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ABSTRACT

The Effect of Public Information on Competition and R&D Investment

by Jim Jin and Michael Tröge*

Using a simple method we show that, in contrast to the case of Bertrand and Cournot competition, better public information about demand reduces the profits of firms playing a R&D cost reduction game. Welfare increases with the precision of public information in the Cournot and cost reduction games, but not in the Bertrand game. We conclude that the provision of public information about demand in the distant future is useful and necessary in order to promote investment in R&D. Information about demand in the short term, however, should not be released.

ZUSAMMENFASSUNG

Die Auswirkungen öffentlicher Informationen auf Wettbewerb und FuE-Investitionen

Mit einer einfachen Methode zeigen wir, daß im Gegensatz zum Bertrand und Cournot Wettbewerb, genauere öffentlich zugängliche Informationen über zukünftige Nachfrage die Gewinne von Firmen reduziert, die Forschungsinvestitionen zur Kostensenkung beabsichtigen. Die Wohlfahrt wird im Falle des Cournot und Forschungswettbewerbs durch bessere öffentliche Informationen erhöht, aber bei Bertrandwettbewerb verkleinert. Wir leiten daraus ab, daß die öffentliche Bereitstellung von Informationen über die zukünftige Nachfrage nützlich und notwendig ist, wohingegen Informationen die aktuelle Nachfrage betreffen nicht veröffentlicht werden sollten.

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1. Introduction

From a game theoretic point of view it is not surprising that in games, unlike in decision problems, more public information may not be beneficial to the players. For example this is mentioned in Ponssard (1976) although he concentrates on the negative value of private information. In applications, however, a negative effect of public information on the players has rarely been observed, and then like in Sakai and Yoshizumi (1991), it has rather been regarded as a pathological property of the model.

In this paper we show that, on the contrary, a negative impact of public information on the players' expected payoff is a robust effect observed in some very common games. We do that by presenting a method by which the value of information to the players can be easily checked for the class of symmetric games with quadratic payoffs. These include the standard competition models. Most interestingly, we do not need any assumptions about the distribution of the signal or the random parameter.

We demonstrate our method by applying it to the case of public information about demand uncertainty in Cournot and Bertrand games as well as to R&D competition followed by Cournot or Bertrand games. The common demand structure of these models allows us to easily derive the welfare implications of information under the same assumptions. We can then compare the private and social impact of public information.

We find that better information increases the expected payoff of firms in price and quantity competition, but lowers their profits in a R&D cost reduction game independently whether it is followed by Cournot or Bertrand competition, as long as the products are not too independent. Social welfare decreases with better information only in the Bertrand game and rises in the other cases. Thus in the Bertrand and cost reduction games, there is a conflict between the interests of the firms and society in collecting public information.

Cournot and Bertrand competition with demand uncertainty have been extensively studied in the information sharing literature (Novshek and Sonnenschein (1982), Vives (1984), Gal-Or (1985), Li (1985), Jin (1994), Raith (1997) and some of the results that we obtain have already been derived as a by-product of this literature. In particular Vives (1984) has already obtained the welfare implications of public information in the Bertrand and Cournot games for the case of signals with normally distributed noise.

But whereas the information sharing literature compares the effect of pooling information and keeping information private, we focus here exclusively on the effect of better versus less precise public information. We believe that this is an important issue which has been somewhat neglected in the literature. As the state interferes in many ways in the collection, release or disclosure of information, the effects of public information have widespread policy implications.

For example in most countries, the state directly generates information that has implications for future demand, by financing institutions doing economic research. The necessity and the usefulness of this activity are often questioned. If we know that information is socially beneficial, we can say that public research is at least not useless. But if this information is at the same time beneficial for the firms, we could expect them to form research cooperations and do this research more efficiently on their own. State intervention would not be necessary.

Our paper shows, that whereas a general conclusion about usefulness and necessity of public research cannot be drawn, we can give a quite simple and intuitive policy advice.

We know that the state should provide information for firms engaged in R&D activities as this rises the welfare and the firms themselves are not interested in public information. However providing information relevant for firms playing a Bertrand game should be avoided. This can easily be achieved by only providing information about the evolution of demand in the long run. As there is a considerable time lag between the R&D decisions and the realization of the cost reduction, whereas the pricing decision is immediate, information about the demand in the more distant future influences the R&D decisions but not the price setting in the short term.

Note that this is a quite unusual characterization of the kind of information that should be provided. Normally public provision of information is justified by the argument that this avoids inefficient duplication of research. Hence, usually fairly general informations concerning a maximum of firms are provided by public institutions. However, our result shows that it may very well be indispensable to provide information concerning only very few firms, as long as these informations concern the long run.

This could be, for example, part of the explanation for the success of Japanese industry. Here, the famous MITI provides long term, very industry specific information. We showed that this will increase the investments in cost reduction as well as public welfare.

Another important situation where information policy plays a role is the release of information. Imagine that the state plans a public order, that is likely to have an important effect on the demand. Should this information be held back, or should the government make early commitments to a decision?

The advise we can give is that information that is thought to influence long term investments in capacity or research should be released early, even if this may not be beneficial to the firms. Information about short term demand changes however, that will only have effects on the price setting behavior of the firms should be held back, as long as the prices are not determined.

