Modifying a One Region Leontief Input-Output Model to Show Sector Capacity Constraints

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A one-region Leontief input-output model may be modified to show sector capacity constraints or "sector destruction." The economist must know the degree of sector destruction, level of imports of the destroyed sector's product, and make certain assumptions about the regional economy. Six cases are presented based on the degree of destruction (complete or partial) and level of imports of the destroyed sector's product (none, sufficient to reach original final demands or insufficient to reach original final demands). A linear programming version of the input-output model is suggested for three of the six cases.

A typical use of input-output models has been for impact analysis. Changes in final demand, an exogenous variable, are estimated, and the effects of these changes on the economy are calculated. There is, however, a special case of impact analysis where the productive capacity of a sector has been curtailed or "destroyed." Although this leads to a reduction in output, it is not caused by a reduction in final demand.[1]

A common example of sector destruction is the cessation of mining in a region due to the depletion of ore. In this case, imports of the destroyed sector's product are unlikely. In other situations, such as the destruction of irrigated agriculture due to water constraints, imports of the product (e.g., feed grains) might increase. In this case it may be inappropriate to use the input-output model to calculate the new sector output. The reduced productive capacity of the destroyed sector now acts as a constraint in the input-output equation system, and imports of the destroyed sector's product may be substituted for the product that was endogenously produced.

The original input-output model, however, need not be abandoned because of these developments; rather, two general approaches may be considered. First, if destruction leads to changes in the direct coefficients, the economy may be modeled by reconstructing the flow matrix. The economist must first determine what structural changes will occur in the economy. For example, he must consider what possible substitutes might be used for the sector's product and the consequences this has on the interindustry transactions. Such a procedure can be hazardous because it relies on predictions that are very difficult to make. Second, if the direct coefficients have not changed, the input-output model may be converted to a linear programming model. Richardson notes that the linear programming version can overcome two problems that we believe are associated with destruction: (1) the existence of bottlenecks in the economy, and (2) substitutes (imports of the destroyed sector's product). This approach is suitable for modeling "short-run"

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[1] Clark, Fletcher and McKinney and Bromley, Blanch and Stoevener have also discussed the problem of estimating output after a reduction in sector capacity has occurred. Their proposals differ from the solution presented here.

changes in which no structural changes have occurred in the regional economy. In short, the choice of approaches depends on the goals of the economist and the assumptions he is willing to make.

The purpose of this article is to show how a Leontief input-output model can be revised to account for sector destruction. Since collecting new data on a region can be time consuming and costly, we will show how the new model can be constructed if the economist knows only the degree of sector destruction and level of imports of the destroyed sector's product (henceforth called "imports"). Six cases are modeled using linear programming or input-output versions of the original model. These cases cover the various situations that could occur in a regional economy after destruction. The article concludes with an application to an actual case of sector destruction that occurred recently in western Nevada.

The Models

Given that information about the regional economy after destruction is limited to the degree of sector destruction and level of imports, three related problems must be resolved before presenting the models. First, how is the destroyed sector's (sector d) final demand affected by destruction? Knowing the post-destruction level of final demand in sector d and the non-destroyed sectors will allow us to solve for sector output in the new models. Second, in the absence of imports of the destroyed sector's commodity, how does destruction affect the non-destroyed sector's final demand? Third, how are the final demand levels of the non-destroyed sectors affected when imports are available? Each problem is discussed below.

1. The effect of destruction on sector d's final demand may be shown using the flow matrix of a conventional input-output model:

$$(1.1)\quad a_{11}X_1^o + \ldots + a_{1d}X_d^o + \ldots + a_{1n}X_n^o + Y_1^o = X_1^o + M_1 = Z_1$$

$$\vdots$$

$$(1.2)\quad a_{d1}X_1^o + \ldots + a_{dd}X_d^o + \ldots + a_{dn}X_n^o + Y_d^o = X_d^o + M_d = Z_d$$

$$\vdots$$

$$(1.3)\quad a_{n1}X_1^o + \ldots + a_{nd}X_d^o + \ldots + a_{nn}X_n^o + Y_n^o = X_n^o + M_n = Z_n$$

$$(1.4)\quad a_{p1}X_1^o + \ldots + a_{pd}X_d^o + \ldots + a_{pn}X_n^o = p_o$$

$$(1.5)\quad a_{v1}X_1^o + \ldots + a_{vd}X_d^o + \ldots + a_{vn}X_n^o = v_o$$

where

a_{ij} = amount of commodity i needed to produce one unit of commodity j (note that the ith commodity is produced in the ith sector)

