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AN EMPIRICAL MODEL
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# Household Labor Force Participation <br> as a Cooperative Game; an Empirical Model 

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## Abstract

In utility consistent models of household behavior, it is usually assumed that the household's preferences can ve described by a single utility function. If the male and female partner have different preferences, however, only under stringent conditions the preferences of the household can be represented by a 'representative' household utility function. Although game theory provides an appropriate theoretical framework for a more general approach, attempts to develop empirical models based on a more general decision making framework have yet not been very succesful.

Bjorn and Vuong estimate game theoretic models of household labor force participation assuming that the equilibrium concept is Nash (Bjorn and Vuong, 1984) or Stackelberg (Bjorn and Vuong, 1985). A drawback of their models is that these equilibrium concepts may yield allocations which are not Pareto optimal. It is not likely, however, that in a household partners will accept an allocation while moving to another allocation would improve the position of both.

In the empirical model in this paper the fundamental assumption is that observed allocations are Pareto optimal outcomes of a game being played in the household. In contrast to the simultaneous models for discrete endogenous variables with structural shift as proposed by Heckman (1976), the present model does not require any logical consistency condition on the parameters in order to be statistically meaningful.

## 1. Introduction

One of the distinctive features of the microeconometric analysis of labor supply during the last decade is the growing complexity of the models being used. Where the literature in early years shows examples of estimating linear labor supply functions using OLS, the more recently published papers present models which take into account the limited dependent nature of the endogenous variable, the non-linearity or even the non-convexity of the budget set, institutional constraints which restrict the choice set of the individual, and random preference variation across individuals (see, for example, the papers by Moffitt (1984) and Hausman (1985).

In addition, there is a tendency to extend models of individual labor supply to models of household labor supply; examples are the articles by Hausman and Ruud (1984), Kooreman and Kapteyn (1986,1987) and Ransom (1987). A first motivation for developing household models rather than individual models is that there is some evidence that the exogeneity assumption on the variable 'other household income' in individual labor supply models is not always tenable (see Smith and Blundell (1986)). More importantly, male and female labor supply decisions within a household are likely to be closely structurally interrelated and a full understanding of labor supply behavior requires to take this relationship into account in setting up the empirical model.

The usual approach to describe the joint determination of male and female labor supply is to specify a household utility function (or a dual representation) with male leisure, female leisure and total household consumption as arguments, and then to derive the corresponding male and female labor supply equations by maximizing the household utility function subject to the household budget constraint ${ }^{1)}$. Although these household models seem to be an improvement over individual models, they introduce the additional problem of a proper representation of the household's preferences and of the desicion making process within the household. For example, if the male and the female partner have different preferences, only under stringent conditions the preferences of the household can be represented by a 'representative' household utility function (cf. Samuelson (1956) and Brown and Chuang (1981)).

Although game theory provides an appropriate theoretical framework for developing more general models of the household's decision process (see, e.g., Manser and Brown (1980) and McElroy and Horney (1980)), the empirical implementation of these game theoretic household models has (yet) not been very succesful. The usual approach in formulating a game theoretic model is to specify individual utility functions and a certain concept of equilibrium. One of the problems in the empirical implementation is that in principle the number of utility parameters to be estimated will be twice as large as compared to models based on household utility functions. The available data do usually not contain sufficient information to identify these paramaters ${ }^{2)}$. A further problem is that, except for some extremely restrictive functional forms for individual preferences and some specific choices of the equilibrium concept, one can generally not derive closed form expressions for the behavioral equations.

The present paper presents and estimates a game theoretic model of household labor supply in a discrete setting (i.e. only the labor force participation is considered). Provided that there is sufficient variation in observed participation across subsamples, identification of the preference ordering of both partners is possible. The model shows some similarities with the models proposed by Bjorn and Vuong (1984,1985). These authors formulate a game theoretic model of household labor force participation using Nash (Bjorn and Vuong (1984)) and Stackelberg (Bjorn and Vuong (1985)) as concepts of equilibrium. Although these papers present an interesting generalization of traditional discrete choice models, the equilibrium concepts adopted suffer from the fact that they may yield allocations which are not Pareto optimal. As has been argued by Manser and Brown (1980) and others it seems more appropriate, however, to employ a cooperative framework within a household context, and hence to employ an equilibrium concept which yields Pareto optimal outcomes only. The model in this paper will be developed along such lines.

The organization of the paper is as follows. Section 2 introduces the model and equilibrium concept. The subject of Section 3 is the empirical implementation and estimation of the model. We also discuss its relation to simultaneous models for discrete endogenous variables with structural shift, as discussed by Heckman (1976) and Schmidt (1981). In contrast to the latter type of models, the present model does not require any logical consistency
condition on the parameters in order to be statistically meaningful. Section 4 presents empirical results; Section 5 concludes.

