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OPTIMAL DELIVERY STRATEGIES FOR HETEROGENEOUS GROUPS OF PORKERS

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Optimal Delivery Strategies for Heterogeneous Groups of Porkers

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Abstract

In this study the question is considered how a farmer should arrange the delivery of groups of porkers, taking into account that each group consists of subgroups differing in growth rates, that the pork price varies with time, and that the next fattening round can only start when all animals in the current round have been delivered for slaughtering. The feeding-regime is assumed to be given, so the central question is how a farmer should react upon the variability of the pork price. For the solution of this problem a Markov decision model is formulated. This model provides a system of critical prices for pork during the periods in which the animals are slaughter-ripe. The comparison of the pork price in a week with the critical level(s) holding for the current group of animals can be of help to the farmer in making his decision whether he should sell a subgroup or combination of subgroups, or proceed with fattening for another period. By exploiting the special structure of the Markov decision problem the optimal delivery strategy can be determined very efficiently. Some examples are presented that show typical courses of the critical levels.

1 Introduction

In this study we deal with the problem at what times a farmer should sell a group of animals to achieve a maximal profit, taking into account that the animals show different rates of growth, that the selling price varies with time, and that the next fattening round can only start when all animals in the current round have been delivered for slaughtering. For convenience the investigation of this general problem is restricted to the fattening of pigs.

(Dis)investment problems related to stocks of "living" commodities (for instance cattle or crops) have been considered by Burt [1], Chavas, Kliebenstein and Crenshaw [2], Feinerman and Siegel [3], Paarsch [4], and Rausser and Hochman [5], among others. By solving a dynamic programming problem or an optimal control problem they derive optimal feeding and selling strategies. In contrast with the present study they restrict their attention to the situation with one growth function. Also, the marketing strategy for the situation where the prices in two consecutive periods are mutually dependent is elaborated in this study. However, no attention is paid to the determination of an optimal feeding-regime.

Nowadays, the production of pork mostly takes place on specialized farms. Such a farm has at its disposal a number of barns, which are divided into compartments. The number of animals that can be placed in a compartment varies from a few tens to well over one hundred, depending on the size of the compartment. Because of health considerations the so-called "all in-all out" system is usually employed for the production of pork. This system means that a compartment is occupied by young pigs at one time, and that all these animals have to be delivered before the next fattening round in this compartment can start. The successive periods of fattening are separated by a short period in which the compartment is cleaned thoroughly. When the young pigs are placed in a compartment, they all have much the same weight. By a balanced feeding-regime the animals are then fattened during a certain period, that is, until they have reached a weight suitable for slaughtering. This feeding-regime is carefully composed from earlier experiences and we suppose that it cannot be changed by the farmer in order to influence the rates of growth. It usually turns out that during the period of fattening the animals in one compartment show a large variation in the growth rates, that is, in the weight gains per kg feed intake. As a consequence, the animals reach a certain weight at different times. For our aim it suffices to distinguish two groups: fast growers and slow growers. However, within these groups there exist no differences in growth properties. As soon as an animal reaches a certain minimal weight, the farmer has the opportunity to sell it to the slaughterhouse. Because of the differences in the weights and the meat qualities of the supplied animals, the slaughterhouses usually do not use one single price but a system of prices. The basic price is paid for an animal with a standard weight and meat quality. Deviations from this standard are taken into account by means of a system of bonuses and (penalty) discounts. In the present study we leave the aspect of quality out of consideration, and we assume that the price per kg is the same for all weights. This price is not constant with respect to time but varies from period to period. In general, the price in an arbitrary period is found in a restricted interval around the price in the preceding period. Therefore we assume that the price of pork in an arbitrary period is a stochastic variable, which only depends on the price in the preceding period. The feed price and interest rate also show variability, but this variability is of a much smaller order of magnitude than that of the pork price. Therefore it is no restriction to regard the feed price and interest rate as deterministic. The heterogeneity of the animals in one compartment together with the "all in-all out" system and the stochastic behaviour of the pork price now in particular raises the question whether it is profitable for the farmer to sell the animals in one compartment at different times.

In this study a micro-economic model is formulated for the decision problem of the farmer, and a method is presented to solve this problem. A preliminary analysis for the case that the pork price is deterministic and constant with respect to time is carried out in Sections 2 and 3. In Section 2 we restrict ourselves to a one-time fattening round. Using a simple model a delivery arrangement that maximizes the net result is derived for both a homogeneous and a heterogeneous group. The situation in which the fattening rounds succeed each other uninterruptedly is discussed in Section 3. In a similar way as in Section 2 an optimal delivery arrangement is derived for both a homogeneous and a heterogeneous group. In Section 4 we then formulate a Markov decision model for the situation where the group of animals in one compartment is homogeneous, but the pork prices in successive periods are mutually independent and identically distributed stochastic variables. By using this model the optimal delivery strategy can be readily determined. This determination boils down to the computation of a critical value for the pork price in each possible period of delivery. For realizations of the pork price above the (current) critical level all animals are sold, whereas the sale is postponed for realizations below this critical level.

By using the techniques developed in Section 4 we discuss in Section 5 the situation where one compartment contains a heterogeneous group of animals, maintaining the assumption of mutually independent and identically distributed pork prices. After these preparations we are in a position to solve the main problem of the present study. In Section 6 the optimal delivery strategy is determined for the situation where the animals in one compartment are heterogeneous, the pork prices in two successive periods are dependent, and the "all in-all out" system of production is pursued. For modelling the dependence of the successive pork prices a first-order Markov scheme is chosen. Finally, in Section 7 we indicate some ways the models discussed in this study can be extended.

2 Preliminary Analysis

We first consider the situation where once a group of N pigs is fattened, all pigs having the same growth properties. At time 0 the (young) pigs are purchased at a price of p_a per animal. At that time they all have the same starting weight, $w(0)$. By a balanced feeding-regime the animals are then fattened during a certain period. Here we assume that this feeding-regime is given and cannot be changed by the farmer. Denoting the feeding ration at time t by $\dot{u}(t) = \frac{du(t)}{dt}$, the weight development of a porker at time t, $\dot{w}(t)$, is described by a growth function h, which is assumed to depend on the weight of an animal and the feeding ration:

$$\dot{w}(t) = h\Big(w(t), \dot{u}(t), t\Big).$$

Note that the weight is added as argument, because a part of the feeding ration is needed for the maintenance of the animal. We assume that the function $u(t)$ is strictly convex and the function $w(t)$ strictly concave, that is,

$$\begin{aligned} \frac{du(t)}{dt} &> 0, & \frac{d^2u(t)}{dt^2} &> 0, \\ \frac{dw(t)}{dt} &> 0, & \frac{d^2w(t)}{dt^2} &< 0. \end{aligned}$$

Finally, the price of feed per kg, p_u, and the price of pork per kg, p_s, are assumed to be given.

When the feeding regime is given, the various prices are fixed and all animals in a group grow according to the same growth function, the farmer only needs to take a decision with respect to the moment of delivery, T. His decision problem can be formulated as follows:

$$\max_T F(T) = -Np_a - Np_u \int_0^T \dot{u}(t)e^{-it}\, dt + Np_s w(T)e^{-iT}, \tag{1}$$

where i stands for the interest rate by which expenses and revenues are discounted to the starting moment. By choosing the moment of delivery according to problem (1) the farmer maximizes the present value of the net profit from a one-time fattening round.

The first-order condition for an optimal delivery moment reads

$$p_s \frac{dw(T)}{dT} = p_u \frac{du(T)}{dT} + ip_s w(T). \tag{2}$$

A solution of this equation represents a maximum if

$$p_s \frac{d^2w(T)}{dT^2} - p_u \frac{d^2u(T)}{dT^2} - ip_s \frac{dw(T)}{dT} < 0.$$

This condition is satisfied since the function $u(t)$ is strictly convex and the function $w(t)$ strictly concave and monotonically increasing. In other words, the equation (2) possesses at most one solution and this solution determines a maximum of problem (1). The economic interpretation of expression (2) is simple. The delivery of the animals is optimal at that time where the marginal profit of delay of the sale, $p_s \frac{dw(T)}{dT}$, equals the marginal cost of delay, consisting of a marginal feeding cost, $p_u \frac{du(T)}{dT}$, and an opportunity cost in the form of interest, $ip_s w(T)$. The unique solution of equation (2), T°, is called the optimal slaughter age or the optimal delivery moment.

