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Forbearance Lending and Soft Budget Constraints in a Model of Multiple Heterogeneous Bank Financing

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Forbearance Lending and Soft Budget Constraints in a Model of Multiple Heterogeneous Bank Financing

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Abstract

Empirical evidence suggests that banks often engage in refinancing of intrinsically insolvent debtors instead of writing off their non-performing loans. Such forbearance lending may induce soft budget constraints for the debtors, as it diminishes their incentives to thwart default. This paper introduces a model of coordination failure to analyze how a relationship bank affects the incidence of forbearance lending and soft budget constraints by signaling its credit decision to other creditors. We find that the relationship bank's signaling ability enhances its incentives to engage in forbearance lending and influences the conditions under which debtors face soft budget constraints.

Keywords: Coordination Failure; Global Games; Asymmetric Information; Forbearance Lending; Soft Budget Constraints

JEL classification: C72, D82, G21, G33

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1 Introduction

After the collapse of the asset price bubble in Japan, banks have been reluctant to write off non-performing loans, even when there was little prospect of debtor firms being able to repay the loans extended. Instead, the banks often engaged in refinancing of economically insolvent debtors, gambling that somehow these firms will recover. A number of studies provide empirical evidence for this phenomenon known as *forbearance lending*. By estimating the loan supply and demand in major Japanese industries, Sekine et al. [2003] find that outstanding loans are apt to increase to firms whose debt-asset ratio exceeds a certain level. Peek and Rosengren [2005] argue that Japanese banks engage in forbearance lending to massage their balance sheets and examine how this behavior impairs economic efficiency. They show that debtor firms are far more likely to be refinanced if they are in poor financial condition, even though these firms continue to perform poorly after receiving additional bank financing. In reference to this misallocation of credit, Caballero et al. [2005] introduce and test a model suggesting that forbearance lending distorts competition throughout the rest of the economy by keeping unprofitable firms in business.

Another issue, referred to as the soft budget constraint problem, can arise from forbearance lending if banks are unable to commit credibly not to refinance a debtor in financial distress. Kornai [1980] originally coined the term *soft budget constraints* to illustrate economic behavior in socialist economies, in which the government is disposed to bail out loss-making firms and thus undermines ex ante incentives of these firms.[1] According to Kornai's definition we say that a debtor firm has a soft budget constraint if it can expect to receive additional credit from a bank and if those expectations diminish the firm's incentives to work against default. Evolving from Schaffer [1989], Dewatripont and Maskin [1995] as well as Berglöf and Roland [1997, 1998] introduced models formalizing the dynamic commitment problem of a bank which gives rise to soft budget constraints. They show that it is an ex post rational choice for a bank to refinance firms with ex ante unprofitable projects if the sunk costs of the prior investment are sufficiently high. This leads to perverse incentives for the debtor firms, as they anticipate the bank's policy of forbearance.

However, neither of these previous models on forbearance lending and soft budget constraints regards the fact that firms typically hold credit relations to diverse

[1] See Maskin [1996] and Kornai et al. [2003] for an elaborate theoretical survey on soft budget constraints.

banking institutions and that coordination failure among these banks can induce inefficient credit terminations.[2] Complementing the work of Morris and Shin [2005] and Takeda and Takeda [2005], we attempt to fill this gap by developing a model of coordination failure among multiple heterogeneous creditors. Assuming that a firm is financed by a single relationship bank and a continuum of arm's length banks, this paper analyzes under which conditions the relationship bank is disposed to refinance an otherwise insolvent firm and in what circumstances such forbearance lending leads to soft budget constraints for the debtor firm. In order to determine the incidence of coordination failure among the banks' credit decisions, we build on the theory of global games as introduced by Carlsson and van Damme [1993] and generalized by Morris and Shin [2003] and Frankel et al. [2003].[3] Based on the assumption that players receive noisy private signals regarding an underlying fundamental state, global games produce a unique equilibrium in coordination games that typically have multiple equilibria under complete information.

Morris and Shin [2005] adopt the equilibrium selection framework of global games to analyze soft budget constraints in a situation where official liquidity provision by the IMF could prevent an international financial crises. In a similar game theoretic context Takeda and Takeda [2005] examine the effects of refinancing provided by a self-interested large creditor on the behavior of small creditors and a debtor firm. However, their model ascribes a rather passive role to the the large creditor. His influence on the firm's fate is limited to the effect that he can merely choose whether and how much to refinance. Small creditors are able to observe the large lender's behavior, but cannot avail this additional information to learn more about the financial state of the firm.

This paper models the relationship bank as a large strategic player, extending the approach of Takeda and Takeda [2005] on two counts. Similar to Berglöf and Roland [1997], we consider the outside option for the relationship bank to invest in new lending. If the debtor firm is in hopeless financial condition even with refinancing, the relationship bank can withdraw its credit and seize the collateral of the firm. Our second contribution is to incorporate social learning in the model. The arm's length banks are able to update their beliefs regarding the financial state of the firm by taking into consideration the relationship bank's roll over or foreclosure decision. Hence, the relationship bank can affect the fate of the debtor firm not

[2] For empirical evidence on the prevalence of multiple heterogeneous bank lending see Detragiache et al. [2000], Elsas [2005], and Ongena and Smith [2000].

[3] Hubert and Schäfer [2002] and Morris and Shin [2004] were the first to apply the global game concept in the context of coordination failure among multiple creditors.

only by providing additional capital, but also by deciding whether to extend or to withdraw its credit and signaling this decision to the arm's length banks.

Our model suggests that it essentially depends on this strategic interaction between the banks if a debtor firm receives forbearance lending and faces a soft budget constraint. We apply a global game as analyzed by Schüle and Stadler [2005] to demonstrate that the relationship bank's size and its informational advantage over the arm's length banks largely affect its signaling ability, which in turn exerts influence on the refinancing decision of the relationship bank and on the debtor firm's incentives to thwart default. Confining our attention to the limiting cases where the relationship bank is arbitrarily better or worse informed than the arm's length banks, we show that the relationship bank's willingness to engage in forbearance lending is increasing in its volume of credit and in its informational advantage. These results are affirmed by strong empirical evidence from Peek and Rosengren [2005]. Analyzing Japanese firm-level panel data, they find that banks are more likely to increase loans to financially distressed firms the greater these banks' proportion of total firm debt is and the closer their affiliations with the debtor firms are. Regarding the debtor firm's incentives to avoid bancruptcy, our model suggests that the effects of anticipated additional bank financing can be both positive or negative, depending on the fundamental state of the firm and the signaling ability of the relationship bank. Under certain conditions, the expectation of refinancing encourages the firm to incur more effort to work against default. However, in a situation where the debtor firm would avoid bancruptcy without assistance, the relationship bank's inability to commit credibly not to grant additional capital diminishes the effort of the firm, inducing debtor moral hazard à la Berglöf and Roland [1997, 1998].