## 2. Household decision making as a cooperative game

Consider a household with a male and a female partner who jointly decide on their labor force participation. Both partners (denoted by $i=m$ and $i=f$, respectively) can take one of two actions: participate ( $y_{i}=1$ ), or not participate $\left(y_{i}=0\right)$. The utility partner $i$ derives from each of the four possible combinations of actions is denoted by $U^{i}\left(y_{m}, y_{f}\right)$. (In the sequel a combination of actions will be called allocation). The preferences of partner $i$ in this model are completely characterized by the order of his or her four utility levels. For each partner 4! different orders of the utility levels are possible. So, for a household there are $(4!)^{2}=576$ possible combinations of utility orders.

Given the preferences of both partners we can define an equilibrium concept. The aim of this is to attach an allocation to each combination of utility orders. (In what follows a combination of utility orders will sometimes be called a game, a partner will sometimes be called a player, and the allocation attached to a game will sometimes be called the outcome of the game). As has been mentioned in Section 1, a cooperative framework seems to be more appropriate in this case. Therefore, we accept the outcome of a game only if it satisfies the requirement of being Pareto optimal.

It is well known that in the type of games described above there may be one or two Nash equilibria or no Nash equilbrium, and that the Nash equilibria may be or may not be Pareto optimal. Therefore, the Nash equilibrium (NE, hereafter) as such is not suitable in the present case. Our choice of equilibrium concept is guided by our wish that the outcome of a game has to be Pareto optimal, and preferably also Nash. This leads to the following behavioral assumptions:

Case 1 (One Nash equilibrium). If the game has exactly one NE and if this NE is Pareto optimal, we assume this to be the outcome of the game (see example 1).

```
UM}(0,0)<\mp@subsup{U}{}{m}(1,0)<\mp@subsup{U}{}{m}(1,1)<\mp@subsup{U}{}{m}(0,1
U}\mp@subsup{\textrm{U}}{}{\textrm{f}}(0,0)<\mp@subsup{\textrm{U}}{}{\textrm{f}}(0,1)<\mp@subsup{\textrm{U}}{}{\textrm{f}}(1,0)<\mp@subsup{\textrm{U}}{}{\textrm{f}}(1,1
```

Example 1. The unicue Nash equi己ibrium (0,1) is Fureio opitimai.

If the unique NE is not Pareto optimal, there exists exactly one allocation, at which both players are better off as compared to the Nash equilibrium ${ }^{3)}$. The household is then assumed to choose this Pareto efficient allocation (see example 2).

$$
\begin{aligned}
& U^{m}(1,0)<U^{m}(0,0)<U^{m}(1,1)<U^{m}(0,1) \\
& U^{f}(0,1)<U^{f}(0,0)<U^{f}(1,1)<U^{f}(1,0)
\end{aligned}
$$

Example 2. Allocation (1,1) is Pareto more efficient than the $N E(0,0)$.

Case 2 (Two Nash equilibria). If the game has two Nash equilibria at least one of these will be Pareto optimal ${ }^{4)}$. If only one NE is Pareto optimal, we assume this to be the outcome of the game (example 3).

```
U'm}(0,0)<\mp@subsup{U}{}{m}(1,0)<\mp@subsup{U}{}{m}(1,1)<\mp@subsup{U}{}{m}(0,1
U'f}(0,0)<\mp@subsup{U}{}{f}(1,1)<\mp@subsup{U}{}{f}(1,0)<\mp@subsup{U}{}{f}(0,1
```

Example 3. Two Nash equilibria ( $(1,0)$ and $(0,1))$, only $(0,1)$ is Pareto optimal.

If both NE are Pareto optimal, the household is assumed to choose one of these with equal probabilities (example 4).

```
Um}(0,0)<\mp@subsup{U}{}{m}(1,0)<\mp@subsup{U}{}{m}(1,1)<\mp@subsup{U}{}{m}(0,1
U'f}(0,0)<\mp@subsup{U}{}{f}(0,1)<\mp@subsup{U}{}{f}(1,1)<\mp@subsup{U}{}{f}(1,0
```

Example 4. Two Nash equilibria ( $(1,0)$ and $(0,1))$, both Pareto optimal.

