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Abstract

If we understand well the individualization of land tenure rules under conditions of growing land scarcity and increased market integration, much less is known about the mode of evolution of the farm-cum-family units possessing the land. Inspired by first-hand evidence from West Africa, this paper argues that these units undergo the same process of individualization governed by the same forces as property rights in land. It provides a simple theoretical account of the coexistence of different forms of family when farms are heterogenous in land endowments and technology is stagnant. The paper also offers analytical insights into the sequence following which such forms succeed each other.

1 Introduction

We have today a solid grasp of why and how land tenure rules evolve at the community level. More precisely, we understand the conditions under which a shift occurs from corporate ownership of land (possibly including the granting of long-term use rights to individual households) to individualized forms of tenure ranging from less to more complete private property rights. In particular, land tenure becomes more individualized when land value increases because externalities are better internalized and stronger incentives to conserve and improve land are thereby provided (Demsetz, 1967; Ault and Rutman, 1979; Feder and Feeny, 1981; Feder and Noronha, 1987; Baland and Platteau, 1998; Platteau, 1996 and 2000: Chaps. 3-4). However, the organizational features of the landholding unit itself evolve over space and time and these variations are far from being understood. What we argue in this paper is that the same force, growing land scarcity, that drives the individualization of land tenure also drives the individualization of the family unit owning and managing the land. Therefore, when private rights in land are well-established, as pressure on land continues to rise as a result of population growth and/or market integration, the individualization process goes on, yet now more at the level of the farm units than at the level of the community where rules governing land allocation and tenure rights are decided.

As epitomized by the past experience of Russia or the recent-day experience of Mali (West Africa), individualization at the farm-cum-family level occurs when either of the two following circumstances arise: (i) the head of a collective farm decides to grant individual plots to members of the household, and these are entitled to keep for themselves the entire proceeds of such plots while they are simultaneously required to work on the collective, family fields; (ii) the head agrees to split the stem household, implying that some members leave with a portion of the land equivalent to a pre-mortem inheritance, in order to form separate, autonomous branch households based on the nuclear family. While the first scenario involves the transformation of a purely collective farm into a mixed farm, the second scenario implies that part of the family land is inherited pre-mortem. The second scenario appears to correspond to a more advanced form of individualization of the farm unit than the first scenario, yet the order in which these two forms should succeed each other as land pressure rises is far from evident.

To explain this evolution, a theoretical framework is needed in which the behaviour of different actors (the head and the members) and their strategic interactions are specified. So far, economists have proposed few theories of the evolution of the farm-cum-family structure, and the available theories aim at explaining either the shift from the collective farm to the mixed form in which individual and collective fields coexist, or the breakup of the collective farms into individual units.

Fafchamps (2001) offers an example of the former by developing a theoretical model to explain the decision of the household head to allocate individual plots to family members. At the core of his model is a problem of commitment. Because the head is unwilling or unable to commit to reward their work on the family field after the harvest, family members are tempted to relax their labour efforts or to divert them to other income-earning activities. To solve this commitment failure problem, the head decides to reward his wife and dependents for their labour on the collective field by giving them individual plots of land and the right to freely dispose of the resulting produce. It is evident that the commitment problem only exists if the short-term gain of deviating from cooperation (which means here reneging on the promise to reward the workers for their effort on the collective field) exceeds the long-term flow of benefits ensuing from a smooth relationship between the household head and the working members. As Fafchamps himself recognizes this condition is restrictive, since the game played within the family is by definition of a long (and indeterminate) duration, and the discount rate of future benefits typically low (future cooperation among close relatives matters a lot). Moreover, even assuming that Fafchamps' hypothesis is valid, it remains unclear why there should be a tendency over time for collective farms to transform themselves into mixed farms. Finally, Fafchamps does not consider a potential break-up of the household accompanied by a (partial or complete) division of the extended family's landholding.

Other authors have tried to explain the coexistence of collective fields and individual plots in agricultural farms, yet agricultural producer cooperatives or quasi-feudal landowner-tenant relationships form the specific context in which their explanations are advanced. Regarding producer cooperatives, emphasis is typically put on the existence of scale economies for certain types of activities, or on the need for insurance and the role of income-pooling (Putterman, 1981, 1985, 1987, 1989; Putterman and DiGiorgio, 1985; Carter, 1987). As for relationships between estate owners and workers, limited liability constraints and the demand for insurance are the main motives prompting the adoption of the mixed farm structure (Allen, 1984; Sadoulet, 1992).

We thus face a relative shortage of pertinent accounts of the existence of individual plots in the precise setting of family farms. Unlike in democratic producer cooperatives, a hierarchical relationship prevails in these farms, and in contrast to feudal or semi-feudal estates (where independent tenants became re-integrated into a seigneurial estate when landlords decided to embark upon direct cultivation of a portion of their land), their transformation typically consists of a shift from the pure collective form to the mixed structure.

Farm breakups, the second form of individualization, are at the center of Foster and Rosenzweig (2002) attempt to explain household-cum-landholding division. They do not allow for individual plots, as for them co-residence implies collective farming only. In their framework, an extended family is composed of several claimants to the land who may decide to split if the benefit of sharing public goods by co-residing is smaller than the loss of efficiency due to decreasing returns to scale in production. There are thus two different ways of explaining the increasing incidence of individual farms: (i) growing disinterest of younger generations in the sort of public goods produced on the collective farm, and (ii) rising importance of decreasing returns to scale as a result of the shift to more land-intensive agricultural techniques.

Clearly related to the latter proposition is the work of Boserup (1965) who attributes the rise of peasant farms to growing land scarcity and the consequent intensification of agricultural techniques. Although formulated by a geographer, the underlying argument has a distinctly economic flavor, hence its large resonance among development economists (Binswanger and Rosenzweig, 1986; Binswanger and McIntire, 1987; Pingali, Bigot and Binswanger, 1987; Binswanger, McIntire and Udry, 1989; Hayami and Otsuka, 1985). As land pressure increases, so the argument runs, farmers are induced to shift to more intensive forms of land use, which implies that they adopt increasingly land-saving and labour-using techniques. An important characteristic of these techniques is that labour quality, which is costly to monitor, assumes growing importance. Given the incentive problems associated with care-intensive activities (sometimes labeled "management diseconomies of scale"), the small family or peasant farm in which a few co-workers (spouses and their children) are residual claimants, appears as the most efficient farm structure.

Although Boserup's story is undeniably appealing, both theoretically and empirically, it cannot apparently account for situations in which an evolution towards more individualized forms of family-cum-farm structures takes place in the absence of noticeable technical progress. Thus, in Russia during the 17-19th centuries, a shift from large and complex agricultural households (married brothers stay together at least till the death of the father) to smaller and more simple ones (married brothers part with each other while the father is still alive, but a household may remain multigenerational) has occurred, a change which historians generally ascribe to the expansion of non-agricultural opportunities rather than to the adoption of new agricultural techniques (Worobec, 1995; Moon, 1999). In the old cotton zone of southern Mali (West Africa), where the authors of this paper did fieldwork, collective farms appear to be increasingly replaced by mixed farms and small farms born of the break-up of large family farms, despite persisting technological stagnation.

Given the incompleteness of the theory proposed by Boserup, we set out to develop an alternative framework susceptible of explaining individualization of family-cum-farm structures in conditions of rising land scarcity and technological stagnation. Toward that purpose, we write a simple model in which the three aforementioned family forms are featured in a static environment characterized by heterogeneous land endowments at farm level. Through comparative statics, we check whether smaller land assets (or growing needs of members) lead to individualization of the farm unit. It is implicitly assumed that adjustment to rising land pressure is easier to achieve through change in the family-cum-farm structure than through demographic change and fertility reduction, or through land markets. While fertility reduction requires a long term horizon, land markets are highly imperfect owing to large transaction costs or because the fear of losing land prevents the supply side of the market from being activated (Basu, 1986). In this context, any change in land allocation is the outcome of a decision regarding the organization of the family farm.

The intuition behind our model is simple. When deciding whether to give individual plots and how large they should be, the family head faces a trade-off between considerations of efficiency in the use of the land and considerations of rent capture. For one thing, production is more efficient on private plots than on the collective field where the moral hazard-in-team problem prevents optimal effort from being applied by family members. Since the head must ensure that family members agree to stay on the family farm while they have outside options available to them, awarding individual plots allows him to more easily satisfy their participation constraints. For another thing, because the head's income entirely comes from the collective produce owing to unenforceable transfers from the private plots, competition between the family field and the individual plots for the allocation of effort is bound to cause a fall in the quantity of harvest appropriable by the head. In the case of a pre-mortem split of the family farm, the total labor force available for work on the collective field also decreases, whereas it is no more incumbent on the household head to provide for the needs of the departed members. Depending on the relative importance of the aforementioned effects, the father may prefer a mixed regime with individual fields to the collective regime, or he may choose to split the family.

The paper is structured as follows. In Section 2, we mention empirical evidence from Russia and southern Mali that is directly relevant for our topic. We focus attention on observations that come in support of the central assumptions underlying the model presented in Section 3. In this section, we first set up the model, define each regime and explore the forces at play when choosing across regimes. In Section 4, we derive analytical results regarding the role of reservation utility and land pressure in regime choice. In Section 5, we present simulation results to illustrate the coexistence of the three regimes and further analyze their occurrence in a reservation utility-land endowment space. Section 6 concludes.

2 A theory of the patriarchal family: supporting evidence

2.1 Farm-cum-family structures in Russia in the sixteenth-nineteenth centuries

Unique material collected by Russian scholars enables us to figure out certain important aspects of the dynamic of household formation in Russia during the pre-industrialization period (17th to 20th century). It shows striking similarities to the situation described below for Mali, as well as some differences. The following account relies mainly on the analysis of the available historical material by Worobec (1995), henceforth labeled WO, and Moon (1999), henceforth labeled MO. When other sources are used, they are referred to in the conventional manner.

Extended households and patriarchy

A large proportion of peasant households were composed of extended families comprising several conjugal units, at least before the emancipation (1861). Pre-mortem household divisions then constituted a departure from the norm (WO: 88, 107), implying that "most newly married couples spent at least the first part of their married lives in the household of one set of parents" (MO: 179). Exceptions to the rule of post-mortem division were mostly found among the households of Siberia and the outlying parts of the forest heartland, which explains why at any given point in time households tended to be smaller and simpler in these areas than in the central black earth region and other steppe regions as well as in the forest heartland (MO: 170).[1]

Extended households were placed under the authority of the head, or patriarch. The patriarch of the Russian peasant household "held absolute power over management of the household economy and the labour input of family members" (WO: 11). This implied that the head could encourage a son to take a job at a domestic industry, in which case he would have "to remit his wages, minus any expenses incurred while he was away on the job, to the household's coffers" (WO: 11).

Household divisions typically took place at the death of the patriarch, often as a result of internal tensions. It is rather easy to understand why families may split after the death of the father. In the words of Christine Worobec: "if a son became household head upon his father's head, he could not command authority over his brothers as had his father, since all brothers were treated equally in the devolution of property. The other brothers were intent on being masters of their own households" (WO: 81). This is confirmed by Moon's account according to which splitting members were typically the younger brothers of men

[1]For a proper analysis of the dynamic analysis of peasant households, David Moon thus emphasizes the need to distinguish between households that divide before and those that divide after the death of the head, rather than between simple and complex households. In other words, the key difference is between two types of household life cycles: the "phases of development" cycle and the"perennial complex household" cycle (MO: 179).

who succeeded to headship on their father's death: "they broke away rather than submit to their elder brother's authority" (MO: 171), and therefore parted with the stem household in order to set up an independent farm unit on a portion of the family land.

Pre-mortem divisions

Pre-mortem divisions were also observed and they often stemmed from suspicions of free riding. Tensions could arise because of the unequal sizes of the different conjugal units forming the joint household. Thus, "a brother resented having to work twice as hard, or so he believed, because one of his brothers had twice as many children" (WO: 81). But there were many other pretexts or reasons nurturing jealous feelings among siblings. In particular, "the relationships between daughters-in-law and mothers-in-law inside households was fraught with tensions and jealousies" (MO: 196).

Especially after the abolition of serfdom and other reforms, in the late nineteenth century, improved outside opportunities in the form of expanding opportunities for wage labour contributed to a surge in pre-mortem fissions and the growth in nuclear family households (WO: 87, 115; see also Waldron, 1997: 71). Household divisions thus increased more rapidly in areas "where a substantial portion of the population derived its income from non-agricultural pursuits" (WO: 105), a phenomenon particularly noticeable in the central non-black earth region and elsewhere in the forest heartland (MO: 176). The tendency for households to split in such conditions was accentuated by the fact that wage-earning members sometimes resented having to pay towards the upkeep of their father's households. If so, they tried to keep all or part of the money for themselves, rather than hand it over to the head, which could lead to severe conflicts and determine them to demand partition "so that they could become the masters of their own households" (MO: 176, 196).

Distribution of individual plots

Individualization of peasant households did not necessarily take the form of a split of the original stem household. Individual plots of land could be awarded to male members who continued to belong to the joint family. These members were expected to continue to help their father and brothers in the cultivation of the household's communal land allotments. The sons were responsible for their share of the tax payments charged on the joint household's land and obliged to help support their parents when they retired. In return, they retained rights to a share of the patrimony, minus whatever property was given them for individual cultivation before the time of bequest. If, on the other hand, they stayed on the farmstead but cultivated in a completely independent manner on a portion of the family's land, they were disqualified for further inheritance (WO: 55).

2.2 A picture of present-day Mali

In 2006 and 2007, we conducted a systematic household survey on a random sample of 502 households belonging to 50 different villages in the districts of Koutiala, San, and Sikasso (South Mali). In this section we use this data and report descriptive evidence regarding the simultaneous presence of the three above-described farm-cum-family structures, the views of local patriarchs on this evolution and the key assumptions that underlie our theoretical approach.

2.2.1 Observations about the transformation of farm-cum-family structures

First, the activity of local land markets is extremely limited in spite of growing land scarcity (in villages land is no more available for outsiders to settle on). The great majority of the land parcels (80%) were inherited (post or pre-mortem), while the remainder were either cleared by the owner a few decades ago (10%), or borrowed by the household.[2] Moreover, the local labour market is hardly developed so that land available per unit of labour is not equalized across farms: farms are heterogeneous in terms of land-labour endowment. On

[2] Land lending is not synonymous of renting in the sense that no cash or in-kind payment is involved. The land is often borrowed over several generations. With increasing land pressure, however, conflicts between owners and borrowers have become more common, frequently because the family which borrowed land a generation ago is reluctant to return it to the owner.

the other hand, household members have outside opportunities available to them, mostly in the form of migration to Malian cities or neighboring countries. Improved communication and increased mobility have contributed during the last decades to enhanced perceptions of potential employment opportunities outside the native village.

Extended households and patriarchy

In our sample, 23% of household heads live with their brothers while, at the other extreme, only 10% have neither brothers nor married sons around (strictly speaking, they are nuclear households). Moreover almost 60% of the household heads are polygamous. On average the sample households count 11 individuals above 12 with a maximum family size of 33.

