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Severance Packages

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ABSTRACT

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Job-to-job turnover provides a way for employers to escape statutory firing costs, as unprofitable workers may willfully quit their job on receiving an outside offer, thus sparing their incumbent employer the firing costs. Furthermore, employers can induce their unprofitable workers to accept outside job offers that they would otherwise reject by offering voluntary severance packages, which are less costly than the full statutory firing cost. We formalize those mechanisms within an extension of the Diamond-Mortensen-Pissarides (DMP) matching model that allows for employed job search and negotiation over severance packages. We find that, while essentially preserving most standard qualitative predictions of the DMP model without employed job search, our model explains why higher firing costs intensify job-to-job turnover at the expense of transitions out of unemployment. We further find that allowing for on-the-job search markedly changes the quantitative predictions of the DMP model regarding the impact of firing costs on unemployment and employment flows: ignoring on-the-job search leads one to strongly underestimate the negative impact of firing costs on unemployment.

JEL Classification: J33, J64, E24

Keywords: firing costs, on-the-job search, mutual consent, minimum wage

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1 Introduction

Job-to-job turnover provides a way for employers to escape firing costs. While the primary effect of firing costs is to induce firms to keep employees in unprofitable matches, such unprofitable employees may willfully quit their job on receiving an outside offer, thus sparing their incumbent employer the firing costs. Furthermore, employers can induce their unprofitable workers to accept outside job offers that they would otherwise reject by offering voluntary severance packages, which are less costly than the full statutory dismissal cost. We formalize those mechanisms within an extension of the Diamond-Mortensen-Pissarides (DMP) matching model (Mortensen and Pissarides, 1994) that allows for employed job search and negotiation over severance packages.

The impact of firing costs on labor market equilibrium has been an object of keen study in the search and matching literature (see the review in Mortensen and Pissarides, 1999, or more recently Ljungqvist, 2002). Firing costs are typically found to discourage both job destruction and job creation, thereby substantially reducing both unemployment outflows and inflows with a resulting effect on the equilibrium unemployment rate which is ambiguously signed and quantitatively negligible.[1]

Yet that large body of research consistently ignores direct job-to-job flows. Job-to-job flows are quantitatively substantial, amounting to between a third and a half of total labor reallocation, and relevant to the issue of firing costs as a worker who quits a job spontaneously is usually not entitled to any redundancy payment and does not make the firm liable to a firing tax. We thus propose to introduce employed job search in a DMP-type model with firing costs and a minimum wage (the role of which we will discuss momentarily). We show that the presence of employed job search strongly affects the response of unemployment flow rates and the unemployment rate to increases in firing costs when the minimum wage is not too low. Specifically, while higher firing costs discourage job destruction much in the same way as they do in the DMP model without employed job search, we find that employed job search largely cushions the adverse impact of firing

[1] Of course this somewhat sweeping summary does not do justice to the vast literature on that subject and exceptions to those findings do exist. A relevant example for our purposes is Cahuc and Zylberberg (1999) who point out that the degree of wage rigidity (specifically, the level of the minimum wage) is a key determinant of the impact of firing costs on labor market equilibrium in the DMP model.

costs on job creation. As a result, firing costs have a quantitatively sizeable negative impact on the unemployment rate in that model. The difference in the economy's behavior brought about by on-the-job search is due to the fact that, as mentioned earlier, job-to-job turnover enable employers to avoid paying the statutory firing cost in some cases.

Aside from on-the-job search, our model has two important specific features. First, as an alternative to the conventional practice of applying the Nash solution, we assume that mutual consent is the basic principle governing wage (re)negotiation in continuing firm-worker matches. Specifically, under that assumption, the wage is only altered when one of the worker's or the firm's outside options becomes binding, in which case the wage is revised up or down by just enough to satisfy whichever constraint is binding. It is an attractive assumption because it features in many labor contracts (see Malcomson, 1997, for empirical evidence), it delivers realistic wage dynamics (Postel-Vinay and Turon, 2010), and it can be justified theoretically in different contexts (see the discussion in footnote 3 below and in Appendix A). Most importantly, as we show in the paper, that assumption lends itself well to the analysis of endogenous severance packages, which arise naturally in the formal bargaining game that we use to deliver the wage determination rules just described.

The second distinguishing feature of our model is, as already mentioned, the presence of a minimum wage. We introduce a minimum wage for two reasons. First, studying the combined impact of firing costs and the minimum wage — two of the most common ingredients of labor market regulation in OECD economies — is an interesting exercise in its own right (Cahuc and Zylberberg, 1999, Boeri and Burda, 2009, Silva, 2010). Second, although so far we have put the emphasis on employers offering their workers severance packages to induce them to quit into alternative jobs, in certain circumstances severance packages may also be used by employers as a means to induce their workers to quit into unemployment (as opposed to firing them and incurring the firing cost). As first shown by Fella (2007) and confirmed in this paper, if employers are allowed to offer severance packages to induce quits into unemployment, then some form of wage rigidity is required for firing costs to have any impact at all on job flows and unemployment. Intuitively, Fella (2007) shows that, if the firing cost is paid by the firm to a third party (tax, red-tape cost...), it is always in the joint interest of the firm-worker match to avoid paying that tax by agreeing on a voluntary separation in which, rather than paying the firing cost (which would then be lost for the match), the firm offers the worker a voluntary severance package equal to the worker's foregone surplus from employment. In so doing, the firm effectively turns the firing tax into a transfer to the worker: the distinction between redundancy tax and transfer is rendered ineffectual by severance packages. As is well known (at least since Lazear, 1990), absent wage rigidity, firing restrictions in the form of firm-worker transfers can be completely offset by contracts between risk-neutral employers and employees. Fella's (2007) contribution is to show that firing *taxes* can also be offset as long as risk-neutral employers and employees are able to negotiate side-payments (severance packages) to induce workers to quit into unemployment without incurring the firing tax. This results no longer holds, however, in the presence of a source of wage rigidity such as an institutional wage floor. We will see that in our model, as in Fella's, the economy finds itself in two very different regimes regarding the effectiveness of firing costs depending on whether the minimum wage is binding at the point of job destruction.

The rest of this paper is organized as follows. Our model is described in the next section, and its solution with exogenous contact rates is discussed in Section 3. The characterization of the full model solution, i.e. with endogenous contact rates is presented in Section 4. A calibration and simulation illustration is carried out in Section 5. Section 6 concludes.

2 The model

2.1 The environment

Technology. We take up all basic assumptions about the workings of the product and labor markets from Mortensen and Pissarides (1994), from which we only depart by allowing employed workers to search on the job. The labor market thus features a unit mass of workers facing a continuum of identical firms producing a unique multi-purpose good sold in a perfectly competitive product market. Workers and firms are infinitely lived, forward-looking, risk-neutral and have a common exogenous discount rate of ρ. Time is continuous and the economy is in steady state.

Workers can either be unemployed or matched with a firm. Firms and workers are matched through a time-consuming search process, modeled in the following conventional way: unemployed (employed) workers receive job offers at a Poisson rate λ_0 (λ_1). We thus assume that all workers search for a (better) job and that their search intensity, although exogenous, depends on their employment status. We should note from the outset that not all offers are conducive to an actual match: workers will only accept an offer whenever it yields greater value than what they enjoy in their current state (more details on this below). Finally, note that we are treating the offer arrival rates λ_0 and λ_1 as exogenous parameters for the time being. We shall endogenize them later in the analysis by means of a matching function.

Firms operate a constant-return technology and are modeled as a collection of job slots which can either be vacant or occupied and producing. The quality of worker-firm matches is heterogeneous and subject to shocks. We define the output flow of a match as $y = p \cdot z$, where p is an aggregate productivity index and z is match-specific productivity, comprised in $[0, 1]$ and subject to shocks. The (Poisson) arrival rate of shocks to match productivity is denoted δ, and the post-shock value of z (z', say) is drawn from a continuous distribution $F(\cdot)$ over $[0, 1]$, independently of the pre-shock value of z.[2] Shocks are also uncorrelated across matches. Finally, following standard practice, we assume that all new matches start out with the highest level of productivity, $z = 1$. Then, like the conventional DMP model, our model features endogenous job destruction: upon drawing a productivity shock below a certain threshold (to be formally defined below), the firm will want to end the employment relationship.

Institutions. One of the objectives of this paper is to analyze the combined impact of firing costs and the minimum wage on labor market equilibrium. We thus assume that firms are required to pay all their employees a wage greater than the institutional wage floor $w_{\min}$, and are liable to a firing cost of T payable upon laying off a worker. It is important to stress that statutory dismissal

[2] This latter assumption is common in this literature, though not necessarily desirable as implying identical destruction hazards for all jobs. We still maintain it, in the interest of tractability.

costs are only applicable to employer-initiated separations to which the worker does not consent.

T and $w_{\min}$ are exogenous policy parameters. In line with much of the literature, it may be easiest in the rest of this paper to think of T as either a firing tax paid or as red tape costs, i.e. as a third-party payment which is a net loss to the firm-worker match rather than a statutory transfer paid by the firm to the worker upon separation. However, given our wage determination rule and the possibility for firms and workers to negotiate severance packages, that distinction has no bearing on the impact of T on equilibrium job flows and unemployment (it only affects the distribution of payoffs). Formal details are given in Appendix A.

2.2 The employment relationship

Notation and working assumptions. Considering a firm-worker match with a current idiosyncratic productivity level of z, we denote the wage flow paid in this match by w and define $J(z,w)$ [resp. $E(z,w)$] as the value of this match to the firm [resp. to the worker]. We also introduce the total (gross) match value: $V(z) = J(z,w) + E(z,w)$. We will justify below that this value is independent of the wage paid. Finally, we denote the worker's lifetime value of unemployment as U and assume that the firm's valuation of a vacant job is zero, as results from free entry on the search market (see below).

We start working under the following set of hypotheses about the impact of wages and productivity on the various value functions: $\partial J/\partial z > 0$, $\partial J/\partial w < 0$, $\partial E/\partial w > 0$, $V'(z) > 0$. All of those hypotheses will be formally confirmed later in the analysis.

