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Depletion and Development: Natural Resource Supply with Endogenous Field Opening

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Abstract

This paper develops a model in which supply of a non-renewable resource can adjust through two margins: the rate of depletion and the rate of field opening. Faster depletion of existing fields means that less of the resource can ultimately be extracted, and optimal depletion of open fields follows a (modified) Hotelling rule. Opening a new field involves sinking a capital cost, and the timing of field opening is chosen to maximize the present value of the field. Output dynamics depend on both depletion and field opening, and supply responses to price changes are studied. In contrast to Hotelling, the long run equilibrium rate of growth of prices is independent of the rate of interest, depending instead on characteristics of demand and geologically determined supply.

JEL-Code: D900, Q300, Q400, Q500.

Keywords: non-renewable resource, depletion, exhaustible, Hotelling, fossil fuel, carbon tax.

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1. Introduction:

How does the supply of a non-renewable resource respond to price changes, and how does the market for such a resource respond to shifts in demand? These questions are important for understanding long-run issues such as the effects of climate policy on the use of fossil fuels, and short-run issues such as the behaviour of commodity prices. At one extreme, the Hotelling (1931) approach treats non-renewable resources as assets which can be depleted at any date, so that prices are linked by inter-temporal arbitrage (the rent increasing at the rate of interest). At the other, some industry experts use extremely low supply elasticities (the US Energy Information Administration uses short-run supply elasticity of 0.02 and long run 0.1, see Smith 2009), implying that opportunities for inter-temporal arbitrage are negligible.[1]

The objective of this paper is to provide a model in which the supply of an exhaustible resource is captured in a richer manner than in the conventional Hotelling approach. The central idea is that supply can adjust through two margins, intensive and extensive. The intensive margin is the rate of depletion of existing open fields (or mines). We posit a relationship between extraction costs and the rate of depletion that can vary between zero and perfect flexibility (the latter being the pure Hotelling case); this endogeneity of extraction costs breaks the rigid link between price growth and the rate of interest. The extensive margin is the development of new fields. Central to our approach is the fact that capital has to be sunk before a new field is opened, a feature that accords with reality, and is a quantitatively important feature of major mining developments and oil investments in offshore and deep fields. Fields differ in capital cost per unit reserve, and it is this that produces, in equilibrium, a sequence of field openings through time.

The supply of the resource depends on choices of how fast to deplete existing fields (the intensive margin) and when to open new fields (the extensive margin). In sharp contrast to the standard approach, the long-run equilibrium of the model has price increasing at a rate that is completely independent of the rate of interest. Extensive margin choices about field opening mean that the rate of price increase depends on characteristics of demand (price elasticity and growth), and characteristics of the geology and technology of supply. This is perfectly consistent with intensive margin choices that are 'Hotelling-like', with depletion rates on individual fields adjusting according to price growth and the rate of interest. The combination of intensive and extensive margin effects also gives different supply responses

[1] Empirical tests have failed to find support for the Hotelling approach. See Chermak and Patrick (2002).

to shocks. For example, demand reduction policies motivated by climate change may bring forward depletion of existing fields (the 'green paradox' noted by Sinn 2008) but will also cause postponement of the development of new fields, so that overall supply and emissions are reduced.

The next section of the paper outlines the model and characterises producers' choices of depletion rates and field opening. In order to model depletion (the intensive margin) in a flexible yet tractable way we assume that extraction costs increase with the rate of depletion.[2] Furthermore, these costs are 'iceberg', using up the resource itself. Both these assumptions seem to be supported in the technical literature on oil extraction (discussed in section 2.2) which suggests that faster depletion means that less of the resource is ultimately recoverable. They are also convenient modelling simplifications which make for a tractable characterisation of the intensive margin and, by allowing aggregation over fields, facilitate analysis of aggregate resource supply.

The extensive margin decision turns on when to sink capital in order to open a new field. This modelling approach is in contrast with much of the literature, where additions to stock are typically modelled as the outcome of a continuous variable (exploration) that adds to the capacity and reduces extraction costs of the existing field (as in Pindyck 1978, Dasgupta and Heal 1979).[3] Existing literature in which there are field set-up costs includes Hartwick et al (1986), Holland (2003), and Livernois and Uhler (1987). Hartwick et al assume zero extraction costs, in which case only one field is operated at any time, and Holland (2003) looks at cases where marginal extraction costs are either constant or infinite. Livernois and Uhler (1987) look at the rate of discovery of new fields with field-specific extraction costs, characterising first order conditions for the problem but doing little subsequent analysis of the equilibrium. We are able to go beyond these models, fully integrating intensive and extensive margin choices.

Section 3 places the intensive and extensive margin choices in the context of a continuum of potential fields and derives aggregate supply. Supply depends on both the rate of change of price (relative to the interest rate), as in the Hotelling model, and on the level of price, operating through the extensive margin and the timing of field openings. Thus, a

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[2] This is more restrictive than much of the literature, in which costs are modelled as a function of extraction and the stock of resource remaining. For example, Pindyck (1978) assumes that costs are proportional to extraction and decreasing in remaining stock. The rate of extraction is the ratio of these variables.

[3] See Krautkraemer (1998) for a survey. Swierzbinski and Mendelsohn (1989) aggregate separate fields, but assuming no fixed costs and constant returns to scale in exploration and extraction.

permanent proportional price reduction postpones field opening, reducing the quantity produced in the short run, raising it in the long run, and reducing the cumulative quantity produced at all future dates. A permanent reduction in the rate of growth of price increases production in the short run (bringing forward depletion of existing fields and, temporarily, field opening), but has a long run negative effect on cumulative quantity supplied.

Section 4 proceeds from analysing the response of supply to a given price path, to the full market equilibrium with price endogenous. The long run rate of change of price is determined by the rate of growth of demand, the price elasticity of demand, and a parameter summarising the geology of supply; it is completely independent of the rate of interest. Reductions in the level or the rate of growth of demand have the effect of reducing the cumulative quantity supplied, even though they may increase the rate of extraction on existing fields. This has implications for our understanding of climate change policy.

2. Field depletion and development:

There is a continuum of fields all of which are known at date 0, and are owned by price-taking profit maximizing agents. Each field contains one unit of the resource, but cannot produce until a field specific fixed cost $Ke^{-\theta T}$, $\theta \geq 0$, has been paid, where $e^{-\theta T}$ captures technical progress in field development that has taken place by date $t = T$, when the cost is paid. K varies across fields, and we will use K as the index of field types, with K running to plus infinity. The number (measure) of fields of type K is $S(K)$.[4]

2.1 Depletion and development

Focusing on a particular field (i.e. taking a particular value of K), output at date t is $xq(z)$, where x is the stock remaining and z is the rate of depletion, defined as the proportionate rate of decline of remaining stock, so $\dot{x} = -xz$. While z is the rate of depletion of the field and xz is the reduction in the stock, $xq(z)$ is the recovered output. The expression $q(z)/z \leq 1$ is the yield curve, giving the fraction of the reduction in stock that is marketable output. All current extraction costs are subsumed in this yield curve. The function $q(z)$ is increasing and concave in z and, if strictly concave, increases in the rate of depletion yield less than proportionate

[4] Assuming each field contains one unit of resource is without loss of generality as K can be interpreted as capital cost per unit capacity. The total stock of resource in fields with capital cost K is $S(K)$.

increases in output, perhaps as too rapid pumping from an oil-field reduces the capacity of the field. We give some examples and further discussion of this relationship in section 2.2.

Profit maximization in a field with fixed cost *K* requires that the opening date, *T*, and subsequent time paths of *z* and *x* are chosen to maximize the present value of profits (evaluated at date $t = 0$ with interest rate *r*),

$$PV \equiv e^{-rT} \int_0^{\infty} p(T+\tau)x(\tau)q(z(\tau))e^{-r\tau}d\tau - Ke^{-(\theta+r)T} \quad (1)$$

subject to

$$\dot{x}/x = -z, \quad \text{and } x(0) = 1, \ x \geq 0. \quad (2)$$

The integral in (1) runs over dates τ measured from when the field is opened, so $t = T + \tau$, and $p(t)$ is the (exogenous) price at date *t*. We assume that, as $t \to \infty$, $p(t)$ converges to constant exponential growth at rate less than or equal to *r*, as is necessary for the objective to be bounded. We denote this limiting rate of change of price $\hat{p}_\infty$.

