CESifo Working Papers
www.cesifo.org/wp

Reciprocity, World Prices and Welfare

Pascalis Raimondos-Møller
Alan D. Woodland

CESIFO WORKING PAPER NO. 3607
CATEGORY 8: TRADE POLICY
OCTOBER 2011

CESifo *Working Paper No. 3607*

Reciprocity, World Prices and Welfare

Abstract

We examine in detail the circumstances under which reciprocity, as defined in Bagwell and Staiger (1999), leads to fixed world prices. We show that a change of tariffs satisfying reciprocity does not necessarily imply constant world prices in a world of many goods and countries. While it is possible to find tariff reforms that are consistent with both reciprocity and constant world prices, these reforms do not follow from the reciprocity condition, but rather from the requirement of unchanged world prices. We propose an alternative reciprocity rule that is guaranteed to raise the welfare of all countries, independently of whether world prices change and independently of the relative numbers of goods and countries.

JEL-Code: F020, F130, F150.

Keywords: GATT, reciprocity, fixed world prices.

Pascalis Raimondos-Møller
Department of Economics
Copenhagen Business School
Porcelaenshaven 16A
Frederiskberg 2000C
Denmark
prm.eco@cbs.dk

Alan D. Woodland
School of Economics
University of New South Wales
Sydney NSW 2052
Australia
a.woodland@unsw.edu.au

September 27, 2011
We would like to acknowledge the comments received from participants at the 2008 Global Economy CESifo conference in Munich, the 2008 ETSG meeting in Warsaw, the 2011 Preferential Trading Agreements workshop in Copenhagen, as well as from seminar participants at Hitotsubashi University and the University of Nottingham. The research was supported by grants from the Australian Research Council and the Danish Social Science Research Council.

1. Introduction

In an important paper in the *American Economic Review*, Kyle Bagwell and Robert Staiger (1999) propose a general equilibrium theory of GATT that rationalizes the use of reciprocity and non-discrimination as the two main pillars of GATT negotiations. Bagwell and Staiger show that multilateral tariff reforms that are based on the rules of reciprocity and non-discrimination remove the well-known terms-of-trade externalities and allow countries to enjoy the positive efficiency gains of their tariff reforms. The main mechanism that leads to this advantageous situation is based on their conclusion that multilateral tariff reforms that adhere to the rule of reciprocity leave *world prices unchanged*. Clearly, an understanding of this argument is essential in appreciating the Bagwell and Staiger theory of GATT negotiations.

Bagwell and Staiger define a tariff reform satisfying reciprocity as one that keeps the value of each country's trade unchanged, where the evaluation is at the initial world prices. Within the context of a two-country, two-product model in which the initial equilibrium is a Nash equilibrium, they demonstrate that a reciprocity-based tariff reform leads to a new equilibrium at which world prices are unchanged. Moreover, they then use the unchanged world price implication to create tariff reforms that yield Pareto improvements in welfare, so both countries gain from the tariff changes. This powerful consequence of reciprocity, viz. that world prices remain unchanged, makes the analysis of the negotiation game between GATT countries more transparent.

Holding world prices fixed has previously appeared as an important insight in the literature of preferential and non-preferential trading agreements. See, for example, Ohyama (1972) and Kemp and Wan (1976) for early contributions, and Kowalczyk and Sjöström (2000), Panagariya and Krishna (2002), Ohyama (2002), Raimondos-Møller and Woodland (2006) and Grinols and Silva (2007) for more recent uses of this insight. However, none of these papers characterize the tariff reforms that preserve the world prices at their initial equilibrium levels. The contribution of Bagwell and Staiger (1999) is quite different as it is argued to provide a simple, and policy relevant, characterization of how to keep world prices constant in a multilateral trading model, viz. by imposing reciprocity.

The present paper contributes to the literature in several different ways. First, the paper addresses the issue of dimensionality and, in particular, examines the conditions needed for this powerful consequence of reciprocity to be extended to a multi-country, multi-good context. We show that, in general, reciprocity by itself *does not imply* fixed world prices. We rigourously demonstrate that Bagwell and Staiger's reciprocity rule for tariff reforms is guaranteed to yield an equilibrium with unchanged world prices only when the world trade matrix satisfies a rank condition and the number of countries is greater than or equal to the number of traded commodities. We show that when the number of traded goods is greater than the number of countries (the empirically relevant case) then there is always a reciprocity-consistent equilibrium with world prices different from those in the initial equilibrium. In undertaking this analysis and providing these results, we provide a clarification of the conditions under which a reciprocity-consistent reform will and will not yield fixed world prices.[1]

Secondly, we also show that even if there exist some tariff reforms that are *consistent* with both reciprocity and fixed world prices, these reforms can only be derived by imposing the extra constraint of fixed world prices. This, in essence, takes us back to the Kemp-Wan-Ohyama insight and its practical difficulties for guiding tariff reforms. We put emphasis on these issues because there seems to be a generally acceptance in the literature that Bagwell and Staiger (1999) show that trade liberalizing reciprocity fixes the world prices. Examples of statements to this effect may be found, amongst others, in Matoo and Olarreaga (2004, p.1), Epifani and Vitaloni (2006, p.428), Anderson and Neary (2007, p.187), Ossa (2011, p.123) and Márzová (2011, p.2).

Our final contribution is to provide a new reciprocity rule[2]. This rule involves countries increasing the volume of trade rather than the value of trade. Importantly, we are able to prove that any differential tariff reform that conforms to this new reciprocity rule is guaranteed to yield a strict Pareto improvement in welfare, irrespective of whether world prices change and independently of the number of countries relative to the numbers of traded goods. Specifically, our reciprocity rule requires all countries to increase their volume of trade by the same proportion. Our reciprocity rule always satisfies the Bagwell and Staiger reciprocity rule, but the reverse is not necessarily true in a multi-good, multi-country setting. Finally, we show that application of our reciprocity rule in the neighborhood of the Nash equilibrium always leads to strict Pareto

[1] Bagwell and Staiger (1999, p. 225, footnote 16) consider an extension of their 2×2 analysis to many goods and many countries that recognizes the issue. See also Bagwell and Staiger (2002, appendix to chapter 5), where a more detailed discussion of the many goods case is provided. Our analysis is meant to complement theirs by providing more precise details and explanation.

[2] Blanchard (2010) and Mrázová (2011) also consider new definitions of reciprocity. In Blanchard (2010), this is needed to take into account the capital mobility externalities that exist in her model. In Mrázová (2011), the new definition of reciprocity takes into account the profit shifting externalities that are present in her model. Note, however, the difference between their new definitions and ours: while theirs are made in order to account for new externalities, ours is not. We have exactly the same setup, and thus the same externalities, as the original Bagwell and Staiger (1999) paper.

improvements in a multi-good, multi-country setting.

