Forward induction and the excess capacity puzzle: An experimental investigation*

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August 27, 2003

Abstract: While the theoretical industrial organization literature has long argued that excess capacity can be used to deter entry into markets, there is little empirical evidence that incumbent firms effectively behave in this way. Bagwell and Ramey (1996) propose a game with a specific sequence of moves and partially-recoverable capacity costs in which forward induction provides a theoretical rationalization for firm behavior in the field. We conduct an experiment with a game inspired by their work. In our data the incumbent tends to keep the market, in contrast to what the forward induction argument of Bagwell and Ramey would suggest. The results indicate that players perceive that the first mover has an advantage without having to pre-commit capacity. In our game, evolution and learning do not drive out this perception. We back these claims with data analysis, a theoretical framework for dynamics, and simulation results.

Keywords: entry, excess capacity, forward induction, equilibrium selection, first-mover advantage.

JEL Classification: C70, C91, D42, L11, L12

* Financial support by the Spanish *Ministerio de Ciencia y Tecnología* (SEC2002-01352) and excellent research assistance by David Rodríguez are gratefully acknowledged.

1. INTRODUCTION

Theoretical industrial organization has argued, since at least Dixit (1980) and going back to Bain (1956) and Modigliani (1958), that excess capacity can be used to deter entry into markets. This issue has received considerable attention in the industrial organization literature, as one of the leading instances of the importance of commitment in sequential games. References to and discussions of Dixit (1980) appear in virtually all the teaching manuals in the area (see e.g. Tirole 1989, Basu 1993, Martin 1993 and Vives 1999). Despite this, there is little empirical evidence that incumbent firms actually hold excess capacity (see, for example, Smiley 1988 and Singh, Utton & Waterson 1998).[1]

Bagwell and Ramey (1996) provide a theoretical rationalization of this fact, based on a new approach to the problem. The specific model they put forward has three principal ingredients. First, it involves a different sequence of moves of the incumbent and the entrant than the one proposed by Dixit. The other two ingredients are the existence of a partially-recoverable capacity or entry cost and the use of forward induction to select among several equilibria. In their model, there are typically monopoly equilibria in which either the incumbent or the entrant captures the market, as well as market-sharing equilibria in which both firms produce positive output levels. Their main result is that forward induction rules out the equilibria where the incumbent invests in capacity and, hence, manages to retain the whole market. The model yields a very suggestive explanation of observed behavior and invites further investigation. However, given the highly stylized nature of the model, and the lack of observability of some of the key variables, a proper test of this model with field data is difficult. We therefore conduct an experiment to study the extent to which this explanation is satisfactory and, more generally, to shed light on the strategic behavior of incumbents and entrants.

In our experiment we use a simple game inspired by (but somewhat different from) the one in Bagwell and Ramey (1996), hereafter B-R. In our game, two firms – an incumbent and a potential entrant – make decisions in three stages. First, the incumbent has the opportunity to partially pre-commit to a given level of capacity, by incurring a certain cost. Then the entrant has the same choice, having observed the incumbent's choice. In the third stage both firms simultaneously decide whether to compete (in prices) in the market, by then paying (the rest of) the capacity cost. There are two pure-strategy equilibrium outcomes: One of the two firms produces and obtains monopoly profit while the other stays out of the market. Both of these situations are equilibrium outcomes resulting from backward induction.

However, only the outcome in which the incumbent leaves the market and the entrant conquers it survives the application of forward induction. In our context, an entrant who (after having observed the incumbent's choice) pre-commits capacity must be signaling that she intends to become the monopolist, as pre-committing and then not producing is a dominated strategy. The entrant could have avoided pre-committing so as not to lose the pre-committed cost. In anticipation of this, the incumbent does not invest in capacity. Thus, in this game the possibility of partial pre-commitment together with the logic of forward induction takes away the advantage that the incumbent has in the standard entry-deterrence model. Hence, in this model the prediction is that, as the field data suggest, incumbents do not invest in capacity to deter entry.

There is one specific difference between the B-R game and ours that should be highlighted here: In our game there are no market-sharing equilibria.[2] The case with market-sharing equilibria can be considered to be the empirically more reasonable one, since what the field evidence suggests is that incumbents cannot use capacity to keep other firms out of the market and not so much that entrants can expel incumbents. It therefore would seem natural to include market-sharing equilibria in the design. However, for evaluating the proposed selection argument, an environment without the possibility for market-sharing is more appropriate. The main reason is that market-sharing involves an element of fairness and this feature could bias data in favor of this outcome. In addition, the exclusion of the possibility of market-sharing eliminates potential coordination problems within the set of those equilibria that survive the proposed selection argument. Our simple set-up allows for a cleaner comparison between the two types of outcomes.

We find that the full B-R prediction does not hold in our laboratory data. In fact, the incumbent becomes the monopolist three times as frequently as the potential entrant. An explanation of the fact that the incumbent tends to win the market may be found in a commonly-held belief by many players that the first mover has a strategic advantage, and thus should become the monopolist in the post-commitment game. In a less restrictive environment with

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[1] See also Geroski (1995) for a discussion of what is known empirically about entry.
[2] This also holds in the B-R game, under specific cost conditions.

ample market-sharing possibilities this first-mover advantage might lead to incumbents capturing a larger part of markets or capturing a given market with smaller expenditures on capacity. At the same time, we find that there is only limited pre-commitment by either the incumbent or the entrant.

As a complement to our main treatment we also conducted sessions with a (more) standard entry game à la Dixit (1980), in which only the incumbent may pre-commit. Here we find a considerably higher rate of incumbent entry deterrence through pre-commitment.[3] The quite moderate level of pre-commitment in our B-R design data is quite suggestive, since it is consistent with the field evidence. However, in which sense is it consistent with the more frequent use of pre-commitment in the Dixit treatment? Perhaps the world is more like the B-R environment, involving capacity decisions by both firms and partially recoverable costs. In addition, behavior in the Dixit treatment may help us to understand behavior in the B-R environment. The pre-conception of the first-mover advantage may be the starting point in both cases. In Dixit the pre-commitment signal is a rather clear one and, hence, is used more frequently; it may be perceived as the way to drive home the point of the incumbent's first-mover advantage. In contrast, in B-R the meaning of the combined signals may seem open to interpretation, and so pre-commitment is used less frequently.

More generally, when forward induction does not clash with a perception of first-mover advantage, as in the case when only the first mover happens to pre-commit or in the game when only one player can pre-commit, then the pre-committed player becomes the monopolist with a very high likelihood. In these cases, the pre-commitment signal is not indispensable but it helps, indicating that most participants do have an understanding of the basic forward-induction force. In fact, we find evidence that the choice of whether or not to participate in the market is strongly dependent on the pre-installation decisions.

The perceived first mover advantage is only part of our explanation of the results. In our game, forward induction selects the same outcome as the iterated deletion of weakly-dominated strategies. Therefore, even if players were boundedly rational, one might expect that the opportunity to play the game repeatedly could lead players to avoid dominated strategies, at least

[3] Mason and Nowell (1998) report results from experiments with a related Dixit-type entry game. Their game is less stylized than ours, so that behavior may be affected by the complexity of the set-up. They find that many incumbents choose an entry-barring output and that many entrants stay out of the market. However, a significant

after enough time of play. An initial perception of a first mover advantage would then vanish over time. It is, however, well known that learning or evolution does not always lead to limiting outcomes that respect the iterated deletion of weakly-dominated strategies.[4] We provide some results that explain why the initial pattern of play is not driven out. We show theoretically that our game has outcomes that do not satisfy the iterated-deletion logic, but are asymptotically present under dynamics where better-performing strategies grow faster than worse-performing ones. We also perform simulations with a learning model (Camerer and Ho 1999) that tracks the behavior of our data.

2. IMPLEMENTATION

2.1. The Game

In our game there are two firms that can produce a homogeneous good with constant, and equal, marginal cost. Production requires the building of a plant or some other initial investment; the total cost of this initial investment is F. The game has three stages: In the first and second stages, the incumbent and the entrant make sequential and observable capacity pre-installation decisions. More precisely, the incumbent first chooses whether or not to irrecoverably sink a fraction $a < 1$ of the fixed cost of production F. After observing the incumbent's choice the entrant then chooses between the same two options. In the third stage, the two firms simultaneously make a decision on whether to compete in the market. This decision involves either paying the remaining part of the full fixed cost, $(1-a)F$, or the whole amount F. Thus, if one of the firms has not pre-committed, it can still pay for the total fixed cost F in the third stage.

The third-stage competition is in prices. As a result, if both firms decide to actually pay (the remainder of) the full capacity cost in the third stage, the resulting price will be equal to the marginal cost. If only one of them chooses to pay the whole cost, then the outcome will be the monopoly outcome. The only relevant actions in the third stage are, hence, whether or not to pay (the remainder of) the whole fixed cost.

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proportion of potential entrants entered when it yielded losses and a substantial proportion of incumbents did not engage in entry deterrence.

[4] See, e.g., Fudenberg and Levine (1998), Samuelson (1997), Vega-Redondo (1996).

There are four possible pre-commitment combinations, which give rise to four possible subgames in the third stage. Figure 1 presents these subgames for our B-R sessions using a general payoff representation in which we have normalized monopoly profits to 1. Earnings from Bertrand competition in the market are zero so that if both firms enter the market they both earn $-F$. Inaction leads to zero profits.

For the incumbent, A denotes no pre-installation and B pre-installation. For the entrant's reactions to A, we denote no pre-installation with C and pre-installation with D, whereas for the reactions to B, the absence of pre-installation is denoted by E and pre-installation by F. This explains the names of the different subgames: AC is the subgame in which no firm pre-commits, AD where only the entrant pre-commits, BE where only the incumbent pre-commits, and BF where both firms pre-commit. We denote by action *I* (in each subgame) the action in which a player pays the whole investment cost in the third stage, and by *NI* the action in which a player does not pay this cost.

Figure 1: Subgames in Bagwell-Ramey Sessions.