The profit maximizing depletion path once the field has been opened is given by the Euler equation

$$\dot{z} = \left[r - \frac{\dot{p}}{p} + z - \frac{q(z)}{q'(z)} \right] \frac{q'(z)}{q''(z)}. \quad (3)$$

This depends on the difference between the rate of interest and rate of price increase, and also on the curvature of $q(z)$, indicating the cost penalty from increasing the rate of depletion.[5]

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[5] The intuition behind Euler equation (3) for optimal depletion is as follows. Suppose that the price is growing at constant rate $\hat{p}$, so $z = z^*$, and consider a perturbation at some date (say date 0)which is an instantaneous increase in extraction δ, offset by a reduction in the next instant which puts the resource stock back on its previous path. If δ is small, the value of the perturbation is

$$p_0 x_0 \left[q(z^* + \delta) - q(z^*) \right] + \frac{p_0}{1 + r - \hat{p}} \left[(1 - z^* - \delta)x_0 q\left(z^* - \delta\right) - (1 - z^*)x_0 q(z^*) \right]$$

The first term is the value of increasing extraction by δ. Stock carried through into the next instant changes from $(1 - z^*)x_0$ to $(1 - z^* - \delta)x_0$ and its value is discounted by the interest rate minus the rate of price growth. To undo the perturbation, the rate of extraction must fall to $z^* - \delta$. Differentiating with respect to δ and evaluating at $\delta = 0$, this expression is $p_0 x_0 \delta \left[q' - \frac{q + (1 - z^*)q'}{1 + r - \hat{p}} \right]$, so the perturbation has zero value if the term in square brackets is zero, this being equation (5).

Notice that z can jump, while the stock variable x cannot. The stock remaining in a field that has been open for τ periods is

$$x(\tau) = \exp\left[-\int_0^{\tau} z(\chi)d\chi\right]. \qquad (4)$$

Concavity of $q(z)$ ensures that differential equation (3) is locally stable (since $z - q(z)/q'(z)$ is decreasing in z), so z converges to stationary value z^* implicitly defined by

$$\hat{p}_\infty - r = z^* - q(z^*)/q'(z^*), \quad \text{or} \quad z^* = \varsigma(r - \hat{p}_\infty), \quad \varsigma' > 0, \qquad (5)$$

where the function $\varsigma(r - \hat{p}_\infty)$ summarizes the long-run relationship. We discuss this further in section 2.2.

The profit maximizing date, T, at which to spend $Ke^{-\theta T}$ and open the field is given by first order condition

$$\frac{\partial PV}{\partial T} = e^{-rT}\left[-r\int_0^{\infty} pxq(z)e^{-r\tau}d\tau + (\theta + r)Ke^{-\theta T} + \int_0^{\infty} \dot{p}xq(z)e^{-r\tau}d\tau\right] = 0. \qquad (6)$$

The intuition is that if the profile of production and costs is shifted back by dT, then the first term is the cost of pushing revenues further away, the second the benefit of moving costs, and the final term is the change in revenue from the fact that output $xq(z)$ is now valued at prices dT later. Rearranging, the date of opening T is given by first order condition

$$\int_0^{\infty} (\dot{p} - rp)xq(z)e^{-r\tau}d\tau + (\theta + r)Ke^{-\theta T} = 0. \qquad (7)$$

To see the implications of this it is easiest to look at (1) and (7) with the assumptions that price is growing at constant rate $\hat{p}$ (taking value p_0 at $t = 0$) and z is at its stationary value z^*. The integral in (1) can then be evaluated as

$$PV = p_0 e^{(\hat{p}-r)T} q(z^*)\int_0^{\infty} e^{(\hat{p}-r-z^*)\tau}d\tau - Ke^{-(\theta+r)T} = e^{-(\theta+r)T}\left[\frac{p_0 e^{(\theta+\hat{p})T} q(z^*)}{z^* + r - \hat{p}} - K\right]. \qquad (8)$$

The first and second order conditions for choice of T are

$$\frac{\partial PV}{\partial T} = (\hat{p} - r)PV + (\theta + \hat{p})Ke^{-(\theta+r)T} = 0, \qquad \frac{\partial^2 PV}{\partial T^2} = -(\theta + \hat{p})(\theta + r)Ke^{-(\theta+r)T} < 0. \qquad (9)$$

If $\theta + r > 0$, the second order condition requires that $\hat{p} + \theta > 0$, and we assume this to be satisfied. From the first order condition, an interior solution requires $r > \hat{p}$, as already assumed; if not it would pay to postpone entry indefinitely getting the dual benefit of later capital cost and higher present value of revenue flow.[6] These conditions imply that the higher is K the later is the field opened, since $\frac{dT}{dK} = -\frac{\partial^2 PV / \partial T \partial K}{\partial^2 PV / \partial T^2}$ and $\frac{\partial^2 PV}{\partial T \partial K} = e^{-(\theta+r)T}(\theta + \hat{p}) > 0$ so $\frac{dT}{dK} > 0$. The implication is that, with a continuum of fields differing only in capital cost per unit reserve, low K fields will be opened first.

2.2 The rate of depletion: discussion

The modeling of extraction costs is drawn from the technical literature on resource depletion, particularly in the oil sector. In this literature the benchmark assumption is that output from a field follows an exponential rate of decline (Adelman 1990, 1993); in our framework this would mean constant z.[7] Varying the rate of depletion has a cost primarily by its impact on total recoverable reserves. This variation is typically achieved by altering the rate of water or gas injection which pressurizes the well, and its effects are geology dependent; Nystad (1985, 1987) categorises fields as 'Hotelling', 'intermediate', and 'geosensitive', in increasing order according to loss of recoverable reserves from faster depletion. We capture this in relationships $q(z)$ and $\dot{x} = -zx$. Concavity means that an increase in output, q, involves a greater than proportionate decrease in remaining (recoverable) reserves, x.

Understanding these relationships is facilitated by working with a particular

[6] And, with prices endogenous, competitive equilibrium would not exist, see Holland (2003).

[7] A constant rate of depletion means exponential decline in remaining stock x, and hence in output $q(z)x$.

functional form that will be used in simulations later in the paper. We suppose that $q(z)$ takes the form

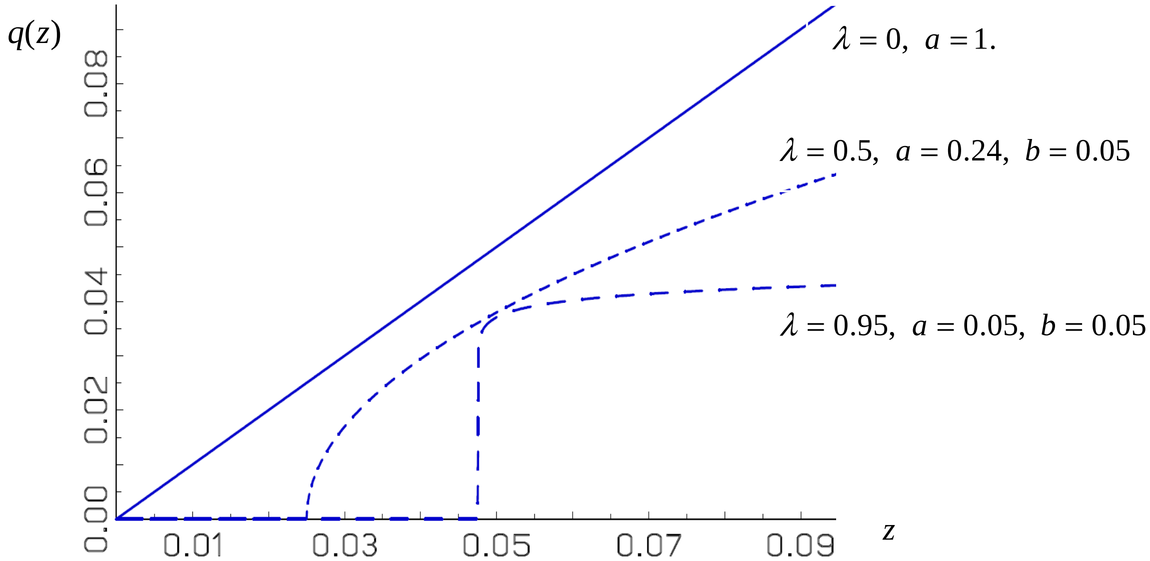
$$q(z) = a(z - b\lambda)^{1-\lambda}, \text{ with parameters } a > 0,\ b \geq 0, \text{ and } \lambda \leq 1. \tag{10}$$

With this specification the Euler condition (3) and long-run value of the rate of depletion are,

$$\dot{z} = \left(\frac{z - b\lambda}{\lambda}\right)\left[\frac{\dot{p}}{p} - r + \frac{\lambda(z-b)}{1-\lambda}\right], \qquad z^* = b + \frac{(1-\lambda)(r - \hat{p}_\infty)}{\lambda}. \tag{11}$$

Examples are given in figure 1. Parameter b gives the minimum rate of extraction below which marketable output is zero. The key parameter is λ which captures the extent to which faster depletion leads to loss of reserves, and hence also the extent to which optimal depletion is sensitive to price. The pure Hotelling case is $\lambda = 0$, (solid line in figure 1) in which the rate of depletion is infinitely sensitive to the gap between the rate of price increase and the rate of interest, so continuing extraction over an interval of time is possible only if these are equal. At the other extreme, as $\lambda \to 1$ with $b > 0$, the optimal rate of depletion is equal to b, and completely independent of the rate of price increase or rate of interest (the long-dashed line has $\lambda = 0.95$). This is consistent with the work of Adelman (1990), who argues that the rate of depletion from a particular reservoir is quite insensitive to price, and well approximated by a constant exponential rate of decline (at rate b in this specification). For cases with intermediate degrees of 'geosensitivity' the extraction path is more tilted towards the present the larger is $r - \hat{p}$.