2. Model of International Trade

To provide a rigorous analysis of the issue, we consider a perfectly competitive general equilibrium model of the world consisting of K nations trading in L internationally tradeable commodities. Following Turunen-Red and Woodland (1991), the model may be expressed as

$$\sum_{k\in K} E_p^k(p^k, u^k) = 0, \tag{1}$$
$$p^\intercal E_p^k(p^k, u^k) = 0,\ k \in K, \tag{2}$$

in terms of the world price vector p ($p^\intercal$ denotes the transpose of a vector), the domestic price vectors p^k for each country $k \in K$ and the utility levels u^k for each country $k \in K$.[3] In this specification, $E^k(p^k, u^k) \equiv e^k(p^k, u^k) - g^k(p^k)$ is the net or trade expenditure function, being the difference between the gross domestic product function g^k and the consumer expenditure function e^k. Also, $E_p^k(p^k, u^k) \equiv m^k$ denotes the gradient of the trade expenditure function with respect to prices and represents the country-specific vector of compensated net import functions.[4]

Equations (1)-(2) consist of the market equilibrium conditions for internationally traded goods and the budget constraints for each country. The market equilibrium conditions express the requirement that the net imports of countries, m^k, sum to the zero vector, meaning that world markets clear. The national budget constraints state that the value (at world prices) of net imports (the balance of trade) must be zero.

It is implicit in this formulation of the model that there is just one consumer in each country, who receives a transfer of the tariff revenue from the government and has utility u^k.[5] Moreover, with the focus being on reciprocity, our model encompasses the most favoured nation (MFN) principle, i.e., a good is facing the same tariff independent of the country of origin. The model is expressed in terms of domestic and world prices. These are connected by tariffs, which may be expressed in specific terms, whence we may write $p^k = p^k(p, t^k) \equiv p + t^k$.[6]

[3] The notation K is used to denote the set of countries as well as the number of countries.

[4] Woodland (1982) spells out the properties of the revenue and expenditure functions.

[5] It is relatively straightforward to extend the model, at the cost of added notational complexity, to handle multi-household economies. In the case of multiple households, Pareto improvements may be ensured by assuming the existence of lump sum income transfers between households and the national governments. Alternatively, under appropriate assumptions, commodity taxes may be used to carry out internal Pareto-improving redistributions. See, for example, Diewert, Turunen-Red and Woodland (1989, 1991).

[6] The model may also be specified in terms of ad valorem tariff rates rather than specific (unit) tariff rates. Nothing of essence is altered by this choice. Indeed, our example introduced further below is specifiied using ad

Reciprocity in Bagwell and Staiger (1999) is defined in terms of the outcome of tariff negotiations. In particular, it is required that the initial world price value of the change in the net trade vector of each country remains zero. The changes in tariffs, which are the policy instruments, have to ensure that the reciprocity conditions hold after all general equilibrium effects have taken place. In our notation, this definition can be formally written as follows.

Definition 1 [Reciprocity]. *A set of tariff changes conforms to the principle of reciprocity if*

$$p_0^{\intercal}(m_1^k - m_0^k) = 0, \qquad k \in K, \tag{3}$$

where subscripts 0 *and* 1 *denote, respectively, pre- and post-tariff reform values.*

Bagwell and Staiger examine the implications of using this reciprocity rule in trade negotiations within a two-good, two-country model. At the same time, they comment about the issues involved in moving to higher dimensions and it is these issues that occupy us in the remainder of this paper. Before moving on to these issues, however, it is useful to provide a diagrammatic account of the essence of their results in the two-dimensional setting and, thereby, provide a platform for our own results.

Figure 1 depicts the offer curves of two countries (1 and 2) that trade two goods internationally and choose tariffs non-cooperatively. The initial equilibrium point is the Nash equilibrium, N_0, where the offer curves intersect and each country's indifference curve is tangential to the other country's offer curve. The shaded 'cigar-shaped' area contained by these indifference curves is the set of trade vectors that yield a Pareto improvement over point N_0.

Figure 1: about here

Bagwell and Staiger define a tariff reform satisfying reciprocity as one that keeps the value of each country's trade unchanged, where the evaluation is at the initial world prices. In the figure, the ray passing through the origin and N_0 indicates the trade vectors that are consistent with a zero trade balance at the initial world prices, p_0. Thus, any expansion of trade along this ray is consistent with the reciprocity rule of Bagwell and Staiger. Thus, it is easy to see in this figure that a negotiated tariff reform that is consistent with reciprocity and market equilibrium expands the trade volumes out along this ray to a point such as N_1, at which the

valorem rates.

equilibrium world prices are unaltered. Moreover, such a tariff change will definitely move the countries within the 'cigar' area of Pareto improvements and so both countries gain from the tariff changes.[7]

The outcomes of the reciprocity rule in this two-dimensional context are clear. Reciprocity implies that the new world prices equal the initial world prices and that, provided the reform is small enough to prevent trade going out of the cigar shaped region, both countries gain in welfare. Bagwell and Staiger argue that the reciprocity reform neutralizes the terms of trade externality by ensuring unchanged world prices. We now proceed to determine how robust these clear-cut results are to the dimensions of the model by considering the general L good, K country case.

3. Reciprocity and World Prices

Using the reciprocity definition of Bagwell and Staiger, our first task is to determine the conditions under which a tariff reform that satisfies reciprocity ensures that the new world price vector is exactly equal to the initial world price vector. To this end, we proceed in several steps.

First, we can write the reciprocity condition in an alternative, but equivalent, way. Using balance of trade conditions $p_0^\intercal m_0^k = 0,\ k \in K$ (see (2)) at the initial equilibrium, equation (3) implies that $p_0^\intercal m_1^k = 0,\ k \in K$. This means that the new trade vectors, evaluated at the initial world prices, also take a value of zero. Combining this last equation with the balance of trade condition $p_1^\intercal m_1^k = 0,\ k \in K$ (see again (2)) in the post-reform situation, we can write $(p_0 - p_1)^\intercal m_1^k = 0, k \in K$. In matrix form, this set of conditions may be written as

$$(p_0 - p_1)^\intercal M_1 = 0, \tag{4}$$

where M_1 is the $L \times K$ matrix of national net imports vectors, m_1^k, in the post-reform situation 1. Accordingly, we have re-written the reciprocity conditions (3) in the form of equation (4). Thus, the reciprocity condition implies that the product of the $L \times 1$ vector, $p_0 - p_1$, and the $L \times K$ world trade matrix, M_1, is the zero vector.

Focusing on (4), we now determine the circumstances under which a solution to this set

[7]The fact that the starting point is a Nash equilibrium is important. If tariffs were not optimally set, the cigar-shaped area of Pareto improvements may be totally to the right or left of the world price ray. For example, if one of the two countries adopted a free trade policy then its indifference curve would be tagential to the ray through the origin and so the Pareto-improving area would be to one side of the ray. In this case, reciprocity would lower the welfare of the free-trading country.

of equations necessarily implies unchanged world prices, i.e., $p_0 = p_1$.[8] Defining $v \equiv p_0 - p_1$, this equation system may be written as $v^\intercal M_1 = 0$ and the question is whether $v = 0$ is the only solution. If $v \equiv p_0 - p_1 = 0$ is the only solution to equation system (4), then reciprocity necessarily implies that world prices do not change as a result of the tariff reform. On the other hand, if solutions with $v \equiv p_0 - p_1 \neq 0$ exist, then reciprocity does not necessarily imply unchanged world prices and, moreover, will generally imply different world prices as a result of the tariff reform.