The organization of the paper is as follows. In the next section we present the common features and the payoff functions of the games we want to analyze. As the calculations become quite involved for the case of cost reduction followed by competition with differentiated goods we demonstrate in the text only the simple case of cost reduction followed by quantity competition with homogenous goods. However, we formulate our results for the general case and prove in the appendix that the results are robust with respect to the introduction of differentiated products and price competition.

In the 3. section we introduce a generalized payoff function, solve this problem and derive then the effect of public information on firms' profit in the special cases.

In the 4. section we derive the welfare effects of public information and in the last section we conclude.

2. The Model

Informational structure: It has become standard in the information sharing literature to model information as a noisy signal of the random parameter. Given the firms' prior of the parameters distribution and the assumptions about the distribution of the noise, Bayesian updating then gives the firm the conditional random distribution of the demand intercept.

It is not necessary to detail this process here. We will just assume that the result of the firms updating process after having received information is a correct estimation $\widetilde{a}$ of the expectation of the demand intercept i.e. $\mathbf{E}(a|\widetilde{a}) = \widetilde{a}$. This doesn't mean that the expected estimation $\widetilde{a}$ conditional on the signal being a will be equal to a, i.e. in general we will not have $\mathbf{E}(\widetilde{a}|a) = a$. For simplicity we will call $\widetilde{a}$ the signal.

It is easy to show (see Appendix A) that with the above definitions a and $\widetilde{a}$ have the following useful properties:

$$\mathbf{E}[a * \widetilde{a}] = \mathbf{E}\left[\widetilde{a}^2\right] \tag{2.1}$$

$$Var[(a - \widetilde{a})] = \mathbf{E}\left[(a - \widetilde{a})^2\right] = \mathbf{E}\left[a^2\right] - \mathbf{E}\left[\widetilde{a}^2\right] \tag{2.2}$$

The value of $Var[(a - \widetilde{a})]$, the ex ante expected squared error of the outcome a after the information of the signal has been taken into account, will be used as a measure of the precision of the signal. A lower value of $Var[(a - \widetilde{a})]$ is equivalent to more public information. Equation 2.2 shows that the variance $\mathbf{E}[\widetilde{a}^2]$ of the signal $\widetilde{a}$ increases with its precision, but will always be lower than the variance of the parameter a, unless the signal is perfect.

Note that we do not assume that all signals are equally informative. After having received certain signals the squared error of the outcome may even increase. But from equation 2.2 it can be seen that this cannot be the case for all signals. As long as the signal has any informational content, ex ante the expected residual variance decreases.

The Demand Structure: We will use a common structure for the demand for the differentiated goods in our three examples. As we want to calculate the consumers' surplus with differentiated products, we assume that the consumers' utility function has the form:

$$U(q_0, q_1, q_2) = q_0 + a\left(q_1 + q_2\right) - \frac{1}{2}\left(bq_1^2 + 2dq_1q_2 + bq_2^2\right) \tag{2.3}$$

with $a > 0$, $b > |d| \geqq 0$. Here q_0 designs the numeraire good, whose price is normalized to 1, and q_1 and q_2 are the quantities of good 1 and 2 consumed. For $b = d$ the goods are homogenous, for $d > 0$, substitutes, for $d < 0$, complements. We assume that consumers' income is sufficiently high, so that the marginal utility of the numeraire good can be treated as constant. Therefore, the inverse demand function can be derived by maximizing the consumers' surplus:

$$p_i = a - bq_i - dq_j \tag{2.4}$$

For $b \neq d$ we can invert the system and obtain the demand function

$$q_i = \alpha - \beta p_i + \delta p_j, \tag{2.5}$$

where $\alpha = a/(b+d)$, $\beta = b/(b^2 - d^2)$, $\delta = d/(b^2 - d^2)$.

Examples: P*rice Competition*

We assume that both firms produce at the constant marginal cost c_0. In a Bertrand game the players decide about the prices p_i, the quantities produced and sold are then determined by the outcome of the random variable. The payoff as a function of the players' action variables and the realization of the random variable can be written as:

$$\begin{aligned} \pi_i^B(p_i, p_j, a) &= (p_i - c_0)(\alpha - \beta p_i + \delta p_j) \qquad (2.6) \\ &= -c_0\alpha + (\alpha + c_0\beta)\, p_i - \beta p_i^2 + \delta p_j p_i - c_0 \delta p_j \end{aligned}$$

Quantity Competition

Here the players decide about the quantities they produce, but the price depends on the random demand. The payoff function in this case is

$$\begin{aligned} \pi_i^C(q_i, q_j, a) &= q_i(a - bq_i - dq_j - c_0) \qquad (2.7) \\ &= (a - c_0)\, q_i - dq_j q_i - bq_i^2 \qquad (2.8) \end{aligned}$$

Cost Reduction Games

We consider a simple two stage process innovation game in the tradition of d'Aspremont and Jaquemin [9] (1988).

In the first stage firms receive a signal of the second period's demand. Based on that information they have to decide on how much they want to invest in R & D in order to decrease their marginal cost of producing in the second period.