Case 3 (No Nash equilibrium). If the game does not have a NE, there may two, three or four Pareto optimal allocations (examples 5, 6 and 7,
respectively). In such a case the household is assumed to choose one of the Pareto optimal allocations with equal probabilities.

```
UM}(0,0)<\mp@subsup{U}{}{m}(1.0)<\mp@subsup{U}{}{m}(1;1)<\mp@subsup{V}{}{m}(0,1
U'f}(0,1)<\mp@subsup{U}{}{f}(0,0)<\mp@subsup{U}{}{f}(1,0)<\mp@subsup{U}{}{f}(1,1
```

Example 5. No Nash equilibrium; two Pareto optimal allocations ( $(1,1)$ and $(0,1))$.

```
Um}(0,0)<\mp@subsup{U}{}{m}(1,1)<\mp@subsup{U}{}{m}(1,0)<\mp@subsup{U}{}{m}(0,1
U'f}(0,1)<\mp@subsup{U}{}{f}(0,0)<\mp@subsup{U}{}{f}(1,0)<\mp@subsup{U}{}{f}(1,1
```

Example 6. No Nash equilibrium; three Pareto optimal allocations ( $(1,1)$, $(0,1)$ and $(1,0))$.

```
Um}(0,0)<\mp@subsup{U}{}{m}(1,1)<\mp@subsup{U}{}{m}(0,1)<\mp@subsup{U}{}{m}(1,0
U
```

Example 7. No Nash equilibrium; all allocations are Pareto optimal.

Table 1 reviews the frequency of occurrence of the different possibilities described above in the total of 576 possible games.

Table 1

As a final remark, we note that to determine the Nash equilibrium only the following utility levels have to be compared:
for the male

$$
U^{m}(1,0) \lesseqgtr U^{m}(0,0) \text { and } U^{m}(1,1) \lesseqgtr U^{m}(0,1)
$$

and for the female

$$
\mathrm{U}^{\mathrm{f}}(0,1) \lesseqgtr \mathrm{U}^{\mathrm{f}}(0,0) \text { and } \mathrm{U}^{\mathrm{f}}(1,1) \lesseqgtr \mathrm{U}^{\mathrm{f}}(1,0)
$$

For each player there are four possible combinations. So, if one would employ the Nash equilibrium (without the requirement that outcomes should be Pareto optimal) only 16 different cases (rather than 576) for the household would have to be considered.

## 3. Empirical implementation and estimation

Following McFadden's random utiliy hypothesis (see, e.g., McFadden (1981)), we assume that the utility level $U^{i}\left(y_{m}, y_{f}\right)$ that individual $i$ attaches to allocation $\left(y_{m}, y_{f}\right)$ can be decomposed into a deterministic component which depends upon a vector $x$ of observed exogenous variables, and a random component $\varepsilon$ which follows some probability distribution:

$$
\begin{equation*}
U^{i}(k, \ell)=x^{\prime} \beta_{k \ell}^{i}+\varepsilon_{k \ell}^{i} \quad i=m, f ; k, \ell=0,1 \tag{3.1}
\end{equation*}
$$

Given the distributional assumptions on the $\varepsilon$ 's, we can calculate the probability that a certain game will be played, i.e. we can evaluate probabilities of the form

$$
\begin{equation*}
P\left[\left\{U^{m}(A)<U^{m}(B)<U^{m}(C)<U^{m}(D)\right\} \text { and }\left\{\left(U^{f}(P)<U^{f}(Q)<U^{f}(R)<U^{f}(S)\right\}\right]\right. \tag{3.2}
\end{equation*}
$$

where ( $A, B, C, D$ ) and ( $P, Q, R, S$ ) are elements from the set of all possible permutations of the elements of $\mathbb{A}=\{(0,0),(0,1),(1,0),(1,1)\}$.

A popular distributional assumption in the discrete choice literature for the random components in (3.1) is the Extreme Value distribution. We assume
that $\varepsilon_{00}^{\mathrm{m}}, \varepsilon_{01}^{\mathrm{m}}, \varepsilon_{10}^{\mathrm{m}}, \varepsilon_{11}^{\mathrm{m}}, \varepsilon_{00}^{\mathrm{f}}, \varepsilon_{01}^{\mathrm{f}}, \varepsilon_{10}^{\mathrm{f}}$ and $\varepsilon_{11}^{\mathrm{f}} \quad$ are independently Extreme Value distributed, i.e.

$$
\begin{equation*}
P\left(\varepsilon_{A}^{i}\langle z)=\exp [-\exp (-z)]\right. \tag{3.3}
\end{equation*}
$$

where $A \in A$ and $i=m, f^{5)}$.
Given these assumptions the following probability can be calculated straightforwardly:

$$
\begin{align*}
& P\{U(A)<U(B)<U(C)<U(D)\}= \\
& \frac{\exp \left(-x^{\prime} \beta_{D}\right)}{\left[\exp \left(-x^{\prime} \beta_{D}\right)+\exp \left(-x^{\prime} \beta_{C}\right)+\exp \left(-x^{\prime} \beta_{B}\right)+\exp \left(-x^{\prime} \beta_{A}\right)\right]} . \\
& \frac{\exp \left(-x^{\prime} \beta_{C}\right)}{\left[\exp \left(-x^{\prime} \beta_{C}\right)+\exp \left(-x^{\prime} \beta_{B}\right)+\exp \left(-x^{\prime} \beta_{A}\right)\right]} . \\
& \frac{\exp \left(-x^{\prime} \beta_{B}\right)}{\left[\exp \left(-x^{\prime} \beta_{B}\right)+\exp \left(-x^{\prime} \beta_{A}\right)\right]} \tag{3.4}
\end{align*}
$$

where, to simplify notation, the superscript i has been omitted ${ }^{6)}$.
Using (3.4) we can calculate the probability that each of the 576 games will be played. The probability that allocation ( $y_{m}, y_{f}$ ) will be observed is now simply the sum of the probabilities of all games with outcome ( $y_{m}, y_{f}$ ).

