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RESEARCH MEMORANDUM

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A TWO-LEVEL PLANNING PROCEDURE WITH
RESPECT TO MAKE-OR-BUY DECISIONS,
INCLUDING COST ALLOCATIONS

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Contents

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Abstract

The paper is concerned with a two-level corporate model. At the lower level, divisions produce for the external market. They require certain technical services. The divisions are coordinated by a central unit at the top level. The central unit must determine the integral plan of technical services i.e. firm-wide optimal make-or-buy decisions, without having complete information on the divisions.
We present a decomposition-based planning procedure during which the central unit gathers information to derive an optimal make-or-buy decision while allocating the costs of internal technical services.

The model formulation has substantial significance for the real-world: a similar organizational structure, information dispersal among subunits and coupling of divisions and departments is commonly observed in existing firms. The resulting planning procedure can be interpreted in terms of planning and budgeting in real organizations.

Key words: Multilevel organization, Decomposition-based planning, Make-or-buy decisions, Cost allocation

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1. Introduction

The approaches to planning and budgeting problems in organizations have become more and more quantitatively oriented. The (seemingly ever) growing computer facilities and the complexity of decision-making processes are very likely to strengthen this tendency in the future. Traditionally, input-ouput analysis and linear programming (LP) are important areas in the field of quantitative methods. E.g. in Tilanus (1976), the fifth paper, by Smits and Verheyen, is devoted to a company-wide budgeting input-output model, and the sixth paper, by Bouma and Bosman, accounts for multiple technologies for products by an LP-formulation. Manes et.al. (1982) treat the historic development from input-output to mixed-integer programming (MIP) models for the so-called reciprocal service cost problem. Their contribution was the incorporation of make-or-buy decisions. One possible extension of the analysis of Manes et.al. is to integrate multiple technologies for products and make-or-buy decisions with respect to technical services in one "integral" model (see Meijboom (1983)).

The latter model as well as its predecessors belong to the class of holistic corporate models: the firm is considered as an entity directed by cq. identified with one single decision-maker. As opposed to the holistic approach, the present paper treats the make-or-buy problem in a decentralized multi-level organization.

In any real firm of a reasonable size, a certain degree of decentralization has taken place. A number of product divisions and service departments occur and the information to make decisions is dispersed among

these subunits. Due to the delegation of tasks and responsibilities, the lower-level managers become local decision-makers, possibly with individual goals and preferences.
The topmanagement of the firm has to coordinate these managers. Furthermore, it usually allocates cost of internal services to users and this might affect the behaviour of the subunits managers (Thomas (1977, pp. 7-8).
Summarizing, many real firms consists of more or less independently acting subunits plus a central unit with a coordinating function. The central unit collects information from subunits to arrive at decisions and will allocate internal costs during this planning process. Or, citing Zimmerman (1979, p. 505): "Cost allocation, managerial behaviour and the structure of the organization, including the incentives facing the managers, are extricably linked".

The paper is organized as follows. The main characteristics of a real firm and the problems to be analysed are introduced in chapter 2. The model of the firm is presented in chapter 3 and the issue of cost allocation is treated in chapter 4.
The search process for an optimal make-or-buy decision can be seen as a planning procedure with practical significance, as during this procedure internal costs are allocated. It is discussed extensively in chapter 5. The mathematical background is described separately, in the appendix.

2. Problem statement

2.1. Introduction

We consider a firm that consists of the following subunits: "divisions", "departments" and a "central unit". After describing each subunit's task, the problem of the firm will be introduced. The organizational structure and the way information is dispersed give rise to a planning procedure to solve this problem.

2.2. Subunits in the firm, organizational structure

The divisions in the firm produce the products that can be sold on the external market. They operate relatively independent of each other and we will view them as profit centers.

Definition 1:

A particular amount of products potentially realizable, by one or more divisions, is called a product mix.

