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ESTIMATING IMPLIED VOLATILITY DIRECTLY FROM "NEAREST-TO-THE-MONEY" COMMODITY OPTION PREMIUMS

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ABSTRACT

Estimates of implied volatility became available with the development of options pricing formulae. However, implied volatility could only be obtained through a cumbersome iterative process. This paper presents an alternative volatility estimator obtained directly from "nearest-to-the-money" option premiums. Comparisons of the two estimators are presented leading to the conclusion that the direct estimator is an adequate substitute for the traditional iterative estimation process.

ESTIMATING IMPLIED VOLATILITY DIRECTLY FROM "NEAREST-TO-THE-MONEY" COMMODITY OPTION PREMIUMS

Option premium bids and offers, presented at the market by prospective participants, contain some probabilistic assessment of individual expectations of future prices. When a large number of options are exchanged, an amalgamation of these assessments is embedded in the prevailing market premium. If this market-wide assessment could be isolated and quantified, then valuable information about price risk could be harnessed to assist decision-makers (King and Fackler).

Numerous studies have explored the issue of futures market efficiency (Kamara; Garcia, Hudson and Waller). If futures markets are efficient, then the prevailing futures price represents today's best estimate of expected price (Fama). With an estimate of the implied variance, which can be gained from the options market, price distributions could be developed. Price forecasts based on these distributions should be a useful management tool because they are derived from the relatively objective pooled probabilistic assessments of all traders at the market.

Estimates of implied volatility became available with the development of options pricing formulae (Black and Scholes; Merton; Black; Gardner) and option trading. Obtaining an estimate of implied volatility is currently cumbersome due to the specific nature of Black's formula and its predecessors. Latané and Rendleman state:

> Although it is impossible to solve the Black-Sholes (B-S) equation for the standard deviation in terms of an observed call price and other variables, one can use numerical search to closely approximate the standard deviation implied by any given option price (p. 370).

The traditional method, which will be referred to as the Iterative Implied Volatility Estimate (IIVE), requires a numeric search to estimate implied volatility. In its current state, the procedure is too intricate for quick calculation and most spreadsheet applications. If a non-iterative method were available to estimate implied volatility with limited loss of precision, then spreadsheet-based probabilistic price forecasts would be easier to develop. Additionally, implied volatility estimates could be obtained more easily in the absence of a computer. The purpose of this study is to present and compare an alternative (direct) method of estimating the volatility implied in options premiums.

Derivation of the Direct Implied Volatility Estimator (DIVE)

Black (1976) specifies the value of a commodity call option and, via put-call parity, the value of a put option could be specified as

(1) $$V_p = e^{-rt}(P_s \cdot F[-d_2] - P_f \cdot F[-d_1]),$$

where

V_p = the value (premium) of the put option;

e = the natural number, 2.71828183;

r = the risk-free short-term interest rate;

t = the time to expiration of the option contract;

P_f = the underlying futures contract price;

$F[d_i]$ = the value of the standard-normal CDF evaluated at d_i;

P_s = the exercise (strike) price of the put option;

$d_1 = [\ln(P_f/P_s) + (.5S^2t)]/S\sqrt{t}$;

$d_2 = [\ln(P_f/P_s) - (.5S^2t)]/S\sqrt{t}$; and,

S^2 = the variance of change in the price of the underlying commodity futures contract.

For "nearest-to-the-money" options, the difference between the futures price (P_f) and the exercise price (P_s) is smaller than any other option trading on the futures contract. It is possible for options to be "at-the-money," at which time the difference is zero. If the futures price, P_f, and the exercise price, P_s, are equal, the terms d_1 and d_2 would simplify to $.5S\sqrt{t}$ and $-.5S\sqrt{t}$, respectively. Substituting these terms into equation (1) and rearranging yields

$$(2) \qquad \frac{V_p}{P_f e^{-rt}} = \{F[.5S\sqrt{t}] - F[-.5S\sqrt{t}]\} ,$$

Symmetry of the standard-normal distribution allows that $F[-d_i] = 1 - F[d_i]$, therefore

$$(3) \qquad \frac{V_p}{P_f e^{-rt}} = \{F[.5S\sqrt{t}] - (1 - F[-.5S\sqrt{t}])\} = 2F[.5S\sqrt{t}] - 1 .$$

and rearranging provides

$$(4)\quad .5\left(\frac{V_p}{P_f e^{-rt}} + 1\right) = F[.5S\sqrt{t}] \quad ,$$

where $F[.5S\sqrt{t}]$ is the area under the standard-normal probability distribution from $-\infty$ to $.5S\sqrt{t}$. Since the values for all terms in the left-hand side expression of Equation (4) are available, "denormalizing" both equations permits directly solving for values of the implied volatility. "Denormalizing" refers to finding the fractile (horizontal axis value) which corresponds to the given probability. In this case, the left-hand side value can be searched-out in a standard-normal distribution table and the corresponding Z-value read off the margins.