Then in the second period the demand function becomes known, and the firm's profit is realized as the outcome of a standard competition model, given the marginal costs determined in the first period. Usually quantity competition with homogenous products is used as endstage of the game, but there are no principal difficulties in using differentiated goods and price competition. We will obtain results for quantity as well as price competition case. As the calculations become quite involved however we will demonstrate our approach in the text only for case of homogenous products and quantity competition. The proofs for price competition and differentiated products can be found in the appendix.

The research and production technology:

Firms produce initially with a constant marginal cost of c_0. Investing $\frac{1}{2}\gamma y_i^2$ in process innovation they are able to reduce this cost by y_i. The convex function reflects the diminishing returns of the R & D technology. To insure the existence of an interior solution we have to make the usual assumption:

$$\gamma > \frac{8}{9b}. \tag{2.9}$$

Payoff function:

We want to calculate the subgame perfect equilibria of this game by backwards induction. In the second stage, the demand function is known. Solving the Cournot game with marginal costs of $c_i = c_0 - y_i$ gives us the firms' profits in the last period. Subtracting the first periods R&D expenditures from this profit will give us the net profit of the firm, depending on the choice of R&D intensity and the realization of the random event:

$$
\begin{aligned}
\pi_i^{R\&D}(y_i, y_j, a) &= \frac{1}{9b}(a - c_0 + 2y_i - y_j)^2 - \frac{1}{2}\gamma y_i^2 \\
&= \frac{1}{9b}(a - c_0)^2 + \frac{4}{9b}(a - c_0)\left(y_i - \frac{1}{2}y_j\right) \\
&\quad -\frac{4}{9b}y_i y_j + \left(\frac{4}{9b} - \frac{1}{2}\gamma\right)y_i^2 + \frac{1}{9b}y_j^2
\end{aligned}
\tag{2.10}
$$

3. Equilibrium

Instead of solving these three problems separately we will find a solution of the following generalized form:

$$
\pi_i(x_i, x_j, a) = \mathcal{F}(a) + \mathcal{G}(a)(x_i + Ax_j) + Bx_j + Cx_i x_j - Dx_i^2 - Ex_j^2 \tag{3.1}
$$

$\mathcal{F}$ may be an arbitrary function but $\mathcal{G}$ has to be of the form $\mathcal{G}(a) = H + K \cdot a$, so that $\mathbf{E}[\mathcal{G}(a)] = \mathcal{G}(\mathbf{E}[a])$ and the calculation rules 2.1 and 2.2 still hold for $\mathcal{G}(a)$. A, B, C, D, E, H, and K are constants. In order to get a unique interior solution we have to assume that $D > 0$, and $C \neq 2D$.

The payoff function of the Bertrand game can be obtained by setting:

$$\mathcal{F}(a) = \frac{-da}{b+e}, \quad \mathcal{G}(a) = \frac{a}{b+e} + d\beta \quad A = 0, \quad B = -c_0\delta,$$
$$C = \delta, \qquad D = \beta, \qquad E = 0,$$

In the Cournot case we have:

$$\mathcal{F}(a) = 0 \quad \mathcal{G}(a) = a - c_0 \quad A = 0, \quad B = 0$$
$$C = -d, \quad D = b \qquad E = 0$$

If we set

$$\mathcal{F}(a) = \frac{1}{9b}(a - c_0)^2, \quad \mathcal{G}(a) = \frac{4}{9b}(a - c_0), \quad A = -\frac{1}{2}, \quad B = 0,$$
$$C = -\frac{4}{9b}, \qquad D = \frac{1}{2}\gamma - \frac{4}{9b}, \qquad E = -\frac{1}{9b},$$

we obtain the cost reduction game.

Solution of the general form: The signal is received before the action is taken, so the players' strategies conditioned on obtaining the signal $\widetilde{a}$ are chosen to maximize the expected payoff

$$\begin{aligned} & \max_{x_i} \mathbf{E}\left(\pi_i\left(x_i, x_j, a\right) | \widetilde{a}\right) \\ = & \max_{x_i} \int \left(\mathcal{F}(a) + \mathcal{G}(a)\left(x_i + Ax_j\right) + Bx_j + Cx_i x_j - Dx_i^2 - Ex_j^2\right) dF\left(a | \widetilde{a}\right). \end{aligned}$$

We can derive under the integral and obtain for the reaction curve of player i:

$$\mathcal{G}(\widetilde{a}) + Cx_j - 2Dx_i = 0$$

We see that for $C > 0$ the decisions are strategic substitutes, in the other case they are strategic complements.

As we have assumed $C \neq -2D$ we can solve for the equilibrium action: .

$$x_i^* (\widetilde{a}) = \frac{\mathcal{G} (\widetilde{a})}{2D - C} \tag{3.2}$$

Reinserting this into equation 3.1, taking the expected value with respect to a and $\widetilde{a}$ and applying the calculation rules 2.1 and 2.2, we obtain for the ex ante expected profit:

$$\begin{aligned} & \mathbf{E} (\pi_i (x_i (\widetilde{a}), x_j (\widetilde{a}), a)) \\ = \; & \mathbf{E} \left[\mathcal{F} (a) + \mathcal{G} (a) \frac{\mathcal{G} (\widetilde{a})}{2D - C} (1 + A) + (C - D - E) \left(\frac{\mathcal{G} (\widetilde{a})}{2D - C} \right)^2 \right] \\ = \; & \mathbf{E} [\mathcal{F} (a)] + \left(\frac{(2A + 1) D - E - AC}{(2D - C)^2} \right) \left(\mathbf{E} \left[\mathcal{G} (a)^2 \right] - K^2 \cdot Var [a - \widetilde{a}] \right) \end{aligned}$$

If the precision of the information rises, $Var [a - \widetilde{a}]$ decreases. This increases the expected payoff iff the factor of the right hand term is positive.

