Calculating incremental risk charges: The effect of the liquidity horizon

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Calculating Incremental Risk Charges: The Effect of the Liquidity Horizon*

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Abstract

The recent incremental risk charge addition to the Basel (1996) market risk amendment requires banks to estimate, separately, the default and migration risk of their trading portfolios that are exposed to credit risk. The new regulation requires the total regulatory charges for trading books to be computed as the sum of the market risk capital and the incremental risk charge for credit risk. In contrast to Basel II models for the banking book no model is prescribed and banks can use internal models for calculating the incremental risk charge. In the calculation of incremental risk charges a key component is the choice of the liquidity horizon for traded credits. In this paper we explore the effect of the liquidity horizon on the incremental risk charge. Specifically we consider a sample of 28 bonds with different rating and liquidity horizons to evaluate the impact of the choice of the liquidity horizon for a certain rating class of credits. We find that choosing the liquidity horizon for a particular credit there are two important effects that needs to be considered. Firstly, for bonds with short liquidity horizons there is a mitigation effect of preventing the bond from further downgrades by trading it frequently. Secondly, there is the possibility of multiple defaults. Of these two effects the multiple default effect will generally be more pronounced for non investment grade credits as the probability of default is severe even for short liquidity periods. For medium investment grade credits these two effects will in general offset and the incremental risk charge will be approximately the same across liquidity horizons. For high quality investment grade credits the effect of the multiple defaults is low for short liquidity horizons as the frequent trading effectively prevents severe downgrades.

*The opinions expressed in the paper are those of the authors and do not necessarily reflect the view of SAS Institute.

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1 Introduction

Trading books in banks consist of positions held with intent to trade or hedge other positions whilst the banking books are usually made of held to maturity items. Therefore market risk and migration risk are more imminent to the trading books. Banks have been calculating internal model market based risk capital charges for many years. The Basel Committee updated its market risk capital requirement in a comprehensive Basel II in 2006 (Basel (2006)). In addition to the general market risk, specific credit spread risk of exposures to idiosyncratic risk of debt securities or equities was added to the requirement as well. The credit risk - including default and rating migration risk - contained in banks trading books were not subject to an internal models approval until 2009 when Basel Committee published a consultation paper introducing the Incremental Risk Charges (IRC) to banks subject to internal models for market risk capital (Basel (2009)). The IRC required is due to the amount of credit exposure in banks trading operations, and, recognizing the fact that credit exposures may give rise to substantial losses. The IRC must be assessed on weekly basis at a 99.9% Value at Risk confidence level using a risk horizon of 1 year. The IRC capital is supposed to complement the current market Value at Risk framework which measures risk on a 10-day holding period at the 99% confidence level. No specific model, approach or method is prescribed for banks in terms of the IRC estimation.

While the IRC requirement of capturing credit default and credit migration risk on a 1 year horizon may be consistent with many banks current internal models for assessing credit risk capital the IRC also requires banks to assign credits to liquidity or trading horizons. The liquidity horizon should reflect the time required to sell or hedge the credit under stressed conditions. In this paper we apply a portfolio credit risk model that can be used to capture the credit migration as well as default risk that is consistent with the IRC requirement. Under this model framework we evaluate the incremental risk charge for corporate bonds with different ratings and liquidity horizons using a multi-factor portfolio credit risk model. The purpose of the paper is to study the sensitivity of the IRC to bond rating and, in particular, bond liquidity horizon. Our findings motivate banks to choose a liquidity horizon for a security that is consistent with both the regulatory view and the market liquidity.

The paper is organized as follows. In section 2 we give an overview of the incremental risk charge addition to the Basel amendment for market risk in the trading book. Section 3 introduces the common multi-factor model for portfolio credit risk by first giving an overview of the foundation univariate and multivariate Merton (1974) model and then proceed to discuss the multi-factor model version we use to model portfolio risk. The multi-factor model was first used in CreditMetrics (1999) and has since then become one of the most popular models for portfolio credit risk. In section 4 we calculate incremental risk charges and in particular analyze the effect of the liquidity horizon for a sample of 28 bonds distributed across 7 different rating classes with different liquidity horizons within each class. Finally, in section 5 we summarize our findings.

2 Incremental risk charge

The incremental risk charge addition, Basel (2009), to the market risk amendment, Basel (1996), seeks to estimate the migration and default risk of traded credit products using a risk horizon of 1 year and the confidence level 99.9%. The 99.9% confidence level and the one year horizon is consistent with the confidence level used in Basel II advanced approaches for traditional banking book credits estimates of capital charges. However, in contrast to Basel II models for banking book, where the advanced model is set to a one-factor model that every bank uses, the incremental risk charge model is an internal model - allowing banks the flexibility to decide which model to use for calculating the portfolio credit risk.

The incremental risk charge includes positions that are subject to risk charges for specific interest rate risk such as corporate bonds however the incremental risk charge captures the exposure migration and default risk holding fixed any market variations such as interest rate risk and spread risk within a rating class. The incremental risk charge is not allowed to capture any diversification effects between market and credit risk. This means that the incremental risk charge is added to the total market risk charge to yield a total market and credit risk charge for items in the trading book.