Letting Z_p represent the denormalized left-hand side value of Equation (4), we have

$$(5)\quad Z_p = DN\left\{ .5\left(\frac{V_p}{P_f e^{-rt}} + 1\right)\right\} = DN\{F[.5S\sqrt{t}]\} = .5S\sqrt{t} \; .$$

Letting S_p represent the volatility measure implied by the put option parameters, and rearranging yields

$$(6)\quad S_p = 2Z_p/\sqrt{t} \quad .$$

The volatility measure implied by the "nearest-to-the-money" call option, S_c, can similarly be derived as

$$(7)\quad Z_c = DN\left\{ .5\left(\frac{V_c}{P_f e^{-rt}} + 1\right)\right\} = DN\{F[.5S\sqrt{t}]\} = .5S\sqrt{t} \; ,$$

where V_c is the call premium and

(8) $S_c = 2Z_c/\sqrt{t}$.

A generalized volatility measure, referred to as the Direct Implied Volatility Estimate (DIVE), of the underlying futures contract may be obtained (following King and Fackler) by taking the simple average of S_c and S_p.

Eliminating the Statistical Table Search

To eliminate the necessity of searching through a statistical table for the appropriate values of Z_c and Z_p, the standard-normal cumulative probability function may be approximated by the formula

$$(9) \quad F[\alpha] = P(Z \le \alpha) = 0.5 + 0.5\left(1 - e^{-(2\alpha^2/\pi)}\right)^{0.5} ,$$

for $\alpha \ge 0$ and $Z \sim N(\mu=0,\ \sigma^2=1)$,

where $F[\alpha]$ is the probability that Z will assume a value of α or less (U.S. Dept. of Commerce, p. 933). Solving for α yields,

$$(10) \quad \alpha = \left\{ \ln\left[\left(\frac{1}{1 - (2F[\alpha] - 1)^2} \right) \right] \cdot \left[\frac{\pi}{2} \right] \right\}^{0.5} .$$

Thus, when the cumulative probability, $F[\alpha]$, is known, the corresponding value of α may be approximated by employing Equation (10). Hence, S_c and S_p may be calculated using

$$(11) \quad S_c = \left(\frac{2}{\sqrt{t}} \right) \cdot \left\{ \ln\left[\left(\frac{1}{1 - (V_c/P_f e^{-rt})^2} \right) \right] \cdot \left[\frac{\pi}{2} \right] \right\}^{0.5} ,$$

and

$$(12)\ S_p = \left(\frac{2}{\sqrt{t}}\right)\cdot\left\{ \ln\left[\left(\frac{1}{1 - (V_p/P_f e^{-rt})^2}\right)\right]\cdot\left[\frac{\pi}{2}\right]\right\}^{0.5},$$

where all variables are defined as before. The Direct Implied Volatility Estimate (DIVE) would be the simple average of S_c from Equation (11) and S_p from Equation (12).

An Example

The direct and iterative methods were applied to the Chicago Board of Trade November, 1988 Soybean options trading on Thursday, August 4, 1988. There were 78 calendar days (0.2137 years) between this date and the final day of trading (October 21, 1988). The closing futures price was \$8.77 per bushel. The "nearest-to-the-money" option exercise price on this date was \$8.75/bu. The settlement premiums for the \$8.75 call and put options were \$0.70/bu and \$0.71/bu, respectively. The interest rate on 13-week treasury bills was 6.94%. Substituting these values into Equations (11) and (12), the directly obtained volatilities implied by the August 4, 1988 options were 44.6297 percent and 43.9990 percent for S_p and S_c, respectively. Thus, the DIVE, $0.5(S_c + S_p)$, for the November, 1988 Soybean option on August 4, 1988 was 44.3143 percent.

For comparison, employing the iterative method yielded implied volatilities of 43.4244 percent and 45.3001 percent for the call and put options, respectively, and the August 4, 1988 IIVE, was 44.3623 percent. On this date, the direct and iterative implied volatility estimates were within five one-hundredths of a percent.