In doing their task, the divisions require commodities called technical services (TS). Every TS can be bought externally, from outside suppliers, but might also be produced internally, in the TS departments.

Definition 2:

A particular combination of make-or-buy decisions, stating which TS will be produced internally and which TS will be bought externally, is called a TS alternative.

Beside production supported by TS-ses, the firm has a sector general services (GS). The GS departments produce common goods from which the firm as a whole benefits. This gives rise to common costs.

The central unit of the firm is responsible for the total net profit i.e. the sum of divisional profits minus internal-TS costs and common costs. Furthermore, the output of the departments is not sold outside the firm, so the central unit must allocate the internal-TS costs and the common costs to the divisions (cf. Kaplan (1982, p. 353)).

The following two-level organizational structure is presumed:

- At the lower level, we have the divisions. Each of them possesses specific knowledge with respect to its production processes, market restrictions etc., not known to other subunits.

- At the higher level, we have the central unit. It directly controls the departments but does not have complete information on the divisions.

The divisional two-level structure is a natural consequence of specialization and decentralization of information in the firm. In particular, the central unit has incomplete information on divisions.

2.3. The problem of the firm

Having outlined the main technological, organizational and financial features of the firm, we can introduce the problem to be analysed.

Problem of the firm:
Which product-mix, TS alternative and cost allocation mechanism will maximize the total net profit of the firm?

Under complete information, the optimal product mix and internal-external alternative can be determined at once. The remaining task is then to allocate costs such that divisional managers have no incentive to object against the firm-wide optimal decisions.

In the present situation however, the information needed to solve the problem of the firm is dispersed among the subunits of the firm. The central unit must gather information in order to solve the problem of the firm. The whole process of solving the problem and then making sure that actions are carried out according to this solution will be called the decision-making process. In this paper, we are mainly concerned with the planning phase.

Definition 3:

Planning is that phase in the decision-making process that involves the search for the decision (or set of decisions) to be made and carried out.

We will propose a planning procedure in which information is exchanged between the two organizational levels during a number of planning sessions. Moreover, common costs and associated interval TS-costs are allocated in each planning session. Now it is an open question what kind of influence these cost allocations have on the speed and the result of the planning procedure.

3. The model of the firm

3.1. Mixed-integer programming formulation

In the preceding chapter, a general firm was outlined. Now we formalize these ideas. We will present the model of a firm with two divisions and three different types of TS-ses.

The problem of finding an optimal product mix and TS alternative can be formulated as a mixed-integer programming (MIP) model:

$$
\begin{array}{llllll}
\text{Maximize} & p_1x_1 - dy_1 & + p_2x_2 - dy_2 & & - cx_0 - dy_0 - F\delta & \\
\text{s.t.} & A_1x_1 - y_1 - z_1 & & & & = 0 \\
& B_1x_1 & & & & \leq b_1 \\
& & A_2x_2 - y_2 - z_2 & & & = 0 \\
& & B_2x_2 & & & \leq b_2 \\
(3.1) & z_1 & + z_2 - z_0 & & & = 0 \\
& & & z_0 - (I-A)x_0 - y_0 & & = 0 \\
& & & x_0 & - M\delta & \leq 0 \\
& & & z_0 & - M\delta & \leq 0 \\
& & & & \text{all } x_n, y_n, z_n & \geq 0 \\
& & & \delta : \text{0-1 vector} & & \\
& & & M : \text{large positive constant} & &
\end{array}
$$

The meaning of the vectors and matrices in this problem formulation will be clarified in the subsequent sections.

3.2. Divisions

The subscripts 1 and 2 indicate the two product divisions. The blocks

$$B_nx_n \leq b_n \ , \ x_n \geq 0 \qquad (3.2)$$

represent local restrictions (e.g. maximum outside supply, limited capacities) and opportunities (e.g. multiple technologies for products) which are only known to the n-th division (n = 1,2) but not to other subunits or even the central unit. Each x_n satisfying (3.2) can be called a divisional product mix. The per-unit contributions of x_n to the profit are contained in the vector p_n.