After these preparations we now consider the situation where once a heterogeneous group of porkers is fattened. Heterogeneity means that the animals in one compartment grow differently, that is, they differ in weight gains per kg feed intake. For convenience it is assumed that out of a total of N animals, a number of N_1 grows relatively fast and a number of N_2 relatively slow. The increase in weight of a fast grower is described by

$$\dot{w}_1(t) = h_1\Big(w_1(t), \dot{u}_1(t), t\Big),$$

and that of a slow grower by

$$\dot{w}_2(t) = h_2\Big(w_2(t), \dot{u}_2(t), t\Big),$$

where the starting weight of a fast grower is the same as that of a slow grower, that is, $w_1(0) = w_2(0)$. We assume that the functions $w_1(t)$ and $w_2(t)$ are both strictly concave, while the functions $u_1(t)$ and $u_2(t)$ are both strictly convex. By heterogeneity within the group is meant that for all $t > 0$,

$$u_1(t) \geq u_2(t) \text{ and } w_1(t) > w_2(t). \tag{3}$$

Because only one round of fattening takes place, so that the "all in-all out" requirement does not influence the delivery times of the two groups, which are therefore independent of each other, the reasoning given above applies to both groups separately. The optimal slaughter age of the fast growers is denoted by T_1° and that of the slow growers by T_2°. Note that inequality (3) does not imply that $T_1^\circ < T_2^\circ$. Because only an arrangement in which the fast growers are delivered before the slow ones is of interest, it is henceforth assumed that $T_1^\circ < T_2^\circ$.

Denote the net profit from a one-time fattening round of a heterogeneous group of porkers by $F(T_1, T_2)$. Then the decision problem of the farmer can be described by the following model:

$$\begin{aligned}\max_{T_1,T_2} F(T_1, T_2) = -Np_a - N_1 p_u \int_0^{T_1} \dot{u}_1(t)e^{-it}\, dt + N_1 p_s w_1(T_1)e^{-iT_1} \\ - N_2 p_u \int_0^{T_2} \dot{u}_2(t)e^{-it}\, dt + N_2 p_s w_2(T_2)e^{-iT_2}\end{aligned}$$

$$\text{subject to } T_1 - T_2 \leq 0. \tag{4}$$

This nonlinear programming problem can be solved by means of the Kuhn-Tucker conditions. To this end we introduce the Lagrangian associated with the decision problem (4), defined by

$$L(T_1, T_2, \lambda) = F(T_1, T_2) + \lambda(T_2 - T_1).$$

Necessary and also sufficient conditions for the determination of the maximum of the programming problem (4) are now given by the following system:

i. The solution has to be feasible:

$$T_1 - T_2 \leq 0; \tag{5}$$

ii. The Lagrange multiplier has to be nonnegative:

$$\lambda \geq 0; \tag{6}$$

iii. The product of constraint and Lagrange multiplier has to be zero:

$$\lambda(T_1 - T_2) = 0; \tag{7}$$

iv. The first-order conditions for a maximum of the Lagrangian $L(T_1, T_2, \lambda)$ with respect to T_1 and T_2 have to be satisfied:

$$\begin{aligned}
\frac{\partial L(T_1, T_2, \lambda)}{\partial T_1} &= -N_1 p_u \frac{du_1(T_1)}{dT_1} e^{-iT_1} + N_1 p_s \frac{dw_1(T_1)}{dT_1} e^{-iT_1} \\
&\quad - iN_1 p_s w_1(T_1) e^{-iT_1} - \lambda = 0, \\
\frac{\partial L(T_1, T_2, \lambda)}{\partial T_2} &= -N_2 p_u \frac{du_2(T_2)}{dT_2} e^{-iT_2} + N_2 p_s \frac{dw_2(T_2)}{dT_2} e^{-iT_2} \\
&\quad - iN_2 p_s w_2(T_2) e^{-iT_2} + \lambda = 0.
\end{aligned} \tag{8}$$

After multiplying by e^{iT_j}, $j = 1, 2$, and substituting

$$C_j(T_j) = p_s \frac{dw_j(T_j)}{dT_j} - p_u \frac{du_j(T_j)}{dT_j} - ip_s w_j(T_j), \tag{9}$$

the last conditions can be shortly represented as follows:

$$\begin{aligned}
&N_1 C_1(T_1) - \lambda e^{iT_1} = 0, \\
&N_2 C_2(T_2) + \lambda e^{iT_2} = 0.
\end{aligned} \tag{8'}$$

The conditions (5)–(8) are sufficient for the determination of the maximum of the programming problem (4), because the function $F(T_1, T_2)$ is strictly concave, which can be easily verified by using the assumptions concerning the functions $u_1(t)$, $u_2(t)$, $w_1(t)$ and $w_2(t)$. So the matrix of second-order partial derivatives of the function $F(T_1, T_2)$ is negative definite. A stationary point of the function $F(T_1, T_2)$, therefore, represents a global maximum and not only a local maximum. Obviously, the last statement also holds for the Lagrangian $L(T_1, T_2, \lambda)$, which implies the sufficiency of the conditions (5)–(8) for the case $\lambda > 0$.

The starting-point for the analysis of the system (5)–(8) is the complementary slackness condition (7). When $\lambda = 0$, the first-order conditions (8′) imply that $C_1(T_1) = C_2(T_2) = 0$. The fast and the slow growers are then delivered at the moments that they reach the optimal slaughter ages. For the fast growers this is the case at time T_1° and for the slow growers at time T_2°, where $T_1^\circ < T_2^\circ$ on account of the assumption made below (3). For the case $\lambda > 0$, and hence $T_1 = T_2$, the system (5)–(8) possesses no solution, since the first equation of (8), or equivalently (8′),

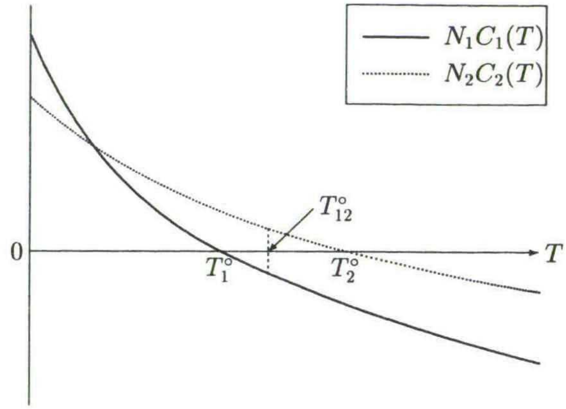

Figure 1: The moment T_{12}°

implies $T_1 < T_1^\circ$ and the second $T_2 > T_2^\circ$, which give a contradiction. Consequently, on fattening a heterogeneous group of animals once, a maximum profit is achieved when each group is delivered at the optimal slaughter age. This conclusion is obvious, because the decisions with respect to the delivery moments of the two groups may be taken independently.

If the farmer decides to deliver not at two times but at one time, then the net result is given by

$$F(T,T) = -Np_a - p_u \int_0^T \Big[N_1\dot{u}_1(t) + N_2\dot{u}_2(t)\Big] e^{-it}\, dt \\ + p_s\Big[N_1 w_1(T) + N_2 w_2(T)\Big] e^{-iT}.$$

This result is maximal when the moment T is chosen such that the marginal cost of delay of the sale of the fast growers equals the marginal profit of acceleration of the delivery of the slow growers, that is,

$$-N_1C_1(T) = N_2C_2(T).$$

The unique solution of this equation is denoted by T_{12}°. Owing to the properties of the functions C_1 and C_2 the optimal delivery moment T_{12}° has to be found between T_1° and T_2°. (For an illustration see Figure 1.) If the fast and the slow growers are delivered at one moment, which differs from the optimal slaughter ages of both groups, the net result is smaller than with a two-time delivery. Without a further specification of the growth functions and the feeding functions, however, it is impossible to derive a readily interpretable expression for the loss in profit resulting from a one-time delivery.

