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A price adjustment process for an economy with a block-diagonal pattern*)

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by

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Abstract

In this paper we deal with the problem of finding an equilibrium price vector in an economy possessing a block-diagonal demand/supply structure. The most appealing example of such an economy concerns an international trade model in which each country has a group of domestic goods traded only within that country, while a group of common goods is traded among all countries. An equilibrium price vector is then defined as a price vector at which for all goods demand equals supply.

Here we present a price adjustment process that can start from an arbitrary starting vector and always reaches such an equilibrium price vector. The successive price adaptions are related to the sign pattern of the excess demand function and their economic interpretation is quite attractive. For example, prices corresponding to goods in excess demand (supply) are increased (decreased).

The process presented here is based on processes developed for solving the so-called Non-Linear Complementarity Problem (NLCP) on a simplotope S. Some adaptions are needed because our problem cannot be stated as a pure NLCP on S. In fact one complementarity constraint is missing.

Keywords: price equilibrium, block-diagonal economy, adjustment process, complementarity.

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1. Introduction

Recently van der Laan and Talman [5] presented convergent price adjustment processes for a Walrasian exchange economy. These processes are both global and universal. So, for any continuous excess demand function the processes converge to an equilibrium price vector from any arbitrarily chosen price vector. This is a substantial advantage above the classical tatonnement process of which we know that it is neither global nor universal, see e.g. Scarf [9], Saari and Simon [8] and Saari [7].

The processes presented in [5] are governed by the excess demand on the markets and the position of the current price vector in the unit price simplex $S^n = \{p \in R^{n+1}_{+} | \Sigma^n_{j=0} p_j = 1\}$ with respect to an arbitrarily chosen starting price vector. By keeping in mind the starting price vector the processes are prevented from cycling, so that convergency is guaranteed. The fact that the price adjustment is governed by excess demands only while additional information about e.g. the gradients of the excess demand function is not needed, allows us to consider these processes as some type of a tatonnement process. We speak about Starting Point Remembering Adjustment (SPRA) processes. In this paper we consider an SPRA-process for an economy with a block-diagonal pattern. Such a pattern occurs in, e.g. international trade models.

In van den Elzen, van der Laan and Talman [2] the SPRA-processes for finding an equilibrium price vector on the n-dimensional unit price simplex S^n are generalized for solving the Non-Linear Complementarity Problem (NLCP) on a simplotope, i.e. a product space of several unit simplices. Let S be a simplotope, i.e.

$$S = \Pi^H_{h=0} S^{n_h} = \{x \in R^M | x = (x_0^T, x_1^T, \ldots, x_H^T)^T \text{ with } x_h \in S^{n_h},\ h = 0,1,\ldots,H\},$$

where $M = \Sigma^H_{h=0}(n_h+1)$. Furthermore, let z be a continuous function from S to R^M for which $x_j^T z_j(x) = 0$ for all j and for all $x \in S$, where $z_j\colon S \to R^{n_j+1}$ and $z(x) = (z_0^T(x), z_1^T(x), \ldots, z_H^T(x))^T$. Then the NLCP on S consists of finding a point x^* in S for which $z(x^*) \leq 0$.

Clearly, the economic equilibrium problem is an NLCP on the particular simplotope $S = S^n$ with z the excess demand function. Another NLCP on S is the problem of finding a Nash equilibrium for a noncooperative more person game. In that case the processes adjust the strategies of the players until an equilibrium strategy vector is found. For a full discussion of this problem we refer to van den Elzen and Talman [3]. The adjustment processes on S described in this latter paper utilize the conditions $x_j^T z_j(x) = 0$ for all j and x. The equilibrium problem for an economy with a block-diagonal pattern has been formulated by van der Laan [4] as a zero point problem on a simplotope. The problem differs from the NLCP by the fact that $x_0^T z_0(x) = 0$ does not hold for all x. Therefore it is not straightforward, whether the processes described in [2] can be applied to this problem. In this paper we want to consider this.

The model and its formulation as a zero point problem on a simplotope is given in section 2. In section 3 we evaluate the existing SPRA-processes on S when applied to the international trade model. It appears that some adaptions are needed. Section 4 deals with a specific example which gives us more insight into the problem. The general case is treated in section 5. The concluding section evaluates the process as a price adjustment process.

2. A block-diagonal economy

In this section we describe an economy possessing a block-diagonal structure. This means that the demand/supply structure is such that we can distinguish between groups of goods traded among a single group of agents and goods traded among all agents.

We consider an economy where agents are divided in H disjunct subsets. Accordingly there exist H groups of non-common goods, each of them consisting of goods traded only within one specific subset of agents. Besides there is a group of common goods traded among all agents. The transactions made in the economy are exchanges of goods based on prices. An equilibrium is a price vector for which demands equal supplies.

The partition of the goods in common goods and non-common goods results in a block-diagonal structure in which each group of non-common goods forms a block. The interpretation of this model as an international trade model is obvious. The common goods are the internationally traded goods. Each group of non-common goods refers to the domestic goods of a certain country, while the related subset of agents is the set of consumers in that country. In the sequel we describe the model of a block-diagonal economy in the context of an international trade model.

Let the groups of non-common goods be indexed by $h = 1,2,\ldots,H$. The same index is used for the related groups of consumers. The index $h = 0$ corresponds to the group of common goods. In the sequel we denote $I_H = \{1,\ldots,H\}$. The number of goods in the h-th group is given by n_h, $h \in I_H$, while the number of common goods is n_0+1. So, the total number of goods is N+1, with $N = \Sigma_{h=0}^{H} n_h$. A price vector $p \in R_+^{N+1}\setminus\{0\}$ can be written as $p = (p_0^\top,p_1^\top,\ldots,p_H^\top)^\top$ with $p_0 \in R_+^{n_0+1}$ and $p_h \in R_+^{n_h}$, $h \in I_H$. Then p_{jk} denotes the price of good k in group j. The excess demand function of country h, $h \in I_H$, is given by the continuous function z^h from $\bar{R}^{N+1}$ into R^{N+1}, where $\bar{R}^{N+1}$ is the subset of R_+^{N+1}, such that for all $p \in \bar{R}^{N+1}$, $(p_0^\top,p_j^\top)^\top$ has at least one positive element for each $j \in I_H$. More precisely, $z^h(p) = ((z_0^h(p))^\top,(z_1^h(p))^\top,\ldots,(z_H^h(p))^\top)^\top$ where $z_0^h(p)$:

$\bar{R}^{N+1} \to R^{n_0+1}$ denotes the excess demand of country h for common goods and $z_k^h(p) : \bar{R}^{N+1} \to R^{n_k}$ the excess demand of the consumers in country h for the domestic commodities of country k with $k,h \in I_H$. Clearly, $z_j^h(p) = 0$ for all $h \in I_H$ and for all $j \neq 0,h$. Because the excess demand of country h only depends on the prices of the common goods and the prices of the goods in country h, $z^h(p)$ can be written as $z^h(p_0,p_h)$. Furthermore we assume that z^h is homogeneous of degree zero in (p_0,p_h) and that each country meets the budget constraint, i.e.

$$p^\top z^h(p) = p_0^\top z_0^h(p_0,p_h) + p_h^\top z_h^h(p_0,p_h) = 0. \tag{2.1}$$

Finally we suppose that there is a positive excess demand for goods with prices equal to zero.

The problem of finding equilibrium prices in this economy can now be stated as the problem of finding a vector $\bar{p} = (\bar{p}_0^\top, \bar{p}_1^\top, \ldots, \bar{p}_H^\top)^\top \in \bar{R}^{N+1}$ such that

$$\begin{aligned} &\text{i)} \quad z_0(\bar{p}) = \Sigma_{h=1}^H \; z_0^h(\bar{p}_0,\bar{p}_h) = 0 \\ &\text{ii)} \quad z_h(\bar{p}) = \; z_h^h(\bar{p}_0,\bar{p}_h) = 0. \end{aligned} \tag{2.2}$$

Observe that (2.1) and (2.2) imply that in equilibrium both $\bar{p}_h^\top z_h^h(\bar{p}_0,\bar{p}_h) = 0$ and $\bar{p}_0^\top z_0^h(\bar{p}_0,\bar{p}_h) = 0$ for all $h \in I_H$. The latter equation says that in equilibrium the balance of payments of each country reveals neither surplus nor deficit. Of course, this does not need to be true outside equilibrium.

To solve the equilibrium problem Mansur and Whalley [6] used a decomposition procedure. Utilizing the block-diagonal structure they compute, for each country, equilibrium prices for its domestic goods if prices for the common goods are given. Given these equilibrium prices for the domestic goods, a zero for the aggregate excess demand for common goods is computed. This procedure is sure to work if each subproblem of finding equilibrium prices for the domestic goods has a unique solution (see [10]).

Also utilizing the block-diagonal structure, van der Laan [4] reformulated (2.2) as a zero point problem on the product space of H+1 unit simplices. This latter problem can be solved with known techniques.

