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ADMISSIBLE TARGET PATHS IN ECONOMIC MODELS

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Admissible target paths in economic models

Abstract

In this paper we solve the problem of which target paths can be tracked exactly, approximately and ultimately respectively, provided the economy on hand is described by a linear time-varying difference equation. Moreover, controllers are derived which yield the desired tracking properties. In particular the weighted minimum variance controller is discussed. The obtained theoretical results are interpreted in a number of economic settings.

I. Introduction

In many (economic) papers the so called target path controllability (also dynamic path controllability or perfect controllability) problem has been discussed. This problem concerns a prespecified time interval. The question is whether any time path of target variables can be attained in this interval by means of a good choice of the policy instruments.
For linear time-invariant systems this problem has been studied in Aoki et al. (1975, 1979), Brocket et al. (1965), Buiter (1979), Maybeck (1982), Preston et al. (1972, 1974, 1982), Tinbergen (1951) and Wohltmann et al. (1981, 1983, 1984); for linear continuous time-varying systems in Albrecht et al. (1986), Grasse (1986) and Wohltmann (1985); for linear discrete time-varying systems in Engwerda (1988b, 1988d) and for nonlinear discrete time-varying systems in Nijmeijer (1988).
From a policy point of view, this target path controllability property of an economy is a very nice one. For, anything a policy maker wishes to achieve in the prescribed time interval can be realized: the designer can always suit the policy maker. If the economy is not target-path controllable, then a natural question posed by the policy maker would be: which target paths can still be tracked. This question is answered in this paper. Any target path that can be tracked, in some sense, is called admissible. A distinction is made between three types of admissibility. First of all we discuss the target paths that can be tracked exactly during a prespecified time interval. Any trajectory belonging to this set is called strongly admissible. Secondly, we distinguish the approximately admissible target paths of level α. Any trajectory belonging to this set can be tracked exactly up to an error not exceeding α over a prespecified time interval. Finally, we discern the target paths that can be tracked asymptotically, i.e. which are reachable in the end. These are called the asymptotically admissible target paths. For each of these concepts, we derive a necessary condition on the dynamic evolution of an admissible target path. Moreover, we provide a method to check whether a predescribed trajectory has any of these three admissibility properties.

If a prespecified target trajectory satisfies any of these three requirements the question arises how an appropriate control can be designed. That is, in which way the instruments should be chosen such that

the prespecified target trajectory is either tracked exactly, or approximately, or ultimately. In section 2 we shall construct for every admissible target path an appropriate control.
This control is composed of an open loop control and a static state-feedback. Unfortunately, the open loop part generally cannot be dispensed of. Since the open loop part extensively uses all information concerning the development of deterministic variables and desired target variables in the future, in section 3 we look for a controller which lacks this amount of information. It turns out that a weighted minimum variance controller has this property if we look for a controller tracking an asymptotically admissible target path. The only prerequisite is that the weight matrix in this controller must be well-chosen. Since this matrix plays such a crucial role in the design, we pay attention to this choice problem in a seperate paragraph.

The paper ends with some conclusions. To help the reader fully understand the most important theoretical results obtained in this paper, we interpret them for section 2 in a seperate subsection 2.2, and illustrate the effect of different choices of the weight matrix in the MV-controller in section 3.3. The paper starts with the introduction of the considered system, some notation and the discussion of several notions which appear throughout the paper.

II. Definitions, Tools and Mathematical Preliminaries

The base system analyzed in this paper is described by the following linear, finite dimensional, time-varying difference equation:

$$
\Sigma_{yd}:\quad \begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) + G(k)d(k);\; x(k_0) = x_0 \\ y(k) &= C(k)x(k), \end{aligned}
$$

where $x(k)$ is an n-dimensional vector consisting of endogenous variables (state variables) observed at time k; $u(k)$ is an m dimensional vector of policy instruments (control variables); $d(k)$ is an s-dimensional vector of deterministic (non-controllable) variables which is assumed to be known at time k; $y(k)$ is an r-dimensional vector of target variables, and x_0 the initial state of the system. We assume that the system parameters (i.e. the matrices $A(k)$, $B(k)$, $G(k)$ and $C(k)$) are bounded in time. The following notation will be used:

$v^*(i)$ denotes a reference value for variable v at time i

$v^T(i)$ denotes the transpose of $v(i)$

$v[k,\ell] := (v^T(k),\dots,v^T(\ell))^T$

$v[k,.] := (v^T(k),v^T(k+1),\dots)^T$

Im A denotes the image of the mapping defined by matrix A, Ker A denotes its kernel