SUBGAME AC

		Entrant	
		NI	**I**
Incumbent	**NI**	0, 0	0, 1
	I	1, 0	$-F$, $-F$

SUBGAME AD

		Entrant	
		NI	**I**
Incumbent	**NI**	0, $-aF$	0, 1
	I	1, $-aF$	$-F$, $-F$

SUBGAME BE

		Entrant	
		NI	**I**
Incumbent	**NI**	$-aF$, 0	0, 1
	I	1, 0	$-F$, $-F$

SUBGAME BF

		Entrant	
		NI	**I**
Incumbent	**NI**	$-aF$, $-aF$	0, 1
	I	1, $-aF$	$-F$, $-F$

Subgame-perfect equilibria. In every subgame, the action pairs (NI, I) and (I, NI) are the only equilibria. Given this, there are a variety of pure-strategy subgame-perfect equilibria in this game. However, some of the outcomes are *not* possible under subgame-perfection with pure strategies. Any firm can guarantee itself zero profits by not pre-committing and then choosing NI in all subgames; thus, no firm can obtain $-aF$ in equilibrium. Also, since the pairs (NI, NI) and (I, I) are not pure strategy equilibria in any subgame, the profit pairs (0, 0) and ($-F$, $-F$) cannot occur as part of a subgame-perfect equilibrium.

But either of the two profit pairs (1,0) and (0,1) is consistent with subgame perfection. For example, if both agents expect (NI, I) in all of the second-stage subgames, it is optimal for the incumbent to not invest, and the entrant will be the monopolist (with or without pre-commitment), with a profit of (0,1). The reverse holds if (I, NI) is expected in all of the second-stage subgames, for a final profit of (1,0). Intuitively, if both firms believe that the incumbent will be the winner in the second stage, this leads to an equilibrium where the incumbent is a monopolist in the second stage and the entrant chooses to not pre-commit and stays out of the market. The reverse happens if the firms believe that the entrant will 'win' after all first-stage outcomes.

2.2. Forward Induction

Matters change under certain refined equilibrium notions of subgame perfection, which select a unique equilibrium from the set. Kohlberg and Mertens (1986) introduced the concept of 'forward induction' with the aim of describing some desirable properties of an equilibrium concept. In general terms, forward induction says that the actions of players have strategic significance, and should be interpreted in this light. It can be described (Battigalli 1996) in the following way: "A player should always try to interpret her information about the behavior of her opponents assuming that they are not implementing 'irrational' strategies."[5] In our case the forward-induction rationality requirements imposed by Bagwell and Ramey seem relatively mild. Players should avoid weakly-dominated strategies, and their opponents should be aware of this, and take it into account when making their decisions.[6]

[5] Forward induction (as with other refined equilibrium concepts, like strategic stability) has, nevertheless, interesting connections with bounded rationality. For a rich discussion of the concept and formal definitions, see Hauk and Hurkens (2002).

[6] An alternative definition by Van Damme (1989, p. 485) states that when: "player i chooses between an outside option or to play a game G of which a unique (viable) equilibrium e^* yields this player more than the outside option, only the outcome in which i chooses G and e^* is played is plausible." It is easy to see that this (stronger) notion yields the same result in this game.

Previous evidence on the predictive value of forward induction is rather mixed. Cooper, DeJong, Forsythe & Ross (1992) analyze experiments involving a choice between an outside option for one of the players and a 2x2 coordination game, with two Pareto-ranked equilibria. They only analyze the case where forward induction and a simple dominance argument lead to the same prediction. Their results are consistent with this kind of forward induction idea. Cooper et al. (1993) present results from an experimental game, where there is an outside option for one of the players and a symmetric Battle-of-the-Sexes game is played if this outside option is foregone. When forward induction coincides with simple dominance the results are again consistent with these notions. However, in a second treatment, an outside option that does not dominate one of the other choices in the Battle-of-the-Sexes was observed to affect play in the same manner as an outside option that does dominate.

Van Huyck et al. (1993) consider an experimental setting in which players participate in an auction for the right to play a coordination game. Their results exhibit two key features: The price in the auction is high enough for a forward induction argument (different from dominance here) to select the Pareto-efficient equilibrium, and subjects' play in the coordination game actually selects this equilibrium. Schotter et al. (1994) study an experimental game for which the application of iterated dominance selects one outcome and obtain results that are not consistent with the predictions of the iterated dominance argument. The results presented in Brandts and Holt (1995) do not support forward induction, except in a very simple game where it is equivalent to the elimination of dominated strategies. Balkenborg (1998) reports results from a game in which backward induction yields an outcome different from that resulting from forward induction arguments; less than 20% of all cases result in the forward-induction outcome.

The focus in our paper is to consider forward induction in relation to a specific and important issue in the area of industrial organization. For this purpose, we have learnt from previous work and use experimental procedures that should give forward induction its best shot. We give subjects considerable experience and put them in both the incumbent's and the entrant's role to help them envision the strategic relations between the two firms.

In our game, forward induction gives the second mover an advantage. The argument goes as follows: At the time of pre-commitment an entrant can guarantee himself a payoff of 0, independently of what the incumbent has done, by not committing and then choosing NI. Thus, any strategy under which a player pre-commits and then chooses NI is weakly dominated, as it yields a lower payoff. Knowing that player 2 does not play dominated strategies, when player 1 observes a pre-commitment by player 2, he must conclude – according to the forward induction logic - that player 2 intends to become the monopolist and will play I, and so player 1 will respond optimally with NI.

As a consequence, player 2 will always (optimally) pre-commit, and then play I. In contrast, pre-commitment does not have the same signaling value for the incumbent firm. An incumbent that pre-committed could, mistakenly, have believed that the entrant was not going to pre-commit (thus, expecting to become the monopolist). So, when faced with the unambiguous subsequent pre-commitment choice of the entrant, the incumbent should yield and leave the monopoly profits to the entrant. An incumbent who has not pre-committed has an even stronger reason to yield in front of a pre-committed entrant. In anticipation of all this, the incumbent does not pre-commit and leaves the market to the entrant.

Therefore, by forward induction, only the outcome (NI, I) is plausible. Taking this into account, the first player will optimally respond by not pre-committing and then choosing NI in all the subgames. So the set of strategies that survive the iterated deletion of weakly-dominated strategies includes only the outcome (NI, I).

2.3. Iterated Deletion of Weakly-dominated Strategies.

In our experiments the game was played using the strategy-elicitation method.[7] This means that the incumbent had to choose whether to pre-install or not and also whether to complete the investment or not for each of the entrant's possible pre-installation decisions in the second stage. Similarly, the entrant had to make a pre-installation decision for the incumbent's two possible pre-installation decisions, as well as a complete investment decision for the two possible resulting pre-installation decisions of the two players.

The corresponding reduced normal form is shown in Table 1; it can be used to further illustrate the selection rationale presented above. Here the incumbent is the row player and the entrant the column player. The labels of the strategies are now those used in the experiment, where the number "1" represents the choice of not completing the investment, *NI*, and the number "2" means completing it, *I*. The reasoning we present holds for all positive values of *a*

[7] This method allows the experimenter to collect data at every decision node. It can be considered to favor somewhat more thoughtful behavior and is, hence, quite appropriate in this context.

and F. However, for ease of exposition, we now also use the same parameter values and payoffs as in the experiment: We chose $a = 1/2$ and $F = 1$. We then transformed the payoffs by adding 1 to each payoff and then multiplying every payoff by 80. Thus, if both players choose *I*, each receives 0. If only one player chooses *I*, he or she receives 160. Choosing *NI* gives a payoff of 80 without pre-installation; however, pre-installing and then choosing *NI* gives a payoff of 40.

Table 1: The Bagwell-Ramey game in reduced normal form.

	CE11	E12	CE21	CE22	CF11	CF12	CF21	CF22	DE11	DE12	DE21	DE22	DF11	DF12	DF21	DF22
A11	80,80	80,80	80,160	80,160	80,80	80,80	80,160	80,160	80,40	80,40	80,160	80,160	80,40	80,40	80,160	80,160
A12	80,80	80,80	80,160	80,160	80,80	80,80	80,160	80,160	160,40	160,40	0,0	0,0	160,40	160,40	0,0	0,0
A21	160,80	160,30	0,0	0,0	160,80	160,80	0,0	0,0	80,40	80,40	80,160	80,160	80,40	80,40	80,160	80,160
A22	160,80	160,80	0,0	0,0	160,80	160,80	0,0	0,0	160,40	160,40	0,0	0,0	160,40	160,40	0,0	0,0
B11	40,80	40,160	40,80	40,160	40,40	40,160	40,40	40,160	40,80	40,160	40,80	40,160	40,40	40,160	40,40	40,160
B12	40,80	40,160	40,80	40,160	160,40	0,0	160,40	0,0	40,80	40,160	40,80	40,160	160,40	0,0	160,40	0,0
B21	160,80	0,0	160,80	0,0	40,40	40,160	40,40	40,160	160,80	0,0	160,80	0,0	40,40	40,160	40,40	40,160
B22	160,80	0,0	160,80	0,0	160,40	0,0	160,40	0,0	160,80	0,0	160,80	0,0	160,40	0,0	160,40	0,0

In this Table, the first (second) number after the last letter indicates whether *I* or *NI* was chosen if the other person did not (did) choose to pre-install. So, for example, A12 means that the incumbent did not pre-install, chose *NI* if the entrant did not pre-install and chose *I* if the entrant did pre-install; DE21 means that the entrant pre-installed if the incumbent did not pre-install, but did not pre-install if the incumbent did; in the third stage, the entrant chose *I* if the incumbent did not pre-install and chose *NI* if the incumbent did pre-install.

First, notice that the incumbent has only one (strictly) dominated strategy: B11 (by A11). This strategy consists of the incumbent pre-committing and then choosing *NI* in all the subgames that can subsequently be reached. The fact that it is dominated simply means that it does not make sense for a player to pre-commit if he does not want to become the monopolist (and so obtain the (*I, NI*) outcome at some point). The entrant has one strictly-dominated strategy (DF11 by CE11), and several weakly-dominated strategies: CF11 (by CE11), CF21 (by CE21), DE11 (by CE11), DE12 (by CE12), DF12 (by CF12) and DF21 (by DE21). All of these dominated strategies correspond to instances where the entrant pre-commits in some (or all) of the cases when he can do so, and then chooses *NI* in the subgame(s) that follow(s) pre-commitment.

Once the weakly-dominated strategies of the entrant have been eliminated, the incumbent has some weakly-dominated strategies: A12 (by A11), A22 (by A21), B12 (by A11) and B22 (by B21). All of these strategies correspond to instances where the incumbent chooses *I* in a subgame where the entrant has chosen to pre-commit. That can only be optimal if the entrant chose *NI* in that subgame. However, such choices by the entrant are dominated, and have already been eliminated.

