Composition of Electricity Generation Portfolios, Pivotal Dynamics and Market Prices*

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Abstract

We use a simulation model to study how the diversification of electricity generation portfolios influences wholesale prices. We find that technological diversification generally leads to lower market prices but that the relationship is mediated by the supply to demand ratio. In each demand case there is a threshold where pivotal dynamics change. Pivotal dynamics pre- and post-threshold are the cause of non-linearities in the influence of diversification on market prices. The findings are robust to our choice of behavioural parameters and match close-form solutions where those are available.

Keywords: Electricity, market power, simulations, technology diversification.

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1 Introduction

Electricity is a non-storable, undifferentiated commodity, delivered into a market with low demand elasticity, high security of supply requirements and wide seasonal variations. As a result, the industry accommodates a wide range of generating technologies, and firms own not one but several plants. Some generators are technologically diversified and own nuclear plants on the base-load as well as high-cost thermal units. For example PG & E, a large US utility, owns hydro, nuclear, thermal and renewable plants (PG & E Corporation, 2006). Others are specialists, focusing on only one technology. Until recently, British Energy's generation portfolio was formed exclusively by eight nuclear generating units (British Energy, 2006).[1]

The effect of technological diversification on prices is an important question in industries with undifferentiated output but different production technologies. A market in which generators are specialised could exhibit more market power because the price-setting part of the merit order is more concentrated. However, in electricity pools, specialised high-cost generators have less incentives to exert market power because they lack base-load plants to reap the benefits. In contrast, diversified firms have incentives to use their high-cost plants to increase market prices and thereby increase the profit on the base-load, but may not have enough price-setting capacity to do so.

In this paper, we address the general questions of "what is the shape of the diversification to prices relationship?" and "what are its determinants?". Specifically, we study different markets where a generation duopoly own varying amounts of base- and peak-load capacity that is bundled into a high- and a low- cost plant. In order to isolate the portfolio effects, we keep market concentration constant, as well as the market base-load and high-cost capacities. Our trading environment is a multi-unit, compulsory, uniform-price auction. This set-up, however, is often characterised by the presence of a manifold of non-Pareto ranked Nash equilibria (von der Fehr and Harbord, 1993). To achieve predictions, we use an inductive selection method based on the adaptive theory of reinforcement learning put forward by Roth and Erev (1995).[2]

Our main policy findings are that more diversification often leads to lower market prices. This relationship, however, is non-monotonic and mediated by the market excess capacity. For each demand to supply ratio, we identify a diversification breaking point, estimated from the simulations, where dynamics change. Up to the breaking point, more intense competition due to

[1] British Energy has recently started to operate one coal-fired plant.

[2] Papers which also use the Roth and Erev method in the electricity context include Nicolaisen et al. (2001), Rupérez Micola and Bunn (forthcoming) and Rupérez Micola et al. (forthcoming).

higher diversification always leads to lower prices. In low-demand situations, prices drop further after the breaking point. In high-demand cases, instead, further diversification leads to higher prices, but prices remain lower or equal to those under perfect specialisation.

We show that the non-monotonic diversification/prices relationship is caused by regime changes in the firms' incentives and market power. Interestingly, the estimated breaking points are shown to (statistically) match the theoretically predicted thresholds at which the number of "pivotal" plants change.[3] In our setup, there is always one and only one pivotal plant under little or no diversification. After the threshold, the number of pivotal plants changes. In low demand situations, the market moves from one to no pivotal plants, which results in further competitive pressures. In high demand cases, there are two peak-load pivotal plants post-threshold, which leads to some implicit coordination and higher prices.

In spite of its importance, the literature on generation portfolios as a source of market power is relatively sparse. Arellano and Serra (2005) show how, in cases where a regulator uses peak-load marginal costs to determine wholesale prices, generators can exercise market power by increasing the share of peak technology in their portfolio. They conclude that market power in this context should not be measured by the traditional price-cost margin or concentration measures, but by the distortion in the composition of the generating portfolio due to regulatory incentives. Bushnell (2003) uses a Cournot model to analyse competition among several firms when each possesses some hydroelectric and thermal generation resources. He concludes that firms may find it profitable to allocate more hydro production to off-peak periods than they would under perfect competition, or if they did not act strategically. Garcia et al. (2005) analyse the price-formation process in an infinite-horizon model where hydroelectric generators engage in dynamic price-based competition and show how simulations with a basic learning algorithm converge to the Markov Perfect Equilibrium. Bunn and Oliveira (2007) also use a simulation to model the interaction between an electricity market and a plant swapping game. They identify a symbiotic interaction between the two markets: initial situations where firms are perfectly diversified evolve, via plant trading, into lower electricity prices than those in which firms were originally specialised.

The remainder of the paper proceeds as follows. In Part 2, we describe the model and simulation procedure. The results are presented in Part 3 and we conclude with a short discussion in Part 4.

[3] A plant is pivotal if the quantity demanded exceeds the sum of production capacities of all other plants and, as a result, the plant is necessary to fulfill demand.

2 Model and Simulation Procedure

2.1 Market structure

Our model incorporates key features of electricity markets in the short run. Two companies that compete for the supply of the market own a mix of low (e.g. nuclear) and high (e.g. thermal) marginal cost capacity.[4] Denoting the generating companies as 1 and 2 and the overall market capacity as K, the capacities of their respective low (l) and high (h) cost plants are

$$k_1^l = k_2^h = (1-\alpha)K/2 \text{ and } k_1^h = k_2^l = \alpha K/2,$$

where $\alpha \in]0, 0.5[$ represents the degree of portfolio diversification. In the case of specialisation ($\alpha = 0$), company 1 is a low-cost specialist and company 2 is specialised in the high-cost technology. Portfolio diversification grows with α, a growing proportion of the base-load generator's capacity is exogeneously replaced with high-cost units. Symmetrically, the generator's high-cost capacity is replaced with base-load. In the case of full diversification ($\alpha = 0.5$), each company holds the same amount of low- and high-cost generating capacity. This formulation isolates the effects of portfolio diversification because, while allowing for different degrees of diversification, the total capacity of each company is kept constant,

$$k_i^l + k_i^h = K/2 \text{ for } i = 1, 2,$$

as are the market aggregates of low- and high-cost capacities,

$$k_1^j + k_2^j = K/2 \text{ for } j = l, h.$$

Marginal costs are assumed to be constant; normalised to 0 for the low-cost plants and equal to c for the high-cost plants, and there are no grid constraints.[5] Although relevant in the long term, we do not deal with entry and exit of firms, capacity expansion, the use of long-term contracts (as in e.g. Baldick et al., 2006), ancillary and capacity payments.

[4] The model could be easily extended to other more realistic market configurations, including all sorts of oligopolies and the existence of a competitive fringe. However, our analysis in a stylised market is more transparent and comparable to previous literature.

[5] The addition of network constraints would undoubtedly make the analysis richer but it would also make it more complicated to dissentangle effects due exclusively to technology diversification from those arising from local market power exerted by relatively small players.

2.2 Market rules

Trading takes place through a multi-unit, compulsory, uniform-price auction. Suppliers submit simultaneous single-price bids at which they are willing to sell up to the capacity of each plant. Each firm, thus, submits a piecewise "step" supply function.[6] Possible bids are bounded between marginal costs and Ψ, with Ψ being the maximum "reasonable" price.[7]

We model the market demand Q as fully inelastic,[8] drawn from a uniform distribution in the interval $[\bar{Q} - \varepsilon, \bar{Q} + \varepsilon]$, where $\bar{Q}$ is its expected value and ε accounts for the small uncertainty typical in day-ahead forecasting.[9] We assume that there is always some system overcapacity, $\bar{Q} + \varepsilon < K$, but demand always exceeds the market aggregate of low-cost capacity, $\bar{Q} - \varepsilon > K/2$, consistent with the normal operations of many de-regulated energy markets.[10]

An independent auctioneer determines the uniform market price P by intersecting the *ad hoc* supply function with the realised demand. She assigns full capacity, $q_i^j = k_i^j$, to the M plants with bids below the market price; the remaining capacity, $q_i^j = Q - \sum_{\{i,j\} \in M} k_i^j$, to the plant with a bid equal to the market price;[11] and zero sales, $q_i^j = 0$, to those bidding above the market price. Profits for each company are

$$\pi_i = P\ q_i^l\ +\ [P - c]\ q_i^h \qquad \text{for } i = 1, 2. \tag{1}$$

2.3 Multiple equilibria and inductive selection

This trading setting often presents a manifold of non-Pareto ranked Nash equilibria (von der Fehr and Harbord, 1993; Crawford et al., 2006). For example, if there is full diversification, having one generator bidding the maximum price for the high-cost unit, while the other generator bids at marginal cost, is part of an equilibrium if demand is relatively high.