To be more precise, let $P_{i j}$ be the probability that for household $i$ the $j$-th game is played $(j=1,576)$. Let $S_{k \ell}$ be the set of all games with outcome $(k, \ell) \quad(k, \ell=0,1)$ and let $I_{k \ell}$ be the set of households for which allocation $(k, \ell)$ is observed. Then the likelihood function of the model is given by:

$$
\begin{align*}
& { }_{i \varepsilon}^{\pi} I_{10}\left({ }_{j}{ }_{\varepsilon}^{\varepsilon} S_{10} P_{i j}\right) \cdot{ }_{i \varepsilon}^{\pi} I_{11}\left({ }_{j} \sum_{\varepsilon} S_{11} P_{i j}\right) \tag{3.5}
\end{align*}
$$

where $\beta$ denotes all the parameters appearing in the model.
At the present stage it seems useful to compare our model with the simultaneous model for dummy endogenous variables, as discussed by Heckman (1976) and Schmidt (1981):

$$
\begin{align*}
& y_{m}^{*}=x^{\prime} \beta_{\mathrm{m}}+\gamma_{\mathrm{m}}^{y_{f}}+\varepsilon_{\mathrm{m}}  \tag{3.6a}\\
& \mathrm{y}_{\mathrm{f}}^{*}=\mathrm{x}^{\prime} \beta_{\mathrm{f}}+\gamma_{\mathrm{f}}^{y_{\mathrm{m}}}+\varepsilon_{\mathrm{f}}  \tag{3.6b}\\
& \mathrm{y}_{\mathrm{i}}=1 \text { if } \mathrm{y}_{\mathrm{i}}^{*}>0  \tag{3.6c}\\
& \mathrm{y}_{\mathrm{i}}=0 \text { if } \mathrm{y}_{\mathrm{i}}^{*} \leq 0 \tag{3.6d}
\end{align*}
$$

$$
i=m, f
$$

A major difficulty with model (3.6) is that is requires $\gamma_{m} \cdot \gamma_{f}=0$ in order to be statistically meaningful ${ }^{7}$ ). With some rare exceptions (Waldman(1981) and Ransom (1987)), such a parameter restriction cannot be motivated from economic theory. In fact, it essentially removes simultaneity from the model.

In the present context $\mathrm{y}_{\mathrm{m}}^{*}$ can be interpreted as the difference between the utility the male attaches to participation and the utility he attaches to non-participation, given $\mathrm{y}_{\mathrm{f}}$, i.e. given the action of the female:

$$
\begin{equation*}
y_{m}^{*}=u^{m}\left(1, y_{f}\right)-u^{m}\left(0, y_{f}\right) \tag{3.7}
\end{equation*}
$$

In view of (3.7), equation (3.6a) can be written as:

$$
\begin{align*}
& U^{m}(1,1)-U^{m}(0,1)=x^{\prime} \beta_{m}+\gamma_{m}+\varepsilon_{m}  \tag{3.8a}\\
& U^{m}(1,0)-U^{m}(0,0)=x^{\prime} \beta_{m}+\varepsilon_{m}
\end{align*}
$$

According to the game theoretic model, the expressions for the differences in utility are (using equation (3.1)):

$$
\begin{align*}
& U^{m}(1,1)-U^{m}(0,1)=x^{\prime}\left(\beta_{11}^{m}-\beta_{01}^{m}\right)+\left(\varepsilon_{11}^{m}-\varepsilon_{10}^{m}\right)=x^{\prime} \beta_{1}^{m}+\varepsilon_{1}^{m}  \tag{3.9a}\\
& U^{m}(1,0)-U^{m}(0,0)=x^{\prime}\left(\beta_{10}^{m}-\beta_{00}^{m}\right)+\left(\varepsilon_{10}^{m}-\varepsilon_{00}^{m}\right)=x^{\prime} \beta_{0}^{m}+\varepsilon_{0}^{m}
\end{align*}
$$

Comparing (3.8) and (3.9) it appears that the game theoretic model has some important advantages over the simultaneous model with structural shift. Firstly, the coefficients in (3.8a) are equal to the coefficients in (3.8b), except for the constant term, whereas in the coefficients in (3.9a) may be different from those in (3.9b). Secondly, the error term in (3.8a) is identical to the error term in (3.8b), whereas in (3.9a) and (3.9b) the errors
terms are allowed to be different. The third and perhaps most important advantage of the game theoretic model is that is does not require any logical consistency restriction on the parameters. The root of the logical consistoncy problem of the simulianenous model with structural shift are equations (3.6c) and (3.6d), which describe the relation between the latent variables and the observed discrete variables. In the game theoretic model this relation is described by a game, which essentially replaces equations (3.6c) and (3.6d) .