The family system functions as a patriarchy, implying that all important decisions are taken by the (male) head, in particular those regarding the way land is allocated and income is distributed on the farm. There are significant facts pointing to the existence of a strongly patriarchal society in southern Mali. Not only do customary inheritance rules exclude female members, but there are also compelling clues attesting to the importance of the authority exercised by the household head. The assumption of patriarchy implies not only that the head decides whether part of the family land will be earmarked for individual plots or not, but also whether some members (and how many) will be allowed to leave the stem household and form separate branch households by using a share of the family land. On the collective field, the head is in complete charge of all important decisions. When individual plots exist, management decisions including the choice of crop and supervision of effort belong to the landholding member, yet the allocation of labor time between the collective field and the individual plot is fixed by the head. Bear in mind, however, that the ability of the head to set the timetable for work on the collective field does not imply that he can control the allocation of effective labour effort between collective and individual activities.

The authority of the head stretches beyond the production sphere. In particular, almost all heads assert that members must seek their approval before taking a loan, and that they often avail themselves of this prerogative to refuse permission. They see themselves as acting on behalf of the family and responsible for its ordered functioning, including the due loans taken by members.

Finally, there is one domain in which household heads admit that their power is limited. This is with respect to consumption choices made by children who have independent incomes (from individual plots) and claim the right to spend them according to their own preferences. In fact, the awarding of individual plots to members goes hand in hand with the devolution of non-food expenditures to them.

Post- and pre-mortem divisions

Family splits occur when some members leave the stem household to form their own independent branch household while the head of the extended family is still alive. About one-fourth of the sample heads belong to that category and most of them have received a fair share of the family's land endowment. In about 60% of the cases, the custom has been followed, implying that at the death of the family head, the eldest living brother or his eldest son (living on the farm) has succeeded him to exert authority over the family and the farm. In the remaining cases, the family has separated at the death of the head. At least part of those break-ups may be termed customary, however. When a patriarch rules over four or five generations of his brothers and his brothers' descendants indeed, the custom is that sons and nephews of the deceased patriarch found branch households under the authority of the oldest sibling. Separation of the household is then accompanied by the division of the family land, and splitting is not, strictly speaking, the outcome of a decision of the household head. Like in the case of Russia, breakup of the stem household may also occur when brothers do not get on well enough to operate together in the absence of the father.

The main reasons given by the heads of branch households to explain why they themselves broke away from the stem household are rising land pressure in the stem household (34% of interpretable answers), and the eruption of conflicts within the family, most often involving their brothers or uncles (again 34%).[3] Other reasons include low production in the stem household, and the existence of special needs that could not be satisfied if the member had stayed with the whole family (expensive medicine to cure a wife, for example). We will focus attention on the first eventuality in which land pressure is the primary cause of family breakups. It must nevertheless be borne in mind that as attested by well-substantiated evidence, village or community-level conflicts, including intra-family disputes, are often caused by acute scarcity, real or anticipated, of available land assets. There may thus be a significant overlap between the tow dominant motives alleged to lie behind household splits (Andre and Platteau, 1998; Haugerud, 1993: 162-176).

Distribution of individual plots

Not only branch households but also mixed farms are found to coexist with traditional collective farms in the Koutiala-San-Sikasso region. In particular, individual plots allotted to male members living on the farm have been observed in about one-fourth of our sample households. It is moticeable that, when this is the case, all male members above a certain age have received a private plot. Moreover, the practice of giving out individual plots is on the increase, and growing land scarcity seems to be associated with this increase: households which have granted individual plots to (male) members turn out to have significantly less land per male member (3.01 ha) than those running pure collective farms (3.67 ha).

Perceptions of the family heads regarding the causes of ongoing transformations

When queried about the reasons underlying the trend toward growing individualization of farm-cum-family structures, whether in the form of mixed farms or broken-up households, the heads whom we interviewed in Mali pointed to increasing land pressure and consumption needs, particularly among the younger generations. As land becomes scarce, so the first argument runs, family heads find it increasingly difficult to provide for the subsistence

[3]These percentages are based on answers given to open questions that we later classified into categories.

needs of the extended family from the collective field. They claim that land scarcity leaves them with no other choice than to let some family members acquire more autonomy through the ability to cultivate individual plots or to form separate branch households. Another oft-heard explanation refers to what senior villagers call "modernity", understood as the greater consumption needs of the young generations. The rhetoric is that, nowadays, young people have new needs, such as a motorbike, nice clothes, sometimes even a cellular phone... Analytically, this change may be captured by an increase in the reservation utility required by household members to continue to work and stay with the head. Because they perceive to have better outside opportunities, typically in the form of migration to Malian cities or neighboring countries, they feel able to demand a higher level of welfare. Improved communication and increased mobility have no doubt contributed to these enhanced perceptions of potential employment opportunities outside the native village. Note the conceptual analogy between the two explanations: an increase in the extent of needs to be satisfied from a given amount of land appears to be the converse of a decrease in the amount of land available to satisfy a given extent of needs. In practice, however, the two outcomes are caused by different forces: rising numbers, on the one hand, and increased market integration, on he other hand.

2.2.2 Evidence in support of the key assumptions behind our theory

The theory developed in the subsequent sections rests on three central assumptions that are derived from key insights obtained in our field research in Mali. The first assumption concerns the patriarchal form of authority that rules over the family farms. Since it has already been amply substantiated above, we focus our attention on the other two assumptions, namely:

- Owing to non-observability of individual labour efforts, incentive problems discourage production on the collective field, especially when effort on this field competes with effort applied to individual plots.[4]

- The whole income accruing to the head is obtained from the output of the collective field because, while this output is observable (as an aggregate) by him, output on individual plots is not easy to monitor. This assumption is consonant with the observations reported in numerous anthropological studies (see Duflo and Udry, 2004, for references).[5]

First, regarding the incentive problems plaguing collective production, many heads have explicitly referred to them while discussing the practice of individual plots. For them, the main shortcoming of such plots is that family members tend to relax their effort on the collective field, thereby impairing yields. Complaints such as "more effort is applied to the individual plots and when members work on the collective plot, they are tired" or "members are prone to keep energy in reserve for their individual plots"[6] are commonly expressed by family heads. They suggest that the granting of individual plots exacerbates the problem of moral-hazard-in-team on the collective field.[7] On the other hand, differentiating payments according to individual effort contributions to collective production is hard not only because such contributions may not be easy to measure but also because as has been stressed by family heads in the interviews, accusations of favoritism or unfair treatment will be inevitably aroused by this practice, leading to vicious intra-family conflicts.

As far as our second assumption is concerned, the crucial fact is that members who cultivate an individual plot tend to keep the entire production for their own consumption, or that of their children. Only 6% of them "helped" the household head in the previous year

[4]Incentive problems also arise if the head observes effort but chooses to use an equal sharing rule to avoid intra-family conflicts.

[5]In their words, "A voluminous literature makes it clear that individuals have substantive control over decision on their plots, and that nominal control over the output from a plot belongs to the cultivators."

[6]In the French parlance used by our interlocutors, members possessing private plots "se réservent".

[7]Another shortcoming of individual plots which has been frequently cited by our respondents is the risk of intra-family tensions and conflicts arising from the coexistence of collective and individual activities. Such risk is linked to the moral-hazard-in-team problem since manifestations of labour shirking may easily prompt accusations of misbehaviour among family members.

through transfers in cash or in kind, and when they do occur such transfers are typically very small. It is revealing that a large majority of household heads admit that members who possess an individual plot have no obligation to transfer income to them. Also revealing is the fact that most household heads consider that, when individual plots are awarded, they are no more responsible for the financing of marriage-related expenditures including brideprice payments. Such expenses now befall the holders of individual plots.[8]

3 A simple model of family farm structure

3.1 Two important clarifications

Before we present the formal structure of our theory, we need to address two important questions the answers to which will condition the way we set up our model. The first question concerns the mode of remuneration of household members for their effort on the collective field, whereas the second question has to do with the nature of the participation constraint in the event of splitting. Let us examine them in turn.

3.1.1 Contractual form

Given the specific context of the family farm, a share system appears as the (second-best) efficient contract even when risk considerations are abstracted from. This conclusion rests on the argument developed by Eswaran and Kotwal (1985), according to which the contract choice problem can be framed as a trade-off between the need to provide tenants with adequate incentive to apply effort, on the one hand, and the need to use the landowner's management skills to the best possible extent, on the other hand. When the relative advantage of the landowner in using his management skills and the relative advantage of the tenant in using his supervision abilities are sufficiently important, Eswaran and Kotwal show that

[8]In terms of the model developed later, granting individual plots relaxes the participation constraint.

the share contract dominates the fixed rent contract (in which the landowner's management skills are poorly used but labour incentives are optimally generated) and the fixed time wage rate system (in which the workers have no incentive to apply any effort but the landowner is optimally induced to use his management skills). In the context of the family farm, Eswaran and Kotwal's argument is especially relevant: not only is the landowner (the family head) physically present on the farm but, far from being passive, he is typically eager to achieve overall supervision of agricultural operations and to take important decisions, such as choosing which crops to grow, fixing the calendar of all the productive tasks involved, setting the days of the week and the hours of the day when members have to work on the collective field, etc.

It could be objected that, since the head is willing and able to supervise agricultural operations, he is also in a position to enforce any effort level that he considers appropriate. The fixed time wage rate contract would then be optimal. However, close supervision of family members is avoided not only because it would involve costly effort for the head but also because it would create a calculative atmosphere that would rouse suspicion and resentment, potentially destroying the delicate balance on which more or less harmonious family relations depend (see Williamson, 1985). Combining self-supervision by members with overall decision-making by the head through the use of a share contract thus appears as the most efficient contractual arrangement.

Under a mixed farm structure, family members apply effort to both the collective field and their private plot, yet when cultivating the latter they also use their own management skills which they have learned while working on the former (bear in mind that members receive a private plot only when they reach adult age). On the private plots, therefore, members exercise management skills which they are prevented from using on the collective field. The question arises as to why the head does not use the management skills of the members instead of his own on the collective field. The answer is that management responsibilities need to be integrated under a single authority: if they were delegated to family members, a serious coordination problem would be created, and the head would be compelled to continuously intervene to settle disruptive conflicts. The implication is that, when a whole team of family members are involved in joint production, the role of unifying management decisions is critical, hence the significant relative advantage of the head in using his management skills when production is a joint activity (for a similar argument made in the context of small-scale marine fishing, see Platteau and Nugent, 1992). There is an additional reason that prompts the head to apply his management skills to the collective field. Since consumption of basic necessities is centralized under his authority in the extended household form, and consumption decisions are closely reflected in production decisions (which crops to produce, when and in what proportions) in semi-autarchic farm households, the head is keen to make his choices prevail in the collective family sphere.

At this juncture, it is interesting to stress that a fixed rent contract is not a feasible arrangement under a mixed farm structure in which effort is more efficiently applied on the individual plots than on the collective field. In other words, the situation in which the head would simultaneously opt for a mixed farm structure (implying division of the available land into both a collective field and individual plots) and charge a fixed rent for the use of the land in the collective field, does not correspond to a Nash equilibrium. Assuming that the head uses a mixed share-cum-fixed-rent contract, he could always raise his rent by marginally decreasing his share of the harvest and increasing the fixed component. This is because he would thereby mitigate the efficiency losses that result from the share system on the collective field. As a result, the value of the share parameter tends to zero, and the fixed component absorbs the whole production on that field, leaving the members forced to achieve their reservation utility entirely from the output of their private plots. (If this were not true, the head could always increase his rent by enlarging the size of the individualized holdings where there is no moral-hazard-in-team problem.) Such an outcome, however, cannot be an equilibrium: since members would not obtain any reward from their work efforts on the collective field, they would not put in any effort and the head would receive zero income. The conclusion is that a mixed farm system may be an equilibrium only if the head's rent is a pure share of collective production. Or, conversely, if the head charges a fixed rent for the use of his land, the farm must have a purely collective form (it must be integrated). A formal proof exists to show this impossibility of the fixed rent contract under a mixed farm structure with heterogeneous production conditions on the collective and private fields (first-best efficiency is achieved only on private fields).[9]

Finally, it must be noted that, when a member leaves the family farm to set up his own farm unit on a portion of the family land, he combines his management skills with own labour effort to produce output. In this respect, his situation is identical to that of the extended family's member who works out his individual plot in a mixed farm system.

3.1.2 Participation constraint in the event of splitting

So far, we have pointed out that household members have a reservation utility that the head must satisfy in order to keep them in the native community. At the same time, we have observed that the standard practice (in both Russia and Mali) in the event of splitting is for the head to let some of his sons set up their own independent farm on the portion of family land to which they are rightfully entitled under the customary patriarchal inheritance system (equal division of the land among all sons). In fact, it is only when a (male) member decides to leave the family against the wish of the living head that he may be disinherited as a punishment for his rebellious decision.[10] In other works, when (male) members leave

[9]It has been shown that the arrangement in which efforts on a collective field are rewarded by free access to a private plot is actually equivalent to a sharecropping contract that would be applied on the whole farm area (Allen, 1984). But this arrangement is not a realistic option since it implies that effort on the collective field can be monitored costlessly so that first-best efficiency is attained on both types of landholdings. It is typically justified by risk-sharing considerations.

[10]Note that, since it is not the outcome of the head's decision, or at least an agreement with him, we do not model this kind of eventuality which in any event appears to be rare in our study area. Our focus is on the question of the optimal farm-cum-family structure that the household head wants to establish or

the stem household with the consent of the head, they obtain their whole inheritance share pre-mortem.

In keeping with this observation, we assume below that when he contemplates the possibility of splitting the stem household, the head compares the rent which he obtains under the status quo (either the pure collective or the mixed farm) with the rent that he would get if he were to let one, two or more male members quit with their rightful share of the family land. The utility thus achieved by an independent (male) member of the family is not smaller than the reservation utility (since this would imply that the farm size is not large enough to enable all dependents to reach the reservation utility in a first best case of productive efficiency.) This causes a discrepancy between the utility of those sons who stay within the stem household and the utility of those who have left to form branch households. The advantage thus gained by the latter may be temporary in so far as the first members to leave are the elder, earlier married sons who are perhaps oing to be succeeded later by their younger brothers. It may also be more apparent than real if the independent sons have a (larger) family to sustain.

Because we want to keep the model as simple as possible - as we shall see, even a simple model of the kind envisaged is not easily tractable - we abstract away from complications arising from the age structure and demographic characteristics of the family. All sons are thus assumed to be identical, and we do not highlight the possible factors justifying differences in (reservation) utility between remaining and departing members.

When we discuss our results, we will nonetheless return to the case that we have just ruled out. The assumption of a uniform participation constraint which holds in all the three regimes implies that, in the event of splitting, a departing member would receive an amount of land just sufficient to afford him the reservation utility. It will be evident that the predictions obtainable under such conditions are much less rich than those derived from

maintain, given that he has to make the best possible living from it while satisfying the reservation utilities of the members.

our basic model.

3.2 The general framework

A household head has N male family members of whom n live and farm with him, and $N-n$ have formed independent households. The male members who left each received an equitable share of the father's total land endowment, $\overline{A}$. This area, $\frac{\overline{A}}{N}$, can be seen as a pre-mortem inheritance transfer. Thus, when the father chooses to let $N-n$ members leave the extended family to form their own separate households, the area remaining for the extended family farm is $A = \frac{n\overline{A}}{N}$. Labor on the stem household's farm is supplied by male members who have stayed with the head. The agricultural production function is $f(a, l)$, where a is land and l is labor. An individual's utility is $x - v(l)$, where x is the production that the individual consumes and l the level of labour he exerts. The function $v(l)$ is the disutility of labor.