Figure 1: **Examples of extraction costs, *q*(*z*)**



While this paper deals with supply coming from many fields, it is worth briefly connecting with the standard model of market equilibrium with a single field. If demand for the resource is iso-elastic, $Q_D = Dp^{-\eta}e^{gt}$ where *D* is a constant, η is the price elasticity of demand, and *g* the exogenous rate of growth of demand, then along the equilibrium path output, *q*(*z*)*x*, must changeat rate $g - \eta\hat{p}$. The rate of change of supply is simply $\dot{x}/x = -z^*$, so, using (11), the equilibrium rate of growth of price is

$$\hat{p} = \frac{\lambda(b+g)+(1-\lambda)r}{\lambda\eta + 1 - \lambda} \quad . \tag{12}$$

This is a simple generalization of the Hotelling model, in which the role of the interest rate depends on parameter λ. λ = 0 this gives the pure Hotelling case, and when λ > 0 the rate of price increase is greater the faster the growth of demand, *g*, the smaller the price elasticity, *η*, and the larger the base rate of depletion, *b*.

2.3 Field development: discussion

The field owner's objective, equation (1), was written in terms of a field of size one ($x(0) = 1$) developed at cost K. Setting the size of each field at unity is a normalization, and the key measure is size per unit capital cost. Uncertainty about recoverable reserves in a new field can be incorporated, providing owners are risk neutral and there are a large number of fields of each type, simply by letting unity be the expected field size. K, the capital cost of a unit of reserve, has empirical counterpart in the oil sector of 'finding and development' (F&D) costs per barrel, and data indicates that these are now the largest part of the sector's costs. F&D costs have risen sharply in recent years, with global average of $21 per barrel over the period 2006-09 (EIA 2011); they are of course field specific and in some cases go much higher (e.g. US F&D costs on offshore projects were $64 per barrel in 2006-08). These costs are several times greater than other production costs ('lifting' costs), running at global average of $11 per barrel (EIA 2011).[8] Furthermore, from an economic standpoint some elements of lifting cost should probably be classified as F&D; for example, some capital equipment may be highly specific to a field but is rented by the firm and counted as 'lifting' not F&D costs.

3. Resource supply

We now move from the decisions taken on a single field to analysis of total supply from all fields. Throughout this section we look at the supply response to an exogenous price path, endogenising price in section 4.

There is a continuum of fields indexed by their capital cost, K, and the measure of fields of type K is $S(K)$. The date of opening a field of type K is given in equation (7), and this gives the type of field opened at date T, i.e. relationship $K(T)$,

$$K(T) = \frac{e^{\theta T}}{(r+\theta)} \int_T^{\infty} (rp - \dot{p}) q(z) x(t,T) e^{-r(t-T)} dt . \qquad (13)$$

Notice that the rate of depletion, z, is a function of time, but is the same for all fields (equation (3)). All open fields are identical apart from the scale factor x giving the remaining stock, and $x(t,T)$ denotes the stock remaining at date t in a field opened at date T ,

[8] Reported lifting and F&D costs both include some tax element, EIA 2011.

$$x(t,T) = \exp\left[-\int_T^t z(\chi)d\chi\right] \ , \quad x(t,t) = 1. \qquad (14)$$

At any instant of time the total capacity that is 'opened' is $\dot{K}S(K)$ and total costs incurred are $\dot{K}S(K)K$. In order to model the evolution of total supply the relationship $S(K)$ needs to be specified. In some of what follows we assume that it is iso-elastic, with $S(K) = K^{\sigma-1}$. Parameter σ may be positive or negative, but we shall generally interpret results taking $\sigma < 0$, meaning that the remaining resource stock is finite, while $\sigma > 0$ means it is infinite.[9] This relationship can easily be given a micro-foundation. The size distribution of oil fields is well approximated by a power law (see the discussion in Laherrere 2000). If the elasticity of capital costs with respect to field size is less than unity and greater than the absolute value of the exponent in the power law, then the relationship $S(K) = K^{\sigma-1}$ with $\sigma < 0$ follows (see appendix for derivation).

We define open reserves at date t, $R(t)$, as the stock remaining in fields that have been opened by that date, i.e.

$$R(t) \equiv \int_{-\infty}^{t} \dot{K}(T)S\big(K(T)\big)x\big(t,T\big)dT \, . \qquad (15)$$

This is the integral over all previous dates of the set of field types that opened at each date, $\dot{K}(T)$, times the number of fields of type K, $S\big(K(T)\big)$, times stock remaining, $x\big(t,T\big)$. R moves according to differential equation

$$\dot{R} = \dot{K}S(K) - zR \, , \qquad (16)$$

derived by differentiating (15) with respect to t and using $x(t,t) = 1$ and $\dot{x} = -zx$ (noting that z is the same on all open fields). The interpretation is straightforward; open reserves change as new fields are opened at rate $\dot{K}S(K)$ and existing ones are depleted at rate z.

Total output at each date is the sum of current extraction from all open fields. Once again, the fact that all open fields are identical, except for the scalar difference in the size of stock remaining, makes this aggregation over open fields straightforward. Total supply, Q_S,

[9] K runs to plus infinity; the stock remaining is finite iff $\sigma < 0$, since $\int_{\overline{K}}^{\infty} S(K)dK = \int_{\overline{K}}^{\infty} K^{\sigma-1}dK = \left[K^{\sigma} / \sigma\right]_{\overline{K}}^{\infty}$

is simply the yield from depletion of the stock of open reserves,

$$Q_S = q(z)R, \tag{17}$$

and its rate of growth is

$$\hat{Q}_S = \frac{q'(z)\dot{z}}{q(z)} + \frac{\dot{K}S(K)}{R} - z. \tag{18}$$

This completes characterization of the supply side of the model, given a price path $p(t)$. To summarize, the supply side is characterized by three variables. The first is z, the rate of depletion, this inducing values of $x(t,T)$ in each field. The second is $K(T)$, the time path of field openings, and the third is $R(t)$, the stock left in open fields this, together with the rate of depletion, determining supply, Q_S. z and K are forward looking decision variables that can jump in response to a shock, although K can only jump upwards (capital costs in field openings are sunk). R is a state variable, depending on both new field openings and past history.

3.1 Long run supply

To analyse the model we suppose first that price grows at a constant exponential rate $\hat{p}$ for all future dates. The rate of depletion is then constant with value $z^* = \varsigma(r - \hat{p})$, and stocks decline exponentially, $x(t,T) = e^{-z^*(t-T)}$ (from (5) and (12)). The path of field openings through time is (from equation (13) with constant growth to evaluate the integral)

$$K(T) = \frac{p_0 e^{(\theta+\hat{p})T} q(z^*)(r-\hat{p})}{(r+\theta)} \int_T^{\infty} e^{(\hat{p}-z^*-r)(t-T)} dt = \frac{p_0 e^{(\theta+\hat{p})T} q(z^*)(r-\hat{p})}{(r+\theta)(z^*+r-\hat{p})} \tag{19}$$

This equation gives $K(0)$ proportional to initial price p_0, and the rate of growth of K equal to the constant, $\hat{K} = \hat{p} + \theta$.