## 4. Estimation results

The model has been estimated using data from a labor mobility survey in The Netherlands, conducted in 1985. The sample contains 849 households.

Since in The Netherlands (and in our sample) almost all males have a paid job, it seems more interesting to model the choice between working full time and working part time for males rather than their choice between working or not working at all. In our empirical analysis we therefore define $\mathrm{y}_{\mathrm{m}}$ and $\mathrm{y}_{\mathrm{f}}$ as follows:
$y_{m}=1$ if the male works at least 38 hours per week
$=0$ if the male works less than 38 hours per week
$y_{f}=1$ if the female works a positive number of hours per week
$=0$ if the female does not work

The 38 hours cut-off point is motivated by the fact that in The Netherlands a full time job usually stands for a working-week of 38 hours. In the total sample of 849 households allocation $(0,0)$ is observed in 141 cases, $(0,1)$ in 82 cases, $(1,0)$ in 377 cases and $(1,1)$ in 249 cases.

The set of explanatory variables is specified in the equations below:

$$
\begin{align*}
\mathrm{U}^{\mathrm{m}}(\mathrm{k}, \ell) & =\beta_{\mathrm{k} \ell}^{\mathrm{m0}}+\beta_{\mathrm{k} \ell}^{\mathrm{m} 1} \cdot \mathrm{~K} 6+\beta_{\mathrm{k} \ell}^{\mathrm{m} 2} \cdot \mathrm{FS}+\beta_{\mathrm{k} \ell}^{\mathrm{m}} \cdot \mathrm{AGEM}+ \\
& +\beta_{\mathrm{k} \ell}^{\mathrm{m} 4} \cdot \mathrm{EDUCM}+\beta_{\mathrm{k} \ell}^{\mathrm{m}} \cdot \mathrm{NLINC}+\varepsilon_{\mathrm{k} \ell}^{\mathrm{m}}  \tag{4.1a}\\
\mathrm{U}^{\mathrm{f}}(\mathrm{k}, \ell) & =\beta_{\mathrm{k} \ell}^{\mathrm{fO}}+\beta_{\mathrm{k} \ell}^{\mathrm{f} 1} \cdot \mathrm{~K} 6+\beta_{\mathrm{k} \ell}^{\mathrm{f} 2} \cdot \mathrm{FS}+\beta_{\mathrm{k} \ell}^{\mathrm{f} 3} \cdot \mathrm{AGEF}+
\end{align*}
$$

$$
\begin{equation*}
+\beta_{\mathrm{k} \ell}^{\mathrm{f} 4} \cdot \mathrm{EDUCF}+\beta_{\mathrm{k} \ell}^{\mathrm{f} 5} \cdot \mathrm{NLINC}+\varepsilon_{\mathrm{k} \ell}^{\mathrm{f}} \tag{4.1b}
\end{equation*}
$$

where
K6: $\quad 1$ if the household contains at least one child younger than 6 , 0 otherwise
FS: Number of persons in the household
AGEM: Age of the male partner
AGEF: Age of the female partner
EDUCM: Education index of the male partner
EDUCF: Education index of the female partner
NLINC: Non-labor income of the household per week ( x .01 ; in Dfl.).

The education index ranges from 1 to 5,1 representing the lowest and 5 the highest level of education.

Although wage rates do not appear in the model directly, the wage effect is captured by including age and education, which have turned out to be important predictors for (potential) wage rates in several studies (see for example Meyer and Wise (1983)).
For reasons of comparability with the simultaneous model with structural shift, we impose restrictions on the $\beta^{\prime}$ s such that, according to ( 3.8 a ) and (3.8b),

$$
\begin{equation*}
\left(\beta_{11}^{i}-\beta_{01}^{i}\right)=\left(\beta_{10}^{i}-\beta_{00}^{i}\right)+\gamma^{i} \quad i=m, f \tag{4.2}
\end{equation*}
$$

Since a normalization of the utility levels is required, we finally impose $\beta_{00}^{m}=\beta_{00}^{f}=0$. After eliminating $\beta_{11}^{i}$ from the likelihood function (3.5) using (4.2), the likelihood is maximized with respect to $\beta_{01}^{\mathrm{m}}, \beta_{10}^{\mathrm{m}}, \gamma^{m}, \beta_{01}^{\mathrm{f}}, \beta_{10}^{\mathrm{f}}$, and $\gamma^{f}$.