The head allocates available land A between a collective field, where the male members work together, and individual fields, where each works individually and for his own benefit. We assume that members operating inside the extended family farm receive an equal treatment with respect to both the division of the produce of the collective field (hence the existence of a moral-hazard-in-team problem) and the apportioning of the land earmarked for individual farming. Therefore, if the head decides to grant individual plots, each member receives $A^I \leq \frac{A}{n}$.

Members consume the whole production of their individual fields, implying that the father's entire consumption R is obtained from his share $\alpha \leq 1$ of the output produced on the collective field, $A - nA^I$, $R = \alpha f(-)$.[11] In keeping with our field observations again, we thus assume that there is no possibility of income transfer from household members to the head. When $A^I = 0$, we say that the farm structure or regime is a pure collective farm,

[11]This implies that the moral-hazard-in-team problem cannot be overcome through the choice of appropriate contracts of a more requiring form. We justify this assumption in Section 5.1.

whereas if $A^I > 0$, it is mixed.

One unit of labor, whether applied on the collective field or on the individual plot, causes the same disutility . Therefore, member's j utility can be written as $x_j - v(l_j^C + l_j^I)$, where x_j is the sum of the share received from the collective field and the production from his individual plot, l_j^C is the level of effort applied to the collective field, and l_j^I that applied to the individual field. Members have an outside option that provides them utility $\underline{u}$, giving rise to a participation constraint.

The problem is a two-stage game. In the first stage, the head chooses α, A^I and n. In the second stage, members observe these choices and individually decide how much effort to apply to the collective field and how much to their individual plot. We restrict our attention to symmetric Nash equilibria in the second stage. This allows us to solve for a single pair (l^C, l^I), and to write the whole problem as follows:

$$\text{Max}_{n,\alpha,A^I} R = \alpha f\left(A - nA^I, nl^C\right) \tag{1}$$

$$\text{s.t.: } \{l^C, l^I\} = \text{Argmax}_{l_j^C, l_j^I} \frac{1-\alpha}{n} f\left(A - nA^I, l_j^C + (n-1)l^C\right) + f\left(A^I, l_j^I\right) - v(l_j^C + l_j^I) \tag{2}$$

$$l^C \geq 0 \text{ and } l^I \geq 0 \tag{3}$$

$$\underline{u} \leq \frac{1-\alpha}{n} f\left(A - nA^I, nl^C\right) + f\left(A^I, l^I\right) - v(l^C + l^I) \tag{4}$$

$$0 \leq \alpha \leq 1 \tag{5}$$

$$0 \leq nA^I \leq A \tag{6}$$

$$A = \frac{n\overline{A}}{N} \tag{7}$$

Total labor on the collective field in the incentive compatibility constraint is written $l_j^C + (n-1)l^C$ to stress that each member takes the behavior of others as given when deciding how much effort to apply to that field.

3.3 Giving out individual plots?

A first question to ask is the following: under which conditions does a household head find it optimal to distribute part of the family land to male members for private use, when n members remain on the farm to cultivate A? The problem is not trivial since there are two forces working in opposite directions. On the one hand, unlike the collective field, individual plots are used efficiently. As a consequence, a smaller amount of land has to be dedicated to meeting the members' reservation utility under a mixed system than under a pure collective regime. On the other hand, incentives to work on the collective field decrease when there is competition between the family field and private plots. This is because the worker is a full residual claimant on the latter whereas on the former he suffers from both the moral-hazard-in-team problem and the disincentive effect of the share system of labour remuneration. Efficiency on the land wherefrom the father derives his income is therefore impaired.

Unfortunately there is no explicit solution for the head's rent in either regime, even when a Cobb-Douglas production function and a quadratic cost of effort are posited. As a result, we cannot directly compare the head's rent between regimes. To understand the underlying logic of the model, however, it is useful to analyze the trade-offs faced by the head when he decides to allocate individual plots. We consider the problem in a sequential manner. First, let us define $\alpha^*(A^I)$ which is the optimal α for a given A^I. Let us now examine how the value function of this degenerate problem varies when A^I changes. If $\frac{\partial V}{\partial A^I}(\alpha^*(A^I)) < 0$ for all A^I such that $0 < A^I \leq \frac{A}{n}$, the head will not allocate individual fields, while if, $\frac{\partial V}{\partial A^I}(\alpha^*(A^I)) > 0$ over some range, the head may choose to allocate individual fields.

Suppose that A^I is fixed. When there exist both a collective field and individual plots, we can replace the members' maximization problem with the first-order conditions with respect to l^C and l^I and write the following Lagrangian (both l^C and l^I are strictly positive):

$$
\begin{aligned}
L\left(l^C, l^I, \alpha\right) = \alpha f\left(A - nA^I, nl^C\right) - \lambda\left(v'(l^C + l^I) - \frac{1-\alpha}{n} f_L(\frac{n\overline{A}}{N} - nA^I, nl^C)\right) \\
-\mu\left(v'(l^C + l^I) - f_L\left(A^I, l^I\right)\right) \\
-\nu\left(\underline{u} - \frac{1-\alpha}{n} f\left(\frac{n\overline{A}}{N} - nA^I, nl^C\right) - f\left(A^I, l^I\right) + v(l^C + l^I)\right)
\end{aligned} \tag{8}
$$

In order to analyze the sign of $\frac{\partial R}{\partial A^I}$, we apply the envelop theorem which yields the following expression:

$$
\begin{aligned}
\frac{\partial V}{\partial A^I} = \frac{\partial L}{\partial A^I} = -n\alpha f_A^C - \lambda(1-\alpha) f_{LA}(A - nA^I, nl^C) + \mu f_{LA}\left(A^I, l^I\right) \\
- \nu(1-\alpha) f_A(A - nA^I, nl^C) + \nu f_A\left(A^I, l^I\right)
\end{aligned} \tag{9}
$$

This expression reveals that as A^I increases (by one unit), the size of the collective field decreases (by n units), and the first term indicates how, everything else being constant, the family head's rent declines with the size of the field from which it is extracted. The second term captures the lower incentives for male members to work on the collective field as A^I increases (we show in appendix, section A.2.1 and A.2.2, that λ is positive). For a given amount of effort, indeed, the marginal product of labour falls when land becomes smaller. The third term reflects the negative impact on R caused by the enlarged size of the individual plots: members have more incentive to spend effort on their individual plot because the marginal productivity of labour has increased for a given amount of effort. As a result, the cost of their effort on the collective field is now higher (we show in appendix, section A.2.1 and A.2.2, that μ is negative).

The last two terms of equation 9 indicate how a change in A^I modifies the participation constraint, and how this affects the head's utility (bear in mind that $\nu \geq 0$ since the head's rent increases if the participation constraint is relaxed). Other things being equal (the distribution of labour efforts being constant), reallocation of land from the collective field to individual plots has the effect of enhancing the ability to produce $\underline{u}$ on the latter and simultaneously decreasing the ability to do so on the former. Measured by the marginal productivity of land in the two locations, this combined effect is positive overall because incentive problems exist on the collective field but not on the individual plots.[12]

It is therefore possible that, over some range of A^I values, $\frac{\partial R^*}{\partial A^I} > 0$, implying that the household head may prefer the mixed regime over the pure collective regime.[13]

3.4 Splitting the family

To understand the effects of splitting the family, we examine the effects of a unit increase in the number of members who stay within the extended family.

Whether in the pure collective or in the mixed regime, if the head decides to keep one more member with him, the impact on his rent can be formally defined as follows:

$$\frac{dR}{dn} = \left(\frac{\partial A}{\partial n} - A^I - n\frac{\partial A^I}{\partial n}\right)\alpha f_A + l^C \alpha f_L + n\frac{\partial l^C}{\partial n}\alpha f_L + \frac{\partial \alpha}{\partial n} f \qquad (10)$$

The four terms have an intuitive interpretation. The first term in both expressions is the *land endowment effect*. When one more member stays on the farm area, the total farm is bigger (when a male member leaves, he receives a fraction $\frac{1}{N}$ of the total land endowment of the family, $\overline{A}$), but the collective field does not increase by this full amount in the mixed regime since the additional member receives and individual plot. Furthermore the head may adjust the size of all individual plots as a response to the change in family size. The second

[12]Indeed, assuming constant returns to scale, we have $f_A(A^I, l^I) > f_A(A - nA^I, nl^C)$. This follows directly from the first order conditions of dependents' labor allocation which implies $f_L(A^I, l^I) < f_L(A - nA^I, nl^C)$, and the assumption of a Cobb-Douglas production function. A fortiori, we then have that: $-\nu(1-\alpha)f_A(A - nA^I, nl^C) + \nu f_A\left(A^I, l^I\right) > 0$.

[13]In fact, if there exists an interior solution to the father's problem, it occurs at a point where the participation constraint binds. Indeed if the sons are able to achieve their reservation utility by just relying on the production of their individual fields, $\nu = 0$, and the head's rent is unambiguously decreasing in the size of individual plots. This case is treated in appendix, section A.2.1.

term is the *labour endowment effect*: the increase in the size of the labour force working on the collective field has a positive direct effect on total production. The last two terms are linked to incentives. We label the third term the *labour incentive effect*, and the fourth term the *compensation effect*. The third term indicates how the individual incentive to work on the collective field is eroded when an additional member stays on the farm, thereby accentuating the moral-hazard-in-team problem. We show in appendix (section A.3.1 for the case of a split occurring in the collective regime, and section A.3.2 for the case of a split occurring in the mixed regime) that, as expected, this term is negative. The fourth term, finally, depicts how the head adapts his rent to the constraint of providing for the subsistence of an additional member. We prove in appendix (sections A.3.1 and A.3.2) that this term is also negative because $\frac{\partial \alpha}{\partial n} < 0$: by way of adjusting to the deteriorated work incentives (on the collective field) and to the necessity of feeding one more mouth, the head allows male members to keep a greater share of the collective field's production.

Reasoning in the converse way, an important lesson to draw from the ambiguous sign of $\frac{\partial R}{\partial n}$ is that, by inducing a son to leave the stem household and form a branch household, the family head is not certain to increase his own income. This is in spite of the fact that he does not have anymore to provide for the consumption needs of the departing son and that the incentives to work on the collective field improve for the members who stay on the farm. There are, indeed effects working in the opposite direction: the departing son stops working on the collective field, and is moreover offered a share of the family land assets that is subtracted from the land available to the residual stem household.

4 The effects of land scarcity and increasing consumption needs

4.1 Analytical results

Recall that one of the main reasons given by local elders for the increasing prevalence of extended family farms with individual fields, and of family splits, is the increase in land pressure. In terms of our model, such increase may be measured by a decrease of the land endowment, for a given family size. The other main reason is that (male) members have greater consumption needs than in the past. This change may be captured by an increase in the reservation utility, $\underline{u}$.

In this section, we test whether these explanations can be supported by our theoretical framework. We examine first how the head's incentive to give out individual plots changes with the family land endowment and the members' reservation utility. We then examine how the head's incentive to split the family is affected by the same variables. In each case we summarize our results in a proposition and refer the reader to the appendix for a presentation of the complete formal proofs. These results are derived on the basis of a Cobb-Douglas production function ($f(a,l) = a^{\epsilon}l^{1-\epsilon}$) and a quadratic cost of effort ($v(l) = \omega l^2$). Before embarking upon such a task, however, we want to establish a set of intermediary results that concern the evolution of the head's share, α. More precisely, we wish to determine how α changes (1) when land becomes more scare and the reservation utility of members increases within the domain of the strictly collective regime or the collective regime, and (2) when the head decides to shift from the former to the latter regime.

We show that, in accordance with intuition, the head lowers his share of collective output when, within the strictly collective or the mixed regime, land becomes more scarce or the members' reservation utility is raised (see appendix B.1 for the detailed proof). Lemma 1 states this result.

Lemma 1 *When, under either the strictly collective or the mixed regime, the family head is confronted with more constraining conditions in the form of a reduction of* $\overline{A}$ *or an increase in* $\underline{u}$*, he responds by decreasing his share of the collective output,* α*. Furthermore, when* $\overline{A}$ *tends to* 0 *or* $\underline{u}$ *tends to* $+\infty$*,* α *tends to zero.*

Much less clear is the case where a regime shift occurs, because two contrary effects are at work when competition emerges between the collective and the individual fields. On the one hand, being keen to mitigate the effects of such a competition, the head raises the share of collective output accruing to members so as to incite them to apply effort to the common field. On the other hand, he wants to make precisely the opposite move in order to make up for the unavoidable decrease in the level of collective output from which he draws his entire income. We show that the second effect outweighs the first, thus establishing the following lemma (proof in appendix B.2:

Lemma 2 *When the family head decides to shift from the purely collective to the mixed regime, he simultaneously raises his own share of the output obtained on the collective field.*

Let us now look at the effect of land endowment on the choice between the mixed and the pure collective regimes. It is stated in Proposition 1.

Proposition 1 *When land is very abundant, the head always prefers a pure collective farm to a mixed structure where male members have individual plots that they cultivate for their own benefit. As land becomes scarce, however, the mixed structure may become more attractive.*

Specifically, if the head of a collective farm is just indifferent between operating the farm as a pure collective unit or as a mixed unit, a marginal decrease in land endowment induces him to strictly prefer the mixed regime over the collective regime. Conversely, a marginal increase in land endowment induces him to strictly prefer the pure collective regime. Since we cannot prove the existence of a point of indifference between the two regimes, we must distinguish two cases. Indeed, as $\overline{A}$ goes from $+\infty$ to 0, either the collective farm remains superior over the full range of land endowments, or the mixed farm dominates below a critical level of land endowment.

Let us now turn to the effect of reservation utility on the choice between the mixed and the pure collective regimes.

Proposition 2 *When the workers' reservation utility is very low, the participation constraints of members are not binding and the head always prefer a pure collective farm to a mixed structure where male member have individual plots that they cultivate for their own benefit. As the reservation utility increases, however, the mixed structure may become more attractive.*

Suppose, in particular, that the head of a collective farm is just indifferent between operating the farm as a pure collective unit or as a mixed unit. A marginal increase in the reservation utility induces him to strictly prefer the mixed regime over the collective regime. As $\underline{u}$ goes from 0 to $+\infty$, either the collective farm remains superior over the full range of reservation utility, or the mixed farm dominates above a critical level of reservation utility.

That the participation constraint is not binding when the reservation utility is very low and the head chooses the pure collective form is not surprising. Since members receive a share of the output obtained on the common field as remuneration for their efforts, it is in the interest of the head to provide them with sufficient incentive to work even if it implies paying them above their reservation utility.

Next, let us consider the effect of land endowment on the choice between splitting the family and keeping it whole.

Proposition 3 *When land is very abundant, the head of a purely collective farm will not accept to let some male members leave with a portion $1/N$ of the land endowment. Conversely, when land is very scarce, the head of a purely collective farm or a mixed farm will choose to split the family and let some members leave with a portion $1/N$ of the land. Furthermore, there exists a unique level of land endowment $0 < \overline{A} < +\infty$ that makes the head of a purely collective farm just indifferent between letting some male members leave with a portion 1/N of the family land, and keeping the family whole.*

Finally, we have to elucidate the effect of reservation utility on the choice between splitting the family and keeping it whole.