The following Proposition establishes the necessary and sufficient conditions for reciprocity to imply unchanged world prices.

Proposition 1. *A multilateral change of tariffs satisfying reciprocity implies that world prices remain unchanged if, and only if, the world trade matrix in the post tariff-change situation has maximal row rank, $L - 1$ (that is, $rank(M_1) = L - 1$). Equivalently, a change in world prices is consistent with a tariff reform satisfying reciprocity if, and only if, $rank(M_1) < L - 1$.*

Proof. See Appendix 1.[9] ■

Analyzing carefully the possibilities that we have regarding the rank conditions in the above proposition, leads us to following corollary.

Corollary 2. *Reciprocity in trade negotiations does not necessarily lead to constant world prices for the (empirically relevant) case where the number of goods is rger than the number of countries. Indeed, in this case there always exists an equilibrium solution for world prices that differ from initial world prices as a result of a tariff reform satisfying reciprocity.*

Proof. See Appendix 2. ■

Bagwell and Staiger are aware of the possibility that reciprocity might not be sufficient to ensure unchanged world prices. They argue, however, that in such cases a supplementary tariff reform will be able to reconstruct the initial world prices and the new domestic prices with no

[8] From (4) is straightforward to see that tariff policy adjustments that preserve world prices will always lead to reciprocity. However, the question that we examine here is the opposite: viz. whether the tariff policy adjustments that satisfy reciprocity necessarily imply fixed world prices.

[9] The essence of the proof relates to the assumed properties of the trade matrix and well-known results from linear algebra concerning the existence of solutions to a set of homogeneous equations, such as (4). Equation system (4) is a set of K homogeneous linear equations in L unknown variables (elements of $v \equiv p_0 - p_1$). The trade matrix cannot have rank greater than $L - 1$ (due to the balance of trade or reciprocity conditions) and this accounts for the reference to the world trade matrix having maximal row rank $L - 1$.

effect on the equilibrium.[10] Let us explain their argument.

As we have seen above (see the discussion prior to (4)), reciprocity implies

$$p_0^{\intercal} m^k(p_1^k, u_1^k) = 0, k \in K, \qquad p_1^k = p_1 + t_1^k. \tag{5}$$

However, (5) can be rewritten as

$$p_0^{\intercal} m^k(p_1^k, u_1^k) = 0, k \in K, \qquad p_1^k = p_0 + t_2^k, \quad t_2^k \equiv p_1 + t_1^k - p_0, \tag{6}$$

where now we define another tariff vector, t_2^k, that replicates the same domestic prices p_1^k and the old world prices p_0.

Clearly, equations (5) and (6) are welfare equivalent, as each set of equations has the same post-reform domestic prices. The only thing that has changed is the definition of domestic prices, with new tariff vectors separating the domestic price vectors from the initial world prices. Thus, all real economic variables are the same and the only thing that has happened is a re-definition of tariffs in one particular way, viz. replicating the new domestic prices and the old world prices. Clearly, since domestic prices and world prices are connected by tariffs, one can *always* choose a particular tariff vector that replicates particular values of domestic and world prices (and thereby tariff revenues).

Does this result vindicate reciprocity as a mechanism for tariff reforms? The answer to this question is "no" if the objective is to obtain a tariff reform that also preserves world prices at their initial values. Because of the results in the previous section, reciprocity failed to preserve world prices at their initial levels. To obtain unchanged world prices an *additional* tariff reform (t_2^k) was necessary and it was constructed explicitly with unchanged world prices as the objective. Accordingly, the important point here is that it is not reciprocity that leads to these new tariffs, but the imposition of the constraint that world prices should remain the same.

Figure 2 attempts an illustration of the issues involved for the case where there are three goods and two countries. This figure has the imports for the three goods of country 2 as the axes.

[10] Specifically, Bagwell and Staiger (1999) write footnote 16, from which we quote: "In the many-good case, however, is also possible that reciprocity can be satisfied even when world prices change. To evaluate this possibility, we note that the restriction of reciprocity can be rewritten as (in our notation) $(p_1 - p_1^k)^{\intercal} m_1^k = (p_0 - p_1^k)^{\intercal} m_1^k$. This indicates that any trade-policy adjustment giving rise to the price vectors p_1 and p_1^k results in the same aggregate tariff revenue as would an alternative tariff-policy adjustment that gave rise to the price vector p_0 and p_1^k, when each adjustment is consistent with the restriction of reciprocity. Since world prices affect welfare only through tariff revenue, we may therefore restrict attention to tariff policy adjustments that preserve the world prices. These properties of reciprocity also extend naturally to a many-country case."

The imports for country 1 are measured negatively on these axes. The initial Nash equilibrium import vector $m_0^2 = -m_0^1$ is denoted by the point N_0 (as in Figure 1). Orthogonal to this point is the initial world price vector p_0, orthogonality being implied by the balance of trade conditions that $p_0^\intercal m_0^k = 0$.

Figure 2: about here

Reciprocity requires that the new import vectors are also orthogonal to the initial world price vector p_0. Geometrically, this requirement is that the new import vectors lie anywhere on the plane defining the set of points orthogonal to p_0. This plane (the dual subspace space for vector p_0) is illustrated in the figure and labeled RP. *Any* new equilibrium import vectors on this plane satisfy reciprocity.

The new equilibrium import vectors (e.g., $m_1^2 = -m_1^1$ denoted by the point R_1) also must satisfy the new balance of trade conditions and so be orthogonal to the new world price vector, p_1. In general, p_1 can be any (semipositive) point on the *plane* orthogonal to the vector OR_1, this plane (the dual subspace for vector OR_1) being illustrated in the figure and labelled R_1^*. One such world price point in R_1^* is provided by the initial price vector, p_0, but it is clearly not the only one. It is evident from the figure that there are an infinity of semipositive world price vectors on the dual subspace R_1^* that are consistent with both the reciprocity-chosen net import vectors m_1^1 (not shown) and m_1^2 (given by point labelled R_1). The essential insight is that the two reciprocity-chosen net import vectors have a *plane* as their dual space; reciprocity, by itself, is not sufficient to ensure that the new world price vector is equal to the initial world price vector.

While reciprocity in the two-good, two-country case required the new import vectors to be on the ray through N_0 in Figure 1 and, hence to have unchanged world prices, that is clearly not the case illustrated in Figure 2 for the three-good, two-country model. Reciprocity now simply requires the new import vectors to be on a plane and does not necessarily imply unchanged world prices. On the other hand, if we were to now include a third country there would be three net import vectors on plane labelled RP. If these point in different directions (so the matrix M_1 has maximal rank $L - 1 = 2$), then their dual space will, indeed, be the initial price vector, p_0; it is the only vector (up to a factor of proportionality, of course) that is orthogonal to all three reciprocity-chosen net import vectors. Thus, when the number of good is matched by the number of countries, reciprocity has enough structure to ensure unchanged world prices.