Y_i^o = final demand for commodity i before destruction (i = 1,2,...,d,...,n)

X_i^o = total production of commodity i before destruction (i = 1,2,..., d,...,n)

a_{pj} = amount of labor needed to produce one unit of commodity j

a_{vj} = amount of other primary inputs needed to produce one unit of commodity j

p_o = total labor purchases

v_o = total other primary inputs purchases

Z_i = total supply of commodity i

M_i = imports of commodity i

For simplicity, we will assume throughout this paper that initially there are no imports of the endogenously produced commodity. Using accounting identities (Chenery and Clark) the row and column sums in the flow matrix for sector d may be specified as:

$$(2)\quad \begin{aligned} & a_{1d}X_d^o + a_{2d}X_d^o + \ldots + a_{dd}X_d^o + \ldots + a_{nd}X_d^o + a_{pd}X_d^o + a_{vd}X_d^o \\ & \text{(column)} \\ = \; & a_{d1}X_1^o + a_{d2}X_2^o + \ldots + a_{dd}X_d^o + \ldots + a_{dn}X_n^o + Y_d^o \\ & \text{(row)} \end{aligned}$$

The degree of destruction is defined by the scalar r, where $0 \leq r \leq 1$. The situation where $r = 0$ is a special case called "total destruction;" that is, all production in sector d ceases. The case $0 < r < 1$ is termed "partial destruction." When sector d is partially destroyed and all direct coefficients are unchanged, Equation (2) becomes:

$$\text{(3)} \quad (a_{1d} + a_{2d} + \ldots + a_{dd} + \ldots + a_{nd} + a_{pd} + a_{vd})rX_d^o \quad \text{(column)}$$
$$= a_{d1}(rX_1^o) + a_{d2}(rX_2^o) + \ldots + a_{dd}(rX_d^o) + \ldots + a_{dn}(rX_n^o) + rY_d^o. \quad \text{(row)}$$

That is, when X_d^o becomes rX_d^o, Y_d^o is decreased to rY_d^o. This assumption implies that sector d's intermediate and final demands are decreased by the same proportion (r).

2. In the absence of imports, the non-destroyed sectors' final demand and output are determined by the assumption of fixed coefficient production functions inherent in the Leontief input-output model. That is, assuming that production in sector d is decreased by 40 percent and the remaining output is distributed in the same proportion as before destruction, production in all other sectors is cut by 40 percent. This result can be seen in Equations (2) and (3). Note in Equation (3) that all sectors (see the "row" side) have been scaled by r. It is evident from the accounting identities that final demand in any non-destroyed sector will now decrease to rY_i^o, just as final demand in the destroyed sector decreased to rY_d^o when X_d^o became rX_d^o.

An exception to this rule occurs if the input-output system is decomposable [Henderson and Quandt, page 370]. That is, some sectors (or a sector) have no transactions with other sectors. In this case there are two or more groups of self sufficient industries, and the final demands of the sectors with no ties to sector d do not need to be scaled by r. Another consideration in the use of scalar r is how industry products were aggregated into sectors. For example, if the product of a destroyed industry was sold entirely as final demand, but grouped in a sector with products sold as intermediate demand, it would be incorrect to scale the final demands of the non-destroyed sectors by r.