Proposition 4 *When the members' reservation utility is very low, the head of a pure collective farm will not accept to let some male members leave with a portion 1/N of the land. Conversely, when the members' utility is very high, the head of a purely collective farm or a mixed farm will choose to split the family and let some members leave with a portion 1/N of the land. Furthermore, there exists a unique level of reservation utility $0 < \underline{u} < +\infty$ that makes the head of a purely collective farm just indifferent between letting some male members leave with a portion 1/N of the family land, and keeping the family whole.*

In short, these four propositions reveal that for large $\overline{A}$ or small $\underline{u}$, the pure collective regime dominates the mixed regime and the head will not split the family. Conversely, for small $\overline{A}$ or large $\underline{u}$, the mixed regime may dominate the collective regime, and whichever of these two regimes prevails, some splitting will occur. While the analytical exploration of the role of land endowment and members' reservation utility confirms our intuition about the role of these two factors, it does not yield a complete set of predictions. For example, we cannot be sure that for small values of $\overline{A}$, a family head operating in the pure collective regime will not choose to shift to the mixed regime before splitting the family. Hence the need to resort to simulation in order to obtain results that allow to examine whether in a $(\overline{A}, \underline{u})$ space, the mixed, the split and the pure collective regimes actually coexist.

4.2 Simulation results

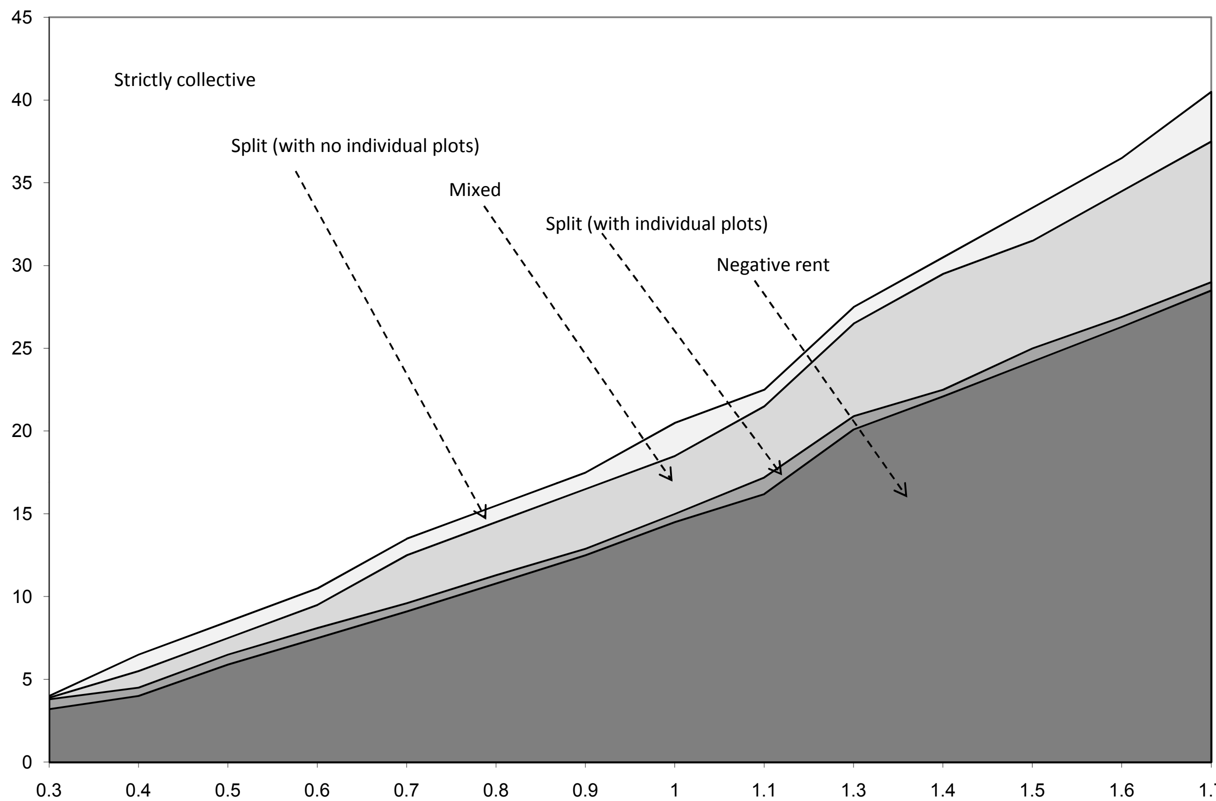

Figure 1: Partition of the land endowment - reservation utility space into regimes.

Our simulation work is summarized in Figure 1 where the family land endowment is measured along the vertical axis and the members' reservation utility along the horizontal axis.[14] What are the main results emerging from this figure?

[14]The simulation is conducted using the software Mathematica. For a given $(\overline{A}, \underline{u}, N)$, for all $1 \leq n \leq N$, we numerically solve for the head's share in both regime. In the mixed regime, when he gives out individual plots and for decreasing sizes of the collective field. Practically, in the case of the results presented below, we decrease the size of the collective field by steps of 0.25. In Mathematica we use the command "FindRoot", to obtain α when the participation constraint binds. For each n, we then compute the head's rent for each size of the collective field and compare it to his rent in the collective regime. For each n we thus know whether

To begin with, the results analytically obtained in the previous section stand confirmed. First, the pure collective regime appears to be superior to all the other regimes in the upper left portion corresponding to small values of $\underline{u}$. Moreover, the triangle-like shape of the strictly collective zone indicates that the smaller the reservation utility $\underline{u}$, the lower the threshold value of $\overline{A}$ above which the pure collective regime dominates the alternative regimes (the zone expands in size as we move to the left in the upper part of the graph). In other words, pure collective farms may subsist even in conditions of acute land scarcity but only provided that exit opportunities for members are sufficiently bad. Conversely, they may withstand the pressure of rising outside opportunities if land is sufficiently abundant.

Second, the area corresponding to the pure collective regime lies entirely above the areas corresponding to the mixed regime, collective farming-cum-splitting and splitting with individual plots. Third, when the head operates his farm under the pure collective regime and $\overline{A}$ becomes sufficiently small (or $\underline{u}$ sufficiently large), he chooses to split the family. And when the head operates a mixed farm and $\overline{A}$ becomes sufficiently small (or $\underline{u}$ sufficiently large), he also chooses to split the family. Fourth, the mixed regime emerges as the optimal farm structure when the reservation utility is not too small and the farmland area is not too large.

The use of simulation also brings to light a number of results that cannot be derived analytically, and therefore add to the knowledge acquired in the previous section. The main finding here concerns the sequence in which optimal regimes succeed each other, as we vary the values of $\overline{A}$ or $\underline{u}$. As $\overline{A}$ is marginally lowered, or $\underline{u}$ is marginally raised, a head operating a pure collective farm may split the family while clinging to collective farming in the remaining portion of the stem household. When $\overline{A}$ is lowered, or $\underline{u}$ raised, to a larger extent still, the head may instead choose the mixed farm in which all members stay in the stem household but obtain access to individual plots. Finally, when the change in $\underline{u}$ or $\overline{A}$ values is made

the head will choose to give out individual plots, and the maximum rent the head can obtain when he keeps n members on the family farm. Comparing the head's rent over the range of n, we determine whether the head prefers to split the family ($n < N$), or not ($n = N$). The parameters used are: $N = 10$, $\varepsilon = 0.7$ and $\omega = 0.5$.

even greater, splitting the family while granting individual plots to the members who stay with the head becomes the optimal regime.

Why is it that as land becomes more scarce, or as exit options of family members improve beyond a point, split-cum-collective farming becomes preferable to collective farming even before the mixed regime (which, on the face of it, is a less individualized form) becomes optimal? This apparently intriguing result has to be seen in the light of the fact that we allow for partial splits of the farm-cum-family, which may prove superior to the mixed farm structure while more complete splits would not. Indeed, the split regime evinces great flexibility inasmuch as the head chooses how many members to let go. Both regimes entail a reduction of the farm area devoted to the collective field so that (some) members can produce on their own plot to meet (part of) their needs. Correspondingly, a portion of the workforce ceases to be available for the collective field. In the split-cum-collective farming regime, this decrease takes on the form of a reduced number of workers with the attendant result that the moral-hazard-in-team problem is mitigated. But this is not the case under the mixed regime. There is thus an obvious tradeoff between the size of the workforce available to work on the collective field (larger under the mixed regime) and the extent of the moral-hazard-in-team problem (also greater under the mixed regime). What our results indicate is that the latter, adverse effect outweighs the former beneficial effect when land is not too scarce (or the reservation utility is not too high), while the reverse is true when land scarcity (or the reservation utility) exceeds a certain threshold.

5 Caveats and alternative frameworks

5.1 Caveats

When effort is continuous, as we have seen, there is no explicit form for the head's income in the mixed regime (even with a Cobb-Douglas production function). Unfortunately, assuming discrete effort levels does not provide clearer insights into the issue at hand. In particular, we remain unable to derive threshold values of land endowment and reservation utility such that the head would be indifferent between the collective and the mixed regimes. The difficulty arises from two sources. On the one hand, the existence of two types of fields between which labor is to be allocated forces us to consider numerous effort configurations, for example: $(0,1), (1,0), (\frac{1}{2}, \frac{1}{2}), (0, \frac{1}{2}), (\frac{1}{2}, 0)$. The problem is then that several cases must be distinguished that give rise to different (explicit) forms of the head's rent under the various regimes. On the other hand, the moral-hazard-in-team problem on the collective field is not well addressed when a few discrete effort levels are considered.

The contract theory literature proposes several solutions to eliminate or mitigate moral-hazard-in-team problems (Bolton and Dewatripont, 2005). We implicitly assume that these solutions which include punishing the team of agents or stimulating competition among them are not feasible in our empirical context. An example of the former solution arises when aggregate output is observable (as in our farm), and this information is used by the head to punish the workers as a whole for excessive shirking. In particular, if observation of aggregate output reveals that first-best effort levels were not applied, the head could refuse any labour payment. Such drastic punishment, however, is not conceivable in a poor economy where farm members critically depend on collective production for their livelihood, and where guaranteeing subsistence to all members is a customary duty of the head of a family. Another important customary duty of the head is to preserve the unity and harmony of the family. This objective makes the second type of solution inapplicable. As we have already pointed out earlier, indeed, differentiation of labour payments among members (assuming that the head has sufficient clues about individual efforts to rank them) is bound to cause frustrations, recriminations and accusations of injustice.

5.2 An alternative framework with an altruistic family head

Relaxing the assumption of a strictly selfish patriarch at the head of the farm and simultaneously removing the participation constraints provide interesting insights into the functioning of our model. More precisely, it reveals that the key feature driving the comparative static results obtained lies in the participation constraints. It is, indeed, the tightening of these constraints under conditions of improved outside opportunities for members, or of growing land scarcity, that induces the family head to put more weight on efficiency considerations so as to be able to satisfy them. As we know, this implies a transformation of the farm-cum-family structure toward more individualized forms. Consider such an alternative framework in which there is no participation constraint, but the head has an altruistic utility function. Altruism can be construed as meaning that the head attaches a positive weight to the members' welfare while making his allocative decisions or, alternatively, that members exert pressure on him to the effect that he takes their interests into account. In the former case, the weight put on the members' welfare reflects the head's degree of altruism while in the latter case it reflects the bargaining power of the members.

When we work out the numerical solutions to this newly defined problem, we find that the three farm-cum-family structures may again arise, yet it is only for relatively high levels of altruism (or members' bargaining strength) that individualized forms are preferred by the head. The second finding, however, contradicts the comparative-static results obtained under the initial model: the farm-cum-family structure chosen is insensitive to variations in land pressure. This is because, when conditions become more stringent, the head now has the ability to transfer part of the welfare loss to the members whereas he had to operate under binding participation constraints in the base model. Thus, if he is sufficiently selfish to prefer the pure collective farm structure, he will stick to it under conditions of increasingly severe land pressure. Efficiency gains are thereby lost in conditions where they matter much, yet this is not the main concern of the head since by accepting a reduction of the collective field, his income loss would be greater than when the farm remains purely collective. In other words, in the absence of participation constraints, an increase in land pressure does not affect the outcome of the trade-off between efficiency in production and the head's ability to extract incomes.

5.3 An alternative framework with a uniform participation constraint

Let us now assume that the participation constraint is uniformly defined across the three regimes that is, in the case of a split, a departing member leaves with just enough land to reach $\underline{u}$ on his new farm. In that case, predictions regarding the transformation of the farm-cum-family structure are much weaker than those achieved in the base model. In particular, if we can still predict that, when land becomes very scarce, the head of a purely collective farm or a mixed farm will choose to split the family (at least in part) it is no more possible to assert that this strategy will never be observed when land is very abundant. The reason is straightforward: if land is abundant and the reservation utility is low, the family head may find it profitable to let some male members to leave the farm. This is because ensuring the leaving members their low reservation utility implies that they will receive a small amount of land and this may well be a low price to pay to improve efficiency by mitigating the moral-hazard-in-team problem.

6 Conclusion

On the basis of a stylized representation of a patriarchal family farm, and in a context of absent land markets, it is possible to use a simple analytical structure to account for possible transformations of a collectively operated farm based upon an extended family unit. More precisely, as land scarcity increases, or as exit options available to family members improve (say, as a result of growing market integration), the pure collective farm will unavoidably become inferior to alternative farm structures from the standpoint of the family head who draws his entire income from a share of the collectively produced harvest. One of these alternative forms is a mixed farm structure combining a collective field with individual plots of land. When a competition thus exists between these two types of fields, the reward function on the collective field is a share contract (the fixed-rent contract is not a Nash equilibrium). Another possible form is a regime in which branch households are formed as a result of the decision of the patriarch to allow the split of the stem household and the concomitant division of the extended family's assets. In the remaining part of the stem household, collective cultivation may be combined with individual fields, but this is not a necessity. As the number of (male) members leaving the stem household may be any number between zero and the total number of them in that household, there is a large variety of alternative forms to the pure collective farm, and each of them needs to be considered in a comparison between possible farm structures.

In spite of the analytical simplicity of the basic farm structure contemplated in our model, a complete comparison turned out to be quite complex and we had to resort to the simulation technique in order to obtain a complete mapping of regime choice into a reservation utility/land endowment space. The most significant result is the following: as land scarcity increases (or as exit options for members improve), splitting the main household while sticking to the pure collective mode of operation in its remaining portion appears to be the first alternative farm organization able to supersede the pure collective farm. It is only at higher levels of scarcity (or exit option levels) that the mixed farm structure becomes the optimal organization from the patriarch's standpoint. And it is at still higher levels that splitting combined with individual plots in the remaining stem household emerges as the best solution.

The above result critically hinges on the existence of participation constraints. In the absence of such constraints, an increase in land pressure does not affect the outcome of the trade-off between efficiency in production and the head's ability to extract incomes. This is evident, for example, when we assume that the family head is altruistic. Other variants of our model have less significant consequences. Thus, assuming that members with individual plots can make income transfers in favor of the family head would, for obvious reasons, make the pure collective farm less appealing than alternative forms. Furthermore, if we assume that disutility of effort is greater on the collective field than on the individual plots, the case of individualization is again strengthened. In the other way around, the presence of scale economies in the production of the collective field and in the consumption of the collective produce would enhance the advantages of the collective farm and enlarge the region of its feasibility. Likewise, the presence of fixed costs, such as storage costs, increases the advantage of mixed farms over branch households as a way of individualizing the collective farm structure as land becomes more scarce. Finally, allowing for dynamic considerations of the sort considered by Boserup could only reinforce our conclusion that rising land pressure leads to more individualized farm-cum-family structures.[15] One of the main merits of our model is actually to show that individualization of farm units can result from land scarcity even in the absence of induced technical change.

[15]In a dynamic setup of the model, one might wish to assume that allowing the departure of members to form separate branch households is a more irreversible step than granting individual plots to staying members. This would obviously reinforce the case for the mixed farm structure.