Both methods permit substantial computational savings when compared with solving the full-dimension problem without using the block-diagonal structure. In the latter case the sum of all prices is normalized to one and the problem is solved by finding a zero point of an excess demand function on S^N.

Here we consider the reformulation of the problem as a zero point problem on the simplotope $S = \Pi_{h=0}^{H} S^{n_h}$. Recall that $S^{n_h} = \{x \in R_+^{n_h+1} \mid \Sigma_{j=1}^{n_h+1} x_j = 1\}$. An element $q \in S$ is written as $q = (q_0^\top, q_1^\top, \ldots, q_H^\top)^\top$ with $q_h \in S^{n_h}$. In the sequel we denote the set $\{(h,0),\ldots,(h,n_h)\}$ by $I(h)$, $h = 0,\ldots,H$. Furthermore $I = \cup_{h=0}^{H} I(h)$.

For each $q \in S$ we define for all $h \in I_H$ price vectors $\pi^h(q_0,q_h) = [(\pi_0^h(q_0,q_h))^\top,(\pi_h^h(q_0,q_h))^\top]^\top \in R^{n_0+1} \times R^{n_h}$ by

$$\pi_{0k}^h(q_0,q_h) = q_{h0}q_{0k} \qquad , (0,k) \in I(0)$$

$$\pi_{hk}^h(q_0,q_h) = q_{hk} \qquad , (h,k) \in I(h)\setminus\{(h,0)\}.$$

Now, let $z(q) = (z_0^\top(q), z_1^\top(q), \ldots, z_H^\top(q))^\top \in R^{N+1}$ be the total excess demand at prices $\pi^h(q)$, $h \in I_H$, i.e.

$$z_0(q) = \Sigma_{h=1}^{H} z_0^h(\pi^h(q_0,q_h))$$

$$z_h(q) = z_h^h(\pi^h(q_0,q_h)) \qquad , h \in I_H.$$

Next we define $\bar{z}(q) = (\bar{z}_0^\top(q), \bar{z}_1^\top(q), \ldots, \bar{z}_H^\top(q))^\top \in \Pi_{h=0}^{H} R^{n_h+1}$ by

$$\bar{z}_{0k}(q) = \alpha z_{0k}(q) \qquad , (0,k) \in I(0),$$

where $0<\alpha<1/H$ is arbitrarily chosen, and for all $h \in I_H$

$$\bar{z}_{h0}(q) = \Sigma_{k=0}^{n_0} q_{0k} z_{0k}^{h}(\pi^{h}(q_0, q_h))$$

$$\bar{z}_{hk}(q) = z_{hk}(q) \qquad , (h,k) \in I(h) \setminus \{(h,0)\}. \qquad (2.3)$$

Observe that for all $h \in I_H$ and for all $q \in S$, (2.1) implies

$$q_h^T \bar{z}_h(q) = \Sigma_{k=0}^{n_0} \pi_{0k}^h z_{0k}^h(\pi^h) + \Sigma_{k=1}^{n_h} \pi_{hk}^h z_{hk}^h(\pi^h) = 0. \qquad (2.4)$$

However, this complementarity condition does not hold for h=0. Then we get

$$q_0^T \bar{z}_0(q) = \alpha \Sigma_{h=1}^{H} \bar{z}_{h0}(q).$$

It follows immediately that $\bar{z}(q^*) = 0$ if and only if $(q_0^*, \bar{q}_1^*, \ldots, \bar{q}_H^*) \in S^{n_0} \times \Pi_{h=1}^{H} R^{n_h}$ is an equilibrium price vector, where $\bar{q}_{hj}^* = q_{hj}^*/q_{h0}^*$, $h \in I_H$ and $j = 1,\ldots,n_h$. This is due to the positiveness of excess demands at zero prices ($q_{h0}^* \neq 0$), and the homogeneity of degree zero. In [4] it is proved that the zero points of $\bar{z}$ correspond to the intersection points of well-defined subsets of S covering the whole set. The artificial α is needed to restrict the set of intersection points to the set of zero points. Then a simplicial algorithm is used to find such an intersection point.

3. Price adjustment processes

Van den Elzen, van der Laan and Talman [2] have presented three SPRA-processes for solving the NLCP on a simplotope S. For all these processes the starting point q^0 can be chosen arbitrarily. Here we assume that this point lies in the interior of S. This starting point is left along one of several rays pointing in different directions. The particular ray along which the process starts is determined by the function value at q^0.

In the first process the ray corresponds to the index $(h,j) \in I$ with $z_{hj}(q^0)$ being maximal over all components of $z(q^0)$. Since I contains $\Sigma_{h=0}^{H}(n_h+1)$ elements the number of rays is equal to $\Sigma_{h=0}^{H}(n_h+1)$. We call this process the sum-process ([2]) or the Maximum Value (MV) process.

In the second process the ray is determined by the set of indices $\{(h,k_h) \in I | h = 0,\ldots,H\}$ with $z_{hk_h}(q^0)$ maximal over all components of $z_h(q^0)$, $h = 0,\ldots,H$. Clearly there are $\Pi_{h=0}^{H}(n_h+1)$ of such sets, and so is the number of rays. We will call this process the product-process ([2]) or the Multiple Maximum Value (MMV) process.

The third will be called the exponent-process ([2]) or the Sign Vector (SV) process. In this process the ray along which the starting point is left depends on the signs of the components of $z(q^0)$. Since for all h and for all $q \in S$, $q_h^T z_h(q) = 0$, we must have that z_h contains at least one negative and at least one positive element. So the number of sign patterns equals $\Pi_{h=0}^{H}(2^{n_h+1}-2)$ and so is the number of rays. Observe that we assume that $z(q^0)$ contains no zero elements.

We now consider the applicability of these processes to the international trade problem. From the starting point q^0 the MV-process follows a path of points q for which the components whose corresponding $\bar{z}$-value is below the maximum are decreased proportionally with respect to q^0, while the components with corresponding $\bar{z}$-value equal to the maximum are relatively higher. More formally, if q is on the path traced by the process then q and $\bar{z}(q)$ satisfy for all $(h,k) \in I$

$$q_{hk} = b_h q^0_{hk} \qquad \text{if } \bar{z}_{hk}(q) < \max_{(i,j) \in I} \bar{z}_{ij}(q)$$

$$q_{hk} > b_h q^0_{hk} \qquad \text{if } \bar{z}_{hk}(q) = \max_{(i,j) \in I} \bar{z}_{ij}(q)$$

with for all $h = 0,\ldots,H$, $0<b_h<1$.
The path always reaches a point q^* for which there exists an element $h \in I_H \cup \{0\}$ with the property that

$$\bar{z}_{hk}(q^*) = \max_{(i,j) \in I} \bar{z}_{ij}(q^*) \text{ for all } (h,k) \in I(h).$$

In [4] it is proved that such a point is a zero point of $\bar{z}$. So, although $\bar{z}$ violates the complementarity condition that $q_h^T\bar{z}_h(q) = 0$ for all $h = 0,1,\ldots,H$ and for all $q \in S$, the MV-process is useful to find a solution point. In [4], this path is followed by simplicial approximation.

However, the description of the path in economic terms is troublesome. For instance, at the starting point the process adapts q^0 by increasing the component q^0_{hk} for which $\bar{z}_{hk}(q^0)$ has the highest value and decreasing all other components q^0_{hj}, $j\neq k$, of q^0_h. So, all components of $\bar{z}(q)$ are compared with each other, not only at q^0 but throughout the whole process. However, that does not only mean that excess demands are compared between different countries, but also that within country h the excess demand for domestic goods $\bar{z}_{hk}$, $k\neq 0$, is compared with $\bar{z}_{h0}$ being the total value of the excess demand for common goods against q_0. This does not seem very attractive. Also the fact that at the starting point q^0_h is adapted for only one h is not very reasonable.

In general the prices of the commodities in some country or on the world market are not adjusted as long as the corresponding excess demands are below the maximum excess demand over all goods, where the excess demand of a country for common goods is measured by the total value against the world market prices q_0. However, it is much more reasonable to suppose that prices on the world market and the domestic markets would be adjusted simultaneously.

This behaviour is revealed by the Multiple Maximum Value process. In this process the starting point q^0 is left by adjusting each

q_h^0, $h \in I_H \cup \{0\}$. For each h the component of q_h^0 corresponding with the maximal element of $\bar{z}_h$ is increased, while all other elements of q_h^0 are decreased. Also this process still works if applied to the international trade model. For more technical details we refer to [2]. Here we only mention that excess demands for domestic goods are again compared with the value of the excess demand for common goods. To overcome this difference in measuring excess demands we want to consider the Sign Vector process.

The SV-process, presented in [2] as the exponent-process, seems very suitable for the international trade model, because not the values but only the signs of the components of the excess demand function are taken into account. So, in this process the comparison between components with different meaning is out of question. However, the process described in [2] utilizes the complementarity conditions obeyed by the excess demand function. As shown in the previous section the function $\bar{z}$ violates the Walras condition $q_0^T\bar{z}_0(q) = 0$ for all q.