The strategy DF22 for the entrant is now weakly dominant among the remaining strategies. This strategy corresponds to the entrant always pre-committing and then choosing *I* in all subgames. This is dominant because the (serially) undominated choice of the incumbent is to play *NI* any time he 'observes' a pre-commitment by the entrant. Finally, since DF22 is dominant, incumbent strategy B21 is sub-optimal, and the only strategies that remain for the incumbent are A11 and A21. So the prediction is that the incumbent will not pre-commit, but that the entrant will pre-commit and then become the monopolist.[8]

It is worth noticing that even without performing this last iteration (that is, with just two rounds of deletion of dominated strategies), the entrant can guarantee his favorite outcome (*NI, I*), as all the equilibria in the game that remains after two rounds of deletion produce that outcome. In some of those equilibria the entrant does not even have to pre-commit. A less stringent and perhaps more robust prediction would, therefore, be that the entrant becomes the monopolist with or without pre-installation.

[8] In general, the order of deletion of weakly dominated strategies might affect the final outcome of a given game. Not so in ours. The reason is that the strategies that are dominated (thus, eliminated) in (our) second round do not affect the relationship between those that are dominated and dominant in (our) first round. For example, after just eliminating DE11, DE12, DF11 and DF12, we can eliminate A12 and A22. Even after this deletion CF11, CF21 and DF21 are still weakly dominated by the same strategies we used.

2.4. The Dixit Game.

In our Dixit games, only the incumbent could pre-install, leading to only the two subgames shown in Figure 2:

Figure 2: Subgames in Dixit Sessions.

SUBGAME A

		Entrant	
		NI	**I**
Incumbent	**NI**	0, 0	0, 1
	I	1, 0	-*F*, -*F*

SUBGAME B

		Entrant	
		NI	**I**
Incumbent	**NI**	-*aF*, 0	0, 1
	I	1, 0	-*F*, -*F*

The forward-induction argument now points to the incumbent's pre-installation decision. The incumbent, at the time of pre-commitment can guarantee himself a payoff of 0, independently of what the entrant does, by not committing and then choosing *NI*. As before, any strategy under which a player pre-commits and then chooses *NI* is weakly-dominated, as it would yield a lower payoff. Knowing that the incumbent does not play dominated strategies, when the entrant observes a pre-commitment by the incumbent, he must conclude that the incumbent will play *I*, and so responds optimally with *NI*. As a consequence, the incumbent will always (optimally) pre-commit, and then play *I*. Therefore, by forward induction, only the outcome (*I, NI*) is plausible. Taking this into account, the entrant will optimally respond by choosing *NI* in the subgames. So the set of strategies that survive the iterated deletion of weakly dominated strategies produces only the outcome (*I, NI*).

The corresponding reduced normal form is shown in Table 2, where, in the strategy labels, the letters A and B refer to pre-installation decisions and the numbers have the natural interpretation. We again chose $a = 1/2$ and $F = 1$, transforming the payoffs by adding 1 to each payoff and then multiplying every payoff by 80.

Table 2: The Dixit game in reduced normal form.

	11	12	21	22
A1	80,80	80,80	80,160	80,160
A2	160,80	160,80	0,0	0,0
B1	40,80	40,160	40,80	40,160
B2	160,80	0,0	160,80	0,0

Strategy B1, the incumbent pre-committing and then choosing *NI*, is strictly dominated by strategy A1. With B1 eliminated, the entrant has two weakly-dominated strategies: 12 (by 11), and 22 (by 21). Once these have been eliminated, incumbent strategies A1 and A2 are weakly dominated by B2. This strategy corresponds to the incumbent pre-committing and then choosing *I*. So the prediction is that the incumbent will pre-commit and keep the market.

2.5. Experimental Design.

We conducted our sessions at Universitat Pompeu Fabra in Barcelona. Recruiting was accomplished via announcements posted in university buildings; participants included students in economics, business, law, political science, and the humanities. There were 12 (different) people in each session; in fact there were two separate groups of six, although this was not mentioned. This segmentation ensures two completely independent observations for each session. A session consisted of 25 periods in which people were matched and randomly re-matched in pairs, within the six-person subgroups. In addition, we alternated participants' roles – if a person was a row player (incumbent) in one period, he or she was a column player (entrant) in the next. We felt that this alternation scheme offered the best chance for people to understand the subtleties involved. The full instructions can be found in Appendix A.

We conducted six B-R sessions using the first of the two games described in the previous section. Payoffs were re-normalized for experimental purposes, and are in pesetas (at the time, \$1 exchanged for approximately 180 pesetas). People received their earnings over 25 periods, plus a show-up fee of 500 pesetas.

As already mentioned, in the B-R sessions play proceeded as follows: Each incumbent stated whether he wished to pre-commit, and also stated a choice (*I* or *NI*) for each of the two cases regarding possible pre-commitment by the entrant. Using the labels of the game above, the incumbent had to choose between A and B, and to indicate his choice in the two possible subgames that could result from the choice between A and B. Each entrant made choices without being informed of the paired incumbent's choices and stated whether she wished to pre-commit if the incumbent had pre-committed and also whether she wished to pre-commit if the incumbent had not pre-committed.[9] Given her own pre-commitment choices, she also stated a choice (*I* or *NI*) for each of the two cases regarding possible pre-commitment by the incumbent. She had to indicate her choice both for A and for B, as well as her choices for the two possible resulting subgames. After the data for the period was collected and matched up, each participant was informed of his or her payoff outcome for that period.

As a control we also conducted three Dixit sessions. As in the B-R sessions, we had 25 periods with random re-matching. Here the incumbent stated his choice concerning pre-commitment, as well as his choice (*I* or *NI*) in the resulting subgame. The entrant stated a choice (*I* or *NI*) if the incumbent had pre-committed and also if the incumbent had not pre-committed. So, overall, we had nine sessions, with a total of 108 participants. Average payoffs for the two-hour sessions were around 2500 pesetas.

3. RESULTS

3.1. Realized Outcomes

We focus first on realized outcomes and move later to the consideration of strategy choices to understand how outcomes eventuate.

[9] We shall henceforth presume that the incumbent is male and the entrant is female, despite the fact that one's gender must therefore change each period.

Table 3 shows the distribution of realized outcomes among the different cells of the B-R game, separately for each of the twelve groups, where we now indicate whether the different subgames involve P(reinstallation) or N(o) P(reinstallation) by the two players[10]:

Table 3: Realized Outcomes in the B-R Sessions.

	Subgame AC (NP,NP)				Subgame AD (NP,P)			
Group	*NI, NI*	*NI, I*	*I, NI*	*I, I*	*NI, NI*	*NI, I*	*I, NI*	*I, I*
1	13	13	13	10	0	5	1	5
2	10	5	20	10	1	6	0	3
3	3	3	22	14	0	14	1	6
4	5	1	7	5	1	2	0	3
5	9	8	29	22	0	0	1	2
6	6	3	22	18	0	3	1	7
7	8	3	41	7	0	1	1	5
8	13	2	49	10	0	0	1	0
9	8	5	11	6	2	13	1	7
10	5	0	31	8	1	1	1	8
11	6	12	11	17	0	5	1	3
12	5	9	5	5	2	8	1	9
Aggregated	91	64	261	132	7	58	10	58
	548 **(60.9%)**				133 **(14.8%)**			

	Subgame BE (P,NP)				Subgame BF (P,P)			
Group	*NI, NI*	*NI, I*	*I, NI*	*I, I*	*NI, NI*	*NI, I*	*I, NI*	*I, I*
1	0	0	6	8	0	1	0	0
2	2	0	11	4	1	1	1	0
3	2	0	6	4	0	0	0	0
4	1	0	43	4	0	0	2	1
5	0	0	0	2	0	0	1	1
6	0	0	11	3	0	0	1	0
7	2	0	5	1	1	0	0	0
8	0	0	0	0	0	0	0	0
9	2	0	10	3	1	3	0	3
10	1	0	12	4	1	0	1	1
11	3	0	7	3	1	3	3	0
12	1	0	12	5	1	8	0	4
Aggregated	14	0	123	41	6	16	9	10
	178 **(19.8%)**				41 **(4.6%)**			

[10] Recall that there were two completely independent six-person groups in each of our nine sessions.

Market capture and pre-commitment. In our context the most fundamental question is, arguably, which of the two players captures the market. The answer represents the bottom line with respect to the forward-induction prediction. If the entrant were to mostly become the monopolist, this would, carried over to a wider context, imply that the incumbent would somehow have to share the market with the entrant. We need to compare the sum of all (NI,I) outcomes with that of all of the (I,NI) outcomes. The result is that the entrant becomes the monopolist only 15% of the time, instead of the predicted 100%. By comparison, the incumbent becomes the monopolist in 45% of the cases. Coordination failure is substantial with no one in the market 13% of the time and both players in the market 27% of the time. Note, however, that together this 40% is still below the rate of coordination on the incumbent becoming the monopolist.

The natural next question is whether the incumbent's preponderance is accomplished through his pre-commitment. Overall, the incumbent pre-installs somewhat more often (24%) than does the entrant (19%).[11] When there is no pre-installation, the incumbent becomes the monopolist 48% of the time (the corresponding figure for the entrant is 12%). When only the incumbent pre-installs, he becomes the monopolist 69% of the time (the corresponding figure for the entrant is 44%), so that pre-commitment does help the incumbent. Taken together these facts allow us to say that the incumbent wins the market frequently and rather effortlessly, more so than the entrant, in contrast to what theory suggests.

It is clear from Table 1 that there is a high degree of variance in behavior across groups. For example, the group 4 incumbents took over the market in 55 (of 75) cases and the group 4 entrants took over the market in 3 cases, while for group 12 these figures were 18 and 25, respectively. Pre-installation behavior also varied considerably by group. For example, group 4 incumbents pre-installed 51 times, while group 8 incumbents never pre-installed; group 12 entrants pre-installed 33 times, while group 5 entrants pre-installed five times (and group 8 entrants pre-installed only once).

[11] Incumbent pre-commitment results in subgames BE or BF (219 of 900 outcomes), while entrant pre-commitment results in subgames AD or BF (174 of 900 outcomes).