We also assume that there are gains to trade on the labor market. More precisely, workers and employers are both willing to consummate a match with the maximum productivity level $z = 1$ and a minimum wage of $w_{\min}$, i.e. formally $E(1, w_{\min}) \geq U$ and $J(1, w_{\min}) \geq 0$.

We now introduce an additional series of definitions for future use. We first define $R_0(w)$, the value of productivity at which, given a wage of w, the firm's valuation of the match is zero, by $J[R_0(w), w] = 0$. Note that, because of layoff costs, this is not the productivity at which the firm decides to terminate the employment relationship. We further denote by $w_0(z)$ the inverse of $R_0(\cdot)$, i.e. the maximum wage that a firm can pay without incurring a loss (in expected present value terms) given a current productivity level of z. We can then *mutatis mutandis* define the threshold productivity $R_T(w)$ as the solution to $J[R_T(w), w] = -T$, and the inverse $w_T(z)$ of the function $R_T(\cdot)$. $R_T(\cdot)$ gives the cutoff productivity value below which the firm is better off firing the worker (thus incurring the layoff cost T) than retaining her/him at the wage w. Our assumptions about the monotonicity of $J(\cdot)$ with respect to both of its arguments ensure the consistency of all these definitions.

Wage determination. Our wage determination mechanism builds on Postel-Vinay and Robin (2002a, b) and Postel-Vinay and Turon (2010). Wages are set by firms. Workers, however, can use market pressure to extract some rent. Indeed they will use outside job offers to obtain wage increases by playing off potential employers against one another, as detailed in the following paragraphs. The principle governing all wage renegotiations is that of mutual consent. That is, either party can only force wage renegotiation to their advantage if the value of their outside option exceeds their current value in the match, giving them a credible threat to leave the match. When renegotiation does take place, the wage is adjusted up or down so as to give the forcing party the exact value of their outside option.[3] In the presence of firing costs, the value of the firm's outside option is the value of a vacant job, i.e. zero, less the firing cost T. Since quits are not legally liable to statutory dismissal costs, the worker's outside option is either the value of unemployment or the value of his/her outside job offer, if any.

Severance packages. The most immediate effect of employment protection is to induce firms to keep employees even when a negative productivity shock leads them to incur losses (up to the level of the firing cost) in the current match. Under the mutual consent rule, as long as these losses are less than the amount the firm would have to pay to fire the worker, the firm has no credible threat to dissolve the match and cannot force the worker to renegotiate the wage downwards.

[3] Different theoretical foundations exist for this rule (see MacLeod and Malcomson, 1993). Models of self-enforcing wage contracts within a match faced with uncertainty about productivity and/or market opportunities deliver a similar wage rule (Thomas and Worrall, 1988). Hall (2005) also considers this wage setting mechanism which he interprets as a social norm favoring wage rigidity. In this particular paper, we justify that rule using an explicit finite-horizon bargaining game spelled out in Appendix A. While we do not mean to tout our own theoretical foundation as being superior to any other, our game has the merit of simplicity.

Thus an obvious, however little explored consequence of the enforcement of statutory redundancy pay is that it leads firms and workers to negotiate severance packages aimed at encouraging the worker to leave when the firm is currently incurring losses. For example, in the event of an outside offer arising, the incumbent employer would be willing to compensate the worker for him/her to accept it, up to the level of the current losses. Assuming that both worker and firm can observe the value of the outside offer, the incumbent firm will offer a severance package to the worker yielding him/her a value of quitting with this offer and this severance package marginally greater than the value of staying with his/her incumbent employer. Employed job search thus allows employers to escape job security regulations, at least to some extent.

Depending on parameter values, severance packages can just as well arise at the firing margin (Fella, 2007). We shall see below that, for given values of the minimum wage and unemployment income, as the firing cost T is increased, it may become both feasible and beneficial for employers to induce workers to quit into unemployment on being hit by a sufficiently adverse productivity shock. Depending on whether this is the case or not, the impact of job security provisions on the equilibrium layoff rate will be very different.

2.3 Outside offers

Given a current wage of w and a current productivity level of z, we denote the continuation values of the match to the firm [resp. the worker] after an outside offer as $\overline{J}_o(z,w)$ [resp. $\overline{E}_o(z,w)$].

When an employed worker is contacted by an outside firm, we assume the sequence of events to be the following. The outside firm gets one chance to make the worker an offer, in the knowledge of current match productivity in the incumbent firm. A bargaining game is then played between the worker and incumbent employer, in which the latter has an opportunity to raise the worker's wage, to offer a severance package or to do nothing, letting the worker either stay in the match with the existing wage, or quit to the poaching firm. Details of a specific strategic game are given in Appendix A and we only give an intuitive description of the outcome of the bargain in the main text.

Denote the value to the worker of the outside offer by E'. Because all new matches start out with the maximum productivity level $z = 1$, the maximum worker's value of the outside offer is $V(1)$, which is also the maximum value of employment as the worker is never able to claim a wage yielding a higher value than this, having no outside offer enabling him/her to do so. The minimum value of the outside offer is in turn constrained by the existence of a minimum wage to be $E(1, w_{\min})$. For future use, we denote by z_m the value of the match productivity such that $E(1, w_{\min}) = V(z_m)$. So the boundaries of the worker's value of the outside offer are summarized as: $V(z_m) \leq E' \leq V(1)$.

If the value of the outside offer is less than the worker's value of the current match, i.e. $E' < E(z, w)$, the worker will opt to stay with the current firm. If, however, the incumbent firm is incurring losses in this match, it will be in its best interest to offer a severance package to the worker to induce him/her to take up the outside offer. The minimum value required by the worker to do so is $E(z, w) - E'$. The firm will find it profitable to offer this amount if it is lower than the current (negative) firm's value of the match. Note that the firm always has the option of firing the worker and pay the layoff premium of T, which therefore is an upper bound on the value of the severance package that the firm is willing to offer.

We first consider the case where the incumbent firm is currently incurring losses at the prevailing wage, i.e. $J(z, w) < 0 \Leftrightarrow E(z, w) > V(z) \Leftrightarrow z < R_0(w) \Leftrightarrow w > w_0(z)$. As the worker is never able to raise his/her wage above $w_0(1)$, the firm cannot be making losses when the match productivity equals 1. If $1 > z \geq z_m$, the poacher will make a maximum offer yielding the worker a value of $V(z)$, which is less than the current worker's value of the incumbent match, $E(z, w)$, since the firm is making losses. So, as it stands, the worker would opt to stay with the incumbent firm. However, as this latter is incurring losses, it is in its interest to induce the worker to leave and it will do so by offering a severance package exactly compensating the worker for the difference between the offer and his/her current value of employment: $E(z, w) - V(z) = -J(z, w)$. As a result, the worker quits her/his job with a continuation value of $\overline{E}_o(z, w) = E(z, w)$, leaving her/his initial employer with a value of $\overline{J}_o(z, w) = J(z, w) < 0$. If $z < z_m$, the poacher's offer cannot be worth less than $V(z_m)$ to the worker because of the binding minimum wage. If that is greater than the value the

		Incumbent employer...	
		incurs losses: $z \leq R_0(w)$	makes a profit: $z > R_0(w)$
$1 \geq z > z_m$	$\overline{E}_o$	$E(z,w)$	$V(z)$
	$\overline{J}_o$	$J(z,w)$	0
	$\overline{V}_o$	$V(z)$	$V(z)$
$z_m \geq z$	$\overline{E}_o$	$\max\{V(z_m); E(z,w)\}$	$V(z_m)$
	$\overline{J}_o$	$-\max\{E(z,w) - V(z_m); 0\}$	0
	$\overline{V}_o$	$V(z_m)$	$V(z_m)$

Table 1: Continuation values after an outside offer

worker gets from the current match, s/he will leave spontaneously. If it is lower, the incumbent firm will offer a severance package of $E(z,w) - V(z_m)$ to induce him/her to leave. The worker thus leaves the initial match with a continuation value of $\overline{E}_o(z,w) = \max\{E(z,w); V(z_m)\}$, while the incumbent employer receives a continuation value of $\overline{J}_o(z,w) = -\max\{E(z,w) - V(z_m); 0\}$. Note that this is greater than $J(z,w)$, so that the worker's receipt of an outside offer is a positive shock for the firm in this instance.

Let us now look at the case where the firm currently makes a profit, i.e. where $z \geq R_0(w) \Leftrightarrow w \leq w_0(z)$. Three situations may arise, depending on the value of z. First, if $z = 1$, then the incumbent and the poaching firm are equally productive. The worker's value is then bid up to $V(1)$, i.e. the worker extracts the whole surplus from either match. The worker then becomes indifferent between staying with her/his initial employer or taking up the poacher's offer, and we assume s/he opts for the former (an innocuous assumption in our context). The continuation value of the initial match will hence be split into $\overline{E}_o[1, w_0(1)] = V(1)$ to the worker and $\overline{J}_o[1, w_0(1)] = 0$ to the incumbent employer. Second, if $1 > z \geq z_m$, the worker will leave the incumbent firm as the poacher now values the worker's employment more than the incumbent. When attempting to retain the worker, the incumbent firm will bid the wage up to a level yielding the worker a value of $V(z)$, which is the maximum offer that the incumbent firm is willing to make. The outside firm (profitably) offers this value in order to bid the worker away from the incumbent. Hence in this case $\overline{E}_o(z,w) = V(z)$

and $\overline{J}_o(z,w)=0$. Finally, if $z<z_m$, that maximum offer is less than the minimum offer that the poacher is allowed to make (because of minimum wage regulations), yielding the worker a value of $E(1,w_{\min})=V(z_m)$. The worker then quits with a value of $\overline{E}_o(z,w)=E(1,w_{\min})$ and the incumbent firm is left with a value of $\overline{J}_o(z,w)=0$.

Table 1 summarizes the continuation values $\{\overline{E}_o,\overline{J}_o,\overline{V}_o\}$ for the worker, the incumbent firm and the match after an outside offer in all cases examined above.

2.4 Productivity shocks and wage cuts

Like outside job offers, productivity shocks can prompt wage renegotiation. The rules for wage renegotiation after a productivity shock are the same as those following an outside job offer, i.e. they are based on mutual consent, and the same bargaining game is played by the worker and the firm. Again details of the game are confined to Appendix A and we only describe the bargain outcome in the main text.