3 Infinite Chain of Fattening Cycles

The preceding section has been concerned with the situation of one fattening cycle. In this section we treat the situation where the fattening cycles succeed each other uninterruptedly. Of course, such a treatment is only meaningful if the net result of each fattening round is nonnegative, which is henceforth assumed. Although the length of the investment period is infinite, so that it is possible to change the capacity of the compartment in which the animals are fattened in course of time, we keep this capacity constant for convenience. Again the farmer has to determine a delivery arrangement for a heterogeneous group of animals that maximizes the net result.

We first consider the situation where the animals placed in a compartment have the same growth properties. Moreover, we suppose that these growth properties do not change from cycle to cycle. With $F(T)$ the net result from an infinite chain of fattening cycles for one compartment, the compartment continually being occupied by a homogeneous group of animals, the optimal time of delivery in each cycle can be determined by means of the following model:

$$\begin{aligned}\max_T F(T) &= \sum_{n=0}^{\infty} e^{-inT}\Big\{-Np_a - Np_u\int_0^T \dot{u}(t)e^{-it}\,dt + Np_s w(T)e^{-iT}\Big\}\\ &= \left(1-e^{-iT}\right)^{-1}\Big\{-Np_a - Np_u\int_0^T \dot{u}(t)e^{-it}\,dt + Np_s w(T)e^{-iT}\Big\}.\end{aligned}$$

(For the various notations we refer to the preceding section.) The first-order condition for the optimal time of delivery reads

$$\begin{aligned}&\left(1-e^{-iT}\right)^{-1}\Big\{-Np_u\frac{du(T)}{dT}e^{-iT} + Np_s\frac{dw(T)}{dT}e^{-iT} - iNp_s w(T)e^{-iT}\Big\}\\ &\quad - ie^{-iT}\left(1-e^{-iT}\right)^{-2}\Big\{-Np_a - Np_u\int_0^T \dot{u}(t)e^{-it}\,dt + Np_s w(T)e^{-iT}\Big\} = 0.\end{aligned}$$

Multiplying this condition by $e^{iT}\left(1-e^{-iT}\right)/N$ and denoting the net result per fattening cycle per animal by $\xi_1(T)$ we have (compare Paarsch [4])

$$p_s\frac{dw(T)}{dT} = p_u\frac{du(T)}{dT} + ip_s w(T) + i\left(1-e^{-iT}\right)^{-1}\xi_1(T). \tag{10}$$

In comparison with the condition (2) for one fattening cycle the condition (10) possesses one additional term, namely $i\left(1-e^{-iT}\right)^{-1}\xi_1(T)$. This term represents the interest on the present value of the net profit per animal over all future cycles. The economic interpretation of expression (10) is as follows. The time of delivery has to be chosen such that the marginal profit of delay of the sale compensates for the extra cost of feed and also the loss of interest. The amount of interest forgone consists of two components, namely the interest on the returns of the sale of the animals that are fattened in the current cycle and the interest on the present value of the net profit per animal over all future cycles. Observe that we can also interpret the second interest component as the (constant) net result per animal per unit of time. For, denoting this net result by η, we have

$$\xi_1(T) = \int_0^T \eta e^{-it}\,dt = \eta\left(1-e^{-iT}\right)/i.$$

Because of the assumption that the net result per cycle is nonnegative, the total marginal cost is here at least equal to that in the situation with one fattening cycle. As a consequence, the optimal length of a fattening cycle for the situation with an infinite number of cycles, T^*, is at most equal to T°, the optimal length with a one-time fattening cycle.

We next proceed with the situation where the animals placed in one compartment have different growth properties. For simplicity we again distinguish only two classes:

fast growers and slow growers. The required notations have been introduced in the preceding section. The decision problem of the farmer for the heterogeneous situation can be described by the following model:

$$\max_{T_1,T_2} F(T_1,T_2) = \left(1-e^{-iT_2}\right)^{-1}\Big\{-Np_a \\ - N_1p_u\int_0^{T_1}\dot{u}_1(t)e^{-it}\,dt + N_1p_sw_1(T_1)e^{-iT_1} \\ - N_2p_u\int_0^{T_2}\dot{u}_2(t)e^{-it}\,dt + N_2p_sw_2(T_2)e^{-iT_2}\Big\}$$

$$\text{subject to } T_1 - T_2 \le 0. \tag{11}$$

The programming problem (11) can be solved in a similar way as problem (4). The Lagrangian associated with the problem (11) is given by

$$L(T_1,T_2,\lambda) = F(T_1,T_2) + \lambda(T_2 - T_1).$$

The Kuhn-Tucker conditions for the nonlinear programming problem (11) now read

$$T_1 - T_2 \le 0; \tag{12}$$

$$\lambda \ge 0; \tag{13}$$

$$\lambda(T_1 - T_2) = 0; \tag{14}$$

$$\frac{\partial L(T_1,T_2,\lambda)}{\partial T_1} = \left(1-e^{-iT_2}\right)^{-1}\Big\{-N_1p_u\frac{du_1(T_1)}{dT_1}e^{-iT_1} + N_1p_s\frac{dw_1(T_1)}{dT_1}e^{-iT_1} \\ - iN_1p_sw_1(T_1)e^{-iT_1}\Big\} - \lambda = 0,$$

$$\frac{\partial L(T_1,T_2,\lambda)}{\partial T_2} = \left(1-e^{-iT_2}\right)^{-1}\Big\{-N_2p_u\frac{du_2(T_2)}{dT_2}e^{-iT_2} + N_2p_s\frac{dw_2(T_2)}{dT_2}e^{-iT_2} \\ - iN_2p_sw_2(T_2)e^{-iT_2}\Big\} \\ - ie^{-iT_2}\left(1-e^{-iT_2}\right)^{-2}\xi_2(T_1,T_2) + \lambda = 0. \tag{15}$$

Here $\xi_2(T_1,T_2)$ stands for the net result per fattening round. The conditions (12)–(15) are necessary for an optimal solution of the problem (11). In general, the Kuhn-Tucker conditions are not sufficient. The maximum of the problem (11), however, can be readily determined by means of the conditions (12)–(15), as is shown below. Multiplying the first relation of (15) by $e^{iT_1}\left(1-e^{-iT_2}\right)$ and the second relation by $e^{iT_2}\left(1-e^{-iT_2}\right)$, and using the substitutions $C_1(T_1)$ and $C_2(T_2)$, introduced in (9), we obtain

$$N_1C_1(T_1) - \lambda e^{iT_1}\left(1-e^{-iT_2}\right) = 0, \\ N_2C_2(T_2) + \lambda e^{iT_2}\left(1-e^{-iT_2}\right) = i\left(1-e^{-iT_2}\right)^{-1}\xi_2(T_1,T_2). \tag{15'}$$

We first consider the case $\lambda = 0$. The equations (15′) then change into

$$N_1C_1(T_1) = 0, \\ N_2C_2(T_2) = i\left(1-e^{-iT_2}\right)^{-1}\xi_2(T_1,T_2). \tag{16}$$

The conditions (16) are the same as the conditions (8′) with $\lambda = 0$ except for the additional term $i\left(1-e^{-iT_2}\right)^{-1}\xi_2(T_1,T_2)$. This term can be interpreted as the (constant) net result per unit of time, as is explained above. The first condition of (16) is satisfied if $T_1 = T_1^\circ$, that is, in each cycle the fast growers are delivered at the optimal slaughter age. From the second condition of (16) it is seen that the slow growers are delivered at that time where the marginal result for the slow growers, $N_2C_2(T_2)$, equals the net result per time unit for the two groups together, $i\left(1-e^{-iT_2}\right)^{-1}\xi_2(T_1,T_2)$. Because of the assumption that the net result per cycle is nonnegative, the delivery of the slow growers takes place before or at the optimal slaughter time T_2°. If the (unique) solution of the conditions (16) also satisfies the feasibility condition (12), that is, the slow growers are not delivered before the fast growers, then this solution satisfies the Kuhn-Tucker conditions (12)–(15), and thus represents a maximal solution of the programming problem (11). In the sequel we denote this maximal solution by (T_1^*, T_2^*). In conclusion, the optimal delivery arrangement corresponding to $\lambda = 0$ satisfies

$$T_1^\circ = T_1^* \leq T_2^* \leq T_2^\circ.$$

This result differs from that found for the situation with one fattening round in that respect that for $\xi_2(T_1^*, T_2^*) > 0$ the delivery of the slow growers is advanced to an earlier time, and this advancement is more significant as the net result per unit of time is greater. For $\xi_2(T_1^*, T_2^*) > 0$ the optimal length of one fattening round in an infinite chain is therefore shorter than T_2°, the optimal length with a one-time fattening round. Note that, if the net result per time unit is so large that $T_2^* = T_1^*$ ($= T_1^\circ$), then both groups are delivered simultaneously at time T_1°, the optimal slaughter age of group 1 animals.