References

Ault, D.E., and G.L. Rutman. 1979. "The Development of Individual Rights to Property in Tribal Africa." *Journal of Law and Economics* 22(1):163-182.

Baland, J.M., and J.P. Platteau. 1998. "Dividing the Commons - A Partial Assessment of the New Institutional Economics of Property Rights." *American Journal of Agricultural Economics* 80:644-650.

Basu, K. 1986. "The Market for Land - An Analysis of Interim Transactions." *Journal of Development Economics* 20:163-177.

Bergstrom, T. 1997. "A survey of theories of the family" *Handbook of Population and Family Economics* ed. Rosenzweig, M.R., O. Stark. Elsevier, Amsterdam.

Binswanger, H., and J. McIntire. 1987. "Behavioural and Material Determinants of Production Relations in Land-Abundant Tropical Agriculture." *Economic Development and Cultural Change* 36(1): 73-99.

Binswanger, H., J. McIntire, and C. Udry. 1989. "Production Relations in Semi-Arid African Agriculture."in Bardhan, P. (ed), The Economic Theory of Agrarian Institutions. Oxford: Clarendon Press. 122-144.

Binswanger, H., and M. Rosenzweig. 1986. "Behavioral and Material Determinants of Production Relations in Agriculture." *Journal of Development Studies* 22(3):503–539.

Bolton, P. and M. Dewatripont. 2004. *Contract Theory* The MIT Press, Cambridge, Massachusetts.

Boserup, E. 1965. *Conditions of Agricultural Growth.* Aldine Publishing Co, Chicago.

Demsetz, H. 1967. "Toward a Theory of Property Rights." *American Economic Review* 57(2):347-359.

Duflo, E. and C. Udry. 2004. "Intrahousehold Resource Allocation in Cote d'Ivoire: Social Norms, Separate Accounts, and Consumption Choices." NBER Working Paper No. 10498

Eswaran M., and A. Kotwal. 1985. "A Theory of Contractual Structure in Agriculture." *American Economic Review* 75(3):352-367.

Fafchamps, M. 2001. "Intrahousehold Access to Land and Sources of Inefficiency: Theory and Concepts." *Access to Land, Rural Poverty and Public Action* ed. de Janvry, A., G. Gordillo, J.P. Platteau and E. Sadoulet. Oxford University Press, Oxford.

Feder, G., and D. Feeny. 1991. "Land Tenure and Property Rights: Theory and Implications for Development Policy." *The World Bank Economic Review* 75(1): 162-77.

Feder, G., and R. Noronha. 1987. "Land Rights Systems and Agricultural Development in Sub-Saharan Africa" *Research Observer* 2(2): 143-169.

Foster, A., and M. Rosenzweig. 2002. "Household Division and Rural Economic Growth." *Review of Economic Studies* 69:839–869.

Hayami, Y., and V. Ruttan. 1985. *Agricultural Development: An International Perspective.* John Hopkins Unversity Press, Baltimore.

Haugerud, A. 1993. *The Culture of Politics in Modern Kenya.* Cambridge: Cambridge University Press.

Meyer, C.A. 1989. Agrarian Reform in the Dominican Republic: An Associative Solution to the Collective/Individual Dilemma. *World Development* 17(8): 1255-1267

Pingali, P., Y. Bigot, and H.P. Binswanger. 1987. "Agricultural Mechanization and the Evolution of Farming Systems in Sub-Saharan Africa." Baltimore and London: The Johns Hopkins University Press.

Platteau, J.-P. and J. Nugent. 1992. Share Contracts and their Rationale: Lessons from Marie Fishing. *Journal of Development Studies* 28(3): 386-422

Putterman, L.. 1983. A Modified Collective Agriculture in Rural Growth-with-Equity: Reconsidering the Private, Unimodal Solution. *World Development* 11(2): 77-100

Appendix

A Analytical framework

A.1 Optimization in the pure collective regime

In this section, we formally derive the Lagrangian multipliers for the maximization problem in the strictly collective regime. The Lagrangian in this case is:

$$L = \alpha f(A, nl) - \beta \left(\frac{1-\alpha}{n} f_L(A, nl) - v'(l) \right) - \gamma \left(u - \frac{1-\alpha}{n} f(A, nl) + v \right) \tag{11}$$

The FOC are (we ignore the arguments of the various functions):

$$\begin{aligned} \frac{\partial L}{\partial \alpha} &= f + \frac{\beta}{n} f_L - \frac{\gamma}{n} f = 0 \\ \frac{\partial L}{\partial l} &= \alpha n f_L - \beta(1-\alpha) f_{LL} + \beta v'' + \gamma(1-\alpha) f_L - \gamma v' = 0 \end{aligned}$$

If the participation constraint is unbinding, then $\gamma = 0$ and $\beta = -\frac{nf}{f_L}$. Using the second equation, we can show that in that case, α is constant:

$$\begin{aligned} 0 &= \alpha n f_L - \beta(1-\alpha) f_{LL} + \beta v'' \\ \Leftrightarrow 0 &= \alpha n f_L + n(1-\alpha)\frac{f f_{LL}}{f_L} - n\frac{f v''}{f_L} \\ \Leftrightarrow 0 &= \alpha n + n(1-\alpha)\frac{f f_{LL}}{f_L^2} - n\frac{f v''}{f_L^2} \end{aligned}$$

With $f(a,l) = a^\epsilon l^{1-\epsilon}$ and $v(l) = \omega l^2$, we have $\frac{f f_{LL}}{f_L^2} = -\frac{\epsilon}{1-\epsilon}$ and $\frac{f v''}{f_L^2} = \frac{1-\alpha}{1-\epsilon}$.[16] Substituting

[16] To obtain the first equality note that with $f(a,l) = a^\epsilon l^{1-\epsilon}$, we have: $f_L = (1-epsilon)a^\epsilon l^{-\epsilon}$ and $f_{LL} = (1-epsilon)\epsilon a^\epsilon l^{-\epsilon-1}$. Therefore $\frac{f f_{LL}}{f_L^2} = -\frac{a^{2\epsilon} l^{-2\epsilon}\epsilon(1-\epsilon)}{(1-\epsilon)^2 a^{2\epsilon} l^{-2\epsilon}} = -\frac{\epsilon}{1-\epsilon}$. To establish the second equality we use the incentive condition: $f_L = \frac{n}{1-\alpha} v' = \frac{n}{1-\alpha} 2\omega l$ and $f = \frac{l}{1-\epsilon} f_L = \frac{2\omega l^2 n}{(1-\alpha)(1-\epsilon)}$ which imply: $\frac{f v''}{f_L^2} = \frac{2\omega n}{(1-\epsilon)(1-\alpha)} \frac{2\omega l^2}{4\omega^2 l^2} = \frac{1-\alpha}{1-\epsilon}$

these expressions in the above equality yields:

$$
\begin{aligned}
0 &= \alpha n - n(1-\alpha)\frac{\epsilon}{1-\epsilon} - n\frac{1-\alpha}{1-\epsilon} \\
\Leftrightarrow \alpha &= \frac{1+\epsilon}{2}
\end{aligned}
$$

If the participation constraint is binding, then the first inequality implies: $\gamma = n + \beta\frac{f_L}{f}$. The second equality can thus be rewritten:

$$\alpha n f_L - \beta(1-\alpha)f_{LL} + \beta v'' + (1-\alpha)f_L n + \beta\frac{f_L^2}{f}(1-\alpha) - nv' - \beta\frac{f_L v'}{f} = 0$$

Replacing v' with $\frac{1-\alpha}{n}f_L$ and solving for β, we obtain:

$$\beta = \frac{f_L(n-1+\alpha)}{(1-\alpha)f_{LL} - v'' - \frac{f_L^2}{f}(1-\alpha)(1-\frac{1}{n})}$$

Finally:

$$\gamma = n - \frac{1}{\frac{v''f}{f_L^2(n-1+\alpha)} + \frac{(1-\alpha)(-f_{LL})f}{f_L^2(n-1+\alpha)} + \frac{(1-\alpha)(1-\frac{1}{n})}{n-1-\alpha}}$$

A.2 Optimization in the mixed regime, for a given A^I

In this section we formally derive the Lagrangian multipliers to the problem described by Equation (8). These multipliers have different expressions depending on whether the participation constraint binds.

A.2.1 Unbinding participation constraint

We show below that if the participation constraint does not bind, then $\frac{\partial V}{\partial A^I} < 0$. If the participation constraint does not bind, $\nu = 0$ and the FOC are:

$$\frac{\partial L}{\partial \alpha} = f(A - nA^I, nl^C) - \frac{\lambda}{n} f_L(A - nA^I, nl^C) = 0 \quad (12)$$

$$\frac{\partial L}{\partial l^C} = \alpha n f_L(A - nA^I, nl^C) - \lambda \left(v''(l^C + l^I) - (1-\alpha) f_{LL}(A - nA^I, nl^C)\right) - \mu v''(l^C + l^I) = 0 \quad (13)$$

$$\frac{\partial L}{\partial l^I} = -\lambda v''(l^C + l^I) - \mu \left(v''(l^C + l^I) - f_{LL}\left(A^I, l^I\right)\right) = 0 \quad (14)$$

In the following, we use the subscript C for the production function on the collective field and I to designate the production function on individual plots. The first equation implies: $\lambda = \frac{nf^C}{f_L^C}$. Substituting λ in the last equation yields: $\mu = \frac{-v'' \frac{nf^C}{f_L^C}}{v'' - f_{LL}^I}$. Since λ is unambiguously positive while μ is unambiguously negative, $\frac{\partial V}{\partial A^I} = -\alpha n f_A^C - \lambda \frac{1-\alpha}{n} f_{LA}^C + \mu \frac{1}{n} f_{LA}^C$ is negative, so that unless the participation constraint binds, it is always optimal for the father to decrease the size of the individual plots, or to increase the size of the collective field.

A.2.2 Binding participation constraint

The FOC of the maximization problem in this case are:

$$\frac{\partial L}{\partial \alpha} = f(A - nA^I, nl^C) - \lambda \frac{1}{n} f_L(A - nA^I, nl^C) - \nu \frac{1}{n} f(A - nA^I, nl^C) = 0 \quad (15)$$

$$\frac{\partial L}{\partial l^C} = \alpha n f_L(A - nA^I, nl^C) - \lambda \left(v''(l^C + l^I) - (1-\alpha) f_{LL}(A - nA^I, nl^C)\right) - \mu v''(l^C + l^I) \quad (16)$$

$$-\nu \left(-(1-\alpha) f_L(A^C, nl^C) + v'(l^C + l^I)\right) = 0 \quad (17)$$

$$\frac{\partial L}{\partial l^I} = 0 \quad (18)$$

$$= -\lambda v''(l^C + l^I) - \mu \left(v''(l^C + l^I) - f_{LL}\left(A^I, l^I\right)\right) - \nu \left(-f_L\left(A^I, l^I\right) + v'(l^C + l^I)\right) = 0 \quad (19)$$

Equation (19) implies: $\mu = -\lambda \frac{v''}{v''-f^I_{LL}}$, since $-f_L\left(A^I, l^I\right) + v'(l^C + l^I) = 0$. Equation (15) implies:

$$\nu = n - \lambda \frac{f^C_L}{f^C} \tag{20}$$

Replacing μ and λ in equation (16) by these expressions yields:

$$\begin{aligned}
& \alpha n f^C_L - \lambda(v'' - (1-\alpha) f^C_{LL}) + \lambda \frac{v''^2}{v'' - f^I_{LL}} - n(-(1-\alpha) f^C_L + v') + \lambda \frac{f^C_L}{f^C}(-(1-\alpha) f^C_L + v') = 0 \\
\Leftrightarrow \quad & \alpha n f^C_L + (n-1)(1-\alpha) f^C_L + \lambda \left(-v'' + (1-\alpha) f^C_{LL} + \frac{v''^2}{v'' - f^I_{LL}} + \frac{(f^C_L)^2}{f^C}(1-\alpha)(-1+\frac{1}{n}) \right) = 0 \\
\Leftrightarrow \quad & \lambda = -\frac{(n-1-\alpha) f^C_L}{-v'' + (1-\alpha) f^C_{LL} + \frac{v''^2}{v''-f^I_{LL}} + \frac{(f^C_L)^2}{f^C}(1-\alpha)(-1+\frac{1}{n})} \\
\Leftrightarrow \quad & \lambda = -\frac{(n-1-\alpha) f^C_L}{(1-\alpha) f^C_{LL} + \frac{v'' f^I_{LL}}{v''-f^I_{LL}} + \frac{(f^C_L)^2}{f^C}(1-\alpha)(-1+\frac{1}{n})}
\end{aligned}$$

This implies $\lambda > 0$, $\mu < 0$. We also know that $\nu > 0$ (property of the Lagrangian multiplier of an inequality). We derive the expression for ν (needed below) from equation (20):

$$\begin{aligned}
\nu &= n + \frac{(f^C_L)^2(n-1+\alpha)}{\frac{v'' f^I_{LL} f^C}{v''-f^I_{LL}} + (1-\alpha) f^C_{LL} f^C + (f^C_L)^2 (1-\alpha)(-1+\frac{1}{n})} \\
\Leftrightarrow \nu &= n - \frac{1}{\frac{v''(-f^I_{LL})}{v''-f^I_{LL}} \frac{f^C}{(f^C_L)^2(n-1+\alpha)} + \frac{(1-\alpha)(-f^C_{LL}) f^C}{(f^C_L)^2(n-1+\alpha)} + \frac{(1-\alpha)(1-\frac{1}{n})}{n-1+\alpha}}
\end{aligned}$$

A.3 Splitting: signing the incentive effects

A.3.1 Splitting under the pure collective regime

In this section, we show that $\frac{\partial l^C}{\partial n} < 0$ and $\frac{\partial \alpha}{\partial n} < 0$ so that both the labor incentive effect and the compensation effect are negative. Consider first the case of an unbinding participation constraint. We can apply the implicit function theorem to the first order condition on labor:

$$F(n, \alpha) = \frac{1-\alpha}{n} f_L(A, nl) - v'(l) = 0 \tag{21}$$

$$
\begin{aligned}
\frac{\partial l}{\partial n} &= -\frac{\frac{\partial F}{\partial n}}{\frac{\partial F}{\partial l}} \\
&= -\frac{-\frac{1-\alpha}{n^2} f_L + \frac{1-\alpha}{n} l f_{LL} + \frac{1-\alpha}{n}\frac{\overline{A}}{N} f_{LA}}{\frac{1-\alpha}{n} n f_{LL} - v''}
\end{aligned}
$$

With a constant return to scale production function, we have $nlf_{LL} + \frac{n\overline{A}}{N} f_{LA} = 0$, thus:

$$
\frac{\partial l}{\partial n} = \frac{\frac{1-\alpha}{n^2} f_L}{\frac{1-\alpha}{n} n f_{LL} - v''}
$$

It is clear that $\frac{\partial l^C}{\partial n}$ is negative. Similarly:

$$
\begin{aligned}
\frac{\partial \alpha}{\partial n} &= -\frac{\frac{\partial F}{\partial n}}{\frac{\partial F}{\partial \alpha}} \\
&= -\frac{-\frac{1-\alpha}{n^2} f_L + \frac{1-\alpha}{n} l^C f_{LL} + \frac{1-\alpha}{n}\frac{\overline{A}}{N} f_{LA}}{-\frac{1}{n} f_L} \\
&= -\frac{1-\alpha}{n}
\end{aligned}
$$

Again, $\frac{\partial l^C}{\partial \alpha}$ is clearly negative.