3. If imports of the destroyed sector's product are available, output of the non-destroyed sectors may increase beyond rX_i^o, and some output may be sold as final demand. We will assume that the levels of final demand for the non-destroyed sectors may not increase beyond the levels that existed before destruction. This assumption is made to correspond to a "short-run" situation; that is, no new market for the commodity may be found. The level of imports that allows the original final demands to be reached will be termed "unlimited imports." The level of import that is not sufficient to reach the original final demands will be termed "limited imports."[2]

The set of all combinations of two types of destruction (partial or complete) and three levels of imports (no imports, unlimited, and limited) gives six cases of sector destruction, each requiring a different model: Case I: Partial destruction - no imports; Case II: Partial destruction - unlimited imports; Case III: Partial destruction - limited imports; Case IV: Total destruction - no imports; Case V: Total destruction - unlimited imports; and, Case VI: Total destruction - limited imports. Cases I and IV use an input-output approach, and cases II, and III, and VI use a linear programming version of the input-output model. Case IV may be modeled with either an input-output or linear programming model.

Dorfman, Samuelson, and Solow and Vandermulen both note that the static Leontief model is a special case of linear programming. Except for cases involving primary resource limitations, the linear programming version of the problem yields results identical to those of a conventional input-output model. And, as is typical of linear programming problems, there are two versions. One could maximize the value of final demands by choosing product prices subject to the condition that prices be at most equal to the cost of primary resources (for example, labor). Al-

[2] It is assumed that imports of the destroyed sector's product are used to meet consumption final demand in the region but not export final demand. For simplicity, Y_d^o is treated throughout the paper as being comprised totally of consumption final demand.

ternatively, one could minimize primary resource costs subject to the conditions that total production of each sector's output is at least equal to their final demands. Both versions of these dual linear programs yield results consistent with conventional input-output analysis. Since this analysis considers cases where imports of endogenous products are potentially limiting, we chose the linear programming approach with labor cost minimization. A linear programming formulation of the input-output model that will be modified to analyze cases II, III, V, and VI is as follows:

(4.1) Minimize $L = a_{p1}X_1 + a_{p2}X_2 + \ldots + a_{pd}X_d + \ldots + a_{pn}X_n + a_1M_1 + \ldots + a_dM_d + \ldots + a_nM_n$

Subject to:

(4.2) $(1 - a_{11})X_1 - a_{12}X_2 - \ldots - a_{1d}X_d - \ldots - a_{1n}X_n + M_1 \geq Y_1$

(4.3) $-a_{d1}X_1 - a_{d2}X_2 - \ldots + (1 - a_{dd})X_d - \ldots - a_{dn}X_n + M_d \geq Y_d$

(4.4) $-a_{n1}X_1 - a_{n2}X_2 - \ldots - a_{nd}X_d - \ldots + (1 - a_{nn})X_n + M_n \geq Y_n$

(4.5) $a_{p1}X_1 + a_{p2}X_2 + \ldots + a_{pd}X_d + \ldots + a_{pn}X_n \leq p_o$

(4.6) $a_{v1}X_1 + a_{v2}X_2 + \ldots + a_{vd}X_d + \ldots + a_{vn}X_n \leq V_o$

(4.7) $X_d \leq rX_d^o$

(4.8) $M_d \leq G^o$

where, L = total labor costs

a_i = an artificial objective function coefficient associated with imports of commodity i; they are set larger than a_{pi} to insure that X_i is used before imports M_i ($i = 1,2,\ldots,d,\ldots,n$)

G^o = specific magnitude of import of commodity d, to be set by the analyst depending on the case studied (e.g., zero, limited, unlimited)

X_i = output of commodity i after destruction of sector d

Y_i = final demand of commodity i after destruction of sector d

Also, note that restrictions (4.7) and (4.8) have no specific counterparts in the conventional input-output model expressed in equation system (1). Restriction (4.7) requires that output of the destroyed sector (X_d) be less than or equal to some previously specified fraction (r) of the original level of output (X_d^o). Restriction (4.8) specifies the upper limit of imports of destroyed sector output (M_d).

Case I: Partial Destruction — No Imports

In this case, $M_d \leq G^o = 0$ after destruction and the output of every sector is multiplied by r. The reduction in output is due to the assumption of fixed coefficient production functions, as expressed in equations (2) and (3). Accordingly, the final demands for all sectors are affected; the new vector of final demands for the Case I model [see equations (1.1) through (1.3)] becomes $(rY_1^o, \ldots, rY_d^o, \ldots, rY_n^o)$. This case does not need to be modeled using a linear programming version of the original model; the input-output model is adequate. Even though sector d is a "bottleneck," the scaling of all final demands reflects the changes caused by destruction. Since there are no imports, the new sector output becomes rX_i^o ($i = 1,2,\ldots, d,\ldots,n$).