As a further illustration of the issues involved we provide a simple numerical example. In this example, there are $K = 2$ countries trading $L = 3$ goods.[11] The countries have fixed endowments of goods and no production. The endowment vectors of the two countries are assumed to be $(0.50, 0.15, 0.05)$ and $(0.25, 0.70, 0.05)$. There is a single consumer in each country, each with the same Cobb-Douglas preferences. The utility functions are $U(c_1, c_2) = (c_1 c_2)^{1/3}$. All tariff revenue is distributed to the consumer as a lump sum. Without loss of generality, good 1 is taken as the numeraire with price equal to unity and it is further assumed that there are no tariffs imposed on this good by any country.

Table 1 presents the equilibria for several different scenarios. Column (1) presents the Nash equilibrium, which we assume is the initial situation prior to the tariff reform. The equilibrium trade pattern involves Country 1 importing goods 1 and 3 and exporting good 2. In this equilibrium, country 1 imposes an ad valorem tariff rate of 54.77% ($t_{21} = 0.5477$) on its imports of good 2 and an export subsidy of 25.6% ($t_{31} = 0.2560$) on good 3. Country 2 taxes its exports of good 2 at the rate 49.34% ($t_{22} = -0.4934$) and has a subsidy rate of 11.19% ($t_{32} = -0.1119$) on imports of good 3.[12] Both countries are worse off in the Nash equilibrium than at free trade (the free trade values are not reported here for simplicity).

Table 1: Example: Equilibia Under Alternative Tariff Policies

		Reciprocity Tariff Reform	
Variable	(1) Nash	(2) Reciprocity	(3) Price Preservation
p_2	1.0911	1.0933	1.0911
p_3	7.0711	7.1237	7.0711
u_1	0.5067	0.5107	0.5107
u_2	0.6608	0.6691	0.6691
t_{21}	0.5477	0.3320	0.3347
t_{31}	0.2560	0.2560	0.2654
t_{22}	-0.4934	-0.4441	-0.4429
t_{32}	-0.1119	-0.1119	-0.1053
m_{12}	0.0835	0.0994	0.0994
m_{22}	-0.0966	-0.1251	-0.1251
m_{32}	0.0031	0.0052	0.0052
TR_1	0.0521	0.0358	0.0358
TR_2	0.0496	0.0565	0.0565

[11] This example is drawn from Table A1 of Kennan and Riezman (1990). Their example has three countries and three goods, so we simply remove the third country to get our example.

[12] By Lerner symmetry we can convert the tariff rates for country 2 such that it imposes no tax on its export good (good 2). These tariff rates for country 2 on the three goods are 0.9740, 0.0 and 0.7531. Thus, country 2 effectively imposes duties of 97.4% and 75.31% on imports of goods 1 and 3.

The equilibrium corresponding to a tariff reform that obeys the Bagwell and Staiger reciprocity condition is presented in column (2) of Table 1.[13] To obtain the results presented, we keep the tariff rates on good 3 as in the Nash equilibrium, and we alter the tariff rate imposed on good 2 by country 2 from $t_{22} = -0.4934$ to $t_{22} = -0.4441$ (a 10% change) and solve the equilibrium conditions and one reciprocity condition for the world prices, utility levels and the tariff rate t_{21}. The resulting tariff reform (only involving good 2 by assumption) obeys the reciprocity conditions for each of the two countries.

It is clearly seen that this reciprocity-compliant reform results in world prices that are different from those observed in the Nash equilibrium. The prices of goods 2 and 3 have both increased as a result of the tariff reform. This result is consistent with Proposition 1 and Corollary 2; since we have that $K = 2 < 3 = L$, a solution with different world prices is assured.

Column 3 of Table 1 provides the equilibrium values of the "constant world price" reform, where new tariff rates are derived *to ensure that world prices remain unchanged.* That is, the new tariffs are obtained as $t_2^k \equiv p_1 + t_1^k - p_0$, where p_0 is the Nash world price vector and p_1 is the world price vector from the reciprocity reform. Naturally, this new equilibrium yields the same real variables (e.g., utility levels and trade flows) and the same tariff revenues (last two rows) as the reciprocity equilibrium.[14]

4. An Alternative Reciprocity Rule

In the present section, we offer an alternative reciprocity rule that is guaranteed to yield strict Pareto improvements in the neighbourhood of the initial Nash equilibrium for any multi-good, multi-country world. In achieving this result, the question of whether this policy reform keeps world prices unchanged does not arise; whether they change or not is quite immaterial to the determination of welfare effects.

This reciprocity rule is defined as follows.

Definition 2. *A set of tariff changes conforms to the principle of reciprocity if*

$$m_1^k = \lambda m_0^k, \ \lambda > 1, \qquad k \in K, \tag{7}$$

[13] There are two reciprocity conditions — one for each country — but one of these conditions is redundant in view of the market equilibrium conditions.

[14] It should be emphasized that reciprocity by itself does not completely specify the required tariff reforms but merely the conditions that the tariff reform must meet. When there are more goods than countries (the empirically relevant case), the reciprocity requirements leave degrees of freedom in the selection of changes to tariffs. We illustrate this point by some further simulations that, for the sake of exposition, we report in Appendix 3.

where subscripts 0 *and* 1 *denote, respectively, pre- and post-tariff reform values.*

According to this new definition of reciprocity, we simply require that all countries increase their net import vectors in the same proportion, defined by the scalar λ.

Clearly, our reciprocity rule implies the Bagwell and Staiger reciprocity rule. This is because (7) implies that $p_0^\intercal(m_1^k - m_0^k) = (\lambda - 1)p_0^\intercal m_0^k = 0$, since the initial balance of trade requires that $p_0^\intercal m_0^k = 0$. The reverse is not generally true as should be evident from the results above. The reverse is true in the two-good, two-country case considered by Bagwell and Staiger.