Consider now the case of a binding participation constraint. We apply the Cramer's rule to the system of equations from which the optimal values for l^C and α are implicitly obtained:

$$
\begin{cases}
F_1 = \frac{1-\alpha}{n} f_L(\frac{n\overline{A}}{N}, nl^C) - v'(l^C) = 0 \\
F_2 = \frac{1-\alpha}{n} f\left(\frac{n\overline{A}}{N}, nl^C\right) - v(l^C) - \underline{u} = 0
\end{cases}
\tag{22}
$$

We have:

$$
\frac{\partial l^C}{\partial n} = -\frac{\det\begin{pmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial n} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial n} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial l^C} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial l^C} \end{pmatrix}}
$$

Assuming that f is homogeneous of degree 1 (or that we have constant returns to scale) greatly simplifies the expressions for $\frac{\partial F_1}{\partial n}$ and $\frac{\partial F_2}{\partial n}$. It implies that f is homogeneous of degree 1 and f_L of degree 0, so that by virtue of Euler's theorem: $f = n\frac{\overline{A}}{N}f_A + nlf_L$ and $n\frac{\overline{A}}{N}f_{LA} + nlf_{LL} = 0$. Thus:

$$\begin{aligned}
\frac{\partial F_1}{\partial n} &= -\frac{1-\alpha}{n^2}f_L + \frac{1-\alpha}{n}\frac{\overline{A}}{N}f_{LA} + l\frac{1-\alpha}{n}f_{LL} \\
&= \frac{1-\alpha}{n^2}(-f_L + Af_{LA} + nlf_{LL}) \\
&= \frac{1-\alpha}{n^2}(-f_L + 0) = -\frac{1-\alpha}{n^2}f_L
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F_2}{\partial n} &= -\frac{1-\alpha}{n^2}f + \frac{1-\alpha}{n}\frac{\overline{A}}{N}f_A + l\frac{1-\alpha}{n}f_L \\
&= \frac{1-\alpha}{n^2}(-f + Af_A + nlf_L) \\
&= \frac{1-\alpha}{n^2}(-f + f) = 0
\end{aligned}$$

Finally:

$$\frac{\partial l^C}{\partial n} = -\frac{-\frac{1-\alpha}{n^2}f_L\frac{f}{n}}{-\frac{f_L}{n}((1-\alpha)f_L - v') + \frac{f}{n}((1-\alpha)f_{LL} - v'')}$$

This expression is unambiguously negative (recall that $v' = \frac{1-\alpha}{n}f_L$ so that $(1-\alpha)f_L - v' > 0$).

Similarly we obtain:

$$\begin{aligned}
\frac{\partial \alpha}{\partial n} &= -\frac{\det\begin{pmatrix} \frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial l^C} \\ \frac{\partial F_2}{\partial n} & \frac{\partial F_2}{\partial l^C} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial l^C} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial l^C} \end{pmatrix}} \\
&= -\frac{-\frac{f_L}{n^2}(f_L(1-\alpha) - v')}{-\frac{f_L}{n}((1-\alpha)f_L - v') + \frac{f}{n}((1-\alpha)f_{LL} - v'')}
\end{aligned}$$

Both the numerator and the denominator are negative so that this expression is unambiguously negative.

A.3.2 Splitting under the mixed regime

In this section, we show that $\frac{\partial l^C}{\partial n}$ and $\frac{\partial \alpha}{\partial n}$ from equation (10) are both negative so that both the labor incentive effect and the compensation effect are negative. For a given A^I, α, l^C and l^I are the (implicit) solution to the following system:

$$\begin{cases} E_1 = \frac{1-\alpha}{n} f_L((\frac{\overline{A}}{N} - A^I)n, l^C + L) - v'(l^C + l^I) = 0 \\ E_2 = f_L(A^I, l^I) - v'(l^C + l^I) = 0 \\ E_3 = \frac{1-\alpha}{n} f\left((\frac{\overline{A}}{N} - A^I)n, nl^C\right) + f\left(A^I, l^I\right) - v(l^C + l^I) - \underline{u} = 0 \end{cases} \tag{23}$$

Like in the case of the pure collective regime, we can use the system of equations that implicitly define α, l^C and l^I in order to find expressions for $\frac{\partial \alpha}{\partial n}$ and $\frac{\partial l^C}{\partial n}$:

$$\frac{\partial \alpha}{\partial n} = -\frac{\det \begin{pmatrix} \frac{\partial E_1}{\partial n} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial n} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial n} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}}{\det \begin{pmatrix} \frac{\partial E_1}{\partial \alpha} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial \alpha} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial \alpha} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}} = -\frac{\text{NUM1}}{\text{DEN}}$$

$$\begin{aligned} \text{NUM1} &= \frac{1}{n^2}(-f_L^C(1-\alpha))(-f_L^C(1-\alpha) + v')(f_{LL}^I - v'') \\ \text{DEN} &= \frac{1}{n}\left((1-\alpha)((f_L^C)^2 - f^C f_{LL}^C) f_{LL}^I + v''(-(1-\alpha)(f_L^C)^2 + f^C((1-\alpha) f_{LL}^C + f_{LL}^I)\right) \\ &+ \frac{1}{n}\left(f_L^C v'(-f_{LL}^I + v'')\right) \end{aligned}$$

To obtain this expression, we used again the fact that f is homogeneous of degree 1 and f_L homogeneously of degree 0. Both the numerator and denominator are unambiguously negative, so that $\frac{\partial \alpha}{\partial n} < 0$.

$$\frac{\partial l^C}{\partial n} = -\frac{\det\begin{pmatrix} \frac{\partial E_1}{\partial \alpha} & \frac{\partial E_1}{\partial n} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial \alpha} & \frac{\partial E_2}{\partial n} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial \alpha} & \frac{\partial E_3}{\partial n} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial E_1}{\partial \alpha} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial \alpha} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial \alpha} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}} = -\frac{\text{NUM2}}{\text{DEN}}$$

$$\text{NUM2} = \frac{1}{n^3}\left(f^C f_L^C(1-\alpha)(f_{LL}^I - v'')\right)$$

NUM2 is unambiguously negative so that $\frac{\partial l^C}{\partial n} < 0$.

It is evident that $\frac{\partial R}{\partial n}$ has an ambiguous sign.

B Analytical results

B.1 Proof of lemma 1

B.1.1 The case of the strictly collective regime

We start with the impact of $\overline{A}$ on α. First we show that α is monotonically decreasing in $-\overline{A}$ (or: $\frac{\partial \alpha}{\partial \overline{A}} > 0$). This implies that α tends to its minimal value when $\overline{A}$ tends to zero. Then we show that for all $\alpha > 0$, there exists a land endowment such that the head would choose α. This implies that the limit of α when $\overline{A}$ tends to 0 cannot be strictly positive: it has to be 0.

As $\overline{A}$ decreases or $\underline{u}$ increases, the participation constraint becomes tighter and eventually binds. When the participation constraint binds, α and l are the solution to the following system:

$$\begin{cases} G_1 = 0 = \frac{1-\alpha}{n} f_L(\frac{n\overline{A}}{N}, nl) - v'(l) \\ G_2 = 0 = \frac{1-\alpha}{n} f\left(\frac{n\overline{A}}{N}, nl\right) - v(l) - \underline{u} \end{cases} \tag{24}$$

To find the sign of $\frac{\partial\alpha}{\partial\overline{A}}$, we apply Cramer's rule:

$$\begin{aligned} \frac{\partial\alpha}{\partial\overline{A}} &= -\frac{\det\begin{pmatrix} \frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial \overline{A}} \\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial \overline{A}} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial \alpha} \\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial \alpha} \end{pmatrix}} \\ &= -\frac{n}{N}\frac{\frac{(1-\alpha)^2}{n} f_{LL} f_A - \frac{n}{N}\frac{1-\alpha}{n} f_A v'' - \frac{1-\alpha}{n} f_{LA}\left((1-\alpha)f_L - v'\right)}{-\frac{1}{n} f\left((1-\alpha)f_{LL} - v''\right) + \frac{1}{n} f_L\left((1-\alpha)f_L - v'\right)} \end{aligned}$$

This expression is unambiguously positive. Assuming $f(a,l) = a^{\epsilon} l^{a-\epsilon}$ and $v(l) = \omega l^2$, we can replace the first equation in the system with $l = A^{\frac{\varepsilon}{1+\varepsilon}} \frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{n(2\omega)^{\frac{1}{1+\varepsilon}}}$

For all $\alpha > 0$, we can then find $\overline{A}$ such that the system is satisfied. Indeed, the second equation can be written:

$$\underline{u} = (\frac{n}{N}\overline{A})^{\frac{2\varepsilon}{1+\varepsilon}} \left(\frac{1-\alpha}{n} \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^{1-\varepsilon} - \omega \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^2 \right),$$

which yields:

$$\overline{A} = \frac{N}{n}\underline{u}^{\frac{1+\varepsilon}{2\varepsilon}} \left(\frac{1-\alpha}{n} \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^{1-\varepsilon} - \omega \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^2 \right)^{-\frac{1+\varepsilon}{2\varepsilon}}$$

Let us now turn to the impact of $\underline{u}$ on α. To prove the Lemma, we just need to show that $\frac{\partial\alpha}{\partial\underline{u}} < 0$, and then the argument developed above applies: For each $\alpha > 0$ there exists a

$\underline{u}$ such that the head would choose α, so that α tends to zero when $\underline{u}$ tends to $+\infty$.

$$\begin{aligned}\frac{\partial \alpha}{\partial \underline{u}} &= -\frac{\det\begin{pmatrix}\frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial \underline{u}} \\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial \underline{u}}\end{pmatrix}}{\det\begin{pmatrix}\frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial \alpha} \\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial \alpha}\end{pmatrix}} \\ &= -\frac{-\left((1-\alpha)f_{LL} - v''\right)}{-\frac{1}{n}f\left((1-\alpha)f_{LL} - v''\right) + \frac{1}{n}f_L\left((1-\alpha)f_L - v'\right)}\end{aligned}$$

This last expression is unambiguously negative.

B.1.2 The case of the mixed regime

To prove the lemma, we use the same arguments as in the pure collective case. We show first that $\frac{\partial \alpha}{\partial \underline{u}} < 0$ and $\frac{\partial \alpha}{\partial \overline{A}} > 0$. We assume the same functional forms as previously. For all given values of A^I, l^C, l^I and α are the solution to the following system:

$$\begin{cases} E_1 = \frac{1-\alpha}{n} f_L((\frac{\overline{A}}{N} - A^I)n, nl^C) - v'(l^C + l^I) = 0 \\ E_2 = f_L(A^I, l^I) - v'(l^C + l^I) = 0 \\ E_3 = \frac{1-\alpha}{n} f\left((\frac{\overline{A}}{N} - A^I)n, nl^C\right) + f\left(A^I, l^I\right) - v(l^C + l^I) = \underline{u} \end{cases} \tag{25}$$

$$
\begin{aligned}
\frac{\partial\alpha}{\partial\overline{A}} &= -\frac{\det\begin{pmatrix} \frac{\partial E_1}{\partial\overline{A}} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial\overline{A}} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial\overline{A}} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial E_1}{\partial\alpha} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial\alpha} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial\alpha} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}} = -\frac{\text{NUM4}}{\text{DENOM}} \\
\text{NUM4} &= \frac{1-\alpha}{N} f^C_{LA}\left(-(f^I_{LL}-v'')((1-\alpha)f^C_L - v')\right) - \left((1-\alpha)f^C_{LL} - v''\right)\left(-\frac{1-\alpha}{N} f_A(f^I_{LL}-v'')\right) \\
&- v''v''\left(\frac{1-\alpha}{N} f_A\right) \\
&= \frac{1-\alpha}{N} f^C_{LA}\left(-(f^I_{LL}-v'')((1-\alpha)f^C_L - v')\right) - \left((1-\alpha)f^C_{LL}\right)\left(-\frac{1-\alpha}{N} f_A(f^I_{LL}-v'')\right) \\
&- v''\left(\frac{1-\alpha}{N} f_A f^I_{LL}\right)
\end{aligned}
$$

Since $\frac{1-\alpha}{n} f^C_L = v'$, we have $(1-\alpha)f^C_L > v'$ and $NUM4$ is unambiguously positive, so that $\frac{\partial\alpha}{\partial\overline{A}} > 0$. When $\overline{A}$ tends to zero, α asymptotically tends to its lower limit.

$$
\begin{aligned}
\frac{\partial\alpha}{\partial\underline{u}} &= -\frac{\det\begin{pmatrix} \frac{\partial E_1}{\partial\underline{u}} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial\underline{u}} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial\underline{u}} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial E_1}{\partial\alpha} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial\alpha} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial\alpha} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}} = -\frac{\text{NUM3}}{\text{DENOM}} \\
\text{NUM3} &= -f^C_{LL} f^I_{LL}(1-\alpha) + ((1-\alpha)f^C_{LL} + f^I_{LL})v''
\end{aligned}
$$

Clearly, $NUM3$ is negative and we showed in section A.3.2 that $DENOM$ is also negative (see section A.3.2). Therefore: $\frac{\partial \alpha}{\partial \underline{u}} < 0$.

The next step is to argue that α's lower limit (when $\underline{u}$ tends to $+\infty$ or $\overline{A}$ tends to zero) cannot be strictly positive. Indeed, for all $\alpha > 0$, we can find $\underline{u}$ such that the system defining l^C, l^I and α holds. To see it, notice that from the difference $E_2 - E_1$ we can extract $l^I(l^C, \alpha)$.[17] Then E_1 defines $l^C(\alpha)$.[18] As a result, we can write $l^I(\alpha)$ and $l^C(\alpha)$ and plug these expressions in E_3. Finally, E_3 defines $\underline{u}(\alpha)$: for all $\alpha > 0$ there is a $\underline{u}$ such that α is a solution to the system. Combined with the fact that $\frac{\partial \alpha}{\partial \underline{u}} < 0$, this implies that the limit of α when $\underline{u}$ tends to $+\infty$ can only be 0 (since $\frac{\partial \alpha}{\partial \underline{u}} < 0$ implies that α tends asymptotically to its limit and the lower limit cannot be strictly larger than 0). This holds true for all possible values of A^I, and, in particular, for the optimal A^I.

We prove in a similar manner that the limit of α when $\overline{A}$ tends to $+\infty$ can only be zero. Holding A^I constant, for any $\alpha > 0$, we can find $\overline{A}$ such that α solves the above system of equations. To see this, we begin by noting that the first equation implies:

$$\left(\left(\frac{\overline{A}}{N} - A^I\right) n\right)^{\epsilon} = 2\omega(l^C + l^I)(nl^C)^{\epsilon} \frac{n}{1-\alpha} \tag{26}$$

If we plug this expression for $\left(\left(\frac{\overline{A}}{N} - A^I\right) n\right)^{\epsilon}$ into E3, we obtain an equation that neither depends on α nor on $\overline{A}$:

$$2\omega n l^C(l^C + l^I) + (A^I)^{\epsilon}(l^I)^{1-\epsilon} - \omega(l^C + l^I)^2 = \underline{u} \tag{27}$$

We can now replace l^C in this equation by $l^C(l^I)$ defined by $E2$[19], so that we obtain an

[17] $l^I(l^C, \alpha) = nl^C(\frac{1-\alpha}{n})^{-\frac{1}{\varepsilon}} \frac{A^I}{\overline{A} - nA^I}$

[18] $l^C = \left(\frac{(1-\alpha)(A - nA^I)^{\varepsilon} n^{-\varepsilon-1}}{2\omega(1 + n(\frac{1-\alpha}{n})^{-\frac{1}{\varepsilon}} \frac{A^I}{A - nA^I})} \right)^{\frac{1}{1+\varepsilon}}$

[19] $l^C = \left(\frac{A^I}{l^I}\right)^{\epsilon} \frac{1}{2\omega} - l^I$

equation that implicitly defines l^I, independently of $\overline{A}$. Thus l^C can also be defined independently of $\overline{A}$, and equation (26) implies that, for all $\alpha > 0$ and A^I, there exists a value of $\overline{A}$ such that α solves the above system of equations. This holds true a fortiori when A^I is allowed to vary.