The matrix A_n contains the per-unit TS requirements and is presumed to have nonnegative entries.

TS-ses can be bought externally (y_n), against market price d, or can be obtained from internal departments (z_n). Internal production of TS-ses is treated in the next section. The allocation of internal TS costs is discussed in chapter 4.

3.3. TS sector

We account for three different types of technical services, denoted by TS_1, TS_2, TS_3. The vectors x_0, y_0, z_0, δ are the decision variables with respect to the internal production of TS-ses. Let $\ell \in \{1,2,3\}$. Now $z_0(\ell)$ is the amount of internally produced TS_ℓ as available for divisions, $x_0(\ell)$ is the total amount of internally produced TS_ℓ, and $y_0(\ell)$ is the amount of externally bought TS_ℓ as used in the internal production of other TS-ses (so not available for divisions).

Internal production of TS_ℓ gives rise to (direct) costs $c(\ell)x_0(\ell) + F_\ell$. Here F_ℓ equals the fixed costs whenever $x_0(\ell) > 0$. Moreover, the internal production of TS_ℓ requires an amount $a_{i\ell}x_0(\ell)$ of TS_i (i = 1,2,3). The input coefficients $a_{i\ell}$ form the square matrix A. The per-unit price for $y_0(\ell)$ is $d(\ell)$ (just as it is for divisions).

The vector δ has two functions. Firstly, it guarantees that, whenever $x_0(\ell) > 0$, the related fixed cost F_ℓ is accounted for in the objective function. Secondly, by the constraints $z_0(\ell) - M\delta(\ell) \leq 0$ and $z_0(\ell) \geq 0$, it is guaranteed that $x_0(\ell) = 0$ implies $z_0(\ell) = 0$. So, if TS_ℓ is not produced internally, then there is no internal flow of TS_ℓ towards divisions; they have to buy TS_ℓ independently from outside the firm.

Finally we require that the central unit cannot obtain external TS-ses via the divisions; it has to buy external TS-ses directly from outside suppliers, without interference of divisions. Therefore z_1, z_2 are non-negative.

3.4. Goal of the firm; common costs

As stated in the objective function of the MIP model, the firm strives after profit maximization. The expression

$$p_1x_1 - dy_1 - p_2x_2 - dy_2 - cx_0 - dy_0 - F\delta =: P$$

can be seen as the gross profit.

Beside production, supported by technical services, the firm has a sector "general services". The occurrence of this kind of activities, from which the firm as a whole benefits, gives lead to common costs, i.e. non-separable costs to be beared by the subunits within the firm. These costs may vary in the long run but, in the short run, they are assumed to be entirely fixed, viz. equal to the (known) constant H.

Therefore they are not included in the objective function of the MIP model. Finally, the net profit P_{net} can be defined as

$$P_{net} := P - H.$$

4. Cost allocation requirements

4.1. Cost allocation defined

We want to investigate how the central unit can allocate the costs of general and (internal) technical services during (the process of solving the MIP problem, i.e. during) the search for an optimal product-mix and internal-external alternative. In the "tradition" of Thomas (1977) and Zimmerman (1979), we explicitly analyse the link between planning and cost allocation as this link is most prevalent in practice. First of all, we have to be more specific on what is meant by a "cost allocation".

Definition 4:

A cost allocation is the efficient partitioning of a cost among a set of cost objects. The term "efficient" expresses that all of the cost is allocated, no more and no less.

In every cost allocation, three elements are important.

1. The total amount of costs to be allocated. In the present case, this amount is equal to the common costs and the cost of internal TS-ses.
2. The cost objects among which the costs are to be allocated. Here the divisions, which are profit centers, are the cost objects.
3. The allocation method or rule that partitions the total costs.