We now show that, in the neighbourhood of the initial Nash equilibrium, our reciprocity reform unambiguously improves welfare in the sense of creating a strict Pareto improvement in welfare. First, we observe that the first order condition for utility maximization by country k at the initial Nash equilibrium is that $\nabla U^k(m_0^k) = \theta_0^k p_0^k$, where $\theta_0^k > 0$ is the Lagrange multiplier; that is, the gradient of the direct trade utility function is proportional to the domestic price ratio.[15] Second, we note that the change in the utility for country k from a tariff reform that moves the import vector in a direction δ ($dm^k = \delta\, d\alpha, d\alpha > 0$), is given by $du^k/d\alpha = D(m_0^k; \delta) = \delta^\intercal \nabla U^k(m_0^k)/\left|\delta\right|$. This directional derivative indicates the gradient of the utility function as imports move away from m_0^k along a ray defined by δ and is evaluated at the initial equilibrium import vector, m_0^k. Third, applying this relationship to our reciprocity based tariff reform, which moves the import vector proportionately outwards (setting $\delta = m_0^k$ and $d\alpha = \lambda - 1$), we obtain that the change in the utility for country k in the direction of the initial import vector is given by $D(m_0^k; m_0^k) = m_0^{k\intercal} \nabla U^k(m_0^k)/\left|m_0^k\right|$.[16] This directional derivative indicates the gradient of the utility function as imports move out along a ray defined by m_0^k and is evaluated at m_0^k. Fourth, using this tariff reform and the first order necessary conditions for the household (first and third results), we obtain that $D(m_0^k; m_0^k) = m_0^{k\intercal} \nabla U^k(m_0^k)/\left|m_0^k\right| = \theta_0^k p_0^{k\intercal} m_0^k/\left|m_0^k\right|$. Fifth, we note that $p_0^{k\intercal} m_0^k = t_0^{k\intercal} m_0^k$ is the tariff revenue that accrues at the initial Nash equilibrium, which is positive. Thus, we have that

$$D(m_0^k; m_0^k) = \theta_0^k p_0^{k\intercal} m_0^k/\left|m_0^k\right| = \theta_0^k t_0^{k\intercal} m_0^k/\left|m_0^k\right| > 0$$

for any differential reciprocity reform $dm^k = m_0^k\, d\alpha, d\alpha = (\lambda - 1) > 0$. Thus, we conclude that a tariff reform satisfying reciprocity in the sense that $m_1^k = \lambda m_0^k,\ \lambda > 1,\ k \in K$, with λ

[15] See Woodland (1980) for further details on the trade utility function and its dual.

[16] From $m_1^k = \lambda m_0^k$, we have that $m_1^k - m_0^k = (\lambda - 1)m_0^k$. For small changes this can be written in differential form as $dm^k = (\lambda - 1)m_0^k$, or as $dm^k = m_0^k d\alpha$, defining $d\alpha = (\lambda - 1)$.

sufficiently close to unity, is strictly Pareto improving in welfare. Accordingly, welfare improves for every country for our small reciprocity-consistent reform. The preceding argument therefore leads to the following proposition.

Proposition 3. *A reciprocity tariff reform of the form $m_1^k = \lambda m_0^k,\ \lambda > 1,\ k \in K$, yields a strict Pareto improvement in welfare for λ sufficiently close to unity.*

This result is very strong. Its only requirements are that the reform is sufficiently small and that the initial equilibrium has every country with positive tariff revenue. The former requirement is, of course, needed because very large quantity reciprocity reforms could easily take the countries well beyond the Pareto improving "cigar". The second requirement is satisfied at any Nash equilibrium and is needed to avoid other situations such as when one country is initially a free trader, in which case any expansion of trade will be clearly welfare reducing. An advantage of this result is that it shows that, under minimal conditions, a quantity reciprocity tariff reform is guaranteed to yield welfare gains for every country. Whether world prices change as a result of the reform is of no consequence.[17] Whether there are more goods than countries is also of no consequence.

If our reciprocity consistent tariff reform has such strong results, why is this not the case for a Bagwell and Staiger reciprocity consistent tariff reform? Using the definition of a directional derivative, the change in utility for country k is given by

$$\begin{aligned} du^k &= D(m_0^k;\delta)d\alpha = \theta_0^k p_0^{k\intercal}\delta / |\delta| \\ &= \theta_0^k p_0^{k\intercal} dm^k / \left|\frac{dm^k}{d\alpha}\right| \\ &= \theta_0^k (p_0 + t_0^k)^\intercal dm^k / \left|\frac{dm^k}{d\alpha}\right| \\ &= \theta_0^k t_0^{k\intercal} dm^k / \left|\frac{dm^k}{d\alpha}\right|, \end{aligned}$$

where the last equality occurs because the tariff reform is assumed to be consistent with Bagwell and Staiger's reciprocity requirement that $p_0^\intercal(m_1^k - m_0^k) = 0$, which in differential form is $p_0^\intercal dm^k = 0$. This expression for the change in utility will be positive if and only if $t_0^{k\intercal} dm^k > 0$, meaning that the reform yields an increase in tariff revenue (for every country) with the evaluation being undertaken at the initial tariff vector. Unfortunately, the reciprocity condition of Bagwell

[17] Our reciprocity reform can be readily constructed to have unchanged world prices, but this is not a requirment.

and Staiger, by itself, is not sufficient to pin down the sign of this term. Additional structure on the tariff reforms, such as those used by Bagwell and Staiger, is needed to ensure welfare improvements.

5. Conclusions

This paper focuses on whether tariff reforms based upon reciprocity imply unchanged world prices or not — an issue raised in the seminal work of Bagwell and Staiger (1999). We show that, in general, it does not. In the empirically relevant context where there are more goods than countries, it is generally the case that a reciprocity-based tariff reform will lead to changes in the world prices. While it is possible to find tariff reforms that are consistent with both reciprocity and unchanged world prices (as argued by Bagwell and Staiger, 1999, footnote 16), we make clear that such reforms do not stem from the reciprocity conditon but rather from the condition that world prices should remain unchanged. In this sense, the characterization that "reciprocity implies fixed world prices" is not accurate. The importance of whether reciprocity fixes or not the world price vector is paramount for the welfare analysis performed in Bagwell and Staiger (1999) and in the literature that followed.

We propose an alternative reciprocity rule that delivers welfare improvements without any condition imposed on the world price vector. Under our reciprocity rule, countries are asked to increase their volume of trade by the same proportion. We are able to show that, starting from the Nash equilibrium, any such marginal reform will lead to strict Pareto improvements: each and every country benefits unambiguously. This result is general and does not depend on whether or not world prices are changed, nor on the dimensions of the world economy.

In comparing the Bagwell and Staiger (1999) reciprocity rule with our reciprocity rule, we see both rules as being within the spirit of the WTO principle of "equal concessions". Under the Bagwell-Staiger rule, tariffs are altered so that, at initial world prices, an increase in a country's import values are matched by an increase in its export values. In the case of our reciprocity rule, an increase in a country's import quantities have to be matched by equi-proportional increases in its export quantities. Arguably, the value rule appears to be more applicable in a trade negotiation situation than the quantity rule. We argue, however, that both reciprocity rules are equally difficult to apply — in both cases the rules are expressed in terms of endogenous variables and, thereby, we have no idea which tariff changes negotiators have to make in order to satisfy the above mentioned reciprocity rules. Future research should focus on characterising the tariff rate changes needed to capture "reciprocity in trade negotiations". Such focus can be found in the piecemeal tariff reform literature.[18]

[18] Ju and Krishna (2000) and Anderson and Neary (2006) have recently made progress in finding tariff reform rules that both increase market access and welfare within a small open economy setup. Extending their work by considering a two country framework with reciprocity conditions could be a promising avenue of research.

1. Appendix 1: Proof of Proposition 1

To prove this proposition, we make use of some well-known results from linear algebra that depend upon the properties of the world trade matrix M_1. To put the resulting condition in context, it is useful to note the general rank properties of this world trade matrix.

