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Wirtschaftswissenschaftliche Fakultät  
der Eberhard-Karls-Universität Tübingen

**Creditor Coordination with  
Social Learning and Endogenous Timing  
of Credit Decisions**

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# Creditor Coordination with Social Learning and Endogenous Timing of Credit Decisions

Tobias Schüle\*

November 2006

## Abstract

In case of multiple source lending even solvent firms may be forced into bankruptcy due to uncoordinated credit withdrawals of their lenders. This paper analyzes whether a debtor firm can thwart such inefficient liquidations by offering creditors the option to delay their foreclosure decision rather than obliging them to simultaneous actions as suggested by MORRIS AND SHIN (2004). With this option, lenders can endogenously determine the timing of their credit decisions, trading off the informational benefit from waiting against the associated cost of delay. Our results state that the option to delay diminishes creditor coordination failure whenever the firm is expected to be in distress.

Keywords: Global Games; Creditor Coordination Failure; Option to Delay; Social Learning

JEL classification: D82, D83, G32, G33

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# 1 Introduction

Particularly in Europe, the business sector is characterized by a large number of small and medium-sized firms, which typically resort to bank debt financing when procuring capital for their investment projects. Since these firms are usually financed by a multitude of bank lenders, it is hard to overstate the importance of creditor coordination failure.<sup>1</sup> Banks may decide to foreclose their loans because they fear that others will also withdraw, even though it would be in their collective interest to roll over the credit. Such uncoordinated withdrawals of bank loans can lead to inefficient project liquidations, forcing even economically solvent firms into bankruptcy.

Despite its considerable relevance, the problem of inefficient creditor coordination has received scant attention in the previous economic literature, since traditional coordination games produced multiple equilibria. Only recently, the risk of creditor coordination failure has been analyzed more elaborately, building on the theory of global games. Global games, introduced by CARLSSON AND VAN DAMME (1993) and generalized by MORRIS AND SHIN (2003) and FRANKEL ET AL. (2003), assume that each player noisily observes the game's payoff structure, which itself is determined by a random draw from a given class of games. Under certain conditions, these assumptions induce a unique equilibrium, so that the incidence of inefficient project liquidations arising from the coordination problem among lenders can be quantified.

The concept of global games has first been applied to credit markets by HUBERT AND SCHÄFER (2002) and MORRIS AND SHIN (2004). They analyze coordination failure among a continuum of homogeneous creditors in a static model, where all lenders have to decide simultaneously at an interim stage of the debtor firm's investment project whether to foreclose or to roll over their loans. Credit decisions are made based on imperfect information regarding a fundamental state, which can be interpreted as a measure of project quality, and even economically sound projects may be doomed to failure if too many creditors foreclose. In this context, MORRIS AND SHIN (2004) and BANNIER AND HEINEMANN (2002) propose that the firm can mitigate the risk of inefficient project liquidations by adjusting the degree of information dissemination. Other studies do not focus on information policy as an instrument of creditor coordination, but analyze to what extent a debtor firm can avoid coordination failure by choosing a heterogeneous creditor structure. These

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<sup>1</sup> See DETRAGIACHE ET AL. (2000) and ONGENA AND SMITH (2001) for empirical evidence on the prevalence of multiple bank lending in European countries.

models assume that over time the firm has established close business relations to a particular bank which is therefore willing to finance a sizeable fraction of the firm's project. TAKEDA (2003) shows that the incidence of inefficient project liquidations is reduced if such a relationship bank jointly finances the firm's project with a continuum of small "arm's length" banks. SCHÜLE AND STADLER (2005) demonstrate that creditor coordination may be even more efficient if the relationship bank is able to signal its credit decision to the small lenders, and ELSAS ET AL. (2004) endogenously determine the relationship bank's optimal proportion of total firm debt. Combining both perceptions regarding a debtor firm's capability to diminish creditor coordination failure, information policy and relationship lending, BANNIER (2006) analyzes the firm's optimal information dissemination strategy in a model with multiple heterogeneous bank lenders. However, at least in some situations it seems questionable whether a debtor firm can actually resort to information policy and borrowing from a relationship bank in order to mitigate the risk of inefficient project liquidations. First, it is doubtful that the firm virtually can control the precision of lenders' private information and thus the extent of uncoordinated credit terminations. Second, especially young and small firms often do not dispose of long-term relations to a particular bank, so that they have to rely exclusively on arm's length debt financing.

This paper introduces a new aspect to the debate on applicable instruments of creditor coordination, as it deals with the question whether a debtor firm can reduce the incidence of coordination failure by offering creditors an alternative debt contract. As a benchmark, we analyze the static global game of MORRIS AND SHIN (2004), who assume that the financing is undertaken via a *standard debt contract*, obliging all lenders to simultaneous credit decisions at an interim stage of the firm's investment project. We then consider the effects of providing creditors with the option to defer their roll over or foreclosure decision. We refer to such a contract as a *leniency debt contract* and examine whether granting the option to delay can serve as an instrument to coordinate lenders more efficiently compared to the benchmark case of standard debt contracting à la MORRIS AND SHIN (2004). Provided with a leniency debt contract, waiting rather than withdrawing the credit early generates an informational benefit via social learning. Lenders who delay their credit decision are able to observe how many creditors have stopped lending before and use this additional information to update their prior beliefs regarding the quality of the firm's project. However, making a better informed decision late in the game may be associated with a cost of delay since foreclosing late rather than early yields a lower payoff. Thus, lenders provided with a leniency debt contract endogenously determine the timing of their credit decisions, trading off the informational benefit from social learning against the expected cost of delay.

Methodically, our dynamic creditor coordination game with endogenous timing of credit decisions and costs of delay is an application of the global game framework analyzed by DASGUPTA (2006).<sup>2</sup> He considers a continuum of players with the option to delay investment in a risky project. Investing late rather than early reduces a player's uncertainty regarding the project quality, but involves a cost of delay by generating a lower payoff if the project succeeds. Hence, our model is different from DASGUPTA (2006) insofar as we assume that the *risky* action (to roll over) is reversible, while the *safe* action (to foreclose) is irreversible. Furthermore, our application to creditor coordination requires that deferring the *safe* action to withdraw the credit is associated with a cost of delay, whereas in DASGUPTA (2006) delaying the *risky* action (to invest) is costly.

These modifications of the payoff structure affect our qualitative results in a limiting case where the incidence of creditor coordination failure can be derived explicitly. As the investment project's level of risk approaches infinity, the extent of coordination failure remains unaltered in our dynamic creditor coordination game compared to the benchmark model of MORRIS AND SHIN (2004), while efficiency increases with the option to delay in Dasgupta's investment game. Away from the limit, for the more relevant case of substantial but finite levels of project risk, it essentially depends on the commonly expected quality of the investment project whether a debtor firm benefits from providing its lenders with a leniency debt contract. Our numerical calculations imply that offering a standard debt contract à la MORRIS AND SHIN (2004) is optimal if the expected project quality is sufficiently high. In contrast, for sufficiently low values of expected project quality, the incidence of inefficient project liquidations can be mitigated by granting lenders the option to delay their credit decisions.