Hence we have proved the following

Proposition 3.1. *The players' expected payoff increases with better information iff:*

$$(2A + 1) D - E - AC > 0 \tag{3.3}$$

Otherwise increased precision of information has a negative effect on the payoff.

In the most cases it is very easy to evaluate expression 3.3 for concrete games. For the Bertrand game, as $A = 0$, $D = \beta$ and $C = \delta$, the inequality 3.3 reduces to $\beta > \delta$. This holds always, as with differentiated goods, demand reacts less to the competitors' price than to the own price.

For the Cournot game as $A = E = 0$, the inequality 3.3 reduces to $b > 0$. So in both cases, more information has a beneficial effect on the firms. We can summarize these results in the following

Corollary 3.2. *The firms' expected profit increases with the precision of information in the Bertrand and Cournot games,*

These results are in line with intuition. More interesting is the analysis of the R&D game. The Cournot case with homogenous products is straightforward. As we have $(2A + 1) = 0$, the inequality 3.3 reduces to :

$$-E - AC = -\frac{1}{9b} > 0, \tag{3.4}$$

which can never be true. In the Appendix B.2 and C.2 we show that the same holds for differentiated products and Bertrand competition in the second stage. Thus we know that:

Corollary 3.3. *In the case of R&D competition followed by Cournot or Bertrand competition the firms profit decreases with better information if the products are not too independent.*

This result may seem counter intuitive. Firms would prefer to stay both uninformed rather than to know more precisely what market they are facing. But in fact it is not really surprising that with more information, competition becomes stronger and the firms' profits decline. Seen from that point of view, it is rather astonishing that this does not happen either in the Cournot or in the Bertrand case.

It is clear that for very weak substitutes the value of information has to become positive for the Cournot as well as for the Bertrand cases. When d approaches zero we will be in the monopolistic case. The game will turn into a decision problem, where more information always has positive effects on the players' profit.

Our formula gives us also the results for negative d, that is for complementary goods. As this case is probably not very relevant for policy implications we will just summarize the results: Information is always positive in Cournot competition. In the case of Bertrand competition the results are ambiguous. Information can be negative for very complementary goods e.g. for d close to $-b$.

It is not easy to get an intuition for why more information may affect payoffs negatively. One could suppose that this has something to do with the reactions of the actors being strategic complements or substitutes, but as the examples of Bertrand and Cournot competition show the value of public information is independent of this property. The mathematical reason is that the players' payoffs become more concave in a, when the players react to information. So even if the ability to react may increase the players' payoff for some signals, ex ante, better signals lower the expected outcome. The economic explanations of this effect, however, may be very different.

Some especially simple cases may help to develop an intuition.

A decision problem with convex externalities:

If in 3.1 we set $\mathcal{F}(a) = 0$, $\mathcal{G}(a) = a$ and $A = B = C = 0$, the payoff function has the form

$$\pi_i(x_i, x_j, a) = ax_i - Dx_i^2 - Ex_j^2$$

and inequality 3.3 becomes $E < C$. In this game the players' reaction curves are constant, that is we now have a decision problem with convex externalities of one player's decision on the other's payoff. Obviously more information for one player increases his reactions and decreases the expected value of the other player's payoff. If $E > C$ this effect is bigger than the expected gain from being able to better predict the outcome.

A team problem:

Setting $\mathcal{F}(a) = 0$, $\mathcal{G}(a) = a$, $A = 1$ and $B = E = 0$ we obtain

$$\pi_i(x_i, x_j, a) = a(x_i + x_j) + Cx_i x_j - Dx_i^2.$$

This is can be interpreted as the payoff function of a team problem. Two agents privately invest the effort x in the random production function $P(x_i, x_j) = 2a(x_i + x_j) + 2Cx_i x_j$ and share the profits equally. The term $-Dx_i^2$ represents the increasing marginal disutility of effort. Equation 3.3 now has the form $3D > C$. So if the actions are strongly complementary, that is when C is big enough, less information may increase the payoffs. In fact here the lack of information helps the players to attain Pareto superior outcomes, on which they would not have been able to coordinate under full information, because they are not Nash equilibria.

4. Welfare

The welfare implications of public information on Bertrand and Cournot games have been derived by Vives (1984), assuming a random signal with normal distribution. These results are still valid in our framework:

Proposition 4.1. *Better information increases welfare in the case of Cournot, but decreases the welfare in Bertrand competition.*

Proof. As we have assumed the marginal utility of the numeraire good for the consumer to be constant, we can calculate the social welfare by subtracting the production costs from the consumers surplus.