A key feature of the incremental risk charge is that banks are allowed to capture the fact that traded credits - in contrast to the banking book positions - may be actively traded during the 1 year risk horizon. The trading horizon is specified by the bank although restrictions on the specification is given. Specifically, the trading horizon or liquidity horizon is subject to a floor of 3 months. Moreover, investment grade credits are expected to be more liquid than non-investment grade and hence have a shorter liquidity horizon. In this setting the liquidity horizon represents the time required to sell the position or to hedge all material risks covered by the incremental risk charge model in a stressed market. The liquidity horizon must be measured under conservative assumptions and should be sufficiently long that the act of selling or hedging, in itself, does not materially affect market prices. Moreover, the liquidity horizon is expected to be greater for positions that are concentrated, reflecting the longer period needed to liquidate such positions. This longer liquidity horizon for concentrated positions is necessary to provide adequate capital against two types of concentration namely issuer concentration and market concentration.

When trading credits at their liquidity horizon the requirement is that the credit is replaced by a credit with the same risk profile such that the initial risk level of the portfolio is maintained. In practice this means that the newly traded credit should have the same initial rating as the original credit but also that the new credit should not change the portfolio features such as concentration. Therefore, in order to preserve the initial risk level from both an exposure and portfolio perspective the newly traded credit should have the same stand-alone characteristics as well as correlation with the rest of the portfolio. That is, concentration and diversification risk is not changed by trading credits.

A bank that applies incremental risk charge to its trading book positions must seek to validate the model as far as possible empirically using back-testing. However, because of the limited availability of the historical loss data, other means of testing the model may be used such as stress testing and scenario testing. The specification of the exposure correlation - driving the concentration risk - is one of the key parameters that needs to be validated.

Currently many banks employ a corporate-wide portfolio credit risk model to evaluate the potential losses in banking and trading book due to credits deteriorating. This means that banks have already specified a portfolio credit risk model that defines the concentration and diversification of the portfolio. It is reasonable to expect that the same model is used in the regular evaluation of banks IRC. However, in banks standard portfolio credit risk model the concept of liquidity horizon and trading of credits is usually not considered. The assumption of a 1 year buy and hold portfolio is frequently used even for traded credits. Therefore, in banks application of the portfolio credit risk model to IRC one of the most crucial parameters is therefore the choice of the liquidity horizons, and hence, there is a need to understand the effect the choice of the liquidity horizon has on the required IRC. Below we employ a multi-factor portfolio credit risk model to a sample of bonds with the purpose of evaluating the effect the choice of the liquidity horizon has on the required capital for default and migration risk.

3 The portfolio credit risk model

In our analysis of incremental risk charge for default and migration risk we use a multi-factor version of the Merton (1974) model. The multi-factor version of a multivariate Merton model was first used in CreditMetrics (1999). The model is in popular use by many banks to estimate portfolio credit risk. Key aspects of the model are its calibration of sensitivity parameters to the systematic factors driving default and migration risk as well as the assessment of the level the unexplained idiosyncratic portion. Another important feature of the model is that it provides a structural explanation to not only default but also credit migrations. All these features together make this class of model especially suitable to meeting the IRC requirement.

Below we first introduce the univariate and multivariate Merton model and then proceed to discuss the multi-factor model version.

3.1 The classical Merton (1974) model

In the Merton (1974) structural bond pricing model the objective is to provide the price of a zero-coupon bond granted to a defaultable firm for a given period of time. That is, to develop a theory of the structure of credit risky discount rates. In the original version of the model there is no market risk involved and the obtained differentials in discount rates are hence solely due to credit risk. In the setup of the model it is supposed that the firm has only two classes of claims

- A single homogenous class of debt.
- The residual claim, equity.

In this simple setup, $V(t)$, the value of the assets of the firm, follows a geometric Brownian motion,

$$dV(t) = \mu V(t)\, dt + \sigma V(t)\, dW(t)$$

and on the liability side of the balance sheet of the firm, the total value is financed by equity, $S(t)$, and one representative zero-coupon (non-callable) debt contract, maturing at time T, with face value K. This gives the identity

$$V(t) = P(t,T;R) + S(t)$$

where $P(t,T;R)$ is the credit risky bond value. We now note the following: If $V(T) \leq K$ the zero-coupon bond is worth $V(T)$ whereas if $V(T) > K$ the zero-coupon bond is worth K. Hence, the value of the risky debt at time T is

$$P(T,T;R) = \min(V(T), K)$$

or,

$$P(T,T;R) = K - \max[K - V(T), 0]$$

where we recognize the last term as the terminal value of a standard Black and Scholes (1973) European put option on the firms assets with strike price K and maturity T. By the no arbitrage principle we have, for $t < T$, that a risk-free debt position $K\exp(-R(T-t))$ is equivalent to a risky debt position, $P(t,T;R)$, with paying interest rate R as the non-credit risky debt, and a long position in a put on the value of the firm, p. We can therefore write

$$K\exp(-R(T-t)) = P(t,T;R) + p$$

and

$$P(t,T;R) = K\exp(-R(T-t)) - p \tag{1}$$

The holders of the risky debt have hence, with paying rate R, issued a put option on the firms assets with strike K. The price of the put option can therefore be interpreted as the cost of eliminating the credit risk, or, the required premium on R for taking on credit risk.