Proposition 1 *Assume that generators are diversified ($\alpha = 0.5$) and the expected demand is high, $\bar{Q} > 3/4$. Then, there is a manifold of non-Pareto ranked Nash equilibria.*

[6] Piecewise supply functions have been introduced, among others, by Hobbs and Pang (2007).

[7] This upper price ceiling can be understood as a limit triggering regulatory intervention or the cost of alternative, expensive, load fuels to which the system administrator could switch at short notice if prices exceed Ψ. It also reflects high cost back-up power generation facilities owned by many industrial users.

[8] The literature has established the extremely low price elasticity of short-term electricity demand, originating, among others, from the lack of real-time metering systems (e.g. Stoft, 2002).

[9] The small uncertainty is introduced for the sake of realism, but its absence would not alter our findings.

[10] For example, the UK energy system includes a reserve margin of about 20% of expected peak demand.

[11] In case of a tie, the selling plant is selected randomly.

Proof. Since the two firms are symmetric, denote the price-setting firm as Firm 1. We are going to show that $(b_1^l, b_2^l, b_1^h, b_2^h) = (0, 0, \Psi, b)$ is a Nash equilibrium for $c \leq b < \widehat{b}$, where $\widehat{b}$ will be defined below. If $b_1^l = b_2^l = 0$, $b_1^h = \Psi$ and $b_2^h < \Psi$ then $P = \Psi$ and $\pi_1 = \Psi\ K/4 + [\Psi - c]$ $(\bar{Q} - 3/4K)$ and $\pi_2 = \Psi\ K/4 + [\Psi - c]\ K/4$. The strategy of Firm 2 is clearly a best response to the strategy of Firm 1 because both quantity and market price are the maximum possible. Setting $b_1^l = \widetilde{b} \geq \overline{b} = b_1^h$ cannot yield higher payoffs for Firm 1 than if $b_1^h = \widetilde{b}$ and $b_1^h = \overline{b}$. Then, by setting $b_1^l = 0$, Firm 1 has no influence in the payoffs. Finally, setting $b_1^h = \Psi$ is a best response as long as

$$\Psi K/4 + \ [\Psi - c]\,(\bar{Q} - 3/4K) > b_2^h K/4 + \ \left[b_2^h - c\right] K/4$$

or

$$b_2^h < \widehat{b} = \frac{\Psi\left(\bar{Q} - K/2\right) + c(K - \bar{Q})}{K/2}.$$

One can easily check that $c < \widehat{b} < \Psi$. ■

Standard comparative statics analyses rely on the Nash specification to determine the solution. Hence, multiple equilibria make it difficult to come up with an answer to our research question, which should be based on a comparative statics exercise with respect to the degree of diversification.

In equilibrium multiplicity cases, a selection method is necessary to choose amongst them. In broad terms, there are two schools of thought in the area of equilibrium selection (Haruvy and Stahl, 2004). On the one hand, we have deductive selection – based on reasoning and coordination in focal points – and, on the other hand, we have inductive selection – based on adaptive dynamics. Until recently, deductive principles have dominated the equilibrium selection literature. Existing deductive mechanisms, however, have been shown to do poorly in experiments (see e.g. Van Huyck et al., 1990). Simple adaptive learning dynamics, instead, often yield good equilibrium predictions (see e.g. Roth and Erev, 1995).

Reinforcement models are widely used adaptive learning mechanisms (see e.g. Nicolaisen et al., 2001, Rupérez Micola and Bunn, 2007, Rupérez Micola et al., 2007, and Veit et al., 2006). They are based on the law of effect, whereby actions that result in positive consequences are more likely to be repeated in the future while those that result in negative consequences are less likely to be replayed. One of the main strengths of reinforcement models is that one does not need to make assumptions on the information that players have about strategies, history of play and payoff structure of the other players. This is especially useful to model very volatile markets such as wholesale electricity.

2.4 Behavioural learning

In order to model learning, we adopt in particular the well-known and practical-to-implement reinforcement learning method put forward by Roth and Erev (1995) –denoted as R-E. The previously described bidding competition is repeated for a finite number of periods. Behavioural learning takes place by repeating the following three steps in each period.

STEP *1: Generators submit price offers for each plant according to a plant-specific probability distribution over the set of possible bids.*

For simplicity, the feasible price offer domain for each plant is approximated by a discrete grid. For each plant, generators choose among S possible prices, equally spaced between the minimum and the maximum price offer. That is, the sets of possible bids for the low- and high-cost plants, B^l and B^h, range from 0 and c,[12] respectively, up to Ψ,

$$B^l = \{s\,(\Psi/S) \mid s = 1, ..., S\}, \quad (2)$$

$$B^h = \{c + s\,(\Psi - c)\,/S \mid s = 1, ..., S\}. \quad (3)$$

Each bid is generated by an "action s". Bids generated from lower actions are more competitive, i.e. closer to marginal costs.

In each round t, each generator i selects an action s for plant j with a likelihood or "propensity" $r^j_{i,s}(t)$. The probability of an action being played is given by its propensity divided by the sum of the propensities of all possible actions,

$$p^j_{i,s}(t) = \frac{r^j_{i,s}(t)}{\sum_{u=1}^{S} r^j_{i,u}(t)}. \quad (4)$$

Propensities for all actions are initialised to the plants' maximum per-period profit, i.e. $r^j_{i,s}(1) = \Psi k^j_i$, so that all actions have the same initial probability, $p^j_{i,s}(1) = \frac{1}{S}$ for all s, i and j.

STEP *2: The auctioneer determines the market price by intersecting the ad hoc supply function with the realised demand.*

As explained above, the auctioneer determines the price and the individual quantities by intersecting the ad hoc supply function with the realised demand, which is assumed to be independently distributed across periods. Subsequently, the price and the individual quantities are communicated independently to each generator.

[12] An alternative is to allow expensive plants to bid below c so that they have to find out for themselves that this is not profitable. This slows the learning process down but does not alter our results. Thus, as most of the electricity simulation literature, we do not allow firms to bid below marginal costs.

STEP *3: Each plant-specific probability distribution is adjusted based on the performance of the bid used.*

At the end of each round, plants reinforce the selected action, a, through an increase in its propensity that is equivalent to the performance of the *company as a whole*, $\pi_i(t)$. Actions that are similar, i.e. $a-1$ and $a+1$, are also reinforced but to a lesser extent, precisely by $(1-\delta)\pi_i(t)$ where $0 < \delta < 1$ ("persistent local experimentation" in the terminology of R-E). All propensities are discounted by γ ("gradual forgetting") and actions whose probability falls below a certain threshold are removed from the space of choice ("extinction in finite time"). The pre-extinction propensities for the following period $r^{j\prime}_{i,s}(t+1)$ are

$$r^{j\prime}_{i,s}(t+1) = \begin{cases} (1-\gamma)\ r^{j}_{i,s}(t) + \pi_i(t) & \text{if } \ s = a \\ (1-\gamma)\ r^{j}_{i,s}(t) + (1-\delta)\ \pi_i(t) & \text{if } \ s = a-1 \text{ or } s = a+1 \\ (1-\gamma)\ r^{j}_{i,s}(t) & \text{if } \ s \neq a-1,\ s \neq a \text{ and } s \neq a+1, \end{cases}$$

and the final propensities, corrected by the extinction feature, are

$$r^{j}_{i,s}(t+1) = r^{j\prime}_{i,s}(t+1) I_{\left\{\frac{r^{j\prime}_{i,s}(t+1)}{\sum_{u=1}^{S} r^{j\prime}_{i,u}(t+1)} > \mu\right\}}, \tag{5}$$

where I is an indicator function that takes value 1 if the condition between brackets is satisfied and zero otherwise.