Bertrand Competition:

Inserting the Bertrand parameters in 3.2 we obtain the price, the firms choose after having received the signal:

$$p_i = \frac{\widetilde{\alpha} + c_0\beta}{(2\beta - \gamma)}$$

Plugging this in the demand function 2.5 we obtain quantities sold. They depend on the signal $\widetilde{\alpha} = \frac{\widetilde{a}}{b+d}$as well as on the actual outcome a:

$$\begin{aligned} q_i &= \alpha + (\gamma - \beta)\frac{\widetilde{\alpha} + c_0\beta}{(2\beta - \gamma)} \\ &= \frac{a}{b+d} - \widetilde{a}\frac{b-d}{b+d} - \frac{c_0 b}{(b+d)(2b-d)} \end{aligned}$$

We can now calculate the expected social welfare as the sum of the consumers surplus and the firms' profits. As the monetary transfer q_0 between consumers and firms cancels out, it suffices to insert the quantities in the net utility function 2.3 and subtract the production costs $2c_0q$. We are not interested in the absolute value of utility, but only want to know how it changes with the precision of the signal. As only the terms with a factor $a\widetilde{\alpha}$ or $\widetilde{\alpha}^2$ depend on the precision of the signal we can summarize the other terms in a constant $\mathcal{K}$.

$$
\begin{aligned}
& \mathbf{E}\left[U\left(q_0, q_1, q_2\right) + q_0 - 2c_0 q\right] \\
= \; & \mathbf{E}\left[2\left(a - c_0\right) q - \frac{1}{2}(2b + 2d)q^2\right] \\
= \; & \mathbf{E}\left[\frac{1}{b+d}\left(-2a\widetilde{a}\left(b-d\right) + 2a\widetilde{a}\left(b-d\right) - \left(\widetilde{a}\left(b-d\right)\right)^2\right)\right] + \mathcal{K} \\
= \; & -\frac{\left(b-d\right)^2}{b+d}\mathbf{E}\left[\widetilde{a}^2\right] + \mathcal{K} \\
= \; & \frac{\left(b-d\right)^2}{b+d} Var\left[a - \widetilde{a}\right] + \mathcal{K}'
\end{aligned}
$$

The factor of $Var\left[a - \widetilde{a}\right]$ is positive. So increasing the variance of the signal increases social welfare, hence lower variance i.e. better information decreases welfare.

Cournot Competition

This is somewhat easier to analyze, as here we can plug in the equilibrium quantities

$$q_i = \frac{\widetilde{a} - c_0}{2b + d}$$

directly in the utility function and obtain:

$$\mathbf{E}\left[U\left(q_1, q_2, q_0\right) + q_0 - 2c_0 q\right] = \mathbf{E}\left[2\left(a - c_0\right)\left(\frac{\widetilde{a} - c_0}{2b + d}\right) - \left(b + d\right)\left(\frac{\widetilde{a} - c_0}{2b + d}\right)^2\right]$$

$$= -\frac{3b+d}{(2b+d)^2} Var\left[a-\widetilde{a}\right] + \mathcal{K}'$$

The factor is negative this time, so welfare increases with decreasing $Var\left[a-\widetilde{a}\right]$. ■

In fact the welfare effects of public information in the Bertrand and Cournot competition are not really caused by the strategic interaction of the players. Setting $d = 0$ shows that information would have the same welfare effects in a price or quantity setting monopoly. With increasing substitutability i.e. for d approaching b from below, information becomes more positive. In the Bertrand case for d close to b the negative impact of information disappears. This is what we would have expected. For Bertrand competition with homogenous products and identical firms, the prices equal the marginal costs and are therefore independent of the demand. Thus information about demand will not have an influence on the market outcome.

The next proposition summarizes our welfare results in the cost reduction games.

Proposition 4.2. *Better information increases welfare in a cost reduction game followed by Cournot or Bertrand competition.*

Proof.

We will again only go through the case of quantity competition with homogenous products and refer the reader to the Appendix B.3 and C.3 for the other cases.

The equilibrium action, i.e. the level of cost reduction chosen by the firms, is:

$$y = \frac{4\left(\tilde{a} - c_0\right)}{9\gamma b - 4}$$

With marginal costs of $c_0 - y$, the Cournot quantities produced are

$$q_i = \frac{a - c_0 + y}{3b}$$

As we want to evaluate social welfare, we also have to subtract the cost reduction investment of both firms γy^2 from the generated surplus:

$$\begin{aligned}
& \mathbf{E}\left[U\left(q_0, q_1, q_2\right) + q_0 - \left(c_0 - y\right)\left(q_1 + q_2\right) - \gamma y^2\right] \\
= \; & \mathbf{E}\left[\frac{4}{9b}\left(a - c_0 + y\right)^2 - \gamma y^2\right] \\
= \; & \mathbf{E}\left[\frac{4}{9b}\left(a - c_0 + \frac{4\left(\tilde{a} - c_0\right)}{9\gamma b - 4}\right)^2 - \gamma\left(\frac{4\left(\tilde{a} - c_0\right)}{9\gamma b - 4}\right)^2\right] \\
= \; & \left[\frac{9b\gamma - 4}{9b\left(9\gamma b - 4\right)^2}\right] 16 \cdot \mathbf{E}\left[\tilde{a}^2\right] + \mathfrak{K}
\end{aligned}$$

From assumption 2.9 we know that $\gamma > 8/9b$, hence the factor is always positive. ■

It is easy to give an economic intuition for this result: Better knowledge of the future leads to a more appropriate investment in cost reduction of both firms, which in turn increases the intensity of competition. Note however that the same kind of intuition did not apply to the simple Cournot and Bertrand games.