We now consider the case $\lambda > 0$. Since the constraint corresponding to problem (11) is active for this case we have $T_1 = T_2$, so that by taking $T_1 = T_2 = T$ in the equations (15′) and then eliminating λ from the resulting equations we obtain

$$N_1C_1(T) + N_2C_2(T) = i\left(1-e^{-iT}\right)^{-1}\xi_2(T,T). \tag{17}$$

Because the functions $C_1(T)$ and $C_2(T)$ are both monotonically decreasing, the equation (17) possesses a unique solution $T_{12}^* > 0$. Additional information concerning the delivery time T_{12}^* can be obtained from the first relation of (15′), which reads with $T_1 = T_2 = T_{12}^*$,

$$N_1C_1(T_{12}^*) = \lambda e^{iT_{12}^*}\left(1-e^{-iT_{12}^*}\right).$$

By using that the function $C_1(T)$ is monotonically decreasing and vanishes for T_1°, the optimal slaughter age of a fast grower, it follows that $T_{12}^* < T_1^\circ$. Relation (17) can now be interpreted as follows. The delivery is optimal at that time where the marginal profit of acceleration of the sale of all animals equals the net result per unit of time. Consequently, for an infinite chain of fattening cycles with a continually heterogeneous group the delivery of the fast and the slow growers at the same time is also a possibility. Finally we note that the two cases $\lambda = 0$ and $\lambda > 0$ lead to mutually disjunct solutions, namely $T_2^* \geq T_1^* = T_1^\circ$ for $\lambda = 0$ and $T_{12}^* < T_1^\circ$ for $\lambda > 0$.

4 Markov Decision Model

Until now we have assumed that all prices remain constant in the course of time. This assumption is not very realistic. In particular, the pork price may show quite some variability. In this section we consider the farming model in which the prices of young pigs and feed, and also the rate of interest are constant, but the price of pork at an arbitrary time is uncertain, or random. For convenience it is assumed that the pork price changes periodically, say every week, and is governed by a discrete probability distribution. In this and the following section we restrict ourselves to a market in which the pork prices in successive periods are independent of each other and identically distributed. The more realistic model with dependent pork prices is considered in Section 6. The price of pork in period t, denoted by $\mathbf{Y}_t$, possesses the following probability distribution:

$$q_j = \Pr\{\mathbf{Y}_t = y_j\}, \tag{18}$$

where the possible prices are numbered in ascending order, that is, $0 < y_1 < y_2 < \cdots < y_m$. Because the price of pork changes only periodically, we formulate the decision problem of the farmer not in continuous but in discrete time. At the start of each period the farmer decides whether to sell animals in that period or not. During each period the farmer has only one opportunity to sell animals. If animals are sold, then they are removed from the compartment immediately, that is, at the beginning of the period. In order to make the decision whether to sell or not, the farmer has to know the weights of the animals and the current price of pork. We suppose that only animals that possess a weight within a certain range, say 80–130 kg, can be sold. Animals with a weight outside this range do not satisfy the quality requirements imposed by the slaughterhouses, and hence yield no profit. If the weight of an animal lies between the minimal and the maximal allowable weight, then we call this animal slaughter-ripe. The price of pork is determined by extraneous circumstances, so the farmer cannot influence this price by the quantity supplied. Finally we assume that the successive cycles are separated by a week in which the compartment is made ready for the next fattening round. The cost for the cleaning of the compartment is given by p_c.

Before answering the question how the farmer has to arrange the delivery of a heterogeneous group, we first derive the decision rule for the situation where the animals have the same growth properties. When a compartment is empty and clean at the beginning of a period, it is filled with N young pigs at a price of p_a per animal. The young pigs have a starting age of x_0 weeks. By a balanced feeding-regime the animals are then fattened during a number of weeks. An animal of age x receives a feeding ration of $u(x)$, which must be purchased at a price of p_u per kg. Without restricting the generality we assume that the growth process is discrete, that is, weight increases occur only at the end of a feeding week. The weight of an animal of age x is denoted by $w(x)$. Further we denote the minimal and maximal weight at which an animal can be slaughtered by w_{min} and w_{max}, and the corresponding minimal and maximal slaughter age by x_{min} and x_{max}. The relation between the age and the weight of an animal is illustrated in Figure 2.

The total financial result is determined by the decisions that the farmer takes in the successive weeks. In each week he decides whether to sell the animals or to postpone the sale and carry on the fattening. We denote the decision taken in week t by a_t. In order to make the decision whether to sell or not, the farmer has to know the weight of the animals and the price of pork. As long as the animals have not reached the minimal slaughter weight the farmer has no choice but to proceed with fattening. The farmer also has no choice if the animals are of such a weight that by feeding them for another week they will exceed the maximal slaughter weight. In

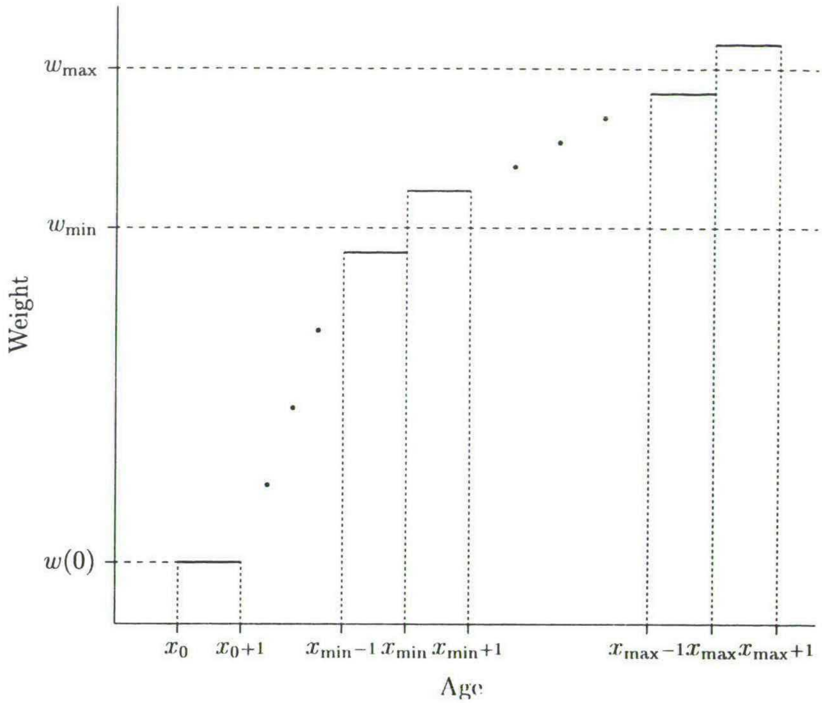

Figure 2: An example of the weight function

this case the animals are sold immediately. In the other weeks the farmer can always choose from two possibilities. He can decide to sell the animals at the current price or he can decide to dispose of the animals in one of the coming weeks at the price valid then but currently unknown. Such a sequence of decisions for the successive weeks is called a strategy and denoted by the symbol π. By terminating each fattening cycle at a suitable moment, that is, by choosing a suitable strategy, the farmer can realize a maximal financial result.