Disregarding the fact that in practice firms assets are in general not tradable we apply standard Black and Scholes reasoning to obtain the value of (1). We arrive at

$$P(t,T;R) = K\exp(-R(T-t))\left[N(d_2) + \frac{V(t)}{K\exp(-R(T-t))}N(-d_1)\right] \tag{2}$$

where N is the cumulative distribution function of the stochastic variable $Z \sim N(0,1)$ and $d_1 = \frac{\ln\left(\frac{V(t)}{K}\right)+\left(R+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$. Clearly $P(t,T;R) \leq K\exp(-R(T-t))$. The above analytical expression of the value of credit risky debt expressed in prices is however simpler to interpret when expressed as an interest rate spread on R, denoting the resulting credit risky interest rate by $\widetilde{R}$. Consider therefore the credit risky equivalent bond value,

$$P\left(t,T;\widetilde{R}\right) = K\exp\left(-\widetilde{R}(T-t)\right) \tag{3}$$

with,

$$\widetilde{R} = -\frac{\ln\left(P\left(t,T;\widetilde{R}\right)/K\right)}{T-t}$$

enabling us to solve for the required $\widetilde{R}$ in (3) yielding

$$\widetilde{R} = R - \frac{1}{(T-t)} \left(\ln \left[N(d_2) + \frac{V(t)}{K \exp(-R(T-t))} N(-d_1) \right] \right). \tag{4}$$

The required credit spread, in excess of R, is then a function of:

- The leverage ratio, $\frac{V(t)}{K \exp(-R(T-t))}$ or inversely of the quasi-debt ratio, $\frac{K \exp(-R(T-t))}{V(t)}$
- The volatility of the firms assets σ (i.e., the firms business risk)
- Maturity of the debt issue (i.e., $(T-t)$)

By declaring a firm to be in default at time T if $V(T) - K < 0$ and using the corresponding risk-neutral asset process

$$dV(t) = RV(t)\,dt + \sigma V(t)\,dW_t$$

we can find

$$P(V(T) < K) = P\left(V(t) \exp\left(\left(R - \frac{\sigma^2}{2} \right)(T-t) + \sigma Z \sqrt{(T-t)} \right) < K \right)$$

where $Z \sim N(0,1)$, and hence

$$P(V(T) < K) = P\left(Z < \frac{\ln \frac{K}{V(t)} - \left(R - \frac{\sigma^2}{2} \right)(T-t)}{\sigma \sqrt{(T-t)}} \right) = N(-d_2)$$

is the risk-neutral probability of default (PD). By rearranging (2) as follows

$$P(t,T) = K \exp(-R(T-t)) - \exp(-R(T-t))\,\mathrm{PD}\,(1-r)$$

where $\mathrm{PD} = N(-d_2)$ is the probability of default, K is the exposure at default (face value of debt) and $(1-r) = \left[1 - \frac{V(t)}{K \exp(-R(T-t))} \frac{N(-d_1)}{N(-d_2)} \right]$ is one minus the recovery rate, r. Hence, we can now interpret the value of risky-debt as the value of secure debt minus the risk-neutral expected loss due to default. We note the following regularities

- PD and Loss Given Default (LGD) are not constant through time.
- PD and LGD are codependent.

By approximating the spread equation (4) as follows

$$\begin{aligned} \widetilde{R} &\approx R - \frac{1}{(T-t)} \left(\frac{V(t)\,N(-d_1)}{K \exp(-R(T-t))} - N(-d_2) \right) \\ &= R + \frac{1}{(T-t)} \mathrm{PD}\,(1-r) = R + \frac{1}{(T-t)} \mathrm{PD} \times \mathrm{LGD}. \end{aligned}$$

We can now price risky debt using the equation

$$P(t,T) \approx \exp(-(R+\pi)(T-t))\,K,$$

where $\pi = \mathrm{PD} \times \mathrm{LGD}$ is the "added particle" due to credit risk[1].

[1] The common approximation $\ln(1+x) \approx x$ is valid here since $\mathrm{PD}\,(1-r) \leq \mathrm{PD}$ is usually very small.

3.2 The multivariate Merton model

In the previous section we derived the Merton bond pricing model which provided us with structural estimates of the bond issuer PD as well as the ultimate losses, should default occur. The probability of default of a single issuer was obtained as,

$$\text{PD} = \Phi\left(-d_2\right)$$

where $d_2 = \frac{\ln\left(\frac{V_t}{B}\right)+\left(R+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} - \sigma\sqrt{T-t}$, B being the nominal debt value, $\frac{V_t}{B}$ the leverage ratio with V_t the value of assets. Further R and σ are respectively the short-rate of interest and the volatility of the asset process.

For a portfolio of N bond issuers we are now concerned with the probability that the sum of N Bernoulli loss indicators, $\{L_i\}_{i=1}^N$ attain the value of $n \leq N$ i.e.,

$$P\left(\sum_{i=1}^{N} L_i = n\right). \tag{5}$$

As an extension of the univariate case consider therefore an N-dimensional geometric Brownian motion process for the asset values, $\{V_{it}\}_{i=1}^N$

$$d\mathbf{V}_t = D\left[\mathbf{V}_t\right]\boldsymbol{\mu}dt + D\left[\mathbf{V}_t\right]\boldsymbol{\sigma}d\mathbf{W}_t \tag{6}$$

where $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_N)'$, $\boldsymbol{\sigma}$ is an N by N square root matrix (i.e., $\boldsymbol{\sigma}\boldsymbol{\sigma}' = \boldsymbol{\Sigma}$ the covariance matrix) given by