$A(k+i,k) := A(k+1-1)\times\dots\times A(k)$ for $i > 0$ and $A(k,k) := I$

$S[k,k-N] := [B(k)\,|\,A(k+1,k)B(k-1)\,|\dots|\,A(k+1,k-N+1)\,|\,B(k-N)]$

$V[k,k-N] := [G(k)\,|\,A(k+1,k)G(k-1)\,|\dots|\,A(k+1,k-N+1)\,|\,G(k-N)]$

$W[k,k+i] := [C^T(k)\,|\dots|\,\{C(k+i)A(k+i,k)\}^T]^T$

$x(k_1,k_0,x_0,u)$ is the state of the system at time k resulting from the initial state x_0 and time k_0 when the control $u[k_0,k-1]$ is applied

$y(k,k_0,x_0,u) := C(k)x(k,k_0,x_0,u)$.

Throughout the paper norms are used. The norm we use is, unless stated differently, as well for vectors as matrices the Euclidean one. That is, if $y = (y_1,\dots,y_r)$ then $\|y\|^2 = \sum_{i=1}^{r} y_i^2$. We proceed with giving a number of elementary lemmas which are used in the forthcoming sections. The first lemma makes use of the unique Moore-Penrose inverse A^+ of a matrix A, which is properly defined e.g. in Lancaster (1985), section 12.8. We state in this section two important properties of it. A proof can be found again in Lancaster (1985), chapter 12.9.

Lemma 1:
Let $A \in \mathbb{R}^{n\times m}$ (i.e. the set of all real n×m matrices). Then the equation $Ax = b$ is solvable iff rank$[A\,|\,b]$ = rank C. In that case, a solution is provided by $x = A^+b$.
Moreover, this solution is uniquely determined iff rank A = m. It then equals $(A^TA)^{-1}A^Tb$.

Lemma 2:

$$\min_x \|Ax-b\| = \|Ax_0-b\|, \text{ where } x_0 = A^+b. \qquad \square$$

Another property we will frequently use concerns the convergence of the solution of an inhomogeneous linear recurrence relation. A necessary condition (which is in general not sufficient) on the inhomogeneous part reads as follows.

Lemma 3:
Let $\|A(k)\| \le c$ for all k, and let $\{e(k)\}$ satisfy

$$e(k+1) = A(k)e(k) + v(k).$$

Then $e(k) \to 0$ implies $v(k) \to 0$.

Proof:
Follows directly from the equality $v(k) = e(k+1) - A(k)e(k)$. $\square$

We conclude this section by introducing the in engineering well-known impulse response matrix $M(k;p,q)$ from $u[k+p+q-2,k]$ to $y[k+p+q-1,k+p]$.
Let $t = k+p$. Then $M(k;p,q)$ equals by definition:

$$\begin{bmatrix} C(t+q-1)B(t+q-2) & C(t+q-1)A(t+q-2)B(t+q-3) & C(t+q-1)A(t+q-1,k+1)B(k) \\ 0 & & \\ \vdots & & \\ 0 \ldots 0 \; C(t)B(t-1) & C(t)A(t-1)B(t-2) \ldots & C(t)A(t,k+1)B(k) \end{bmatrix}$$

This matrix tells us what the effect of the policy instruments is on the target variables, that is, with $x_0 = 0$ we have that $y[k_0+p+q-1,k_0+p] = M(k_0;p,q)u[k_0+p+q-2,k_0]$.

The matrix plays a fundamental role in the target path controllability problem (see e.g. Engwerda (1988d)). In the remainder of this paper we shall see that this is also the case for the problems we consider here.

III. Characterization of admissible trajectories

1. Formal definitions and results

In the introduction of this paper we intuitively introduced the concepts of strongly, approximately and asymptotically admissible target paths. We shall formalize them now.

Definition 1:

Let $-\infty < k_0 \le k < \ell \le \infty$, and x be the state of Σ_{yd} at k_0. A reference trajectory $y^*[k,\ell]$ is called

- strongly admissible for x at k_0 if there exists a control sequence $u[k_0,\ell-1]$ such that $y[k,\ell] = y^*[k,\ell]$;
- approximately admissible of level α for x at k_0 if there exists a control sequence $u[k_0,\ell-1]$ such that $\|y[k,\ell]-y^*[k,\ell]\| \le \alpha$.