For the determination of the optimal fattening strategy we make use of a Markov decision model. In accordance with the terminology of this method we introduce the stochastic process $\{(\mathbf{X}_t, \mathbf{Y}_t), t = 0, 1, 2, \dots\}$, where $\mathbf{X}_t$ stands for the age of an animal and $\mathbf{Y}_t$ for the price of pork in week t. The age varies from x_0 up to $x_{\max}$ and the pork price from y_1 up to y_m. Suppose that in an arbitrary week the system is in state (x, y_i) and the farmer chooses action a. Then the farmer receives a reward $r(x, y_i; a)$ in that week and the system changes into a new state $(\hat{x}, y_j)$. The probability that such an event occurs is denoted by $p_{x,y_i;\hat{x},y_j}(a)$. If the farmer employs the strategy π, that is, in week t he chooses action a_t, and the system is initially in state (x, y_i), then the expected total discounted return is given by

$$v_\pi(x, y_i) = \mathrm{E}_\pi\left\{ \sum_{t=0}^{\infty} r(\mathbf{X}_t, \mathbf{Y}_t; a_t)\alpha^t \;\middle|\; \mathbf{X}_0 = x, \mathbf{Y}_0 = y_i \right\},$$

where α stands for the weekly rate of discount. Note that E_π represents the conditional expectation, given that strategy π is employed. The farmer now attempts to maximize the expected total discounted return when starting with a new group of animals and the initial pork price is y_i. Thus the problem faced by the farmer is to find a strategy π^* that maximizes $v_\pi(x_0, y_i)$, that is,

$$v_{\pi^*}(x_0, y_i) = \max_\pi v_\pi(x_0, y_i).$$

To determine the strategy π^* we apply the optimality principle of Bellman. This principle states that when starting in state (x, y_i) and taking action a there, one cannot do better than following the optimal strategy starting from the new state. In other words, if in any week the system is in state (x, y_i), then the expected total discounted return from this week onwards is maximized by employing the strategy π^*. This argument leads to the following optimality equation (see Ross[6]):

$$v_{\pi^*}(x, y_i) = \max_a \left\{ r(x, y_i; a) + \alpha \sum_{(\hat{x}, y_j)} p_{x,y_i;\hat{x},y_j}(a) v_{\pi^*}(\hat{x}, y_j) \right\}, \tag{19}$$

where the summation is over all possible states $(\hat{x}, y_j)$.

For the decision problem of the farmer the optimality equation (19) can be specified as follows. If the animals are not yet slaughter-ripe, that is, $x_0 \leq x < x_{\mathsf{min}}$, then we have

$$v_{\pi^*}(x, y_i) = -Np_u u(x) + \alpha \sum_{j=1}^m q_j v_{\pi^*}(x+1, y_j).$$

If the animals have gained sufficient weight, then the farmer can decide to proceed with fattening or he can decide to sell and start a new fattening round. So for $x_{\mathsf{min}} \leq x \leq x_{\mathsf{max}}$ we have

$$\begin{aligned} v_{\pi^*}(x, y_i) = \max \Big\{ & -Np_u u(x) + \alpha \sum_{j=1}^m q_j v_{\pi^*}(x+1, y_j), \\ & Nw(x) y_i - p_c - Np_a + \alpha \sum_{j=1}^m q_j v_{\pi^*}(x_0, y_j) \Big\}, \end{aligned}$$

where the first possibility cancels for $x = x_{\mathsf{max}}$, because animals of age x_{max} will exceed the maximal slaughter weight w_{max} by feeding them for another week (compare Rausser and Hochman [5]). Hence selling the animals is optimal if

$$Nw(x)y_i - p_c - Np_a + \alpha \sum_{j=1}^m q_j v_{\pi^*}(x_0, y_j) > -Np_u u(x) + \alpha \sum_{j=1}^m q_j v_{\pi^*}(x+1, y_j),$$

so there exists a critical value of the pork price for each age at which the animals can be delivered. If in a week the animals have age x and the pork price is above the corresponding critical level, then the animals are sold immediately, whereas the sale is postponed if the opposite is true.

5 Delivery Strategy for a Heterogeneous Group

In this section we consider the question how the farmer should organize the delivery in case the compartment contains a (homogeneous) group of fast growers and a (homogeneous) group of slow growers. It is assumed that out of a total of N animals placed in a compartment, a number of N_1 grows relatively fast and a number of N_2

relatively slow. A fast grower of age x receives a feeding ration of $u_1(x)$ and a slow grower of age x a feeding ration of $u_2(x)$. The weight of a fast grower of age x is denoted by $w_1(x)$ and that of a slow grower of age x by $w_2(x)$. At the beginning of each cycle all animals have the same weight, that is, $w_1(x_0) = w_2(x_0)$. The fast growers reach the minimal weight for which a positive pork price holds at the age of $x^{(1)}_{\min}$ weeks, while they can be delivered at latest at the age of $x^{(1)}_{\max}$ weeks. With the delivery of the slow growers the farmer can only start at a later age, namely at the age of $x^{(2)}_{\min}$ $(> x^{(1)}_{\min})$. At latest these animals leave the farm at the age of $x^{(2)}_{\max}$ $(> x^{(1)}_{\max})$. In the sequel it is assumed that $x^{(1)}_{\max} \geq x^{(2)}_{\min}$, which is the most common configuration. We again restrict ourselves to a market in which the pork prices in successive weeks form a sequence of independent and identically distributed stochastic variables with common distribution q_j, introduced in (18).

In Sections 2 and 3 we have seen that each of the two groups within the heterogeneous livestock is delivered in its entirety, usually at different times for both groups. Also for the farming model in which the price of pork varies from week to week each of the two (homogeneous) groups is always sold in its entirety. For, if it is advantageous to sell one animal from a homogeneous group in a certain week, then it is also advantageous to sell the other animals from that group in this week, and nothing can be gained by spreading the selling of the animals over several periods. Whether the periods in which the two groups are sold differ from each other, cannot be said in advance. Surely, one would expect that the fast growers are always delivered before or at the same time as the slow growers, but this need not be the case. The feeding and weight progresses could be such that in fact the opposite occurs, for instance if group 2 animals grow relatively very slowly. Here we leave this theoretical possibility out of consideration and we assume that the growth functions and the feeding functions have such courses that the fast growers are never delivered after the slow ones.

For the determination of the optimal delivery strategy for a heterogeneous group of porkers we again formulate a Markov decision model. To this end we introduce the stochastic process $\{(\mathbf{X}_t, \mathbf{C}_t, \mathbf{Y}_t), t = 0, 1, 2, \dots\}$, where $\mathbf{X}_t$ stands for the age of an animal, $\mathbf{C}_t$ for the composition of the livestock and $\mathbf{Y}_t$ for the price of pork in week t. The composition indicates whether both groups are present ($\mathbf{C}_t = 12$) or whether only group 2 is present and group 1 is already sold ($\mathbf{C}_t = 2$). Denote the expected total discounted return by $v_\pi(x, c, y_i)$, given that the farmer employs the delivery strategy π and the system is initially in state (x, c, y_i). The weekly rate of discount is given by α. The farmer now attempts to find a strategy that maximizes $v_\pi(x_0, 12, y_i)$. The optimality equations can be written down as follows (compare the similar derivation given in the preceding section). We need to distinguish 4 different cases:

i. If the animals are not yet slaughter-ripe, the farmer can only proceed with fattening. So for $x_0 \leq x < x^{(1)}_{\min}$ we have

$$v_\pi(x, 12, y_i) = -N_1 p_u u_1(x) - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_j v_\pi(x+1, 12, y_j).$$

ii. If the fast growers have gained sufficient weight but the slow growers are still too light, the farmer can choose between two possibilities. He can decide to proceed with the fattening of both groups or he can decide to sell the fast growers and proceed with the fattening of only the slow growers. His choice of course depends on the value of the pork price. So for $x_{\min}^{(1)} \le x < x_{\min}^{(2)}$ we have

$$v_\pi(x, 12, y_i) = \max\Big\{-N_1 p_u u_1(x) - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_j v_\pi(x+1, 12, y_j),$$
$$N_1 w_1(x) y_i - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_j v_\pi(x+1, 2, y_j)\Big\}.$$