Case II: Partial Destruction — Unlimited Imports

In this case, G^o is a large number equaling or exceeding the level of imports needed to meet all original final demands. Endogenous production of d meets rX_d^o, while M_d is used to meet the original levels of final demands for all sectors; that is, $(1 - r)Y_1^o + \ldots + (1 - r)Y_d^o + \ldots + (1 - r)Y_n^o$. The new vector of final demands becomes $(Y_1^o, \ldots, rY_d^o, \ldots, Y_n^o)$. Although the import restriction is still (4.8), the value of M_d used in the linear programming model should be $M_d - (1 - r)Y_d^o$ because we are interested in calculating the new value of X_d that is produced endogenously. The $(1 - r)Y_d^o$ portion does not need to be included because it goes directly to the d^{th} sector's final demand for consumption and therefore does not enter the interindustry transactions in the regional economy.

Case III: Partial Destruction — Limited Imports

In this case, imports are available but not in sufficient quantity to meet all of the sector's original final demands. The linear programming formulation for this case is nearly identical to the case of unlimited imports (Case II), except that the level of imports is smaller and the final demands for the non-destroyed sectors must be scaled by a value, s, where $r < s < 1$.

Also, in the absence of any relevant information, M_d is assumed to be distributed between Y_d^o and total intermediate demand of sector d $(\sum_j a_{dj} X_j^o)$ on the basis of the proportion of X_d^o originally going to Y_d^o. For example, if originally

$$X_d^o = a_{d1}X_1^o + a_{d2}X_2^o + \ldots + a_{dd}X_d^o + \ldots + a_{dn}X_n^o + Y_d^o,$$

the proportion of M_d going to Y_d becomes

$$\frac{Y_d^o}{X_d^o},$$

and the proportion used as the intermediate demand is

$$1 - \frac{Y_d^o}{X_d^o}.$$

The value of M_d entered in the model is equal to $(1 - Y_d^o/X_d^o)\, M_d$. The final demand of the destroyed sector becomes rY_d^o. The (Y_d^o/X_d^o) portion of imports (M_d) goes directly to sector d's final demand and therefore does not enter the interindustry transactions. This is one of the more problematical assumptions in this paper for two reasons. First, unlike technical coefficients, the magnitude of the proportions is not fixed by production processes. Second, we have assumed that before destruction $M_d = 0$. There is no way to proportionately increase the M_d levels across intermediate and final demands by assuming the same M_d distribution after destruction. In short, other assumptions may be used in this instance; we present these assumptions in the absence of other information about the regional economy.

To run the linear programming model, G^o is set equal to a level of import of the destroyed sector's output that is insufficient to meet original final demands. The destroyed sector's output is restricted by $X_d \leq rX_d^o$.

The magnitude of s is estimated by the following iterative procedure: First, s is set equal to any value between r and 1.0 before running the linear programming model. If the artificial variable associated with the destroyed sector's final demand equation (see inequation 4.3) enters the basis, the value of s is too high. If the slack variable associated with the restriction $M_d \leq G^o$ enters the basis, the value of s is too low. Adjustments in s are made, depending on whether the artificial or slack enters the basis. This procedure is continued until either the artificial or slack variable in the basis is sufficiently "small." The analyst must judge what value he finds acceptable; the authors suggest that a "small" value occurs when sector outputs in the basis are relatively insensitive to small changes in s.

Case IV: Total Destruction — No Imports

Since $r = 0$ in total destruction, all production in the economy ceases due to the assumption of fixed coefficient production functions. This is evident from Equations (2) and (3). This is not true, however, for a decomposable input-output system.

Case V: Total Destruction — Unlimited Imports

Sector outputs may be calculated in this case using either an input-output or linear programming model. In the input-output version, the d^{th} row and column are deleted [see equations (1.1) through (1.5)] and the remaining final demands are left unchanged. Sector output may be calculated with the new $(n-1) \times (n-1)$ matrix using the input-output algorithm. In the linear programming version, $rY_d^o = 0$, and all other final demands are unchanged. As in Case II, G^o is a large number equaling or exceeding the level of imports needed to meet all original final demands. The level of M_d should be entered in the model as $(M_d - Y^o_d)$. Using the linear programming approach, this case is implemented by specifying $r = 0$, and $X_d \leqslant rX^o_d$.