While constant future growth of prices implies that $z = z^*$ is constant and K grows at a constant rate, the behaviour of R depends on the history of past field opening and the total capacity of fields of each type, $S(K)$, as given by equation (15). If $S(K)$ is iso-elastic, $S(K) = K^{\sigma-1}$, as discussed above, then the differential equation for open reserves, (16) becomes

$$\dot{R} = \dot{K}K^{\sigma-1} - z^* R, \tag{20}$$

which, with z^* and $\hat{K}$ constant, has explicit solution,

$$R = \frac{\hat{K}K^{\sigma}}{z^* + \sigma\hat{K}} + e^{-z^* t}\left[R_0 - \frac{\hat{K}{K_0}^{\sigma}}{z^* + \sigma\hat{K}}\right] \tag{21}$$

where K_0 and R_0 are the values of K and R at date zero. The effect of these initial values goes to zero with $e^{-z^* t}$, so R converges asymptotically to value given by $R / K^{\sigma} = \hat{K} / \left(z^* + \sigma\hat{K}\right)$. The long run rate of change open reserves is therefore $\hat{R} = \sigma\hat{K} = \sigma(\hat{p} + \theta)$ so, with $\sigma < 0$, open reserves decline exponentially. Furthermore, since $Q_S = q(z^*)R$, output is declining at the same rate. We summarize these properties as follows.

Proposition 1:

If price is growing at constant rate $\hat{p}$ at all future dates and $\hat{p} + \theta > 0, r > \hat{p}$, then:

i) z, the rate of depletion of each field is constant, and is faster the larger is $r - \hat{p}$ (equation 5).

ii) K, the sunk cost per unit reserve incurred on fields opened at each date, is proportional to p_0 and increasing at rate $\hat{K} = \hat{p} + \theta$ (equation 19).

If, additionally, the number of fields of type K is $S(K) = K^{\sigma-1}$, with $\sigma < 0$ (corresponding to a finite stock of the resource) then:

iii) The rate of growth of open reserves and of supply converge asymptotically to $\hat{Q}_S = \hat{R} = \sigma(\hat{p} + \theta) < 0$.

iv) On the long run (asymptotic) growth path values of R and Q are given by

$$R = \frac{K^{\sigma}(\hat{p} + \theta)}{z^* + \sigma(\hat{p} + \theta)}, \qquad Q_S = q(z^*)R. \tag{22}$$

Proposition 1 makes clear the different behaviour of the intensive and extensive margin. The intensive margin (the rate of depletion) depends on the rate of change of the price, not the price level, in the usual Hotelling manner. The extensive margin, the date at which new fields are opened, depends on the level of the price, as well as its rate of change. So too do open reserves and the level of output at each date. Comparative dynamics across asymptotic growth paths indicates that a higher initial price, p_0, is associated with more fields having been opened at each date (higher K, equation 19) and, if $\sigma < 0$, lower open reserves and supply of output at each date (equation 22). The intuition is that a higher level of prices means that more fields have been opened and (partially) depleted so current output is lower.

More interesting – and more insightful – than the asymptotic behaviour of supply is the response of supply to unanticipated permanent changes in p_0 and in $\hat{p}$ to which we now turn. To investigate this we suppose that the economy is initially on the long run path described above, this determining values of z, K, R and Q_S as given in proposition 1. How does supply respond to unanticipated change in p_0 and in $\hat{p}$ occurring at date $t = 0$? z and K are choice variables which can jump (the latter, upwards only). The motion of R, the stock of open reserves, is given by (20); it cannot jump independently, although a jump in K at date zero will cause a discrete change in the stock of open reserves.

3.2 Price level changes.

Suppose that an unanticipated upwards jump in p occurs at date 0 and lifts the price path by the same proportion at all future dates. Since this is a price level (not growth) effect it his has no effect on the rate of depletion (intensive margin, equations (3), (5)), in which price enters only in the form of future price growth; in a pure Hotelling model this change would have no effect whatsoever on supply. However, an increase in p_0 affects the extensive margin through the timing of field openings, causing an equi-proportionate increase in K as given by equation (19). This is illustrated in the top left panel of figure 2a below, for a jump of 20% in the price level (parameter values are in the appendix). The horizontal axis is time and the vertical is $\ln(K)$. The solid line is the path without the price change, and the dashed is with the change. There is an upwards shift but no change in the subsequent rate of growth of K.

An upwards jump in K means that a discrete number of new fields are opened as the shock occurs but, if $\sigma < 0$, fewer fields are opened at every date thereafter. (The number of fields opened is $\dot{K}S(K) = \hat{K}K^{\sigma}$, and while $\hat{K}$ is constant K^{σ} has fallen). This jump and subsequently lower rate of field opening works through into the stock of open reserves and hence output through equations (20) and (21). R jumps and then converges asymptotically to $R / K^{\sigma} = \hat{K} / \left(z^* + \sigma\hat{K}\right)$; the right hand side of this expression is unchanged, but since K^{σ} is lower, so too is R, giving the path illustrated on the top right panel of figure 2a.

The corresponding path of output is in the lower left panel, proportional to the path of open reserves since the rate of depletion is constant. A permanent proportional price increase therefore elicits a positive short to medium run supply response which turns negative as fewer new fields are being opened. The elasticity of asymptotic supply with respect to the price level is σ, as can be seen by noting that K is proportional to the price level (equation (19)), while asymptotic R and Q_S are proportional to K^{σ} (equation (22)). While the short run price elasticity of supply is positive, the long run supply elasticity is therefore negative (if $\sigma < 0$). The short and long run supply responses can be combined by looking at the change in supply cumulated from the date of the shock; this is illustrated in the final panel of figure 2a, expressed as a proportion of cumulated output on the initial path. An increase in price causes a permanent increase in cumulative output, although the proportionate increase goes asymptotically to zero.

A downwards price jump (-20% all dates) is illustrated on figure 2b, and is not completely symmetric to a price increase because there is no possibility of field closure. The shift in K (top left panel) is therefore a horizontal shift, and there is a period in which no new fields are opened. During this period open reserves fall, as does output. Once field openings resume output and open reserves recover, coming to lie above what they otherwise would have been, mirroring the long run effects of a price increase. The price decrease reduces cumulative output at all dates. We summarize these effects in proposition 2.

Proposition 2:

A permanent proportionate change in the price ($\hat{p}$ constant and unchanged) has no effect on the rate of depletion or the long run rate of growth of supply. A price increase brings forward the opening of fields. Supply increases before eventually falling below what it otherwise would have been (with long run price elasticity of supply of σ). Cumulative supply is increased at all dates. A price decrease has reverse effects, leading to a reduction in cumulative supply at all dates.