5. Concluding remarks

Whereas better information always increases the profit of firms playing a standard competition game we have shown that in a cost reduction game better information is detrimental to firms' profits. However welfare is still increased by better information. Remarkably the results do not depend on any distributional assumptions nor on the specification of the competition model chosen in the second stage of the cost reduction game. Thus we can claim our policy recommendations to be quite general. Of course the notion of short and long term still depends very much on the specific situation one is looking at. In some industries cost reduction may essentially consist in buying better machinery and therefore be achieved very fast, whereas for example in the chemical industry there is definitely a huge time lag between the decision to innovate a production process and the beginning of the production.

We have concentrated our attention in this paper on the incentives of firms to commonly acquire information and not considered the possibility of private information acquisition. Our argument for the public provision of long term information relies on the assumption that the acquisition of private information is insufficient or inefficient. The complete analysis of the firms' incentives to acquire private information is beyond the scope of this paper. However, we know from Ponssard (1976) that they are not always positive. The fact that a competitor knows that a firm has private information may lead to a lower profit for this firm. This suggests that the incentives may sometimes be insufficient.

But there is also a simpler way to complete the argument justifying the public provision of information. It is not unreasonable to assume that the market research technologies of the firms produce highly correlated signals. Then parallel acquisition of private information is definitely inefficient. As we have shown the firms cannot be expected to form research cooperations, so only the state can efficiently provide information

In this paper, we have restricted ourselves to the analysis of information with respect to demand uncertainty. The generalization to other random variables that commonly affect the players is straightforward. Using inequality 3.3, it is very easy to see that information about common cost shocks is positive for firms engaged in Bertrand or Cournot competition. A welfare analysis shows that, analogous to the demand case, information about cost is detrimental for social welfare in Bertrand competition. This might be an explanation why central banks are so secretive about the refinancing interest rate that they are going to offer to banks.

Of course our method can also be used to test the impact of public information in other symmetric games with quadratic payoff. It may reassure the reader that in most of the cases we have analyzed information seems to have a positive effect.

Appendix

A. Proof of equations 2.1 and 2.2

Let $f(a,\widetilde{a})$ be the density function of the joint probability distribution of a and $\widetilde{a}$. The marginal distribution $\widetilde{f}(\widetilde{a})$ of a can then be calculated as $\widetilde{f}(\widetilde{a}) = \int f(a,\widetilde{a})\,da$ and the density of a conditional on $\widetilde{a}$ is $f(a|\widetilde{a}) = f(a,\widetilde{a})/\widetilde{f}(\widetilde{a})$. The assumption $\mathbf{E}(a|\widetilde{a}) = \widetilde{a}$ translates into $\int af(a|\widetilde{a})\,da = \int af(a,\widetilde{a})\,da/\widetilde{f}(\widetilde{a}) = \widetilde{a}$.

In order to show equation 2.1 we write:

$$\begin{aligned}
\mathbf{E}[a*\widetilde{a}] &= \iint a\widetilde{a}f(a,\widetilde{a})\,da\,d\widetilde{a} \\
&= \int \widetilde{a}\left(\int af(a,\widetilde{a})\,da\right)d\widetilde{a} \\
&= \int \widetilde{a}^2\widetilde{f}(\widetilde{a})\,d\widetilde{a} \\
&= \mathbf{E}\left[\widetilde{a}^2\right]
\end{aligned}$$

Equation 2.2 can the be proved as follows:

$$\begin{aligned}
Var[(a-\widetilde{a})] &= \mathbf{E}\left[(a-\widetilde{a})^2\right] - [\mathbf{E}(a-\widetilde{a})]^2 \\
&= \mathbf{E}\left[a^2 - 2a\widetilde{a} + \widetilde{a}^2\right] \\
&= \mathbf{E}\left[a^2\right] - 2\mathbf{E}[a*\widetilde{a}] + \mathbf{E}\left[\widetilde{a}^2\right] \\
&= \mathbf{E}\left[a^2\right] - \mathbf{E}\left[\widetilde{a}^2\right] \ \blacksquare
\end{aligned}$$

B. Cost reduction followed by quantity competition with differentiated products

B.1. Payoff Function

Solving the Cournot game with differentiated products and with marginal costs of $c_i = c_0 - y_i$ and subtracting the first periods R&D expenditures gives us the net profit of the firm, depending on the choice of R&D intensity and the realization of the random event:

$$
\begin{aligned}
\pi_i^{R\&D+C}(y_i, y_j, a) &= b\left(\frac{(2b-d)(a-c_0)+2by_i-dy_j}{4b^2-d^2}\right)^2 - \frac{1}{2}\gamma y_i^2 \\
&= b\left(\frac{(2b-d)(a-c_0)}{4b^2-d^2}\right)^2 + 4b^2\frac{(2b-d)(a-c_0)}{(4b^2-d^2)^2}\left(y_i - \frac{d}{2b}y_j\right) \\
&\quad - \frac{4b^2 d}{(4b^2-d^2)^2}y_i y_j + \left(\frac{4b^3}{(4b^2-d^2)^2} - \frac{1}{2}\gamma\right)y_i^2 + \frac{bd^2}{(4b^2-d^2)^2}y_j^2
\end{aligned}
$$