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix}$$

with $i,j:th$ element σ_{ij}. Here $\left(\sum_{j=1}^N \sigma_{ij}^2\right)^{\frac{1}{2}} = 1\ \forall i$. Further $D\left[\mathbf{x}\right]$ is an N by N diagonal matrix with the vector $\mathbf{x}$ on the diagonal and $\mathbf{W}_t = (W_{1t}, \ldots, W_{Nt})'$ are independent standard Wiener processes. However $\overline{\mathbf{W}}_t = \boldsymbol{\sigma}\mathbf{W}_t$ are N correlated Wiener processes. In particular the instantaneous correlation between $\overline{W}_{it}$ and $\overline{W}_{jt}$ is given by

$$\begin{aligned} \rho_{ij}dt &= E\left(d\overline{W}_{it}d\overline{W}_{jt}\right) - E\left(d\overline{W}_{it}\right)E\left(d\overline{W}_{jt}\right) \\ &= E\left(\sum_{k=1}^{N}\sigma_{ik}dW_{kt}\sum_{l=1}^{N}\sigma_{jl}dW_{lt}\right) \\ &= E\left(\boldsymbol{\sigma}_i' d\mathbf{W}_t\left(\boldsymbol{\sigma}_j' d\mathbf{W}_t\right)'\right) \\ &= \boldsymbol{\sigma}_i'\boldsymbol{\sigma}_j. \end{aligned}$$

In full analogy with the probability of default in the univariate case we here find for an issuer i

$$P\left(V_{iT} < B_i\right) = P\left(V_{it}\exp\left(\left(\mu_i - \frac{\sum_{j=1}^N \sigma_{ij}^2}{2}\right)(T-t) + \sum_{j=1}^{N}\sigma_{ij}Z_j\sqrt{(T-t)}\right) < B\right)$$

with $Z_j \sim N\ (0,1)$ yielding,

$$\begin{aligned} & P\left(\frac{\sum_{j=1}^{N} a_{ij} Z_j}{\left(\sum_{j=1}^{N} \sigma_{ij}^2\right)^{\frac{1}{2}}} < \frac{\ln \frac{B_i}{V_{it}} - \left(\mu_i - \frac{(\sigma_i)^2}{2}\right)(T-t)}{\left(\sum_{j=1}^{N} \sigma_{ij}^2\right)^{\frac{1}{2}} \sqrt{(T-t)}}\right) \qquad (7) \\ = \ & P\left(\widetilde{Z}_i < \frac{\ln \frac{B_i}{V_{it}} - \left(\mu_i - \frac{(\sigma_i)^2}{2}\right)(T-t)}{\left(\sum_{j=1}^{N} \sigma_{ij}^2\right)^{\frac{1}{2}} \sqrt{(T-t)}}\right) = \Phi\left(-d_2^i\right) \end{aligned}$$

where $a_{ij} = \frac{\sigma_{ij}}{\left(\sum_{j=1}^{N} \sigma_{ij}^2\right)^{\frac{1}{2}}}$ and $\widetilde{Z}_i = \frac{\sum_{j=1}^{N} a_{ij} Z_j}{\left(\sum_{j=1}^{N} \sigma_{ij}^2\right)^{\frac{1}{2}}} \sim N\ (0,1)$.

To estimate the probability defined in (5) requires the calibration of the asset process parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$. However, as in the univariate model, we in practice face the problem that asset values are non-traded and hence unobserved. In practical implementation of the multivariate Merton model one therefore use equity data as a proxy for asset values i.e., assuming that the correlations and volatilities of asset returns and equity returns are comparable.

3.3 Multifactor model for returns and rating migration

A popular version of the multivariate Merton model (cf. CreditMetrics (1999)) employs a factor model as an approximation to the issuers vector asset process. That is the standardized returns for issuer i, $\widetilde{Z}_i$, is driven by a multi-factor model. The factors are observed indices such as country indices, sector indices and other global economic factors. The returns of an issuer i is described by the following linear multi-factor model

$$\widetilde{Z}_i = \sum_{j=1}^{N^*} \beta_{ij} Z_j + \lambda_i \varepsilon_i$$

where the $Z_j : s$ are now interpreted as credit factors i.e., $Z_j \sim N\left(\mu_j, \sigma_j\right)$ and with β_{ij} the sensitivity of issuer i to the $j : th$ index factor. The $\varepsilon_i : s$ are assumed to be independent and identically distributed standard normal variables which are independent of the $Z_j : s$. Since the $Z_j : s$ are not necessarily standardized we can obtain the standardized $\widetilde{Z}_i : s$ as

$$\begin{aligned} \hat{Z}_i &= \phi_i \left(\sum_{j=1}^{N^*} \beta_{ij} Z_j\right) + \sqrt{1-\lambda_i} \varepsilon_i \\ \phi_i &= \sqrt{\frac{\lambda_i}{\boldsymbol{\beta} \boldsymbol{\Sigma} \boldsymbol{\beta}'}} \end{aligned}$$

where $\boldsymbol{\Sigma}$ is the covariance matrix of $(Z_1, ..., Z_N)$ and $\boldsymbol{\beta} = (\beta_1, ..., \beta_N)^2$. In this model the default threshold, as given by the debt level, B_i, in the Merton model is here obtained empirically

[2] Note here that the horizon of the covariance matrix, $\boldsymbol{\Sigma}$, must be consistent with the simulation horizon of the standardized factor, $\hat{Z}_i$. For example if $\boldsymbol{\Sigma}$ is calculated on monthly data and the simulation horizon is yearly then $\boldsymbol{\Sigma}$ must be scaled by $\sqrt{12}$ for consistency.

using the observed default frequency. That is for a given observed default probability, p, the threshold is obtained as $N^{-1}(p)$ where N^{-1} is the inverse normal distribution function. For different classes of rating categories we have $\hat{K}$ distinct transition probabilities to the $\hat{K}$ classes i.e., for an issuer belonging to class k we have $p_{k1}, p_{k2}, ..., p_{k\hat{K}}$. In this case the $\hat{K}$ thresholds are obtained from the $\hat{K}$ transition probabilities and the realized rating is determined by the realized return, $\hat{Z}_i$, and the thresholds.