Figure 2a: Price increase

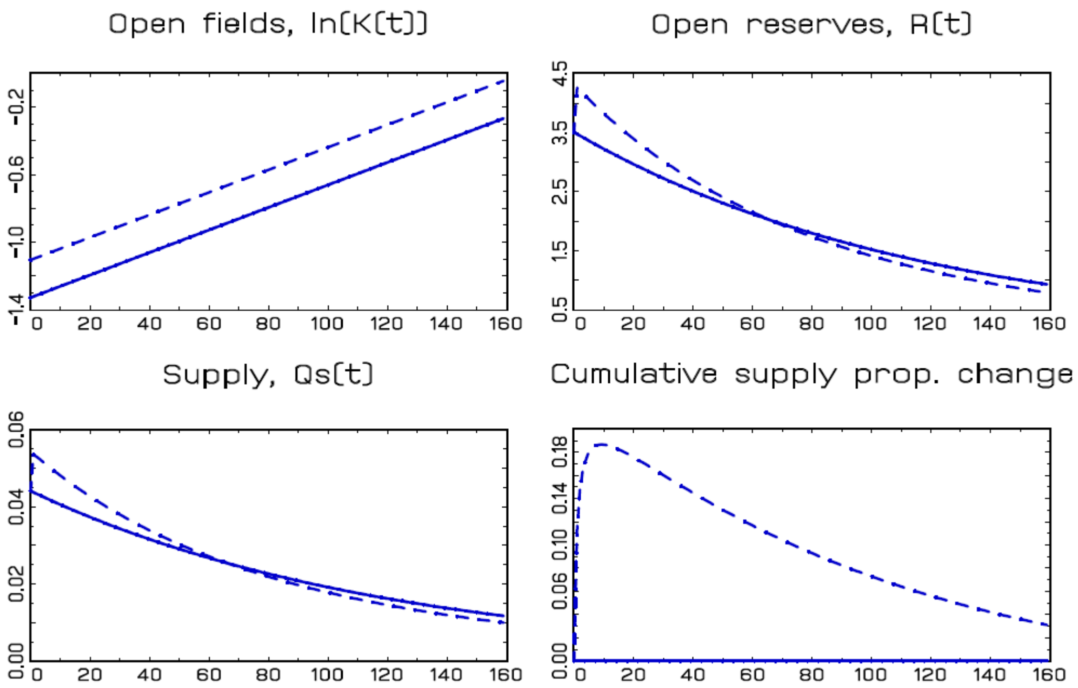
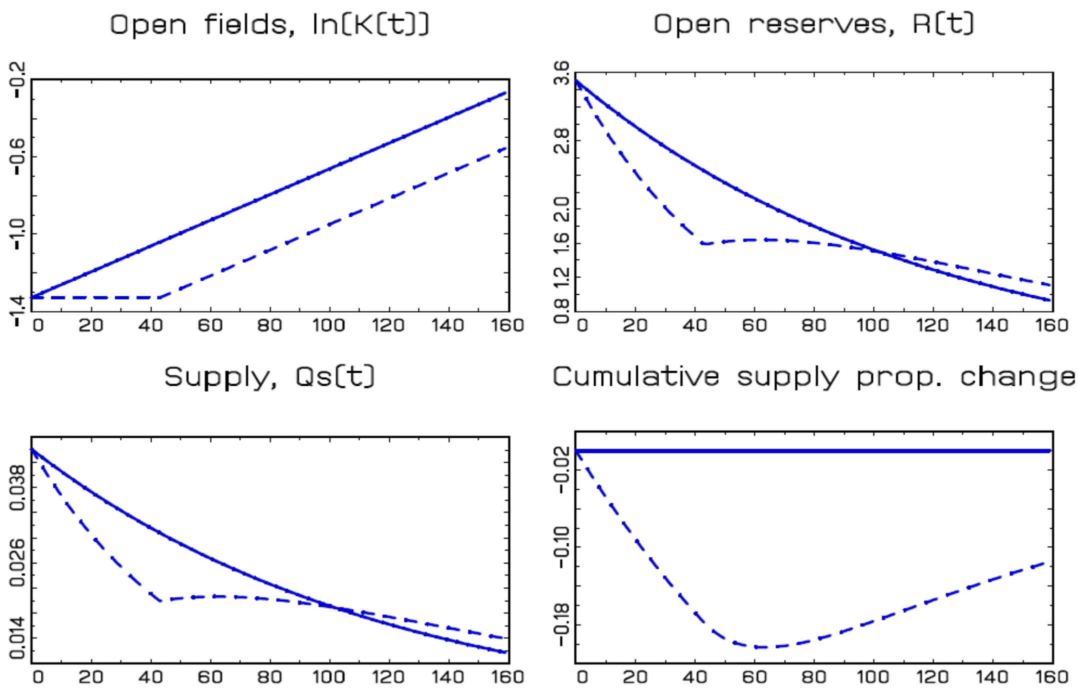


Figure 2b: Price decrease



Solid line: original path. Dashed line: new path.

3.3 Price growth:

We now turn from a change in the level of price to a change in its rate of growth. At the intensive margin, a permanent increase in price growth causes an immediate and permanent fall in the rate of depletion, z (equation (5)). Intensive margin effects on other variables are illustrated by the short-dashes in figure 3a, constructed with the decrease in z but holding the time path of K unchanged; solid lines are the original paths. Slower depletion means less supply from a given quantity of open reserves but more open reserves at all future dates, so a short run reduction in supply is followed by higher supply in future, the Hotelling-like response that would be expected. Cumulative output is reduced for a period but then becomes larger than it otherwise would have been.[10]

The extensive margin now operates in a similar manner to the intensive as higher future prices creates an incentive to postpone field opening. Field opening is reduced (or ceases altogether) for a period, and then resumes at a faster rate, since $\hat{K} = \hat{p} + \theta$. The tension between these forces can be seen by using equation (5), $z^* + r - \hat{p} = q(z^*)/q'(z^*)$, in equation (19) to give

$$K(T) = \frac{p_0 e^{(\theta+\hat{p})T} q(z^*)(r-\hat{p})}{(r+\theta)(z^*+r-\hat{p})} = \frac{p_0 e^{(\theta+\hat{p})T}\left[q(z^*) - z^* q'(z^*)\right]}{(r+\theta)} \qquad (23)$$

and differentiating with respect to T giving

$$\frac{dK(T)}{d\hat{p}} = \frac{p_0 e^{(\theta+\hat{p})T}}{(r+\theta)}\left[\left\{q(z^*) - z^* q'(z^*)\right\}T - z^* q''(z^*).\frac{dz^*}{d\hat{p}}\right]. \qquad (24)$$

This expression is negative for small T (since $q'' < 0$ and $dz^*/d\hat{p} < 0$) and positive for large T, when the first term in the square brackets comes to dominate. There is therefore a period in which field openings are reduced (or cease altogether), following which more fields are opened at each date and the new path overtakes the old.

[10] This long run increase is because slower depletion uses up less of the resource in extraction costs.

This is illustrated in the top left panel of figure 3a. Faster price growth increases the value of opening fields in the future, causing opening to pause for a period but then to continue more rapidly giving the crossing identified in equation (24). The long-dashes in the top right and bottom left hand panels give paths of R and Q_S when both intensive and extensive margin effects operate. The pause in field opening causes a decline in open reserves and larger initial fall in output. But following the pause, faster field opening eventually leads to higher open reserves, higher output, and a positive effect on cumulative supply. The effect of the extensive margin change is therefore to amplify intensive margin changes, as seen most clearly for the bottom right hand panel, giving the cumulative supply response.

Figure 3b gives the effects of a permanent reduction in the rate of growth of price. This increases the rate of depletion and brings forward field opening, giving the K crossing that we noted above (top left hand panel). The top right and bottom left panels give the paths of R and Q_S, once again giving initial path (solid), intensive margin only (K constant, short dash) and full adjustment (long dash). Faster depletion alone (short dash) gives a fall in open reserves at all dates, associated with higher output in the short run and lower output in the long run. Combining this with the change in field openings (long dash), the effect is magnified with a larger output increase in the short run, but a sharper fall in the long run. Cumulative output is raised for a short period, but then permanently reduced as lower prices have a major impact in reducing field openings (bottom left panel). We summarize results in proposition 3:

Proposition 3:

A permanent increase in the rate of growth of price tilts production to the future. Depletion of existing fields is slowed down, and opening of new fields postponed. Supply is reduced for a period, after which it overtakes its previous level. The converse holds for a permanent decrease in the rate of price growth.