The SV-process leaves the starting point by increasing all components with positive function value and by decreasing all components with negative function value. If the complementarity conditions hold there is for all $h \in I_H \cup \{0\}$ always at least one component with positive function value and at least one component with negative function value, which allows for a movement described above. However, if the complementarity condition does not hold for certain h, all components of q_h^0 may have positive (or negative) function values and no movement is possible because the sum of the components must be equal to one. On the other hand the international trade model possesses some additional interdependences between $\bar{z}_0$ and the components $\bar{z}_{h0}$. As an example we mention that $\bar{z}_{h0}$ can't be positive for all $h \in I_H$ if $\bar{z}_0 = 0$, because for at least one country the value of the excess demand for common goods must be nonpositive if the common goods markets are in equilibrium.

In section 5 we adapt the SV-process to the international trade model. In the next section we first consider a simple example to get a better understanding for the general case.

4. An example

In this section we want to consider an "international trade problem" with only one country, two common goods and one domestic good. Of course, the distinction between common and non-common goods makes no sense in this case and we can solve the equilibrium price vector problem by searching for such a vector of prices on the 2-dimensional unit price simplex S^2. However, here we formulate the problem on the simplotope $S = S^1 \times S^1$ with the two common goods indexed by (0,0) and (0,1) and the non-common good indexed by (1,1). It will appear that this artificial example provides useful insight for attacking the general problem with more countries.

Our problem concerns the search for a vector q^* in S at which the "excess demand" $\bar{z}(q^*) = 0$. Recall from session 2 that $\bar{z}$ is a function from S to R^4 with

$$\bar{z}_{0j}(q) = z_{0j}(\pi(q)) \qquad ,j=0,1$$

$$\bar{z}_{10}(q) = q_{00}z_{00}(\pi(q)) + q_{01}z_{01}(\pi(q))$$

$$\bar{z}_{11}(q) = z_{11}(\pi(q))$$

where $\pi(q)$ is the vector of prices $(\pi_{00},\pi_{01},\pi_{11})^T$ defined by

$$\pi_{0j}(q) = q_{10}q_{0j} \qquad ,j=0,1$$

$$\pi_{11}(q) = q_{11},$$

and with $(z_{00},z_{01},z_{11})^T$ the excess demand at prices $\pi(q)$. Since the process only considers the signs of the components of $\bar{z}$, we don't need the artificial α present in the definition of $\bar{z}_0$, i.e. α can be set equal to one.

In this section we don't want to confuse the reasoning with many definitions and formulas. For the moment it suffices to know that the SV-process considers vectors in the intersections of corresponding

Primal (P) and Dual (D) subsets of S. For certain sign vectors $s = (s_{00}, s_{01}, s_{10}, s_{11})^T$ with $s_{hk} \in \{-1,0,+1\}$ for all (h,k) we define a primal set P(s) and a corresponding set D(s). Let q^0 be the starting point of the process. We assume that q^0 lies in the interior of S and hence $q^0_{hk} > 0$ for all (h,k). Now, suppose that some point q lies in the intersection of P(s) and D(s) for certain sign vector s. Then the first set gives information about the position of q with respect to q^0 while the fact that $q \in D(s)$ says that the sign pattern of $\bar{z}(q)$ equals s. We can restrict our attention to those sign vectors s for which there could exist a q in S with $\text{sgn } \bar{z}(q) = s$. As mentioned before this set of so-called allowed sign vectors differs from the collection of possible sign vectors in case of an NLCP on S. This implies that we have to adapt the sets P(s) to our international trade problem.

We now consider the particular example. For $\bar{z}_1$ the complementarity condition holds, i.e.

$$
\begin{aligned}
q_1^T\bar{z}_1(q) &= q_{10}\bar{z}_{10}(q) + q_{11}\bar{z}_{11}(q) \\
&= q_{10}q_{00}z_{00}(\pi(q)) + q_{10}q_{01}z_{01}(\pi(q)) + q_{11}z_{11}(\pi(q)) \\
&= \pi_{00}z_{00}(\pi) + \pi_{01}z_{01}(\pi) + \pi_{11}z_{11}(\pi) = 0,
\end{aligned}
$$

because of Walras' law. So, for any $q \in S$ we have that $\bar{z}_{10}(q)\bar{z}_{11}(q) \leq 0$, i.e. the two components are not both positive or negative. With respect to $\bar{z}_0$ the complementarity condition does not hold and hence all sign patterns are possible. On the other hand, it follows from the definition of $\bar{z}_{10}(q)$ that $\bar{z}_{10}(q) > 0$ (< 0) if both $\bar{z}_{00}(q)$ and $\bar{z}_{01}(q)$ are nonnegative (nonpositive). If q is in the interior of S and $\bar{z}(q) \neq 0$ it follows that the collection of possible sign patterns is given by the set T with

$$
T = \left\{ \begin{bmatrix}+\\+\\+\\-\end{bmatrix}, \begin{bmatrix}-\\-\\-\\+\end{bmatrix}, \begin{bmatrix}+\\-\\+\\-\end{bmatrix}, \begin{bmatrix}+\\-\\-\\+\end{bmatrix}, \begin{bmatrix}-\\+\\+\\-\end{bmatrix}, \begin{bmatrix}-\\+\\-\\+\end{bmatrix}, \begin{bmatrix}+\\0\\+\\-\end{bmatrix}, \begin{bmatrix}-\\0\\-\\+\end{bmatrix}, \begin{bmatrix}0\\+\\+\\-\end{bmatrix}, \begin{bmatrix}0\\-\\-\\+\end{bmatrix}, \begin{bmatrix}+\\-\\0\\0\end{bmatrix}, \begin{bmatrix}-\\+\\0\\0\end{bmatrix} \right\}.
$$

We now define for each $s \in T$ a primal set P(s) such that the collection of sets P(s) covers $S = S^1 \times S^1$. This covering is given in figure 4.1. The reasoning behind this figure is that prices induced by a vector $q \in P(s)$ are relatively higher (lower) than in q^0 if the corresponding component of s is positive (negative). Prices corresponding to zero components of s may vary between these relative bounds.

For example consider the primal set P(-,-,-,+) induced by $s = (-1,-1,-1,+1)^T$. For a q in this set we have that $q_0 \leqq q_0^0$ while $q_{10} < q_{10}^0$ and $q_{11} > q_{11}^0$. Hence $\pi_{0j}(q) < \pi_{0j}(q^0)$, j=0,1 and $\pi_{11}(q) > \pi_{11}(q^0)$. Since we are interested in points q in the intersection of corresponding sets P(s) and D(s) this reflects the fact that $\bar{z}_{0j}(q) < 0$, j=0,1, and $\bar{z}_{11}(q) > 0$ if $q \in D(s)$. So the prices of the common goods (0,0) and (0,1) should be decreased and the price of the non-common good (1,1) should be increased. Furthermore, q_{10} should be decreased which reflects a revaluation of the domestic currency of country 1 in order to get quit of the surplus on its balance of payments. Observe that $q_0 \leqq q_0^0$ implies $q_0 = q_0^0$ because the sum of the components of any vector q_0 is equal to one.

For some $q \in$ P(-,+,+,-) we have that $q_{00}/q_{01} < q_{00}^0/q_{01}^0$, while $q_{10}/q_{11} > q_{10}^0/q_{11}^0$. So $\pi_{00}(q)/\pi_{01}(q)$ is lower than at q^0. If $q \in D(s)$ this reflects the fact that $\bar{z}_{00}(q) < 0$ and $\bar{z}_{01}(q) > 0$. Furthermore $q_{10}/q_{11} > q_{10}^0/q_{11}^0$ implies that $\pi_{00}(q) + \pi_{01}(q) = q_{10} > q_{10}^0 = \pi_{00}(q^0) + \pi_{01}(q^0)$, while $\pi_{11}(q) = q_{11} < q_{11}^0 = \pi_{11}(q^0)$, which reflects that the value $\bar{z}_{10}(q)$ of the excess demands for common goods is positive if $q \in D(s)$.

We see that the regions P(s) with $s_{jk} = 0$ for some (j,k) are the convex hull of P(s') and P(s") with $s'_{jk} \in \{-1,+1\}$ and $s''_{jk} = -s'_{jk}$. For instance P(-,0,-,+) is the convex hull of P(-,-,-,+) and P(-,+,-,+). Since $\bar{z}_{10}(q) = 0$ iff $\bar{z}_{11}(q) = 0$ because of the complementarity condition, we have that P(-,+,0,0) is the convex hull of P(-,+,-,+) and P(-,+,+,-). Analogously we can combine the regions P(0,-,-,+), P(-,-,-,+) and P(-,0,-,+) to one region P(0,0,-,+) if we have to solve an ordinary NLCP on S with $q_0^T z_0(q) = 0$ for all q (see [2]). So, in fact the regions P(s) with $s_0 = 0$ are split up in order to deal with the fact that for the international trade problem the complementarity condition does not hold for $h = 0$.