The remainder of this paper is organized as follows. Section 2 sets up the model, introducing the timing of events and the information available at all stages in the static benchmark game and in the dynamic game with the option to delay. In Section 3 we briefly discuss the incidence of coordination failure when creditors are provided with a standard debt contract. Section 4 solves for the equilibrium of the dynamic creditor coordination game, when creditors obtain a leniency debt contract. Comparing the risk of inefficient project liquidations in both games, we provide implications on the optimal debt contract in Section 5. Finally, Section 6 concludes.

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<sup>2</sup> Alternative global games with endogenous timing of actions and learning have been analyzed by HEIDHUES AND MELISSAS (2003), XUE (2003), and BRINDISI (2005).

## 2 The model

The model considers a simple economy consisting of a debtor firm and a continuum of ex ante identical, risk neutral arm's length lenders. Resorting to debt financing from the continuum of creditors, the firm decides to set up a risky investment project which matures in period  $T$ . Whether the project succeeds and loans can be repaid at maturity decisively depends on a fundamental state  $\theta \in \mathbb{R}$ , to which we refer as project quality, and on the credit decisions of lenders. Each creditor finances a loan which is secured on collateral and has a face value normalized to 1. Provided with a standard debt contract, creditors have the option to foreclose their loans prematurely in period  $t_1 < T$  and seize the collateral  $\kappa_1 \in (0, 1)$ . Alternatively, the debtor firm can offer a leniency debt contract, granting the option to delay credit decisions. Then, lenders may withdraw their credit either in  $t_1$  or in a later period  $t_2 \in (t_1, T)$ , if they are willing to foreclose at all. As foreclosing a loan in  $t_2$  merely generates a payoff of  $\kappa_2 \in (0, \kappa_1)$ , protracting the decision to stop lending is associated with a cost of delay.<sup>3</sup>

Whether a lender decides to withdraw his credit prematurely or to roll over is determined by his expectations of the project quality  $\theta$ , which is drawn from the commonly known distribution  $N(\mu, \frac{1}{a})$  and is not revealed until the project matures in period  $T$ . In addition to the expected project quality  $\mu \in \mathbb{R}$  and the project's level of risk  $\frac{1}{a} \in \mathbb{R}^+$ , every creditor  $i$  observes a noisy private signal  $x_i$  regarding  $\theta$  previous to his credit decision in  $t_1$ :

$$x_i = \theta + \frac{\varepsilon_i}{\sqrt{b}}, \quad (1)$$

where  $\varepsilon_i$  is a random variable distributed i.i.d.  $N(0, 1)$  and independent of  $\theta$  and  $b > 0$  is a scale factor reflecting the precision of  $x_i | \theta$ . Provided with the option to delay, lenders who have rolled over their loans in  $t_1$  receive an additional private signal  $y_i$  before making their final credit decision in period  $t_2$ :

$$y_i = \Phi^{-1}(1 - \ell_1) + \frac{\eta_i}{\sqrt{c}}, \quad (2)$$

where  $c > 0$  is a constant and the idiosyncratic random variable  $\eta_i \sim N(0, 1)$  is i.i.d. across creditors and independent of  $\varepsilon_i$ . As  $\ell_1$  denotes the fraction of creditors who decided to withdraw their credit in  $t_1$ , the signal  $y_i$  can be interpreted as a noisy statistic based on the proportion of "active lenders".<sup>4</sup> Hence, exercising the

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<sup>3</sup> Note that all payoffs are stated in expected terminal wealth, so that discounting does not complicate our analysis. Hence, the restriction  $\kappa_2 < \kappa_1$  is equivalent to the assumption that creditors discount and receive a constant payoff at the time they withdraw their credit.

option to delay generates an informational benefit which has to be balanced with the potentially incorporated costs of delaying the foreclosure decision.

By assumption, lenders withdrawing their loans prematurely cause disruption to the debtor firm's investment project, such that the project is doomed to failure whenever  $\ell > \theta$ , where  $\ell \in [0, 1]$  denotes the total mass of foreclosing creditors. In this case, the firm is forced into bankruptcy, implying that the loans of creditors who have rolled over cannot be refunded. In contrast, if the project succeeds ( $\ell \leq \theta$ ), the firm remains in operation and is able to repay all loans at full face value. Thus, a creditor's payoff from extending his credit until maturity in period  $T$  is given by

$$u(\theta, \ell) = \begin{cases} 1 & \text{if } \theta \geq \ell \\ 0 & \text{if } \theta < \ell. \end{cases}$$

Clearly, the unconstrained efficient outcome of this creditor coordination game would have all lenders withdrawing their credit in  $t_1$  whenever  $\theta < 0$  and not at all otherwise, independent of the debt contract offered by the firm. However, with imperfect information on  $\theta$  creditors cannot coordinate on this efficient equilibrium, so that even economically sound projects with  $\theta \geq 0$  may be liquidated. Below, we examine whether the debtor firm can mitigate the risk of such inefficient project liquidations by offering lenders a leniency debt contract with the option to delay rather than a standard debt contract as proposed by MORRIS AND SHIN (2004).

### 3 The standard debt contract with simultaneous credit decisions

To set a benchmark, we first discuss the static creditor coordination game in which the debtor firm offers a standard debt contract, obliging all lenders to simultaneous credit decisions in period  $t_1$ . MORRIS AND SHIN (2004) show that this global game has a unique equilibrium, provided that the firm's investment project is sufficiently risky relative to the precision of creditors' private information. The equilibrium is then characterized by trigger strategies, such that each lender rolls over his loan

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<sup>4</sup> We assume that creditors observe the statistic  $y_i$  with some idiosyncratic noise in order to ensure the existence of a unique equilibrium. However, when determining the incidence of creditor coordination failure, we focus on the case of perfect observation of the past ( $c \rightarrow \infty$ ) as is common in the literature on herds and cascades (see e.g. BIKHCHANDANI ET AL. (1992)). In this limit, the monotone transformation of  $\ell_1$ ,  $\Phi^{-1}(1 - \ell_1)$ , is equivalent to observing  $\ell_1$  and thus without loss of generality.



whenever he obtains a private signal  $x_i$  greater than a trigger value  $x^*$  and withdraws credit otherwise. Since private signals are correlated with  $\theta$ , this implies that the project fails whenever a quality lower than the fundamental threshold  $\theta^*$  is realized.