A high proportion of both entrants and incumbents chose to pre-install no more than 10% of the time; however, there is considerable variation across the population. Comparing rates for each individual in B-R, 35 individuals (of 72) were more likely to pre-install as an entrant than as an incumbent, while this was reversed for 30 individuals; there was no difference in pre-installation rates for seven participants. While few participants (19% of the incumbents and 3% of the entrants) pre-installed more than half of the time in B-R, nearly 40% of the participants did so (as the incumbent) in the Dixit sessions.

Comparisons across subgames. We now focus on different comparisons involving observed frequencies in the different subgames, which we use to more clearly highlight the degree of support for the general notion of pre-commitment as a tool for market control. We start by observing that the incumbent's pre-installation decision has only a very minor impact on the entrant's pre-installation rate: The entrant chooses to pre-install 18.7% of the time when the incumbent pre-installs, compared to19.5% of the time when the incumbent does not pre-install.

At this point one might be tempted to jump to the conclusion that the strategic principles put forward in game-theoretic analysis have no effect, even in its most basic form. Nevertheless, pre-commitment by at least one firm occurs frequently enough in our data to warrant an examination of how firms' pre-commitment patterns and their relation to final investment (entry) decisions affect subsequent choices. We next compare decisions within and across subgames, and find that players' decisions on whether to compete in the market are quite sensitive to pre-installation choices of themselves and their counterparts.

Table 4 shows the proportion of eventual entry (choices of *I* strategies) into the market for each player, contingent on pre-installation decisions. Focusing first on comparisons within rows in the table, we observe the following pattern: When only one of the players pre-installs that player completes the investment more frequently and, hence, can be thought of having an advantage in capturing the market. When neither pre-installs the incumbent completes the investment more frequently than the entrant; when both pre-install, the investment rate is somewhat higher for the entrant.

[12] In these Figures, each category on the horizontal axis shows the highest percentage. Thus, the first column means that nearly 50% of incumbents pre-installed between 0 and 10% of the time, inclusive. Rates for entrants are aggregated across whether or not the incumbent chose to pre-install.

Table 4: Market entry conditional on pre-installation (strategies).

Players pre-installing	Incumbent's investment %	Entrant's investment %
Incumbent only (BE)	92.1%	23.0%
Neither (AC)	71.7%	35.8%
Both (BF)	46.3%	63.4%
Only the entrant (AD)	51.1%	87.2%

Comparing the proportions along the columns, it is easy to see that a player is most likely to invest when she alone has pre-installed and is much less likely to invest when only her counterpart has pre-installed. Investment rates are at intermediate levels when neither player pre-installs or when both players pre-install. Overall, aggregate choices of both first-mover and second-mover (recall that all subjects are sometimes first-movers and sometimes second-movers) were affected by pre-installation decisions. However, since the frequency of entrant pre-installation is low, this sensitivity to pre-installation choices may just not be enough to yield the Bagwell-Ramey predictions.

First-mover advantage. Some of the regularities that we have just discussed suggest that there is a perceived first-mover advantage in this game. Observe the large difference in investment rates (71.7% vs. 35.8%) when neither player pre-commits; in addition, note the difference in these rates when only the other player has chosen to pre-install, 51.1% for the incumbent compared to 23.0% for the entrant. In both cases, the investment rate for incumbents is more than double the investment rate for entrants. Thus, while in the strategic analysis the asymmetry of positions favors the entrant, in our data it appears to help the incumbent. It does seem that there is a contingent of people who (correctly) believe that the second mover has the advantage; this perception may prevail in groups 1, 11, and 12, for example.[13] However, for the majority of players the pattern is *as if* the incumbent had a 'natural' first-mover advantage, something not captured by the strategic analysis presented above.

[13] Note that the highest levels of 'confusion' are observed when only the second mover pre-installs, with an average of 1.38 market participants. By contrast, the norms seem tightest (# of participants closest to the cooperative rate of 1.00) when neither player pre-installs or when both players pre-install, with only 1.06 and 1.10 market participants, respectively.

This interpretation may be confronted with the data from the Dixit sessions, in which both the strategic analysis and the notion of a first-mover advantage reinforce each other. Table 5 shows the distribution of realized outcomes for these sessions:

Table 5: Realized Outcomes in the Dixit Sessions.

	Game A (NP)				Game B (P)			
Group	*NI, NI*	*NI, I*	*I, NI*	*I, I*	*NI, NI*	*NI, I*	*I, NI*	*I, I*
1	3	4	22	12	1	0	27	6
2	4	0	48	5	0	0	14	4
3	7	13	1	5	0	0	47	2
4	5	5	12	8	0	0	44	1
5	7	6	28	10	3	3	15	3
6	9	3	36	8	2	0	15	2
Aggregated	35	31	147	48	6	3	162	18
	261 **(58.0%)**				189 **(42.0%)**			

Overall the incumbent and the entrant become the monopolists in 69% and 8% of the cases. Recall that here only the incumbent can pre-commit, by choosing subgame B; in this subgame, the incumbent becomes the monopolist in 88% and the entrant in only 1% of the instances. Even without pre-installation, the entrant tends to yield to the incumbent, who becomes the monopolist 54% of the time, compared to 10% for the entrant. The orderings of these percentages is qualitatively similar to those in the B-R sessions (with no entrant pre-installation). The relevant rates are here 69% and 0% when the incumbent pre-installs (BE in the B-R sessions) and 48% and 12% when she doesn't (AC). The first mover chooses to pre-install 42% of the time (189/450), almost twice as often as in the B-R sessions. The pattern of conditional investment decisions is similar to that in the B-R sessions: when the incumbent does not pre-install, incumbent and entrant complete the investment in 75% and 30% of the cases, whereas with pre-installation these figures are 95% and 5%.

Statistical analysis. While we cite summary statistics for our data, we must be careful when performing a statistical analysis. Each observation of an interaction between an incumbent and an entrant is far from independent, given the 25 observations for each participant and the high degree of interaction within each group. However, since each group never interacts with any other, one clean and conservative statistical approach is to consider only group-level data when performing non-parametric tests.

Table 6 provides Wilcoxon signed-rank tests for a variety of comparisons. In all cases the null hypothesis is the equality of the number of outcomes. For all instances for which we indicate a prediction in the table the alternative hypothesis is the one that corresponds to the direction of that prediction. In these cases we use one-tailed tests to reflect these *ex ante* hypotheses. In the cases where we do not have a theoretical prediction, we use two-tailed tests:

Table 6: Non-parametric tests of Behavior across Subgames, by Group

B-R Sessions

Comparison of Rates	Prediction	Outcomes, by Group	p-value
Across Roles			
I_{ALL} vs. E_{ALL}	E > I	11I – 1E	0.998
I_{AC} vs. E_{AC}	-	9I – 2E	0.005*
I_{AD} vs. E_{AD}	E > I	8E – 2I	0.005
I_{BE} vs. E_{BE}	I > E	10I – 0E	0.001
I_{BF} vs. E_{BF}	E > I	3E – 4I	0.437
P_I vs. P_E	$P_E > P_I$	7I – 5E	0.765
Within Roles			
I_{AC} vs. I_{AD}	$I_{AC} > I_{AD}$	7AC – 5AD	0.170
I_{AC} vs. I_{BE}	$I_{BE} > I_{AC}$	9BE – 2AC	0.003
I_{AC} vs. I_{BF}	-	7AC – 3BF	0.348*
I_{AD} vs. I_{BE}	$I_{BE} > I_{AD}$	10BE – 1AD	0.002
I_{AD} vs. I_{BF}	$I_{BF} > I_{AD}$	5AD – 4BF	0.674
I_{BE} vs. I_{BF}	$I_{BE} > I_{BF}$	7BE – 1BF	0.008
I_{AC+AD} vs. I_{BE+BF}	$I_{BE+BF} > I_{AC+AD}$	10B – 1A	0.007
E_{AC} vs. E_{AD}	$E_{AD} > E_{AC}$	11AD – 1AC	0.001
E_{AC} vs. E_{BE}	$E_{AC} > E_{BE}$	8AC – 3BE	0.103
E_{AC} vs. E_{BF}	-	5AC – 3BF	0.547*
E_{AD} vs. E_{BE}	$E_{AD} > E_{BE}$	10AD – 1BE	0.001
E_{AD} vs. E_{BF}	$E_{AD} > E_{BF}$	8AD – 2BF	0.010
E_{BE} vs. E_{BF}	$E_{BF} > E_{BE}$	7BF – 3BE	0.138
E_{AC+AD} vs. E_{BE+BF}	$E_{AC+AD} > E_{BE+BF}$	9A – 2B	0.009

*Two-tailed test

Dixit Sessions

Comparison of Investment Rates	Prediction	Outcomes, by Group	p-value
Across Roles			
I_{ALL} vs. E_{ALL}	I > E	6I – 0E	0.016
I_A vs. E_A	I > E	5I – 1E	0.047
I_B vs. E_B	I > E	6I – 0E	0.016
Within Roles			
I_A vs. I_B	$I_A < I_B$	6B – 0A	0.016
E_A vs. E_B	$E_A > E_B$	5A – 1B	0.078

In this Table, I and E refer to the respective investment rates for the incumbent and the entrant, and P refers to pre-commitment rates. Subscripts refer to subgames.

The comparison in the very first row, I_{ALL} vs. E_{ALL}, pertains to which of the two players becomes the monopolist. The 11I-1E comparison means that the incumbent became the monopolist more frequently (overall) than did the entrant in 11 of the 12 groups, providing very little support for the alternative hypothesis. For all other across-role comparisons we compare, for each group, the number of times that the incumbent invested vs. the number of times the entrant invested. So, for example, the 9I-2E in the second row under *Across Roles* means that the incumbent invested more frequently than did the entrant in 9 of 11 groups.

For within-role comparisons, we consider the incumbent and entrant investment rates for each group. So, for example, the 9BE-2AC in the second row under *Within Roles* means that the incumbents invested more frequently in subgame BE than in subgame AC for nine of 11 groups. While we have data for 12 groups, there are often ties and/or cases where investment rates in a category cannot be calculated (a zero divisor), so the number of comparisons is frequently less than 12.

The tests confirm most of the descriptive analysis presented above. First, the group tests confirm that the incumbent is significantly more likely to control the market in both the B-R and the Dixit sessions. While this is in accord with the theoretical predictions when only the incumbent can pre-install, it is not consistent with the predictions when the entrant has the final pre-installation decision.