In analogy with the Merton model the value of a zero-coupon credit with face value K is calculated as

$$P(t, T, k) = \exp\left(-\left(\pi(k)\right)(T-t)\right)K,$$

where the interest rate for credits in class k, $\pi(k)$, is determined exogenously from observed interest rates in the market for similar rated credits.

4 Calculation of incremental risk charges

The calculation of incremental risk charges for a portfolio of credits involves a portfolio credit risk model for calculation of losses as well as the capability of trading credits at their corresponding liquidity horizon. For the portfolio credit risk model we use a multi-factor model for each of the issuers to describe the concentration and idiosyncratic risks of the portfolio. At the liquidity horizon of a bond, the bond is traded for another bond, with the same risk characteristics i.e., with the same initial rating and multi-factor model. The multi-factor model remains the same between traded bonds to ensure that portfolio level risk characteristics remain constant i.e., concentration risk. At a particular horizon the valuation of the bonds use a credit discount rate that is contingent on the particular realized rating class and the loss is measured vs. the corresponding credit holding fixed the initial rating at the same horizon. This way of measuring loss means that the realized loss is only due to downgrades and defaults and not due to discounting effects. This is consistent with the fact that incremental risk charge should not measure market risk effects.

In our analysis of incremental risk charges we simulate losses at the horizons 3, 6, 9 and 12 months and the bonds with liquidity horizon less than 12 months are traded for a bond with the same risk characteristics as the original bond had at initially. This means that if the bond had a rating of k at the start of the analysis the bond is replaced by a bond with rating k at each of the liquidity horizons.

4.1 Bond data and model

The sample data consists of 28 zero-coupon bonds that are distributed across 7 non-default categorical rating classes corresponding to Moody's categories of ratings i.e., Aaa, Aa, A, Baa, Ba, B, and, Caa. A particular bond in a rating class can belong to the different liquidity horizons of 3, 6, 9 and 12 months respectively. A bond with liquidity horizon 3 months is traded at all the possible horizons i.e., 3, 6 and 9 months while a bond with 6 months liquidity horizon is traded at 6 month horizon. Similarly a bond with 9 month liquidity horizon is traded at 9 month horizon and a bond with 12 month liquidity horizon is a buy and hold bond under the risk horizon of 1 year. In our sample portfolio all bonds have a face value of 100 units of currency and the maturity term is set to 4 years. In the valuation of the bonds we use a discounting rate that is contingent on the rating of the bond. In particular the 7 non-default rating classes have associated a credit adjusted interest rate that is used for discounting. Since all the bonds have a maturity term of 4 years and the analysis horizon is confined to one year it suffices to specify the interest rates used for discounting between 3 and 4 years. Table 1 displays the discounting interest rates used at 3 and 4 years maturity horizon respectively. Discounting rates between 3 and 4 years are obtained using linear interpolation. In case of default we assume a recovery of 25% of the exposure amount[3]. The exposure amount is here measured as the value holding fixed the initial rating at that particular horizon. The analysis of the incremental risk charges for the 28 bonds use a transition matrix to describe the probabilities of migrating from one class to another. Our base transition matrix is the Moody's average one year transition matrix between 1920-1996, Moody's (1997), displayed in Table 2. The transition matrix has empirical transition probabilities for the Moody's rating classes Aaa, Aa, A, Baa, Ba, B and Caa, as well as the default state, D. The matrix has been estimated conditional on no withdrawal of rating and hence contains no category attributed to non-rated exposures as is usually the case. We refer to Moody's, (1997) for details on the construction of the empirical transition probabilities. In our analysis though we need the 3 month transition matrix to evaluate the transition probabilities for all the 3 month liquidity horizons i.e., 0-3 months, 3-6 months, 6-9 months and 9-12 months. We therefore construct a generator matrix from the Moody's transition matrix (cf. Israel, Rosenthal and Wei, 2001). The generator matrix, $\mathbf{Q}$, is obtained from the transition probability matrix, $\mathbf{A}$, such that

$$\mathbf{Q} = \sum_{k=0}^{n} (-1)^k \frac{\mathbf{D}^{k+1}}{k+1}$$

where $\mathbf{D} = \mathbf{A} - \mathbf{I}$, with $\mathbf{I}$ the identity matrix. The required power, r, of the initial transition matrix $\mathbf{A}$ is then calculated as $\mathbf{B} = \exp(r\mathbf{Q})$ using the Taylor expansion

$$\mathbf{B} = \mathbf{I} + \sum_{k=1}^{n} \frac{(r\mathbf{Q})^k}{k!}.$$

In this way we obtain transition probabilities for the 3 months horizon as well as longer liquidity horizons such as 6 months and 9 months from the generator matrix $\mathbf{Q}$. Table 3 displays the generator matrix, $\mathbf{Q}$, obtained from the Moody's one year average transition probability matrix for the Moody's rating classes Aaa, Aa, A, Baa, Ba, B and Caa, as well as the default state, D.