As a necessary condition for an equilibrium in trigger strategies, the marginal creditor who receives the critical signal  $x^*$  must be indifferent between foreclosing his loan in  $t_1$  and rolling over, i.e.

$$\kappa_1 = Pr(\theta \geq \theta^* | x^*).$$

Since the posterior beliefs of a lender  $i$  who has observed the realization of the private signal  $x_i$  are given by

$$\theta | x_i \sim N\left(\frac{a\mu + bx_i}{a+b}, \frac{1}{a+b}\right), \quad (3)$$

the indifference condition for the marginal creditor can be rewritten as

$$x^* = \frac{a+b}{b}\theta^* - \frac{a}{b}\mu + \frac{\sqrt{a+b}}{b}\Phi^{-1}(\kappa_1). \quad (4)$$

The second condition necessary to derive the equilibrium thresholds  $x^*$  and  $\theta^*$  reflects that the investment project is at the margin of success and failure at the state  $\theta$  for which  $\theta = \ell$ . Due to the assumed independence of private signals and the continuum of creditors, the mass of foreclosing lenders  $\ell$  is equivalent to the probability that an individual lender withdraws his credit, i.e.

$$\ell = Pr(x_i < x^* | \theta) = \Phi\left(\sqrt{b}(x^* - \theta)\right).$$

Thus, the critical project quality  $\theta^*$  is implicitly given by the *critical mass condition*

$$\theta^* = \Phi\left(\sqrt{b}(x^* - \theta^*)\right). \quad (5)$$

Finally, substituting the creditors' *cutoff condition* (4) into the *critical mass condition* (5) delivers an equation purely in terms of  $\theta^*$ :

$$\theta^* = \Phi\left(\frac{a}{\sqrt{b}}(\theta^* - \mu) + \sqrt{\frac{a+b}{b}}\Phi^{-1}(\kappa_1)\right). \quad (6)$$

As a sufficient condition for the uniqueness of equilibrium, consider that the expression on the right-hand side of equation (6) must have a slope of less than 1 everywhere. Deriving the right-hand side with respect to  $\theta^*$ , it can easily be seen that a sufficient condition for a unique equilibrium is  $a < \sqrt{2\pi b}$ . Hence, as long as the creditors' prior information regarding the project quality  $\theta$  is sufficiently diffuse relative to their private signals  $x_i$ , equation (6) delivers a unique  $\theta^* \in (0, 1)$ , quantifying the risk of inefficient project liquidations by the set of states  $\theta \in [0, \theta^*]$ .

## 4 The leniency debt contract with the option to delay credit decisions

We now augment the static creditor coordination game of MORRIS AND SHIN (2004) as analyzed above to examine how efficiency is affected if creditors are provided with a leniency debt contract, granting each lender the option to delay his credit decision. The information system of this dynamic game, given by (1) and (2), implies that deferring the credit decision rather than withdrawing the credit in period  $t_1$  generates an informational benefit which may offset the costs associated with delaying the foreclosure decision. As a creditor  $i$  who waits until  $t_2$  observes a private signal  $y_i$  in addition to his first period signal  $x_i$ , the information held by this creditor in  $t_2$  can be specified by a sufficient statistic  $s_i(x_i, y_i)$ . We can thus look for equilibria where lenders act according to trigger strategies around thresholds  $(x_D^*, s_D^*)$ , such that:

- A creditor  $i$  forecloses his loan in  $t_1$  if and only if  $x_i < x_D^*$ . Otherwise he chooses to wait.
- A creditor  $i$  who has exercised the option to delay forecloses his loan in  $t_2$  if and only if  $s_i < s_D^*$ .

Assuming such trigger strategies, the proportion of creditors who withdraw their credit in period  $t_1$  at any state  $\theta$  is given by

$$\ell_1 = Pr(x_i < x_D^* | \theta) = \Phi\left(\sqrt{b}(x_D^* - \theta)\right).$$

Substituting  $\ell_1$  into equation (2) demonstrates that the second period signal  $y_i$  actually provides a lender  $i$  with additional information regarding the unknown project quality  $\theta$ :

$$y_i = \sqrt{b}(\theta - x_D^*) + \frac{\eta_i}{\sqrt{c}}.$$

Note that in equilibrium observing  $y_i$  is equivalent to observing an exogenous signal  $z_i = \frac{y_i}{\sqrt{b}} + x_D^*$ , where  $z_i$  can be rewritten as

$$z_i = \theta + \frac{\eta_i}{\sqrt{bc}}.$$

Since  $z_i | \theta$  is distributed  $N(\theta, \frac{1}{bc})$ , applying Bayes' Rule to update the creditors' previous beliefs  $\theta | x_i$  as given by (3) delivers

$$\theta | x_i, z_i \sim N\left(\frac{a\mu + bx_i + bc z_i}{a + b + bc}, \frac{1}{a + b + bc}\right).$$

Finally, resubstituting  $z_i = \frac{y_i}{\sqrt{b}} + x_D^*$ , we get

$$\theta|x_i, y_i \equiv \theta|s_i \sim N\left(s_i, \frac{1}{a+b+bc}\right), \quad (7)$$

where

$$s_i = \frac{a\mu + bx_i + \sqrt{bc}y_i + bcx_D^*}{a+b+bc}$$

denotes the sufficient statistic for  $(x_i, y_i)$ .

Having derived the posterior beliefs of creditors who exercise their option to delay, we are now in a position to establish necessary conditions for an equilibrium in trigger strategies. If lenders follow trigger strategies as outlined above, the total mass of creditors who foreclose their loans prematurely at any fundamental state  $\theta$  is given by  $Pr(x_i < x_D^*|\theta) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta)$ . Thus, the debtor firm's project succeeds if and only if

$$\theta \geq Pr(x_i < x_D^*|\theta) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta).$$

However, since the decisions of a creditor to withdraw his credit or to roll over in the two periods are not independent in the dynamic game with the option to delay, it is not apparent that there exists a critical  $\theta_D^*$  above which the investment project succeeds and below which it fails. Lemma 1 verifies that such a threshold  $\theta_D^*$  really exists.

**Lemma 1.** *Define*

$$G(\theta) = Pr(x_i < x_D^*|\theta) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta) - \theta.$$

*Then,  $G(\theta)$  is strictly decreasing and crosses zero exactly once.*

Proof. See the Appendix.

Given Lemma 1, we can express the *critical mass condition* of the dynamic creditor coordination game as

$$\theta_D^* = Pr(x_i < x_D^*|\theta_D^*) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta_D^*). \quad (8)$$

The *cutoff condition* for creditors considering to exercise their option to delay in  $t_1$  states that lenders trade off the proceeds from foreclosing early against the expected benefit of waiting and then acting optimally:

$$\kappa_1 = Pr(s_i < s_D^*|x_D^*)\kappa_2 + Pr(\theta \geq \theta_D^*, s_i \geq s_D^*|x_D^*). \quad (9)$$

Finally, the marginal creditor who has rolled over his loan in  $t_1$  must be indifferent between withdrawing his credit in period  $t_2$  and continuing lending until the project matures:

$$\kappa_2 = Pr(\theta \geq \theta_D^* | s_D^*).$$

Using (7), this *cutoff condition* for lenders in  $t_2$  can be rewritten as

$$s_D^* = \theta_D^* + \frac{\Phi^{-1}(\kappa_2)}{\sqrt{a+b+bc}}. \quad (10)$$

As a first step to solve the system of equations (8) - (10), note that substituting the threshold  $s_D^*$  as given by (10) into the *critical mass condition* (8) yields an equation merely in  $x_D^*$  and  $\theta_D^*$ :

$$\theta_D^* = Pr(x_i < x_D^* | \theta_D^*) + Pr(x_i \geq x_D^*, s_i < \theta_D^* + M | \theta_D^*),$$

where  $M = \frac{\Phi^{-1}(\kappa_2)}{\sqrt{a+b+bc}}$ . Lemma 2 states that as long as the debtor firm's investment project is sufficiently risky, this equation implicitly defines  $\theta_D^*$  as a smooth increasing function of  $x_D^*$  with a bounded derivative.