Irrespective of the signaling effect, incorporating the relationship bank's option to withdraw its credit enables us to confirm a result of Berglöf and Roland [1997]. An increase in the firm's collateral lowers the relationship bank's incentives to engage in forbearance lending and hardens the budget constraint of the debtor firm in the sense that it reduces the severity of debtor moral hazard.

The remainder of this paper is organized as follows. In Section 2 we set up the model and describe the timing of events. Section 3 solves the coordination game among the banks. We analyze under which conditions the relationship bank is disposed to engage in forbearance lending and when the debtor firm exerts an effort to avoid bankruptcy. Section 4 examines the effects of refinancing on the behavior of the firm. Section 5 concludes.

2 The Model

The model considers a simple economy with three types of risk neutral agents: a firm, a relationship bank, and a continuum of arm's length banks. The firm decides to set up a risky investment project financed by the banks. A proportion $\lambda \in (0,1)$ of the required credit is provided by the relationship bank. Loans financed by the continuum of arm's length banks are negligibly small individually but amount to a combined mass of $1-\lambda$. Success or failure of the project are uncertain at the time of investment. If the project fails, the firm is forced into bancruptcy and is not able to refund the loans. If the project succeeds, the debtor firm remains in operation and banks receive the full face value of a loan, normalized to 1. At an interim stage before the project matures, both the relationship bank and the arm's length banks have the option to withdraw their credit and seize the collateral $\kappa \in (0,1)$. In order to meet the claims of foreclosing banks, the debtor firm can draw on available cash of $\theta \equiv \psi + e$. The variable $\psi \in \mathbb{R}$ denotes the underlying fundamental state of the investment project and can be interpreted as a measure of project quality, while $e \geq 0$ represents a costly restructuring effort undertaken by the firm to enhance its liquidity.

By assumption, the debtor firm can avoid the project's failure on its own and is able to repay the loans at maturity whenever $\theta \geq \ell$, where $\ell \in [0,1]$ denotes the proportion of loans that are foreclosed at the interim stage. Hence, success of the project is certain in case of high liquidity $\theta \geq 1$, and we say that the firm is insolvent if the project is doomed to failure without foreign support, i.e. if $\theta < 0$. In the intermediate range $0 \leq \theta < 1$, the debtor firm is solvent but illiquid and its fate lies entirely in the hands of the banks. While the arm's length banks can affect the project's success merely by their decision whether to roll over their loans or to foreclose, the relationship bank has an accessory influence capability. It is able to provide additional liquidity $m \geq 0$ to the firm in order to reduce the probability of default. With this refinancing from the relationship bank, the project succeeds whenever

$$\theta + m \geq \ell \ .$$

It is assumed that the additional credit m cannot be refunded by the firm, so that the relationship bank's costs of refinancing amount to m.

We can now describe our setup more formally by supplying the sequence of moves in the game, the information available at all stages, and the payoffs of all interested parties.

Period 0:

- The firm invests the bank loans in the project.
- Nature chooses the fundamental state ψ of the project. ψ becomes the firm's private information.
- The firm chooses the restructuring effort e, taking into account the value of ψ and the costs $c(e)$ associated with restructuring. Let $c(e)$ be any strictly increasing function with $c(0) = 0$.
- From the banks' perspective, $\theta = \psi + e$ is a random variable with an improper uniform prior over the real line, i.e. banks do not possess any public information on the firm's liquidity.[4] However, they receive private signals regarding the available cash of the firm. The relationship bank observes the realization of the noisy signal

$$y = \theta + \tau\eta \,, \tag{1}$$

where $\tau > 0$ is a scale factor indicating the amount of noise and η is a random variable drawn from the standard normal distribution. Likewise, an arm's length bank i receives the private signal

$$x_i = \theta + \sigma\varepsilon_i \tag{2}$$

with the scale factor $\sigma > 0$ and $\varepsilon_i \sim$ i.i.d. $N(0,1)$.

Period 1:

- Considering the private signal y, the relationship bank makes its roll over or foreclosure decision. If it decides not to foreclose on its loans, the bank chooses an amount of refinancing $m \geq 0$ so as to maximize its payoff

$$u(\theta, m, \ell) = \begin{cases} 1 - m & \text{if } \theta + m \geq \ell \\ -m & \text{if } \theta + m < \ell \end{cases}$$

from rolling over a loan. The amount of refinancing m is announced publicly and hence is common knowledge among all.

[4] As Morris and Shin [2003] point out, improper uniform priors are well behaved, as far as we are concerned only with conditional beliefs, and can be interpreted as the limiting case where the information in the prior density becomes diffuse.

- The arm's length banks are able to observe the relationship bank's behavior and make their own roll over decisions based on this observation and on their private signals x_i. Their payoff from extending the credit is given by

$$v(\theta, m, \ell) = \begin{cases} 1 & \text{if } \theta + m \geq \ell \\ 0 & \text{if } \theta + m < \ell \, . \end{cases}$$

Period 2:

- The project matures. Payoffs of the firm and of banks that have rolled over their loans are realized.

- Clearly, the payoff of the debtor firm has to be an increasing function of its liquidity θ. For the sake of simplicity, we assume without further loss of generality that the firm's payoff is identical to the payoff of an arm's length bank except for the costs of restructuring $c(e)$:

$$v(\theta, m, \ell) - c(e) = \begin{cases} 1 - c(e) & \text{if } \theta + m \geq \ell \\ -c(e) & \text{if } \theta + m < \ell \, . \end{cases}$$

3 The Equilibrium

In this section we solve the model by backwards induction, analyzing successively the banks' roll over strategies, the refinancing decision of the relationship bank, and the restructuring effort undertaken by the firm. This enables us to determine how signaling by the relationship bank affects its willingness to engages in forbearance lending and under which conditions this behavior gives rise to soft budget constraints for the debtor firm.