In the simulation of the standardized return of an issuer for each of the bonds 1 to 28 we use the same multi-factor model. The model is a four-factor model with factors, Z_1, Z_2, Z_3 and Z_4 respectively. The four systematic variables are jointly normally distributed with monthly covariance matrix given in Table 4. The systematic factor parameters or so-called loadings are given by $0.6231, 0.33, 0.0268$, and, 0.0201 for the factors Z_1, Z_2, Z_3 and Z_4 respectively. The idiosyncratic normal random variable has parameter 0.9. This gives a multi-factor model R^2 of $1 - 0.9^2 = 0.19$[4]. In the application of the model we simulate 100,000 samples of the

[3] We intentionally set the recovery rate low so to ensure that the default state is the maximum loss state for all bonds. The choice of a low recovery rates will simplify the interpretation of the results.

[4] In this paper we do not study the portfolio effects of IRC calculations. We have therefore choosen a

Table 1 Credit adjusted interest rates for the 7 non-default Moody's rating classes, Aaa, Aa, A, Baa, Ba, B, and, Caa respectively, for maturity terms 3 and 4 years

Rating Category	Interest Rate (3 Years)	Interest Rate (4 Years)
Aaa	0.02651775	0.02934361
Aa	0.02687823	0.02990976
A	0.02784931	0.03124889
Baa	0,02933442	0.03306486
Ba	0.03176641	0.03581472
B	0.05451719	0.05929989
Caa	0.12388839	0.12019516

Table 2 Moody's 1920-1996 average one-year transition matrix for the non-default rating grades Aaa, Aa, A, Baa, Ba, B and Caa, and, the default state, D

	Aaa	Aa	A	Baa	Ba	B	Caa	D
Aaa	0.9218	0.0651	0.0104	0.0025	0.0002	0	0	0
Aa	0.0129	0.9162	0.0611	0.007	0.0018	0.0003	0	0.0007
A	0.0008	0.025	0.9135	0.0511	0.0069	0.0011	0.0002	0.00014
Baa	0.0004	0.0027	0.0422	0.8916	0.0525	0.0068	0.0007	0.0031
Ba	0.0002	0.0007	0.0044	0.0511	0.8708	0.0557	0.0046	0.0125
B	0	0.0004	0.0014	0.0069	0.0652	0.852	0.0354	0.0387
Caa	0	0.0003	0.0004	0.0037	0.0145	0.06	0.783	0.1381
D	0	0	0	0	0	0	0	1

factor model for each of the issuers. This is done for all the liquidity horizons of 3, 6, 9 and 12 months using an arithmetic Brownian motion model for the systematic factors Z_1, Z_2, Z_3 and Z_4, and, a normal idiosyncratic variable that is specific to each issuer. The realized loss of the 28 bonds is aggregated across the liquidity horizons such that for a bond with a 3 month liquidity horizon the loss is aggregated at horizons 3, 6, 9 and 12 months whereas for a bond with liquidity horizon 12 months the loss is measured as the loss at the 12 month horizon.

4.2 Analysis results

For our 28 sample bonds we calculate the IRC at the 99.9% Value at Risk level by aggregating the loss across the liquidity horizons. The incremental risk charge results are presented in

stylized model of correlation such that all issues share the same multi-factor model for the systematic factors. At the same time the multi-factor model for the issuers have been choosen such that it is empiricially realistic on the issue level - ensuring that the issue level results are realistic.

Table 3 The transition generator matrix obtained from Moody's 1920-1996 average one-year transition matrix for the non-default rating grades Aaa, Aa, A, Baa, Ba, B and Caa, and, the default state, D

	Aaa	Aa	A	Baa	Ba	B	Caa	D
Aaa	-0.081773619	0.07560427	0.006003093	0.00013346	3.0173E-07	2.63646E-05	1.53069E-06	4.59901E-06
Aa	0.01156699	-0.088779698	0.072929533	0.002030102	0.002208302	0	8.70017E-06	3.607E-05
A	0.000972872	0.026312935	-0.090179981	0.055385913	0.005357302	0.002076315	0	7.46447E-05
Baa	0	0.001712761	0.055737266	-0.12446407	0.058023925	0.006935059	0.001059708	0.000995352
Ba	1.30434E-06	0.001087549	0.003450218	0.053100269	-0.155604472	0.079932231	0.003861997	0.014170903
B	0	0.001080588	0.001060703	0.004323729	0.064858352	-0.187323273	0.02646484	0.089535061
Caa	0	0.004971885	0.004650809	0.009388565	0.027274856	0.070929805	-0.360754688	0.243538769
D	0	0	0	0	0	0	0	0

Table 4 Covariance matrix for the systematic factors in the multi-factor model for the bonds

	Z_1	Z_2	Z_3	Z_4
Z_1	0.007191	0.004117	0.003889	0.002926
Z_2	0.004117	0.004845	0.003942	0.003037
Z_3	0.003889	0.003942	0.006162	0.002911
Z_4	0.002926	0.003037	0.002911	0.002469

Table 5. Table 5 displays the obtained incremental risk charge, the maximum potential loss and the realized loss ratio for each liquidity horizon, for the bonds in rating classes Aaa to Caa. While the absolute IRC level in Table 5 is of immediate interest it is also interesting to focus on the realized loss percentage of the IRC loss to maximum loss. The ratios show the true risk percentage that is realized, at the 99.9% Value at Risk level, in the IRC model for the bonds rating grade and liquidity horizon. Because of our choice of a low recovery rate of 25% in case of default the maximum loss is interpreted as the loss obtained when the maximum number of defaults that can happen is realized. This means that a bond with a liquidity horizon of 3 months can default up to 4 times whereas a bond with liquidity horizon of 6 or 9 months can default up to 2 times and, finally, a buy and hold bond, having liquidity horizon 12 months can only default once during the risk horizon of 1 year[5].