Because of the mutual consent rule, productivity shocks can only lead to downward wage revisions (or layoffs). Indeed a positive productivity shock does not affect the worker's outside option, and therefore does not give her/him an opportunity to force her/his employer to accept a wage increase.[4] Now not all productivity shocks will cause a wage cut: again because of the mutual consent restriction, the productivity shock has to be bad enough that the firm is better off firing the worker (firing cost notwithstanding) than keeping her/him in employment at the current wage rate. This happens whenever the firm would incur losses greater than T under the current wage w and the new productivity draw z' (say), i.e. formally whenever $z'<R_T(w)$.

When this is the case, the wage is revised down to the point where the firm's outside option (to fire the worker) is met, i.e. to the point where the firm's valuation of the match given the new productivity value z' equals $-T$, the value of firing the worker. Formally, the renegotiated wage is

[4] We are taking a bit of a shortcut here. This statement is only true to the extent that the worker is never tempted to quit into unemployment. Given the minimum wage $w_{\min}$, this formally translates into assuming that $E(z,w_{\min})\geq U$ for all "viable" values of z, i.e. values of z such that the match is not dissolved (more on this below). Were this not the case, firms would sometimes have to raise their worker's wage following an adverse productivity shock in order to keep them from quitting into unemployment. While this latter situation is undeniably a theoretical possibility, covered in the renegotiation game analyzed in Appendix A, we regard it as factually implausible and rule it out in the main body of the paper.

set to $w_T(z')$.

Of course it may happen that the minimum wage makes this legally impossible. For instance it may happen that the firm incurs losses greater than T, even when paying the minimum wage $w_{\min}$. In this case (which formally translates as $z' < R_T(w_{\min})$), the match is dissolved. Yet the situation regarding match dissolution is further complicated by the possibility of negotiating severance packages. We now analyze the job destruction rule in detail.

2.5 Job destruction

Two regimes. As argued above, if productivity falls short of $R_T(w_{\min})$, then the firm is better off firing the worker and paying T than maintaining the match. As a result, any job drawing a productivity level below $R_T(w_{\min})$ will inevitably be destroyed. Whether or not jobs are maintained at all productivity levels above $R_T(w_{\min})$ depends on the worker's valuation of those jobs as we now explain.

First, if $E[R_T(w_{\min}), w_{\min}] > U + T$, then the worker strictly prefers employment at the minimum wage and any productivity level above $R_T(w_{\min})$ to being laid off with the maximum severance payment of T. In this case the firm can never profitably (i.e. at a cost less than T) induce the worker to quit into unemployment if $z' \geq R_T(w_{\min})$, and will find it optimal to fire the worker as soon as $z' < R_T(w_{\min})$. In this situation, separations are privately inefficient as the match surplus is strictly positive at $R_T(w_{\min})$.[5]

If conversely $E[R_T(w_{\min}), w_{\min}] \leq U + T$, then there is scope for the firm and the worker to negotiate a severance package smaller than T that will induce the worker to quit into unemployment. Indeed consider a match where the going wage is $w\,(\geq w_{\min})$ and a productivity draw of $z'\,(\geq R_T(w_{\min}))$ such that $U < E(z', w) \leq U + T$ and $J(z', w) < 0$. The firm thus incurs losses and would like the worker to leave, but this latter is not going to do so spontaneously because $E(z', w) > U$. The compensation it would take to induce the worker to leave is $E(z', w) - U \leq T$ and the employer's value of inducing a quit is therefore $-E(z', w) + U = J(z', w) - [V(z') - U]$. Assuming that the firm and the worker can sign an agreement whereby the worker accepts to vol-

[5]It equals $V[R_T(w_{\min})] - U = E[R_T(w_{\min}), w_{\min}] - U + J[R_T(w_{\min}), w_{\min}] = E[R_T(w_{\min}), w_{\min}] - U - T > 0$.

untarily leave the firm with a severance package of $E(z', w) - U$, it will be in the parties' joint interest to do so as soon as the productivity shock falls in a range such that $V(z') < U$, i.e. falls short of the threshold R_S defined by $V(R_S) = U$, such that the match surplus is zero. Separations are privately efficient in this case.[6]

Summary of the job destruction rule. We use the generic notation D to designate the job destruction threshold, i.e. the level of idiosyncratic productivity below which matches break up. The above developments make it clear that the labor market will be in one of two regimes, with either $D = R_T(w_{\min})$ or $D = R_S$. In the former regime, separations are privately inefficient and directly governed by a binding minimum wage and mandatory severance pay. In the latter regime, separations are privately efficient given T and $w_{\min}$, and the impact of these two policy instruments on the job destruction rate is only indirect (see below). For want of a better denomination, we shall refer to the case $D = R_T(w_{\min})$ as the "inefficient separation regime" and to the case $D = R_S$ as the "efficient separation regime".

Considerations in the previous paragraph imply that the job destruction threshold D can be generically defined over both regimes as $D = \max\{R_T(w_{\min}), R_S\}$. Section 3 below establishes formal definitions of $R_T(w_{\min})$ and R_S, analyzes the conditions under which either regime will prevail and studies the impact of T and $w_{\min}$ under both regimes.

2.6 Continuation values after a productivity shock

Given an initial wage of w, we denote the expected values of the match to the firm [resp. the worker, the match] after a shock to z as $\overline{J}_s(w)$ [resp. $\overline{E}_s(w)$, $\overline{V}_s$]. The generic analytical expressions of those continuation values are somewhat cumbersome (due to the fact that one has to consider different regimes depending on the relative ranking of various productivity thresholds). Fortunately, we do not need to spell them out as the ensuing analysis will only make use of a few relatively simple special cases. At this point we only bring up the obvious (yet important) remark that these

[6] Relating to footnote 4 above, we implicitly assume throughout the analysis that the minimum wage is high enough that the worker is never willing to quit into unemployment without severance compensation. At this point this formally amounts to assuming that $E(R_s, w_{\min}) > U$.

continuation values do not depend on any particular value of z (which immediately flows from the assumption of independent sequential productivity draws).

2.7 Bellman equations

We end our layout of the model by formally writing down the Bellman equations solved by the various value functions defined and used above.

$$(\rho + \delta + \lambda_1) J(z, w) = pz - w + \delta \overline{J}_s(w) + \lambda_1 \overline{J}_o(z, w) \tag{1}$$

$$(\rho + \delta + \lambda_1) E(z, w) = w + \delta \overline{E}_s(w) + \lambda_1 \overline{E}_o(z, w) \tag{2}$$

$$(\rho + \delta + \lambda_1) V(z) = pz + \delta \overline{V}_s + \lambda_1 \overline{V}_o(z) \tag{3}$$

$$(\rho + \lambda_0) U = b + \lambda_0 V(z_m) \tag{4}$$

These definitions will be used for derivations below. A series of properties of these functions is established in Appendix B.

3 Model solution: the case of exogenous contact rates

In order to gain some insight about the equilibrium impact of the minimum wage and firing costs on labor market performance, we begin by treating the firm-worker contact rates, λ_0 and λ_1, as exogenous parameters. Again, those will be endogenized later on. The purpose of this first step in the analysis is to focus on the partial impact of T and $w_{\min}$ on the job separation margin.[7]

3.1 The job destruction thresholds

The inefficient separation regime: $D = R_T(w_{\min})$**.** By definition of the inefficient separation regime, the job destruction condition is given by $D = R_T(w_{\min}) \Leftrightarrow J(D, w_{\min}) = -T$. Substituting this definition into the Bellman equation (1) defining the firm's valuation of a job, $J(\cdot)$, is shown in Appendix C to translate into the following job destruction condition:

$$D + \frac{\delta}{\rho + \delta + \lambda_1} \int_D^1 \overline{F}(x)\, dx = \frac{w_{\min}}{p} - \frac{T}{p}(\rho + \lambda_1) \tag{JD$_i$}$$

where, in standard fashion, a bar over a CDF denotes the corresponding survivor function.

[7] Once again, as stated in footnotes 4 and 6, we assume throughout the analysis that parameters are such that the worker is never willing to quit into unemployment without severance compensation, i.e. that $E(D, w_{\min}) > U$.

The efficient separation regime: $D = R_S$**.** In the efficient separation regime, the firm and the worker agree to separate when the joint value of the match reaches the value of unemployment:

$$V(R_S) = U \tag{JD$_e$}$$

The formal definition of R_S is somewhat more involved than that of D in the inefficient separation regime, and we therefore leave it in Appendix C, equations (A.6) to (A.8). Inspection of those three equations, from which the parameter T is absent, readily confirms the following result by Fella (2007) in our environment — which differs from his by the presence of on-the-job search and wage renegotiation by mutual consent:

Proposition 1 *In the efficient separation regime, firing costs have no impact on the job destruction threshold,* R_S*.*

This result is formally established in Appendix C. The underlying intuition is spelled out in Fella (2007). If the firing cost is a tax, paid by the firm to a third party, actually paying that tax is in the interest of none of the parties in a joint-surplus-maximizing firm-worker match. Now institutions provide firm-worker matches with a way of dodging that tax: because the firing tax only applies to layoffs and not to quits, if the firm's value of the match becomes negative, so that the firm would like to terminate the match, it will induce a quit by offering the worker a voluntary severance package (equal to the worker's foregone surplus from employment) rather than fire the worker and incur the firing tax. In so doing, the firm effectively turns the firing tax into a transfer to the worker. In the efficient separation regime, the minimum wage is low enough to allow for that transfer to be fully priced out (in expectation) by adjusting the wage over the lifetime of the match. Things are different in the inefficient separation regime, where the minimum wage prevents the firm and the worker from fully pricing out severance packages. In that case the firing cost, even though it is still turned into a transfer to the worker, has bite.

3.2 Conditions under which either regime prevails

Given fixed contact rates λ_0 and λ_1, which one of the inefficient or efficient separation regime prevails depends on the implemented mix of employment protection, minimum wage and unemployment compensation following a general pattern that we summarize in the ensuing two propositions[8].