The choice or design of the allocation rule is usually difficult in the presence of fixed and hence non-separable costs. Nevertheless, the following properties are considered most desirable:

- the allocation of costs of internal TS-ses to a particular division is proportional to the use of internal TS-ses by that division (cf. Kaplan (1982, p. 355)).
- on behalf of the firm as a whole, the cost allocations should be such that the maximum net profit remains the same.

These requirements could be referred to as "fairness" and "optimality preservation", respectively. Because the common costs are essentially non-separable, it will be difficult to allocate them in a fair way.

4.2. The reciprocal allocation method

The reciprocal allocation method (cf. Kaplan (1982, p. 363-372)) is an allocation method in which each type of TS allocates its variable and fixed costs to other TS-ses and to divisions on the basis of usage. If one divides the total redistributed costs of each TS by its production volume, one obtains a per-unit price to be charged to users of that service. Now we summarize the main formules of the method.

Suppose one has at hand $\bar{x}_0$, $\bar{y}_0$, $\bar{z}_0$, $\bar{\delta}$ which are feasible with respect to the MIP model (3.10).
Define the gross amount of TS-ses $\bar{z}$ as $\bar{z} = \bar{x}_0 + \bar{y}_0$ and the corresponding diagonal matrix Z by $Z_{\ell\ell} = \bar{z}(\ell)$. For explanatory reasons, we assume $\bar{x}_0(\ell)\bar{y}_0(\ell) = 0$ and $\bar{z}(\ell) > 0$ $(\ell = 1,2,3)$.
Let $\bar{F}'$ and $\bar{c}'$ be the (row) vectors of fixed costs and per-unit direct costs. So $\bar{F}(\ell) = F(\ell)\bar{\delta}(\ell)$ and $\bar{c}(\ell) = c(\ell)$ if $\bar{x}_0(\ell) > 0$, $\bar{c}(\ell) = d(\ell)$ if $\bar{y}_0(\ell) > 0$.
The associated input-output matrix $\bar{A}$ is

$$\bar{A}_{*\ell} := A_{*\ell}, \text{ if } \bar{x}_0(\ell) > 0 \; ; \; \bar{A}_{*\ell} := 0, \text{ if } \bar{x}_0(\ell) = 0$$

Now we can give the main formula of the reciprocal allocation method: the <u>total redistributed costs</u> are the components of the row vector G' given by

$$G' := (\bar{c}' + \bar{F}'Z^{-1})(I-\bar{A})^{-1}Z \tag{4.1}$$

If we multiply both sides of (4.1) with Z^{-1}, we obtain the <u>per-unit redistributed costs</u> w', i.e.

$$\bar{w}' := \bar{G}'Z^{-1} = (\bar{c}' + \bar{F}'Z^{-1})(I-\bar{A})^{-1} \tag{4.2}$$

In the following chapter we will see that this price can be used <u>during</u> the process of finding an optimal product mix and internal-external alternative.

4.3. Allocation of common costs

As noted earlier, the allocation of common costs will always be subject to a certain degree of arbitrariness. Here we mention one possibility, namely adding H to one of the components of $\bar{F}'$.

5. A planning procedure including cost allocation

5.1. Introduction

In the previous chapter we have developed a MIP model that represents the problem of finding an optimal product-mix and TS alternative. The mathematical solution algorithm (see Appendix) has a useful economic interpretation: it can be seen as a multilevel planning procedure. Moreover, the cost allocation requirements can readily be incorporated in the procedure. In this paper we will extensively describe and discuss this planning procedure.

It is worthwhile to view the task of the central unit as a "make-or-buy problem in extensive form". This task consists of the following three problems:

[P1] Which TS-ses must be produced internally?

[P2] How much is required of each internal TS?

[P3] How should the amount of interal TS-ses be divided over the divisions?

The decision variables related to [P1], [P2] and [P3] are δ, z_0 and z_n, n = 1,2, respectively.