Table 2 presents the estimation results ${ }^{8)}$.

Table 2. Estimation Results

| parameter | estimate | t-value |
| :---: | :---: | :---: |
| $\beta_{01}^{m}$ |  |  |
| constant | 52.67 | 1.59 |
| K6 | -2.53 | -1.58 |
| FS | 0.06 | 0.07 |
| AGEM | -0.83 | -1.65 |
| EDUCM | -1.42 | -1.31 |
| NLINC | -0.64 | -1.19 |
| $\beta_{10}^{\text {m }}$ |  |  |
| constant | -1.01 | -0 72 |
| K6 | 0.33 | 2.04 |
| FS | -0.01 | -0.03 |
| AGEM | 0.02 | 1.16 |
| EDUCM | 0.46 | 4.18 |
| NLINC | -0.11 | -1.33 |
| $\gamma^{m}$ | -2.17 | $-6.77$ |
| $\beta_{01}^{f}$ |  |  |
| constant | -12.12 | -2. 25 |
| K6 | -0.57 | -3.32 |
| FS | -0.78 | -4.56 |
| AGEF | -0.02 | -1.54 |
| EDUCF | 0.57 | 5.10 |
| NLINC | 0.01 | 0.08 |
| $\beta_{10}^{f}$ |  |  |
|  |  |  |
| constant | 2.56 | 129 |
| K6 | -4.14 | -2.79 |
| FS | 0.03 | 0.06 |
| AGEF | -0.07 | -1.91 |
| EDUCF | -0.23 | -0 80 |
| NLINC | 0.44 | 1.30 |
| $\gamma^{f}$ | 14.32 | 2.67 |
| log likelihood | -892.9 |  |

First of all we note that both $\gamma_{m}$ and $\gamma_{f}$ are significantly different from zero, i.e. our results reject the restrictions on preferences that would be imposed by the logical consistency condition for the simultaneous model with
structural shift. Notice that $\gamma_{m}$ and $\gamma_{f}$ have opposite signs. The negative value for $\gamma_{m}$ indicates that labor force participation of the wife makes the husband more inclined to work part time rather than full time. For females we observe a difforent effect. A fuli time joi of the husband makes the wifemore inclined to work than a part time job of the husband.

In discussing the estimation results for the $\beta^{\prime} s$, we choose to focus primarily on $U^{i}(1,0)-U^{i}(0,1)$, i.e. the difference between the utility partner $i$ attaches to 'male works full time, female does not work' and the utility he/she attaches to 'male works part time, female works'. For the male the presence of young children, his age and his level of education have a significant positive effect on this difference. So, for example, the older the male, the more he prefers to be the only breadwinner in the household. For females, the size of the family has a significant positive effect on the utility difference, whereas the effect of her education level is significantly negative. So, the smaller the family and the higher her level of education, the more the wife prefers to share working outdoors with her husband. The effect of having young children turns out to be more subtle. The presence of a young child decreases the utility the female attaches to the allocation 'male works part time, female works', but also the utility she attaches to 'male works full time, female does not work'.

To see how the effects ultimately affect the participation probabilities, we have calculated these probabilities for different types of households; see table 3 (remember that for males 0 means working full time and 1 working part time).

Table 3. Estimated participation probabilities for different households ${ }^{\text {a) }}$


AGEM $=25$
AGEF $=25$
EDUCM =
EDUCF $=1$
$\mathrm{K} 6=0$
FS $=2$

AGEM $=25$
AGEF $=25$
EDUCM $=1$
0.46
0.06
0.39
0.10
0.49
0.15

EDUCF $=1$
$\mathrm{K} 6=1$
$\mathrm{FS}=3$
AGEM $=40$
AGEF $=40$

| EDUCM | 1 | 0.38 | 0.05 | 0.49 | 0.08 | 0.58 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | 0.13

EDUCF $=1$
$\mathrm{K} 6=1$
$\mathrm{FS}=3$
AGEM $=25$
AGEF $=25$
EDUCM $=1$
$0.45 \quad 0.03$
0.47
0.05
0.52
0.08

EDUCF $=1$
$\mathrm{K} 6=1$
$\mathrm{FS}=4$
AGEM $=25$
AGEF $=25$
EDUCM $=4$
0.03
0.23
0.12
0.62
0.74
0.85

EDUCF $=3$
$\mathrm{K} 6=0$
$\mathrm{FS}=2$
AGEM $=50$
AGEF $=50$
$\begin{array}{lllllll}\text { EDUCM } & \begin{array}{lllll}4 & 0.11 & 0.15 & 0.22 & 0.52\end{array} & 0.74 & 0.67\end{array}$
EDUCF $=3$
$\mathrm{K} 6=0$
$\mathrm{FS}=2$
a) NLINC has been set to its sample mean.