Proposition 2 *For a given value of the minimum wage* $w_{\min}$ *and the unemployment income* b*, there exists a cutoff value* $T^c(w_{\min}, b) \geq 0$ *of the statutory redundancy pay (possibly equal to zero) such that the inefficient separation regime prevails for all* $T \leq T^c(w_{\min}, b)$ *while the efficient separation regime prevails for all* $T > T^c(w_{\min}, b)$*.*

Given the minimum wage and unemployment benefits, the inefficient separation regime tends to prevail under low values of the firing cost. As the firing cost is increased, it becomes increasingly worthwhile for firms to negotiate whatever side payment fully compensates the worker for quitting into unemployment as match productivity hits low levels in order to avoid paying the full firing cost.

Similar results hold for the other two policy tools, $w_{\min}$ and b:

Proposition 3 *Fix the statutory redundancy pay at a given value* T*. Then, for a given value of the unemployment income* b *(respectively, the minimum wage* $w_{\min}$*), there exists a cutoff value* $w^c_{\min}(T, b)$ *of the minimum wage (respectively, a cutoff value of the unemployment income* $b^c(T, w_{\min})$*) such that the efficient separation regime prevails for all* $w_{\min} \leq w^c_{\min}(T, b)$ *(respectively, all* $b \geq b^c(T, w_{\min})$*), while the inefficient separation regime prevails for all* $w_{\min} > w^c_{\min}(T)$ *(respectively, all* $b < b^c(T, w_{\min})$*).*

Again intuitively, given the level of firing costs, the inefficient separation regime tends to prevail under high values of the minimum wage and low values of the unemployment income. As either the minimum wage is reduced or the unemployment benefit level is increased, the gap between the worker's valuations of employment at the minimum wage *vis à vis* unemployment narrows, and eventually falls short of the firing cost T, at which point the negotiation of severance packages lower than T at the job destruction margin becomes feasible.

[8] Proofs of both propositions can be found in Appendix D.

3.3 Comparative statics

We confine our analysis of comparative statics to the inefficient separation regime as firing costs do not affect outcomes in the efficient separation regime. The job destruction condition (JD_i) establishes an equilibrium relationship between D and λ_1, which is parameterized by the policy tools $(T, w_{\min})$. It is straightforward to show by differentiation that, given λ_1, firms will adopt a lower D in the face of higher firing costs and a higher D if the minimum wage is raised. Unsurprisingly, our model thus confirms the standard qualitative predictions about the response of job destruction behavior to these specific policy changes.

What is more interesting within the context of our model is to look at the interaction between firing costs and on-the-job search (as measured by its intensity λ_1) in shaping the job destruction rule. The slope of (JD_i) in a (D, λ_1) space turns out to be of ambiguous sign for positive firing costs. Absent such costs, a ceteris paribus increase in λ_1 unambiguously reduces the firm's valuation of a job (at any productivity level) as it raises both the worker's quit rate and the probability of having to transfer some of the match surplus to the worker in response to an outside job offer, leading the firm to discount the positive option value of the job (showing in the integral term in (JD_i)) more heavily. Absent firing costs, a higher λ_1 thus clearly leads to an increase in the job destruction threshold D. When firing costs are in place though, the event of the worker quitting is beneficial to the firm in some cases. As we saw in subsection 2.2, any firm incurring losses is indeed happy to let its worker go at a cost lower than T: the opportunity arises when the worker receives an outside offer which s/he will accept, possibly subject to receiving a severance package which is always less than the statutory firing cost T. In short, employed job search provides a channel through which employers can escape job security provisions. And of course, the higher the firing cost, the greater scope there is for this type of situation to arise. As a result, for any positive value of T, the job destruction schedule becomes downward sloping for high values of λ_1.

The flip side of this argument is that employed job search impacts the sensitivity of the job destruction threshold to changes in job security provisions. When faced with an increase in T, firms weigh the cost of hoarding labor to a further extent (by lowering D) against the associated benefit of not having to pay the higher firing cost. More specifically, the direct cost of lowering D is to keep currently less productive jobs occupied, thus incurring greater current losses. This is mitigated by a positive effect of lowering D, which is to increase the job's option value (which accounts for the possibility of the job becoming profitable again in the future). However, on-the-job search weakens this positive option value effect as it makes it more likely that the worker will leave the firm before the job becomes productive again. Therefore the total cost of cutting the job destruction threshold is ceteris paribus higher when employees search more intensely. The immediate benefit of further labor hoarding, on the other hand, is to trade the certainty of having to pay the firing cost now for the possibility of paying it at some future date. But again this latter possibility is less likely to materialize in the presence of on-the-job search, as it may cause the worker to leave the job at no (or little) cost for the firm. Hence the total benefit of labor hoarding also increases with employed job search intensity.

Because both the cost and benefit of reducing the job destruction threshold in the face of an increase in T are higher when employees search more intensely, the net impact of on-the-job search on the responsiveness of D to firing costs is generally ambiguous. For large enough values of T though, labor hoarding becomes the more attractive option when workers are searching on the job and the job destruction threshold will be lower than in the absence of employed job search.

To summarize, in the absence of firing costs, the job destruction threshold will be higher with on-the-job search than without whereas the opposite is true under large enough firing costs. As a consequence, when contrasting an equilibrium with zero firing costs with one with large firing costs, we predict a greater decrease in the job destruction rate when employed job search is allowed.

4 The full model: endogenous contact rates

The analysis so far has kept the firm-worker contact rates (λ_0, λ_1) exogenous. While it allowed us to preserve some analytical tractability (most of which we shall lose in the rest of this paper) and therefore to analyze the impact of policy on the job destruction margin with some degree of generality, it is clearly unsatisfactory from an equilibrium viewpoint as it amounts to shutting down important aspects of the firms' labor demand behavior. We thus now extend the model to allow for endogenous contact rates. In so doing we shall use a matching function, thus continuing to stick to the DMP tradition.

4.1 The matching process

Search is random and firms advertise new jobs in the knowledge that the pool of job seekers comprises both unemployed and employed workers. As seen above, all employed workers are assumed to engage in on-the-job search. Their search intensity, relative to that of unemployed workers, is denoted $s_1 = \lambda_1/\lambda_0$. We assume the matching technology to exhibit constant returns to scale, which leads to the following expressions for the two job offer rates: $\lambda_0 = \chi \cdot \theta^\alpha$ and $\lambda_1 = s_1 \cdot \lambda_0$, where $\theta = \frac{v}{u+s_1(1-u)}$ denotes labor market tightness. u, $1-u$ and v denote the stocks of unemployment, employment and vacancies, respectively, χ captures matching efficiency and α denotes the matching elasticity with respect to vacant jobs.

The rate at which a firm contacts an unemployed [employed] worker is $q_0(\theta)$ [$q_1(\theta)$], defined as follows (we will see shortly that these contacts are not necessarily turned into a match): $q_0(\theta) = \frac{\lambda_0}{\theta} \cdot \frac{u}{u+s_1(1-u)}$ and $q_1(\theta) = \frac{\lambda_0}{\theta} \cdot \frac{s_1(1-u)}{u+s_1(1-u)}$.

4.2 Job creation

We assume free entry of vacancies in the search market so that the value of a vacant job is zero. Under our maintained assumption that $E(1, w_{\min}) > U$, unemployed workers accept all job offers made to them as they yield a value that is greater than the value of being unemployed. As seen in Section 2.3, the optimal offer made to attract an employed worker is to give the worker a value of $V(z)$, where z is the productivity of the worker's old match. The expected profit of a vacancy will thus depend on the distribution of z in filled jobs. We will denote $L(\cdot)$ the steady-state distribution of productivity in currently filled jobs and derive it as follows by writing the flow balance for the stock of matches with productivity above z:

$$\overline{L}(z)\,\delta\left[F(z) - F(D)\right] = L(z)\left[\delta\overline{F}(z) + \delta F(D) + \lambda_1\right],$$

which implies:[9]

$$L(z) = \frac{\delta}{\delta + \lambda_1} \left[F(z) - F(D) \right]. \tag{5}$$

Employed workers accept offers yielding a value (including the severance package if any) greater than the maximum value of employment that their current employer is prepared to offer them. Workers currently employed in a match with a productivity less than z_m will be successfully attracted with a wage offer of $w_{\min}$, leaving the advertising firm with a value of the new match of $J(1, w_{\min})$. Workers currently employed in a match with a productivity z in the range $[z_m, 1]$ will require a wage offer that yields them a value of $V(z)$ to quit their existing employer (with or without a severance package from that employer, as is detailed above in Section 2.3). This leaves the advertising firm with a value of the new match of $V(1) - V(z)$. Combining all these facts, we can write the asset price equation for vacant jobs as:

$$0 = -k + q_0(\theta) \cdot J(1, w_{\min}) + q_1(\theta) \cdot \left[J(1, w_{\min}) L(z_m) + \int_{z_m}^{1} (V(1) - V(x)) \, dL(x) \right]$$

where k denotes advertising costs. Using (5) and integration by parts, we obtain the following job creation condition:

$$0 = -k + \frac{p}{\rho + \delta} \cdot \left[q_0(\theta)(1 - z_m) + q_1(\theta) \frac{\delta}{\delta + \lambda_1} \int_{z_m}^{1^-} (F(x) - F(D)) \, dx \right] \tag{JC}$$

Condition (JC) equates the marginal flow cost k of posting a vacancy to the marginal expected flow benefit of doing so. Looking at the expected benefit terms, the first of those reflects the expected benefit from contacting an unemployed worker whereas the second one relates to contacts with already employed workers. The total expected benefit from posting a vacancy decreases with labor market tightness θ for the following two reasons. First, the matching rates of vacancies are decreasing functions of θ. Second, the expected value of a new hire is negatively related to θ as a higher θ causes a higher λ_1 which in turn yields an increase in the mass of filled jobs at the top productivity $z = 1$. Besides, the right-hand side of equation (JC) decreases with D as a higher job destruction threshold raises the mean productivity in filled jobs in which productivity is greater

[9] Note the existence of a mass at the productivity of new matches ($z = 1$) equal to $\frac{\lambda_1 + \delta F(D)}{\delta + \lambda_1}$.

than z_m. The job creation condition (JC) thus yields a negatively-sloped relationship between θ and D.[10]

The level of firing costs T does not affect the relationship between θ and (z_m, D) embodied in (JC), so increases in T will only affect equilibrium labor market tightness through their impact on the thresholds z_m and D. Finally, let us note that, absent employed job search, all contacts made by vacant jobs occur with unemployed workers and that these leave, on average, a larger share of the new match value $V(1)$ to the firm. As the search intensity of the employed, s_1, increases, the expected benefit of posting a vacancy decreases, so a greater s_1 unambiguously discourages job creation, all other things equal.