**Lemma 2.** Assume  $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$ . Then, for any  $x_D^*$ , there is a unique  $\hat{\theta}(x_D^*)$ , such that  $G(\hat{\theta}, x_D^*) = 0$ , where

$$G(\theta, x_D^*) = Pr(x_i < x_D^* | \theta) + Pr(x_i \geq x_D^*, s_i < \theta_D^* + M | \theta) - \theta.$$

Moreover,  $\frac{d\hat{\theta}}{dx_D^*} \in (0, \frac{b}{a+b})$ .

Proof. See the Appendix.

Using Lemma 2 and equation (10), the *cutoff condition* (9) of creditors in period  $t_1$  can be expressed purely in terms of  $x_D^*$ :

$$\kappa_1 = Pr(s_i < \theta_D^*(x_D^*) + M | x_D^*) \kappa_2 + Pr(\theta \geq \theta_D^*(x_D^*), s_i \geq \theta_D^*(x_D^*) + M | x_D^*).$$

As we show in the Appendix, this equation has a unique solution, provided that  $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$ . By means of Lemma 2, this implies that there also exists a unique solution  $(x_D^*, s_D^*, \theta_D^*)$  to the system (8) - (10). We can thus state:

**Proposition 1.** When creditors act according to trigger strategies, the dynamic game with the option to delay credit decisions has a unique equilibrium provided that  $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$ .

Proof. See the Appendix.

While this uniqueness result holds for general values of  $c$ , we have to focus on the limiting case when creditors in  $t_2$  observe the fraction of active lenders with vanishing noise ( $c \rightarrow \infty$ ) in order to identify the incidence of inefficient project liquidations. In this limit, lenders who exercise their option to delay the credit decision essentially face no uncertainty regarding the project quality  $\theta$  in period  $t_2$ . The *critical mass condition* (8) then reduces to

$$1 - \theta_D^* = (1 - \kappa_2) \Phi \left( \sqrt{b}(\theta_D^* - x_D^*) \right), \quad (11)$$

whereas equation (9) can be rewritten as<sup>5</sup>

$$1 - \kappa_1 = (1 - \kappa_2) \Phi \left( \sqrt{a+b} \left( \theta_D^* - \frac{a\mu + bx_D^*}{a+b} \right) \right). \quad (12)$$

Rearranging (12),

$$x_D^* = \frac{a+b}{b} \theta_D^* - \frac{a}{b} \mu + \frac{\sqrt{a+b}}{b} \Phi^{-1} \left( \frac{\kappa_1 - \kappa_2}{1 - \kappa_2} \right), \quad (13)$$

and substituting into equation (11), the critical project quality  $\theta_D^*$  is implicitly given by

$$1 - \theta_D^* = (1 - \kappa_2) \Phi \left( \sqrt{\frac{a+b}{b}} \Phi^{-1} \left( \frac{1 - \kappa_1}{1 - \kappa_2} \right) - \frac{a}{\sqrt{b}} (\theta_D^* - \mu) \right). \quad (14)$$

Similar to the benchmark static game, in case of leniency debt contracting the risk of inefficient project liquidations is specified by the interval  $\theta \in [0, \theta_D^*]$ , where  $\theta_D^* \in (0, 1)$ .

## 5 Implications on the optimal debt contract

Having derived implicit solutions for the equilibrium project quality thresholds  $\theta^*$  and  $\theta_D^*$ , we analyze in this section what kind of debt contract the debtor firm should offer in order to mitigate creditor coordination failure. As mentioned above, the incidence of inefficient project liquidations is given by the intervals  $[0, \theta^*)$  and  $[0, \theta_D^*)$ , respectively. Offering a leniency debt contract with the option to delay credit decisions instead of a standard debt contract à la MORRIS AND SHIN (2004) therefore reduces the risk of uncoordinated credit withdrawals if and only if  $\theta_D^* < \theta^*$ .

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<sup>5</sup> See the Appendix for a formal derivation of the equations (11) and (12).

## 5.1 Comparison of coordination failure in the limit as $a \rightarrow 0$

Both, the static creditor coordination game and the dynamic game with the option to delay required that the debtor firm's investment project is sufficiently risky in order to ensure the uniqueness of equilibrium. We now focus on the extreme case where the project's level of risk approaches infinity ( $a \rightarrow 0$ ), so that the prior distribution  $\theta \sim N(\mu, \frac{1}{a})$  converges to an improper uniform prior over the real line. This property allows for a characterization of the thresholds  $\theta^*$  and  $\theta_D^*$  of the respective games in closed form.

Provided with a standard debt contract, uncoordinated credit withdrawals of lenders lead to a failure point  $\theta^*$  as implicitly defined by equation (6). In the limit as  $a \rightarrow 0$ , the right-hand side of this equation simplifies to  $\Phi(\Phi^{-1}(\kappa_1))$ , implying that

$$\theta^* = \kappa_1.$$

If the firm offers a leniency debt contract instead, the critical state  $\theta_D^*$  at which the project is at the margin of failure and success is implicitly given by equation (14), which reduces to

$$\theta_D^* = \kappa_1$$

in the limit as  $a \rightarrow 0$ . Hence, whenever the investment project conducted by the debtor firm is arbitrarily risky, the incidence of inefficient project liquidations is determined by  $\theta \in [0, \kappa_1)$  in the static benchmark game as well as in our dynamic creditor coordination game, and thus independent of the debt contract offered by the firm.

Let us examine this limiting result in more detail by providing some insight into the decision strategies of creditors in the static game and in the dynamic game with the option to delay, respectively. If lenders are provided with a standard debt contract and thus simultaneously decide on rolling over or foreclosing their loans, we know from equation (4) that

$$x^* = \frac{a+b}{b}\theta^* - \frac{a}{b}\mu + \frac{\sqrt{a+b}}{b}\Phi^{-1}(\kappa_1).$$

Clearly, a higher expected project quality  $\mu$  shifts the trigger signal  $x^*$  to the left as it increases the lenders' incentives to extend their credit. Considering the critical signal  $x_D^*$  of the dynamic game as given by equation (13),

$$x_D^* = \frac{a+b}{b}\theta_D^* - \frac{a}{b}\mu + \frac{\sqrt{a+b}}{b}\Phi^{-1}\left(\frac{\kappa_1 - \kappa_2}{1 - \kappa_2}\right),$$

it is easy to see that the same intuition applies to the credit decisions of lenders in period  $t_1$  if they are provided with a leniency debt contract. In contrast, creditors who exercise their option to delay and additionally observe the second period signal  $y_i$  essentially face no uncertainty regarding the project quality  $\theta$  in the limit as  $c \rightarrow \infty$ , and thus follow strategies independent of the prior mean  $\mu$ . Hence, for a finite level of project risk  $1/a$ , the strategies of all lenders in the static game are affected by the ex ante expected project quality  $\mu$ , whereas some creditors provided with a leniency debt contract follow mean independent strategies. However, as  $a \rightarrow 0$ , the strategies of *all* creditors are mean independent in *both* games, finally implying that the risk of inefficient project liquidations does not depend on the debt contract offered by the firm.