One of the things table 3 shows is that the presence of young children in the household does not only decrease the participation probability for the
female, but also makes the husband more likely to have a part time rather than a full time job, especially in younger families. Households with a full time working male and a working female are most frequently observed among the higher educated and swaller families.

The generality of the model allows to test for equality of the systematic part of male and female preferences ${ }^{9}$ ). On the basis of a likelihood ratio test the joint hypothesis $\beta_{01}^{m}=\beta_{01}^{f}, \quad \beta_{10}^{m}=\beta_{10}^{f}$ and $\gamma^{m}=\gamma^{f}$ is rejected decisively ${ }^{10)}$. This result indicates that employing a more general decision making framework may not only be theoretically more appealling, but also empirically important in describing household labor supply decisions.

## 5. Conclusions

We have presented and estimated a simultaneous model for discrete choice, rooted within a utility maximization framework where different members of the decision making unit are allowed to have different preferences.

A distinctive property of the model is that observed allocations are assumed to be Pareto optimal outcomes of a game being played in the household, which seems to be more realistic in a household context than the use of noncooperative equilibria (such as Nash or Stackelberg). In contrast to simultaneous models for discrete choice with structural shift, the model in this paper does not require any logical consistency restriction on the parameters to be statistically meaningful.

Although the empirical specification has been relatively simple, the results suggest that the model combines some desirable theoretical properties with the capability of appropriately describing observed data on household labor force participation.

## Footnotes

1) It may be useful to note that such model can be interpreted as the reduced form of a system of two simultaneous equations, the first equation being the conditional male labor supply equation given fixed female labor supply and the second equation being the conditional female labor supply equation given fixed male labor supply.
2) Kooreman and Kapteyn (1985) use survey information on actual working hours and on desired working hours of respondents to arrive at identification in a game theoretic household labor supply model. Their procedure, however, requires strong assumptions on the interpretation of the responses to the survey questions.
3) Let ( $k, \ell$ ) be a Nash Equilibrium (NE) which is not Pareto optimal $(k, \ell=0,1)$. Because it is a NE we have $U^{m}(k, l)>U^{m}(1-k, \ell)$ and $U^{f}(k, l)>U^{f}(k, 1-$ $\ell)$. So, the allocation at which both players are better off than at $(k, \ell)$ is (1-k,1- ).
4) Suppose $(k, \ell)$ is one of the two NE $(k, \ell=0,1)$. Then we have

$$
\begin{equation*}
U^{m}(k, \ell)>U^{m}(1-k, \ell) \tag{F1}
\end{equation*}
$$

and

$$
\begin{equation*}
U^{f}(k, \ell)>U^{f}(k, 1-\ell) \tag{F2}
\end{equation*}
$$

so that the other $N E$ must be $(1-k, 1-\ell)$. This implies
$U^{m}(1-k, 1-\ell)>U^{m}(k, 1-\ell)$
and

$$
\begin{equation*}
U^{f}(1-k, 1-\ell)>U^{f}(1-k, \ell) \tag{F4}
\end{equation*}
$$

Allocations ( $1-\mathrm{k}, \ell$ ) and ( $k, 1-\ell$ ) cannot be Pareto more efficient than ( $k, \ell$ ) in view of (F1) and (F2), whereas ( $1-\mathrm{k}, \ell$ ) and ( $k, 1-\ell$ ) cannot be Pareto more efficient than ( $1-\mathrm{k}, 1-\ell$ ) in view of (F3) and (F4). If

$$
\begin{equation*}
U^{i}(k, \ell)>U^{i}(1-k, 1-\ell) \quad i=m, f \tag{F5}
\end{equation*}
$$

then only $(k, \ell)$ is Pareto optimal $((1-k, 1-\ell)$ if the inequality is reversed). If

$$
U^{i}(k, \ell)>U^{i}(1-k, 1-\ell) \quad \text { and } U^{j}(k, \ell)<U^{j}(1-k, 1-\ell) \quad i \neq j
$$

then both $(k, \ell)$ and $(1-k, 1-\ell)$ are Pareto optimal.
5) Another possibility is to assume that the $\varepsilon$ 's follow a multivariate normal distribution. In that case the evaluation of probabilities of the form (3.1) requires higher order numerical integration. The computational burden of estimation would then become so large, that a supercomputer would be needed. This will be left for future research.
6) The fact that $\varepsilon_{A}^{i}$ has a non zero mean $\left(E \varepsilon_{A}^{i}=.577216\right)$ is innocent as it only affects the constant term in $\beta_{A}^{i}$.
7) This can be seen as follows. The model implies the following conditions on ( $\varepsilon_{m}, \varepsilon_{f}$ ) for the different regimes:

$$
\begin{aligned}
& \left\{\begin{array} { l } 
{ y _ { m } = 1 } \\
{ y _ { f } = 1 }
\end{array} \quad \text { implies } \quad \left\{\begin{array} { l } 
{ x ^ { \prime } \beta _ { m } + \gamma _ { m } + \varepsilon _ { m } > 0 } \\
{ x ^ { \prime } \beta _ { f } + \gamma _ { f } + \varepsilon _ { f } > 0 }
\end{array} \quad \text { or } \quad \left\{\begin{array}{l}
\varepsilon_{m}>-x^{\prime} \beta_{m}-\gamma_{m} \\
\varepsilon_{f}>-x^{\prime} \beta_{f}-\gamma_{f}
\end{array}\right.\right.\right. \\
& \left\{\begin{array} { l } 
{ y _ { m } = 1 } \\
{ y _ { f } = 0 }
\end{array} \quad \text { implies } \quad \left\{\begin{array} { l } 
{ x ^ { \prime } \beta _ { m } } \\
{ x ^ { \prime } \beta _ { f } + \gamma _ { f } + \varepsilon _ { f } \leq 0 }
\end{array} \quad \text { or } \quad \left\{\begin{array}{l}
\varepsilon_{m}>-x^{\prime} \beta_{m} \\
\varepsilon_{f} \leq-x^{\prime} \beta_{f}-\gamma_{f}
\end{array}\right.\right.\right. \\
& \left\{\begin{array} { l } 
{ y _ { m } = 0 } \\
{ y _ { f } = 1 }
\end{array} \quad \text { implies } \quad \left\{\begin{array} { l } 
{ x ^ { \prime } \beta _ { m } + \gamma _ { m } + \varepsilon _ { m } \leq 0 } \\
{ x ^ { \prime } \beta _ { f } + \varepsilon _ { f } > 0 }
\end{array} \quad \text { or } \quad \left\{\begin{array}{l}
\varepsilon_{m} \leq-x^{\prime} \beta_{m}-\gamma_{m} \\
\varepsilon_{f}>-x \beta_{f}
\end{array}\right.\right.\right. \\
& \left\{\begin{array} { l } 
{ y _ { m } = 0 } \\
{ y _ { f } = 0 }
\end{array} \quad \text { implies } \quad \left\{\begin{array} { l } 
{ x ^ { \prime } \beta _ { m } } \\
{ x ^ { \prime } \beta _ { f } }
\end{array} + \varepsilon _ { m } \leq 0 \quad \text { or } \quad \left\{\begin{array}{l}
\varepsilon_{m} \leq-x^{\prime} \beta_{m} \\
\varepsilon_{f} \leq-x^{\prime} \beta_{f}
\end{array}\right.\right.\right.
\end{aligned}
$$

In figure 1, the conditions for the different regimes have been represented in the $\left(\varepsilon_{m}, \varepsilon_{f}\right)-p l a n e$ for $\gamma_{m}>0$ and $\gamma_{f}>0$. Clearly, the conditions define sets in the $\left(\varepsilon_{m}, \varepsilon_{f}\right)-p l a n e$ which are neither mutually exclusive nor exhaustive. As
a consequence, the probabilities of observing each of the regimes do not add up to one.


Figure 1. Regions in the $\left(\varepsilon_{\mathrm{m}}, \varepsilon_{\mathrm{f}}\right)$-plane
8) The model is not identified if there are no explanatory variables (other than the constant term). In that case there are six parameters to be estimated, while there are only three independent probabilities to be explained. A formal proof of identification in the case with explanatory variables can be given along the lines set forth in Rothenberg (1971) but will not be presented here.
 model reduces to a multinomial logit model.
10) The test statistic follows a $x^{2}(13)$ distribution and is computed at 69.2. The critical levels for $5 \%$ and $1 \%$ are 22.4 and 27.7 , respectively.

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```
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    Local Times of Bernoulli Walk
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    Some methodological issues in the implementation
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        Sampling for Quality Inspection and Correction: AOQL Performance
        Criteria
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    afhankelijkheden
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249 Peter M. Kort
    The Influence of a Stochastic Environment on the Firm's Optimal Dyna-
    mic Investment Policy
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    Preliminary version
    The reaction of the firm on governmental policy: a game-theoretical
    approach
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    Higher order moments of bilinear time series processes with symmetri-
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    Evaluatie van marketing-activiteiten
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    DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen
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    On the identifiability of household production functions with joint
    products: A comment
255 B. van Riel
    Was er een profit-squeeze in de Nederlandse industrie?
256 R.P. Gilles
    Economies with coalitional structures and core-like equilibrium con-
    cepts
```

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Computation of an industrial equilibrium
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Association schemes
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Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining

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Competition, Risk Neutrality and Loan Commitments
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Preference Interdependence and Habit Formation in Family Labor Supply
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Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

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```
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    Note
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    negativity constraints
```


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Estimation of time dependent parameters in lineair models
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