Case VI: Complete Destruction — Limited Imports

With total destruction and limited imports, the d^{th} row and column are deleted [see equations (1.1) through (1.5)] and the remaining final demand elements are scaled by s; again, $r < s < 1$. In this case, G^o is set equal to a level of import of the destroyed sector's output insufficient to meet original final demands for all sectors. Restriction (4.7) becomes $X_d \leqslant 0$. The iterative procedure for Case III is used here.

Application

To apply the procedure described above, we used a 40 sector input-output model of western Nevada [Ching]. Partial destruction of the mining sector is due to the planned closing of Anaconda Corporation's Lyon County mining operation. The mining sector in the western Nevada model is typical in that it exhibits weak intersectoral ties. While the mining sector does make substantial purchases of utilities and certain services, major purchases are from the household sector. Similarly, there are only limited purchases by the other endogenous sectors of the mining sector's output. In the case of a partially destroyed mining sector, imports of mining products are not likely to occur; and, Case I is the only case applicable to the situation in western Nevada. The remaining cases are presented as illustrations.

We estimate that the partial destruction scalar for the mining sector are 0.3. All six cases are discussed below. Table 1 shows the total regional output, employment, and r and s values for each case.

In Case I the new employment and output levels are simply the original regional employment and output multiplied by the scalar of destruction (r= .3). This result holds either with the linear programming model or the input-output model.

TABLE 1. Results of Empirical Example

Case	Regional Output (Million Dollars)	Regional Employment (Full Time Equivalents)	r	s
ORIGINAL CASE	1253.3	67,180	NA[a]	NA
I: No imports, partial destruction	376.0	20,154	0.3	NA
II: Unlimited imports, partial destruction	1233.7	66,303	0.3	NA
III: Limited imports partial destruction	1145.27	61,512	0.3	0.928
IV: No imports, complete destruction	0	0	0.0	NA
V: Unlimited imports, complete destruction	1225.3	65,928	0.0	NA
VI: Limited imports, complete destruction	762.1	41,007	0.0	0.622

[a]Not Applicable

Cases II and III were investigated using a linear programming algorithm. For Case II, M_d was set equal to an arbitrarily large number. This specification allowed us to determine the amount of imports needed to meet all original final demands. For Case III, M_d was set equal to a value below this import level; the resulting s scalar was equal to 0.928. The iterative procedure for estimating s was relatively easy to use. In Cases III and VI, we were able to converge on a satisfactory value of s in five or less iterations.

The interpretation of the total destruction cases (IV, V and VI) is similar to the partial destruction cases, except that $r = 0$ and $X_d \leq 0$. The levels of imports in Cases III and VI are equal.

It is very important to note in Cases II, III, V, and VI that the traditional multiplier analysis does not hold. The imported commodity is not endogenously produced, and therefore does not directly generate output, employment and income in the region. Employment was calculated in Table 1 by multiplying each sector's output by the ratio of the sector's predestruction employment expressed in Full Time Equivalents to predestruction output.

Conclusion

Short-run changes occurring in a regional economy after sector destruction may be modeled by revising the original input-output model; linear programming or new input-output versions of the original model can be used. These procedures are possible even if the economist knows only the degree of destruction and the level of imports, and if he is willing to make certain assumptions. Two major assumptions are (1) the direct coefficients remain unchanged, and (2) postdestruction final demand levels do not exceed predestruction levels. The six cases presented here show how new final demands may be estimated and how sector output may be calculated. Cases I through VI were applied to an instance of sector destruction that occurred recently in western Nevada.

One of the more important aspects of the models presented here is their use in policy considerations. The option of being able to vary degrees of destruction and levels of imports would allow decision makers to predict the outcome of policies on output, income, and employment. Note that the traditional multiplier analysis for estimating changes in output, income and employment would be of little value because of the effects of destruction and imports.

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