2}-p_s)}\left(1-\frac{\lambda+\rho}{\lambda+\rho+\phi\pi_{ns|e}\nu_e+(1-\phi)\pi_{ns|ne}\nu_{ne}}\right)\right]^{-1}.\end{aligned}$$

For the non-educated workers

$$\begin{aligned}\tilde{w}_{ne} &= w(s,ne)\frac{p_s - p_{e1}}{(p_s - p_{e1}) + (1 - p_{e2})}\left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{s|ne}\nu_{ne}}\right) + \\ & \quad w(ns,ne)\frac{1 - p_{e2}}{(p_s - p_{e1}) + (1 - p_{e2})}\left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{ns|ne}\nu_{ne}}\right)\end{aligned}$$

And so

$$\begin{aligned}w_{ne} &= \tilde{w}_{ne}\left[\frac{p_s - p_{e1}}{(p_s - p_{e1}) + (1 - p_{e2})}\left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{s|ne}\nu_{ne}}\right) + \right. \\ & \quad \left. \frac{1 - p_{e2}}{(p_s - p_{e1}) + (1 - p_{e2})}\left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{ns|ne}\nu_{ne}}\right)\right]^{-1}\end{aligned}$$