These three steps are repeated for a finite number of periods T. Although generators refine their strategies through learning, there always remains some degree of uncertainty. We therefore perform many simulation runs and we define convergence in terms of the across-run average standard deviation of prices. We require the finite number of periods T to be such that the initial average standard deviation is reduced below a given threshold. Our simulations produce a large dataset, which is described in the following subsection, and analysed econometrically in the subsequent section.

2.5 Simulation parameters and dataset

The end-user reasonable price ceiling is set at $\Psi = 200$, with a discrete grid of $S = 100$ possible prices. Total capacity is set to $K = 300$, so that each generator's capacity is $K/2 = 150$. Marginal costs for the high-cost plants are $c = 100$ and zero for the low-cost plants.

We perform simulations for a discrete grid of fourteen expected demand cases, $\bar{Q} = \{160, 170..., 290\}$, corresponding to market expected excess capacity of 46.66% through 3.33%, with a small uncertainty ($\varepsilon = 5$). If, for example, $\bar{Q} = 240$ then $Q \sim U[235, 245]$. For each instance, we consider fifty one diversification levels, $\alpha = \{0,\ .01,\ .02,\ ...,\ .50\}$. Further, we check the robustness of the analysis to changes in R-E parameters by taking nine combinations of the learning parameters, $\gamma = \{0.0025, 0.005, 0.0075\}$ and $\delta = \{0.25,\ 0.50,\ 0.75\}$, with $\mu = 0.0005$ throughout.

For each specification, we have performed fifty simulation runs. We consider that convergence is attained if the average standard deviation is reduced by 1/4. This occurs around period 200. We allow each simulation to run for 500 periods.[13] We then build a dataset consisting of average prices for the last 200 periods (301 to 500), for each simulation run, α, $\bar{Q}$, γ and δ.[14] Our dataset includes $50 \times 51 \times 14 \times 3 \times 3 = 321,300$ observations. As representative cases, we focus on the demand cases of $\bar{Q} = \{240, 180 \text{ and } 280\}$, with expected excess capacities of 20%, 40% and 6.66%, respectively. In those examples, we approximate power systems under normal operations, spare and tight capacity conditions.

As a robustness check, we have run additional simulations for $\bar{Q} = 100$ and $\bar{Q} = 300$ (for various specifications of the other parameters), where von der Fehr and Harbord's (1993) price predictions would be unique and equal to 0 and 200, respectively. Simulated prices evolve in the direction of their prediction and the 95% confidence intervals of observed prices include in both cases the predicted prices.[15]

3 Results

3.1 Diversification/prices relationship

Table 1 summarises the relationship between α and stationary market prices, with $\bar{Q}$ as a covariate, and fixed effects for δ and γ. The results show a positive relationship between demand and prices, as expected. The relationship between the diversification parameter, α, and market prices is negative and strongly significative. Diversification leads to lower market prices. The inclusion of fixed effects suggests that, overall, the results are robust to the R-E parameter specifications.[16]

[13] For each specification, the one-lag with trend Augmented Dickey-Fuller test-statistic for the price series is lower than -10. Given that the 95% critical value is -3.43, the null hypothesis of non-stationarity is always rejected.

[14] Increasing substantially the length of the simulation, e.g. to 1,000 periods and using the averages between 801 and 1,000, did not change the nature of our results.

[15] Figures are available upon request from the authors.

[16] Figures with average prices under each R-E specification are available from the authors. Both the positive demand-to-prices and negative diversification-to-prices effects are clear. The figures are also remarkably similar,

Table 1: Parameter estimates diversification, demand and prices

Dependent variable: Market Price		
Effect	**Estimate**	**t-value**
Intercept	113.70	802.29
α	-0.0485	-123.38
$\bar{Q}$	0.2343	347.33
R^2	49 %	

However, the idea that diversification always leads to lower prices is too naive. To see why, Figures 1-3 present the mean price as a function of portfolio diversification for the three representative cases, $\bar{Q} = \{240, 180 \text{ and } 280\}$ (with $\gamma = 0.005$ and $\delta = 0.50$). In all figures when $\alpha = 0$, one firm is exclusively on the base-load and the other on the peak-load. Remember that portfolio diversification grows with α, as a growing proportion of the base-load generator's capacity is replaced with high-cost units, and vice-versa, the generator's high-cost capacity is increasingly replaced with base-load. At the other end, when $\alpha = 0.50$, both firms own one half of each technology. Besides the mean price, we also represent the set of prices within two standard deviations from the mean, corresponding approximately to the 95% confidence intervals.

In Figure 1 ($\bar{Q} = 240$), prices start from a mean of 169.3 and are reduced very slightly as the two firms' portfolios become more balanced. When $\alpha = 0.40$, the price is 168.1, and increases thereafter, to 173.7. Portfolio diversification seems to have a small downward price effect before $\alpha = 0.40$, but a clear upward effect occurs thereafter. Figures 2 ($\bar{Q} = 180$) and 3 ($\bar{Q} = 280$) reinforce the view of demand, or its analogue "excess capacity", mediating on the influence of portfolio diversification. In the spare capacity situation, Figure 2, prices are lower than in the baseline case, with an average specialization price of 157.6. The relationship between prices and diversification is also different, flat until $\alpha = 0.17$, where prices start to decrease markedly. The end of the decrease is at $\alpha = 0.23$, with average price 145.2, which remains stable for further diversification cases. Prices are higher in the tight capacity situation (Figure 3) but they seem to follow a similar pattern to the baseline case. They start at 175.7 and stay flat until around $\alpha = 0.15$, where there is an increase to about 179.2. Beyond $\alpha = 0.15$, prices are flat once again, albeit at the higher level.

which suggest that the qualitative nature of the results is not affected by the choice of R-E parameters, at least within our ranges.

Taken together, Table 1 and Figures 1 to 3 suggest that the shape of the portfolio diversification to market prices relationship is generally decreasing, with two caveats. First, the relationship is not monotonic and second, there is a significant variation in its shape, depending on $\bar{Q}$. Structural breaks occur for $\alpha = 0.40$ for $\bar{Q} = 240$, between $\alpha = 0.17$ and $\alpha = 0.23$ for $\bar{Q} = 180$ and $\alpha = 0.14$ for $\bar{Q} = 280$. We hence obtain three preliminary stylised facts regarding the diversification to prices relationship:

1. Diversification leads to lower prices in general;

2. The market demand influences both absolute price levels and the shape of the diversification to prices relationship;

3. The diversification to prices relationship is not monotonic but seems to present structural stability breaks.

In the remainder of the paper, we further explore those findings. We first show that the diversification/prices relationship presents non-linearities in all demand cases.

3.2 Structural breaking points and market prices

For each demand and R-E combination, we estimate a simple piecewise linear model between the level of diversification and the prices, using dummy variables. The model is uniquely specified by the choice of some threshold value α_v,

$$P_i = \beta_0 + \beta_1 D_i + \beta_2 \alpha_i + \beta_3 D_i \alpha_i + u_i, \tag{6}$$

where $D_i = 0$ if $\alpha_i < \alpha_v$, and $D_i = 1$ when $\alpha_i \geq \alpha_v$. That is, pre- and post- breaking point regression estimates are specified, respectively, by

$$E(P_i | D_i = 0, \alpha_i) = \beta_0 + \beta_2 \alpha_i \text{ and } E(P_i | D_i = 1, \alpha_i) = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)\alpha_i.$$

The simulated breaking point is then defined as the threshold value that generates the best-fit regression.