Furthermore the central unit intends to allocate the costs of internal TS-ses plus the common costs. So the fourth problem is:

[P4] How should the (fixed) common costs and the (fixed and variable) costs of internal TS-ses be allocated to the divisions?

These four problems, though separately postulated, are interrelated. The planning procedure to be described decouples [P1] and [P2] from [P3] and [P4]. The main steps, or sessions, of the procedure are concerned with the question: how much should be produced internally of which type of TS? Each main step, in turn, consists of a number of so-called substeps in which some constant amount of internal TS-ses is divided over the divisions while, at the same time, the associated costs are allocated to users of these services. Now we provide for a detailed description.

5.2. The main steps of the procedure

During the main steps of the planning procedure information is exchanged between the top level and the second level in the organization with respect to the total amount of TS-ses to be produced internally. Based on the current divisional information as available at the top level, the central unit computes a particular TS alternative (represented by $\bar{\delta}^t$) and a particular production volume of internal TS-ses (represented by $\bar{z}_0^t$) plus an "optimistic" estimate $\bar{P}^t$ for the maximum attainable profit P^{opt}. This estimate is optimistic in the sense that the actual maximum attainable profit cannot exceed this estimate. A

natural test for optimality is now:
"If the divisions use up the amount $\bar{z}_0^t$ of internal TS-ses, is the sum of their profits equal to $\bar{P}^t$?"

Before actually performing this test, the central unit must gather additional information. This involves the optimal distribution of the amount of internal TS-ses $\bar{z}_0^t$ among the divisions, while simultaneously generating a marginal valuation $\hat{s}^t$ for $\bar{z}_0^t$. These two issues embodie a subproblem, to be discussed in the next section.

For the moment, suppose this subproblem has been solved and let $\hat{P}^t$ be the maximum attainable profit, given the amount $\bar{z}_0^t$ of internal TS-ses. If the divisions appear to be able to realize the estimate $\bar{P}^t$, i.e. $\hat{P}^t = \bar{P}^t$, the central unit concludes that the current volume of internal TS-ses is optimal.
If not so, the marginal valuation $\hat{s}^t$ and reported profit $\hat{P}^t$ form sufficient new information for the central unit to derive a more realistic estimate $\bar{P}^{t+1}$ ($< \bar{P}^t$) for the maximum attainable profit, with associated TS alternative $\bar{\delta}^{t+1}$ and updated production volume of internal TS-ses $\bar{z}_0^{t+1}$.

5.3. The substeps of the procedure, cost allocations

Now we will analyse the subproblem in more detail. The question is: given some $\bar{z}_0^t$, i.e. some particular amount of internally available TS-ses, find the maximum attainable profit by the divisions (to be denoted by $\hat{P}^t$) and the associated marginal valuation of $\bar{z}_0^t$ (to be denoted by $\hat{s}^t$)?

Formally the central unit would like to solve

$$\begin{array}{llll}
\text{Maximize} & p_1x_1 - dy_1 & + p_2x_2 - dy_2 & \\
\text{s.t.} & A_1x_1 - y_1 - z_1 & & = 0 \\
 & B_1x_1 & & \leq b_1 \\
 & & A_2x_2 - y_2 - z_2 & = 0 \\
 & & B_2x_2 & \leq b_2 \\
 & z_1 & + z_2 & = \bar{z}_0^t \\
 & & \text{all } x_n, y_n, z_n & \geq 0
\end{array} \qquad (5.1)$$

and is particularly interested in the optimal solution value $\hat{P}^t$ and an optimal dual variable associated to the constraint $z_1 + z_2 = \bar{z}_0^t$. Apparently, the common use of internal TS-ses is the only interdependency between the divisions. The problem of finding the optimal common use of $\bar{z}_0^t$ can be attacked by applying a price or resource directive planning procedure (Dirickx and Jennergren (1979, Chapter 6)). In the sequel, a resource directive approach is described. The steps, or sessions, of this "subprocedure" are called substeps, as they take place within a particular main step.