Lemma 4. *Properties of the world trade matrix. The world trade matrix, M, has the following rank properties: (i) $rank(M) \leq K-1$ due to the world market equilibrium conditions, (ii) $rank(M) \leq L-1$ due to the balance of trade conditions in the new situation (or the reciprocity conditions) and (iii) $rank(M) \leq \min(L-1, K-1)$, since the rank of any matrix must be smaller than or equal to the lower of its dimensions.*

Proof. (i) The world market equilibrium conditions require that $M\iota = 0$, where ι is a vector of ones. This means that M cannot have full column rank and so $rank(M) \leq K-1$. (ii) The balance of trade conditions require that $p^\intercal M = 0$, where p is the world price vector. This means that M cannot have full row rank and so $rank(M) \leq L-1$. (iii) The rank of a matrix is the minimum of its row and column ranks. This means that $rank(M) \leq \min(L-1, K-1)$. ■

Proof. (of Proposition 1) The task is to determine conditions under which equation (4) has a solution with $p_1 = p_0$. Using a result from Hadley (1965, p.173-174), a necessary and sufficient condition for the system of K homogeneous linear equations $v^\intercal M_1 = 0$ to have a non-trivial solution $v \neq 0$ is that $rank(M_1) < L$. Since $rank(M_1) \leq L-1 < L$ from property (ii) of Lemma 4 above, it follows that a non-trivial solution $v = p_0 - p_1 \neq 0$ always exists. For example, the reciprocity conditions for situation 1 imply that $v = p_0$ is a solution and so $v = \lambda p_0$ is also a solution for any $\lambda > 0$. Thus, the equilibrium solution for p_1 can be written as $p_1 = (1-\lambda)p_0$. However, this is uninteresting from an economics viewpoint, since it says that one price vector is a multiple of the other. Normalizing the price of one good to be unity (the numeraire), $\lambda = 0$ is required and so the solution for v becomes trivial. Accordingly, we restrict attention to price systems that contain a numeraire good whose price is set to unity and which is not subject to a tariff in either situation.

Without loss of generality, we choose the first good as the numeraire, normalize the first element of vector $v = p_0 - p_1$ to 0 and assume that no tariffs are allowed to be imposed on the numeraire good. Since $v_1 = 0$, the equation system $v^\intercal M_1 = 0$ may be written as $\widetilde{v}^\intercal \widetilde{M_1} = 0$, where $\widetilde{v}$ is the $(L-1)$-dimensional vector of price differences for non-numeraire goods and $\widetilde{M_1}$ is the $(L-1) \times K$ dimensional trade matrix for non-numeraire goods. Hadley's result quoted above now implies that $\widetilde{v}^{\intercal}\widetilde{M_1} = 0$ has a non-trivial solution $\widetilde{v} \neq 0$ if and only if $rank(\widetilde{M_1}) < L-1$. In words, the necessary and sufficient condition for reciprocity to allow $p_1 \neq p_0$ is that the rank of the trade matrix in the new situation is less than the number of traded goods less one (that is, less than the number of non-numeraire goods).

A corollary to Hadley's theorem quoted above (Hadley, 1965, p.174) is that a necessary and sufficient condition for $\widetilde{v}^{\intercal}\widetilde{M_1} = 0$ to *only* have the trivial solution $\widetilde{v} = 0$ is that $rank(M_1) = L$. Applying this result to the problem at hand, we can write that

$$\widetilde{v}^{\intercal}\widetilde{M_1} = 0 \Rightarrow \widetilde{v}^{\intercal} = 0 \quad (p_1 = p_0) \text{ if, and only if, } rank(\widetilde{M_1}) = L-1. \tag{A.1}$$

This completes the proof. ■

2. Appendix 2: Proof of Corollary 2

There are two possibilities regarding the rank condition: (1) $rank(M_1) = L-1$ and (2) $rank(M_1) < L-1$.

First, if $rank(M_1) = L-1$ then Proposition 1 states that a tariff change obeying the reciprocity conditions ensures that the resulting world price vector is exactly the same as in the initial equilibrium ($p_1 = p_0$). This outcome means that there are no terms of trade externalities arising from the multilateral tariff reform. The second possibility is that $rank(M_1) < L-1$. In this case, Proposition 1 states that reciprocity is not sufficient to ensure unchanged world prices as a result of the tariff change. In other words, under this rank condition, the equation system $(p_1 - p_0)^{\intercal}M_1 = 0$ *always* has a solution such that $p_1 \neq p_0$ (in addition to the solution with $p_1 = p_0$, which always exists). When this situation arises, terms of trade externalities do, indeed, arise from the multilateral tariff reform.

Using the above results, we now consider two possible cases based upon the relative numbers of goods and countries.

Case α: $L \leq K$, i.e. the number of goods is smaller than, or equal to, the number of countries. In this case $rank(M_1) \leq \min(L-1, K-1) \leq L-1 \leq K-1$, which is consistent with either possibilities in the above Proposition. If the trade matrix M_1 has maximal row rank ($rank(M_1) = L-1$) then reciprocity does imply the world prices are unchanged as a result of the tariff reform.[19]

[19] However, it is possible that the trade matrix has lower rank ($rank(X_1) < L-1$), in which case a difference

Case β: $L > K$, i.e. the number of goods is larger than the number of countries. Here $rank(M_1) \leq \min(L-1, K-1) \leq K-1 < L-1$ and this inequality clearly violates the rank condition in Proposition 1. As a consequence, a multilateral tariff reform that obeys reciprocity does not necessarily imply unchanged world prices. Indeed, in this case equilibrium solutions with post-reciprocity-reform world prices different from the initial world prices always exist.

This completes the proof. ■

3. Appendix 3: Example

We made the point in the text that when there are more goods than countries (the empirically relevant case), the reciprocity requirements leave degrees of freedom in the selection of changes to tariffs. This point can be effectively made by referring back to the simulation example.

To implement the reciprocity-compliant tariff reform in the above example, the tariff rates t_{21} and t_{22} were altered, leaving other tariff rates unchanged. We needed to change (at least) one tariff and needed another tariff to change endogenously to ensure that reciprocity was satisfied. There was a choice of which tariffs to leave unchanged, which to change endogenously and which to be the policy change tariff. Put another way, there may be many (an infinity of) tariff reforms that satisfy reciprocity and, hence, there is some arbitrariness in a reciprocity-compliant tariff reform.

This important point may be further illustrated by extending the above example. To this end, we allow the t_{22} rate to fall from the Nash rate towards zero and compute the corresponding t_{21} values that satisfy reciprocity. By performing this calculation over a grid of t_{22} rates, we thus obtain a *reciprocity locus* in tariff rate space, i.e., a (t_{22}, t_{21}) locus that satisfies reciprocity but without preserving the world prices at their initial Nash equilibrium values (see Figure 3). We also plot the utility levels relative to their Nash values for the two countries along the reciprocity locus in Figure 4.