Definition 2 *A structural breaking point* $\widehat{\alpha_v}$ *satisfies* $F(\widehat{\alpha_v}) \geq F(\alpha_v)$ *for any threshold* α_v*, where* $F(\alpha_v)$ *denotes the F-statistic obtained from a piecewise linear regression with threshold* α_v*.*

Figure 4 provides the estimated relationship between diversification and prices of the best-fit regression, for each demand and R-E combination. Each line corresponds to an R-E parameter combination and each panel corresponds to a different demand specification. If $160 \leq \bar{Q} \leq 220$

(*low* demand) the negative price effects of diversification are strengthened at the breaking point. If demand is *intermediate*, $\bar{Q} = \{230, 240, 250\}$, there are positive price effects at the breaking points, which continue until full diversification. If demand is large, $\bar{Q} = \{260, 270, 280, 290\}$, there is little excess capacity and prices are already approaching the simulation's maximum price. Under those conditions, the breaking point coincides with large price jumps followed by flat diversification/prices relationships.

These results suggest that:

4. The dynamics pre- and post-breaking point are responsible for the nonlinearities in the influence of diversification on market prices;

5. In spare capacity cases (i.e. $\bar{Q} = \{160, ...220\}$) prices drop at the breaking point;

6. When capacity is tight (i.e. $\bar{Q} = \{230, ...290\}$), on the other hand, prices increase at the breaking point.

3.3 Pivotal regime switching point: theory and simulation

In this section, we explore further the non-linearities. We show that the simulated structural breaks coincide with pivotal regime switching points (e.g. Genc and Reynolds, 2005; Entriken and Wan, 2005; Perekhodtsev et al., 2002). We also show how those depend on the industry's excess capacity.

Definition 3 *A high-cost plant is pivotal if the quantity demanded exceeds the sum of production capacities of all other plants. A level of diversification α_t is a switching point if the number of pivotal plants for $\alpha < \alpha_t$ is different than that for $\alpha \geq \alpha_t$.*

For example, when $\bar{Q} = 240$ we have that $\alpha_t = 0.40$ because the number of pivotal plants changes from one to two at this level. If $\alpha < 0.40$ the peak-load plant of Firm 2 is the only pivotal since $k_1^l + k_2^l + k_1^h < 240$ and $k_1^l + k_2^l + k_2^h > 240$.On the other hand if $\alpha > 0.40$ both peak-load plants are pivotal because $k_1^l + k_2^l + k_1^h < 240$ and $k_1^l + k_2^l + k_2^h < 240$.Similarly, if $\bar{Q} = 280$, there is a switching point at $\alpha_t = 0.13$, where the number of pivotal plants increases from one to two. In contrast, if $\bar{Q} = 180$, the number of pivotal plants is reduced from one to none at the switching point, which is $\alpha_t = 0.20$. More generally, the level of excess capacity generates two regimes and we can use the four plants' capacity values (k_1^l, k_2^l, k_1^h and k_2^h) to identify the pivotal regime changes under each $\bar{Q}$ assumption. The closed-form values are presented in the following proposition.

Proposition 4 *(a) If $K/2 < \bar{Q} \leq 3K/4$, the switching point is $\alpha_t = (2\bar{Q} - K)/K$, at which the number of pivotal plants is reduced from one to none.*
(b) If $3K/4 < \bar{Q} \leq K$, the switching point is $\alpha_t = 2(K - \bar{Q})/K$, at which the number of pivotal plants is increased from one to two.

Table 2 summarises the switching point for each demand level, together with the capacities of each plant at this level and the number of pivotal plants before and after the threshold.

Table 2: Pivotal regime change thresholds as a function of demand ($\bar{Q}$).

			Capacities at $\alpha = \alpha_t$				Nº pivotal	
K	$\bar{Q}$	α_t	k_1^l	k_2^l	k_1^h	k_2^h	$\alpha < \alpha_t$	$\alpha \geq \alpha_t$
300	160	**0.07**	140	10	10	140	1	0
300	170	**0.13**	130	20	20	130	1	0
300	180	**0.20**	120	30	30	120	1	0
300	190	**0.27**	110	40	40	110	1	0
300	200	**0.33**	100	50	50	100	1	0
300	210	**0.40**	90	60	60	90	1	0
300	220	**0.47**	80	70	70	80	1	0
300	230	**0.47**	80	70	70	80	1	2
300	240	**0.40**	90	60	60	90	1	2
300	250	**0.33**	100	50	50	100	1	2
300	260	**0.27**	110	40	40	110	1	2
300	270	**0.20**	120	30	30	120	1	2
300	280	**0.13**	130	20	20	130	1	2
300	290	**0.07**	140	10	10	140	1	2

That leads to the following stylised facts:

7. For each demand assumption, there is a switching point where the number of pivotal plants changes;

8. Within our parameter boundaries, there is always one pivotal plant before the switching point;

9. In relative spare capacity cases (i.e. $\bar{Q} = \{160, ...220\}$) there are no pivotal plants after the switching point;

10. When capacity is relatively tight (i.e. $\bar{Q} = \{230, ...290\}$), there are two pivotal plants after the switching point.

Notice that the theoretical pivotal switching points for $\bar{Q} = 240$, $\bar{Q} = 180$ and $\bar{Q} = 280$ are visually close to the structural regime change thresholds in the diversification/prices relationships of Figures 1 to 3. We now need to derive confidence intervals for the estimated values to formally compare the theoretical and the estimated breaking points. We approximate the distribution of the structural breaks through a bootstrap procedure. For each demand (fourteen cases) and R-E (nine) assumptions, we extract 99 random subsamples of $1,020$ observations stratified for α (we use twenty observations for each α out of the fifty available). We use Definition 2 to obtain the sub-samples' structural breaking points.

In Figure 5, we represent the mean (squares) and the 95% confidence intervals of the structural breaking points. Also in the same figure, we present the theoretical switching points (diamonds), as described in Table 2. Each panel corresponds to one R-E combination and for each panel we represent the simulated and theoretical breaking points (vertical axis) for each level of demand (horizontal axis). Theoretical thresholds and simulation breaking points co-move following an inverted-V shape: they increase when the change is from one to zero pivotal players, and decrease when we move from one to two pivotal players. Confidence intervals are narrow in general.[17] Overall, the fit between the close-form and (endogenously obtained) simulated results is very good. Out of 126 comparisons, only 3 theoretical switching points fall outside the confidence intervals and 123 fall inside. That leads us to the following stylised fact:

11. There is strong correspondence between simulated breaking points and analytically derived switching points.

3.4 Diversification and latent intensity of competition

In the simulation environment it is possible to inspect the probability priors from which bids are chosen. It is therefore possible to study how market structures (excess demand, generation diversification, etc.) influence the firms' "competitive attitude" and not only market outcomes.[18]

[17] Breaking points are more difficult to identify for $\bar{Q} = 230$, $\bar{Q} = 240$, and $\bar{Q} = 290$. That might, in part, be due to the location of the corresponding α_t in the extremes. For $\bar{Q} = 230$ and $\bar{Q} = 240$ one has $\alpha_t = 0.47$, which in the estimation would correspond to a linear model for all observations in $\alpha < 0.47$ and a second regime for only $\alpha = \{.48, .49\}$. Similarly, for $\bar{Q} = 290$ we have $\alpha_t = 0.07$, and there is one regime for $\alpha < 0.7$ and another for $\alpha \geq 0.7$. The flexibility in the two-regime model is thus less present in those cases.

[18] We have also analysed latent competition intensities in Rupérez Micola and Bunn (forthcoming) and Rupérez Micola et al. (forthcoming).

Through their trading interaction and the R-E algorithm, firms learn to prioritise those bidding strategies that achieve higher payoffs and choose them more often. Price regularities follow once marginal supply patterns are established.