If the fast growers are already sold while the slow growers are not yet slaughter-ripe, the farmer must proceed with fattening the remaining animals. Hence for $x_{\min}^{(1)} < x < x_{\min}^{(2)}$,

$$v_\pi(x, 2, y_i) = -N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_j v_\pi(x+1, 2, y_j).$$

iii. If both groups are slaughter-ripe, the farmer can choose from three possibilities. He can proceed with fattening both groups, he can sell the fast growers and proceed with fattening the slow growers, or he can sell both groups. So for $x_{\min}^{(2)} \le x \le x_{\max}^{(1)}$ we have

$$v_\pi(x, 12, y_i)$$
$$= \max\Big\{-N_1 p_u u_1(x) - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_j v_\pi(x+1, 12, y_j),$$
$$N_1 w_1(x) y_i - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_j v_\pi(x+1, 2, y_j),$$
$$\big(N_1 w_1(x) + N_2 w_2(x)\big) y_i - p_c - N p_a + \alpha \sum_{j=1}^{m} q_j v_\pi(x_0, 12, y_j)\Big\},$$

where the first possibility cancels for $x = x_{\max}^{(1)}$, because the sale of the fast growers can no longer be postponed.

iv. If the fast growers are already sold and the slow growers are slaughter-ripe, the farmer can decide to proceed with fattening or he can decide to sell. So for $x_{\min}^{(2)} \le x \le x_{\max}^{(2)}$ we have

$$v_\pi(x, 2, y_i) = \max\Big\{-N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_j v_\pi(x+1, 2, y_j),$$
$$N_2 w_2(x) y_i - p_c - N p_a + \alpha \sum_{j=1}^{m} q_j v_\pi(x_0, 12, y_j)\Big\},$$

where of course the first possibility cancels for $x = x_{\max}^{(2)}$.

From the above system of optimality equations it is easily seen that there exist three critical levels for the pork price. The first level holds for only the fast growers at each slaughter-ripe age and is denoted by $\gamma_1(x)$. If in a week the animals have age x and the pork price is above the critical level $\gamma_1(x)$, then the fast growers are sold immediately. Whether the slow growers are also sold depends on the second critical level, denoted by $\gamma_{12}(x)$. This critical level holds for the two groups together at each age at which both the fast and the slow growers are slaughter-ripe. If in a week the animals have age x and the pork price is above the critical level $\gamma_{12}(x)$, then both groups are sold immediately. It is obvious that $\gamma_{12}(x) \geq \gamma_1(x)$. In case the fast growers are already sold, the decision whether to sell the remaining slow growers depends on the third critical level. This critical level holds for the slow growers at each slaughter-ripe age and is denoted by $\gamma_2(x)$. For realizations of the pork price above the critical level $\gamma_2(x)$, given that the remaining slow growers have age x, these animals are sold immediately and a new fattening round is started.

Several numerical methods are available for the computation of the critical levels $\gamma_1(x)$, $\gamma_{12}(x)$ and $\gamma_2(x)$. The Markov decision problem formulated above can be solved by the value-iteration method, by the strategy-iteration method and by linear programming. A detailed discussion of these methods can be found in Ross[6] or Tijms[7]. For the present problem we have used the strategy-iteration method to compute the optimal solution. In this method the special structure of the problem can be exploited to reduce the system of linear equations to be solved in each iteration step to a considerably smaller system of linear equations on only the (initial) states (x_0, y_i), $i = 1, \ldots, m$. If $x_0 < x^{(1)}_{\text{min}}$, the resulting system can be further reduced to a (simple) linear equation, because the values $v_\pi(x_0, 12, y_i)$ with π an arbitrary strategy do not depend on i. The developed algorithm turns out to be very efficient.

To conclude this section we present a numerical example that gives some insight in the critical prices as functions of the age. The example concerns the following model:

$$\begin{aligned}
&N = 100, N_1 = 40, N_2 = 60, p_a = 125, p_u = 0.45, p_c = 250, \alpha = 0.996;\\
&y_i = 2 + 0.25(i-1), \quad q_i = 1/9, \quad i = 1, \ldots, 9;\\
&x_0 = 5, w_{\text{min}} = 80, w_{\text{max}} = 130, x^{(1)}_{\text{min}} = 12, x^{(1)}_{\text{max}} = 21, x^{(2)}_{\text{min}} = 14, x^{(2)}_{\text{max}} = 27;\\
&u_1(x) = 0.1x^2 - 0.1x + 10, \qquad x_0 \leq x \leq x^{(1)}_{\text{max}} - 1;\\
&u_2(x) = 0.05x^2 - 0.28x + 9.35, \qquad x_0 \leq x \leq x^{(2)}_{\text{max}} - 1;\\
&w_1(x) = -0.2x^2 + 11.67x - 28.38, \qquad x_0 \leq x \leq x^{(1)}_{\text{max}};\\
&w_2(x) = -0.13x^2 - 8.88x - 16.18, \qquad x_0 \leq x \leq x^{(2)}_{\text{max}};
\end{aligned}$$

Note that the pork price is uniformly distributed on the equidistant points $2, 2.25, 2.5, \ldots, 3.75, 4$. In Figures 3 and 4 we display the numerical results obtained for this model. These typical results clearly illustrate the effects of the age on the critical prices and need no further discussion. Worth mentioning is only that the numerical procedure requires a total of 5 iterations and a total computation time of 3 seconds.

6 Dependent Pork Prices

In the preceding sections we have assumed that the pork prices in successive weeks are independent and identically distributed stochastic variables. This assumption is not very realistic. In general, the pork price in an arbitrary week only slightly differs from the price in the week just expired. In this section we consider the farming model in which the pork prices in successive weeks are dependent. We restrict ourselves to a first-order dependence, that is, the pork price in an arbitrary week only depends on the price in the week just expired. So, in contrast with the models considered in the preceding sections, knowledge of the price in the current week gives information on the price in the next week. For convenience we assume that the pork price in any week possesses a discrete distribution. The possible pork prices are given by $y_1, y_2, \ldots, y_m$