Figure 3 and 4: (about here)

As the subsidy rate t_{22} is reduced from the Nash value of $t_{22} = -0.4934$ to $t_{22} = -0.2$ moving

between initial and new world prices may ensue. It is noteworthy that this rank situation is only consistent with a trade matrix that is degenerate in the sense that it has less than maximal rank determined by theory. If such degeneracies are ruled out, then reciprocity necessarily implies unchanged world prices when $L \leq K$.

from left to right in Figure 3, the tariff rate t_{21} falls from its Nash value of $t_{21} = .5477$ towards $t_{21} = -0.2045$. Figure 4 shows that both countries gain from the reciprocity-based tariff reforms along the reciprocity locus until around $t_{22} \simeq -.38$, after which country 1's utility falls as the locus moves out of the Pareto improvement set.

Figure 5: (about here)

For each equilibrium on the reciprocity locus we now compute the supplementary tariff reform needed to preserve the world prices at their Nash values. It is important to note that this supplementary tariff reform requires all tariff rates to change, and is therefore more complex than the reciprocity reform alone. A complete summary of these results is provided in Figure 5, which plots the four tariff rates required to preserve world prices at the initial Nash levels relative to their values along the reciprocity-based tariff reform locus against the grid of reciprocity values for t_{22}. Each curve provides a measure of the additional (supplementary) tariff reform needed on top of the initial reciprocity-based reform to preserve world pries at initial Nash levels. The graph shows that t_{21} and t_{22} have to be further increased, while t_{31} and t_{32} have to be decreased (recalling that they were held constant in our chosen reciprocity-based reform).

In short, these figures illustrate the important points that (i) the implementation of reciprocity generally does not completely define the required tariff reform but allows degrees of freedom in the selection of which tariffs to change and by how much, and (ii) that this supplementary tariff reform requires all tariffs to be altered. This supplementary reform is not an implication of reciprocity, but is defined by the requirement of fixed world prices.

References

[1] Anderson, J. and P. Neary, 2007, Welfare versus market access: the implications of tariff structure for tariff reform, *Journal of International Economics* 71, 187-205.

[2] Bagwell, K. and R. Staiger, 1999, An economic theory of GATT, *American Economic Review* 89, 215-248.

[3] Bagwell, K. and R. Staiger, 2002, *The Economics of the World Trading System*, Cambridge, MIT Press.

[4] Blanchard, E., 2010, Reevaluating the role of trade agreements: does investment globalization make the WTO obsolete?, *Journal of International Economics* 82, 63-72.

[5] Diewert, W.E., Turunen-Red, A.H. and A.D. Woodland, 1989, Productivity and Pareto-improving changes in taxes and tariffs, *Review of Economic Studies* 56, 199–215.

[6] Diewert W.E., Turunen-Red, A.H. and A.D. Woodland, 1991, Tariff reform in a small open multi-household economy with domestic distortions and nontraded goods, *International Economic Review* 32, 937-57.

[7] Epifani, P. and J. Vitaloni, 2006, "GATT-think" with asymmetric countries, *Review of International Economics* 14, 427-444.

[8] Grinols, E. and P. Silva, 2007, An enhancement of modern free trade area theory, *Oxford Economic Papers* 59, 219-225.

[9] Hadley, G., 1965, *Linear Algebra*, Reading, Mass., Addison-Wesley Publishing Company.

[10] Kennan, J. and R. Riezman, 1990, Optimal tariff equilibria with customs unions, *Canadian Journal of Economics*, 23, 70-83.

[11] Ju, J. and K. Krishna, 2000, Welfare and market access effects of piecemeal tariff reform, *Journal of International Economics* 51, 305-316.

[12] Kemp, M.C. and H.Y. Wan Jr., 1976, An elementary proposition concerning the formation of customs unions, *Journal of International Economics* 6, 95–97.

[13] Kowalczyk, C. and T. Sjöström, 2000, Trade as transfers, GATT and the core, *Economic Letters* 66, 163-169.

[14] Matoo, A. and M. Olarreaga, 2004, Reciprocity across modes of supply in the WTO: a negotiating formula, *The International Trade Journal* 18, 1-22.

[15] Mrázová, M., 2011, Trade negotiations when market access matters, mimeo, downloadable from http://www.monikamrazova.com/TradeNeg.pdf.

[16] Ohyama, M., 1972, Trade and welfare in general equilibrium, *Keio Economic Studies* 9, 37–73.

[17] Ohyama, M., 2002, The economic significance of the GATT/WTO rules, in: A.D. Woodland, Editor, *Economic Theory and International Trade: Essays in Honour of Murray C. Kemp*, Edward Elgar, Cheltenham, UK.

[18] Ossa, R., 2011, A new trade theory of GATT/WTO negotiations, *Journal of Political Economy* 119, 122-152.

[19] Panagariya, A. and P. Krishna, 2002, On necessarily welfare-enhancing free trade areas, *Journal of International Economics* 57, 353–367.

[20] Raimondos-Møller, P. and A.D. Woodland, 2006, Non-preferential trading clubs, *Journal of International Economics* 68, 79-91.

[21] Turunen-Red, A.H. and A.D. Woodland, 1991, Strict Pareto-improving multilateral reforms of tariffs, *Econometrica* 59, 1127-1152.

[22] Woodland, A.D., 1980, Direct and indirect trade utility functions, *Review of Economic Studies* 47, 907-926.

[23] Woodland, A.D., 1982, *International Trade and Resource Allocation*, Amsterdam, North Holland.