B.2 Proof of lemma 2

If the head prefers the mixed regime, we know that the participation constraint is binding for the optimal A^I in that regime (appendix A.2.1). To show that $\alpha^{mix} > \alpha^{col}$, we argue that $\alpha^{mix} \leq \alpha^{col}$ is impossible.

If $\alpha^{mix} \leq \alpha^{col}$ then the participation constraint is not binding in the mixed regime since dependents utility is strictly greater in the mixed regime than in the collective regime (where it is greater or equal to $\underline{u}$). To see it, consider the out-of-equilibrium situation where dependents would apply the same effort in the mixed regime than in the collective regime and would apply it to the same extent on the collective field and on their private plots. In that case, overall farm production is the same in both regimes but the part retained by the dependents is greater in the mixed regime (so that their utility is larger): per unit of land under collective production they get at least the same income as before while they are full claimants on the area under individual production and can thus extract more income from it. We know that this is not an equilibrium situation and that by reallocating effort so as to equalize their marginal income from the collective and their individual plot, they can further increase their utility. It is thus clear that if $\alpha^{mix} \leq \alpha^{col}$, the participation constraint is not binding in the mixed regime, which would never be chosen.

B.3 Proof of proposition 1

We first compute the impact of an increase in $\overline{A}$ on the head's rent under each regime to compare the expressions obtained. In the mixed regime, for a given A^I:

$$\frac{\partial R}{\partial \overline{\overline{A}}} = \frac{\partial L}{\partial \overline{\overline{A}}} = \alpha \frac{n}{N} f_A^C + \lambda \frac{1-\alpha}{n} \frac{n}{N} f_{LA}^C + \nu \frac{1-\alpha}{n} \frac{n}{N} f_A^C$$

Since $\nu = n - \lambda \frac{f_L^C}{f^C}$ (Section A.2.2), we can write:

$$\frac{\partial R}{\partial \overline{\overline{A}}} = \frac{n}{N} \alpha f_A^C + \lambda \frac{1-\alpha}{N} f_{LA}^C + (n - \lambda \frac{f_L^C}{f^C}) \frac{1-\alpha}{N} f_A^C$$

Or:

$$\frac{\partial R}{\partial \overline{\overline{A}}} = \frac{n}{N} f_A^C + \lambda \frac{1-\alpha}{N} f_{LA}^C (1 - \tau_{LA})$$

where $\tau_{LA} = \frac{f_A f_L}{f f_{LA}}$ is the substitution elasticity of production factors. Because $\tau_{LA} = 1$ in the case of the Cobb-Douglas function, the above expression reduces to:

$$\frac{\partial R}{\partial \overline{\overline{A}}} = \frac{n}{N} f_A^C$$

A unit increase in the total family endowment increases the area of the farm by $\frac{n}{N}$ and the impact on the head's rent is equal to $\frac{n}{N}$ times the marginal productivity of land on the collective field.

The same holds in the collective regime. If the participation constraint is unbinding, then $\gamma = 0$ and $\beta = -\frac{nf}{f_L}$ (section A.1)) and:

$$\begin{aligned}
\frac{\partial R}{\partial \overline{\overline{A}}} &= \frac{\partial L}{\partial \overline{\overline{A}}} = \alpha \frac{n}{N} f_A - \beta \frac{1-\alpha}{N} f_{LA} \\
&= \frac{n}{N} f_A - \frac{1-\alpha}{N} f_{LA} \frac{nf}{f_L} \\
&= \frac{n}{N} f_A
\end{aligned}$$

To obtain the last simplification in the above expression, we use again the fact that $f_A = \frac{f_{LA}}{f f_L}$. If the participation constraint is binding we know $\gamma = n + \beta \frac{f_L}{f}$ (section A.1) and :

$$
\begin{aligned}
\frac{\partial R}{\partial \overline{A}} &= \frac{\partial L}{\partial \overline{A}} = \alpha \frac{n}{N} f_A - \beta \frac{1-\alpha}{N} f_{LA} + \gamma \frac{1-\alpha}{N} f_A \\
&= \frac{n}{N} f_A + \frac{1-\alpha}{N} \beta \left(f_{LA} - \frac{f_L f_A}{f} \right) \\
&= \frac{n}{N} f_A
\end{aligned}
$$

In both regimes, as expected, the head's rent is monotonically increasing in $\overline{A}$.

Consider a given farm area A and a given family size n. If total effort is smaller in the mixed regime $l^C + l^I \leq l$, then:

$$
\frac{nl}{\mathring{A}} > \frac{nl^C}{\mathring{A}} + \frac{nl^I}{\mathring{A}} \tag{28}
$$

$$
\frac{nl}{\mathring{A}} > \frac{nl^C}{A^C} \frac{A^C}{\mathring{A}} + \frac{l^I}{A^I} \frac{nA^I}{\mathring{A}} \tag{29}
$$

The incentive compatibility constraint in the mixed regime ($\frac{1-\alpha}{n} f_L^C = f_L^I$) implies $\frac{l^I}{A^I} = \frac{nl^C}{A^C} \left(\frac{n}{1-\alpha} \right)^{\frac{1}{\epsilon}}$, thus $\frac{l^I}{A^I} > \frac{nl^C}{A^C}$, or $\frac{l^I}{A^I} = \frac{nl^C}{A^C} + k$, with $k > 0$. Thus inequality (29) becomes:

$$
\begin{aligned}
\frac{nl}{\mathring{A}} &> \frac{nl^C}{A^C} \left(\frac{A^C}{\mathring{A}} + \frac{nA^I}{\mathring{A}} \right) + k \frac{nA^I}{\mathring{A}} \\
\frac{nl}{\mathring{A}} &> \frac{nl^C}{A^C} + k \frac{nA^I}{\mathring{A}} \\
f_A &> f_A^C
\end{aligned}
$$

If total effort is greater in the mixed regime, that is if $l^C + l^I > l$:

$$
v'(l^C + l^I) > v'(l) \tag{30}
$$

$$
\Leftrightarrow \frac{1-\alpha^{mix}}{n} f_L^{Cmix} > \frac{1-\alpha^{col}}{n} f_L^{col} \tag{31}
$$

If $\alpha^{mix} > \alpha^{col}$, then $1-\alpha^{mix} < 1-\alpha^{col}$ and inequality 31 implies:

$$\begin{aligned} f_L^{C mix} &> f_L^{col} \\ \Leftrightarrow (1-\epsilon)\left(\frac{A^C}{nl^c}\right)^{\epsilon mix} &> (1-\epsilon)\left(\frac{\mathring{A}}{nl}\right)^{\epsilon col} \\ \Rightarrow \left(\frac{A^C}{nl^c}\right)^{mix} &> \left(\frac{\mathring{A}}{nl}\right)^{col} \\ \Leftrightarrow \left(\frac{nl^c}{A^C}\right)^{mix} &< \left(\frac{nl}{\mathring{A}}\right)^{col} \\ \Rightarrow f_A^{C mix} &< f_A^{col} \end{aligned}$$

If $\alpha^{mix} \leq \alpha^{col}$, we have shown above (see section B.2) that the participation constraint is not binding in the mixed regime, so that this is not a relevant case to examine. Finally, we have established that wherever the mixed regime is relevant (in the sense that there exists A^I such that the participation constraint is binding in the mixed regime and it is not trivially inferior to the collective regime), the head's rent is monotonically increasing in both regimes and it increases faster in the collective regime than in the mixed.

Suppose that the head is indifferent between the mixed and the collective regime. As shown above, a marginal decrease in $\overline{A}$ decreases his rent to a greater extent in the collective than in the mixed regime when A^I is maintained constant, so that the mixed regime is strictly preferred. This holds true a fortiori when the head can change A^I. If we consider a marginal increase in $\overline{A}$, the head's rent increases more in the collective than in the mixed regime when A^I is maintained constant. If A^I is allowed to vary, could the head's rent increase to a greater extent in the mixed regime? The answer is negative because a marginal increase in $\overline{A}$ has a greater impact on the head's rent in the collective than in the mixed regime for all relevant A^I.[20]

[20] To see this, consider a marginal increase in $\overline{A}$ from A_1 to A_2. Call $A^{I*}(\overline{A})$ the optimal size of individual plot when total land endowment is $\overline{A}$, $R^{col}(\overline{A})$ the head's rent in the collective regime and $R^{mix}(\overline{A}, A^{I*}(\overline{A}))$, his rent in the mixed regime. We know: $R^{mix}(A_2, A^{I*}(A_2)) - R^{mix}(A_1, A^{I*}(A_2)) < R^{col}(A_2,) - R^{col}(A_1)$. In addition, by definition, it is true that: $R^{mix}(A_2, A^{I*}(A_2)) - R^{mix}(A_1, A^{I*}(A_1)) < R^{mix}(A_2, A^{I*}(A_2)) -$

Let A_{min} be the level of land endowment such that the head's rent is null in the collective regime. As $\overline{A}$ goes from A_{min} to $+\infty$, either the collective regime dominates everywhere, or there exists a level of land endowment $\mathring{A}$ such that the head is just indifferent between the mixed and the collective regimes. Then, for $\overline{A} < \mathring{A}$, the head prefers the mixed regime, while for $\overline{A} > \mathring{A}$, the head prefers the collective regime.[21]

B.4 Proof of proposition 2

We first show that, if $\underline{u}$ tends to 0, the collective regime dominates the mixed regime. We then examine the influence of $\underline{u}$ on the head's propensity to give out individual fields when he is just indifferent between both regimes.

When $\underline{u}$ tends to zero, the participation constraint becomes unbinding in the mixed regime for all A^I. To prove this result, let us show that, if the incentive constraints are satisfied, the participation constraint is automatically satisfied for values of $\underline{u}$ very close to zero. With the Cobb-Douglas production function and the polynomial cost of effort ($v(l) = \omega l^2$), the incentive constraints are:

$$\tfrac{1-\alpha}{n}(1-\varepsilon)(A - nA^I)^\varepsilon (nl^C)^{-\varepsilon} = 2\omega(l^C + l^I), \text{ and } (1-\varepsilon)(A^I)^\varepsilon (l^I)^{-\varepsilon} = 2\omega(l^C + l^I).$$

$R^{mix}(A_1, A^{I*}(A_2))$. It follows that: $R^{mix}(A_2, A^{I*}(A_2)) - R^{mix}(A_1, A^{I*}(A_1)) < R^{col}(A_2,) - R^{col}(A_1)$. Even when the father adjusts A^I in the mixed regime, therefore, his rent does not increase as much as in the collective regime.

[21] It is indeed not possible that in the range A_{min} and $\mathring{A}$, there exist a land endowment a where the mixed regime is irrelevant in the sense that the participation constraint would be unbinding and the collective is chosen. This would imply that at a, the participation constraint is unbinding in the mixed regime, while at $\mathring{A}$ it is binding. This is impossible since a decrease in $\overline{A}$ tightens the participation constraint.

Adding the two expressions, we get:

$$
\begin{aligned}
\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^\varepsilon(nl^C)^{-\varepsilon}+(1-\varepsilon)(A^I)^\varepsilon(l^I)^{-\varepsilon} &= 4\omega(l^C+l^I)\\
\Leftrightarrow (l^C+l^I)\left(\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^\varepsilon(nl^C)^{-\varepsilon}+(1-\varepsilon)(A^I)^\varepsilon(l^I)^{-\varepsilon}\right) &= 4\omega(l^C+l^I)^2\\
\Leftrightarrow \frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^\varepsilon n^{-\varepsilon}(l^C)^{1-\varepsilon}+(1-\varepsilon)(A^I)^\varepsilon(l^I)^{1-\varepsilon} &\\
+l^I\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^\varepsilon(nl^C)^{-\varepsilon}+l^C(1-\varepsilon)(A^I)^\varepsilon(l^I)^{-\varepsilon} &= 2\omega(l^C+l^I)^2+l^C2\omega(l^C+l^I)\\
&\quad +l^I2\omega(l^C+l^I)
\end{aligned}
$$

Since the incentive constraints on the individual and collective fields imply: $\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^\varepsilon(nl^C)^{-\varepsilon}=(1-\varepsilon)(A^I)^\varepsilon(l^I)^{-\varepsilon}=2\omega(l^C+l^I)$, the last two terms on both sides of the previous equality cancel out and we obtain:

$$
\begin{aligned}
\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^\varepsilon n^{-\varepsilon}(l^C)^{1-\varepsilon}+(1-\varepsilon)(A^I)^\varepsilon(l^I)^{1-\varepsilon} &= 2\omega(l^C+l^I)^2\\
\Rightarrow \frac{1-\alpha}{n}(A-nA^I)^\varepsilon n^{1-\varepsilon}(l^C)^{1-\varepsilon}+(A^I)^\varepsilon(l^I)^{1-\varepsilon}-\omega(l^C+l^I)^2 &> 0
\end{aligned}
$$

The LHS is the level of utility achieved by a member. Since it is strictly greater than zero, we conclude that, for values of $\underline{u}$ very close to zero, the participation constraint is automatically satisfied. The mixed regime never dominates if the participation constraint is unbinding for all A^I, since the father's rent is then monotonically decreasing in A^I (see section A.2.1). As a result, when $\underline{u}$ tends to zero, the head of a collective farm never finds it optimal to grant individual plots.

Furthermore, when $\underline{u}$ tends to zero, the member's participation constraint is unbinding in the collective regime. To show it, we use a similar argument than in the case of the mixed regime. The incentive constraint is: $\frac{1-\alpha}{n}(1-\varepsilon)(A)^\varepsilon(nl)^{-\varepsilon}=2\omega l$. This expression is equivalent to $\frac{1-\varepsilon}{n}\frac{1-\alpha}{n}(A)^\varepsilon(nl)^{1-\varepsilon}=2\omega l^2$, which implies: $\frac{1-\varepsilon}{n}\frac{1-\alpha}{n}f>\omega l^2$ and finally $\frac{1-\alpha}{n}f-v(l)>0$. Thus, for values of $\underline{u}$ very close to zero, the participation constraint is automatically satisfied.