For the investment grade credits in Table 5 i.e., bonds 1-12 belonging to rating class Aaa, Aa or A we observe that the incremental risk charge is smallest for the shortest liquidity horizon of 3 months. For the bonds in rating grade Aaa the IRC is seen to increase substantially when going from 3 months liquidity horizon to a 6 month liquidity horizon. However, for the 9 and 12 month horizon the IRC decreases significantly compared to the 6 month liquidity horizon, although not being as low as for the 3 month liquidity horizon. For bonds rated in class Aa the IRC increase when moving from liquidity horizon of 3 months to 6 and 9 months. But in contrast to the bonds in rating class Aaa the maximum IRC is achieved for the bonds in rating class Aa when the bond is a buy and hold bond. The bonds rated in rating category A have the smallest IRC level with the shortest liquidity horizon of 3 months and is then constant on a higher level for liquidity horizons of 6, 9 and 12 months. We also observe for the investment grade bonds that the realized loss ratio increases with decreasing bond rating quality, and, in general with increasing liquidity horizon.

To understand the obtained results for the investment grade bonds it is useful to consider the two effects that are involved in the determination of the IRC for a particular bond. Firstly, for a bond which trade frequently one expects a multiple default risk effect since the newly traded bond is exposed to default and migration risk. However, for the high investment grade credits which have a relatively small probability of default we should expect this effect to be rather small even for high loss quantiles such as at the 99.9% IRC level. The second effect, working in opposite direction to the multiple defaults effect, is the positive effect of frequent trading to prevent further downgrades and hence trading frequently in general mitigates the losses for investment grade credits.

Returning to the analysis of the premium rated bonds i.e., the bonds in rating class Aaa we observe that the incremental risk charge increase as we move from the liquidity horizon

[5]In general the loss ratio increases with the liquidity horizon because the longer a bond is held the more likely it defaults and hence the maximum loss is realized. The worse the rating grade of the bond the closer the loss ratio is to 100% as the probability of realizing the maximum loss is high.

of 3 months to 6 and, then, decrease as we move from liquidity horizon 6 months to 9 and 12 months. This result can be interpreted in the context of the two offsetting effects of short liquidity horizons yielding a positive probability of multiple defaults (or severe migrations) and, at the same time, potentially mitigating default by frequent trading. The bond with liquidity horizon 3 months benefit from the mitigating effect of frequent trading as the short-term 3 month default probability is effectively null. However, at the 6 month horizon, as the IRC increase as well as the realized loss ratio, it seems that there is indeed a multiple defaults effect. This is because the 6 months liquidity horizon involves two 3 month periods and hence the 6 months liquidity horizon can effectively give rise to multiple defaults by first migrating in the first 3 months and then defaulting at the second 3 month period that is within the 6 months horizon. The bond with liquidity horizon 9 months benefit from the second trading period being relatively short at 3 months and hence a default in both the 9 month and 3 month horizon bond is not possible since the 3 month default probability is effectively null. For the bond with liquidity horizon 12 months i.e., the buy and hold bond the IRC is the same as for the bond with a 9 months liquidity horizon. However, the realized loss ratio has increased from the 9 months bond. On absolute level though, since the 9 months liquidity horizon bond has the same IRC as the corresponding 12 months buy and hold bond, the second liquidity period of the 9 months liquidity horizon bond of 3 months contributes very little to IRC.

For the bonds in rating class Aa we note that IRC increases as the liquidity horizon increases and hence the dominating effect is the positive effect of mitigating losses by trading frequently. We also observe that the realized loss ratio increases as the liquidity horizon increases such that the bonds in rating category Aa in general benefits from frequent trading - the mitigating effect of frequent trading being stronger than the multiple defaults effect. For the bonds in rating category A we also see an increasing realized loss ratio as the liquidity horizon increase though the actual IRC level remains the same across liquidity horizons of 6, 9 and 12 months. The 3 months liquidity horizon bond having the lowest level of IRC for the bonds rated A.

For the medium investment grade bonds in rating class Baa we note that the incremental risk charge level is constant over the 3, 6, 9 and 12 month liquidity horizons. However, the realized loss ratio increase as we increase the liquidity horizon. In particular, the bond with liquidity horizon 3 months has a realized loss ratio of 25% whereas the bonds with liquidity horizon 6 and 9 months have a realized loss ratio of 50%, and, the buy and hold bond with 12 months liquidity horizon has a realized loss ratio of 100%. For the bonds in rating class Baa the constant level of IRC across liquidity horizons can be attributed to the two effects of multiple defaults and the effect of the trade truncating the possibility of further downgrades approximately offsetting each other. The bonds rated in rating category Ba shows an increase in the realized loss ratios compared to the Baa rated bond. We also note that the bonds rated Ba is the first rating category that has the minimum IRC level at the 12 months liquidity horizon i.e., for the buy and hold bond. This is consistent with that, on absolute IRC level, the effect of multiple defaults being stronger than the mitigating effect of frequent trading for this rating category.