In the course of the substeps, the TS alternative $\bar{\delta}^t$ and the amount $\bar{z}_0^t$ of internally available TS-ses do not change. During the process of computing this latest $\bar{\delta}^t$ and $\bar{z}_0^t$, the central unit also derives the <u>total</u> amount of internal TS-ses $\bar{x}_0^t$ and the amount of external TS-ses $\bar{y}_0^t$ as required in the production of $\bar{x}_0^t$. Of course, these $\bar{x}_0^t$, $\bar{y}_0^t$, $\bar{z}_0^t$, $\bar{\delta}^t$ satisfy the requirements of the MIP model. According to the reciprocal allocation method (section 4.2), per-unit prices for internal TS-ses can be computed (formula (4.2)), eventually covering common costs as well (see section 4.3). The price vector $\bar{w}^t$ remains constant during the substeps of main step t.

At the beginning of each substep, the central unit computes how the amount $\bar{z}_0^t$ can be fully distributed over the divisions. In other words, $\bar{z}_0^t$ is splitted into $\tilde{a}_1^s, \tilde{a}_2^s \geq 0$ with $\tilde{a}_1^s + \tilde{a}_2^s = \bar{z}_0^t$. At the same time, upperbounds $\tilde{P}_n^s$ (n = 1,2) for each division's maximum attainable profit are computed. Now each division is asked to report its own maximum attainable profit $\hat{P}_n^s$ under the restrictions:

1. that its part $\tilde{a}_n^s$ of internal TS-ses is fully used up, and
2. that it pays the price $\bar{w}^t$ for these internal TS-ses.

Formally, each division solves (n = 1,2):

$$\begin{aligned}
\text{Maximize } & p_n x_n - d y_n - \bar{w}^t z_n \\
\text{s.t. } \quad & A_n x_n - y_n - z_n = 0 \\
& B_n x_n \leq b_n \\
& z_n = \tilde{a}_n^s \\
& x_n, y_n, z_n \geq 0
\end{aligned} \tag{5.2}$$

Let $\hat{P}_n^s$ be the optimal solution value of (5.2), i.e. the maximum attainable profit given $\tilde{a}_n^s$, and let γ_n^s be the marginal valuation associated to $\tilde{a}_n^s$. The central unit will now check if

$$\hat{P}_n^s = \tilde{P}_n^s \,, \quad n = 1,2.$$

If this is so, the central unit has solved the subproblem. It has found the optimal distribution of the (temporarily fixed) amount of internal TS-ses $\bar{z}_0^t$ and easily derives the associated maximum attainable profit $\hat{P}^t$ given $\bar{z}_0^t$ as well as the marginal valuation of $\bar{z}_0^t$ as required in the main step of the procedure. Otherwise, for those n with

$$\hat{P}_n^s < \tilde{P}_n^s ,$$

γ_n^s and $\hat{P}_n^s$ form sufficient new information to derive improved upperbounds $\tilde{P}_1^{s+1}$, $\tilde{P}_2^{s+1}$ and associated distribution $\tilde{a}_1^{s+1}, \tilde{a}_2^{s+1}$. The price $\bar{w}^t$ remains the same as $\bar{z}_0^t$ has not changed.

6. Discussion

As can be seen from (5.2), each division is <u>obliged</u> to use up its amount $\tilde{a}_n^s$ of internal TS-ses, so that the costs allocated to such a division will be constant, viz. $\bar{w}^t \hat{z}_n = \bar{w}^t \tilde{a}_n^s$, independent of the divisional profit. On the other hand, because the division considers $\bar{w}^t$ as a per-unit charge for internal TS-ses, its marginal valuation of $\tilde{a}_n^s$ will turn out to be exactly $\bar{w}^t$ lower than in the case without cost allocation (i.e. $\bar{w}^t = 0$). As a result, the information exchange between division and central unit is not essentially disturbed, because the central unit immediately "reconstructs" the correct marginal valuation.