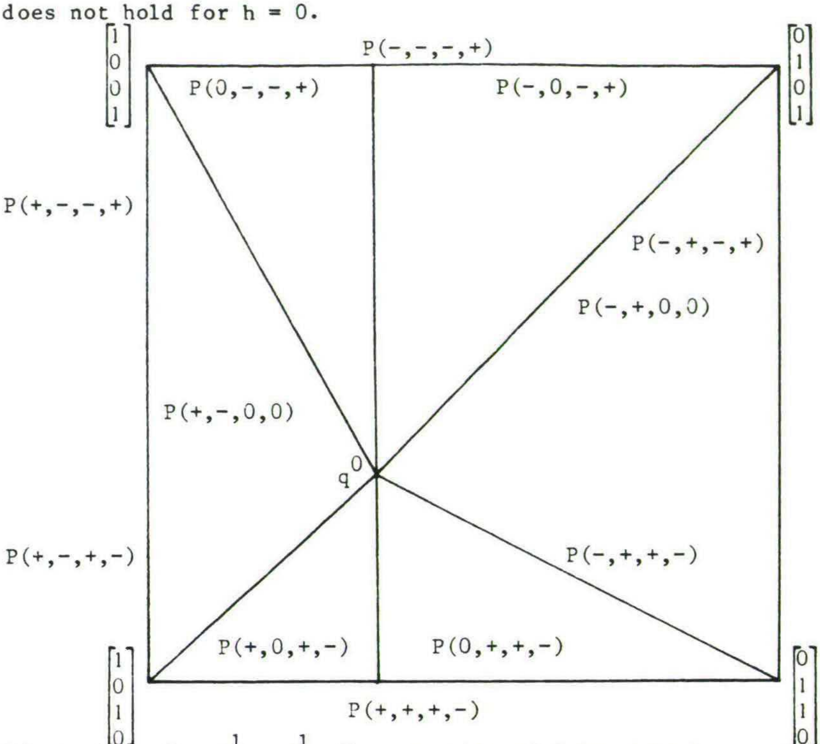

Figure 4.1. $S = S^1 \times S^1$. The covering of S by primal sets $P(s)$, $s \in \mathcal{T}$.

This is the main idea behind the process. One of the consequences is that the price ratios between common goods are not adjusted when the common goods are all in excess demand or all in excess supply.

The collection of dual sets $D(s)$, $s \in \mathcal{T}$, where $D(s) = C\ell\{q \in S \mid \text{sgn}\, \bar{z}(q) = s\}$, and where $C\ell$ stands for closure, is illustrated in figure 4.2. The set $D(s)$ with $s_{jk} = 0$ for some (j,k) is the intersection of the two sets $D(s')$ and $D(s'')$ with $s'_{jk} \in \{+1,-1\}$ and $s''_{jk} = -s'_{jk}$. For instance $D(0,+,+,-)$ is the intersection of the sets $D(+,+,+,-)$ and $D(-,+,+,-)$, while $D(+,-,0,0)$ is the intersection of $D(+,-,+,-)$ and $D(+,-,-,+)$. Observe that $D(+,-,+,+)$ and $D(+,-,-,-)$ are empty because of the complementarity condition $q_1^T\bar{z}_1(q) = 0$ for all q.

Figure 4.2 shows the typical shapes of the sets $D(s)$. For instance consider the point q. Since $q_{10} \ll q_{11}$ and $q_{00} \sim q_{01}$ we have that $\pi_{00}, \pi_{01} \ll \pi_{11}$, i.e. the common goods are cheap relatively to the non-common good, so that we may expect that $\bar{z}_{00}, \bar{z}_{01} > 0$ and hence $\bar{z}_{10} >$

0, $\bar{z}_{11} < 0$. So, it is very reasonable that q indeed lies in D(+,+,+,-). Similar arguments hold for other points q in S.

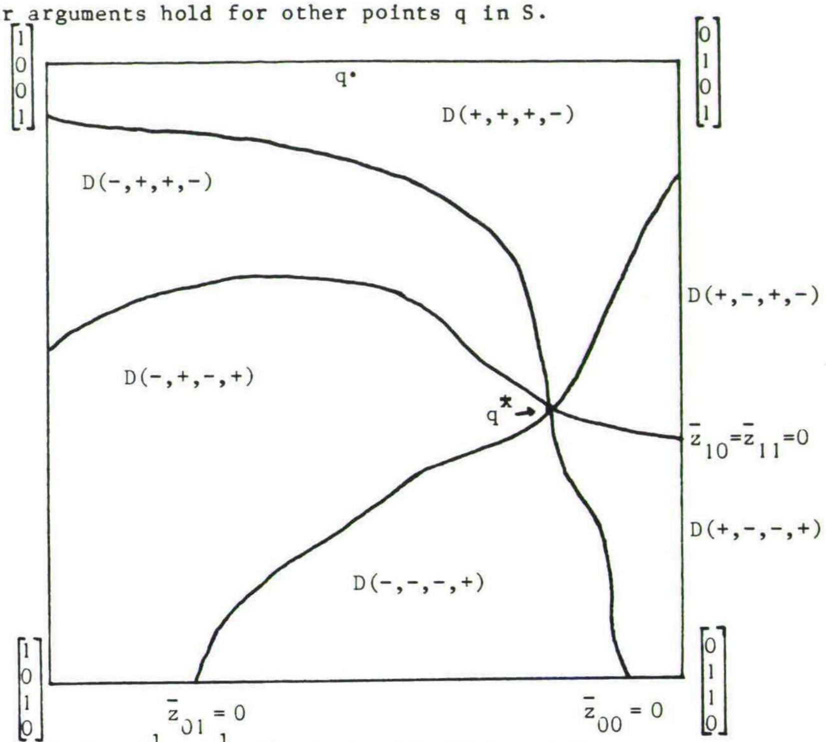

Figure 4.2. $S = S^1 \times S^1$. The dual sets D(s), $s \in \mathcal{T}$.

Starting in some point q^0, the SV-process follows a path of points in corresponding primal and dual sets until an equilibrium point q^* is found. This is illustrated in figure 4.3 for the typical example of figure 4.2 with the starting point as chosen in figure 4.1. Since $q^0 \in$ D(-,+,-,+) the SV-process leaves q^0 along the ray P(-,+,-,+) tracing a path of points q in P(-,+,-,+) ∩ D(-,+,-,+) until the point $\bar{q} \in$ D(-,+,0,0) is reached. Then the process continues by tracing a path of points in the intersection of P(-,+,0,0) and D(-,+,0,0) until the equilibrium point q^* is found. The process always finds such an equilibrium point.

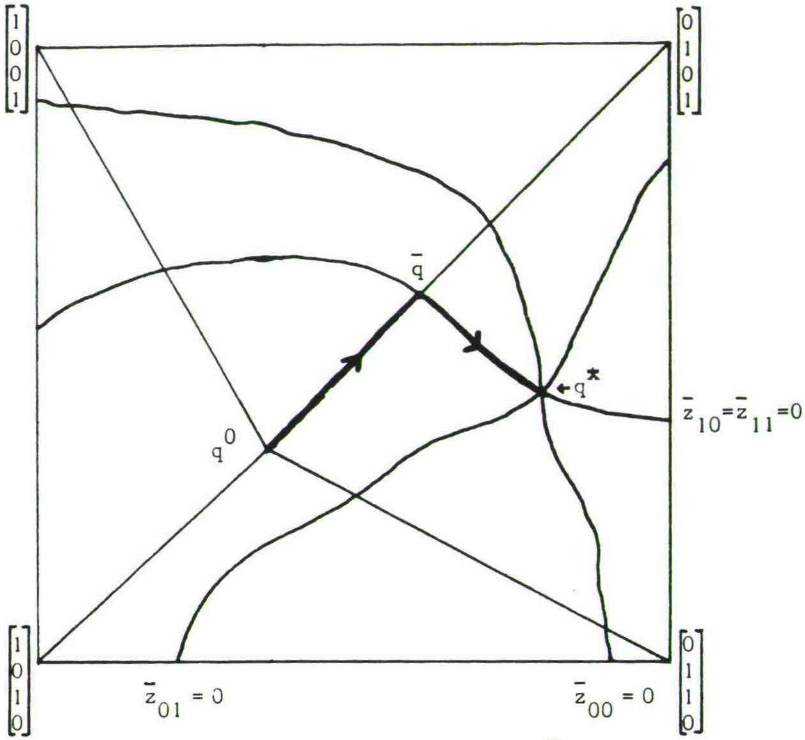

Figure 4.3. The path followed by the SV-process, $(q^0 \in D(-,+,-,+))$.