Nevertheless, the group tests also confirm that investment rates are highly sensitive to pre-installation decisions. In the B-R game, while the incumbent is significantly more likely to control the market when the entrant does not pre-install (subgames AC and BE), the entrant is significantly more likely to control the market when she pre-installs and the incumbent does not (subgame AD); there does not appear to be a consensus when both pre-install. Within-role comparisons are particularly strong when they reflect a difference in the pre-installation decision for that role, and no more than one party pre-installs, we see that $I_{BE} > I_{AC}$, $I_{BE} > I_{AD}$, $E_{AC} < E_{AD}$, and $E_{BE} < E_{AD}$, all at easily significant levels. People clearly understand why they themselves have pre-installed.

However, the effects are not as strong for within-role comparisons that reflect whether or not the other party has pre-installed. Both the E_{BE} vs. E_{AC} and the I_{AC} vs. I_{AD} comparisons show that there is some (not-quite-significant) tendency to respect the other firm's pre-installation choice. The only significant within-role comparisons involving the BF subgame are the cases where the role player has pre-installed (I_{BE} vs. I_{BF} and E_{AD} vs. E_{BF}), suggesting that a player who understands the logic of pre-installation is more likely to be sensitive to the pre-installation decisions of others.

The group tests also confirm that the incumbent is significantly more likely to invest after she has pre-installed than after she has not pre-installed ($p = 0.007$), and that the entrant is significantly less likely to invest after the incumbent pre-installs ($p = 0.009$). However, there is no significant difference in pre-commitment on a group basis.

In all six Dixit sessions, the incumbent invests more than the entrant when he has pre-installed; without pre-installation, this still holds in five of the six sessions. Similarly, the incumbent invests more often in all sessions when he has pre-installed, while the entrant invests less in five of the six sessions when the incumbent pre-installs.

3.2. Strategy Choices

Our results may look somewhat puzzling given the theoretical discussion in section 2. After all, the solution via deletion of dominated strategies looks very sensible, given that only a few rounds of elimination are necessary. One obvious explanation is that the rationality of experimental subjects is lower than what is necessary for achieving the theoretical outcome. But, one may wonder just how irrational these agents are. Up to now we have provided a rather synthesized view of the results. Since we elicit full strategies for each player, we can also examine the frequency of play for each strategy. The strategy choices as well as the corresponding expected choice for each B-R session are shown in Table 7:

Table 7: Strategy Choices in B-R Sessions.

	Complete 1st mover strategies							
Session	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	A11	*A12*	*A21*	*A22*	*B11*	*B12*	*B21*	*B22*
1	42	7	41	25	1	1	11	22
2	22	2	18	45	3	0	4	56
3	20	7	26	78	0	1	3	15
4	21	6	33	81	2	0	5	2
5	22	13	29	44	1	3	16	22
6	33	13	26	27	0	5	26	20
Aggregate	160	48	173	300	7	10	65	137

	Complete 2nd mover strategies							
Session	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	CE11	*CE12*	*CE21*	*CE22*	*CF11*	*CF12*	*CF21*	*CF22*
1	53	9	19	32	2	2	1	4
2	66	3	16	20	1	0	2	0
3	63	13	21	31	1	2	2	1
4	106	12	9	13	2	0	0	0
5	59	4	16	5	2	12	3	6
6	33	6	29	10	1	4	11	8
Aggregate	380	47	110	111	9	20	19	19
Session	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
	DE11	*DE12*	*DE21*	*DE22*	*DF11*	*DF12*	*DF21*	*DF22*
1	1	1	14	4	1	0	3	4
2	4	0	13	5	1	0	1	18
3	0	1	7	2	1	0	3	2
4	1	0	2	1	0	1	3	0
5	2	0	18	5	2	2	4	10
6	2	1	8	8	0	4	3	22
Aggregate	10	3	62	25	5	7	17	56

To obtain a more in-depth view of subjects' behavior we need to look at the expected *ex post* payoffs of complete strategy choices, as well as the frequency of these choices. Table 8 displays this information, where strategies have been ordered by the frequency of their choice. Using the information in Tables 7 and 8 we can now discuss incumbents' and entrants' strategy choices and afterwards move to an analysis of why the iterated elimination of dominated strategies does not fully work in our experiment.

Table 8: *Ex post* Expected Earnings for each Strategy

Bagwell-Ramey Sessions

Incumbent Strategies			**Entrant Strategies**		
Strategy	Frequency	Exp. Earnings	Strategy	Frequency	Exp. Earnings
A22	.333	85.5	CE11	.422	80.0
A21	.192	97.5	CE22	.123	40.0
A11	.178	80.0	CE21	.122	56.4
B22	.152	108.8	DE21	.069	78.7
B21	.072	100.3	DF22	.062	72.0
A12	.053	68.0	CE12	.052	63.6
B12	.011	40.0	DE22	.028	62.2
B11*	.008	40.0	CF12	.022	73.3
			CF22	.021	49.8
			CF21*	.021	46.7
			DF21*	.019	68.9
			DE11*	.011	49.7
			CF11*	.010	70.3
			DF12*	.008	43.1
			DF11*	.006	40.0
			DE12*	.003	33.3

*Indicates that a strategy is at least weakly dominated in the first round of iterations.

Incumbent strategies. Tables 7 and 8 reveal that the predicted strategies are not the most frequently chosen ones and that neither the predicted ones nor the most frequent ones are the most profitable ones. Recall that the two incumbent strategies that should be played according to forward induction are A11 and A21. However, the most commonly-used incumbent strategy (33.3%, by far the highest) is A22 (do not pre-install and always invest later). Strategy B22 is used roughly as often as the predicted ones.

A22 has an expected *ex post* payoff of 85.5, better than the safe payoff of 80 that is available by not pre-installing and never investing (A11). Choosing A22 could indicate that the incumbent thinks that it is likely that the second player thinks that the first mover has the advantage. However, the first mover could increase own payoffs by either choosing A21, which only differs from A22 in that the incumbent chooses not to complete the investment if he observes pre-installation by the second player. He would earn even more by just pre-installing and playing B21 or, even better, B22 (so certainly play *I* if the second player does not pre-install). Pre-installing really clears the way for the incumbent's market dominance here.

A11 is chosen almost 18% of the time. This strategy is supported in the B-R equilibrium. A21 is like A11 except that it chooses *I* if player 2 does not pre-install. It does better than the more aggressive A22 strategy, as the expected payoff from playing *I* in AD is only 21.6, compared to 80 for *NI*. In fact, many people follow these incentives – A21 is played 19% of the time, and does best of all the no-pre-installation strategies.

The most profitable strategy, B22, is chosen 137/900 or 15.2% of the time. Perhaps its frequency could increase over time. The number of times it is chosen (grouped in ascending 5-period intervals) is 20, 26, 27, 30, and 34, so that it is gradually growing in frequency; the 34 B22 choices in the last five periods represents close to 20% of all choices in the final five periods. B21 is like B22 except that it chooses *NI* if the second player pre-installs. It is chosen 65/900, or 7% of the times. It has a reasonable expected payoff, 100.3, and its frequency seems 'stable' over time, with aggregate choices being 15, 9, 13, 14, and 13.

A12, B11, and B12 are played with such low frequencies, combined 65/900 or 7%, that one may view them as errors or experiments. Perhaps reassuringly, the only directly-dominated choice, B11, is the least used one.

Entrant strategies. The most common entrant strategy, used in 42.2% of the cases, is CE11, which consists of never pre-installing, then always playing *NI* and, hence, staying out of the market. It was chosen more than three times as frequently as any other strategy, and in fact this strategy pays the best *ex post* against the observed aggregate first-mover strategy. It is also the completely safe strategy, with a guaranteed payoff of 80. Looking at both players at the same time, the most frequent strategy combination is the one where the incumbent does not pre-commit and goes for the market regardless of the entrant's behavior and where the entrant never pre-installs and yields the market, regardless of what the incumbent does.

The next most common entrant strategies are CE21 and CE22 (110/900 or 12.2%, 111/900 or 12.3% respectively). CE21 prescribes the following: Do not pre-install, choose *I* if the first-mover does not pre-install, but choose *NI* if he does. CE21 does not do well, getting an expected payoff of 56.4, since only 208/900 of the incumbents who do not pre-install yield in subgame AC. This is even worse with CE22 (the same as CE21, except that it plays *NI* against a pre-committed incumbent), yielding 40.0.[14] How do they do these strategies do over time? The 5-period frequencies for CE21 are 25, 16, 23, 22, and 24. Regarding CE22 over time, the 5-period frequencies are 31, 27, 18, 18, and 17. There is no obvious trend in the first case, while the latter appears to be a downward trend.

DF22 is the prediction under the iterated elimination of weakly-dominated strategies; always pre-install and play *NI* in the subgames that follow. This is optimal provided that the first mover chooses *NI* in all subgames. It happens very infrequently, 56/900 or 6.2% of the time. This strategy has an *ex post* expected payoff of 72, given the observed distribution of first-mover strategies

DE21 is the strategy where the second mover pre-installs if and only if the first mover doesn't. It then chooses *I*, if the first mover does not install, and *NI* when the first mover pre-installs. This strategy is chosen 62/900 or 6.9%. It does almost as well as the completely safe strategy of CE11, and becomes somewhat more popular over time – the 5-period frequencies are 5, 16, 10, 16, and 15.

The strategies that start with CF represent a pre-installation as a second mover only if the first mover has pre-installed. They collectively happen only 67/900 times or 7.4% of the time. The strategies DE11, DE12, DE22, DF11, and DF12 account for a total of 50/900 or 5.6%. None of these strategies are very profitable.

Why does the iterated elimination not work? Recall that the only dominated strategy for player 1 is B11. For player 2, the strategies that are dominated in the first round are CF11, CF21, DE11, DE12, DF11, DF12, and DF21. None one of these are played very often. The highest frequency is 19/900; collectively this is 70/900, which is still quite low, particularly when taking into consideration that they might have been used to test the response of player 1 to different modes of pre-commitment.