3.1 Roll Over Decision of the Banks

In order to solve the subgame where the banks make their roll over or foreclosure decisions conditional on the debtor firm's liquidity and the amount of refinancing granted by the relationship bank, we adopt a global game as analyzed by Schüle and Stadler [2005]. Within the class of switching-strategies this sequential-move game has a unique equilibrium, characterizing the behavior of the banks by the 5-tuple $(\hat{y}, \underline{x}, \overline{x}, \underline{\theta}, \overline{\theta})$. In equilibrium, the relationship bank decides to foreclose on

its loans if and only if the private signal y is below the threshold signal $\hat{y}$. Arm's length banks observe the decision of the relationship bank and use this additional information to update their beliefs regarding the firm's liquidity. Thus, an arm's length bank i withdraws its credit only for private signals $x_i < \underline{x}$ if it has observed the relationship bank rolling over and refinancing, but it will even stop lending for signals below the upper threshold $\overline{x}$ if the relationship bank has foreclosed before. Since the private signals of the banks are correlated with the firm's liquidity θ, there exist threshold values $\underline{\theta}$ and $\overline{\theta}$ corresponding to the respective critical signals $\underline{x}$ and $\overline{x}$, so that the success of the project entirely depends on the relationship bank's behavior whenever $\underline{\theta} \leq \theta < \overline{\theta}$.

To derive these equilibrium thresholds, first consider the decision of the relationship bank. As its expected payoff from rolling over and refinancing is given by

$$Pr(\theta \geq \underline{\theta}|y) - m = \Phi\left(\frac{y - \underline{\theta}}{\tau}\right) - m \ ,$$

the critical signal $\hat{y}$ has to solve the indifference condition

$$\Phi\left(\frac{\hat{y} - \underline{\theta}}{\tau}\right) - m = \kappa \ ,$$

so that

$$\hat{y} = \underline{\theta} + \tau\Phi^{-1}(\kappa + m) \ . \tag{3}$$

If the relationship bank forecloses on its loans, the project succeeds if and only if $\theta \geq \overline{\theta}$. Thus, the relevant threshold signal $\overline{x}$ for the arm's length banks is implicitly defined by the cutoff condition

$$Pr(\theta \geq \overline{\theta} \mid y < \hat{y},\ x_i = \overline{x}) = \kappa \ . \tag{4}$$

In this case, the proportion of loans foreclosed at the interim stage amounts to $\ell = \lambda + (1 - \lambda)Pr(x_i < \overline{x}|\theta)$. Since the relationship bank stops lending, it has no incentive to refinance and thus the liquidity threshold $\overline{\theta}$ for a successful completion of the project solves the critical mass condition

$$\overline{\theta} = \lambda + (1 - \lambda)Pr(x_i < \overline{x}|\theta = \overline{\theta}) \ . \tag{5}$$

If the relationship bank decides to roll over and refinance instead, due to a private signal $y \geq \hat{y}$, the threshold signal $\underline{x}$ where the arm's length banks are indifferent between premature foreclosure and continued lending is given by

$$Pr(\theta \geq \underline{\theta}|y \geq \hat{y}, x_i = \underline{x}) = \kappa \tag{6}$$

and the corresponding critical mass condition is

$$\underline{\theta} + m = (1 - \lambda) Pr(x_i < \underline{x} | \theta = \underline{\theta}) \; . \tag{7}$$

To obtain the threshold values characterizing the equilibrium of this subgame, the equations (3) to (7) have to be solved simultaneously. While explicit solutions are not procurable for general parameter values, we are able to derive the equilibrium thresholds in the two limiting cases where the relationship bank is arbitrarily better and worse informed, respectively, than the arm's length banks.

PROPOSITION 1 *The firm's liquidity thresholds contingent on the relationship bank's roll over or foreclosure decision and the amount of refinancing m converge to*

$$\underline{\theta} = -m \tag{8}$$

$$\overline{\theta} = 1 \tag{9}$$

in case of an arbitrarily better informed relationship bank ($\sigma/\tau \to \infty$), and they tend to

$$\underline{\theta} = \kappa(1 - \lambda) - m \tag{10}$$

$$\overline{\theta} = \kappa(1 - \lambda) + \lambda \tag{11}$$

if the relationship bank is arbitrarily worse informed ($\sigma/\tau \to 0$).

PROOF. *See the Appendix.*

This proposition implies that there are two ways for the relationship bank to exert influence on the success of the firm's investment project. *Directly*, granting additional bank financing might bail out a debtor firm in financial distress by lowering the relevant threshold $\underline{\theta}$. In addition to this *direct* influence capability analyzed by TAKEDA AND TAKEDA [2005], our model suggests that the relationship bank can affect the fate of the debtor firm also *indirectly* by signaling its roll over or foreclosure decision. As the liquidity thresholds in Propostion 1 indicate, this ability to coordinate the behavior of arm's length banks is considerably influenced by the relationship bank's size λ and by its informational advantage σ/τ.

As $\sigma/\tau \to \infty$, the thresholds $\underline{\theta}$ and $\overline{\theta}$ converge to their lower and upper bounds, respectively, indicating that an infinitely better informed relationship bank *indirectly* exerts the maximum influence on the arm's length banks' roll over strategy. They follow the credit decision of the relationship bank blindly, irrespective of their own

private signals regarding θ and regardless of the relationship bank's size λ. Anticipating this pure herding behavior among the arm's length banks, the relationship bank acts as if it was the only lender. But even an informationally disadvantaged relationship bank in the opposite extreme case when $\sigma/\tau \to 0$ has an impact on the arm's length banks' roll over or foreclosure decision. However, since the relationship bank does not possess any informational signaling ability in this case, it can affect the equilibrium outcome of the game *indirectly* only in as much as its size λ is relevant. As $\lambda \to 1$, the relationship bank is essentially the only lender and the liquidity thresholds of the debtor firm correspond to those in case when $\sigma/\tau \to 0$. In contrast, as $\lambda \to 0$, the firm's liquidity thresholds $\underline{\theta}$ and $\overline{\theta}$ coincide except for the amount of refinancing m, implying that an arbitrarily worse informed relationship bank can influence the success of the project merely *directly* by providing additional bank financing. This special case of our analysis corresponds to the benchmark model of Takeda and Takeda [2005] which abstracts from signaling.

3.2 Refinancing by the Relationship Bank and Implications on Forbearance Lending

As we have derived the liquidity thresholds for the project's success contingent on the relationship bank's roll over or foreclosure decision and the amount of additional bank financing m, we are now in a position to analyze the refinancing strategy of the relationship bank. Taking into consideration the two limiting cases of an arbitrarily better and worse informed relationship bank allows us to determine under which conditions refinancing of financially distressed debtor firms actually degenerates into forbearance lending.