The panels in Figure 6 depict the end-of-simulation individual latent probability distributions from which firms choose bids. On the horizontal axes, strategies are identified with numbers ranging from 1 for the more competitive to 100 for the highest possible bid. Cumulative probabilities are calculated on the vertical axes for each element of the action space. The concentration of probabilities is largely invariant across a large number of periods once the market reaches convergence, so the distributions on the last trading period are an indication of the plants' long-term mixed strategies. Probabilities concentrated on lower and higher actions result on the plants bidding more and less competitively, respectively, and curve movements to the upper-left and lower-right corners suggest that the market becomes more and less competitive.

The curves summarise end-of-simulation cumulative bidding probabilities under $\bar{Q} = 180$, and $\bar{Q} = 280$ for specialisation ($\alpha = 0$), diversification ($\alpha = 0.5$) and at the breaking points ($\alpha_t = 0.20$ and $\alpha_t = 0.13$, respectively, see Table 2), averaged across the 50 simulation runs for $\delta = 0.5$ and $\gamma = 0.005$.

Figure 6 offers a number of general insights linking individual probability distributions to market outcomes. Base-load plants trade more competitively than peak-load plants. Under $\alpha = 0$, Firm 1 and Firm 2's bids are very different - lower for the base-load specialist. However, when $\alpha = 0.50$, bidding priors for each plant type are very similar but different across plant type. Thus, there is a clear identification between generation technology and the competitiveness of the plant's trading prior.

Moreover, we find evidence suggesting a link between portfolio diversification, learning, trading behaviour and market outcomes. For a given α, a demand increase from $\bar{Q} = 180$ to $\bar{Q} = 280$ has the effect of making the bids less competitive (i.e. lower right movements). Moreover, trading priors shift in the competitive direction (upper left) when diversification grows from $\alpha = 0$ to α_t with resulting lower bids and, hence, market prices. However, when the movement is from α_t to $\alpha = 0.5$, the curves move to the lower-right corner, which suggests a less competitive attitude, with resulting higher prices.

3.5 Plant size and diversification

Our portfolio configurations do not vary only in terms of their degree of diversification but also in the size of the power plants assigned to each firm. It is fair to ask to what extent the two effects interact. To answer this question, we run new simulations with marginal costs equal to zero for all plants. Figures 7a, 7b and 7c provide the market price difference in the representative cases ($\bar{Q} = \{240, 180, 280\}$) between the main simulations, i.e. under technological diversification (different marginal costs), and those with different plant sizes but only one technology, i.e. where marginal costs are zero for all plants. The figures, thus, separate the plant size and technological aspects.

The difference attributable to technology diversification is positive throughout. Moreover, the relationship between α and prices presents a visually identifiable kink at the pivotal dynamics' switching point ($\alpha_t = 0.40, 0.20, 0.13$ for $\bar{Q} = 240, 180, 280$, respectively). In all cases, the effect of technology diversification is stronger before the switching point. After a drop at the switching point, the effect remains stable in the intermediate and tight demand conditions ($\bar{Q} = \{240, 280\}$) but is reduced even further as α increases in the case of spare capacity ($\bar{Q} = \{180\}$). In short, technology diversification causes an additional change in the price dynamics, which comes on top of plant size effects.

4 Discussion

We study the relationship between the degree of diversification in electricity generation portfolios and the firms' ability and incentives to influence prices. The setting, a version of von der Fehr and Harbord's (1993) electricity market, describes some aspects of energy trading well but includes a very large number of non-Pareto ranked pure strategy equilibria. Thus, computational learning algorithms offer a number of conceptual and practical advantages for economic analysis. We choose the R-E inductive equilibrium selection algorithm and, rather than focusing on pure specialisation and diversification, we also analyse a wide range of intermediate combinations.

Our main research question concerns the shape of the diversification versus market price relationship. The simulations suggest that this is often decreasing, but that demand levels influence both its shape and the price levels. The relationship is not monotonic but it includes structural stability breaks. For a wide range of parameters, we have also identified a strong correspondence between close-form switching points and those endogenously obtained in the simulations.

It is well-known that a market where generators are specialised can exhibit more market power because its price-setting players are more concentrated. However, specialised high cost generators have less incentive to exert market power because they lack base-load plants. In contrast, diversified firms have the incentives, but may not have enough price-setting capacity. Our research contributes to clarify the influence of generation technologies with a characterisation of the role played by pivotal players. The composition of a firm's generation portfolio is a market power instrument because it modifies pivotal dynamics and changes the intensity of competition. A firms' ability to influence the market price grows with its size and position in the supply stack. Pivotal players have more market power than non-pivotal. In low-demand cases, there is a regime-switching point of the diversification level where the market moves from one to no pivotal plants. At that point, the formerly pivotal plant suffers a sudden loss of market power and prices drop. In high-demand cases, there is a regime-switching point where the market setting changes from one to two pivotal players. As a result, the new pivotal plant experiences a sudden increase in its degree of market power, which is not compensated by a decrease in the previously unique pivotal plant's market power. More balanced bargaining power between the two peak-load plants then facilitates some implicit cooperation and prices increase.

Although the results are stylised and do not scale up directly to any particular real world situation, they suggest a number of general policy implications. In demand/supply situations which are usual in Western energy markets (around 20% excess capacity), the relationship between diversification and prices is V-shaped in duopoly. With some diversification, competition at the margin leads to lower prices until prices increase at the pivotal threshold. Under perfect diversification, all generators hold the same technological portfolio and bidding coordination opportunities are highest. Prices are then higher than in the intermediate cases, but lower than under specialisation.

Our results might also help explain some features of the electricity time-series, e.g. high volatility and seasonality. Low- and high-demand periods might not only lead to more or less supply competition but also change its nature. When demand is low there are no pivotal plants but in high-demand cases there are two of them. Since electricity demand is extremely price inelastic and a function of temperature, exogenous weather patterns might explain changes in the firms' trading behaviour, which in turn might contribute to cause the dramatic regime switching observed in econometric studies of electricity prices (for example, Karakatsani and Bunn, 2004).

Our results, however, rely on a number of assumptions. First, they stem from the R-E algorithm, which is only one of the models one could use. R-E reinforcement learning is shown to be a fruitful alternative where standard theoretical methods are impractical. Moreover, where there are unique theoretical predictions (e.g. switching points, von der Fehr and Harbord's (1993) predictions for high and low demand/supply ratios), R-E simulations match them well.

Moreover, our robustness tests suggest that different R-E versions do not qualitatively change our conclusions. However, it might also be possible to use a different behavioural model, given that related algorithms also perform well against theoretical predictions (e.g. Day and Bunn, 1999; García et al., 2005). Moreover, R-E simulation agents are naive, and one might expect that experimental studies will yield outcomes more predictive of real-life electricity markets.

Hence, our quantitative results should be taken as indicators of the direction and relative importance of the effects, rather than of their magnitude. Second, the various simulation parameters – including number of firms, technology stocks, etc.– were defined as exogenous and independent of one another. It is possible that in real markets they would be endogenously determined and simulations might also contribute to study their reciprocal dynamics, as in Bunn and Oliveira (2007). Here, we have focused on a stylised, relatively standard market model, whose outcomes are more directly comparable to those of close-form approaches.