The cost allocations in this form do not influence (e.g. speed up) the planning procedure in comparison with the case without cost allocation. However, the planning procedure is realistic, because in practice (the topmanagement of) many firms allocate (full!) costs (e.g. see Zimmerman (1979)). The present paper gives conditions under which full costing can be applied without provoking sub-optimality.

Summarizing, we have presented a planning procedure for a divisionalized organization that faces make-or-buy decisions for certain technical services. Decentralization features are explicitly accounted for and a well-known and useful cost allocation mechanism is integrated in the planning process. The underlying model as well as the proposed procedure capture elements of the practice of decision-making. Taking the occurrence of full-costing practices for granted in real-world firms, it is the question whether full costing can improve the planning process. The present contribution is a step towards solution of this problem, as conditions are given under which full-costing at least does not damage the planning process.

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Appendix. The solution algorithm for the MIP problem

A1. Introduction

This section is concerned with a solution algorithm for the MIP problem (3.1). The presentation does not provide for a complete development of the algorithm. For the mathematical correctness, in particular the convergence properties, we refer to Lasdon (1970) and Dirickx and Jennergren (1979).

Roughly speaking, the algorithm proceeds as follows.
We partition the variables z_0, x_0, y_0, δ from the remaining variables. Then a two-level solution algorithm very similar to Benders' partitioning method (see Lasdon (1970, pp. 370-381) is applied, in which z_0, x_0, y_0, δ are iteratively held fixed. The iterations of this algorithm are called the main iterations.
The associated subproblem has a block-angular structure and is solved by a decomposition method. This is, in turn, an iterative two-level algorithm, whose iterations are called subiterations.

A2. The main iterations

Each iteration starts with the solution of a so-called restricted upper problem (RUP). This yields a (trial) solution, say $\bar{p}^t$, $\bar{z}_0^t$, $\bar{x}_0^t$,

$\bar{y}_0^t$, $\bar{\delta}^t$. This solution is optimal if $\bar{P}^t = -c\bar{x}_0^t - d\bar{y}_0^t - F\bar{\delta}^t + s\bar{z}_0^t + v_1b_1 + v_2b_2$ for all (u_1,v_1,u_2,v_2,s) satisfying

$$\begin{array}{llll} u_nA_n + v_nB_n & \geq p_n & & \\ -u_n & \geq -d & n = 1,2 & \\ -u_n \qquad + s & \geq 0 & & \\ \qquad v_n & \geq 0 & & \end{array} \qquad \text{(A1)}$$

Therefore we solve the (dual) subproblem (A2), i.e.

$$\text{Minimize } v_1b_1 + v_2b_2 + s\bar{z}_0 \text{ subject to (A1)} \qquad \text{(A2)}$$

Let $(\hat{u}_1^t,\hat{v}_1^t,\hat{u}_2,\hat{v}_2^t,\hat{s}^t)$ be an optimal dual solution to (A2) with solution value $\hat{P}^t$. Now we must check if $\bar{P}^t = \hat{P}^t$.

If this condition is fullfilled, then the trial solution $(\bar{P}^t,\bar{z}_0^t,\bar{x}_0^t,\bar{y}_0^t,\bar{\delta}^t)$ is globally optimal.

Otherwise, if $\bar{P}^t > \hat{P}^t$, the RUP has to be extended with the constraint

$$P \leq -cx_0 - dy_0 - F\delta + \hat{s}^t z_0 + \hat{v}_1^t b_1 + \hat{v}_2^t b_2$$

and then be resolved, thus leading to $\bar{P}^{t+1}$, $\bar{z}_0^{t+1}$ etc.

The iterations of this procedure are called the main iterations. The only problem left is the solution of the subproblem (A2).