First, consider the case where the relationship bank obtains much more precise private information regarding the debtor firm's liquidity than the arm's length banks ($\sigma/\tau \to \infty$). Taking into account its private signal y, the relationship bank chooses an amount of additional bank financing $m \geq 0$ to maximize the expected payoff

$$Pr(\theta \geq \underline{\theta}|y) - m = \Phi\left(\frac{y+m}{\tau}\right) - m \tag{12}$$

whenever rolling over and refinancing is profitable. To ensure that a solution for the above optimization problem exists, assume that the relationship bank's signal y is sufficiently precise, $\tau \in (0, \frac{1}{\sqrt{2\pi}}]$. Then, maximization of the expected payoff (12) with respect to m yields

$$m(y) = -y + R\,, \tag{13}$$

where $R \equiv \tau\phi^{-1}(\tau) > 0$.[5] Since $m \geq 0$ by definition, equation (13) implies that the relationship bank has no incentive to grant additional credit for sufficiently high private signals $y \geq R$. However, it also abstains from refinancing if foreclosing on the loans is a dominant strategy, i.e. if the expected payoff from rolling over and choosing the optimal amount of refinancing is lower than the debtor firm's collateral,

$$\Phi\left(\frac{y + m(y)}{\tau}\right) - m(y) < \kappa \, .$$

Rearranging this condition implies that the relationship bank stops lending and does not engage in additional financing of the debtor firm if

$$y < R - S \, ,$$

where $S \equiv \Phi(R/\tau) - \kappa$. Hence, in the limiting case when $\sigma/\tau \to \infty$, the optimal amount of refinancing m as a function of the relationship bank's private signal y can be expressed as

$$m^*(y) = \begin{cases} 0 & \text{if } y < R - S \\ -y + R & \text{if } R - S \leq y < R \\ 0 & \text{if } y \geq R \, . \end{cases} \tag{14}$$

In case of good expectations regarding the firm's liquidity, $y \geq R$, the relationship bank extends its loans but does not provide additional capital since the costs of refinancing exceed its benefits in terms of a higher probability of success. In contrast, low private signals $y < R - S$ imply that the expected payoff from rolling over and refinancing is lower than the collateral κ. In this case it is a dominant strategy for the relationship bank to stop lending and abstain from refinancing. However, the relationship bank has an incentive to roll over its loans and grant additional credit $m > 0$ for private signals $R - S \leq y < R$ whenever $S > 0$. That is, the relationship bank engages in refinancing for intermediate values of y if and only if the debtor firm's collateral κ is sufficiently low, $\kappa < \Phi(\phi^{-1}(\tau))$. Beyond this, we can infer from $\partial S/\partial\kappa < 0$ that the relationship bank is more likely to provide additional capital the lower the collateral it could seize in case of premature foreclosure.

By our definition, refinancing degenerates into forbearance lending if the relationship bank is disposed to bail out economically insolvent debtor firms with $\theta < 0$. To derive more clear-cut results regarding the incidence of forbearance lending, consider the case where the private signal of the relationship bank becomes arbitrarily precise.

[5] Proof. See the Appendix.

As $\tau \to 0$, we obtain $y \to \theta$, $R \to 0$, and $S \to 1-\kappa$, so that the refinancing strategy (14) transforms to

$$m^*(\theta) = \begin{cases} 0 & \text{if } \theta < \kappa - 1 \\ -\theta & \text{if } \kappa - 1 \leq \theta < 0 \\ 0 & \text{if } \theta \geq 0 \, . \end{cases} \tag{15}$$

This result makes intuitive sense. In the case when $\sigma/\tau \to \infty$, an infinitely precise informed relationship bank is willing to bail out the debtor firm if it is insolvent, but not hopelessly so. If $\kappa - 1 \leq \theta < 0$, it is a rational choice for the relationship bank to engage in forbearance lending, since the costs of refinancing are overcompensated by the increase of the expected payoff from rolling over. However, there is no need to refinance the investment project of a solvent firm, since the informational signaling ability of the relationship bank completely averts coordination failure among the arm's length banks. Thus, additional bank financing necessarily degenerates into forbearance lending if the relationship bank is much better informed than the arm's length banks.

To analyze the effects of signaling by the relationship bank on the incidence of forbearance lending, compare these results with the opposite extreme case where $\sigma/\tau \to 0$. The informationally disadvantaged relationship bank chooses the amount of refinancing $m \geq 0$ that maximizes

$$Pr(\theta \geq \underline{\theta}|y) - m = \Phi\left(\frac{y - \kappa(1-\lambda) + m}{\tau}\right) - m$$

whenever rolling over and refinancing yields an expected payoff not lower than κ. By similar reasoning as above, the incidence of additional bank financing contingent on the relationship bank's private signal y can be derived as

$$m^*(y) = \begin{cases} 0 & \text{if } y < \kappa(1-\lambda) + R - S \\ \kappa(1-\lambda) - y + R & \text{if } \kappa(1-\lambda) + R - S \leq y < \kappa(1-\lambda) + R \\ 0 & \text{if } y \geq \kappa(1-\lambda) + R \, . \end{cases} \tag{16}$$

This solution indicates that an infinitely worse informed relationship bank also engages in refinancing for intermediate values of the private signal y if $S > 0$. Thus, irrespective of the relationship bank's signaling ability, additional credit is granted to a firm in financial distress only if its collateral κ is sufficiently small. A comparison of (14) and (16) shows to what extent signaling by the relationship bank exerts influence on the conditions for additional bank financing. Since a relatively uninformed relationship bank can affect the credit decision of the arm's length banks

only in as much as its size λ is relevant, the interval of private signals y where refinancing takes place is shifted to the right by $\kappa(1-\lambda)$. As $\lambda \to 0$, the signaling effect of the arbitrarily worse informed relationship bank vanishes, so that our refinancing strategy (16) coincides with the result of Takeda/Takeda (2005) who disregard the strategic interaction between banks in their model.

As the relative precision of information σ/τ and the volume of credit λ influence the conditions under which a relationship bank engages in refinancing, these factors also have an impact on the incidence of forbearance lending. In the limit as $\tau \to 0$,[6] additional financing by an arbitrarily worse informed relationship bank is given by

$$m^*(\theta) = \begin{cases} 0 & \text{if } \theta < \kappa(2-\lambda) - 1 \\ \kappa(1-\lambda) - \theta & \text{if } \kappa(2-\lambda) - 1 \leq \theta < \kappa(1-\lambda) \\ 0 & \text{if } \theta \geq \kappa(1-\lambda)\ . \end{cases} \tag{17}$$

Obviously, the limited signaling ability of an informationally disadvantaged relationship bank compared to the case when $\sigma/\tau \to \infty$ diminishes its incentives to bail out an intrinsically insolvent debtor firm with $\theta < 0$. Since the relationship bank does not posses any informational signaling ability as $\sigma/\tau \to 0$, the signaling effect is merely a result of its size λ. Thus, the relationship bank engages in forbearance lending if and only if its credit as proportion of total firm debt is sufficiently large, $\lambda > 2 - \frac{1}{\kappa}$. Otherwise forbearance lending is not profitable for the relationship bank since its limited capability to coordinate the credit decisions of arm's length banks causes too severe coordination failure.