In order to illustrate how mean independence induces an identical risk of inefficient project liquidations in both games, let us compare the mass of lenders withdrawing their credit prematurely at the critical state  $\theta^*$  in the static game with the mass of foreclosing creditors at the critical state  $\theta_D^*$  in the dynamic game. First consider the case of standard debt contracting, obliging lenders to simultaneous credit decisions in period  $t_1$ . Using the definition of the trigger signal  $x^*$  as given by (4), the mass of creditors who decide to foreclose their loans can be expressed as

$$Pr(x_i < x^* | \theta^*) = \Phi \left( \frac{a}{\sqrt{b}}(\theta^* - \mu) + \sqrt{\frac{a+b}{b}} \Phi^{-1}(\kappa_1) \right). \quad (15)$$

Similarly, using equation (13), the mass of lenders who withdraw their credit in period  $t_1$  in the dynamic game with the option to delay is given by

$$Pr(x_i < x_D^* | \theta_D^*) = \Phi \left( \frac{a}{\sqrt{b}}(\theta_D^* - \mu) + \sqrt{\frac{a+b}{b}} \Phi^{-1} \left( \frac{\kappa_1 - \kappa_2}{1 - \kappa_2} \right) \right). \quad (16)$$

On the one hand, we expect this mass to be lower than the mass of creditors foreclosing in  $t_1$  in the static game since creditors provided with a leniency debt contract have another opportunity to foreclose their loans in  $t_2$  and seize the collateral  $\kappa_2$ . On the other hand, in case of leniency debt contracting a creditor mass of

$$Pr(x_i \geq x_D^*, s_i < s_D^* | \theta_D^*) = Pr(x_i \geq x_D^* | \theta_D^*) \kappa_2, \quad (17)$$

exercises its option to delay and stops lending in period  $t_2$ , thus causing additional disruption to the debtor firm's project.<sup>6</sup> Whether the total mass of foreclosing lenders at the critical state in the dynamic game, given by the sum of (16) and

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<sup>6</sup> The decomposition of the product term arises because as  $c \rightarrow \infty$ ,  $Cov(x_i, s_i | \theta) \rightarrow 0$ , since  $s_i \rightarrow \theta$ . See section A.4 of the Appendix for a formal derivation of equation (17).

(17), exceeds the total mass of foreclosing lenders at the critical state of the static game, given by (15), obviously depends on the parameters of the prior distribution  $\theta \sim N(\mu, \frac{1}{a})$ .

However, in the limit as  $a \rightarrow 0$  the decision strategies of all creditors are independent of the prior mean  $\mu$ , so that the total mass of lenders who withdraw their credit merely depends on the payoffs of the respective games. Considering the benchmark static game, equation (15) reduces to

$$\lim_{a \rightarrow 0} Pr(x_i < x^* | \theta^*) = \kappa_1,$$

whereas the mass of creditors who do not exercise their option to delay in the dynamic game is given by

$$\lim_{a \rightarrow 0} Pr(x_i < x_D^* | \theta_D^*) = \frac{\kappa_1 - \kappa_2}{1 - \kappa_2}.$$

Thus, the debtor firm "gains" a creditor mass of

$$\kappa_1 - \frac{\kappa_1 - \kappa_2}{1 - \kappa_2} = \frac{1 - \kappa_1}{1 - \kappa_2} \kappa_2$$

in period  $t_1$  by offering a leniency debt contract instead of a standard debt contract. However, as  $a \rightarrow 0$ , equation (17) becomes

$$\lim_{a \rightarrow 0} Pr(x_i \geq x_D^*, s_i < s_D^* | \theta_D^*) = \left(1 - \frac{\kappa_1 - \kappa_2}{1 - \kappa_2}\right) \kappa_2 = \frac{1 - \kappa_1}{1 - \kappa_2} \kappa_2,$$

implying that the benefits from offering a leniency debt contract in  $t_1$  are just balanced by the loss of creditors in period  $t_2$ . It is no coincidence then, that the thresholds  $\theta^*$  and  $\theta_D^*$  of the respective games coincide when lenders follow mean independent strategies due to diffuse prior information regarding the project quality  $\theta$ .

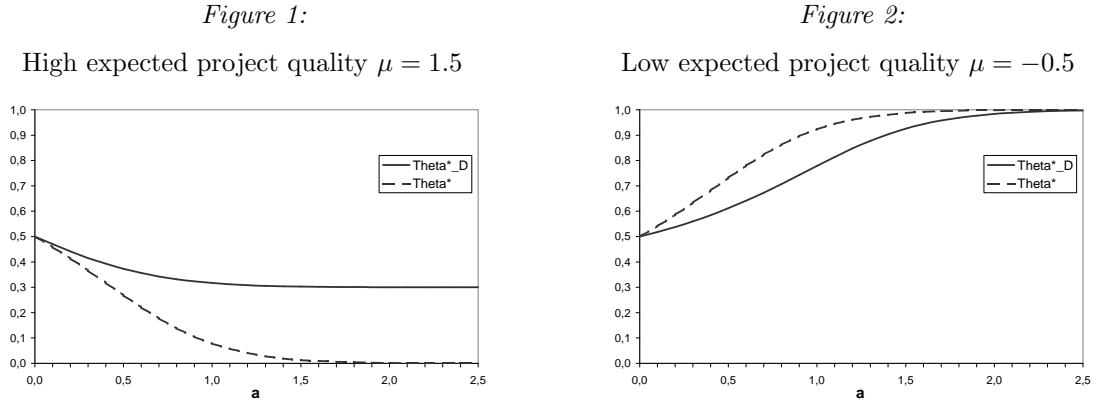
## 5.2 Comparison of coordination failure away from the limit

While the result of debtor firms not being able to affect the incidence of creditor coordination failure by debt contracting in the limit as  $a \rightarrow 0$  is rather disconcerting, the above discussion indicates that offering a leniency debt contract instead of a standard debt contract à la MORRIS AND SHIN (2004) may well have an effect for the more relevant case of substantial but finite levels of project risk ( $a \rightarrow 0$ ). Using numerical methods, we thus examine to what extent the incidence of inefficient project liquidations is influenced by a decreasing level of project risk in the static as well as in the dynamic creditor coordination game. For all numerical calculations, let  $\kappa_1 = 0.5$  and  $\kappa_2 = 0.3$ , implying that the lenders' cost of delay, given by  $k \equiv \kappa_1 - \kappa_2$ ,



amounts to 0.2.<sup>7</sup> As the idea of debtor firms being able to control for the precision of creditors' private signals  $x_i$  is indeed precarious and our results rather depend on the ratio of  $a$  to  $b$  than on the absolute values of these precisions, we fix  $b = 1$  while varying the prior precision  $a$  in the range where the uniqueness of equilibrium can be guaranteed:  $a \in (0, \sqrt{2\pi})$ .

Figures 1 and 2 depict the results for low ( $\mu = -0.5$ ) and high ( $\mu = 1.5$ ) expected project qualities, whereby our choice of means is determined by their distance from the crucial region of  $\theta$ ,  $\theta \in (0, 1)$ .