The reader may convince himself by drawing some other figures. One other example is given in figure 4.4. Here the starting point q^0 lies in a subset of $D(-,-,-,+)$ being a hole in the set $D(-,+,-,+)$. Observe that $[q^0,a) \subset PD(-,-,-,+)$, $[a,b] \subset PD(-,0,-,+)$, $(b,c) \subset PD(-,+,-,+)$ and $[c,q^*) \subset PD(-,+,0,0)$ where $PD(.) = P(.) \cap D(.)$.

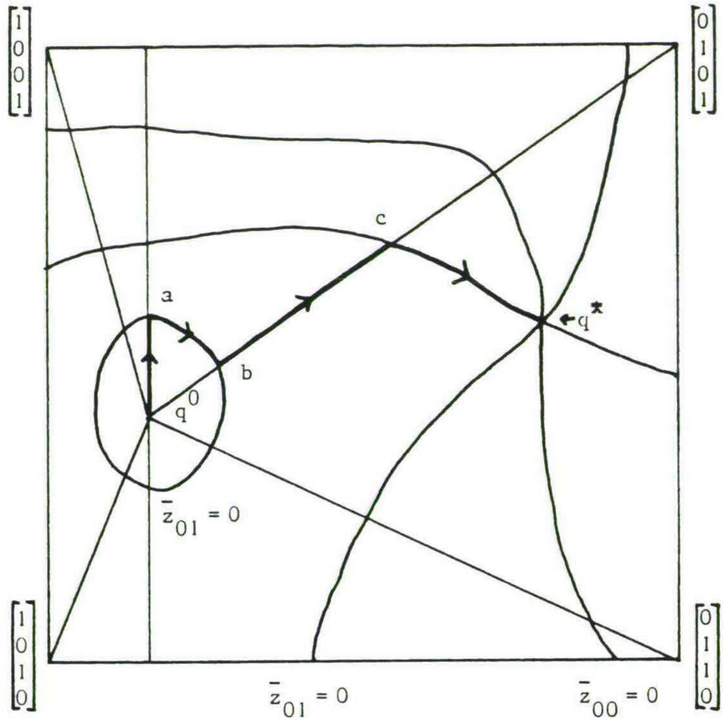

Figure 4.4. The path followed by the SV-process, ($q^0 \in D(-,-,-,+)$).

5. The general case

In this section we consider the SV-process for the international trade model given in section 2. Recall that there are H countries (groups of agents) with corresponding sets of domestic commodities. These countries are indexed by $h = 1,\dots,H$. Furthermore there is an international market $h = 0$ for common goods. These common goods are indexed by $(0,k)$, $k = 0,\dots,n_0$, while the domestic goods of country h are indexed by (h,k), $k = 1,\dots,n_h$; $h = 1,\dots,H$.

As shown in section 2 we have to design a procedure that reaches an equilibrium point q^* in S for which $\bar{z}(q^*) = 0$, where $\bar{z}$ is a suitable "excess demand" function from S to $\Pi_{h=0}^{H} R^{n_h+1}$ related to the excess demands $z_{hk}(\pi)$ by $\bar{z}_0(q) = z_0(\pi)$ and $\bar{z}_h(q) = (q_0^T z_0^h(\pi), z_h^T(\pi))^T$ with $\pi = (\pi^1, \dots, \pi^H)$ and $\pi^h = (q_{h0}q_0^T, q_{h1}, \dots, q_{hn_h})^T$, $h \in I_H$. Recall that $z_0(\pi) = \Sigma_{h=1}^{H} z_0^h(\pi)$ with z_0^h the excess demand for common goods by country h, and that $z_h(\pi) = z_h^h(\pi)$ denotes the excess demand for domestic goods of country h. The excess demand function $\bar{z}$ satisfies the complementarity condition $q_h^T\bar{z}_h(q) = 0$ for $h = 1,\dots,H$ because of the budget constraint of each country. For $h = 0$ holds that $q_0^T\bar{z}_0(q) = \Sigma_{h=1}^{H} \bar{z}_{h0}(q)$ and hence $q_0^T\bar{z}_0(q) > 0$ (< 0) implies that $\bar{z}_{h0}(q) > 0$ (< 0) for at least one $h \in I_H$. In particular this holds if the elements of $\bar{z}_0$ are all positive (negative).

These considerations restrict the set T of feasible sign patterns of $\bar{z}$. This set of feasible sign patterns induces a covering of S into primal sets. A primal set states conditions on the location in S of its elements. The corresponding dual set is induced by the sign pattern of the function values of its elements.

Let s be a sign vector in $\Pi_{h=0}^{H} R^{n_h+1}$, i.e. $s_{jk} \in \{-1,0,+1\}$ for all $(j,k) \in I = \cup_{h=0}^{H} I(h)$. For each s we define

$$I_h^-(s) = \{(h,k) \in I(h) \mid s_{hk} = -1\},$$

$$I_h^0(s) = \{(h,k) \in I(h) \mid s_{hk} = 0\},$$

and

$$I_h^+(s) = \{(h,k) \in I(h) | s_{hk} = +1\} \ , \ h = 0,1,\ldots,H.$$

Furthermore we define $I^\alpha(s)$ as the union of $I_h^\alpha(s)$, $\alpha \in \{-1,0,+1\}$, over all $h \in I_H \cup \{0\}$. Each sign vector s in T can be written as $s = (s_0^T, s_1^T, \ldots, s_H^T)^T$. With $s_j > 0$ we mean $s_{jk} > 0$ for all $k \in \{0,\ldots,n_j\}$. If $s_{jk} \geq 0$ for all k and $s_{jk} > 0$ for at least one k we write $s_j \geq 0$, while $s_j \geqq 0$ means that s_j is a nonnegative vector. Accordingly we define $<$, $\leq$ and $\leqq$.

We require an element of T to satisfy the following conditions.

T1. there exists an $h \in I_H \cup \{0\}$, for which $I_h^+(s) \neq \emptyset$ and $I_h^-(s) \neq \emptyset$.

T2. for all $h \in I_H$, $I_h^+(s) = \emptyset$ iff $I_h^-(s) = \emptyset$.

T3. if $s_0 = 0$ then $s_{h0} = 0$ for all $h \in I_H$.

T4. if $s_0 \geq 0$ ($s_0 \leq 0$) then $s_{h0} = +1$ ($s_{h0} = -1$) for at least one $h \in I_H$.

Condition T1 is implied by T2, T4 and the fact that $s \neq 0$. T2 reflects the complementarity condition holding for all $h \in I_H$. The fact that $q_0^T \bar{z}_0(q) = \Sigma_{h=1}^H \bar{z}_{h0}(q)$ for all q implies T4. The only additional restriction on T is imposed by T3. It would be possible that $s_{h0}s_{k0} < 0$ for two indices $h,k \in I_H$, while $s_0 = 0$. This restriction of feasible sign vectors will be motivated later on.

For each $s \in T$ we define a primal set P(s). Let $q \in P(s)$ and recall that $q^0 \in S$ is an arbitrarily chosen starting point in the interior of S. Then q must satisfy the following conditions, where $0 \leq b \leq b_0 \leq 1$, and for all $h \in I_H \cup \{0\}$, $a_h \geq 1$.

P1. for all $h \in I_H$ if $s_0 \leq 0$, and for all $h \in I_H \cup \{0\}$ otherwise,

$$q_{hk} = a_h q_{hk}^0 \quad \text{if } s_{hk} = +1$$

$$bq_{hk}^0 \leq q_{hk} \leq a_h q_{hk}^0 \quad \text{if } s_{hk} = 0$$

$$bq^0_{hk} = q_{hk} \qquad \text{if } s_{hk} = -1.$$

P2. if $s_0 < 0$ then $q_0 = q^0_0$.

P3. if $s_0 \leq 0$ and $s_{0k} = 0$ for at least one k,

$$b_0 q^0_{0k} \leq q_{0k} \leq a_0 q^0_{0k} \qquad \text{if } s_{0k} = 0$$

$$b_0 q^0_{0k} = q_{0k} \qquad \text{if } s_{0k} = -1.$$

These conditions look very complicated. To provide a better insight we will make some remarks. First, suppose that for all h, $s_{hk} = +1$ for just one k say k_h, while $s_{hj} = -1$ for all other $(h,j) \in I(h)$. Then the set P(s) is the line segment from q^0 to the vertex of S with $v_{hk_h} = 1$ and $v_{hj} = 0$ for all $j \neq k_h$, $h = 0,\ldots,H$. Observe that along this ray the ratio between prices of commodities with corresponding negative s-component does not change, since all of them are decreased with the uniform factor 1-b. Of course we need a not uniform factor a_h to increase the prices of commodities $(h,j) \in I(h)$ for which $s_{hj} = +1$ in order to keep the sum of the components q_{hj}, $j = 0,\ldots,n_h$, equal to one.