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Figure 1: Mean plus and minus two standard deviation of prices (Q=240)

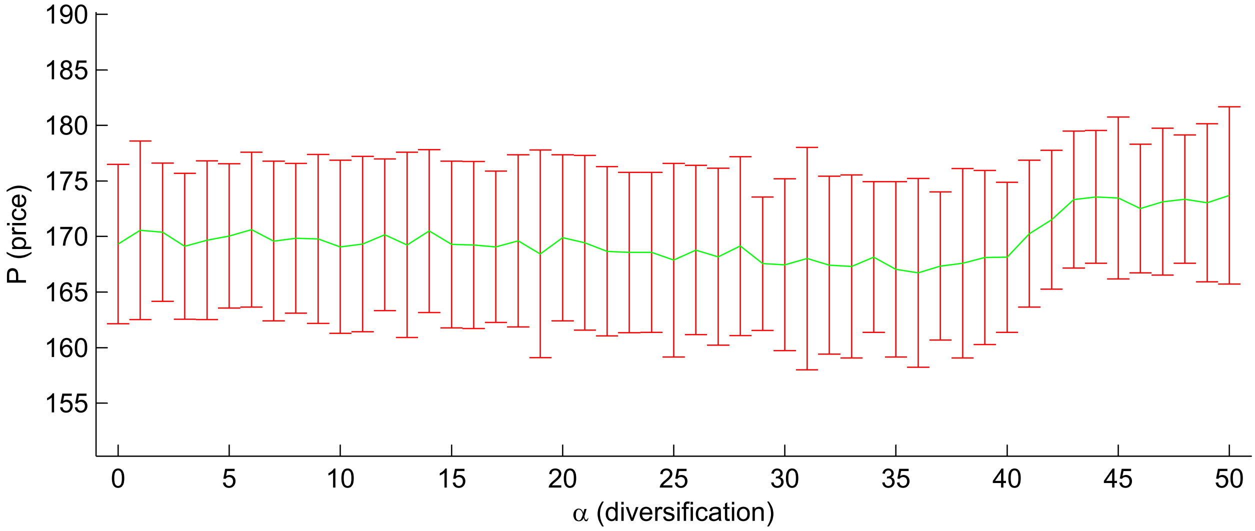


Figure 2: Mean plus and minus two standard deviation of prices (Q=180)

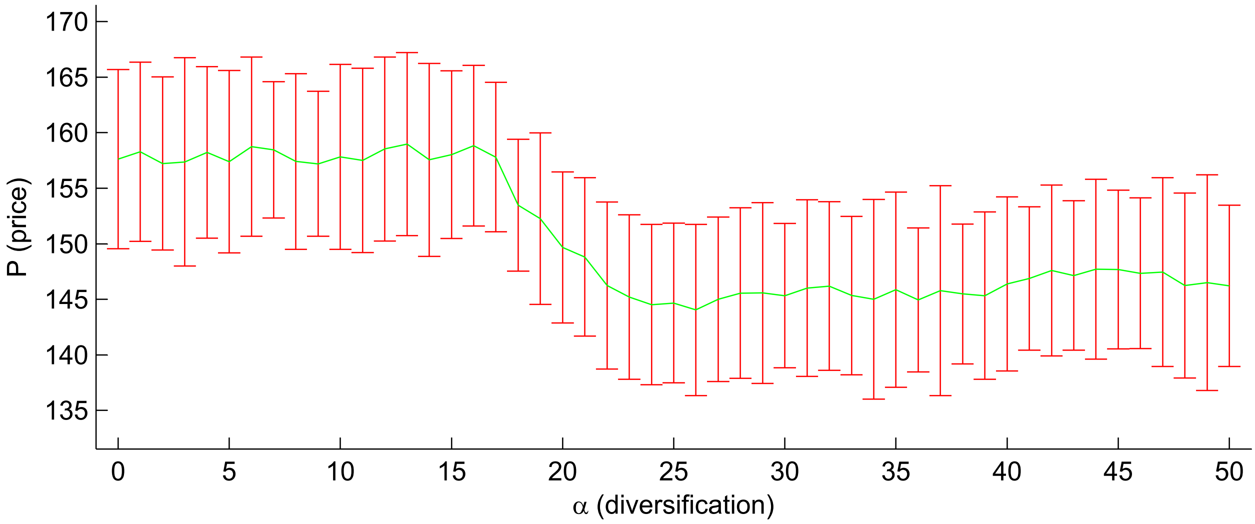


Figure 3: Mean plus and minus two standard deviation of prices (Q=280)

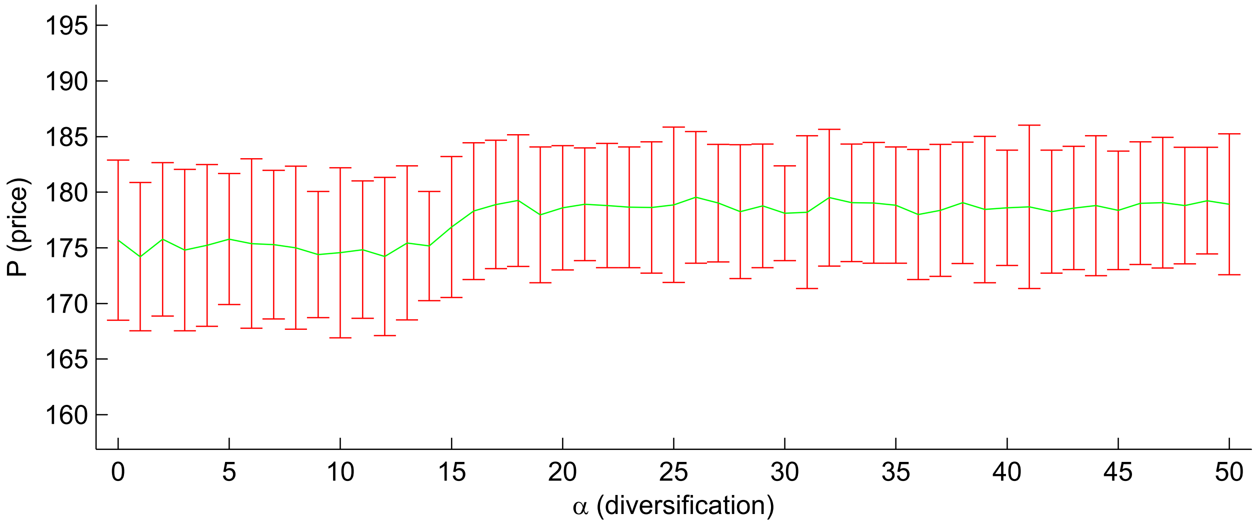
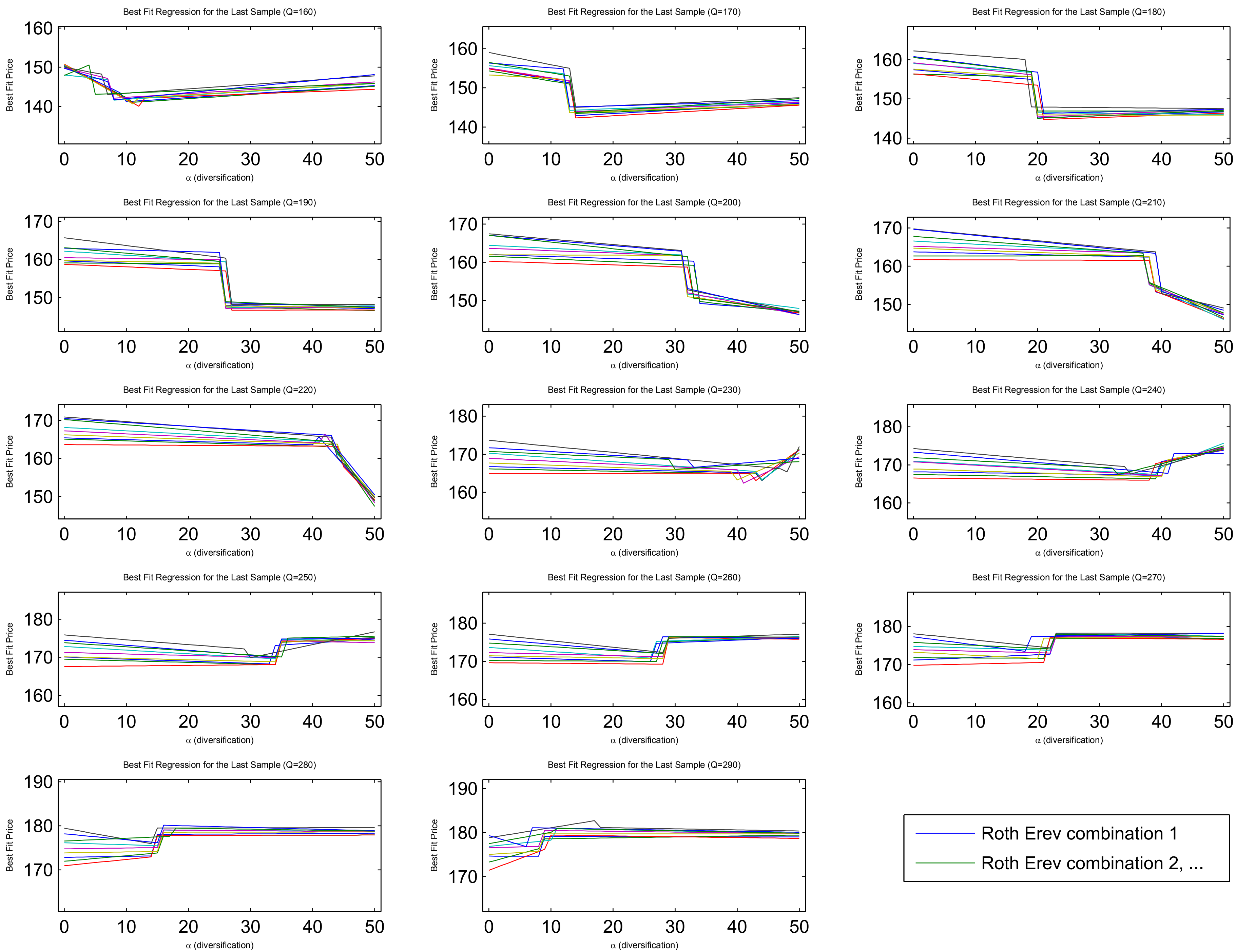