Both figures approve our result that in the limit as  $a \rightarrow 0$ , the risk of inefficient project liquidations is given by  $\theta \in [0, \kappa_1)$ , independent of the debt contract offered by the firm. However, as the project's level of risk  $1/a$  decreases, creditors in period  $t_1$  put more weight on their prior information regarding  $\theta$  relative to the information contained in their private signals  $x_i$ . For high values of the expected project quality (Figure 1), this implies that all lenders in the static benchmark game become more optimistic as  $a$  increases, leading to a declining mass of foreclosing creditors and thus to a decrease in  $\theta^*$ . The same intuition applies to the decision strategies of lenders in the first period of the dynamic game. But provided with a leniency debt contract, creditors who exercise their option to delay follow mean independent strategies in  $t_2$  and thus are less optimistic, causing more disruption to the firm's investment project overall ( $\theta_D^* > \theta^*$ ). Hence, a firm conducting a project with high expected quality  $\mu$  benefits from offering a standard debt contract instead of a leniency debt contract with the option to defer credit decisions, independent of the cost of delay and the project's level of risk. In contrast, similar arguments imply that granting creditors the option to delay rather than providing them with a standard debt contract is beneficial whenever the debtor firm has access only to projects with sufficiently low

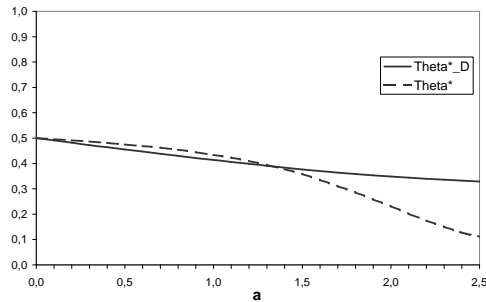
<sup>7</sup> We have checked many different values of  $\kappa_1 \in (0, 1)$  and  $k \in (0, \kappa_1)$ , but did not find evidence that our results are affected qualitatively by varying costs of delay and collateral values.

expected quality (Figure 2). In this case, a declining level of project risk  $1/a$  fosters the pessimism of creditors acting in period  $t_1$  of the static benchmark game and therewith leads to an increase of  $\theta^*$ . Whereas this intuition also applies to lenders in  $t_1$  in the dynamic game, the strategies of creditor who choose to protract their credit decision are independent of the low prior mean  $\mu$ , thus less pessimistic, implying that overall less creditors withdraw their loans if they are provided with a leniency debt contract ( $\theta_D^* < \theta^*$ ).

For an intermediate value of expected project quality ( $\mu = 0.6$ ), lying inside the crucial region  $\theta \in (0, 1)$  where uncoordinated credit withdrawals may lead to inefficient project liquidations, Figure 3 illustrates that the effect of the prior mean  $\mu$  may be not dominant, so that the firm's choice of the optimal debt contract becomes a nontrivial decision.

Figure 3:

Intermediate expected project quality  $\mu = 0.6$



## 6 Concluding remarks

This paper has introduced a new aspect to the debate on how debtor firms can mitigate the risk of inefficient project liquidations arising due to uncoordinated credit withdrawals of their lenders. Adopting a dynamic global game as analyzed by DASGUPTA (2006), it examines whether a firm can diminish creditor coordination failure by granting its lenders the option to delay their roll over or foreclosure decisions rather than obliging them to simultaneous credit decisions as suggested by MORRIS AND SHIN (2004). In this respect, our analysis of efficient debt contracting complements the previous literature which exclusively concentrates on the firm's information policy and the possibility of relationship lending as applicable instruments of creditor coordination.

Our model implies that creditors endogenously determine the timing of their credit decisions, trading off the benefits from waiting and gathering more accurate infor-

mation against the potentially incorporated cost of delaying the foreclosure decision, whenever the debtor firm offers a leniency debt contract with the option to delay. Comparing this dynamic creditor coordination game to the static benchmark game of MORRIS AND SHIN (2004) who assume financing via a standard debt contract, binding creditors to simultaneous roll over or foreclosure decisions, enabled us to provide implications on the firm's choice of the optimal debt contract.

In the limit when the investment project conducted by the debtor firm is arbitrarily risky, so that the equilibrium of the model can be analyzed in closed form, our results state that the risk of inefficient project liquidations remains unaffected of the debt contract offered by the firm. In contrast, resorting to numerical calculations for the more relevant case of substantial but finite levels of project risk, we have demonstrated that in general granting lenders the option to delay does exert decisive influence on their ability to coordinate credit decisions. Whenever the expected quality of the firm's investment project is sufficiently sound, providing lenders with an option to delay is detrimental for efficient creditor coordination, implying that the firm should adhere to a standard debt contract à la MORRIS AND SHIN (2004). However, when the debtor firm is expected to be severely in distress, it can reduce the incidence of uncoordinated credit withdrawals by offering its lenders a leniency debt contract with the option to delay credit decisions. Hence, just in those situations in which the issue of creditor coordination failure becomes most prominent our paper suggests an alternative way to mitigate the risk of inefficient project liquidations, complementing the commonly discussed instruments in terms of information policy and relationship lending.

## Appendix

### A.1. Proof of Lemma 1

Using equation (1) and rewriting (2) as  $y_i = \sqrt{b}(\theta - x^*) + \frac{\eta_i}{\sqrt{c}}$ , the sufficient statistic  $s_i = \frac{a\mu + bx_i + \sqrt{bc}y_i + bcx_D^*}{a+b+bc}$  can be expressed as  $s_i = \frac{a\mu}{a+b+bc} + \frac{b(1+c)}{a+b+bc}\theta + \frac{\sqrt{bc}}{a+b+bc}\gamma$ , where  $\gamma = \frac{\varepsilon_i}{\sqrt{c}} + \eta_i$ . Then,  $s_i < s_D^*$  is equivalent to  $\gamma < \frac{a+b+bc}{\sqrt{bc}}s_D^* - \frac{a\mu}{\sqrt{bc}} - \frac{b(1+c)}{\sqrt{bc}}\theta$  and we can rewrite:

$$G(\theta) = \Phi(A(\theta)) + \int_{A(\theta)}^{\infty} \int_{-\infty}^{B(\theta)} f(\varepsilon, \gamma) d\gamma d\varepsilon - \theta,$$

where  $A(\theta) = \sqrt{b}(x_D^* - \theta)$  and  $B(\theta) = \frac{a+b+bc}{\sqrt{bc}}s_D^* - \frac{a\mu}{\sqrt{bc}} - \frac{b(1+c)}{\sqrt{bc}}\theta$ . Using Leibnitz' rule, differentiation under the double integral delivers

$$G'(\theta) = A'(\theta)\phi(A(\theta)) - A'(\theta) \int_{-\infty}^{B(\theta)} f(A(\theta), \gamma) d\gamma + B'(\theta) \int_{A(\theta)}^{\infty} f(\varepsilon, B(\theta)) d\varepsilon - 1.$$

Denoting by  $\phi(\cdot)$  the standard normal PDF of  $\varepsilon$ , and by  $\hat{\phi}(\cdot)$  the (non-standard) normal PDF of  $\gamma$ , we can express the joint densities as

$$\begin{aligned} f(\varepsilon = A(\theta), \gamma) &= \phi(a(\theta))f(\gamma|\varepsilon = A(\theta)) \\ f(\varepsilon, \gamma = B(\theta)) &= \hat{\phi}(B(\theta))f(\varepsilon|\gamma = B(\theta)). \end{aligned}$$