Each sign vector s with $s_{hk} \neq 0$ for all $(h,k) \in I$ induces a ray. If neither $s_0 < 0$ nor $s_0 > 0$ such a ray points to a point $\bar{q}$ of S with

$$\bar{q}_{hk} = 0 \qquad \text{if } s_{hk} = -1$$

and

$$\bar{q}_{hk} = \bar{a}_h q^0_{hk} \qquad \text{if } s_{hk} = +1$$

where $\bar{a}_h = (\Sigma_{(h,k) \in I^+_h(s)} q^0_{hk})^{-1}$. If $s_0 > 0$ or $s_0 < 0$ this remains true for all $\bar{q}_h$, $h \neq 0$, while $\bar{q}_0 = q^0_0$. The latter fact also induces the factor $b_0 \neq b$ if $s_0 \leq 0$ with $s_{0k} = 0$ for at least one k. If $s_{hk} = 0$, the variable q_{hk} varies between the lower bound bq^0_{hk} (or $b_0 q^0_{hk}$) and the upper bound $a_h q^0_{hk}$. So, the dimension of P(s) increases with one if s_{hk}

goes from nonzero to zero. However, if for some h, s_{hk_h} becomes equal to zero while $s_{hj} = 0$ for all $j \neq k_h$, the dimension does not increase because we have the additional restriction $\Sigma_{k=0}^{n_h} q_{hk} = 1$. So we obtain that

$$\dim P(s) = 1 + \Sigma_{h=0}^{H}(|I_h^0(s)| - k_h(s)),$$

where $k_h(s) = 1$ if $|I_h^0(s)| = n_h + 1$ and $k_h(s) = 0$ otherwise.

In figure 4.1 we have drawn the sets P(s) in case $S = S^1 \times S^1$. Observe that these sets induce a subdivision of S. However, this is not true in the general case. For instance, let s be a sign vector with $s_{0h} = +1$ for just one h, say h_0, and $s_{0j} = 0$ for all $j \neq h_0$. Then P1 says that for some q in P(s)

$$q_{0h_0} = a_0 q_{0h_0}^0$$

and

$$bq_{0j}^0 < q_{0j} < a_0 q_{0j}^0.$$

Let λ_j, $j \neq h_0$ be such that $q_{0j} = \lambda_j q_{0j}^0$ and define $\lambda_k = \min_j \lambda_j$. Then with $b_0 = \lambda_k > b$ we have that q is also in $P(\bar{s})$ with $\bar{s}_{0k} = -1$, $\bar{s}_{0j} = 0$ for all $j \neq k$ and $\bar{s}_h = s_h$, $h = 1,\ldots,H$. Let $S_0^\alpha = \{s_0 | s_{0h} = \alpha$ for just one h and $s_{0j} = 0$ for all $j \neq h\}$, $\alpha = -1,+1$. Then for some $s_1,\ldots,s_H$ such that $s = (s_0^T, s_1^T, \ldots, s_H^T)^T \in \mathcal{T}$, we have that

$$\mathop{}_{s_0 \in S_0^{+1}} P(s_0,s_1,\ldots,s_H) = \mathop{}_{s_0 \in S_0^{-1}} P(s_0,s_1,\ldots,s_H).$$

The sets P(s) are induced by the starting point q^0. In fact these sets reveal the position of their elements with respect to q^0.

We now come to the other feature of the SV-process, namely the sign pattern of the excess demand function $\bar{z}$. We define for each sign pattern s that can occur the dual set D(s) by

$$D(s) = C\ell\{q \in S \mid \mathrm{sgn}\ \bar{z}(q) = s\}.$$

Of importance are now the intersections PD(s) of primal sets P(s) and corresponding dual sets D(s). Under some regularity conditions the union PD of all these intersections, i.e. $PD = \cup_{s \in T} PD(s)$ consists of a collection of disjoint paths and loops. Just one of these paths connects q^0 with a solution point q^* for which $\bar{z}(q^*) = 0$. This path is followed by the SV-process. All other paths in PD connect two solution points, each of them corresponding to an equilibrium price vector. If the model has a unique solution, say q^*, then PD contains only one path, namely the path from q^0 to q^*.

The regularity conditions concern the behaviour of $\bar{z}$. Recall that $\bar{z}$ is a continuous function from $S = \Pi_{h=0}^{H} S^{n_h}$ to $\Pi_{h=0}^{H} R^{n_h+1}$ with for all $q \in S$,

$$q_j^T \bar{z}_j(q) = 0 \qquad , \ j \in I_H$$

and (5.1)

$$q_0^T \bar{z}_0(q) = \Sigma_{h=1}^{H} \bar{z}_{h0}(q).$$

First we assume that $\bar{z}$ is continuously differentiable. Secondly we need that, for each s, the set $D(s) = \{q \in S \mid \mathrm{sgn}\ \bar{z}(q) = s\}$ is a collection of connected m-dimensional manifolds, where m corresponds to the number of zeroes in s in the following way. From (5.1) we may suppose that D(s) is a collection of connected manifolds of dimension $d(s) = \Sigma_{h=0}^{H}(n_h - |I_h^0(s)| + \ell_h(s))$, where $\ell_h(s) = k_h(s)$ for $h = 1,\ldots,H$ while $\ell_0(s) = 1$ if $s_0 = 0$ and $s_{h0} = 0$ for all $h = 1,\ldots,H$, and $\ell_0(s) = 0$ otherwise. The regularity conditions assume that this supposition is true. So, d(s) must be equal to the dimension $\Sigma_h n_h$ of S minus the number of zeroes in s plus the number of identities in system (5.1), i.e. the number of equations with all corresponding components of $\bar{z}$ equal to zero. Recall that $\dim P(s) = 1 + \Sigma_{h=0}^{H}(|I_h^0(s)| - k_h(s))$. Since condition T3 implies that $s_{h0} = 0$ for all $h = 1,\ldots,H$, if $s_0 = 0$, it follows that $\ell_h(s) = k_h(s)$ for all h if $s \in T$. So, for all $s \in T$, $P(s) \cap D(s)$ is either a collection of manifolds of dimension $d = \dim P(s) + d(s) - \dim S = 1 + \Sigma_{h=0}^{H}(|I_h^0(s)| - k_h(s)) + \Sigma_{h=0}^{H}(n_h - |I_h^0(s)| + \ell_h(s)) - \Sigma_{h=0}^{H} n_h = 1$, or $P(s) \cap D(s)$ is empty.

Finally, the function must possess some kind of nondegenerated behaviour. In particular we need that $|I^0(s^0)| = 0$ where $s^0 = \text{sgn}\ \bar{z}(q^0)$. We may suppose that this is true for almost all $q^0 \in \text{int}\ S$. This assumption quarantees that the ray along which the starting point will be left is uniquely determined by s^0. The nondegeneracy assumption imposes further conditions on changes in the sign pattern of $\bar{z}$. Formally, for some sign vector s, let $q^1, q^2, \ldots$, be a sequence of points in an area PD(s) converging to $\tilde{q}$, with $\text{sgn}\ \bar{z}(q^t) = s$, $t = 1,2,\ldots$, and $\text{sgn}\ \bar{z}(\tilde{q}) = \tilde{s} \neq s$. We assume that neither q^t, $t = 1,2,\ldots$, nor $\tilde{q}$ contains zero components. Since $\bar{z}(q)$ is continuous, we can only have that in the limit point $\tilde{q}$ non-zero elements of s change into zero. Now, the nondegeneracy assumption says that $d(\tilde{s}) = d(s) - 1$. So, for some s and $q^1, q^2, \ldots$, a sequence of points in D(s) converging to $\tilde{q}$ with $\text{sgn}\ \bar{z}(\tilde{q}) = \tilde{s}$, one of the following cases can occur.

N1. There is an element $(0,p) \in I(0)$ such that $\tilde{s}_{0p} = 0$ while $s_{0p} \in \{-1,+1\}$. Besides $\tilde{s}_{k\ell} = s_{k\ell}$ for all $(k,\ell) \neq (0,p)$ with at least one element $(0,k) \in I(0)$ for which $\tilde{s}_{0k} \neq 0$.

N2. There is a $j \in I_H$ and just one element $(j,p) \in I(j)$ such that $\tilde{s}_{jp} = 0$, $s_{jp} \in \{-1,+1\}$, while $\tilde{s}_{k\ell} = s_{k\ell}$ for all $(k,\ell) \neq (j,p)$. Besides there is a pair $\{(j,\ell),(j,r)\} \subset I(j)$ such that $\tilde{s}_{j\ell} = +1$ and $\tilde{s}_{jr} = -1$.

N3. There is a $j \in \{0,1,\ldots,H\}$ and two elements $(j,p),(j,r) \in I(j)$ such that $\tilde{s}_{jp} = \tilde{s}_{jr} = 0$, $s_{jp}s_{jr} = -1$, while $\tilde{s}_{j\ell} = s_{j\ell} = 0$ for all $\ell \neq p,r$ and $\tilde{s}_{hk} = s_{hk}$ for all other $(h,k) \in I$. In case $j = 0$, $s_{h0} = 0$ for all $h \in I_H$.