4.3 Equilibrium: determination and comparative statics

Details of the derivation of equilibrium are reported in Appendix C. The key equilibrium outcomes, i.e. D, z_m and θ are obtained as the solution to a system of three equations: (A.5), (JC) and (JD$_i$) [resp. (JD$_e$)] in the inefficient [resp. efficient] separation regime. The job offer rates λ_0 and λ_1 and the steady-state unemployment rate $u = \frac{\delta F(D)}{\lambda_0 + \delta F(D)}$ can then be deduced from equilibrium labor market tightness θ.[11]

We now examine the comparative statics properties of these solutions. Our two policy tools of interest impact job creation in a conventional way: raising either the minimum wage $w_{\min}$ or the the firing cost T decreases the expected value of the newly created job, decreasing D for given values of θ, thus shifting the job creation schedule downwards.

As we saw in section 3.3, at given labor market tightness θ, the job destruction threshold D decreases with an increase in firing costs T and increases with an increase in the minimum wage $w_{\min}$ in the inefficient separation regime. These facts combined with the downward shift

[10] This claim is also straightforward to prove formally by differentiation of (JC).

[11] As we saw in subsection 3.3, for small values of firing costs T the job destruction rule defines an increasing schedule in the (D, θ) space under the inefficient separation regime. This combined with the negative slope of the job creation schedule established in the previous subsection ensures the uniqueness of equilibrium when T is small. For larger values of T however, the slope of the inefficient job destruction schedule may become negative, so, if the inefficient regime is still prevalent, this opens the possibility of multiple equilibria arising. Things get even less transparent under the efficient separation regime, where equilibrium is defined by a nonrecursive system of three nonlinear equations ((A.5), (JC) and (JD$_i$)). While the issue of multiple equilibria is a potential concern, we shall nonetheless leave it aside from the analysis, relying on the fact that multiple equilibria never occurred in any of the calibration exercises that we tried.

of the job creation schedule allow us to predict that an increase in T will result in a decrease in the job destruction threshold D and an ambiguously-signed change in θ, while an increase in the minimum wage will result in a drop in labor market tightness θ and an ambiguous net change in job destruction. It is difficult to say more analytically about the comparative static properties of our model's equilibrium. The combined effect of policy on job creation and job destruction on equilibrium unemployment is ambiguously signed.[12] We thus now turn to simulations.

5 Calibration and comparative statics

The simulation exercises that we now describe are intended to illustrate the role of on-the-job search in the response of the labor market to changes in firing costs and the minimum wage. As will be seen shortly, this response is markedly different with and without employed job search.

5.1 Baseline simulation

Table 2 reports the values that we assign to the model's parameters and the functional form that we adopt for the sampling distribution of productivity shocks. Flow parameters are expressed in monthly values. We set the annual discount rate to 5%, which translates into the monthly value shown in the table. We assume that productivity shocks occur every three years on average. While this may seem infrequent, we should recall that these shocks capture profound and long-lasting changes to match productivity that have the potential of causing a match breakup. The aggregate productivity index p is normalized to 1. The sampling distribution of match-specific productivity shocks $F(\cdot)$ is assumed to be uniform over the $(0, 1)$ interval. The matching elasticity with respect to vacancies, α is set at 0.2, which lies in the range of estimated values reported by Petrongolo and Pissarides (2001). The parameters relating to advertising costs and matching efficiency k and χ are simply calibrated so as to ensure a monthly finding rate consistent with typical UK facts. In order to highlight the contrast between labor market responses to changes in institutions with

[12] Another important property that no longer holds in general under endogenous contact rates is the pattern of regime prevalence described in Propositions 2 and 3. Because λ_0 and λ_1 both respond to policy changes in equilibrium, the monotonicity properties of the match surplus that were used in the proofs of these Propositions are lost. While any pattern can potentially emerge with endogenous contact rates, the labor market will still behave according to Propositions 2 and 3 in our simulations below.

ρ	δ	k	α	χ	p	b	$w_{\min}$
0.0043	1/36	22	0.2	0.3	1	-0.3	0.65
$F(\cdot)$ uniform over $(0,1)$.							

Table 2: Baseline model calibration

and without employed job search, we carry out the simulations with two benchmark values of the relative matching efficiency: $s_1 = 0$ and $s_1 = 0.15$. The former value yields a labor market without employed job search while the latter value delivers a labor market where about a third a labor reallocation relates to direct job-to-job moves.

As for our institutions of interest, we set the minimum wage at $w_{\min} = 0.65$ and we allow the firing cost to vary between 0 and 12 months of the maximum productivity. We also set the unemployment income to $b = -0.3$. Such a low value is needed to ensure positive match surpluses (at least over some range of productivity values). It can be justified if one interprets b as unemployment income net of specific costs incurred by the unemployed (search costs, stigma...).

Figure 1 shows the response of various basic labor market indicators to a variation in the statutory dismissal cost T (x-axis, expressed in months of match output at maximum productivity $z = 1$), under both values of the relative matching efficiency for the employed detailed above. We predict monthly job destruction rates of around 1.5% and a job finding rate for the unemployed of about 18%, which yields a predicted unemployment rate of around 8%, all in line with typical UK figures.

5.2 The impact of firing costs

Job creation and job destruction under both regimes. What is immediately striking in Figure 1 is the obvious regime switch occurring at a value of T equal to 4-5 months of the maximum productivity. In spite of contact rates being endogenous in the simulations, the market clearly behaves as described in Propositions 2 and 3: for a given value of $w_{\min}$ the inefficient separation regime prevails at relatively low values of T, while the market stays in the efficient separation regime

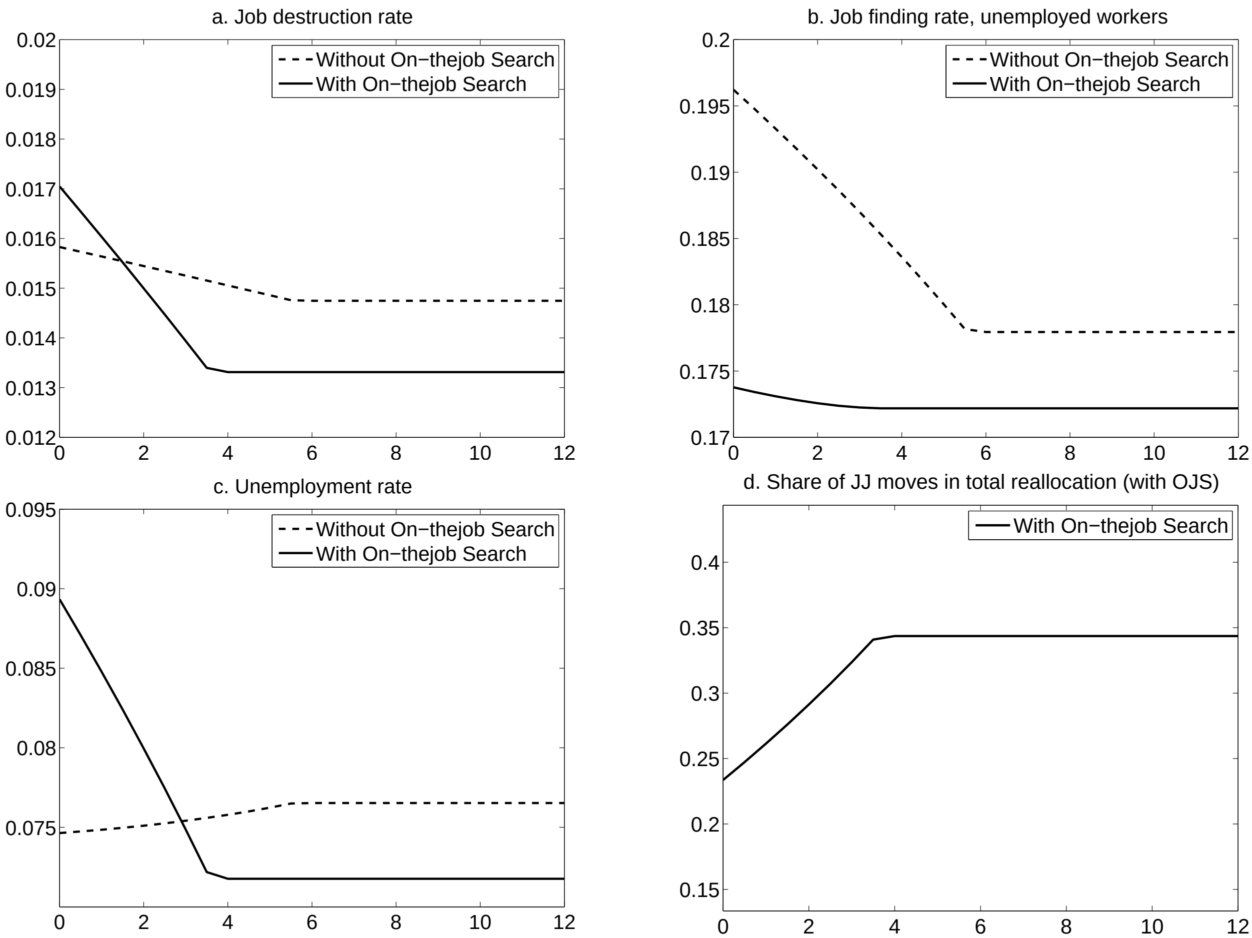

Figure 1: Simulation results for $w_{\min} = 0.65$ and variable firing costs T (x-axis).

for a redundancy pay beyond a certain size. Note that the threshold between the two regimes is lower in the presence of employed job search, so that a market where employed workers' search intensity is greater will be more likely to be in the efficient separation regime, for a given level of the institution mix of T and $w_{\min}$.