Figure 4



Best Fit Regression for the Last Sample (Q=160)
Best Fit Regression for the Last Sample (Q=170)
Best Fit Regression for the Last Sample (Q=180)
Best Fit Regression for the Last Sample (Q=190)
Best Fit Regression for the Last Sample (Q=200)
Best Fit Regression for the Last Sample (Q=210)
Best Fit Regression for the Last Sample (Q=220)
Best Fit Regression for the Last Sample (Q=230)
Best Fit Regression for the Last Sample (Q=240)
Best Fit Regression for the Last Sample (Q=250)
Best Fit Regression for the Last Sample (Q=260)
Best Fit Regression for the Last Sample (Q=270)
Best Fit Regression for the Last Sample (Q=280)
Best Fit Regression for the Last Sample (Q=290)
Best Fit Price
α (diversification)
Roth Erev combination 1
Roth Erev combination 2, ...

Figure 5

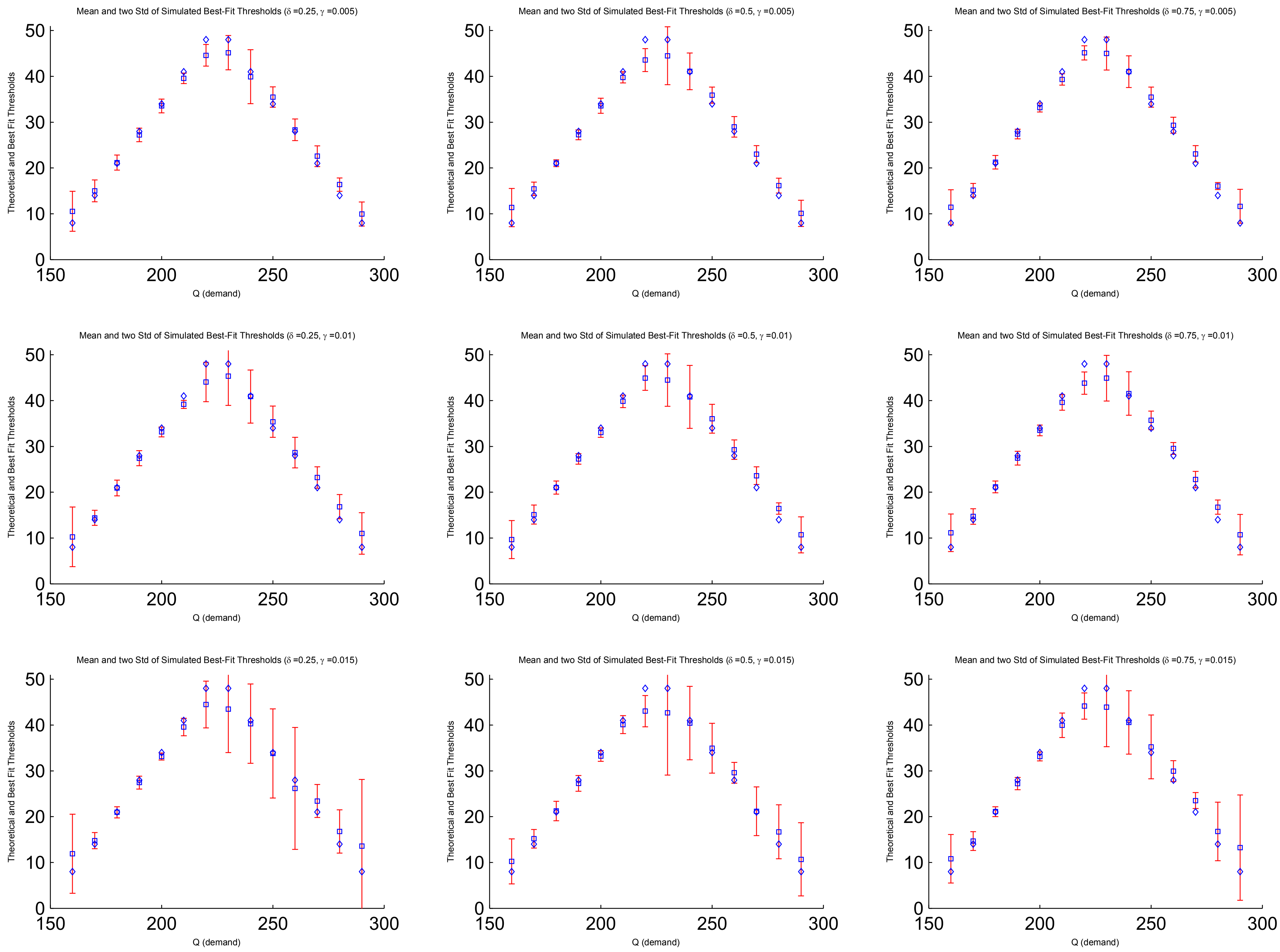

Mean and two Std of Simulated Best-Fit Thresholds (δ =0.25, γ =0.005)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.5, γ =0.005)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.75, γ =0.005)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.25, γ =0.01)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.5, γ =0.01)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.75, γ =0.01)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.25, γ =0.015)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.5, γ =0.015)
Mean and two Std of Simulated Best-Fit Thresholds (δ =0.75, γ =0.015)
Theoretical and Best Fit Thresholds
Q (demand)
0
10
20
30
40
50
150
200
250
300