Although we had to restrict our analysis of the refinancing decision to the limiting cases where the relationship bank is arbitrarily better or worse informed relative to the arm's length banks, our findings indicate that the relationship bank's signaling ability exerts considerable influence on the incidence of forbearance lending. Comparing our results with the benchmark model of Takeda and Takeda [2005], we find that the relationship bank's ability to signal its roll over decision reduces the costs of forbearance lending as it dilutes coordination failure among the arm's length banks. Hence, an intrinsically insolvent debtor firm can rather expect to be rescued from bancruptcy if the credit granted by the relationship bank is large as a proportion of total firm debt and if the relationship bank is relatively well informed compared to the arm's length banks. In the extreme cases where the relationship bank can completely avert coordination failure either by its informational signaling

[6] As $\tau \to 0$ in the case of an arbitrarily worse informed relationship bank, σ has to converge to 0 with an infinitely faster rate to ensure that $\sigma/\tau \to 0$ holds.

ability ($\sigma/\tau \to \infty$) or by its size ($\lambda \to 1$), granting additional bank financing to a debtor firm necessarily degenerates into forbearance lending.

3.3 The Restructuring Effort of the Firm

To determine the restructuring effort undertaken by the debtor firm in equilibrium, we confine our attention to the limiting case of an infinitely precise informed relationship bank. Taking the limit as $\tau \to 0$ facilitates our analysis to the effect that the investment project succeeds if and only if the relationship bank rolls over its loans and grants the optimal amount of refinancing.[7] Anticipating the subsequent behavior of the banks, the debtor firm chooses the restructuring effort e so as to maximize its payoff

$$u(e) = \begin{cases} 1 - c(e) & \text{if } y \geq \hat{y} \\ -c(e) & \text{if } y < \hat{y} \ . \end{cases}$$

Again, we consider the two limiting cases where the relationship bank obtains arbitrarily more and less precise signals than the arm's length banks in order to analyze to what extent the signaling effect has an impact on the restructuring effort of the firm.

First, suppose that the relationship bank is very accurately informed compared to the arm's length banks ($\sigma/\tau \to \infty$). As $\tau \to 0$, we obtain $y \to \theta = \psi + e$ and $\hat{y} \to \kappa - 1$, so that the optimization problem of the debtor firm can be expressed as

$$\max_{e \geq 0} \quad u(e) = \begin{cases} 1 - c(e) & \text{if } \psi + e \geq \kappa - 1 \\ -c(e) & \text{if } \psi + e < \kappa - 1 \ . \end{cases} \tag{18}$$

Considering without further loss of generality the case of the linear cost function $c(e) = e$, the firm's optimal effort level as a function of the fundamentals ψ can be explicitly derived as

$$e^*(\psi) = \begin{cases} 0 & \text{if } \psi < \kappa - 2 \\ \kappa - 1 - \psi & \text{if } \kappa - 2 \leq \psi < \kappa - 1 \\ 0 & \text{if } \psi \geq \kappa - 1 \ . \end{cases} \tag{19}$$

[7] Deviating from the limit as $\tau \to 0$ would merely complicate the analysis without changing our results qualitatively. We would have to consider four cases: 1. the relationship bank rolls over and the project succeeds, 2. the relationship bank rolls over and the project fails, 3. the relationship bank stops lending and the project succeeds, 4. the relationship bank stops lending and the project fails.

The debtor firm embarks on a costly restructuring effort only for intermediate values of project qualities, and the effort level diminishes linearly in ψ. If the underlying fundamental state of the project is sound, $\psi \geq \kappa - 1$, the debtor firm anticipates that refinancing and continued lending by the banks are sufficient for the project to succeed even without restructuring. In contrast, if $\psi < \kappa - 2$, the project is doomed to failure even with an effort, so that the debtor firm abstains from restructuring.

Analogously, we can determine the optimal restructuring effort of the debtor firm in case of an arbitrarily worse informed relationship bank when $\sigma/\tau \to 0$. In the limit as $\tau \to 0$, the firm has to choose e to solve

$$\max_{e \geq 0} \quad u(e) = \begin{cases} 1 - c(e) & \text{if } \psi + e \geq \kappa(2-\lambda) - 1 \\ -c(e) & \text{if } \psi + e < \kappa(2-\lambda) - 1 \, . \end{cases} \tag{20}$$

For the linear cost function $c(e) = e$, the optimal effort level is given by

$$e^*(\psi) = \begin{cases} 0 & \text{if } \psi < \kappa(2-\lambda) - 2 \\ \kappa(2-\lambda) - 1 - \psi & \text{if } \kappa(2-\lambda) - 2 \leq \psi < \kappa(2-\lambda) - 1 \\ 0 & \text{if } \psi \geq \kappa(2-\lambda) - 1 \, . \end{cases} \tag{21}$$

Again, the firm has an incentive to invest in restructuring if and only if its project is of intermediate quality, and the restructuring effort is linearly decreasing in ψ.

Comparing the solutions (19) and (21), we find that restructuring by the firm occurs in both cases with the same probability ex ante, before nature chooses the project's fundamental state ψ. However, as the signaling ability of the relationship bank affects coordination among the arm's length banks, it also exerts influence on the conditions under which the debtor firm embarks on a costly restructuring effort. The firm has an incentive to work against default in case of lower quality projects if the relationship bank's size λ and its informational advantage σ/τ are large, i.e. if signaling by the relationship bank diminishes coordination failure among the credit decisions of the arm's length banks.

4 Implications on Soft Budget Constraints

Having derived the equilibrium behavior of the relationship bank and the debtor firm in the last section, we are now able to analyze in what circumstances anticipated refinancing by the relationship bank actually leads to a soft budget constraint for the firm. By our definition, a firm has a soft budget constraint if and only if it can

expect to receive financial support from the relationship bank and if this expectation diminishes the debtor firm's incentives to embark on a costly restructuring effort. To determine the incidence of such debtor moral hazard we have to compare the effort level of a firm with anticipated additional bank financing to a world in which the relationship bank can credibly commit not to refinance. Again, we are particularly interested to what extent the signaling ability of the relationship bank in terms of its size and the relative informational advantage impacts our results. Thus, we consider both limiting cases of an arbitrarily better and worse informed relationship bank when analyzing soft budget constraints of the debtor firm.