This payoff function can be generated by inserting the following list of parameters in the general form:

$$
\begin{aligned}
&\mathcal{F}(a) = b\left(\tfrac{(2b-d)(a-c_0)}{4b^2-d^2}\right)^2, && \mathcal{G}(a) = 4b^2\tfrac{(2b-d)(a-c_0)}{(4b^2-d^2)^2}, \\
&A = -\tfrac{d}{2b}, && B = 0, \\
&C = -\tfrac{4b^2 d}{(4b^2-d^2)^2}, && D = \tfrac{1}{2}\gamma - \tfrac{4b^3}{(4b^2-d^2)^2}, \\
&E = -\tfrac{bd^2}{(4b^2-d^2)^2}, &&
\end{aligned}
$$

In order to insures the existence of an interior solution we have to assume $D > 0$, i.e. the condition 2.9 now has the form:

$$\gamma > \frac{8b^3}{\left(4b^2 - d^2\right)^2}, \tag{B.1}$$

B.2. Firms profits

In order to analyze the effect of information on the firms profits, we have to evaluate the inequality 3.3. With the above parameters it be written in the form:

$$\left(1 - \frac{d}{b}\right)\left(\frac{1}{2}\gamma\left(4b^2 - d^2\right)^2 - 4b^3\right) - bd^2 > 0 \tag{B.2}$$

For homogenous products, i.e. $d = b$ this reduces to $-b^3 > 0$, and we get the result of corollary 3.3. However when d decreases, this result will be inverted, as with assumption B.1 the second factor $\frac{1}{2}\gamma\left(4b^2 - d^2\right)^2 - 4b^3$ of equation B.2 is positive.

B.3. Welfare

The equilibrium cost reduction has the form:

$$y = \frac{4b^2\left(2b - d\right)\left(\widetilde{a} - c_0\right)}{\gamma\left(4b^2 - d^2\right)^2 - 4b^2\left(2b - d\right)} \tag{B.3}$$

The Cournot quantities produced with marginal costs of $c_0 - y$ are :

$$q_i = \frac{a - c_0 + y}{2b + d}$$

Hence we obtain for the expected social welfare:

$$
\begin{aligned}
& \mathbf{E}\left[U\left(q_0, q_1, q_2\right)+q_0-\left(c_0-y\right)\left(q_1+q_2\right)-\gamma y^2\right] \\
= & \mathbf{E}\left[\frac{(3b+d)}{(2b+d)^2}\left(2ay+y^2\right)-\gamma y^2\right]+\mathcal{K} \\
= & \frac{(3b+d)}{(2b+d)^2}\left(\frac{\left(\gamma\left(4b^2-d^2\right)^2-2b^2(2b-d)\right) 8b^2(2b-d)}{\left(\gamma\left(4b^2-d^2\right)^2-4b^2(2b-d)\right)^2}\right) \mathbf{E}\left[\widetilde{a}^2\right] \\
& -\frac{\gamma\left(4b^2(2b-d)\right)^2}{\left(\gamma\left(4b^2-d^2\right)^2-4b^2(2b-d)\right)^2} \mathbf{E}\left[\widetilde{a}^2\right]+\mathcal{K}'
\end{aligned}
$$

Welfare is increasing with better information iff the factor of $\mathbf{E}\left[\widetilde{a}^2\right]$ is positive. This is the case iff

$$
\gamma > \frac{2b^2(3b+d)}{\left(4b^2-bd-d^2\right)(2b+d)^2} \tag{B.4}
$$

For $d=b$ we get the same inequality as in paragraph 4. For $d=0$, with B.1 $\gamma > \frac{8b^3}{\left(4b^2-d^2\right)^2}=\frac{8}{16b}$ we obtain

$$
16b^4\gamma-6b^3>2b^3>0.
$$

So more information increases the social welfare even in monopolistic situations.

When considering general $d \in [-b, b]$, we have to take into account that for complementary goods, i.e. for $d<0$, the equilibrium cost reduction may not be negative. From equation B.3 we obtain the additional regularity condition

$$
\gamma > \frac{4b^2(2b-d)}{\left(4b^2-d^2\right)^2} \tag{B.5}
$$

For $d<0$ this condition is stronger than 2.9 and we have to make this additional assumption. For substitute goods 2.9 implies B.5. Inserting γ from B.5 in B.4 we obtain

$$4b^2 - bd - d^2 > 0$$

which always holds. So more information always increases welfare for all $d \in [-b, b]$.