So one could argue that subjects do largely avoid weakly-dominated strategies. But, crucially for the lack of empirical success of the B-R predictions, the subsequent round of deletion fares much more poorly in the data. The first and the fourth most used strategies for player 1 are A22 and B22 (frequencies 300/900 and 137/900), which are dominated if the first

[14] Note that CE22 is the strategy chosen by entrants who believe that the entrant has an advantage and should not even have to pre-install to capture the market. It is the complement to incumbent strategy A22, where the incumbent is always aggressive but doesn't pre-install, apparently perceiving a first-mover advantage. CE22 was chosen 111 times, compared to 300 times for A22.

round of deletion goes through. So given that those strategies dominated in the first round are used so rarely, how do these relatively common other strategies survive? The answer is that the strategies for player 2 that make this domination apparent are also quite infrequent.

To make this discussion more concrete, take as an example B22, which is dominated (in the second round) by B21. The payoffs of these two strategies of player 1 differ only against the following strategies for player 2: CF11, CF12, CF21, CF22, DF11, DF12, DF21 and DF22. B22 does worse than B21 when paired with CF12, CF22, DF12, and DF22, and does better when paired with the other ones: CF11, CF21, DF11, and DF21. But CF11, CF21, DF11, and DF21 are all weakly dominated, which is why B22 disappears under iterated deletion. However, in practice the frequencies of these eight strategies for player 2 are all quite small, so the payoffs of both B22 and B21 are quite similar, even at the aggregate level, and given the individual uncertainty are probably indistinguishable for most players. Moreover, the frequency of CF12, CF22, DF12, and DF22 must collectively be at least three times greater than that of CF11, CF21, DF11, and DF21, in order for the expected payoff of B21 to even be larger than that of B22. And this is not the case, so in fact the expected aggregate payoff of B22 is a bit better that that of B21. Again, this is all aggregated data; at the individual level, the difference must be difficult to discern.

Something similar happens when looking at A22, which is dominated (in the second round) by A21. The only difference is that in this case, the aggregate payoff is somewhat higher for A21. But once again the differences are modest, so that at the individual level they are swamped by the uncertainty. So this can explain why strategies that are dominated in the second round do not disappear. After that the third round of deletion would not be optimal.

Hence, a pre-conception, like the notion of the first mover having an advantage can completely stall the progress of iterated deletion. But even if agents are (mildly) boundedly rational, it may be possible for them to learn to avoid dominated strategies via the repeated interaction. In the next section we will show that this argument is misleading.

4. DYNAMICS AND SIMULATIONS

We mentioned earlier that it is now widely recognized that learning by boundedly rational agents does not necessarily eliminate weakly dominated strategies. As the intuitive arguments suggest, under learning or evolution a strategy that does worse than another one will tend to be observed less frequently. But if the strategy against which the dominated strategy does poorly is also decreasing over time (so that the advantage of the dominating one becomes smaller as well), the decrease of the dominated strategy will be slower and slower, so that it can stabilize at a positive level.

We will now show, first with a theoretical framework for deterministic dynamics, and then through simulation under stochastic dynamics, that the equilibria with iteratively dominated strategies can survive in the long run under learning in this game, and that models of learning can track observed behavior in the lab reasonably well. The analytical and simulation results are complementary. The analytical results with deterministic dynamics are rather general (within their class) but only suggest a possibility, namely, that given the "right" initial conditions, iteratively weakly dominated strategies may survive in the limit as time goes to infinity.[15] The simulations show that even in small populations, with a short time-horizon, particular stochastic learning models have many of the features of our data, in particular that iteratively weakly dominated strategies can survive for the duration of our experiment.

Deterministic dynamics. We must first introduce some notation: Let $x_i^{s_i}$ be the probability assigned by the player i to strategy s_i. Let $x_i \in \Delta_i$ be a mixed strategy for agent i, where Δ_i is the simplex which describes player i's mixed-strategy space. We formalize the behavior of each player in terms of the mixed strategy he adopts at each point in time, so the vector $x(t) = (x_1(t), x_2(t))$ will describe the state of the system at time t,[16] defined over the simplex $\Delta = \Delta_1 \times \Delta_2$ of which Δ^0 is the relative interior.

Assumption d.1. The evolution of *x(t)* is given by a system of continuous-time differential equations:

$$x_i^{s_i}(t) = D_i^{s_i}(x(t)).$$

[15] Deterministic dynamics can (and perhaps should) be interpreted as limits of stochastic dynamics for large populations (Cabrales 2000) or for slow adaptation (Börgers and Sarin 1997).

We require that the autonomous system satisfies the standard regularity conditions; i.e. D must be (i) Lipschitz continuous with (ii) $\sum_{s_i \in S_i} D_i^{s_i}(x(t)) = 0$. Furthermore, D must also satisfy the following requirements:

Assumption d.2. D is a regular (payoff) *monotonic* selection dynamic. More explicitly, let $g_i(s_i, x(t)) \equiv \dot{x}_i^{s_i}(t) / x_i^{s_i}(t)$ denote the growth rate of strategy s_i. Then, for all s_i, s_i' and all $x(t)$, it must be true that

$$sign\left[g_i(s_i, x(t)) - g_i(s_i', x(t))\right] = sign\left[u_i(s_i, x(t)) - u_i(s_i', x(t))\right].$$

Assumption d.2 merely says that a strategy that has a higher payoff, given the current state of the population grows faster (decreases more slowly) than a strategy with a lower payoff.

Assumption d.3. $x(0) \in \Delta^0$.

This assumption is a technical necessity because regular dynamics are such that a strategy with zero initial weight will have zero weight at all subsequent times. So a weakly-dominant strategy will have no power against dominated ones, when the strategies of other players against which it does well are never used. This assumption guarantees that the survival of dominated strategies does not arise simply due to an initial non-existence of those strategies.

We will now show that the elements in one of the subgame-perfect equilibrium components which does not survive iterated deletion are limit points of the dynamics from some interior solution. To state the theorem we introduce more notation. By Lipschitz continuity there is a constant $K > 0$, such that, for all s_i, x_{-i}, x'_{-i} we have that:

$$\left|g_i(s_i, x_{-i}) - g_i(s_i, x'_{-i})\right| \leq K\left|x_{-i} - x'_{-i}\right|$$

where the $|.|$ denotes the norm of a vector. This in turn implies that when $u_i(s_i, x_{-i}) - u_i(s_i', x_{-i}) < -1$, there exists some h such that $g_i(s_i, x_{-i}) - g_i(s_i, x'_{-i}) \leq -h$.

[16] As is common in the evolutionary literature (*x(t), y(t)*) can also be interpreted as the proportions of people playing each strategy when a game is repeatedly played by a randomly-matched large population.

Proposition 1. *Assume that*

$$2\frac{K}{h}\frac{1 - \sum_{s_i \in S_2^*}\frac{x_2^{s_i}(0)}{x_2^{CE11}(0)}}{Max_{s_i \neq CE12^*} x_2^{s_i}(0)} x_2^{CE12}(0) > \frac{9}{16} \text{ and that } 2\frac{K}{h}\frac{1 - \sum_{s_i \in \{A21,A22\}}\frac{x_1^{s_i}(0)}{x_1^{A22}(0)}}{Max_{s_i \in \{A21,A22\}} x_1^{s_i}(0)} x_1^{A22}(0) > \frac{7}{8}$$

(a) *For all* $s_i \in \{CE11, CE12, CF11, CF12\}$, *and for all t,*

$$x_2^{s_i}(t) < \exp(-ht)\frac{x_2^{s_i}(0)}{x_2^{CE11}(0)}$$

(b) *For all t,* $x_2^{CES_1Z_2}(t) > \frac{9}{16}$.

(c) F*or all* $i \in \{A21, A22\}$, *and for all t,*

$$x_1^{s_i}(t) < \exp(-ht)\frac{x_1^{s_i}(0)}{x_1^{A22}(0)}$$

(d) *For all t,* $x_1^{A22}(t) > \frac{7}{8}$.

Proof: See Appendix B.

Simulation results. We next present simulation results based on the Camerer and Ho (1999) learning model. Figures 3 and 4 show in parallel the observed and predicted frequencies of use for the eight different incumbent strategies, while Figures 5 and 6 show observed and frequencies of use for the 16 different entrant strategies. The graphs show the average of 1000 simulation runs.[17]

[17] We chose the following parameter values: phi= 0.9 and lambda=1.2.