A3. The subiterations

For each fixed $\bar{z}_0^t$, the primal subproblem is:

$$
\begin{array}{llll}
\text{Maximize} & p_1x_1 - dy_1 & + p_2x_2 - dy_2 & \\
\text{s.t.} & A_1x_1 - y_1 - z_1 & & = 0 \\
 & B_1x_1 & & \leq b_1 \\
 & & A_2x_2 - y_2 - z_2 & = 0 \\
 & & B_2x_2 & \leq b_2 \\
 & z_1 & + z_2 & = \bar{z}_0^t \\
 & & \text{all } x_n, y_n, z_n & \geq 0
\end{array}
\qquad (A3)
$$

This block-angular LP problem can be solved by price or resource directive decomposition algorithms. Below the latter approach is outlined. We apply a variant of Benders' method as described in Dirickx and Jennergren (1979, pp. 66-69).

In each iteration, a restricted subproblem (RSP) is solved, which leads to a (trial) solution say $\tilde{P}_1^s, \tilde{a}_1^s, \tilde{P}_2^s, \tilde{a}_2^s$. For these values $\tilde{a}_1^s, \tilde{a}_2^s$, two lower subproblems are solved:

$$\begin{aligned} \text{Maximize } & p_n x_n - d y_n \\ \text{s.t. } & A_n x_n - y_n - z_n = 0 \\ & B_n x_n \leq b_n \\ & z_n = \tilde{a}_n^s \\ & x_n, y_n, z_n \geq 0 \end{aligned}$$

Let $(\hat{x}_n^s, \hat{y}_n^s, \hat{z}_n^s; \hat{\alpha}_n^s, \hat{\beta}_n^s, \hat{\gamma}_n^s)$ be a primal-dual pair of optimal solutions. Let $\hat{P}_n^s := p_n \hat{x}_n^s - d\hat{y}_n^s$ $(n = 1,2)$.

If for each n $\tilde{P}_n^s = \hat{P}_n^s$, then optimality of the SP has been achieved. (Now it is not difficult to derive optimal dual variables $\hat{u}_1^t, \hat{v}_1^t, \hat{u}_2^t, \hat{v}_2^t, \hat{s}_2^t$ as required for the solution (A2).)

If for some n $\tilde{P}_n^s > \hat{P}_n^s$, constraints have to be added to the RSP, which is then to be resolved. We call the iterations of the just outlined procedure subiterations. Note that these subiterations are required within a particular main iterations in order to solve the SP of that main iteration.

A4. Modification of the algoritm, cost allocation

Consider (A3), to be referred to as the original subproblem, for certain fixed $\bar{z}_0^t$. Choose some w which has the same dimension as $\bar{z}_0^t$. If we add the term

$$w(\bar{z}_0^t - z_1 - \dots - z_n) \tag{A4}$$

in the objective function, the optimal solution value will not change. Furthermore, let s^* be an optimale dual variable associated to the constraint

$$z_1 + \dots + z_n = \bar{z}_0^t$$

in the "perturbed" version of the subproblem, then $s^* + w$ is an optimal dual variable associated to the same constraint in the original subproblem.

Altogether, the addition of the term (A4) does not really distort the optimum of the original subproblem. If we keep w fixed during the subiterations, the solution algorithm for (A3) as proposed in the previous section, will still yield an optimal solution to (A3), while the dual information is easily corrected in order to be used in the main iterations. Now w can be taken such that

1. all (fixed and variable) costs of internal costs are equal to $w\bar{z}_0^t$;
2. all internal TS-costs plus the common costs H are equal to $w\bar{z}_0^t$.

It depends on which cost components the central unit wants to allocate, whether 1. or 2. (or even another alternative) is chosen. So this section merely provides for allocation opportunities than for allocation rules. Finally, the proposed method fails if all components of $\bar{z}_0^t$ are zero. The only internal costs are the common costs H but we cannot allocate them.

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