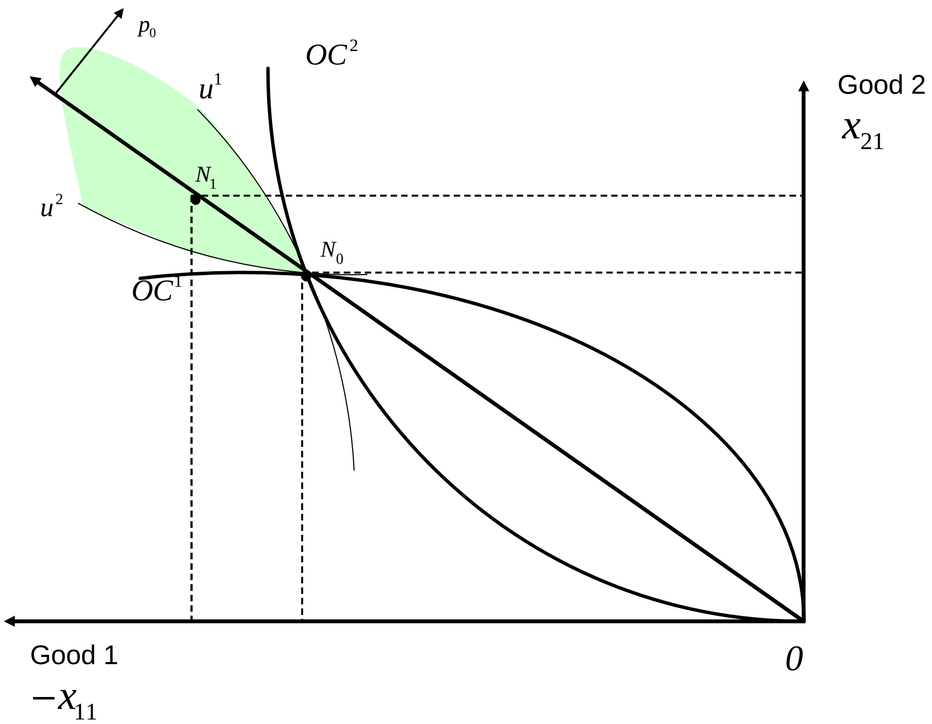

Figure 1: World price preserving reciprocity

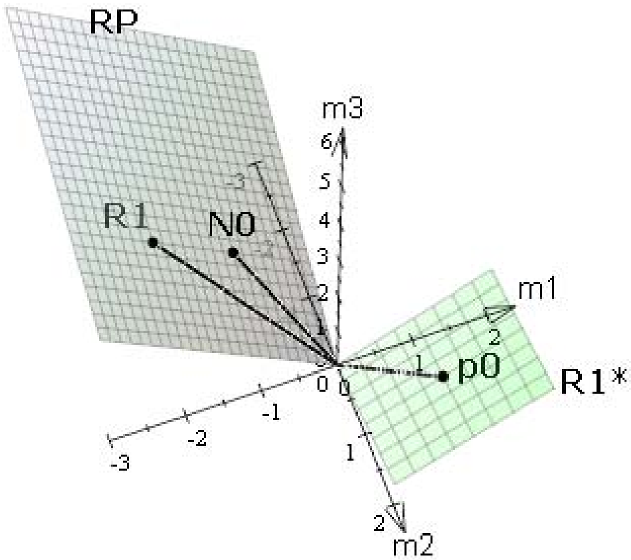

Figure 2: Reciprocity and World Prices with 3 Goods and 2 Countries

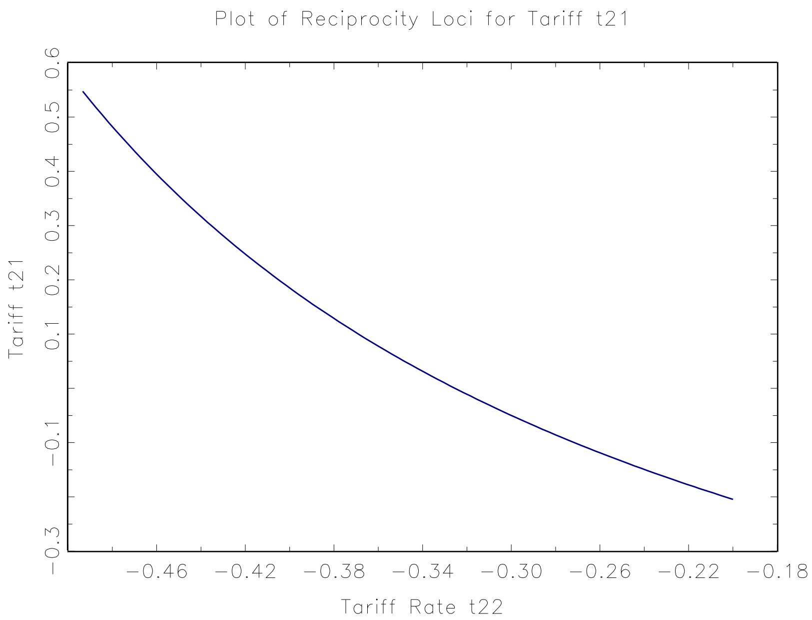

Figure 3: Reciprocity Locus

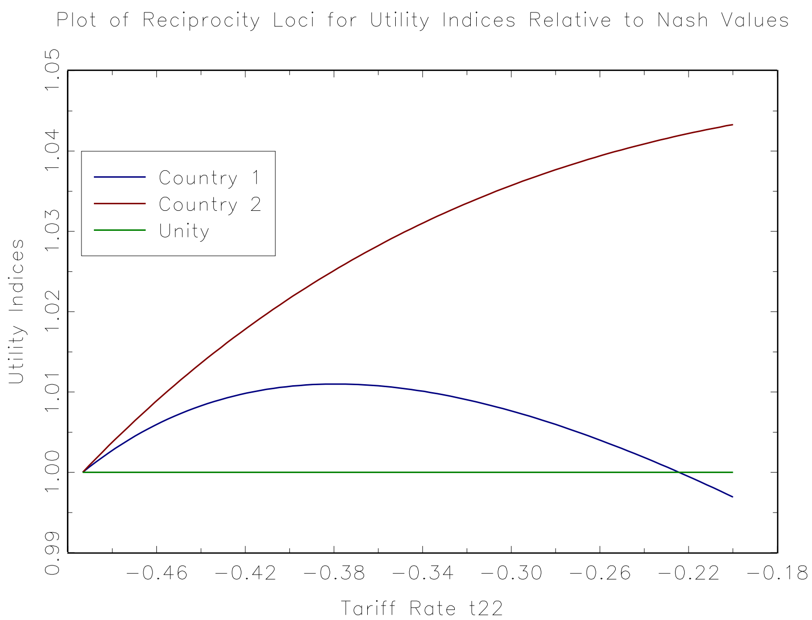

Figure 4: Utilities along Reciprocity Locus Relative to Nash Levels

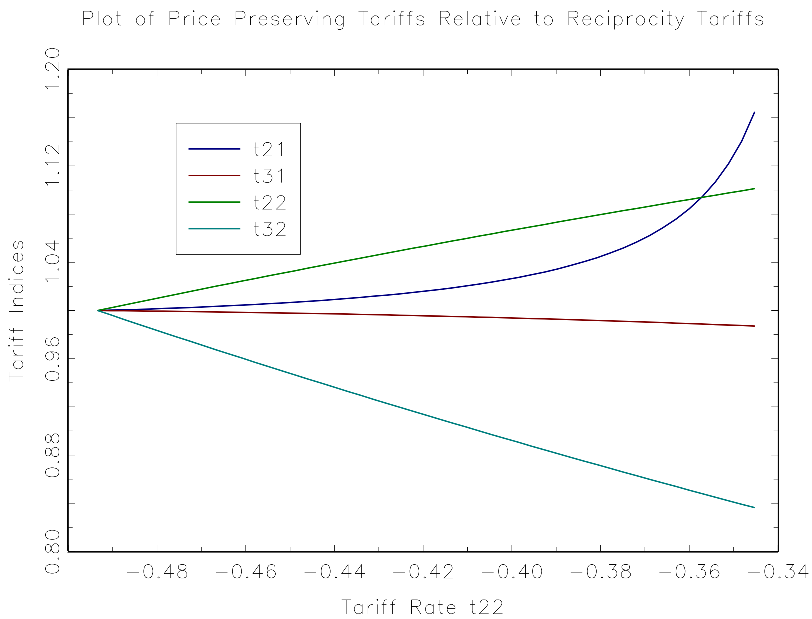

Figure 5: World Price Preserving Tariff Rates Relative to Reciprocity Rates