Under the inefficient separation regime and in the absence of employed job search, the qualitative predictions of our model regarding the impact of statutory firing costs on job destruction and job creation rates are standard. Higher firing costs slow down both of these, unambiguously reducing unemployment flows but having a small and ambiguously-signed net effect on the unemployment rate. In fact, this effect turns out to be slightly positive in this simulation. By contrast, with on-the-job search, we observe a much sharper decline of the job destruction rate as severance pay increases, as was predicted in section 3. From a quantitative standpoint, the impact of T on job destruction is magnified by the presence of on-the-job search, thus suggesting that the firing-tax avoidance effect of employed job search is quantitatively more important than the discount effect on the option value (see the discussion in subsection 3.3). The picture relating to the unemployed job finding rate also shows a sharp difference with the simulated results obtained without on-the-job search: increases in severance pay have hardly any impact on the job finding rate when employed job search is allowed. This difference in responsiveness of job creation to variations in T comes from the steeper response of the job destruction threshold D to changes in firing costs when employed job search is present. Indeed, a decrease in D renders a contact made with an employed worker more attractive in expectation terms as it shifts the distribution of idiosyncratic productivity in filled jobs to the left. In other words, it increases the probability that an advertising employer will only have to offer $V(z_m)$ to attract an employed worker upon meeting one (and decreases the probability of being left with a zero share of the match value, i.e. of making contact with an employed worker in an existing match with maximum idiosyncratic productivity $z = 1$). As a consequence, the overall impact of increases in firing costs on the unemployment rate is much more marked than with than without on-the-job search as the substantial decrease displayed in the bottom-left panel of Figure 1 illustrates. Increasing the statutory severance pay from 0 to five months' maximum productivity is predicted to cut the unemployment rate by slightly over 1.5 percentage point.

This, however, only relates to an economy in the inefficient separation regime. As is obvious from Figure 1, things change dramatically as the economy enters the efficient separation regime, at which point firing costs cease to have any impact at all.

Job-to-job reallocation. The job-to-job switching rate increases as statutory firing costs are raised. The job switching rate equals the job offer arrival rate λ_1 times the probability that the typical outside offer be accepted by its recipient, $L(1^-) = \delta \overline{F}(D) / (\delta + \lambda_1)$ — see (5). As the arrival rate of offers is virtually unaffected by firing costs and D decreases markedly, the overall impact is an increase in the job-to-job turnover rate. This combined with the fact that the unemployment rate decreases with T implies that total job-to-job reallocation increases while unemployment flows decrease, so that the *share* of worker reallocation due to direct job-to-job moves increases markedly with increases in T, as illustrated in the bottom-right panel of Figure 1. This is consistent with the prediction that job security provisions "redistribute employment opportunities from unemployed to employed workers" (Kugler and Saint-Paul, 2004). It also matches empirical evidence gathered by Boeri (1999), who reports shares of job-to-job reallocation within total labor reallocation for a sample of OECD (mostly European) countries in the range of 30-50% and increasing with the tightness of employment legislation.

6 Conclusion

Adding on-the-job search and negotiation over severance packages alters markedly the quantitative predictions of the standard DMP model without employed job search regarding the impact of firing costs on employment flows and the unemployment rate. In our simulations, increasing the firing costs from 0 to 5 months of the maximum productivity reduces the unemployment rate by one and a half percentage points, whereas a typical calibration of the DMP model without on-the-job search would predict an ambiguous and quantitatively negligible effect on the unemployment rate. Furthermore, past a certain level of firing cots, further increases in said costs become marginally ineffective, as in Fella (2007). This occurs because, as firing costs are increased beyond a threshold, it becomes profitable for firms to induce workers to quit into unemployment by offering suitable compensation. That threshold is an increasing function of the minimum wage, so that firing costs are less likely to impact unemployment when the minimum wage is high. This concurs with arguments made in Garibaldi and Violante (1998) or Cahuc and Zylberberg (1999), whereby the degree of wage rigidity is a key determinant of the effectiveness of firing restrictions policies.

The main distinguishing feature of our analysis is that we take into account the possibility of on-the-job search and job-to-job quits. The role that job quitters play in our framework becomes important in situations where the firm incurs losses (in the presence of firing restrictions), poaching firms are constrained in the offers they can make (in the presence of a wage floor) and the incumbent firm and worker pair are able to negotiate over the amount of a severance package (under the mutual consent rule). We argue that the extent of job-to-job reallocation is a key feature of the economy's response to a change in the policy choice in terms of combined firing cost and minimum wage. Of course, it may be that firing costs also impact on employed search intensity if, for example, only tenured employees are entitled to job protection. This feedback channel is not explored in this paper and is a possible avenue of extension of this research.

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APPENDIX

A Strategic bargaining games

In this appendix we give formal details of the strategic bargaining game that is played by parties in a match after the occurrence of either a productivity shock or an outside job offer. While the same game is played in both situations, payoffs and equilibrium play depend on whether the game was triggered by a productivity shock or an outside offer. We begin by describing the former case.

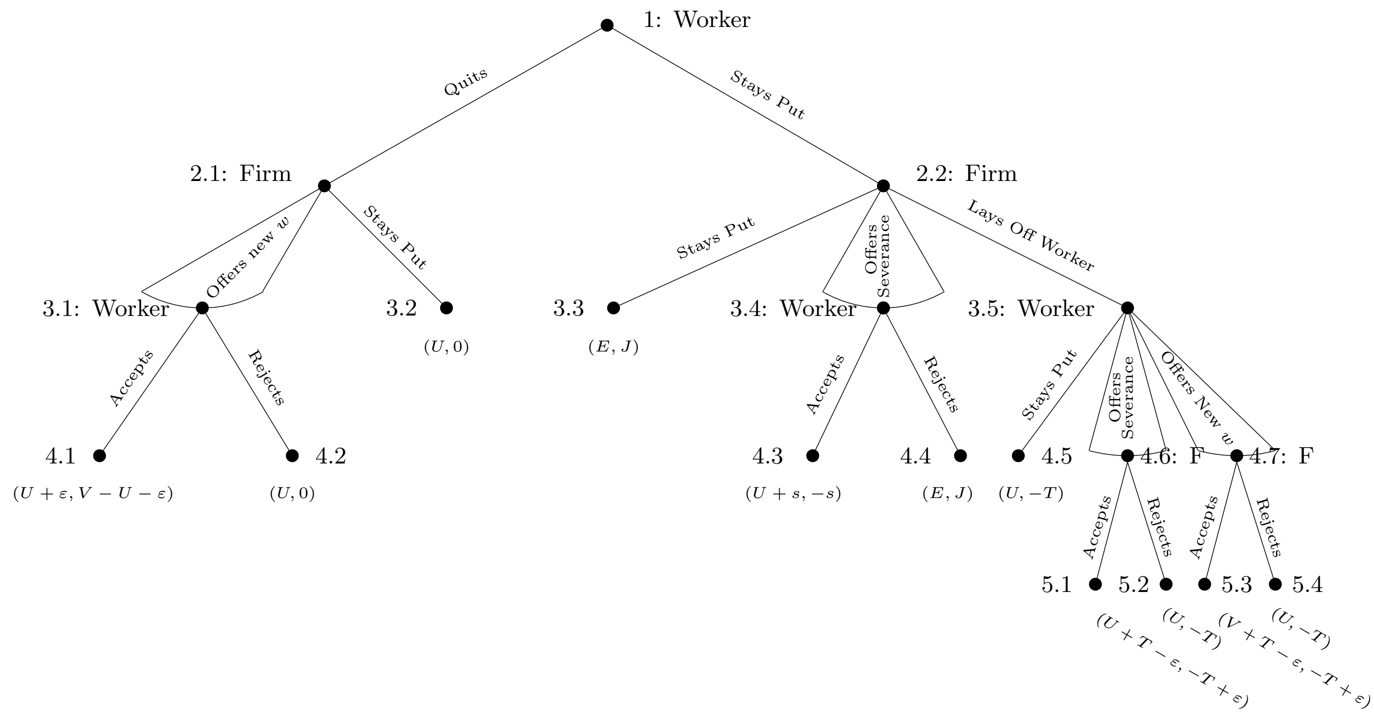

Figure A.1: Renegotiation game played after productivity shock.

A.1 Renegotiation following a productivity shock

Consider an ongoing match drawing a new productivity shock x, which, given the wage w inherited from previous periods, yields values of E and J to the worker and the firm, respectively. These values are known to both parties. The game represented in extensive form in Figure A.1 ensues. Following the occurrence of a productivity shock, the worker has no outside offer in hand, so his outside option is to quit into unemployment, which has value U. For the firm, the outside option is worth $-T$ if the firm has to lay off the worker to exit the relationship, as the value of holding a vacant job is zero. If the worker quits, the firm's value is zero as it is not paying the firing cost. In our initial description of the renegotiation game in Figure A.1 we ignore the minimum wage, which we will reintroduce at a later stage.

Each party has the opportunity to prompt a negotiation. A key aspect of the game is that, in the event of a party prompting negotiation, the other party gets *one single chance to respond*. The worker has first move, and triggering negotiation for the worker means quitting the match (node 1 in Figure A.1. For the firm it can mean laying off the worker or offering them an agreed separation with a side-payment s to be determined in equilibrium (node 2.2).

If the worker announces a quit, the firm's response (node 2.1) is either to let him do so (leading to node 3.2) or to try and retain him (leading to node 3.1), which will be achieved by offering a wage increase yielding the worker a value of employment in the current match of $U + \varepsilon$, where ε is an arbitrarily small, strictly positive number. At node 3.1, the worker simply accepts or rejects the firm's new offer.

If the worker does not threaten to quit the job, the firm can do one of three things (node 2.2): do nothing (leading to status quo payoffs: node 3.3), offer a severance package s (leading to node 3.4), or layoff the worker (leading to node 3.5). If the firm offers a severance package (node 3.4), the worker may either accept or reject the offer. In the latter case, status quo prevails and the firm does not get another chance to layoff in this round of negotiation (the game ends at node 4.4). Finally, when faced with the threat of a layoff (node 3.5), the worker may of course do nothing and accept to be laid off (leading to node 4.5), but he also gets a chance to respond by either offering a severance deal (leading to node 4.6) which the firm subsequently accepts or rejects, or by offering to take a wage cut and continue the relationship (leading to node 4.7) which the firm subsequently accepts or rejects.