N4. There is a $j \in I_H$ with $s_{j0} \in \{-1,+1\}$ and an element $(0,p) \in I(0)$ with $s_{0p} = s_{j0}$ while $\tilde{s}_{0p} = \tilde{s}_{j0} = 0$ and $\tilde{s}_{0\ell} = s_{0\ell} = 0$ for all $\ell \neq p$, $\tilde{s}_{h0} = s_{h0} = 0$ for all $h \neq 0,j$, and $\tilde{s}_{hk} = s_{hk}$ for all other elements $(h,k) \in I$. Besides there are two elements $(j,r),(j,t) \in I(j)$ such that $s_{jr}s_{jt} = -1$.

N5. There is an element $(0,p) \in I(0)$ such that $s_{0p} \in \{-1,+1\}$, $\tilde{s}_{0p} = 0$ and $s_{0k} = 0$ for all $k \neq p$, while $s_{j0} = 1$ and $s_{h0} = -1$ for two pairs of indices $(j,0)$ and $(h,0) \in I$. For all elements $(k,r) \in I$ with $(k,r) \neq (0,p)$, $\tilde{s}_{kr} = s_{kr}$.

N6. There are two elements $j,h \in I_H$ with $s_{j0} = 1$, $s_{h0} = -1$ and $\tilde{s}_{j0} = \tilde{s}_{h0} = 0$, while $\tilde{s}_{p\ell} = s_{p\ell}$ for all other $(p,\ell) \in I$. Furthermore $s_0 = 0$ and for both j and h there are two elements $(j,\ell),(j,r)$ and (h,s), (h,t) respectively such that $s_{j\ell} = s_{hs} = +1$ and $s_{jr} = s_{ht} = -1$.

In all cases described above we have that $d(\tilde{s}) = d(s) - 1$. For N1-N4 both s and $\tilde{s}$ belong to T, unless $\tilde{s} = 0$, and we also have that $\dim P(\tilde{s}) = \dim P(s) + 1$. If along the path followed by the process the sign pattern of $\bar{z}$ changes according to one of the cases N1-N4 while $\tilde{s} \neq 0$, the process continues by following the curve $P(\tilde{s}) \cap D(\tilde{s})$. Of course, if $\tilde{s} = 0$, a solution point q^* is found. In case N6 we have that $s \notin T$. Hence this case cannot occur on the path of points followed by the process. The pattern s of N6 appears as $\tilde{s}$ in N5. So, suppose that along the path N5 occurs. Then $\tilde{s}$ does not obey condition T3 and $P(\tilde{s})$ is not defined. We are now ready to clarify the reason of condition T3. Suppose we define $P(\tilde{s})$ by P1, P2 and P3. Then with s and $\tilde{s}$ obeying N5, we have that $\dim P(\tilde{s}) = \dim P(s)$, while $d(\tilde{s}) = d(s) -1$. Hence $D(\tilde{s}) \cap P(\tilde{s})$ should be a collection of points instead of a collection of 1-dimensional manifolds. Below we describe how the process continues if N5 occurs.

First we introduce some additional notation. When s and $\tilde{s}$ are related to each other as described in one of the cases N1-N6, we denote $s \leftarrow \tilde{s}$ ($\tilde{s}$ conforms to s). If $s \leftarrow \tilde{s}$ while s and $\tilde{s}$ are both in T, we denote $s \leftarrow\!o\; \tilde{s}$ ($\tilde{s}$ strictly conforms to s). Now, for some $s \in T$, we consider the disjoint set of smooth paths and loops $PD(s) = P(s) \cap D(s)$. All end points of the paths in PD(s) lie on the boundary of PD(s) given by

$$\text{bd } PD(s) = [(\text{bd } P(s)) \cap D(s)] \cup [P(s) \cap (\text{bd } D(s))].$$

From the definition of P(s) it is straightforward to derive that

$$\text{bd } P(s) = S(s) \cup \Big(\bigcup_{\tilde{s} \leftarrow o\, s} P(\tilde{s})\Big),$$

where $S(s) = P(s) \cap \text{bd } S = \{q \in P(s) | q_{jk} = 0 \text{ for all } (j,k) \in I^-(s)\}$. On the other hand we have that

$$\text{bd } D(s) = \bigcup_{s \leftarrow \tilde{s}} D(\tilde{s}).$$

We remark that a path in PD(s) can never reach S(s). Suppose it does, then the path reaches a point q with $q_{jk} = 0$ for any index (j,k) for which $s_{jk} = -1$. However, then $\bar{z}_{jk}(q) > 0$ which contradicts that $s = \text{sgn } \bar{z}(q)$.

Now suppose that an end point $\tilde{q}$ of a path in PD(s) lies in bd D(s), i.e. there exists an $\tilde{s}$ with $s \leftarrow \tilde{s}$ such that $\tilde{q} \in D(\tilde{s})$, and hence $\tilde{s} = \text{sgn } \bar{z}(\tilde{q})$. If $\tilde{s} = 0$ a solution point is found. Otherwise either $s \leftarrow\!o\ \tilde{s}$, i.e. $P(\tilde{s})$ exists, or not. In the first case $\dim P(\tilde{s}) = \dim P(s) + 1$ and $\tilde{q}$ is an end point of a path in $PD(\tilde{s})$ on bd $P(\tilde{s})$, namely P(s). If $P(\tilde{s})$ does not exist we must have that $\tilde{s}_0 = 0$ and $\tilde{s}_{h0}\tilde{s}_{k0} = -1$ for two elements (h,0),(k,0) in I while $s_{0p} \neq 0$ for just one element (0,p) (case N5). From the definition of P(s) it follows that there exists an element $(0,\ell)$, $\ell \neq p$, such that $\tilde{q} \in P(\hat{s})$ with $\hat{s}_{hk} = s_{hk}$ for all $(h,k) \neq (0,p), (0,\ell)$, while $\hat{s}_{0p} = 0$ and $\hat{s}_{0\ell} = -s_{0p}$. It follows immediately that also $\hat{s} \leftarrow \tilde{s}$ and $\tilde{q}$ is also an end point of a path in $PD(\hat{s})$, where $\dim P(\hat{s}) = \dim P(s)$ and $d(\hat{s}) = d(s)$. Concluding we have that if $\tilde{q}$ is an end point of a path in PD(s) in bd D(s), while $\tilde{q}$ is not a solution point, then $\tilde{q}$ is also an end point of a path in either $PD(\tilde{s})$ or $PD(\hat{s})$ with $\tilde{s} = \text{sgn } \bar{z}(\tilde{q})$ and $\hat{s}$ as indicated above.

Next suppose that $\tilde{q}$ is an end point of a path in PD(s) in bd P(s). Then there exists an $\tilde{s}$ such that $\tilde{s} \leftarrow\!o\ s$ and $\tilde{q} \in P(\tilde{s})$ with $\dim P(\tilde{s}) = \dim P(s) - 1$. In this case $\tilde{q}$ is also an end point of a path in $PD(\tilde{s})$. Linking together all paths PD(s) we get a collection of paths and loops in $PD = \bigcup_{s \in T} PD(s)$. This collection contains just one path connecting the starting point q^0 and a solution point q^*.

6. Description of the process as a price adjustment process

In section 5 we defined a process that leads from an arbitrary starting vector q^0 in the interior of S to a vector q^* which is uniquely connected with a price equilibrium vector in an international trade model. Here we look for an economic interpretation of the path generated by that process. The excess demand function z is transformed into a function $\bar{z}$, while a price vector p is transformed into a vector q in S. By successive adaptions of q in relation to $\bar{z}(q)$, the process reaches a vector q^* at which $\bar{z}(q^*) = 0$. In section 2 it is shown that q^* determines a p^* at which $z(p^*) = 0$. When speaking about the price adjustment process the reader should keep this in mind.

Because the process focusses its attention on $\bar{z}$ and q we first present a meaning for both quantities. Recall from section 2 that the countries are indexed by h, h = 1,...,H. Each country h has n_h non-common goods while $n_0 + 1$ common goods are traded among all countries. So, the total number of goods equals N+1, where $N = \Sigma_{h=0}^{H} n_h$. Furthermore, the non-common goods of country h are indexed by $(h,1),\ldots,(h,n_h)$ while the common goods are indexed by $(0,0),\ldots,(0,n_0+1)$. Each vector $q \in S = \Pi_{h=0}^{H} S^{n_h}$ can be written as $q = (q_0^{\top}, q_1^{\top}, \ldots, q_H^{\top})^{\top}$ with $q_h \in S^{n_h}$, h = 0,1,.. ...,H. An element q_{0k}, $(0,k) \in I(0)$, can be regarded as being the price of the k-th common good denoted in an international monetary unit (e.g. ECU's). For each h, an element q_{hk}, $(h,k) \in I(h) \setminus \{(h,0)\}$ denotes the price of good k in country h counted in the currency of that country. Finally, each element q_{h0}, $h \in I_H$, can be seen as the value of one unit of the international currency in terms of the domestic currency of country h. Therefore, an increase of q_{h0} corresponds to a devaluation of the currency of country h.