For the speculative grade rating classes i.e., bonds in rating classes B and Caa Table 5 shows that it is always beneficial to assume a long liquidity horizon if one wants to minimize the level of IRC. This is due to the fact that for these bonds the probability of default is severe and hence the probability of multiple defaults and losses if the credit is allowed to trade is quite likely - especially at the high 99.9% level Value at Risk at which the IRC is measured. The realized loss ratio is 100% or close to 100% for all the bonds in rating categories B and Caa. The preference for a long liquidity horizon for speculative grade credits, to minimize the IRC level, is also consistent with the guidance set forth by regulators that they do expect that lower rated bonds should have a longer liquidity horizon than investment grade bonds. Banks preferred assignment of longer liquidity horizons for non-investment grade bonds is therefore consistent with regulators view.

To summarize our above analysis findings we note that in the calculation of incremental risk charges there are two important effects that needs to be considered. Firstly, for bonds with short liquidity horizons there is a mitigation effect of preventing the bond from further downgrades by trading it frequently. Secondly, there is the effect of the possibility of multiple defaults. Of these two effects the multiple default effect will generally be more pronounced for non investment grade credits as the probability of default is severe even for short liquidity periods and hence incremental risk charges will generally increase the shorter the liquidity horizon. For medium investment grade credits these two effects will in general offset and the incremental risk charge will be approximately the same across liquidity horizons. For investment grade credits the effect of the multiple defaults is low for short liquidity horizons as the frequent trading effectively prevents severe downgrades. Not surprisingly, this finding about preferred liquidity horizons for investment grade and non-investment grade credits, in the context of IRC, coincides with results in the credit spread term structure modeling literature. For example, Merton (1974), Sarig and Warga (1989), Fons (1994), Longstaff and Schwartz (1995) and especially Jarrow et al. (1997). That is, that the investment grade credits have increasing credit spreads and the non-investment grades or speculative grades have downward sloping spreads reflecting the survival contingent effects.

5 Summary and conclusions

The incremental risk charge calculations required by regulators represent a substantial challenge for banks in adapting their current portfolio credit risk models for traded credits to incorporate assumptions about liquidity horizons. The assigned liquidity horizon for any particular credit represents the banks view on the time required to fully hedge or sell the credit without any significant negative liquidity effects on the price. Moreover, the assigned liquidity horizon should, according to regulators, be valid even under stressed conditions. Using the experience of the recent crisis this requirement has the practical implication that only investment grade credits can in effect be considered to have relatively short liquidity horizons whereas medium grade credits should have fairly long liquidity horizons, and, finally non-investment or speculative grade credits need effectively be considered as buy and hold securities. As we have demonstrated in this paper the market and regulatory rationale for assigning stressed liquidity horizons to credits is aligned with banks desire to minimize the IRC add-on to the total market and credit risk regulatory capital charge for the trading book. Specifically, there are two important effects at play that determine the IRC for a particular credit rating and liquidity horizon. That is, the mitigation effect of preventing the bond from further downgrades by trading it frequently and the multiple default effect obtained from frequent trading. In case of medium rating grade credits these effects approximately offset each other so that

Table 5 Maximum loss, incremental risk charge and realized loss ratio for the sample bonds 1 - 28 rated in Moody's categories Aaa, Aa, A, Baa, Ba, B, and Caa with different liquidity horizons of 3, 6, 9 and 12 months

Bond	**Rating Class**	**Liquidity Horizon (Months)**	**Maximum Loss**	**99.9% IRC Loss**	**Loss Ratio %**
1	Aaa	3	266.74	1.52	0.57
2	Aaa	6	133.37	10.05	7.53
3	Aaa	9	133.37	2.27	1.70
4	Aaa	12	66.68	2.27	3.40
5	Aa	3	266.14	9.85	3.70
6	Aa	6	133.07	9.85	7.40
7	Aa	9	133.07	10.96	8.24
8	Aa	12	66.53	26.9	40.4
9	A	3	262.8	25.79	9.81
10	A	6	131.4	65.70	50
11	A	9	131.4	65.70	50
12	A	12	65.70	65.70	100
13	Baa	3	259.9	64.98	25
14	Baa	6	129.96	64.98	50
15	Baa	9	129.96	64.98	50
16	Baa	12	64.98	64.98	100
17	Ba	3	236.6	76.21	32.2
18	Ba	6	118.30	118.30	100
19	Ba	9	118.30	76.21	64.4
20	Ba	12	59.15	59.15	100
21	B	3	185.43	165.70	89
22	B	6	92.72	92.72	100
23	B	9	92.72	92.72	100
24	B	12	46.36	46.36	100
25	Caa	3	105.56	105.56	100
26	Caa	6	52.79	52.79	100
27	Caa	9	52.79	52.79	100
28	Caa	12	26.39	26.39	100

the IRC level remains approximately constant across the choice of liquidity horizon. However, for non investment grade credits the multiple default effect is stronger such that, in general, one should expect a lower IRC for a conservative assumptions about the liquidity horizon i.e., by assuming the credit is a buy and hold. For investment grade credits the mitigation effect from frequent trading is generally stronger than the multiple defaults effect due to low default probabilities, and hence, a short liquidity horizon is preferred for high-quality credits as this assumption in general gives a lower IRC.

While it is noticeable that the interest of regulators to keep liquidity horizons short only for investment grade credits is aligned with banks incentives of how to allocate the liquidity horizons across different credit grades it is not surprising. Indeed, this finding, in the context of IRC, that the best allocation of the liquidity horizon across credit qualities, with high quality credits having short liquidity periods and low quality credits having long liquidity horizons, coincides with the empirical credit spread term structure. That is, that the investment grade credits have increasing credit spreads and the speculative grades have downward sloping spreads reflecting the survival contingent effects.

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