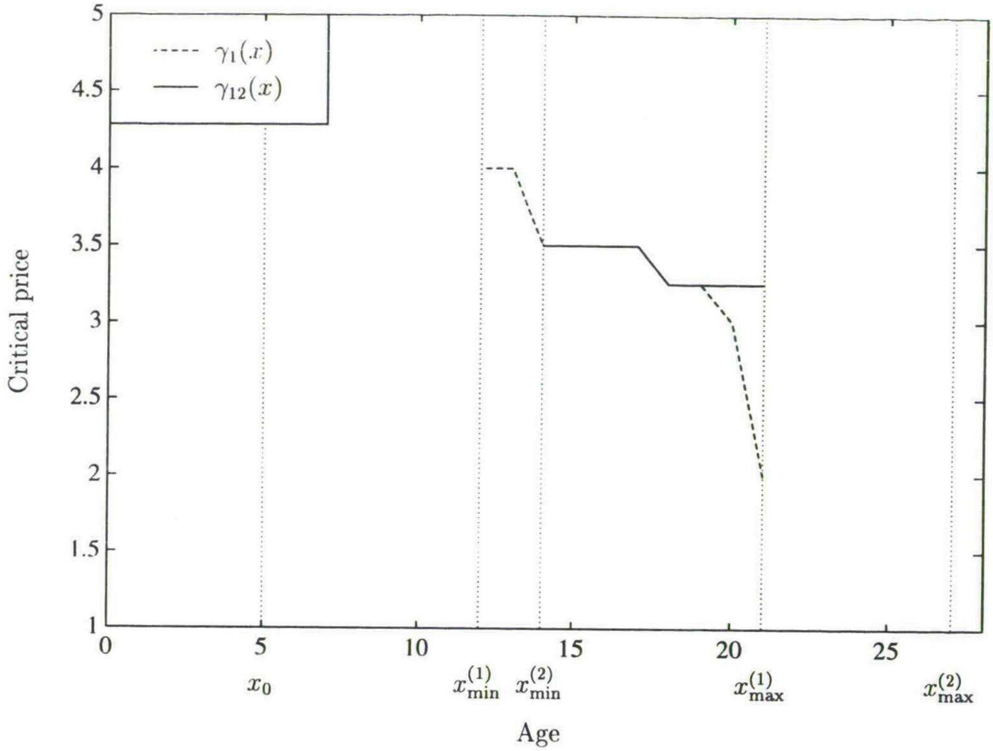

Figure 3: Critical prices for the two groups together

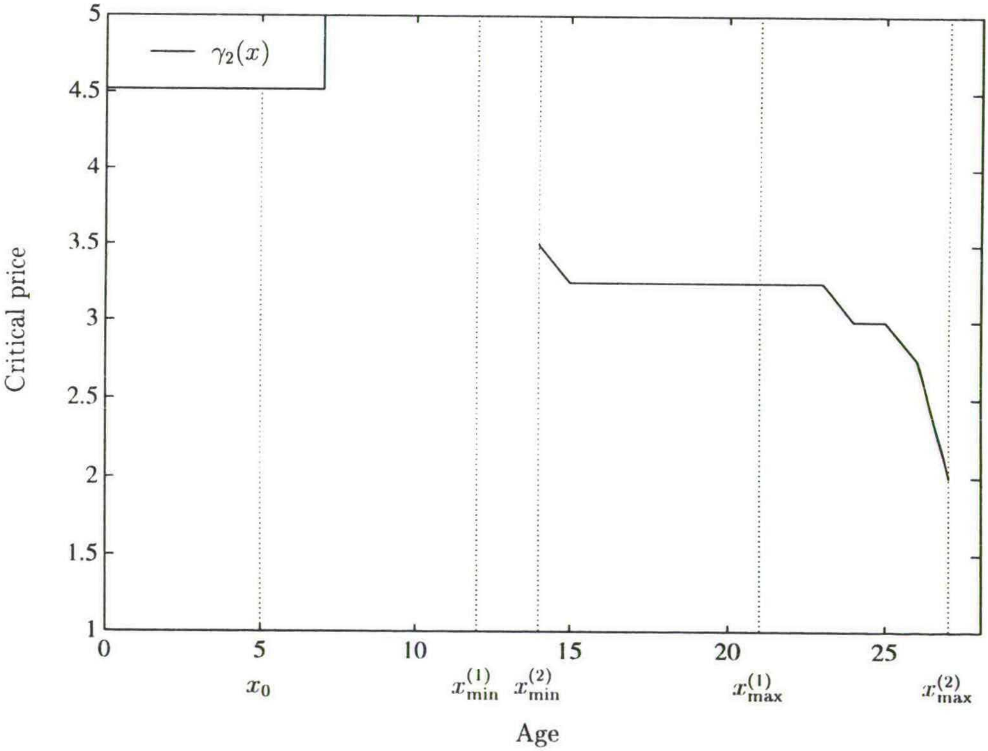

Figure 4: Critical price for only group 2

with $y_1 < y_2 < \cdots < y_m$. Further we assume that the transition probabilities are independent of time. Denoting the pork price in week t by $\mathbf{Y}_t$, it is easily seen that the process $\{\mathbf{Y}_t, t = 0, 1, \dots\}$ constitutes a Markov chain with stationary transition probabilities. These transition probabilities are given by

$$q_{ij} = \Pr\{\mathbf{Y}_{t+1} = y_j \mid \mathbf{Y}_t = y_i\}.$$

With the exception of the pork price, the model being considered in this section is the same as that in the preceding section. Once again, the farmer wants to determine a delivery strategy for a heterogeneous group of porkers that maximizes his net result. To solve this problem we make use of a Markov decision model. Suppose the farmer employs the strategy π, which consists of a sequence of selling decisions for the successive weeks. The decision in an arbitrary week depends on the current state of the system described by the age of an animal, the composition of the livestock and the pork price. The corresponding stochastic process is denoted by $\{(\mathbf{X}_t, \mathbf{C}_t, \mathbf{Y}_t), t = 0, 1, \dots\}$. Starting with a new group of animals of heterogeneous composition the farmer wants to maximize the present value of all future returns discounted with a weekly rate of α. Denote the expected total discounted return by $v_\pi(x, c, y_i)$, given that the farmer employs the strategy π and the system is initially in state (x, c, y_i). The optimality equations are derived in a similar way as in the preceding section. Note that the introduction of dependence between the pork prices in two successive weeks does not result in fundamental changes in the structure of the system of optimality equations. For the four cases that need to be distinguished within a cycle of maximal length the optimality equations read:

i. If both groups are not yet slaughter-ripe, the farmer can only proceed with fattening. So for $x_0 \le x < x^{(1)}_{\min}$ we have

$$v_\pi(x, 12, y_i) = -N_1 p_u u_1(x) - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x+1, 12, y_j).$$

ii. If the fast growers are slaughter-ripe but the slow growers not yet, the farmer can proceed with fattening both groups or he can sell the fast growers and proceed with fattening the slow growers. So for $x^{(1)}_{\min} \le x < x^{(2)}_{\min}$ we have

$$\begin{aligned} v_\pi(x, 12, y_i) = \max\Bigg\{ & -N_1 p_u u_1(x) - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x+1, 12, y_j), \\ & N_1 w_1(x) y_i - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x+1, 2, y_j) \Bigg\}. \end{aligned}$$

If the fast growers are already sold and the slow growers are not yet slaughter-ripe, the farmer must proceed with fattening the remaining animals. Hence for $x^{(1)}_{\min} < x < x^{(2)}_{\min}$,

$$v_\pi(x, 2, y_i) = -N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x+1, 2, y_j).$$

iii. If both groups are slaughter-ripe, the farmer can proceed with fattening both groups, or he can sell the fast growers and proceed with fattening the slow growers, or he can sell both groups. So for $x_{\min}^{(2)} \leq x \leq x_{\max}^{(1)}$ we have

$$\begin{aligned}
&v_\pi(x,12,y_i) \\
&\quad = \max\Bigg\{ -N_1 p_u u_1(x) - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x+1,12,y_j), \\
&\qquad\qquad N_1 w_1(x) y_i - N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x+1,2,y_j), \\
&\qquad\qquad \big(N_1 w_1(x) + N_2 w_2(x)\big) y_i - p_c - N p_a + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x_0,12,y_j) \Bigg\},
\end{aligned}$$

where the first possibility cancels for $x = x_{\max}^{(1)}$.

iv. If the fast growers are already sold and the slow growers are slaughter-ripe, the farmer can proceed with fattening or he can sell. So for $x_{\min}^{(2)} \leq x \leq x_{\max}^{(2)}$ we have

$$\begin{aligned}
v_\pi(x,2,y_i) = \max\Bigg\{ &-N_2 p_u u_2(x) + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x+1,2,y_j), \\
&N_2 w_2(x) y_i - p_c - N p_a + \alpha \sum_{j=1}^{m} q_{ij} v_\pi(x_0,12,y_j) \Bigg\},
\end{aligned}$$

where the first possibility cancels for $x = x_{\max}^{(2)}$.

It is easily seen that the above system of optimality equations does not necessarily result in critical levels for the pork price. However, delivery strategies to be employed in practice should be based on a comparison of the current pork price with certain critical levels. Therefore we restrict ourselves to delivery strategies of this form. The critical prices are again denoted by $\gamma_1(x)$, $\gamma_{12}(x)$ and $\gamma_2(x)$. For an interpretation of these critical prices see the preceding section.

To conclude this section we present a numerical example that sheds some additional light on the critical levels $\gamma_1(x)$, $\gamma_{12}(x)$ and $\gamma_2(x)$. The critical levels are computed by the strategy-iteration method (see preceding section). Note that in each iteration step we need to solve a system of m linear equations, because the reduction to a single linear equation is not possible for the model with dependent pork prices, because the values $v_\pi(x_0,12,y_i)$ now depend on i. The numerical example concerns the following model:

$$N = 100, N_1 = 40, N_2 = 60, p_a = 125, p_u = 0.45, p_c = 250, \alpha = 0.996;$$

$$y_i = 2 + 0.25(i-1), \qquad i = 1,\ldots,9;$$

$$q_{ij} = \begin{cases} 0.2 & \text{if } i = 1,2; j = 1,\ldots,5; \\ & \text{if } i = 3,\ldots,7; j = i-2, i-1, i, i+1, i+2; \\ & \text{if } i = 8,9; j = 5,\ldots,9; \\ 0 & \text{otherwise;} \end{cases}$$

$$x_0 = 5, w_{\min} = 80, w_{\max} = 130, x_{\min}^{(1)} = 12, x_{\max}^{(1)} = 21, x_{\min}^{(2)} = 14, x_{\max}^{(2)} = 27;$$

$$\begin{aligned}
u_1(x) &= 0.1x^2 - 0.1x + 10, & x_0 \leq x \leq x_{\max}^{(1)} - 1; \\
u_2(x) &= 0.05x^2 - 0.28x + 9.35, & x_0 \leq x \leq x_{\max}^{(2)} - 1; \\
w_1(x) &= -0.2x^2 + 11.67x - 28.38, & x_0 \leq x \leq x_{\max}^{(1)}; \\
w_2(x) &= -0.13x^2 - 8.88x - 16.18, & x_0 \leq x \leq x_{\max}^{(2)};
\end{aligned}$$