Figure 6

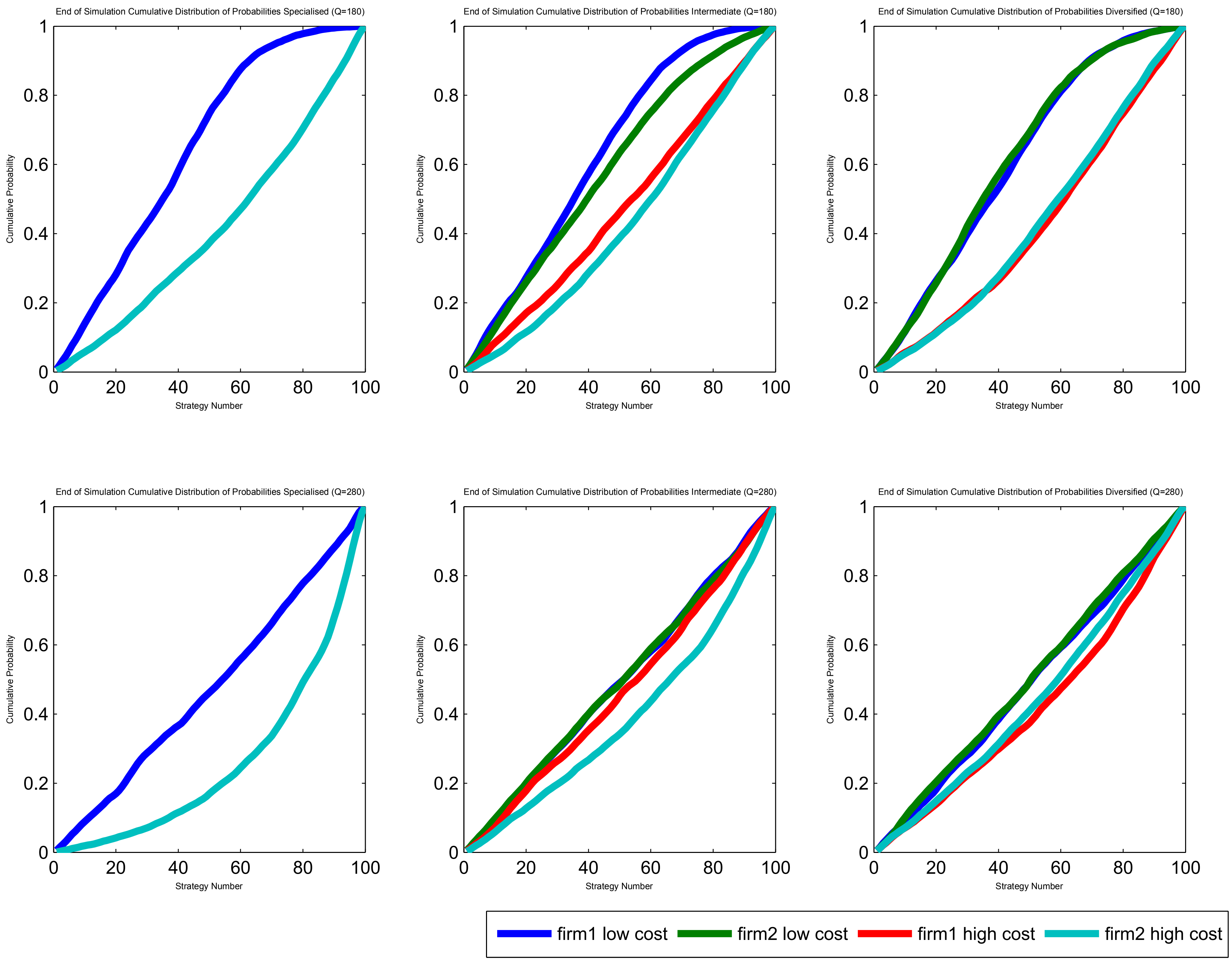

End of Simulation Cumulative Distribution of Probabilities Specialised (Q=180)
End of Simulation Cumulative Distribution of Probabilities Intermediate (Q=180)
End of Simulation Cumulative Distribution of Probabilities Diversified (Q=180)
End of Simulation Cumulative Distribution of Probabilities Specialised (Q=280)
End of Simulation Cumulative Distribution of Probabilities Intermediate (Q=280)
End of Simulation Cumulative Distribution of Probabilities Diversified (Q=280)
Cumulative Probability
Strategy Number
firm1 low cost
firm2 low cost
firm1 high cost
firm2 high cost

Figure 7a: Difference of prices high-cost=100 minus high cost=0(Q=240)

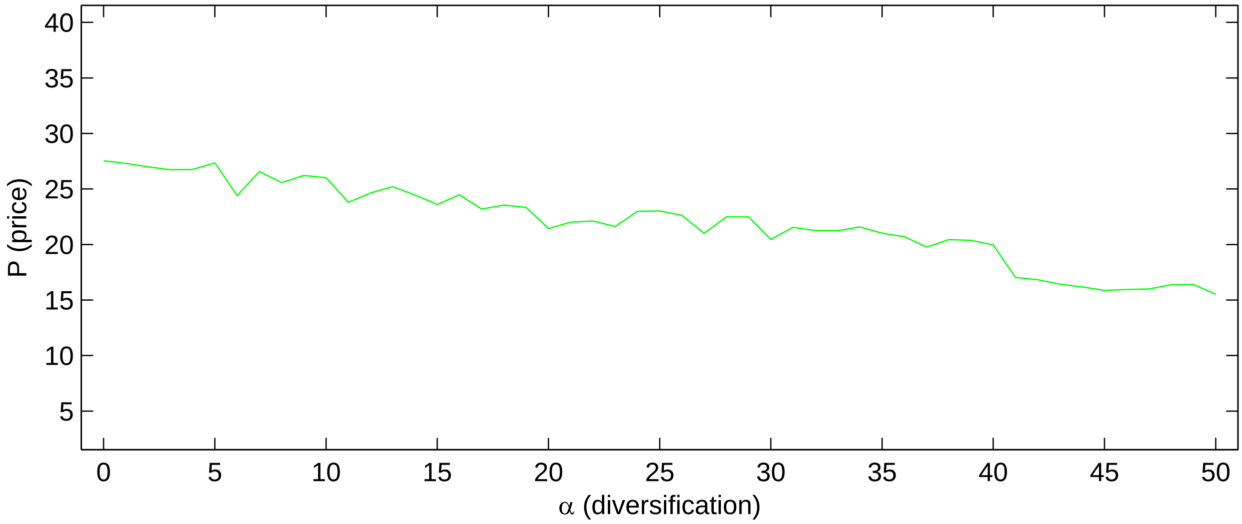


Figure 7b: Difference of prices high-cost=100 minus high cost=0(Q=180)

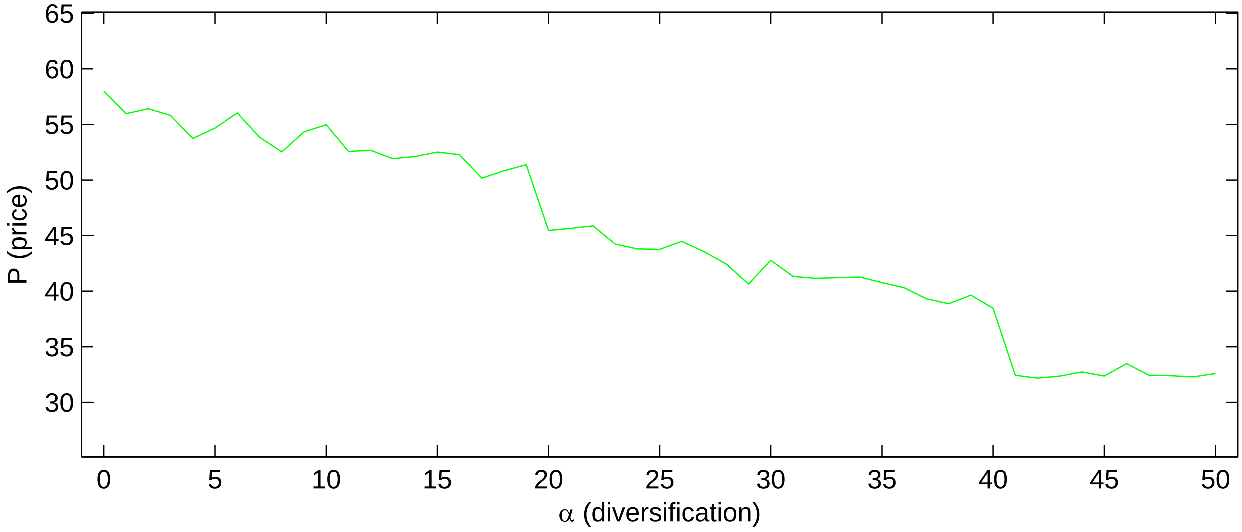


Figure 7c: Difference of prices high-cost=100 minus high cost=0(Q=280)