Next we examine the function $\bar{z}$. An element $\bar{z}_{hk}$, $(h,k) \in I(0) \cup (\cup_{h=1}^{H} I(h) \setminus \{(h,0)\})$ denotes the excess demand for good (h,k). The elements of $\bar{z}_0$ are the excess demands for the respective common goods, being those excess demands summed over all countries. For each $h \in I_H$, $\bar{z}_{h0}$ is equal to the total value of the excess demand for common goods in country h, in terms of the international monetary unit. In other words, $\bar{z}_{h0}$ denotes the deficit on the balance of payments of country h counted in the international currency. So, if $\bar{z}_{h0} < 0$, country h faces a surplus on its balance of payments. The price vector π^h that determines the excess demands in country h consists of the domestic prices of the non-common goods in that country and the common-good-prices denoted in domestic currency. The latter prices are calculated by multiplying the prices of the common goods denoted in the international standard with its exchange rate related to the domestic currency.

Generally speaking, the SV-process gives higher prices to the goods for which $\bar{z}$ is positive, while prices related to negative $\bar{z}$-values are decreased. Analogously we have that the exchange rate of the international standard in terms of the currency of country h is related to the balance of payments of that country. A further feature of the process is that the in- and decreases of the prices are related to their value at the starting point. This remembering of the starting point is such that it prevents the process from cycling. The process can start at any arbitrarily chosen vector q^0 in the interior of S. Here we assume that q^0 itself is not a solution to the problem.

The process leaves q^0 by increasing relatively the prices corresponding to positive $\bar{z}$-values, while prices belonging to negative $\bar{z}$-values are relatively decreased. This means that prices corresponding to non-common goods with positive excess demand are increased, which is economically reasonable. If the excess demand for a common good is positive, the corresponding price in the international currency is increased, making the good ceteris paribus more expensive. We may expect a decrease of the excess demand. Furthermore, if country h faces a budget of balance deficit ($\bar{z}_{h0}(q^0) > 0$) then the currency of country h is devaluated. This means that the value of that currency when expressed in units of the international currency decreases, making the common goods more expensive for country h. This may have a positive impact on the balance. Observe that positive excess demands for common goods are attacked by two means, i.e. directly by increasing the corresponding prices and indirectly via decreases of the exchange rate in countries facing a balance deficit. The reasonings in case of negative $\bar{z}$-values are reversals of those given above.

It can occur that the elements of $\bar{z}_0(q^0)$ are all positive (negative). Then the prices of the common goods are not changed. In this case those excess demands (supplies) change by adaptions of the exchange rates. When the excess demand for each common good is positive (negative) there must be at least one country with a deficit (surplus) on its balance of payments. The corresponding currency is devaluated (revaluated), which has a negative (positive) impact on the demand for common goods.

In all cases the relative decreases of prices (including the exchange rates of the international currency) are equal, while the relative increases are equal per country respectively group of common goods but may differ among this group and the countries.

In the sequel we say that a good $(j,k) \in I \setminus \cup_{h=1}^{H} \{(h,0)\}$ is in equilibrium at price vector q if $\bar{z}_{jk}(q) = 0$. The balance of payments belonging to country h is in equilibrium if $\bar{z}_{h0}(q) = 0$. Country $h \in I_H$ is said to be in equilibrium if $\bar{z}_h(q) = 0$.

The process continues as described above till it reaches a vector x for which $\text{sgn } \bar{z}(x) \neq \text{sgn } \bar{z}(q^0)$, i.e. at which a component $\bar{z}_{hk}(x)$ of $\bar{z}(x)$ becomes zero. So, either a good or a balance becomes in equilibrium. The process takes care of that equilibrium. The way in which this is done can differ. If $h \in I_H$ with $n_h > 1$ or if $h = 0$ while $\bar{z}_0(q^0) \not> 0$ or $\not< 0$ (cases N1, N2), then the corresponding price x_{hk} is increased (decreased) away from the relative minimum (maximum) depending on whether $\bar{z}_{hk}$ was negative (positive). In case $h \in I_H$ and $n_h = 1$ also the other element of $\bar{z}_h$ must become zero (N3) and country h is in equilibrium. Besides the two components of $\bar{z}_h$ must have had opposite sign. Then the process keeps country h in equilibrium by increasing the component of x_h whose corresponding $\bar{z}$-value was negative, away from the relative minimum and by decreasing the other component of x_h away from the relative maximum. Finally, if $\bar{z}_{0k}(x)$ becomes zero while $\bar{z}_0(q^0) > 0$ (< 0) then $\bar{z}_{0k}$ is kept equal to zero by decreasing (increasing) the corresponding price away from its starting value, while prices of all other international goods are increased (decreased) relatively equal away from their starting values.

In general the process generates price vectors x in S such that the prices corresponding to non-common goods in excess supply or balances revealing a surplus are all relatively equally (to q^0) decreased, while prices corresponding to positive elements of $\bar{z}_h(x)$, $h \in I_H$, are relatively increased. These relative increases are equal per country. Prices related to a non-common good or a balance in equilibrium may vary between those bounds. The same holds for the common-good-prices if sgn $\bar{z}_0(x) > 0$. This means that if e.g. the excess demand for each common good is positive, all international prices are kept equal to their starting values. The same holds if the excess demand for each common good is negative. Finally we consider the case in which the excess demands for common goods are all zero or negative with at least one excess demand equal to zero. Then the prices corresponding to common goods with a negative excess demand are all relatively (to q^0) equal, while all other common-good-prices are relatively higher. The rate with which the first prices are decreased is smaller than or equal to the uniform rate holding for the countries.

Given a point x whose related vector $\bar{z}(x)$ reveals a sign pattern s, the process continues along a path of vectors x for which sgn $\bar{z}(x)$ = s as described above, until this path reaches the boundary of D(s) or P(s).

The first thing happens when one of the cases N1, N2, N3, N4, N5 occurs (see section 5). Then at most two goods or balances reach an equilibrium situation. The cases in which one good or balance reaches an equilibrium situation and the cases in which a country reaches an equilibrium are already described in the beginning of this exposure. However it is also possible that the last two excess demands for common goods which are not in equilibrium get into equilibrium. This can only happen if all the balances of payments are in equilibrium (N3). The resulting adaptions are equal to those sketched for comparable situations of the countries. Also it can occur that the last excess demand for a common good and the last balance of payments which are not in equilibrium get into equilibrium (N4). Before they must have had equal signs. The adaptions are again comparable with those mentioned before. Finally we consider the case when the last common good, say (0,k), gets into equilibrium while there is at least one positive and one negative balance among the countries. If (0,k) was in excess supply then the excess demand for the common good related to the relatively highest common-good-price is made positive. The rate with which the latter price is relatively (to q^0) increased serves further on as the upper bound for the prices of the common goods. In case $\bar{z}_{0k}$ was positive the excess demand for the common good related to the relatively lowest common-good-price is made negative. The rate between the latter price and its starting value serves further on as the lower bound for the common goods. This bound is greater than or equal to the uniform lower bound holding for the countries.

The process reaches the boundary of an area P(s) if one or two prices corresponding to a $\bar{z}$-element in equilibrium reach their lower or upper bound. If this occurs for one price, that price is fixed at its upper (lower) bound while the corresponding good or balance is allowed to become in excess demand (supply) or in deficit (surplus). If a country is in equilibrium and one of its prices reaches its lower bound then the corresponding $\bar{z}$-value is made negative while the good (balance) corresponding to the relatively highest price of that country is made in excess demand (deficit). This can also happen if the common goods are in equilibrium but then all the balances must be in equilibrium. The last case in which two prices related to $\bar{z}$-elements in equilibrium reach a boundary simultaneously can happen if all common goods and all balances are in equilibrium, i.e. when the exchange rate of the international monetary standard and some domestic currency reaches its upper (lower) bound. Then this rate is fixed at that upper (lower) bound while the corresponding balance is made in deficit (surplus). Besides the common good related to the relatively highest (lowest) common-good-price is made in excess demand (supply). The way in which this is done is already described before.

Observe that reaching the boundary of a P-area corresponds to disturbing an equilibrium position of a good or even of a country. At first sight this may seem somewhat strange, but this is needed to quarantee convergency. In fact the process, starting from an arbitrarily chosen vector q^0, reaches a vector q^* at which $\bar{z}(q^*) = 0$ by a sequence of adaptions described above. The vector $q = (q_0^\top, q_1^\top, \ldots, q_H^\top)^\top$, with $q_0 = q_0^*$ and $q_h = (q_{h1}^*, q_{h2}^*, \ldots, q_{hn_h}^*)^\top \cdot \frac{1}{q_{h0}^*}$ for $h \in I_H$, is then the equilibrium price vector of the economy.

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