Figure 3a: Faster price growth

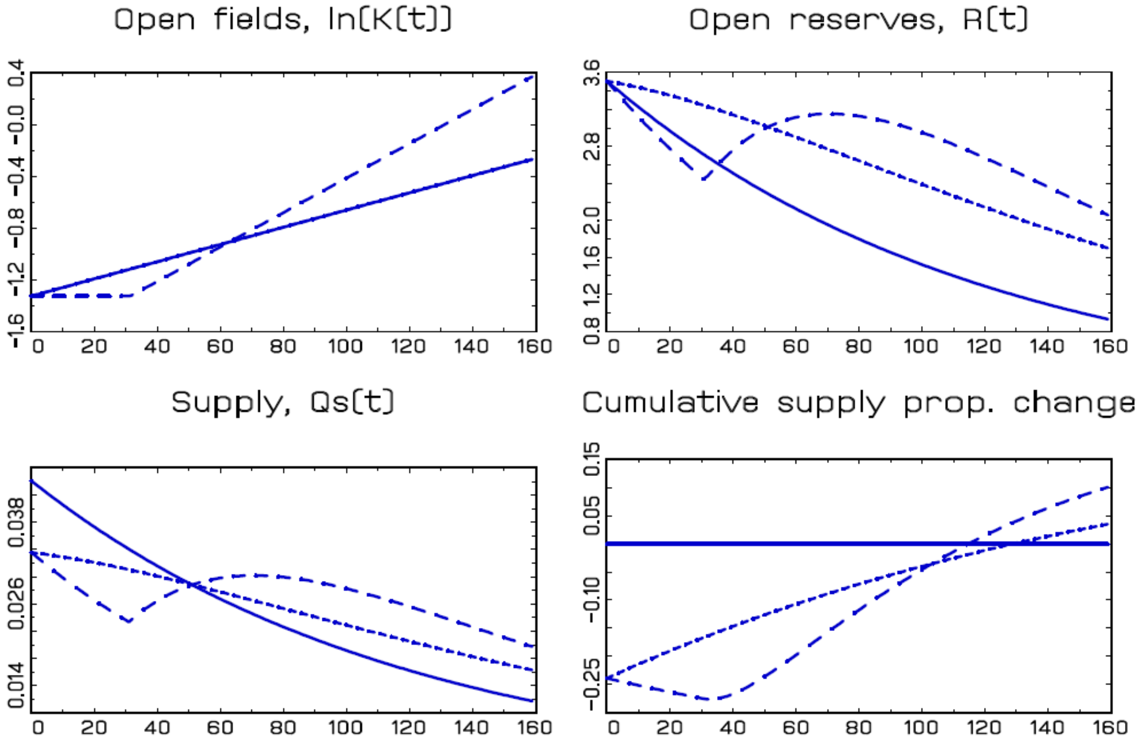
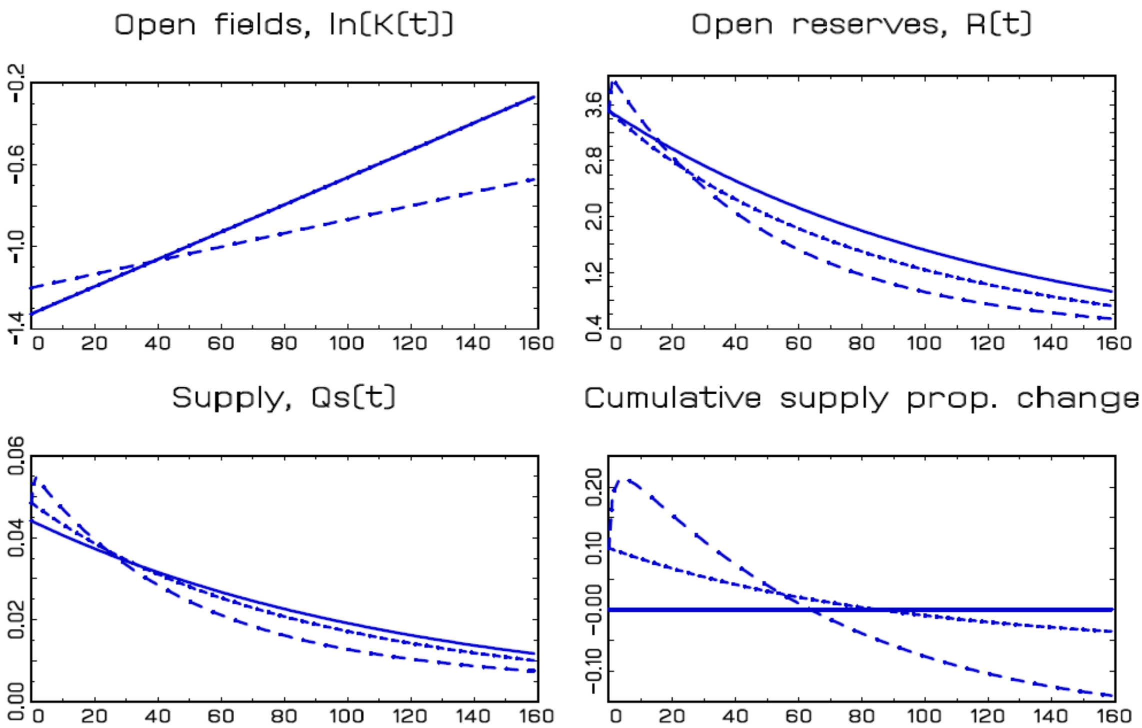


Figure 3b. Slower price growth



Short dash: Intensive margin, *K* constant, *z* adjusts.
Long dash: Intensive and extensive margin, *K* and *z* adjust.

4. Market equilibrium:

We now go from looking at the response of supply to price, to the full market equilibrium with price endogenous. The demand curve is assumed to have constant price elasticity $\eta \geq 0$, exogenous rate of growth g, and level parameter D,

$$Q_D = Dp^{-\eta} e^{gt} \text{ , so } \quad \hat{Q}_D = g - \eta \hat{p} . \tag{25}$$

The equilibrium price path comes from equating Q_D to Q_S.

4.1 Constant growth.

Section 3 established that if price is growing at a constant rate the long run rate of growth of supply is constant at $\hat{Q}_S = \sigma(\hat{p} + \theta)$ (proposition 1). Equating this with the rate of growth of demand, the equilibrium rate of growth of price is

$$\hat{p} = \frac{g - \sigma\theta}{\eta + \sigma} . \tag{26}$$

Recalling that σ is the (asymptotic) price elasticity of supply, this expression links a demand shift (demand growth g) to price change via elasticities of supply and demand in the usual way. In the present context, a number of points are noteworthy.

First, in contrast to the standard Hotelling approach, the equilibrium rate of price increase is independent of the rate of interest. The model gives a Hotelling-like result (equation (12)) if the extensive margin is completely fixed (no new fields open, and supply response comes only from altering depletion of existing fields). However, once the extensive margin is included in the supply response the long run rate of growth of price depends on demand and supply elasticities in a familiar way, and not at all on the interest rate.

Second, the necessary condition for our characterization of the date of field opening to be a profit maximum is that $\hat{p} + \theta > 0$ (section 2.1). With $\hat{p}$ given endogenously by (26), this condition could fail for two distinct reasons. One is that g is substantially negative (with denominator of (26) positive) in which case demand is falling too fast to support the positive price growth necessary to induce delay in field opening.[11] The other is that $\eta + \sigma < 0$ (with numerator of (26) positive). This could arise if σ < 0 in which case, as already noted, the *long run* price elasticity of supply is negative. We impose the condition that $\eta + \sigma > 0$, failing which the second order condition for field opening is not satisfied.

Equilibrium values of other variables in the system follow directly from the price growth given by (26) together with proposition 1. The long run rates of growth of open fields, open reserves, and output are

$$\hat{K} = \frac{g + \eta\theta}{\eta + \sigma}, \qquad \hat{Q} = \hat{R} = \frac{\sigma(g + \eta\theta)}{\eta + \sigma}. \tag{27}$$

The initial price equates supply and demand so, using (19) and (22) in (25), satisfies

$$D = p_0^{\eta+\sigma} \frac{(\hat{p} + \theta)q(z^*)}{z^* + \sigma(\hat{p} + \theta)} \left[\frac{q(z^*)(r - \hat{p})}{(r + \theta)(z^* + r - \hat{p})} \right]^{\sigma}. \tag{28}$$

The following proposition summarizes these properties of the long run equilibrium.

Proposition 4:

On the long run (asymptotic) path the rate of growth of price is independent of the rate of interest, and given by $\hat{p} = (g - \sigma\theta)/(\eta + \sigma)$. The elasticity of the equilibrium price with respect to the level of demand is $1/(\eta + \sigma)$. On this path the rate of depletion is constant, and output is declining at rate $\sigma(g + \eta\theta)/(\eta + \sigma)$.

This describes the long run equilibrium path but, as before, it is more interesting to investigate responses to exogenous changes. We look first at shocks to the level of demand, and then to its rate of growth.

[11] A high value of θ, the rate of technical change on *K*, supports postponement of field opening.

4.2 Proportional change in demand:

Consider a change in the level of demand at all dates, i.e. a shift in *D.* We know from the preceding sub-section that there is no effect on long rate rates of growth of *p*, Q_S, *R*, or on *z*, although there is a change in the price level. If there were no extensive margin effects (the path of *K* held constant) then there would be no short-run effects either; all quantities would be unaffected and the demand change would be shifted wholly to the price level. However, the extensive margin is sensitive to the level of prices, as well as their rate of change; a change in the price level changes the timing of field opening, this changing supply and inducing a transitional dynamic response.

Figure 4 illustrates the effect of a permanent decrease in demand (*D* falling to 75% of its previous value), with all variables now expressed relative to the initial constant growth path. The top right hand panel gives the price path. The short dashed line gives the price path in the absence of extensive margin effects: a one-off drop to 0.866 = $0.75^{1/\eta}$ of its previous value. Including extensive margin effects, the long dashed line indicates a larger ultimate price drop, asymptoting to 0.68= $0.75^{1/(\eta+\sigma)}$ of its previous value. The dynamics associated with this take the following form. There is an immediate cessation of field opening (top left), so a period in which supply is less than it otherwise would have been (below unity, bottom left). This mitigates the price fall (top right). Postponement of field openings means that, beyond some date, supply becomes greater than it otherwise would have been and price correspondingly lower. However, combining effects, cumulative supply is lower at all dates.