Suppose that the relationship bank is significantly better informed than the arm's length banks. If the firm is not able to anticipate refinancing, the lower liquidity threshold $\underline{\theta}$ for the project's success converges to 0 as $\sigma/\tau \to \infty$, and thus the debtor firm's optimization problem (18) transforms to

$$\max_{e \geq 0} \quad u(e) = \begin{cases} 1 - c(e) & \text{if } \psi + e \geq 0 \\ -c(e) & \text{if } \psi + e < 0 \, . \end{cases} \tag{22}$$

The difference between (18) and (22) is that the minimum level of liquidity $\psi + e$ required to avoid the project's default has to be larger without anticipated refinancing by the relationship bank. Again considering the case of the linear cost function $c(e) = e$, the debtor firm's effort decision contingent on the project quality ψ can be derived as

$$\hat{e}^*(\psi) = \begin{cases} 0 & \text{if } \psi < -1 \\ -\psi & \text{if } -1 \leq \psi < 0 \\ 0 & \text{if } \psi \geq 0 \, . \end{cases} \tag{23}$$

A comparison of (19) and (23) indicates that the expectation of being refinanced by an arbitrarily better informed relationship bank exerts influence on the debtor firm's restructuring effort whenever $\kappa - 2 \leq \psi < 0$. However, in our model the relationship bank's inability to commit credibly not to bail out a firm in financial distress does not necessarily lead to soft budget constraints as in Dewatripont and Maskin [1995] or Berglöf and Roland [1997, 1998]. In fact, if $\kappa - 2 \leq \psi < -1$ the debtor firm's incentives to thwart default are higher compared to a situation in which the firm cannot expect to receive additional credit. Anticipating the support of the relationship bank, the debtor firm feels encouraged to incur a costly restructuring effort that is just sufficient to avoid bancruptcy. Without anticipated refinancing however, the costs of preventing the project's failure would be prohibitive, so that the firm would abstain from restructuring and default. In contrast, a debtor firm

expecting additional financing from an arbitrarily better informed relationship bank faces a soft budget constraint whenever $-1 \leq \psi < 0$. In this case the project quality ψ lies in the region where the debtor firm would avoid bankruptcy by its own effort without assistance, but the anticipation of refinancing by the relationship bank reduces the firm's incentives to work against default.

To investigate the effects of the relationship bank's signaling ability on the incidence of soft budget constraints let us compare these results with the case of an infinitely worse informed relationship bank. As $\sigma/\tau \to 0$, $\underline{\theta}$ converges to $\kappa(1-\lambda)$ if the relationship bank can commit credibly not to refinance the debtor firm, and thus, as $\tau \to 0$, (20) transforms to

$$\max_{e \geq 0} \quad u(e) = \begin{cases} 1 - c(e) & \text{if } \psi + e \geq \kappa(1-\lambda) \\ -c(e) & \text{if } \psi + e < \kappa(1-\lambda) \ . \end{cases} \tag{24}$$

If the cost function is given by $c(e) = e$, the solution to this optimization problem amounts to

$$\hat{e}^*(\psi) = \begin{cases} 0 & \text{if } \psi < \kappa(1-\lambda) - 1 \\ \kappa(1-\lambda) - \psi & \text{if } \kappa(1-\lambda) - 1 \leq \psi < \kappa(1-\lambda) \\ 0 & \text{if } \psi \geq \kappa(1-\lambda) \ . \end{cases} \tag{25}$$

Comparing the debtor firm's effort decision (21) and (25) in case of an infinitely worse informed relationship bank implies that the bank's inability to commit credibly not to grant additional credit again exerts ambiguous influence on the firm's restructuring effort. For intermediate values of the project quality, $\kappa(2-\lambda) - 2 \leq \psi < \kappa(1-\lambda) - 1$, anticipated refinancing by the relationship bank prompts the debtor firm to embark on a costly restructuring effort, averting bancruptcy of the firm. However, if $\kappa(1-\lambda) - 1 \leq \psi < \kappa(1-\lambda)$ the firm faces a soft budget constraint, as the relationship bank's incapability to commit credibly not to pursue a policy of forbearance diminishes the effort level of the debtor firm.

Table 1:

Effects of anticipated refinancing by the relationship bank

Anticipated refinancing leads to	The relationship bank is relatively informed $(\sigma/\tau \to \infty, \tau \to 0)$	uninformed $(\sigma/\tau \to 0, \tau \to 0)$
Increased effort	$\kappa - 2 \leq \psi < -1$	$\kappa(2-\lambda) - 2 \leq \psi < \kappa(1-\lambda) - 1$
Soft budget constraints	$-1 \leq \psi < 0$	$\kappa(1-\lambda) - 1 \leq \psi < \kappa(1-\lambda)$

Table 1 depicts to what extent the relationship bank's signaling ability exerts influence on the debtor firm's incentives to exert a costly restructuring effort. Note that the length of the respective intervals in the table is equal and independent of the relationship bank's size λ. Thus, neither the relative precision of information σ/τ nor the volume of credit λ impact the ex ante probability of soft budget constraints and increased effort by the debtor firm, before nature chooses the project quality ψ. However, the ability of the relationship bank to coordinate the roll over and foreclosure decisions of the arm's length banks essentially affects the conditions under which a firm faces a soft budget constraint or is disposed to increase its effort. As the table reveals, the regions in which anticipated refinancing influences the level of the firm's restructuring effort are shifted to the right by $\kappa(1-\lambda)$ in the case when $\sigma/\tau \to 0$. Hence, if the relationship bank can essentially diminish coordination failure among the arm's length banks by signaling its roll over decision, soft budget constraints rather appear in situations where we expect them to occur: when firms conduct unprofitable projects and thus are intrinsically insolvent.

Our model implies that soft budget constraints arise due to a dynamic commitment problem of the relationship bank. Thus, the relationship bank would have to improve the credibility of its commitment not to engage in refinancing in order to harden a firm's budget constraint. Berglöf and Roland [1998] argue that the collateral of the debtor firm can serve as a commitment device for a bank if it restores the bank's credibility not to pursue a policy of forbearance. In their model, a sufficiently high value of collateral necessarily gives rise to a hard budget constraint, inducing the firm to exert high effort. As our model incorporates the relationship bank's option to foreclose on its loans at an interim stage and seize the collateral κ, we are able to confirm and enhance this finding of Berglöf and Roland [1998]. Irrespective of the relationship bank's signaling ability, our results on the effort decision of the firm state that the "softness" of the debtor firm's budget constraint, i.e. the severity of debtor moral hazard $\triangle e = \hat{e}^* - e^*$, is decreasing in κ. An increase in the debtor firm's collateral κ leads to greater incentives for the relationship bank to refrain from refinancing and to foreclose on its loans instead. Anticipating this behavior, a highly collateralized firm incurs more restructuring effort to avoid bankruptcy, thus diluting the severity of debtor moral hazard. In the limit as $\kappa \to 1$, it is a strictly dominant strategy for the relationship bank to stop lending and abstain from refinancing, so that the debtor firm faces a hard budget constraint as in Berglöf and Roland [1998].