Comparison of the Iterative and Direct Estimators

Tests of the closeness of the DIVE and IIVE estimates over a larger sample period were conducted. Two strategies were utilized to compare the volatility estimates: (1) direct comparison of the volatility measures, DIVE and IIVE, and (2) comparison of the errors between actual and predicted premiums using five-day moving averages of the DIVE and IIVE. The data employed in the tests were obtained from the Chicago Board of Trade (CBOT) Research Division and span the period from the first trading day in April through the option expiration date for the 1986 and 1987 November Soybean and December Corn options trading on the CBOT. The decision to begin the data series in April was somewhat arbitrary. It was chosen because planting decisions would be arrived at for both crops in most areas and considered an appropriate time when producers would begin to consider forward pricing strategies. For 1988, April through June 30 premiums for both contracts were used. Data beyond this date in 1988 were unavailable when the analysis was conducted.

DIVE vs. IIVE: A Direct Comparison

Initially, the DIVE and IIVE were computed for the nearest-the-money settlement put and call options on each trading day during the three study periods. Relevant statistics for directly comparing the two measures were then calculated for each study period and for the aggregate study period 1986-88 (Table 1).

Table 1. Comparison of the DIVE and the IIVE[a,b,c]

Time Period		December Corn	November Soybeans
1986	Mean Difference	0.7527	0.4018
	Standard Error	0.1694	0.0606
	Mean Squared Error	4.9877	0.6713
	t Statistic	-1.4598	-9.8773 *
	Observations	155	140
1987	Mean Difference	0.6405	0.6113
	Standard Error	0.1792	0.1713
	Mean Squared Error	5.6115	4.3928
	t Statistic	-2.0062 *	-2.3107 *
	Observations	163	143
1988	Mean Difference	-0.1031	0.0717
	Standard Error	0.0250	0.0188
	Mean Squared Error	0.0494	0.0272
	t Statistic	-35.8763 *	-49.2783 *
	Observations	63	63
1986-1988	Mean Difference	0.5973	0.4283
	Standard Error	0.1036	0.0744
	Mean Squared Error	4.4380	2.0921
	t Statistic	-3.8860 *	-7.6868 *
	Observations	381	346

[a] Mean Difference was computed on daily estimates of DIVE minus IIVE for the sample periods and represents percentage point difference in the implied volatility estimates.

[b] Mean squared error was computed under the assumption that the IIVE was the true measure of implied volatility.

[c] t statistic is calculated to test the null hypothesis that the absolute value of the mean difference is greater than or equal to one percent. This hypothesis is rejected if the T value is less than -1.96 for $\alpha = .025$ (* indicates rejection).

For the four study periods, the mean difference between the two volatility measures (DIVE minus IIVE) for both corn and soybeans implies that the DIVE tends to exceed the IIVE. However, the difference is, on average, less than one percentage point. In all samples except the 1986 corn contract the absolute value of the mean difference was significantly less than one percent at the 97.5% level of confidence. Based on this comparison, it may be appropriate to consider the DIVE sufficiently close to the IIVE to warrant its use as a substitute estimator of implied volatility.

The difference between the direct and iterative estimates increased substantially in the last two weeks of trading prior to option expiration for both contracts in 1986 and 1987. Caution should be exercised with using the direct estimator in the last days before expiration. The errors from the final trading days were included in the mean difference estimates. Had these days been omitted, the mean differences would have been lower in 1986 and 1987.

Mean squared error (MSE) was computed under the assumption that IIVE is the true measure of implied volatility. The MSE provides a measure of dispersion of the estimator around the actual value of the parameter (Kmenta, p. 156), and thus provides another tool by which the relative closeness of IIVE and DIVE may be evaluated.

DIVE vs. IIVE: For Predicting Premiums

The second line of testing pursued involved predicting nearest-the-money put and call option premiums by using 5-day moving averages of DIVE and IIVE. Prediction errors (predicted premiums less actual premiums) were then calculated for both estimators and relevant statistics derived.

Of primary interest is the relative efficiency of the DIVE compared to the IIVE for predicting premiums. The measure employed to assess their relative efficiency was the ratio of the mean squared prediction errors (Table 2). For all periods and for all option contracts studied, the nearest-the-money premiums predicted by employing the DIVE were close to those predicted by employing the IIVE. In fact, over the aggregate study period, the DIVE predictions marginally outperformed the IIVE predictions.

Observation of the mean prediction errors (Table 3) finds few instances where either predictor over- or underestimates the option premium, on average, in excess of $0.005 per bushel. For 1988, the DIVE predictor underestimated the option premiums for corn and soybeans by $0.007 to $0.028 per bushel. During the same period, the IIVE predictor underestimated these same premiums by $0.008 to $0.03 per bushel. Promisingly, the DIVE predictors performed well over the aggregate period (1986-88), with the largest mean errors, $0.0015 and $0.004 per bushel occurring for the call option premium predictions for corn and soybeans, respectively. In contrast, the IIVE predictors for the aggregate period did not fare as well as the DIVE for puts or calls for either commodity.