The main message concerns the equilibrium path of supply, particularly cumulative supply. Without the extensive margin, a demand change would have no effect whatsoever on output. With the extensive margin operating, a reduction in demand cuts supply in the short run, raises it in the long run, and has a negative impact on cumulative quantity supplied at all dates.

4.3 Change in rate of growth of demand:

A permanent change in demand growth affects both the long run growth of variables and transitional dynamics. Long run growth rates can be found explicitly (appendix table 1) and the full dynamic story is illustrated in figure 5, for a reduction in demand growth.

Inter-temporal substitution creates an incentive to shift both depletion and field opening from the future to the present, but this is combined with a price level effect that deters field opening. If adjustment were to take place only at the intensive margin, then the path of supply would be unambiguously tilted towards the present (short dashes). Price growth is slower, and the increase in present supply leads to an immediate fall in price. The extensive margin of field opening responds both to this fall in the price level, and to the slow future growth of prices. There is unambiguously slower growth of K, but the impact effect is ambiguous: lower price induces postponement (as in figure 2b) and lower price growth induces opening (as in figure 3b). These effects net out to close to zero in our example (figure 5, top left panel).

Combining these elements gives the U-shaped path of output (bottom left). In the short run, the faster extraction of open fields dominates, this giving the supply increase. In the medium run supply is lower because open fields have been depleted faster and because fewer new fields have been opened. In the long run supply turns up, because the high $S(K)$ field types, opening of which was postponed, are coming on stream. Looking at cumulative supply, we see that adding the extensive margin effect mitigates the shift in supply towards the present; cumulative supply is raised for a shorter period, beyond which it is associated with larger reductions in cumulative output and cumulative stock of resource extracted.

Figure 4: Decrease in demand: relative to constant growth path

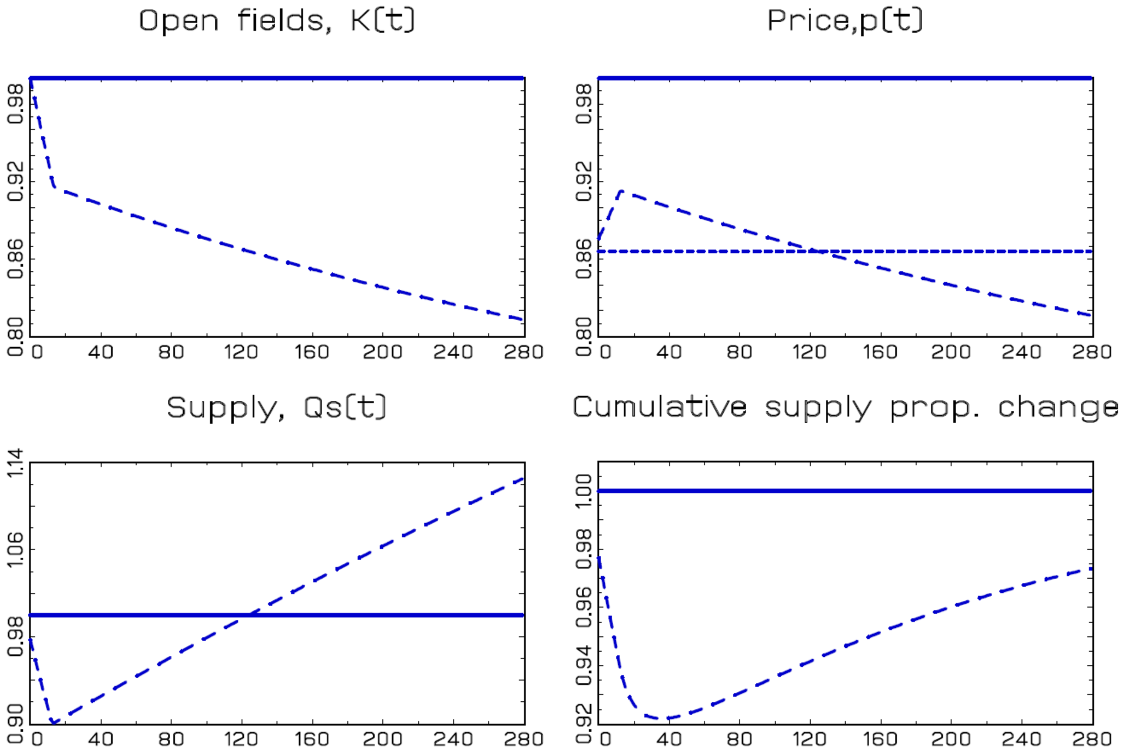
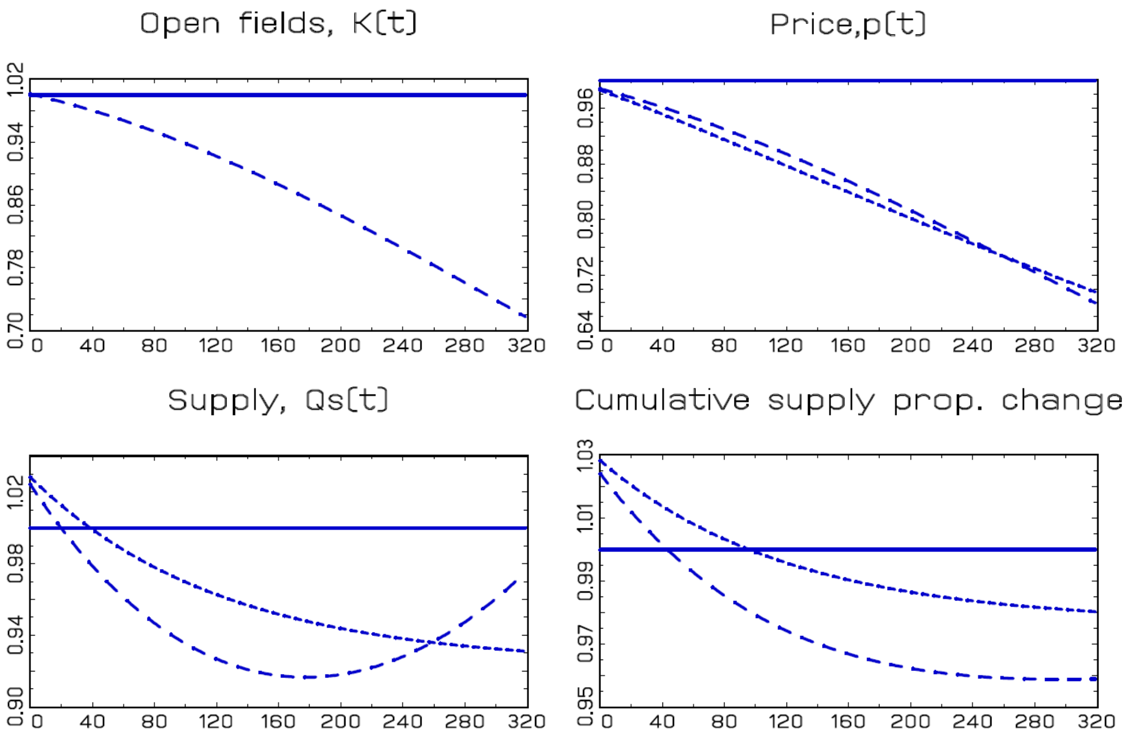


Figure 5: Slower growth of demand: relative to constant growth path



Short dash: Intensive margin, *K* constant, *z* adjusts:
Long dash: Intensive and extensive margin, *K* and *z* adjust.