5 Concluding Remarks

This paper has introduced a model of coordination failure among multiple heterogeneous creditors to account for the appearance of forbearance lending and soft budget constraints in credit markets. Under certain conditions it might be a rational choice for a bank to refinance an intrinsically insolvent debtor firm instead of writing off its non-performing loans. This inability to commit credibly not to engage in forbearance lending often gives rise to a soft budget constraint, as it reduces the firm's incentives to thwart default.

Our model endogenously determined the equilibrium behavior of a relationship bank, a continuum of arm's length banks, and a debtor firm under incomplete private information regarding the firm's financial condition. Augmenting the game theoretic framework of Morris and Shin [2005] and Takeda and Takeda [2005], we modeled the relationship bank as a strategic player which can affect the fate of the debtor firm by foreclosing or extending its credit, refinancing the firm, and signaling its decisions to the arm's length banks. We adopted the global game framework from Schüle and Stadler [2005] to analyze how signaling by the relationship bank impacts the severity of coordination failure among the arm's length banks. This enabled us to examine to what extent the relationship bank's attributes exert influence on the incidence of forbearance lending and soft budget constraints. We found that a relationship bank is more likely to engage in forbearance lending the greater its credit as a proportion of total firm debt is and the more precise its private information compared to the arm's length banks' information is. Analyzing the credit market in Japan, Peek and Rosengren [2005] find strong empirical evidence for our results. They show that financially distressed firms predominantly receive refinancing from their main banks, which typically are in close relation to their debtor, and that additional bank financing is more likely to occur the greater these banks' proportion of total firm debt is. Regarding the effects of refinancing by the relationship bank on the firm's incentives to exert a costly restructuring effort, we were able to specify the findings of Takeda and Takeda [2005]. In our model it does not only depend on the fundamental state of a debtor firm whether anticipated additional bank financing increases the firm's effort to avoid bankruptcy or induces a soft budget constraint, but also on the relationship bank's signaling ability. Irrespective of this signaling effect, our results state that the debtor firm's value of collateral crucially impacts the decisions of the relationship bank and the firm. Confirming the findings of Berglöf and Roland [1998], we found that a higher collateral value diminishes the relationship bank's willingness to engage in forbearance lending and hardens the firm's budget constraint as it dilutes the severity of debtor moral hazard.

Summarizing our results, we believe that the signaling ability of a large and powerful lender can exert considerable influence on the occurrence of soft budget constraints and forbearance lending. In reference to the empirical prevalence of relationship lending, this signaling effect should thus be regarded when analyzing incentive problems in credit markets with asymmetric information.

Appendix

A.1 Proof of Proposition 1

From (1) and (2), the private signal of the relationship bank can be rewritten as

$$y = x_i + \tau\eta - \sigma\varepsilon_i \ . \tag{A1}$$

Using the equations (3) and (A1), an arm's length bank's posterior probability assessment of the project's success, conditional on the signal x_i and observing the relationship bank continuing lending, can be expressed as

$$\begin{aligned} Pr\left(\theta \geq \underline{\theta} \mid y \geq \hat{y}, x_i\right) &= Pr\left(x_i - \sigma\varepsilon_i \geq \underline{\theta} \mid x_i + \tau\eta - \sigma\varepsilon_i \geq \underline{\theta} + \tau\Phi^{-1}(\kappa + m)\right) \\ &= Pr\left(\varepsilon_i \leq \frac{x_i - \underline{\theta}}{\sigma} \middle| \tau\eta - \sigma\varepsilon_i \geq \underline{\theta} - x_i + \tau\Phi^{-1}(\kappa + m)\right), \end{aligned}$$

so that the cutoff condition (6) transforms to

$$\frac{Pr\left(\varepsilon_i \leq \frac{\underline{x}-\underline{\theta}}{\sigma}, \tau\eta - \sigma\varepsilon_i \geq \underline{\theta} - \underline{x} + \tau\Phi^{-1}(\kappa + m)\right)}{Pr\left(\tau\eta - \sigma\varepsilon_i \geq \underline{\theta} - \underline{x} + \tau\Phi^{-1}(\kappa + m)\right)} = \kappa \ . \tag{A2}$$

By similar reasoning, the indifference condition (4) in case of premature foreclosure by the relationship bank can be rewritten as

$$\frac{Pr\left(\varepsilon_i \leq \frac{\overline{x}-\overline{\theta}}{\sigma}, \tau\eta - \sigma\varepsilon_i < \underline{\theta} - \overline{x} + \tau\Phi^{-1}(\kappa + m)\right)}{Pr\left(\tau\eta - \sigma\varepsilon_i < \underline{\theta} - \overline{x} + \tau\Phi^{-1}(\kappa + m)\right)} = \kappa \ . \tag{A3}$$

Explicit solutions for the equations (A2) and (A3) cannot be derived for general parameter values, but for the limiting cases of an arbitrarily better ($\sigma/\tau \to \infty$) or worse ($\sigma/\tau \to 0$) informed relationship bank.

First, consider the case when $\sigma/\tau \to \infty$. Rewrite equation (A2) as

$$\frac{Pr\left(\varepsilon_i \leq \frac{\underline{x}-\underline{\theta}}{\sigma}, \frac{\tau}{\sigma}\eta - \varepsilon_i \geq \frac{\underline{\theta}-\underline{x}}{\sigma} + \frac{\tau}{\sigma}\Phi^{-1}(\kappa + m)\right)}{Pr\left(\frac{\tau}{\sigma}\eta - \varepsilon_i \geq \frac{\underline{\theta}-\underline{x}}{\sigma} + \frac{\tau}{\sigma}\Phi^{-1}(\kappa + m)\right)} = \kappa$$

and take the limit as $\sigma/\tau \to \infty$:

$$\frac{Pr\left(\varepsilon_i \leq \frac{\underline{x}-\underline{\theta}}{\sigma}, -\varepsilon_i \leq \frac{\underline{\theta}-\underline{x}}{\sigma}\right)}{Pr\left(\varepsilon_i \leq \frac{\underline{x}-\underline{\theta}}{\sigma}\right)} = 1 > \kappa \ . \tag{A4}$$