Since  $A'(\theta) = -\sqrt{b}$  and  $B'(\theta) = -\frac{b(1+c)}{\sqrt{bc}}$ , we can now rewrite  $G'(\theta)$  as

$$-\sqrt{b}\phi(A(\theta)) \left[ 1 - \int_{-\infty}^{B(\theta)} f(\gamma|\varepsilon = A(\theta)) d\gamma \right] - \frac{b(1+c)}{\sqrt{bc}} \hat{\phi}(B(\theta)) \int_{A(\theta)}^{\infty} f(\varepsilon|\gamma = B(\theta)) d\varepsilon - 1.$$

Clearly,  $G'(\theta) < 0$ . Furthermore,  $\lim_{\theta \rightarrow -\infty} G(\theta) = \infty$  and  $\lim_{\theta \rightarrow \infty} G(\theta) = -\infty$ , which completes the proof of Lemma 1.  $\blacksquare$

### A.2. Proof of Lemma 2

As above, the sufficient statistic  $s_i$  can be rewritten as  $s_i = \frac{a\mu}{a+b+bc} + \frac{b(1+c)}{a+b+bc}\hat{\theta} + \frac{\sqrt{bc}}{a+b+bc}\gamma$ , where  $\gamma = \frac{\varepsilon_i}{\sqrt{c}} + \eta_i$ . Writing  $s_D^* = \hat{\theta} + M$ ,  $s_i < s^*$  implies that  $\gamma < \frac{a}{\sqrt{bc}}(\hat{\theta} - \mu) + \frac{a+b+bc}{\sqrt{bc}}M$ . Define

$$B(\hat{\theta}) = \frac{a}{\sqrt{bc}}(\hat{\theta} - \mu) + \frac{a+b+bc}{\sqrt{bc}}M.$$

Since  $B'(\hat{\theta}) = \frac{a}{\sqrt{bc}}$ , using the proof of Lemma 1 gives us

$$\begin{aligned} \frac{\partial G(\hat{\theta}, x_D^*)}{\partial \hat{\theta}} &= -\sqrt{b}\phi(A(\hat{\theta}, x_D^*)) \left[ 1 - \int_{-\infty}^{B(\hat{\theta})} f(\gamma|\varepsilon = A(\hat{\theta}, x_D^*))d\gamma \right] \\ &+ \frac{a}{\sqrt{bc}}\hat{\phi}(B(\hat{\theta})) \int_{A(\hat{\theta}, x_D^*)}^{\infty} f(\varepsilon|\gamma = B(\hat{\theta}))d\varepsilon - 1. \end{aligned}$$

Now define

$$\begin{aligned} P_1 &= \int_{B(\hat{\theta})}^{\infty} f(\gamma|\varepsilon = A(\hat{\theta}, x_D^*))d\gamma \\ P_2 &= \int_{A(\hat{\theta}, x_D^*)}^{\infty} f(\varepsilon|\gamma = B(\hat{\theta}))d\varepsilon. \end{aligned}$$

Note that  $P_2 < 1$  and  $\hat{\phi}(\cdot) < \frac{\sqrt{c}}{\sqrt{2\pi}\sqrt{1+c}}$ , since the variance of  $\gamma$  amounts to  $\frac{1+c}{c}$ . Thus,  $a < \sqrt{2\pi b(1+c)}$  is a sufficient condition for  $\frac{\partial G(\hat{\theta}, x_D^*)}{\partial \hat{\theta}} < 0$ . In contrast,

$$\frac{\partial G(\hat{\theta}, x_D^*)}{\partial x_D^*} = \sqrt{b}\phi(A(\hat{\theta}, x_D^*))P_1 > 0.$$

Applying the implicit function theorem,

$$\frac{d\hat{\theta}(x_D^*)}{dx_D^*} = -\frac{\frac{\partial G(\hat{\theta}, x_D^*)}{\partial x_D^*}}{\frac{\partial G(\hat{\theta}, x_D^*)}{\partial \hat{\theta}}},$$

defining  $Q = \frac{\partial G(\hat{\theta}, x_D^*)}{\partial x_D^*} > 0$ , and rewriting

$$\frac{d\hat{\theta}(x_D^*)}{dx_D^*} = \frac{Q}{Q - \frac{a}{\sqrt{bc}}\hat{\phi}(B(\hat{\theta}))P_2 + 1},$$

it is easy to check that

$$\frac{d\hat{\theta}(x_D^*)}{dx_D^*} < \frac{b}{a+b}$$

holds whenever  $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$ . Since this implies that  $a < \sqrt{2\pi b(1+c)}$ , the proof is complete.  $\blacksquare$

### A.3. Proof of Proposition 1

Using Lemma 2 and equation (10), we can write  $s_D^* = \theta_D^*(x_D^*) + M$ , where  $M = \frac{\Phi^{-1}(\kappa_2)}{\sqrt{a+b+bc}}$ . Thus, the creditors' *cutoff condition* in  $t_1$  can be expressed purely in terms of  $x_D^*$ :

$$\kappa_1 = Pr(s_i < \theta_D^*(x_D^*) + M|x_D^*)\kappa_2 + Pr(\theta \geq \theta_D^*(x_D^*), s_i \geq \theta_D^*(x_D^*) + M|x_D^*)$$

Write  $x$  for  $x_D^*$  and let

$$G(x) = Pr(s_i < \theta_D^*(x) + M|x)\kappa_2 + Pr(\theta \geq \theta_D^*(x), s_i \geq \theta_D^*(x) + M|x) - \kappa_1.$$

Note that

$$Pr(\theta \geq \theta_D^*(x)|x) = 1 - \Phi\left(\sqrt{a+b}\left(\theta_D^*(x) - \frac{a\mu + bx}{a+b}\right)\right).$$

Define  $A(x)$  as

$$A(x) = \sqrt{a+b}\left(\theta_D^*(x) - \frac{a\mu + bx}{a+b}\right)$$

and note that, given  $x$ ,

$$s_i = \frac{a\mu + bx + bc\theta + \sqrt{bc}\eta_i}{a+b+bc}.$$

Rearranging terms, this can be rewritten as

$$s_i = \frac{a\mu + bx}{a+b} + \frac{bc}{a+b+bc}\left[\frac{z}{\sqrt{a+b}} + \frac{\eta_i}{\sqrt{bc}}\right],$$

where  $z = \sqrt{a+b}\left(\theta - \frac{a\mu + bx}{a+b}\right)$  is distributed  $N(0,1)$  conditional on  $x$ . Let  $\gamma = \frac{z}{\sqrt{a+b}} + \frac{\eta_i}{\sqrt{bc}}$ . Then,  $s_i < \theta_D^*(x) + M$  is equivalent to

$$\gamma < \frac{a+b+bc}{bc\sqrt{a+b}}A(x) + \frac{\sqrt{a+b+bc}}{bc}\Phi^{-1}(\kappa_2).$$