Summary and Conclusions

Commodity options trading and Black's option pricing formula provide an opportunity to extract futures contract price volatility information from the options market. In the past, the volatility

Table 2. The Relative Accuracy of DIVE-predicted Premiums versus IIVE-predicted Premiums

Time Period	December Corn		November Soybeans	
	Put	Call	Put	Call
		Mean Squared Error Ratio[a,b]		
1986	1.014149	1.036823	1.037881	1.017068
1987	1.010784	1.005379	0.977625	0.990863
1988	0.972377	0.982952	0.989261	0.988323
1986-1988	0.983913	0.991390	0.994440	0.993094

[a]Statistics presented in this table are based on comparisons of actual premiums and those predicted using five-day moving averages of the DIVE and the IIVE.

[b]Mean Squared Error (MSE) ratios indicate the efficiency of one predictor in relation to another. In this case, the MSE of the DIVE is the numerator and MSE of the IIVE is the denominator. An MSE ratio equal to one indicates that both predictors are equivalent; greater than one indicates that the DIVE is less efficient; and, less than one indicates that the DIVE is more efficient.

Table 3. Mean Errors in Prediction Premiums Using the DIVE and IIVE[a,b]

Time Period		December Corn		November Soybeans	
		Put	Call	Put	Call
		$/bu of premium			
1986	DIVE	0.000066	0.000176	0.002746	0.001205
	t	0.117 *	0.361 *	1.897 *	0.575 *
	IIVE	-0.000870	-0.000759	0.000444	-0.001097
	t	-1.566 *	-1.600 *	0.0308 *	0.528 *
		n=150		n=135	
1987	DIVE	0.000725	0.000484	-0.000563	0.000575
	t	1.362 *	0.734 *	-0.309 *	0.173 *
	IIVE	-0.000171	-0.000411	-0.002700	-0.001562
	t	-0.321 *	-0.625 *	-1.478 *	-0.469 *
		n=158		n=138	
1988	DIVE	-0.007306	-0.010989	-0.010053	-0.028171
	t	-2.399 *	-3.256 *	-1.832 *	-3.270
	IIVE	-0.007986	-0.011669	-0.011419	-0.029538
	t	-2.607	-3.463	-2.087	-3.436
		n=58		n=58	
1986-1988	DIVE	-0.000818	-0.001460	-0.000876	-0.004205
	t	-1.368 *	-2.179	-0.637 *	-1.835 *
	IIVE	-0.001696	-0.002338	-0.002946	-0.006275
	t	-2.837	-3.508	-2.148	-2.746
		n=366		n=331	

[a] Statistics presented in this table are based on comparisons (predicted minus actual) of actual premiums and premiums predicted using 5-day moving averages of the DIVE and IIVE.

[b] t test null hypothesis is mean prediction error = 0. The critical value for rejection is + or - 1.96 for the .05 level of significance (* indicates failure to reject at this level).

measure implied by Black's formula was obtained by employing an iterative calculation process; the alternative was to calculate the historical standard deviation of past futures contract prices over an arbitrarily chosen period of time. The purpose of this effort was to present a direct method for calculating implied volatility and to test this method against the more traditional iterative method.

Testing was conducted to compare the direct implied volatility estimator (DIVE) against the iterative implied volatility estimator (IIVE). The results indicated that, for a simple measure of implied volatility, the DIVE does not yield exactly the same volatility estimate as the IIVE, though the mean differences arrived at in the samples were very small. The fact that they did not yield exactly the same estimate was anticipated due to the underlying assumptions necessary to derive the DIVE. However, the mean differences over the sample periods is considered small enough to not substantially mislead a user of the direct estimator.

Measures of implied volatility are often used by traders and researchers, in conjunction with Black's formula, to calculate the fair market value of an option (the premium). Thus, the accuracy of DIVE-predicted option premiums was compared to that of IIVE-predicted premiums. The results from the sample periods were favorable for the direct estimator. This further supports the conclusion that the direct estimator presented is sufficient to serve as a substitute for the traditional iterative procedure.

Given these results, employing the DIVE as a measure of futures contract price volatility is considered warranted. One note of concern was the increased difference between the two estimators during the last few weeks of an option contract's maturity. At that time, the IIVE may be a more appropriate volatility measure than the DIVE. With this potential restriction aside, use of the DIVE may reduce implied volatility calculation time and cost substantially since an iterative search is no longer involved. With the conclusion that the DIVE is an appropriate substitute estimator of implied volatility, spreadsheet based price distributions are possible and could be created to assist in market risk management.

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