It follows from (A4) that the critical signal of an arm's length bank which has observed the relationship bank rolling over its loans, tends to

$$\underline{x} \to -\infty \ . \tag{A5}$$

Hence, the probability in equation (7) is equal to 0, implying that (8) holds. By the same token, equation (A3) can be transformed to

$$\frac{Pr\left(\varepsilon_i \leq \frac{\overline{x}-\overline{\theta}}{\sigma}, \frac{\tau}{\sigma}\eta - \varepsilon_i < \frac{\overline{\theta}-\underline{x}}{\sigma} + \frac{\tau}{\sigma}\Phi^{-1}(\kappa+m)\right)}{Pr\left(\frac{\tau}{\sigma}\eta - \varepsilon_i < \frac{\overline{\theta}-\underline{x}}{\sigma} + \frac{\tau}{\sigma}\Phi^{-1}(\kappa+m)\right)} = \kappa \ ,$$

and reduces to

$$\frac{Pr\left(\varepsilon_i \leq \frac{\overline{x}-\overline{\theta}}{\sigma}, \varepsilon_i > \frac{\underline{x}-\overline{\theta}}{\sigma}\right)}{Pr\left(\varepsilon_i > \frac{\underline{x}-\overline{\theta}}{\sigma}\right)} = 0 < \kappa \tag{A6}$$

in the limit as $\sigma/\tau \to \infty$. It follows from (A6) that the critical signal of an arm's length bank, having observed the relationship bank foreclosing on its loans, tends to

$$\overline{x} \to \infty \ . \tag{A7}$$

Hence, the probability in equation (5) is equal to 1, implying that (9) holds.

Consider the opposite extreme case with $\sigma/\tau \to 0$. Equation (A2) can be rewritten as

$$\frac{Pr\left(\varepsilon_i \leq \frac{\underline{x}-\underline{\theta}}{\sigma}, \eta - \frac{\sigma}{\tau}\varepsilon_i \geq \frac{\underline{\theta}-\underline{x}}{\tau} + \Phi^{-1}(\kappa+m)\right)}{Pr\left(\eta - \frac{\sigma}{\tau}\varepsilon_i \geq \frac{\underline{\theta}-\underline{x}}{\tau} + \Phi^{-1}(\kappa+m)\right)} = \kappa \ .$$

Taking the limit as $\sigma/\tau \to 0$ yields

$$\frac{Pr\left(\varepsilon_i \leq \frac{\underline{x}-\underline{\theta}}{\sigma}, \eta \geq \frac{\underline{\theta}-\underline{x}}{\tau} + \Phi^{-1}(\kappa+m)\right)}{Pr\left(\eta \geq \frac{\underline{\theta}-\underline{x}}{\tau} + \Phi^{-1}(\kappa+m)\right)} = \kappa$$

and reduces to

$$Pr\left(\varepsilon_i \leq \frac{\underline{x}-\underline{\theta}}{\sigma}\right) = \Phi\left(\frac{\underline{x}-\underline{\theta}}{\sigma}\right) = \kappa \tag{A8}$$

since the error terms ε_i and η are independent. Rearranging (A8) provides an arm's length bank's critical signal if it observes the relationship bank rolling over:

$$\underline{x} = \underline{\theta} + \sigma\Phi^{-1}(\kappa) \ . \tag{A9}$$

Substituting (A9) into equation (7) implies that (10) must hold:

$$\begin{aligned}
\underline{\theta} &= (1-\lambda)Pr(\underline{\theta} + \sigma\varepsilon_i < \underline{\theta} + \sigma\Phi^{-1}(\kappa)) - m \\
&= (1-\lambda)\ \Phi\left(\Phi^{-1}(\kappa)\right) - m \\
&= \kappa(1-\lambda) - m \ .
\end{aligned}$$

Equation (A3) can be rewritten as

$$\frac{Pr\left(\varepsilon_i \leq \frac{\overline{x}-\overline{\theta}}{\sigma}, \eta - \frac{\sigma}{\tau}\varepsilon_i < \frac{\theta-\overline{x}}{\tau} + \Phi^{-1}(\kappa+m)\right)}{Pr\left(\eta - \frac{\sigma}{\tau}\varepsilon_i < \frac{\theta-\overline{x}}{\tau} + \Phi^{-1}(\kappa+m)\right)} = \kappa \ ,$$

reducing to

$$Pr\left(\varepsilon_i \leq \frac{\overline{x}-\overline{\theta}}{\sigma}\right) = \Phi\left(\frac{\overline{x}-\overline{\theta}}{\sigma}\right) = \kappa \tag{A10}$$

in the limiting case with $\sigma/\tau \to 0$. Solving (A10) for $\overline{x}$,

$$\overline{x} = \overline{\theta} + \sigma\Phi^{-1}(\kappa) \ , \tag{A11}$$

and substituting this threshold signal of an arm's length bank in case of premature foreclosure by the relationship bank into (5) yields equation (11):

$$\begin{aligned}\overline{\theta} &= \lambda + (1-\lambda)Pr(\overline{\theta} + \sigma\varepsilon_i < \overline{\theta} + \sigma\Phi^{-1}(\kappa)) \\ &= \lambda + (1-\lambda)\ \Phi\left(\Phi^{-1}(\kappa)\right) \\ &= \kappa(1-\lambda) + \lambda \ . \end{aligned}$$

Q.E.D.

A.2 Derivation of the amount of refinancing $m(y)$ as $\sigma/\tau \to \infty$

Maximization of (12) with respect to m yields the first order condition

$$\frac{1}{\tau}\ \phi\left(\frac{y+m}{\tau}\right) - 1 \stackrel{!}{=} 0 \ . \tag{A12}$$

As follows from the properties of the standard normal density, a solution to (A12) exists if and only if $\tau \leq \frac{1}{\sqrt{2\pi}}$. Taking this into account when solving the first order condition for m, we obtain equation (13):

$$m(y) = -y + \tau\phi^{-1}(\tau) \ .$$

Using the extensive form of the standard normal density, $\phi(z) = \frac{1}{\sqrt{2\pi}}\ e^{-z^2/2}$, the second order condition of the relationship bank's maximization problem can be expressed as

$$-\frac{2}{\tau\sqrt{2\pi}}\ e^{-\frac{(y+m)^2}{2\tau^2}}\ \frac{y+m}{2\tau^2} < 0 \ . \tag{A13}$$

Obviously, (A13) holds if and only if $y+m > 0$, i.e. the relationship bank's optimal amount of refinancing m has to be larger than $-y$. Thus, it follows from (13) that

$$\tau\phi^{-1}(\tau) > 0$$

must hold.

Q.E.D.

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