Now let

$$B(x) = \frac{a+b+bc}{bc\sqrt{a+b}}A(x) + \frac{\sqrt{a+b+bc}}{bc}\Phi^{-1}(\kappa_2)$$

and rewrite

$$\begin{aligned} G(x) &= Pr(\gamma < B(x))\kappa_2 + Pr(z \geq A(x), \gamma \geq B(x)) - \kappa_1 \\ &= Pr(z < A(x), \gamma < B(x))\kappa_2 + Pr(z \geq A(x), \gamma < B(x))\kappa_2 \\ &\quad + Pr(z \geq A(x), \gamma \geq B(x)) - \kappa_1. \end{aligned} \tag{18}$$

Differentiating under the double integral and rearranging, we get:

$$G'(x) = B'(x)\hat{\phi}(B(x))[\kappa_2 - P_2] - A'(x)\phi(A(x))P_1,$$

where  $P_1$  and  $P_2$  are defined as follows:

$$P_1 = \int_{B(x)}^{\infty} f(\gamma|z = A(x))d\gamma$$

$$P_2 = \int_{A(x)}^{\infty} f(z|\gamma = B(x))dz.$$

Using standard computations to derive conditional distributions of normal random variables (see e.g. MITTELHAMMER (1996)), we know that:

$$z|\gamma = B(x) \sim N\left(A(x) + \frac{\sqrt{a+b}}{\sqrt{a+b+bc}}\Phi^{-1}(\kappa_2), \frac{a+b}{a+b+bc}\right).$$

Thus,

$$P_2 = \int_{A(x)}^{\infty} f(z|\gamma = B(x))dz = \kappa_2,$$

and therefore  $G'(x)$  reduces to

$$G'(x) = -A'(x)\phi(A(x))P_1.$$

Under the conditions of the theorem, Lemma 2 states that  $\frac{d\hat{\theta}(x)}{dx} < \frac{b}{a+b}$ . Thus,  $A'(x) < 0$ , implying that  $G'(x) > 0$ . Moreover, note that  $\lim_{x \rightarrow -\infty} G(x) = \kappa_2 - \kappa_1 < 0$  and  $\lim_{x \rightarrow \infty} G(x) = 1 - \kappa_1 > 0$ . Hence, there exists a unique solution  $(x_D^*, s_D^*, \theta_D^*)$  to the three necessary conditions (8) to (10) for an equilibrium in trigger strategies.

Finally, fixing  $\theta_D^*$ , the indifference condition for creditors in  $t_1$  as given by (18) depends on  $x$  only via the functions  $A(x) = \sqrt{a+b}(\theta_D^* - \frac{a\mu+bx}{a+b})$  and  $B(x) = \frac{a+b+bc}{bc\sqrt{a+b}}A(x) + \frac{\sqrt{a+b+bc}}{bc}\Phi^{-1}(\kappa_2)$ . If  $\theta_D^*$  is fixed,  $A(x, \theta_D^*)$  clearly is strictly decreasing in  $x$  for all  $b > 0$ , so that creditors who receive signals  $x < x_D^*$  choose to foreclose in  $t_1$ , and they choose to delay the foreclosure decision whenever  $x \geq x_D^*$ . Therefore, the proof is complete.  $\blacksquare$

#### A.4. Formal derivation of equations (11) and (12)

First consider the derivation of equation (11). Applying Lebesgue's theorem of dominated convergence, we can write:

$$Pr(x_i \geq x_D^*, s_i < s_D^* | \theta_D^*) = \int_{x_D^*}^{\infty} Pr(s_i < s_D^* | \theta_D^*, x_i) f(x_i | \theta_D^*) dx_i.$$

By definition  $s_i = \frac{a\mu+bx_i+\sqrt{bc}y_i+bcx_D^*}{a+b+bc}$ . Given  $x_i$  and  $\theta_D^*$ , and substituting  $y_i = \sqrt{b}(\theta_D^* - x_D^*) + \frac{\eta_i}{\sqrt{c}}$ , this transforms to  $s_i = \frac{a\mu+bx_i+bc\theta_D^*+\sqrt{bc}\eta_i}{a+b+bc}$ . Using equation (10),  $s_i < s_D^*$  then implies that

$$\eta_i < \frac{a+b}{\sqrt{bc}}\theta_D^* + \sqrt{\frac{a+b+bc}{bc}}\Phi^{-1}(\kappa_2) - \frac{a}{\sqrt{bc}}\mu - \sqrt{\frac{b}{c}}x_i.$$

As  $c \rightarrow \infty$ , the right-hand side converges pointwise to  $\Phi^{-1}(\kappa_2)$  and therefore

$$Pr(s_i < s_D^* | \theta_D^*, x_i) \rightarrow \Phi(\Phi^{-1}(\kappa_2)) = \kappa_2.$$

Thus,

$$Pr(x_i \geq x_D^*, s_i < s_D^* | \theta_D^*) \rightarrow Pr(x_i \geq x_D^* | \theta_D^*) \kappa_2 = \Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right) \kappa_2,$$

implying that equation (8) reduces to

$$\theta_D^* = 1 - \Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right) + \Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right) \kappa_2.$$

Rearranging terms, we finally get

$$1 - \theta_D^* = (1 - \kappa_2) \Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right).$$

Now, consider the derivation of equation (12). By Lebesgue dominated convergence,

$$Pr(\theta \geq \theta_D^*, s_i \geq \theta_D^* | x_D^*) = \int_{\theta_D^*}^{\infty} Pr(s_i \geq s_D^* | \theta, x_D^*) f(\theta | x_D^*) d\theta.$$

Given  $x_D^*$  and  $\theta$ , it is easy to see that  $s_i = \frac{a\mu + bx_i + bc\theta + \sqrt{bc}\eta_i}{a+b+bc}$ . Thus,

$$s_i \geq s_D^* \Leftrightarrow \frac{a\mu + bx_i + bc\theta + \sqrt{bc}\eta_i}{a+b+bc} \geq \theta_D^* + \frac{1}{\sqrt{a+b+bc}} \Phi^{-1}(\kappa_2),$$

which reduces to

$$\eta_i \geq \sqrt{bc}(\theta_D^* - \theta) + \frac{a+b}{\sqrt{bc}}\theta_D^* + \sqrt{\frac{a+b+bc}{bc}}\Phi^{-1}(\kappa_2) - \frac{a}{\sqrt{bc}}\mu - \frac{b}{\sqrt{bc}}x_i.$$

As  $c \rightarrow \infty$ , the right-hand side of this inequality tends to  $-\infty$  if  $\theta > \theta_D^*$ , and to  $\infty$  if  $\theta < \theta_D^*$ . Hence,

$$Pr(s_i \geq s_D^* | x_D^*) \rightarrow \begin{cases} 1 & \text{if } \theta > \theta_D^* \\ 0 & \text{if } \theta < \theta_D^*. \end{cases}$$

Thus,

$$Pr(\theta \geq \theta_D^*, s_i \geq s_D^* | x_D^*) \rightarrow Pr(\theta \geq \theta_D^* | x_D^*).$$

This implies that equation (9) reduces to  $\kappa_1 = Pr(\theta < \theta_D^* | x_D^*) \kappa_2 + Pr(\theta \geq \theta_D^* | x_D^*)$ , or in other words,

$$1 - \kappa_1 = (1 - \kappa_2) \Phi\left(\sqrt{a+b}\left(\theta_D^* - \frac{a\mu + bx_D^*}{a+b}\right)\right).$$

■



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