Figure 3: Observed Incumbent Strategies

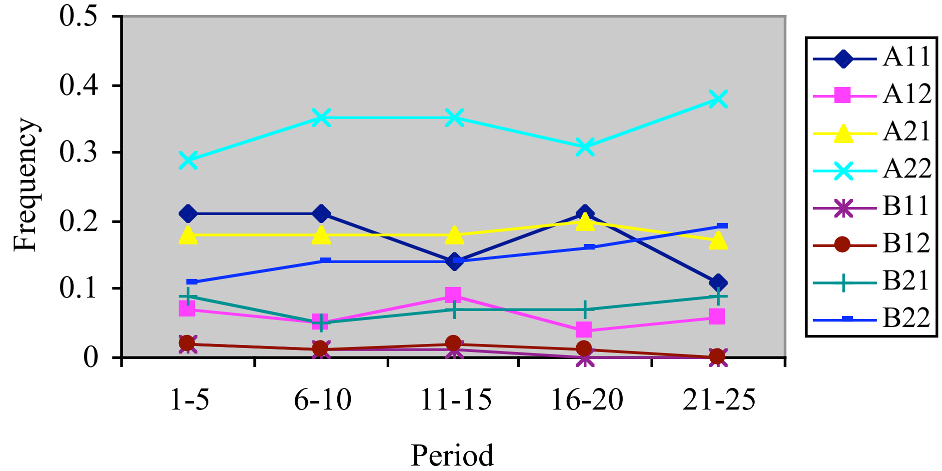


Figure 4: Simulated Incumbent Strategies

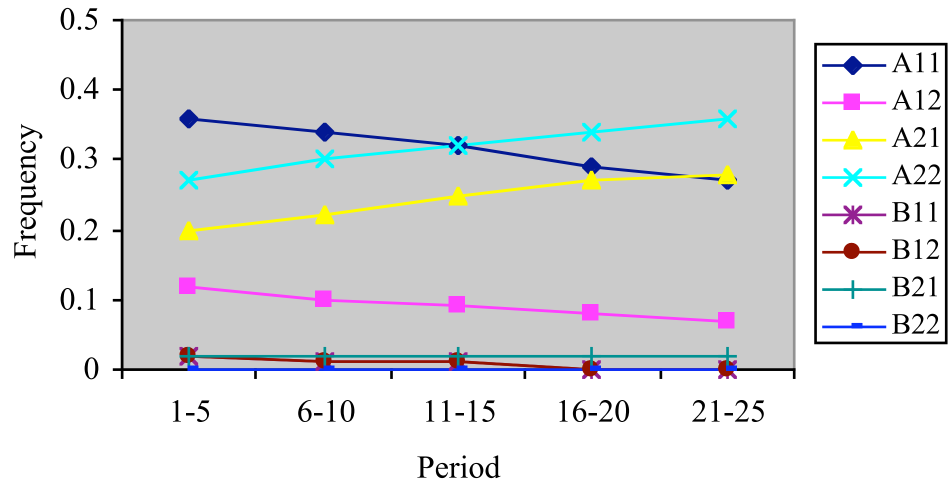


Figure 5: Observed Entrant Strategies

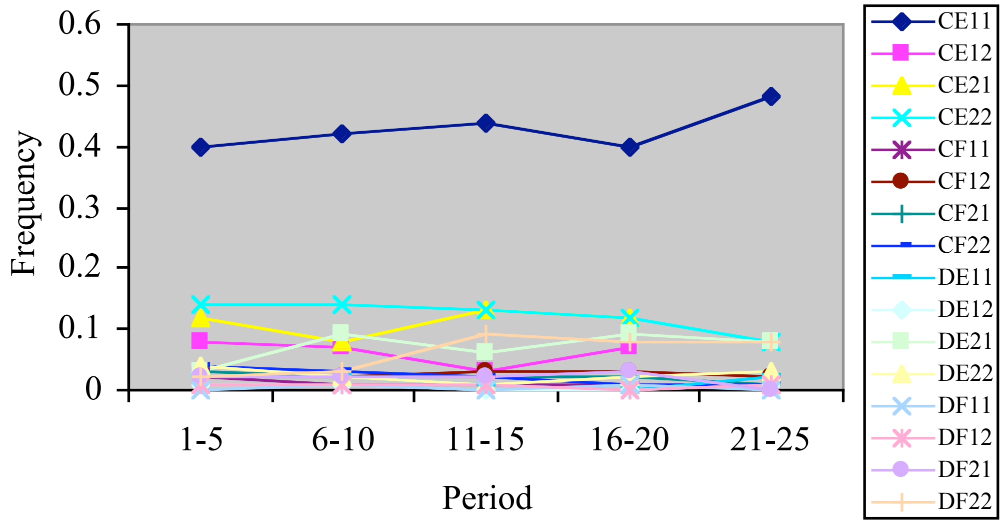
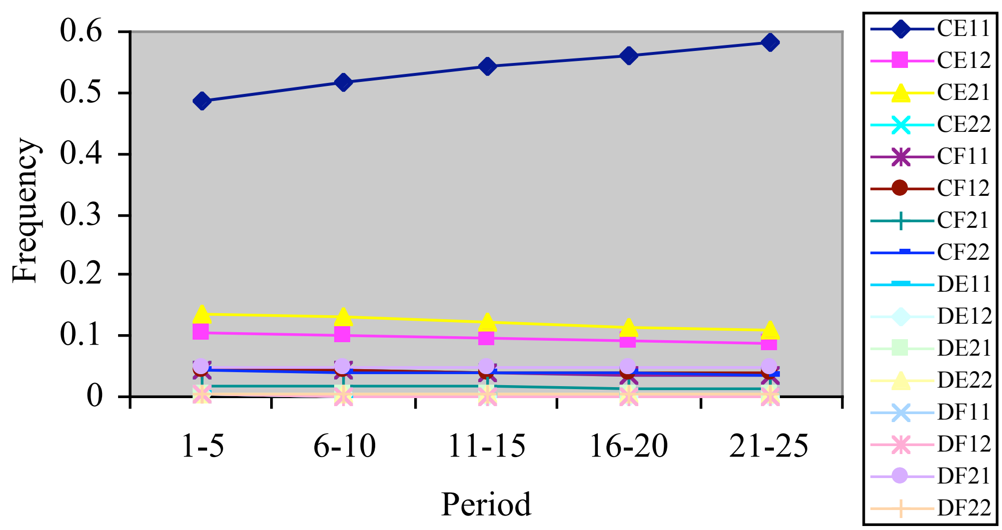


Figure 6: Simulated Entrant Strategies



Observe how the simulation model correctly captures many of the main features of the data. In terms of the levels, of the four strategies (A11, A21, A22, B22) for the incumbent with frequency above 10% in the data at round 25, three of them (A11, A21, A22) also have frequency above 10% in the simulations. The only strategy of the entrant (CE11) with frequency above 10% in the data also has frequency above 10% in the simulations. In terms of the dynamics, notice, for example, that the general trends of A11 (moderately downwards) and A22 (slightly upwards) are well captured. The simulations also capture the moderate upward trend for the entrant's strategy CE11. We also wish to highlight that, consistent with the intuitive argument presented above, B21 does not grow at the expense of B22. Similarly A21 does not grow at the expense of A22.

5. CONCLUSION

There is a puzzle in industrial organization: Despite the theoretical arguments advanced for entry deterrence by incumbents, there is little empirical evidence that firms behave in this manner. Bagwell and Ramey (1996) propose an alternative model of the timing in the entry game, providing a 'last-mover advantage' through the logic of forward induction. It is suggested that this is the reason that incumbent firms do not engage in entry deterrence.

We study behavior in a simplified version of the B-R model and compare it to observations from the simpler Dixit game. Our work is, hence, an experimental investigation of a theoretical rationalization of field evidence. We find that in both games the first mover tends to capture the market. This does not require a strong propensity towards pre-commitment by the first mover in either case, although a strong minority of incumbents do engage in frequent pre-installation in the Dixit game. Nevertheless, pre-commitment by the incumbent helps profitability.

In the B-R game, the strategy in which the incumbent does not pre-install, but nevertheless enters the market regardless of the entrant's pre-installation decision, is by far the most common (chosen fully 1/3 of the time). The logic of forward induction makes some inroads against an *a priori* perception of a first-mover advantage, but generally must surrender to it – the strategy leading to the highest expected payoff for an entrant in the B-R game involves always staying out of the market. On the other hand, forward induction and such a perception work together in the Dixit game, and the incumbent enjoys an overwhelming advantage.

Just to posit the existence of a perceived first-mover advantage is not a sufficient explanation of our data; one must also clarify why learning does not do away with this pre-conception. Our theoretical result with deterministic dynamics and our stochastic simulation model results provide a rationale for why the initial behavior does not change too much over time.

Thus we feel that we have obtained a rather complete explanation of why forward induction is only weakly supported in our data. It is not simply based on the notion that people have limitations in their depth of reasoning. Rather, matters start with a pre-conception that subsequent experience with the environment is unable to erase.

One may wonder about the origins of a perceived first-mover advantage. Our result bears relation to some recent experimental findings reported in Weber and Camerer (2002) and Muller and Sadanand (2003). These studies show that when simple two-person games that are 'simultaneous' in terms of information are played sequentially, the first mover tends to do better than when both players make actual simultaneous choices. Note that this cannot be explained just by the fact that the temporal asymmetry in moves allows for better coordination and that players use any available clue to facilitate coordination. If this were the driving force, then a second player advantage could arise just as well. There appears to be something special about moving *first*.

Huck and Müller (2000) report another instance of a first-mover advantage that transcends game-theoretic prediction. They study experimentally the Bagwell (1995) result that the commitment value of moving first is severely undermined if the observability of the earlier move is even slightly in doubt and find that people do not ignore prior moves even with imperfect observability.

Our environment is somewhat different since the second mover does observe (at least in principle) the first mover's choice. However, perhaps there is something more general in the psychology of reasoning about timing that favors earlier movers. Weber and Camerer (2002) suggest that, consistent with some evidence from psychological experiments, players may be better at reasoning backward, about events known to have already happened, than reasoning forward. Description of possible outcomes of previously-occurring events is often richer and more complex than description of later-occurring events. Weber and Camerer conjecture on p.29 that "the past is easier to 'imagine' than the future."

What can we say about the excess capacity puzzle that brought us here? While our evidence does not support the B-R prediction, it suggests, perhaps paradoxically, that the B-R environment may be the better one to study the matter at hand. Both firms have an opportunity for capacity investment but neither uses it very much, resulting in a prevailing tendency of market dominance by the incumbent.

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APPENDIX A - INSTRUCTIONS

Thank you for participating in this experiment. For your participation you will receive a positive amount of money. This quantity will be determined by the way in which your decisions relate to those of the other players. You will, in any case, receive 500 pesetas, that is in addition to the amount of money you make during the experiment.

In this experiment there will be 25 rounds. In each round you will be paired with a different person in the room. You will not be informed about the identity of the person that you are paired with in any of the rounds, neither during the experiment nor afterwards. Neither will anybody know that they are or have been paired with you at any moment.

In each round there will be a Player 1 and a Player 2. The players determine which game is played in each round. **Whether you are Player 1 or Player 2 will be determined randomly in each round,** so that you will be almost for sure in each of the roles at some point in the session. Player 1 decides whehterh she chooses action A or action B. Player 2 chooses action C or action D for the case where Player 1 has chosen A, and action E or action F for the case where Player 1 has chosen B.

For the case in which Player 1 has chosen action A, and Player 2 has chosen C for the case in which 1 chooses A, the relevant game is AC.
For the case in which Player 1 has chosen action A, and Player 2 has chosen D for the case in which 1 chooses A, the relevant game is AD.
For the case in which Player 1 has chosen action B, and Player 2 has chosen E for the case in which 1 chooses B, the relevant game is BE.
For the case in which Player 1 has chosen action B, and Player 2 has chosen F for the case in which 1 chooses B, the relevant game is BF

All the payoffs you can see in the tables are in pesetas; the first of the two numbers that can be seen in acell corresponds to the payoff for Player 1 and the second number corresponds to the payoff for Player 2.

GAME AC

		Player 2	
		S1	**S2**
Playerr 1	**R1**	60,60	60,120
	R2	120,60	0,0

GAME AD

		Player 2	
		U1	**U2**
Player 1	**T1**	60,30	60,120
	T2	120,30	0,0

GAME BE

		Player 2	
		Z1	**Z2**
Player 1	**W1**	30,60	30,120
	W2	120,60	0,0

GAME BF

		Player 2	
		Y1	**Y2**
Player 1	**X1**	30,30	30,120
	X2	120,30	0,0

.