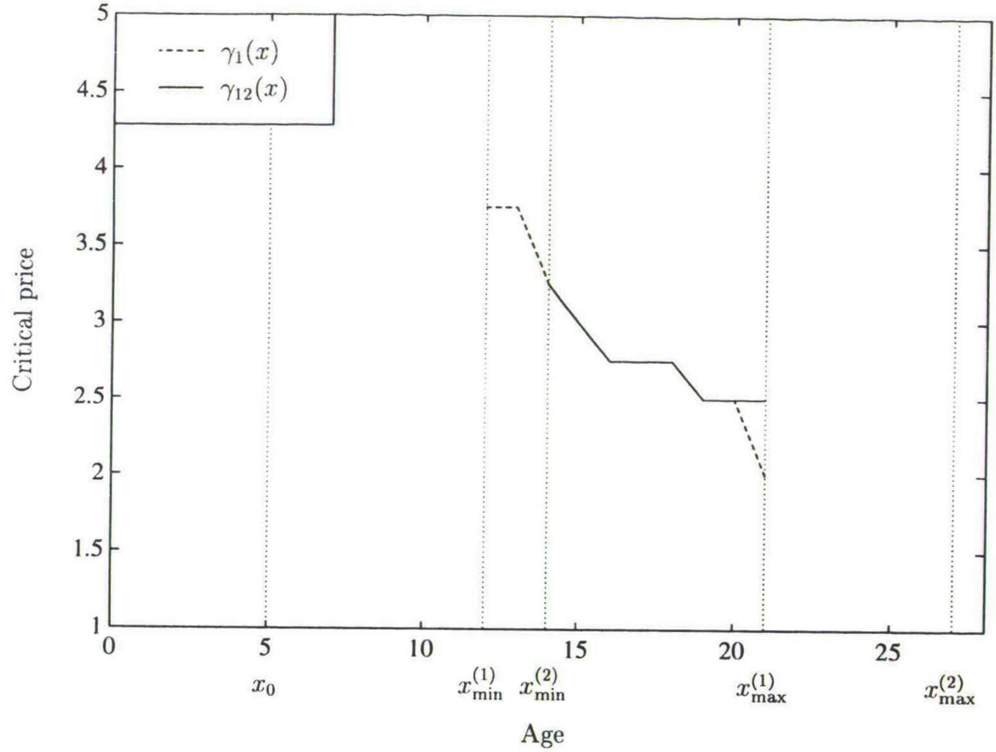

Figure 5: Critical prices for the two groups together

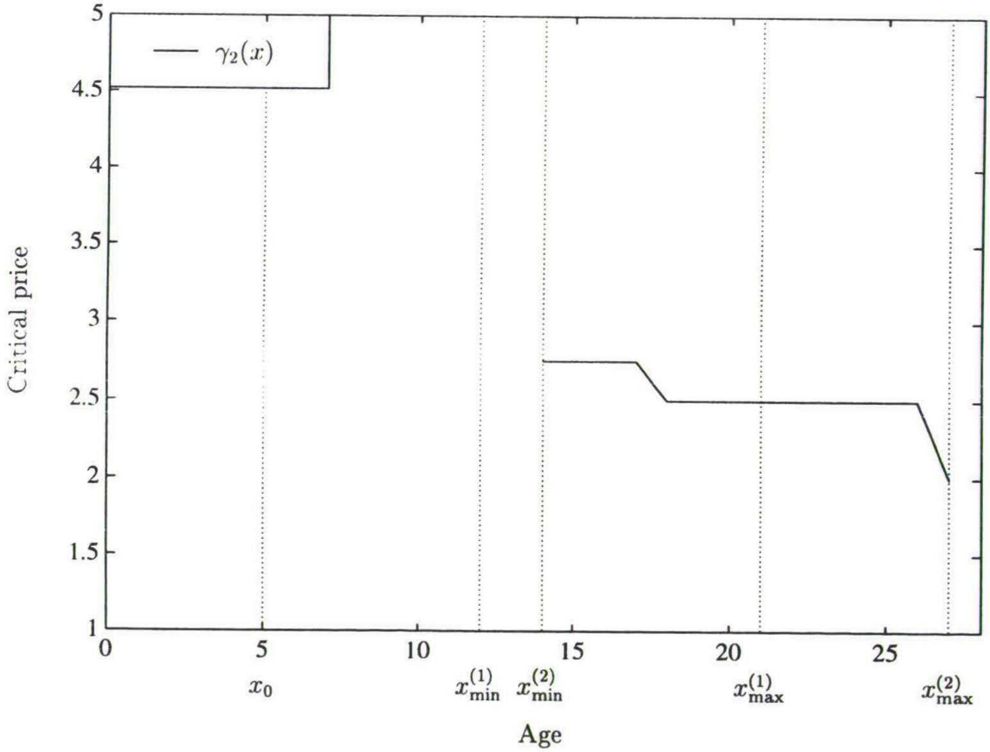

Figure 6: Critical price for only group 2

In Figures 5 and 6 we display the numerical results obtained for this model. These typical results clearly illustrate the effects of the age on the critical prices and need no further discussion. Note that the numerical procedure to compute the optimal strategy for this model requires a total of 5 iterations and a total time of 2 seconds.

7 Conclusion

In this study we formulate a Markov decision model by which the optimal delivery strategy for a heterogeneous group of porkers can be determined. In formulating the model it is assumed that the feeding-regime is given and cannot be changed by the farmer. For simplicity we restrict our attention to a group of porkers consisting of two homogeneous subgroups, that is, the animals in each subgroup have the same growth properties. The farmer starts a fattening round with only young pigs and he does not fill the vacant places created by the selling of a subgroup until the current round is terminated. Further it is assumed that only the pork price is random.

The model formulated in this study applies to an individual firm only. In contrast with the sector as a whole such a firm cannot influence the pork price by the quantity supplied. The central question of this study is how a farmer striving after a maximal financial result should react upon the variability of the pork price in exploiting the firm.

The optimal delivery strategy can be characterized by a system of critical prices. A critical price holds for a (homogeneous) group at a slaughter-ripe age or for a combination of (homogeneous) groups at a slaughter-ripe age. If in a week the pork price exceeds the critical level for a group or a combination of groups, then it is worth while to sell that group or combination of groups in this week. If on the other hand the pork price lies below the critical level for a certain group, then the sale of this group should be postponed to a later time. So the critical prices can be characterized as a micro-supply function.

We have paid no attention to the possibility that by choosing a suitable feeding-regime the farmer can influence the weight progresses of the animals within certain bounds. Here it is assumed that the feeding-regime is fixed. The motivation for this is that in practice the farmers usually follow the feeding schemes that are advised by agricultural experiment stations. However, by speeding up or slowing down the tempo of feeding the farmer could anticipate a favourable or an unfavourable development of the pork price. The possibility of controlling the weight progresses by the feeding-regime can be included in the model by changing the state description to the triplet (w_1, w_2, y), where w_i stands for the weight of group i animals for $i = 1, 2$, and y for the current price of pork, and by allowing for different feeding rations at each age.

In this study we distinguish two groups, namely fast and slow growers. It is assumed that all animals within a group show the same weight progress. The variability that in reality occurs within a group is neglected. Without introducing additional complications this variability can be taken into account by enlarging the number of distinct groups. Of course, this generalization leads to a larger dimension of the resulting Markov decision model. Also the fineness of the lattice that is used to characterize the distribution of the pork price influences the dimension of the Markov decision model. In the application of the proposed model one has to decide upon the number of different prices.

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