The game is solved in standard fashion by backward induction and we skip the details of the solution (available on request). Equilibrium strategies as well as the equilibrium value of the severance package s offered by the firm at node 2.2 depend on initial match values E and J and their position relative to the parties' outside option payoffs. When the firm offers an agreed separation (implying the worker quitting with a severance package s and the firm not having to pay the layoff tax T), the worker may reject it if his value of staying in the match with the status quo, E, is greater than the value of leaving with the severance package, $U + s$. Thus the minimum package s that the firm has to offer for the worker to accept it depends on the worker's current value of employment E. If $E \leq U$, then a severance package of ε is enough to induce the worker to leave the match. If $E > U$, a severance pay of $E - U + \varepsilon$ is just enough for the worker to strictly prefer the agreed separation to the status quo. Note, however, that the firm will never offer a severance package in excess of T as this is the amount it would need to pay for a layoff. So s is capped at T. The relationship between the required s and E is summarized as:

$$s = \begin{cases} T & \text{if } E > U + T \\ E - U + \varepsilon & \text{if } U < E \leq U + T \\ 0 + \varepsilon & \text{if } E \leq U \end{cases}$$

Given those values of s, Table A.1 summarizes equilibrium play and payoffs for a continuing firm-worker match hit by a productivity shock, in all possible cases regarding initial match values.

Binding minimum wage. As mentioned above, the results in Table A.1 were obtained under the assumption of no binding minimum wage, so that a worker's wage may be low enough to make unemployment

		$J < -T$	$-T < J < 0$	$J > 0$
$V \geq U$	$E > U$	2.2, 3.5, 4.7, 5.3 wage decrease $\{V + T - \varepsilon, -T + \varepsilon\}$	2.2, 3.3 status quo $\{E, J\}$	2.2, 3.3 status quo $\{E, J\}$
	$E < U$	—	—	2.1, 3.1, 4.1 wage increase $\{U + \varepsilon, V - U - \varepsilon\}$
$V < U$	$E > U$	2.2, 3.4, 4.3 if $U < E \leq U + T$ 2.2, 3.5, 4.6, 5.1 if $E > U + T$ agreed separation $\{U + s, -s\}$, $s = \min(T, E - U + \varepsilon)$	2.2, 3.4, 4.3 agreed separation $\{E + \varepsilon, -(E - U + \varepsilon)\}$	—
	$E < U$	2.2, 3.4, 4.3 agreed separation $\{U + \varepsilon, -\varepsilon\}$	2.2, 3.4, 4.3 agreed separation $\{U + \varepsilon, -\varepsilon\}$	2.1, 3.2 quit $\{U, 0\}$

Table A.1: Equilibrium play and payoffs after a productivity shock

preferable to employment at that wage, for a given productivity value x. In that case, the worker either quits or agrees to a separation with an infinitesimal severance pay if the match becomes unprofitable (bottom row of Table A.1), and the worker is able to claim a pay raise by credibly threatening to quit into unemployment if the match is still profitable (second row of the top panel of Table A.1). Those cases may be ruled out in the presence of a minimum wage that prevents the worker's wage from dropping low enough to equate E to U.

Also in the presence of a minimum wage, the wage may not be allowed to drop enough to reduce the firm's losses to $-T + \varepsilon$ (top row of Table A.1, when firm's losses exceed T and the worker prefers to stay in employment, $V > U$). In the notation introduced in the main text, this happens when the new productivity draw, x, falls short of $R_T(w_{\min})$. In that case, the match is dissolved (even though it has positive surplus, $V > U$), and an agreed separation occurs with a side payment of $s = \min\{E - U + \varepsilon, T\}$.

If T is a transfer. When T is a tax, the sum of payoffs upon an agreed separation is U whereas it is only $U - T$ if a layoff goes ahead. Hence it is always jointly profitable to agree on a severance deal rather than resort to a layoff, and layoffs never occur in equilibrium (Fella, 2007). On the other hand, if T is a transfer, the sum of payoffs upon separation is always U, be it a layoff or a separation. Layoffs are no longer jointly inefficient and occur in some cases. This is the only substantive change to equilibrium brought about by assuming that T is a transfer from the firm to the worker rather than a tax.

Suppose that T is now paid directly from the firm to the worker upon layoff. The only things that change in Figure A.1 are the payoffs in case of a layoff, i.e. at nodes 4.5, 5.2 and 5.4. If T is a transfer, the worker receives $U + T$ rather than U at all three of those nodes. Solution of the game by backward induction, however, reveals that the payoffs reported in Table A.1 still apply in the case where the firing cost is a transfer. The only difference to the case where T is a tax is that, at node 4.6, the worker now rejects the severance deal offered by the firm and gets laid off, receiving a payoff of $U + T$ instead of $U + T - \varepsilon$. So when T is a transfer, a fraction of the separations are now labeled as layoffs, as opposed to all separations being agreed on in the case where T is a tax. Apart from that, equilibrium strategies and payoffs are the same in both cases.

A.2 Renegotiation following an outside job offer

Consider an ongoing match with productivity z and wage w (yielding values of E and J to the worker and the firm, respectively) in which the worker receives an outside offer with value E'.[13] All of those values are known to both parties. The game represented in extensive form in Figure A.2 ensues. Apart from the fact that the worker's outside option is now to accept the outside offer, which has value $E' > U$, the game is identical to the one that is played when renegotiating after a productivity shock (Figure A.1). Also in Figure A.2, the branch of the game starting at note 2.2 with the firm laying off the worker was deliberately omitted, as it is never played in equilibrium. Indeed, unless an adverse productivity shock occurs, it cannot be optimal for the firm to fire the worker (otherwise the firm would have done so already).

Once again, the game in Figure A.2 is solved by backward induction. Equilibrium payoffs and the implied optimal value E' of the poacher's offer are discussed in the main text (see Table 1). Formal details of the game's solution are available on request.

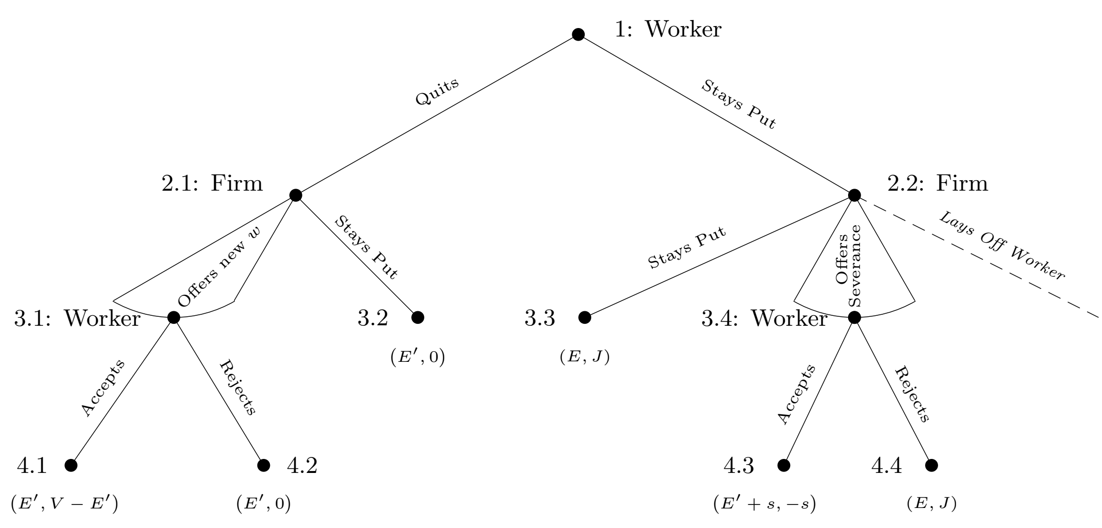

Figure A.2: Renegotiation game played after outside job offer.

[13]Note that, when the worker receives an outside offer, he is in a continuing match, so $E > U$ and $J > -T$.

B Properties of the value functions

In this appendix we establish a set of properties of the value functions defined in equations (1) - (4) that will be useful for the rest of the analysis.

Derivatives and monotonicity. From Table 1, we know that $\overline{V}_o(z) = \max\{V(z); V(z_m)\}$ where, from (3):

$$V'(z) = \begin{cases} \dfrac{p}{\rho+\delta} & \text{if } z > z_m \\ \\ \dfrac{p}{\rho+\delta+\lambda_1} & \text{if } z_m \geq z \end{cases} \tag{A.1}$$

The combination of Table 1 and the value of $V'(\cdot)$ just derived allows us to express the partial derivatives $\left\{\frac{\partial \overline{E}_o}{\partial z}; \frac{\partial \overline{J}_o}{\partial z}\right\}$ as reported in Table A.2.

	$z \leq R_0(w)$	$z > R_0(w)$
$1 \geq z > z_m$	$\{V'(z); 0\}$	$\{\partial E/\partial z; \partial J/\partial z\}$
$z_m \geq z$	$\{0; 0\}$	$\{0; 0\}$ if $V(z_m) \geq E(z,w)$ $\{\partial E/\partial z; -\partial E/\partial z\}$ if $V(z_m) < E(z,w)$

Table A.2: $\left\{\partial \overline{E}_o/\partial z; \partial \overline{J}_o/\partial z\right\}$.

Expressions for the partial derivatives $\left\{\frac{\partial E}{\partial z}; \frac{\partial J}{\partial z}\right\}$ then follow from the Bellman equations (1) - (4). The various cases are covered in Table A.3.

		$z \leq R_0(w)$	$z > R_0(w)$
$1 \geq z > z_m$	$\partial E/\partial z$	0	$\frac{p\lambda_1}{(\rho+\delta)(\rho+\delta+\lambda_1)}$
	$\partial J/\partial z$	$\frac{p}{\rho+\delta}$	$\frac{p}{\rho+\delta+\lambda_1}$
$z_m \geq z$	$\partial E/\partial z$	0	0
	$\partial J/\partial z$	$\frac{p}{\rho+\delta+\lambda_1}$	$\frac{p}{\rho+\delta+\lambda_1}$

Table A.3: $\{\partial E/\partial z; \partial J/\partial z\}$.