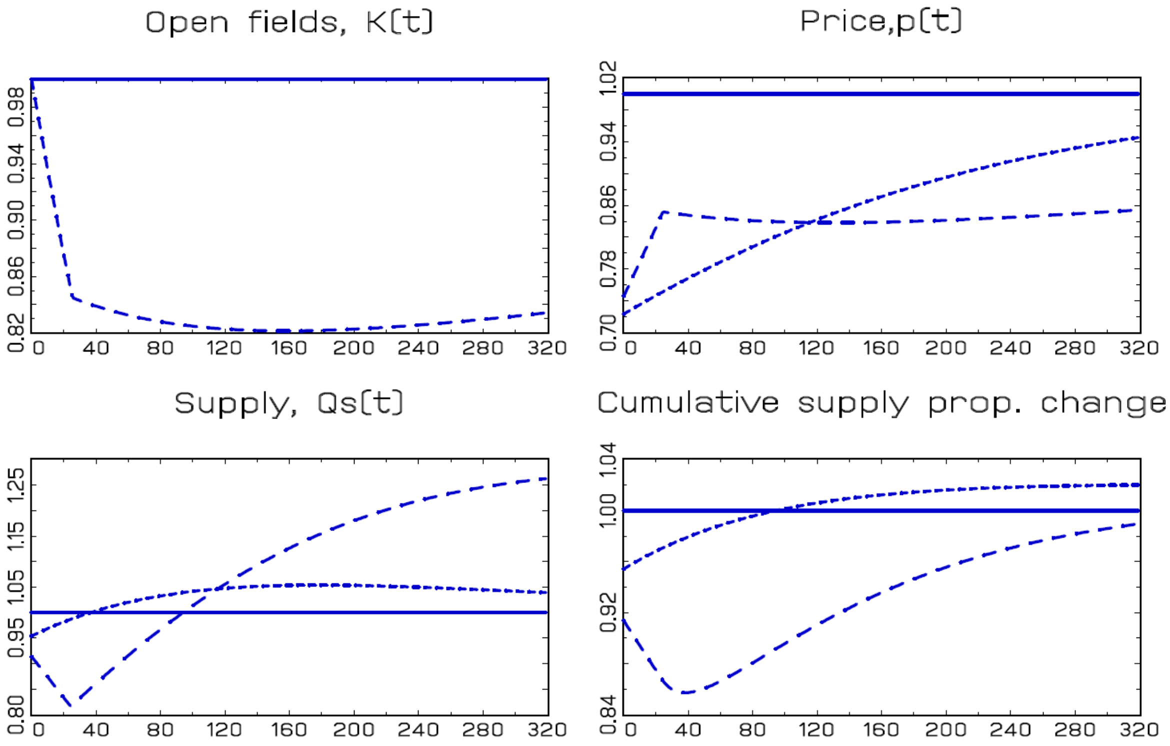
5. Carbon taxation with endogenous field opening:

The equilibrium impact of climate change measures such as a carbon tax depend on both the demand and supply responses of fossil fuel. Much of the climate change literature has concentrated on demand reduction, while Sinn (2008) has used a simple model of resource depletion to argue that supply conditions may create a 'green paradox'; carbon taxes or other measures to reduce demand might be ineffective or, if they are expected to become more severe in future, have perverse effects, bringing forwards extraction from the far future to the nearer future. How does this work when both extensive and intensive margin effects are present?

Policy measures that lead to permanent proportionate demand reduction cause an immediate and continuing reduction in the cumulative quantity of the resource supplied (section 4.2), as the lower price delays field opening and postpones production. This is in contrast to the case when the extensive margin effect is absent, in which policy has no effect on quantities produced. Policy measures that reduce the rate of growth of demand (section 4.3) bring forward extraction from existing fields, this raising current output. This is offset by the price level effect which postpones field opening. Output therefore falls faster, and the cumulative output increase is smaller, and positive for a shorter period of time, when the extensive margin effect is present.

Demand shifts could be implemented by a tax on resource use, such as an emissions tax. For a proportionate decrease in demand this would require a constant ad valorem tax (demand iso-elastic) while, if the rate of growth of demand is to be reduced, the tax rate would need to increase exponentially. Figure 6 looks at an alternative case in which an emissions tax is imposed at date 0 and then held constant in perpetuity, (therefore declining relative to the resource price). As before, short-dashed lines give the effect when only the intensive margin operates. The producer price falls on impact, but then converges back to its previous level (as the relative value of the tax diminishes). This reduces the rate of extraction, giving the short run fall in supply followed by long run increase. However, when the extensive margin operates (long dashes) the producer price fall leads to a period in which no new fields are opened, and hence a much larger fall in supply. As usual, this is a postponement of field opening, so supply rises in future. Once again, the key point is that the price level effect of demand reduction policy postpones field openings and thereby has a negative impact on supply in the near future.

Figure 6: Constant specific resource tax: relative to constant growth path



Short dash: Intensive margin, *K* constant, *z* adjusts:
Long dash: Intensive and extensive margin, *K* and *z* adjust.

6. Concluding comments

The paper has developed a model of the supply of a non-renewable resource in which the empirically compelling fact that large sunk costs are associated with the development of new mines or fields is put centre stage. The model encompasses both depletion of existing fields and the development of new fields, thereby providing a modest step towards greater reality. New insights come from the approach. The most fundamental is that while the rate of interest may matter for depletion rates and short run transitional dynamics, it has no impact on the long run behaviour of resource prices; long run price growth depends on demand and underlying supply considerations (the geology of available fields). The approach also provides perspective on some 'paradoxes' that have gained recent attention. For example, emissions taxes may tend to bring forward depletion of existing resources, but they also discourage the development of new fields, so are likely to have to the desired effect of pushing production into the future, reducing cumulative output and any associated stock of emissions.

The approach suggests a number of extensions and applications. For example, we have assumed throughout that future price paths are known with certainty and that owners of fields will postpone opening until the date at which the present value of the field is maximized. Allowing price uncertainty and placing the field opening decision in a stochastic context is clearly important. Lags in opening fields will introduce a more complex dynamic response to shocks. The development of substitutes provides a further supply margin. On the applied side, the model provides a relatively tractable framework for thinking about a number of practical and policy issues. The paper discusses some of the issues to do with fossil fuel supply and climate change. The model also provides a framework for analysis of rent taxes (royalties, production sharing arrangements and corporate income taxes) which have to balance the need to capture rent with incentives for field development

Appendix:

In the text field size is normalized at unity, fields vary in capital cost K, with the number of fields of type K denoted $S(K)$. This can be derived from the following alternative set up. Suppose that fields are ordered by size, s, with $m(s)$ fields of size s, $m' < 0$. $m(s)$ is follows a power law, so $m(s) = s^{\alpha}$, $\alpha < 0$. The total capacity of fields of size s is $sm(s) = s^{1+\alpha}$. The capital cost of a field of size s is $k(s)$, and we suppose $k(s) = s^{\kappa}$, $0 < \kappa < 1$, so costs are increasing and strictly concave in field size; the capital cost of one unit of capacity on a field of size s is $s^{\kappa-1}$, i.e. $K = s^{\kappa-1}$. Since the capacity associated with fields of size s is $S = s^{1+\alpha}$, we have, eliminating s, $S(K) = K^{(1+\alpha)/(\kappa-1)}$. Thus, $\sigma - 1 = (1+\alpha)/(\kappa-1)$ and hence $\sigma = (\kappa+\alpha)/(\kappa-1)$, which is negative if $\kappa < 1$ and $\kappa+\alpha > 0$.

Appendix:

Parameter values, figures 2, 3, and 4:

$r = 0.02$; $g = 0.005$; $\eta = 2$; $\sigma = -1.25$; $a = 0.1$; $b = 0.005$; $\lambda= 0.5$.

Long run equilibrium $\hat{p} = 0.067$ (exogenous in figures 2 and 3).

Figure 2: initial price p_0 raised by 20%, reduced by 20%.

Figure 3: $\hat{p}$ doubled to 0.01, halved to 0.0025

Figure 4: demand, D, cut by 25%

Figure 5: growth rate g halved to 0.0025

Figure 6: Constant specific tax at 30% of initial price (eg carbon price \$50, oil price \$70, 0.43 tonnes of CO_2 per barrel of oil).

Table 1:Asymptotic growth rates for a reduction in the rate of growth of demand $g_N < g_I$

	Initial, g_I		New, g_N Intensive margin only		New, g_N Intensive & extensive margin		
$\hat{K}$	$\frac{g_I+\eta\theta}{(\eta+\sigma)}$	=	$\frac{g_I+\eta\theta}{(\eta+\sigma)}$	>	$\frac{g_N+\eta\theta}{(\eta+\sigma)}$	>	0
$\hat{Q}_S$	$\frac{\sigma(g_I+\eta\theta)}{(\eta+\sigma)}$	=	$\frac{\sigma(g_I+\eta\theta)}{(\eta+\sigma)}$	<	$\frac{\sigma(g_N+\eta\theta)}{(\eta+\sigma)}$	<	0
$\hat{Q}_D$	$g_I-\eta\hat{p}$	=	$g_N-\eta\hat{p}$	<	$g_N-\eta\hat{p}$	<	0
$\hat{p}$	$\frac{g_I-\sigma\theta}{\eta+\sigma}$	>	$\frac{g_N-\sigma\theta+\sigma(g_N-g_I)/\eta}{\eta+\sigma}$	>	$\frac{g_N-\sigma\theta}{\eta+\sigma}$	>	0

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