We now turn to the analysis of the impact of a marginal change in $\underline{u}$ on the head's rent in both regimes. The envelop theorem implies that a marginal increase in $\underline{u}$ decreases the father's rent by γ in the pure collective regime and by ν in the mixed regime (since we know that the optimal A^I in the mixed regime is such that the participation constraint binds, cf footnote 3.3.), where the Lagrangian multipliers have a parallel expression:

$$\gamma = n - \frac{1}{\frac{v'' f^s}{(f_L^s)^2(n-1+\alpha^s)} + \frac{(1-\alpha^s)(-f_{LL}^s)f^s}{(f_L^s)^2(n-1+\alpha^s)} + \frac{(1-\alpha^s)(1-\frac{1}{n})}{n-1+\alpha^s}} \tag{32}$$

$$\nu = n - \frac{1}{\left(\frac{-f_{LL}^I}{v''-f_{LL}^I}\right)\frac{v''(f^C)^n}{((f_L^C)^n)^2(n-1+\alpha^m)} + \frac{(1-\alpha^m)(-(f_{LL}^C)^m)(f^C)^m}{((f_L^C)^m)^2(n-1+\alpha^m)} + \frac{(1-\alpha^m)(1-\frac{1}{n})}{n-1+\alpha^m}} \tag{33}$$

With a Cobb-Douglas production function ($f(A,l) = A^{\varepsilon}l^{1-\varepsilon}$) and a polynomial cost of effort ($v(l) - \omega l^2$), we have: $\frac{-ff_{LL}}{f_L^2} = \frac{\varepsilon}{1-\varepsilon}$ and $\frac{fv''}{f_L^2} = \frac{1-\alpha}{1-\varepsilon}$. Using these relationships, the above expressions become:

$$\gamma = n - \frac{1}{\frac{(1-\alpha^s)}{(1-\varepsilon)(n-1+\alpha^s)} + \frac{(1-\alpha^s)\varepsilon}{(1-\varepsilon)(n-1+\alpha^s)} + \frac{(1-\alpha^s)(1-\frac{1}{n})}{n-1+\alpha^s}}$$

$$\nu = n - \frac{1}{\left(\frac{-f_{LL}^I}{v''-f_{LL}^I}\right)\frac{(1-\alpha^m)}{(1-\varepsilon)(n-1+\alpha^m)} + \frac{(1-\alpha^m)\varepsilon}{(1-\varepsilon)(n-1+\alpha^m)} + \frac{(1-\alpha^m)(1-\frac{1}{n})}{n-1+\alpha^m}}$$

When the head is just indifferent between the pure collective and the mixed regimes, we know that $\alpha^m > \alpha^s$ (appendix B.2) so that, $\gamma > \nu$. At a point of indifference, a marginal increase in the reservation utility therefore has a greater (negative) impact in the pure collective regime than in the mixed regime. The head would thus strictly prefer the mixed regime. Conversely, a marginal decrease in the reservation utility induces the head to strictly prefer the collective regime.

Finally, as $\underline{u}$ goes from 0 to $+\infty$, either the collective regime dominates everywhere, or an indifference level of utility exists, and the mixed regime dominates for utility levels higher than that threshold level, while the collective regime dominates for smaller levels.

B.5 Proof of proposition 3 and proposition 4

To analyze how a marginal change in $\underline{u}$ or in $\overline{A}$ changes the incentive to split the family, we examine the conditions under which $\frac{\partial R}{\partial n} > 0$.

B.5.1 Land endowment, reservation utility, and the decision to split in the pure collective regime

We know that, as $\underline{u}$ tends to zero, the participation constraint becomes unbinding (section B.4).To obtain an expression for $\frac{\partial R}{\partial n}$ when the participation constraint does not bind, we replace $\frac{\partial \alpha}{\partial n}$ and $\frac{\partial l}{\partial n}$ in equation 10 by the expression obtained in section A.3.1, and we use $\beta = -\frac{nf}{f_L}$. We then find:

$$\begin{aligned}\frac{\partial R}{\partial n} &= \alpha(\frac{\overline{A}}{N} f_A + l f_L) + \beta \frac{1-\alpha}{n^2} f_L - \beta \frac{1-\alpha}{n} \left(\frac{\overline{A}}{N} f_{LA} + l f_{LL} \right) \\ &= \frac{\alpha}{n} f + \beta \frac{1-\alpha}{n^2} f_L \\ &= \frac{2\alpha - 1}{n} f\end{aligned}$$

We know that, when the participation constraint is not binding, $\alpha = \frac{1+\epsilon}{2}$ (section A.1). Finally:

$$\frac{\partial R}{\partial n} = \frac{\epsilon}{n}$$

We can thus conclude that $\frac{\partial R}{\partial n}$ is unambiguously positive, meaning that it is never desirable for the head to let one son leave the farm with some land. By implication, when $\underline{u}$ tends to zero, the head will never choose to split the family.

Conversely when $\underline{u}$ tends to $+\infty$, the participation constraint is binding and we can show that the family head will choose to split the family. Again, to obtain an expression for $\frac{\partial R}{\partial n}$ we replace $\frac{\partial \alpha}{\partial n}$ and $\frac{\partial l}{\partial n}$ in equation 10 by the expression obtained in section A.3.1. We find that:

$$\begin{aligned}\frac{\partial R}{\partial n} &= \alpha(\frac{\overline{A}}{N} f_A + l f_L) - \frac{f\alpha f_L \frac{1-\alpha}{n^2} f_L + f\frac{f_L}{n^2}(f_L(1-\alpha)-v')}{\frac{f_L}{n}((1-\alpha)f_L - v') - \frac{f}{n}((1-\alpha)f_{LL} - v'')} \\ &= \frac{\alpha}{n} f - \frac{f f_L^2 \frac{1-\alpha}{n^2}(\alpha+1-\frac{1}{n})}{\frac{f_L}{n}((1-\alpha)f_L - v') - \frac{f}{n}((1-\alpha)f_{LL} - v'')}\end{aligned}$$

from which we infer that:

$$\begin{aligned}\frac{\partial R}{\partial n} &> 0 \\ \Leftrightarrow \frac{\alpha}{n} &> \frac{f_L^2 \frac{1-\alpha}{n^2}(\alpha+1-\frac{1}{n})}{\frac{f_L}{n}((1-\alpha)f_L - \frac{1-\alpha}{n} f_L) - \frac{f}{n}((1-\alpha)f_{LL} - v'')} \\ &> \frac{\frac{1}{n}(\alpha+1-\frac{1}{n})}{(1-\frac{1}{n}) - \frac{f f_{LL}}{f_L^2} + \frac{f v''}{f_L^2(1-\alpha)}}\end{aligned}$$

We again use $\frac{-f f_{LL}}{f_L^2} = \frac{\varepsilon}{1-\varepsilon}$ and $\frac{f v''}{f_L^2} = \frac{1-\alpha}{1-\varepsilon}$. This considerably simplifies the previous expression since we now obtain the following condition:

$$\begin{aligned}\frac{\partial R}{\partial n} &> 0 \\ \Leftrightarrow \alpha &> \frac{\alpha+1-\frac{1}{n}}{\frac{2}{1-\varepsilon} - \frac{1}{n}} \\ \Leftrightarrow \alpha &> \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon} - \frac{1}{n}}\end{aligned}$$

This condition is increasingly difficult to satisfy as n increases (for all n: $\frac{\partial \psi}{\partial n} > 0$, with $\psi = \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon} - \frac{1}{n}}$), which is intuitive. Once the family is really large, it becomes less interesting for the head to keep it whole.

Furthermore, when $\underline{u}$ gets very large, then α tends to 0 (proof in section B.1) and we have:

$$\begin{aligned} \frac{\partial R}{\partial n} &< 0 \\ \Leftrightarrow 0 &> \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon}-1} \end{aligned}$$

This last inequality holds for all $n > 1$, and suggests that the family head will always split the family if $\underline{u}$ is infinitely large

We have just shown that the head of a collective farm chooses to split the family when $\underline{u}$ tends to $+\infty$ while he prefers to keep the family whole when $\underline{u}$ tends to 0. Since the father's rent is monotonically decreasing in $\underline{u}$, there must exist a unique level of $\underline{u}$ so that the head is just indifferent between splitting and not.

Let us now show that when land is very abundant, the participation constraint does not bind. We argue that for any given reservation utility $\underline{u}$, we can find a land endowment large enough to make the dependent's utility exceed $\underline{u}$. The incentive constraint implies $l = \left(\frac{(1-\alpha)(1-\epsilon)}{2\omega n}\right)^{\frac{1}{1+\epsilon}} n^{\frac{1-\epsilon}{1+\epsilon}} A^{\frac{\epsilon}{1+\epsilon}}$. Using this expression, we can write a dependent's utility as:

$$\begin{aligned} u &= \frac{1-\alpha}{n} A^{\epsilon}(nl)^{1-\epsilon} - \omega l^2 \\ &= \frac{1-\alpha}{n} A^{\epsilon} n^{1-\epsilon} \left(\frac{(1-\alpha)(1-\epsilon)}{2\omega n}\right)^{\frac{1-\epsilon}{1+\epsilon}} n^{\frac{(1-\epsilon)^2}{1+\epsilon}} A^{\frac{\epsilon(1-\epsilon)}{1+\epsilon}} - \omega \left(\frac{(1-\alpha)(1-\epsilon)}{2\omega n}\right)^{\frac{2}{1+\epsilon}} n^{\frac{2(1-\epsilon)}{1+\epsilon}} A^{\frac{2\epsilon}{1+\epsilon}} \\ &= n^{\frac{2(1-\epsilon)}{1+\epsilon}} A^{\frac{2\epsilon}{1+\epsilon}} \left(\frac{1-\alpha}{n}\right)^{\frac{2}{1+\epsilon}} \left(\left(\frac{1-\epsilon}{2\omega}\right)^{\frac{1-\epsilon}{1+\epsilon}} - \omega \left(\frac{1-\epsilon}{2\omega}\right)^{\frac{2}{1+\epsilon}}\right) \end{aligned}$$

We know that, if the participation constraint is not binding, $\alpha = \frac{1+\epsilon}{2}$ (section A.1). We thus have to ask whether, for large enough A, $u \geq \underline{u}$ with $\alpha = \frac{1+\epsilon}{2}$. The above equation implies that this is the case for A such that:

$$A > \underline{u}\left(\frac{1-\epsilon}{2n}\right)^{-\frac{1}{\epsilon}} n^{-\frac{1-\epsilon}{\epsilon}} \left(\left(\frac{1-\epsilon}{2\omega}\right)^{\frac{1-\epsilon}{1+\epsilon}} - \omega\left(\frac{1-\epsilon}{2\omega}\right)^{\frac{2}{1+\epsilon}}\right)^{-\frac{1+\epsilon}{2\epsilon}}$$

In particular, when $\overline{A}$ tends to $+\infty$, this condition will be satisfied for all $\underline{u}$ so that the participation constraint is unbinding. We have shown above that, if the participation constraint does not bind, it is never optimal to split the family. Therefore, when $\overline{A}$ tends to $+\infty$, the head will not split the family.

Conversely, when land is very scarce, the participation constraint binds and we have shown that:

$$\frac{\partial R}{\partial n} > 0 \Leftrightarrow \alpha > \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon} - \frac{1}{n}}$$

When $\overline{A}$ gets very small, then α tends to 0 (proof in section **??**) and we have:

$$\frac{\partial R}{\partial n} < 0 \Leftrightarrow 0 > \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon} - 1}$$

This last inequality holds for all $n > 1$, which suggests that the family head will always split the family if $\overline{A}$ is close to zero.

Since the father's rent is monotonically increasing in $\overline{A}$, there exists a level of $\overline{A}$ such that he is just indifferent between splitting the family and keeping it whole.

B.5.2 Land endowment, reservation utility, and the decision to split the family in the mixed regime

We know that, if $\overline{A}$ tends to $+\infty$ or $\underline{u}$ tends to zero, the collective regime always dominates the mixed regime. We therefore focus on the head's propensity to split in the mixed regime when $\overline{A}$ tends to zero or $\underline{u}$ tends to $+\infty$.

Let us derive an expression for $\frac{\partial R}{\partial n}$ in the case where the participation constraint binds (which is the case when $\overline{A}$ tends to 0 or $\underline{u}$ tends to $+\infty$). Replacing $\frac{\partial \alpha}{\partial n}$ and $\frac{\partial l}{\partial n}$ in equation (10) by the expressions obtained in section A.3.2, we have:

$$\frac{\partial R}{\partial n} = \frac{\alpha}{n} f^C$$
$$+ \frac{f^C f^C_L (1-\alpha)(f^C_L - v')(f^I_{LL} - v'')}{n\left((1-\alpha)((f^C_L)^2 - f^C f^C_{LL}) f^I_{LL} + v''(-(1-\alpha)(f^C_L)^2 + f^C((1-\alpha) f^C_{LL} + f^I_{LL}) + f^C_L v'(-f^I_{LL} + v'')\right)}$$

Thus:

$$\begin{aligned} \frac{\partial R}{\partial n} &> 0 \\ \Leftrightarrow \alpha &> \frac{f^C_L (1-\alpha)(f^C_L - v')(f^I_{LL} - v'')}{(1-\alpha)((f^C_L)^2 - f^C f^C_{LL}) f^I_{LL} + v''(-(1-\alpha)(f^C_L)^2 + f^C((1-\alpha) f^C_{LL} + f^I_{LL}) + f^C_L v'(-f^I_{LL} + v'')} \end{aligned}$$

With the Cobb-Douglas production function and the polynomial cost of effort, we again have: $\frac{-ff_{LL}}{f_L^2} = \frac{\varepsilon}{1-\varepsilon}$ and $\frac{fv''}{f_L^2} = \frac{1-\alpha}{1-\varepsilon}$. As a result:

$$\begin{aligned}
\frac{\partial R}{\partial n} &< 0 \\
\Leftrightarrow \alpha &< \frac{f_L^C(1-\alpha)(f_L^C - v')(f_{LL}^I - v'')}{f_{LL}^I (f_L^C)^2 \frac{1-\alpha}{1-\varepsilon} - v''(f_L^C)^2 \frac{1-\alpha}{1-\varepsilon} + v'' f^C f_{LL}^I + (f_L^C)^2 \frac{1-\alpha}{n}(-f_{LL}^I + v'')} \\
&< \frac{(f_L^C)^2(1-\alpha)(1-\frac{1-\alpha}{n})(f_{LL}^I - v'')}{(f_L^C)^2 \frac{1-\alpha}{1-\varepsilon}(-f_{LL}^I + v'')\left(-\frac{1}{1-\varepsilon} + \frac{1}{n}\right) + v'' f^C f_{LL}^I} \\
&< \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{v'' f^C f_{LL}^I}{(f_L^C)^2 (1-\alpha)(f_{LL}^I - v'')}} \\
&< \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{\frac{(1-\alpha)(f_L^C)^2}{f^C v''} - \frac{(1-\alpha)(f_L^C)^2}{f_{LL}^I f^C}}} \\
&< \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{(1-\varepsilon) - \frac{f^I}{f^C} \frac{\frac{n^2}{1-\alpha}(f_L^I)^2}{f_{LL}^I f^I}}} \\
&< \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{(1-\varepsilon) + \frac{f^I}{f^C} \frac{n^2}{1-\alpha} \frac{1-\varepsilon}{\varepsilon}}}
\end{aligned}$$

When $\overline{A}$ tends to 0 or $\underline{u}$ tends to $+\infty$, α tends to 0 (proof in section B.1.2) and we have:

$$\frac{\partial R}{\partial n} < 0 \Leftrightarrow 0 < \frac{1 - \frac{1}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{(1-\varepsilon) + \frac{f^I}{f^C} n^2 \frac{1-\varepsilon}{\varepsilon}}}$$

This last inequality holds for all $n > 1$. We may therefore conclude that, when land is very scarce or the reservation utility is very large, the father will choose to split the family under the mixed regime.