APPENDIX B – PROOFS

<u>Proof of Proposition 1</u>: Suppose that (a) is the statement that stops being true the earliest, that it does so for strategy s_i and that the boundary time is *t'*. Then it must be that:

$$x_2^{s_i}(t') = \exp(-ht')\frac{x_2^{s_i}(0)}{x_2^{CE12}(0)}.$$

Note that $u_2(s_i, x_1) \le 4x_1^{A22} + 16(1 - x_1^{A22}) \le 11/2$, and also that $u_2(CE12, x_1) \ge 8(x_1^{A12} + x_1^{A22}) \ge 7$, where the last inequality in both cases, holds because by (d) and the assumption here for all $t < t'$, $x_1^{A22}(t) \ge 7/8$.

Thus, $u_2(s_i, x_1(t)) - u_2(CE12, x_1(t)) < -1$, so by continuity and assumption d.2 there exists $h_{(a)}$ such that $g_i(s_i, x_1(t)) - g_i(CE12, x_1(t)) < -h$ for all $t<t'$. Then, by integrating, we have that $\frac{x_2^{s_i}(t')}{x_2^{CE12}(t')} < \exp(-ht')\frac{x_2^{s_i}(0)}{x_2^{CE12}(0)}$, which implies

$x_2^{s_i}(t') < \exp(-ht')\frac{x_2^{s_i}(0)}{x_2^{CE12}(0)} x_2^{CE12}(t') \le \exp(-ht')\frac{x_2^{s_i}(0)}{x_2^{CE12}(0)}$, a contradiction.

Suppose, then, that (b) is the statement that stops being true the earliest, that it does so for strategy s_i and that the boundary time is *t'*. Then it must be that: $x_2^{CES_1Z_2}(t) = \frac{9}{16}$. As before, we will reach a contradiction, but before we will prove the following:

<u>Claim:</u> *For all* $s_i \in \{CE11, CF11, CF12\}$

$g_i(CES_1Z_2, x_1(t)) - g_i(s_i, x_1(t)) \ge -2K(1 - x_1^{AR_1T_2}(t) - x_1^{AR_2T_2}(t))$.

<u>Proof of Claim</u>: For all $s_i \in \{CE11, CF11, CF12\}$, since $u_2(s_i, \hat{x}_1)) = u_2(CE12, \hat{x}_1)$, for all $\hat{x}_1$ such that $\hat{x}_1^{A21} + \hat{x}_1^{A22} = 1$, we have that $g_i(CE12, \hat{x}_1(t)) = g_i(s_i, \hat{x}_1(t))$. So, by Lipschitz-continuity we have that for all x_1

$$g_i(CE12, x_1(t)) - g_i(CE12, \hat{x}_1(t)) \ge -2K\left|x_1(t) - \hat{x}_1(t)\right|$$

$$g_i(s_i,\hat{x}_1(t)) - g_i(s_i,x_1(t)) \ge -2K\left|x_1(t) - \hat{x}_1(t)\right|$$

But since $g_i(CE12,\hat{x}_1(t)) = g_i(s_i,\hat{x}_1(t))$, adding the previous two inequalities yields the result.

Now, by the claim, $g_i(CE12,x_1(t)) - g_i(s_i,x_1(t)) \ge -2K(1 - x_1^{A12}(t) - x_1^{A22}(t))$, but by assumption, for all *t*<*t'* $x_1^{s_i}(t) < \exp(-h_{(c)}t)\frac{x_1^{s_i}(0)}{x_1^{A22}(0)}$ for all $s_i \in \{A21, A22\}$ so that

$g_i(CE12,x_1(t)) - g_i(s_i,x_1(t)) \ge -2K\left(\exp(-h_{(c)}t)\frac{\sum_{s_i\in\{A21,A22\}} x_1^{s_i}(0)}{x_1^{A22}(0)}\right)$. Then by integration we have that

$\frac{x_2^{CE12}(t')}{x_2^{CE12}(0)}\frac{x_2^{s_i}(0)}{x_2^{s_i}(t')} > 2\frac{K}{h_{(c)}}\frac{\sum_{s_i\in\{A21,A22\}} x_1^{s_i}(0)}{x_1^{A22}(0)} \ge 2\frac{K}{h_{(c)}}$. This implies that

$\frac{x_2^{CE12}(t')}{x_2^{CE12}(0)} > 2\frac{K}{h_{(c)}}\frac{x_2^{s_i}(t')}{x_2^{s_i}(0)}$ and by summing over all

$s_i \in S_2^* \equiv \{CES_1Z_1, CE12, CF11, CF12\}$ we have that:

$$\frac{x_2^{CE12}(t')}{x_2^{CE12}(0)} > 2\frac{K}{h}\frac{\sum_{s_i\in S_2^*} x_2^{s_i}(t')}{Max_{s_i\in S_2^*} x_2^{s_i}(0)} = 2\frac{K}{h}\frac{1-\sum_{s_i\in S_2^*} x_2^{s_i}(t')}{x_2^{s_i}(0)} \ge 2\frac{K}{h}\frac{1-\sum_{s_i\in S_2^*}\frac{x_2^{s_i}(0)}{x_2^{CE12}(0)}}{Max_{s_i\in S_2^*} x_2^{s_i}(0)}$$

where the last inequality is true by (a). This yields a contradiction by the assumption about the initial conditions.

Suppose that (c) is the statement that stops being true the earliest, that it does so for strategy s_i and that the boundary time is *t'*. Then it must be that:

$$x_1^{s_i}(t') = \exp(-h_{(c)}t')\frac{x_1^{s_i}(0)}{x_1^{A22}(0)}.$$

Note that $u_1(s_i,x_1) \le 16(1 - x_2^{CF12}) \le 7$ and also that $u_1(A22,x_1) \ge 16x_2^{CF12} \ge 9$, where the last inequality in both cases, holds because by (d) and the assumption here for all *t* < *t'*, $x_1^{CF22}(t) \ge 9/16$.

Thus, $u_1(s_i, x_2(t)) - u_1(A22, x_2(t)) < -1$, so by continuity and assumption d.2 there exists $h_{(a)}$ such that $g_i(s_i, x_2(t)) - g_i(A22, x_2(t)) < -h_{(a)}$ for all $t<t'$. Then, by integrating, we have that $\frac{x_1^{s_i}(t')}{x_1^{A22}(t')} < \exp(-ht')\frac{x_1^{s_i}(0)}{x_1^{A22}(0)}$, which implies

$x_1^{s_i}(t') < \exp(-ht')\frac{x_1^{s_i}(0)}{x_1^{A22}(0)} x_1^{A22}(t') \leq \exp(-ht')\frac{x_1^{s_i}(0)}{x_1^{A22}(0)}$, a contradiction.

Suppose, then, that (d) is the statement that stops being true the earliest, that it does so for strategy s_i and that the boundary time is t'. Then it must be that:

$x_1^{A22}(t') = 9/16$. As before, we will reach a contradiction, but before we will prove the following:

Claim: $g_i(A22, x_2(t)) - g_i(A21, x_2(t)) \geq -2K\left(1 - \sum_{s_i \in S^*_2} x_2^{s_i}(t)\right)$.

Proof: Since $u_1(A21, \hat{x}_2)) = u_2(A22, \hat{x}_2)$, for all $\hat{x}_2$ such that $\sum_{s_i \in S^*_2} \hat{x}_2^{s_i}(t) = 1$, we have that $g_i(A22, \hat{x}_2(t)) = g_i(A21, \hat{x}_2(t))$. So, by Lipschitz-continuity we have that for all x_2

$$g_i(A22, x_2(t)) - g_i(A22, \hat{x}_2(t)) \geq -2K|x_2(t) - \hat{x}_2(t)|$$
$$g_i(A21, \hat{x}_2(t)) - g_i(A21, x_2(t)) \geq -2K|x_2(t) - \hat{x}_2(t)|$$

But since $g_i(A22, \hat{x}_2(t)) = g_i(A21, \hat{x}_2(t))$, adding the previous two inequalities yields the result.

Now, by the claim, $g_i(A22, x_2(t)) - g_i(A21, x_2(t)) \geq -2K\left(1 - \sum_{s_i \in S^*_2} x_2^{s_i}(t)\right)$, but by assumption, for all $t<t'$ $x_2^{s_i}(t) < \exp(-ht)\frac{x_2^{s_i}(0)}{x_2^{CE12}(0)}$ for all $s_i \notin S_2^*$ so that

$g_i(A22, x_2(t)) - g_i(A21, x_2(t)) \geq -2K\left(\exp(-ht)\frac{\sum_{s_i \notin S_2^*} x_2^{s_i}(0)}{x_2^{CE12}(0)}\right)$. Then, by integration we have that

$\frac{x_1^{A22}(t')}{x_1^{A22}(0)}\frac{x_1^{A22}(0)}{x_1^{A22}(t')} > 2\frac{K}{h}\frac{\sum_{s_i \notin S_2^*} x_2^{s_i}(0)}{x_2^{CE12}(0)} \geq 2\frac{K}{h}$. This implies that

$\frac{x_1^{A22}(t')}{x_1^{A22}(0)} > 2\frac{K}{h}\frac{x_1^{A21}(t')}{x_1^{A21}(0)}$. By adding up over all $s_i \in \{A21, A22\}$ we have that

$$\frac{x_1^{A22}(t')}{x_1^{A22}(0)} > \frac{K}{h}\frac{\sum_{s_i \in \{A21,A22\}} x_1^{s_i}(t')}{Max_{s_i \in \{A21,A22\}} x_1^{s_i}(0)} = 2\frac{K}{h}\frac{1 - \sum_{s_i \notin \{A21,A22\}} x_1^{s_i}(t')}{Max_{s_i \in \{A21,A22\}} x_1^{s_i}(0)} \geq 2\frac{K}{h}\frac{1 - \sum_{s_i \notin \{A21,A22\}} \frac{x_1^{s_i}(0)}{x_1^{A22}(0)}}{Max_{s_i \in \{A21,A22\}} x_1^{s_i}(0)},$$

where the last inequality is true by (c). This yields a contradiction by the assumption about the initial conditions.

Since this exhausts all cases the results follows.