C. Cost reduction followed by Bertrand competition

C.1. Payoff function

Solving the Bertrand game with differentiated products and marginal costs of $c_i = c_0 - y_i$ and subtracting the first periods R&D expenditures yields a payoff function of the form

$$\pi_i^{R\&D+B}(y_i, y_j, a) = b\frac{\left[a(b-d)(2b+d) - (2b^2-d^2)(c_0-y_i) + bd(c_0-y_j)\right]^2}{(b^2-d^2)(4b^2-d^2)^2} - \frac{1}{2}\gamma y_i^2$$

The parameters used in the inequality 3.3 are

$$\begin{aligned} A &= -\tfrac{bd}{2b^2-d^2}, & C &= -\tfrac{2b^2d(2b^2-d^2)}{(b^2-d^2)(4b^2-d^2)^2}, \\ D &= \tfrac{1}{2}\gamma - \tfrac{b(2b^2-d^2)^2}{(b^2-d^2)(4b^2-d^2)^2}, & E &= -\tfrac{b^3d^2}{(b^2-d^2)(4b^2-d^2)^2}, \end{aligned}$$

The existence of an interior solution has to be insured by the assumption

$$\gamma > \frac{2b(2b^2-d^2)^2}{(b^2-d^2)(4b^2-d^2)^2} \tag{C.1}$$

C.2. Firms profit

The inequality 3.3 now writes

$$\left(1-\frac{2bd}{2b^2-d^2}\right)\left(\gamma-\frac{2b\left(2b^2-d^2\right)^2}{\left(b^2-d^2\right)\left(4b^2-d^2\right)^2}\right)>0$$

With the regularity assumption C.1 we obtain

$$2b^2-d^2>2bd$$

For small or negative d the inequality obviously holds. So information is certainly positive if the firms produce nearly independent or complementary products. For $\sqrt{3}-1<\frac{b}{d}<1$ however, this is no longer true. In this case information is harmful for the firms.

C.3. Welfare

With known a and cost $c_i=c_0-y_i$ the equilibrium prices in the last stage are:

$$p_i=\frac{\left(b-d\right)\left(2b+d\right)a+2b^2c_i+bdc_j}{4b^2-d^2} \tag{C.2}$$

The net profits can easily represented as a function of the equilibrium prices:

$$\pi_i=\frac{b}{\left(b^2-d^2\right)}\left(p_i-c_i\right)^2-\frac{1}{2}\gamma y_i^2 \tag{C.3}$$

From equation C.2 we can see that $\frac{d(p_i-c_i)}{dy_i}=\frac{2b^2-d^2}{(b^2-d^2)}$. So deriving equation C.3

with respect to y_i gives us the first order condition:

$$\frac{d}{dy_i}\mathbf{E}\left[\pi_i|\widetilde{a}\right] = \frac{2b}{(b^2-d^2)}\left(\mathbf{E}\left[p_i|\widetilde{a}\right] - c_i\right)\frac{2b^2-d^2}{(b^2-d^2)} - \gamma y_i = 0$$

We know that the solutions are symmetric, so we can simplify equation C.2 with $c_i = c_j = c$ to

$$\mathbf{E}\left[p_i|\widetilde{a}\right] - c = \frac{(b-d)\left(\widetilde{a}-c\right)}{2b-d}$$

and obtain from equation C.3

$$\frac{2b\left(2b^2-d^2\right)}{(2b-d)\,(b+d)\left(4b^2-d^2\right)}\left(\widetilde{a}-c_0+y\right) - \gamma y = 0 \qquad \text{(C.4)}$$

If we set $S := \frac{2b\left(2b^2-d^2\right)}{(2b-d)(b+d)(4b^2-d^2)} > 0$, the equilibrium cost reduction, depending on the signal can be written as:

$$y^* = \frac{S}{\gamma - S}\left(\widetilde{a}-c_0\right)$$

It is easy to see that with assumption C.1 we have $\gamma > S$, so we only obtain positive equilibrium cost reductions.

With symmetry the corresponding quantities are easily calculated :

$$q = \frac{b}{(2b-d)\,(b+d)}\left(a - c_0 + \frac{S}{\gamma - S}\left(\widetilde{a}-c_0\right)\right) \qquad \text{(C.5)}$$

We can now plug the quantities and marginal costs in the utility function and apply our calculation rules:

$$
\begin{aligned}
&\mathbf{E}\left[2\left(a-c_0+y^*\right)q-(b+d)\,q^2-\gamma y^{*2}\right] \\
=\;& \widetilde{a}^2\left[\frac{2bS\,(2\gamma-S)\,(2b-d)-b^2S\,(2\gamma-S)}{(-\gamma+S)^2\,(2b-d)^2\,(b+d)}-\frac{\gamma S^2}{(\gamma-S)^2}\right]+\mathcal{K}' \\
=\;& \widetilde{a}^2\left[\frac{b}{(2b-d)\,(b+d)}\frac{(3b-2d)}{(2b-d)}\frac{S\,(2\gamma-S)}{(\gamma-S)^2}-\frac{\gamma S^2}{(\gamma-S)^2}\right]+\mathcal{K}'
\end{aligned}
$$

As $\frac{(3b-2d)}{(2b-d)}>1$ and $\frac{b}{(2b-d)(b+d)}>\frac{2b\left(2b^2-d^2\right)}{(2b-d)(b+d)(4b^2-d^2)}=S$ we can estimate the term in the squared brackets as

$$
\frac{b}{(2b-d)\,(b+d)}\frac{(3b-2d)}{(2b-d)}\frac{S\,(2\gamma-S)}{(\gamma-S)^2}-\frac{\gamma S^2}{(\gamma-S)^2}>\frac{S^2\,(\gamma-S)}{(\gamma-S)^2}>0
$$

So information is positive for welfare.

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