An important property derived from Table A.3 is that the workers' valuation of a job, $E(\cdot)$, is independent of match productivity z whenever the latter is less than z_m.

Ranking of various productivity thresholds. The following ranking of some important productivity thresholds will be used in subsequent developments.

Lemma 1 *$R_T(w_{\min}) < R_0(w_{\min}) < z_m \leq 1$ and $R_S < R_0(w_{\min})$.*

Proof. In the first series of inequalities, the last inequality is almost by definition as $V(1) > E(1, w_{\min}) = V(z_m)$ (the inequality there holds true because otherwise there would be no trade on the labor market). Then, because $E(\cdot)$ is increasing in z over $[z_m, 1]$ (see Table A.3), we have that $E(z_m, w_{\min}) < E(1, w_{\min}) = V(z_m)$. Hence $J(z_m, w_{\min}) = V(z_m) - E(z_m, w_{\min}) > 0$, which proves that a firm paying the minimum wage in a match with productivity z_m makes a profit, hence $R_0(w_{\min}) < z_m$ (the second inequality). The first inequality $R_T(w_{\min}) < R_0(w_{\min})$ is a direct consequence of $\partial J/\partial z$ being positive throughout.

Now turning to the second claim $R_S < R_0(w_{\min})$, we have that $J(R_S, w_{\min}) = V(R_S) - E(R_S, w_{\min}) = U - E(R_S, w_{\min})$ (by the definition of R_S). This last term is negative by assumption (as workers are never willing to quit into unemployment). □

C Derivation of the job destruction thresholds

The inefficient separation regime: $D = R_T(w_{\min})$. By definition of the inefficient separation regime, the job destruction condition is given by $D = R_T(w_{\min}) \Leftrightarrow J(D, w_{\min}) = -T$. Combining this condition with the Bellman equation (1) defining the firm's valuation of a job $J(\cdot)$ in (1), we get:

$$-(\rho + \delta + \lambda_1) T = pD - w_{\min} + \delta \overline{J}_s(w_{\min}) + \lambda_1 \overline{J}_o(D, w_{\min}). \tag{A.2}$$

From Table **??**, the firm's value of the continuing match after an outside offer, $\overline{J}_o(D, w_{\min})$ is zero as $E(D, w_{\min}) < V(z_m)$. The firm's expected value of the continuing match after a shock to match quality becomes:

$$\overline{J}_s(w_{\min}) = -T \cdot F(D) + \int_D^1 J(x, w_{\min})\, dF(x) = -T + \frac{p}{\rho + \delta + \lambda_1} \int_D^1 \overline{F}(x)\, dx \tag{A.3}$$

where the second equality is obtained with integration by parts and the partial derivative values in Table A.3 (note that this integration by parts also uses the property $z_m \geq R_0(w_{\min})$, from Lemma 1.). The job destruction rule (JD_i) then obtains by substitution of $\overline{J}_o$ and $\overline{J}_s$ into (A.2).

$$D + \frac{\delta}{\rho + \delta + \lambda_1} \int_D^1 \overline{F}(x)\, dx = \frac{w_{\min}}{p} - \frac{T}{p}(\rho + \lambda_1) \tag{A.4}$$

where, in standard fashion, a bar over a CDF denotes the corresponding survivor function.

The definition of z_m, $V(z_m) = E(1, w_{\min})$, implies that $V(1) - V(z_m) = J(1, w_{\min})$. The derivative $V'(z)$ derived in (A.1) imply in turn that $V(1) - V(z_m) = \frac{p(1-z_m)}{\rho+\delta}$, while the partial derivatives $\partial J/\partial z$ given in Table A.3 further imply that $J(1, w_{\min}) = J[R_T(w_{\min}), w_{\min}] + \frac{p(1-R_T(w_{\min}))}{\rho+\delta+\lambda_1} = -T + \frac{p(1-R_T(w_{\min}))}{\rho+\delta+\lambda_1}$. Combining both expressions, we obtain a solution for z_m:

$$z_m = \frac{\lambda_1 + (\rho + \delta) R_T(w_{\min})}{\rho + \delta + \lambda_1} + (\rho + \delta) \frac{T}{p}. \tag{A.5}$$

The efficient separation regime: $D = R_S$. The formal definition of D applying in this case is slightly more involved and will require solving the three equations derived below for the unknowns R_S, z_m, $V(z_m)$.

First, from (4), the value of unemployment is written as:

$$U = \frac{1}{\rho + \lambda_0}\left[b + \lambda_0 V(z_m)\right]$$

Derivatives given in Table A.3 further imply that $V(z_m) = V(R_S) + \frac{p(z_m - R_S)}{\rho+\delta+\lambda_1}$. Combining this with the job destruction condition in the efficient regime $V(R_S) = U$, we obtain a first expression for $V(z_m)$:

$$\rho V(z_m) = b + \frac{\rho + \lambda_0}{\rho + \delta + \lambda_1} \cdot p(z_m - R_S) \tag{A.6}$$

Second, the value of employment when first hired from unemployment (recall that new matches start with a productivity of 1) is obtained from Bellman equation (2) applied to this specific case:

$$(\rho + \delta + \lambda_1) E(1, w_{\min}) = w_{\min} + \delta F(R_S) E(R_S, w_{\min}) + \delta \int_{R_S}^{1} E(x, w_{\min}) \, dF(x) + \lambda_1 V(1),$$

which, after some algebra,[14] gives us our second expression for $V(z_m)$:

$$\rho V(z_m) = w_{\min} + \frac{\lambda_1 p}{(\rho + \delta + \lambda_1)(\rho + \delta)} \cdot \left[(1 - z_m)(\rho + \lambda_1) + \delta \int_{z_m}^{1} \overline{F}(x) \, dx\right] \tag{A.7}$$

Finally, using Bellman equation (3) to write the value of the surplus at z_m:

$$(\rho + \delta) V(z_m) = p z_m + \delta F(R_S) U + \delta \int_{R_S}^{1} V(x) \, dF(x),$$

we obtain our third equation:[15]

$$\frac{\rho(\rho + \delta + \lambda_0)}{\rho + \lambda_0} V(z_m) = p z_m + \frac{\delta b}{\rho + \lambda_0} + \frac{\delta p}{\rho + \delta + \lambda_1} \int_{R_S}^{z_m} \overline{F}(x) \, dx + \frac{\delta p}{\rho + \delta} \int_{z_m}^{1} \overline{F}(x) \, dx \tag{A.8}$$

The three equations (A.6), (A.7) and (A.8) pin down the three unknowns $V(z_m)$, R_S and z_m. It is clear from these equations that R_S does not depend on T as firing costs are absent altogether from the system of equations. This contrasts with the inefficient separation regime seen above where T clearly impacts on $R_T(w_{min})$ as equation (A.4) above shows.

D Proofs of Propositions 2 and 3

As discussed in Section 3, the inefficient separation regime prevails whenever $V(D) > U$ where D, the job destruction threshold, equals $R_T(w_{\min})$ and is defined in (A.4). We first differentiate (A.4) to obtain the

[14] First substitute $E(1, w_{\min}) = V(z_m)$, then use integration by parts in the integral taking the value of $\partial E(x, w_{\min}) / \partial x$ from Table A.3.

[15] Again, this involves integration by parts in the integral taking the value of $V'(x) / \partial x$ from (A.1).

following partial derivatives:

$$\left.\frac{\partial D}{\partial T}\right|_{\mathrm{JD}_i} = -\frac{1}{p} \cdot \frac{(\rho+\lambda_1)(\rho+\delta+\lambda_1)}{\rho+\delta F(D)+\lambda_1} \tag{A.9}$$

$$\left.\frac{\partial D}{\partial w_{\min}}\right|_{\mathrm{JD}_i} = \frac{1}{p} \cdot \frac{\rho+\delta+\lambda_1}{\rho+\delta F(D)+\lambda_1} \tag{A.10}$$

$$\left.\frac{\partial D}{\partial b}\right|_{\mathrm{JD}_i} = 0. \tag{A.11}$$

Substitution into (A.5) and differentiation further yields:

$$\left.\frac{\partial z_m}{\partial T}\right|_{\mathrm{JD}_i} = \frac{1}{p} \cdot \frac{(\rho+\delta)\,\delta F(D)}{\rho+\delta F(D)+\lambda_1} \tag{A.12}$$

$$\left.\frac{\partial z_m}{\partial w_{\min}}\right|_{\mathrm{JD}_i} = \frac{1}{p} \cdot \frac{\rho+\delta}{\rho+\delta F(D)+\lambda_1} \tag{A.13}$$

$$\left.\frac{\partial z_m}{\partial b}\right|_{\mathrm{JD}_i} = 0. \tag{A.14}$$

Now evaluating $V(z)$ at $z = D = R_T(w_{\min})$ using the definitions (3) and (4) and rearranging using (JD_i) and (A.1) leads to:

$$(\rho+\delta F(D)+\lambda_1)\,V(D) = w_{\min} - b - (\rho+\lambda_1)\,T - \frac{\lambda_0-\lambda_1}{\rho+\delta+\lambda_1}\left[\frac{\lambda_1 p\,(1-D)}{\rho+\delta+\lambda_1} + (\rho+\delta)\,T\right] + \frac{\delta\lambda_1 p}{(\rho+\delta)(\rho+\delta+\lambda_1)}\int_{z_m}^{1} \overline{F}(x)\,dx. \tag{A.15}$$

The right hand side in (A.15) is a function of the policy tools T, $w_{\min}$ and b, both directly and through the threshold values D and z_m, which we denote by $N(T, w_{\min}, b)$. Using the partial derivatives derived above in (A.9)-(A.14), tedious but straightforward algebra allows to establish that $\partial N/\partial T < 0$, $\partial N/\partial w_{\min} > 0$, and $\partial N/\partial b < 0$ (we do not report the derivations here but keep them available on request). It is likewise fairly straightforward to show that $\lim_{T\to+\infty} N(\cdot) = \lim_{b\to+\infty} N(\cdot) = -\infty$ and that $\lim_{w_{\min}\to+\infty} N(\cdot) = +\infty$. This proves Propositions 2 and 3.