A reference trajectory $y^*[k_0,.]$ is called asymptotically admissible for x at k_0 if there exists a control sequence $u[k_0,.]$ such that $\|y(k)-y^*(k)\| \to 0$ for $k \to \infty$.

In each of these three cases we call u(.) a successful control.

In the remainder of this subsection we derive a necessary condition on the dynamic evolution of an admissible target path, a criterion to check whether a prespecified trajectory is admissible and a successful control for an admissible target path. These subjects are treated separately for strongly, approximately and asymptotically admissible target paths.
We start with the strongly admissible trajectories.
The first, rather trivial, observation is that these trajectories evolve in time in accordance with Σ_{yd}.

Theorem 4:

A reference trajectory $y^*[k,\ell]$ is strongly admissible for x at k_0 iff there exists a $u^*[k_0,\ell-1]$ such that

$$y^*(t) = C(t)x^*(t), \; k \le t \le \ell;$$

$$x^*(t+1) = A(t)x^*(t) + B(t)u^*(t) + G(t)d(t);$$

$x^*(k_0) = x,\ k_0 \le t \le \ell-1.$

Proof:

The sufficiency of the condition is trivial. That the condition is also necessary is seen by the following reasoning.

Consider time k. Let $y^*(k)$ be strongly admissible.

Then, by definition, there exists a $u^*[k_0,k-1]$ such that $y(k) - y^*(k) = 0$. Since $y(k) = C(k)x(k,k_0,x,u^*)$, this proves the necessity part of the theorem.

□

In order to study the subject of how strong admissibility of a prespecified reference trajectory can be checked, we introduce the notion of command error flow. The command error flow is defined as the diffence between the flow of Σ_{yd} from k_0 up to k (i.e. the evolution of the targets if no control is applied to the system) and the target reference value at k. Formal,

Definition 5:

The command error flow at k from k_0, $z_{k_0}(k)$, is defined as:

$$z_{k_0}(k) := y^*(k) - C(k)A(k,k_0)x(k_0) - C(k)V[k-1,k_0]d[k-1,k_0].$$

Theorem 6:

A reference trajectory $y^*[k_0+p,k_0+p+q-1]$ is strongly admissible at k_0 iff either one of the following two equivalent conditions is satisfied:

i) $$z_{k_0}[k_0+p,k_0+p+q-1] = M(k_0;p,q)M^+(k_0;p,q)z_{k_0}[k_0+p,k_0+p+q-1];$$

ii) $$z_{k_0}[k_0+p,k_0+p+q-1] \in \text{Im } M(k_0;p,q).$$

Moreover, the following control is successful:

$$u[k_0,k_0+p+q-2] = M^+(k_0;p,q)z_{k_0}[k_0+p,k_0+p+q-1].$$

Proof:

Easy calculations show that $y[k_0+p,k_0+p+q-1] - y^*[k_0+p,k_0+p+q-1]$ equals

$$M(k_0;p,q)u[k_0,k_0+p+q-2] - z_{k_0}[k_0+p,k_0+p+q-1].$$

From this equality, it is clear that the command error is zero on the interval $[k_0+p,k_0+p+q-1]$ iff the command error flow on this interval belongs to the image of matrix $M(k_0;p,q)$. Application of lemma 1 yields then that condition ii) is necessary and sufficient for the existence of a solution of the problem. Moreover, this lemma brings in the successfulness of the stated control.

Finally, substitution of this control in the above mentioned equality shows that condition i) is also necessary for the strong admissibility of the reference path $y^*[k_0+p,k_0+p+q-1]$.

Obviously, this condition is also sufficient. □

We shall now perform this analysis for approximately admissible target paths. The results will turn out to be more complicated than the corresponding results for the strongly admissible target paths. This is due to the fact that command errors can compensate each other when time passes. Nevertheless, the main conclusion will be that the system structure of Σ_{yd} plays a dominant role in the development of an approximately admissible trajectory. The results are summarized in the theorems 7 and 8.

Theorem 7:

A reference trajectory $y^*[k,\ell]$ is approximately admissible for x at k_0 of level α iff there exist Δ, $u[k_0,\ell-1]$, $v[k_0,\ell-1]$ and $w[k_0,\ell-1]$ such that

$$y^*(t) = C(t)x^*(t) - w(t), \quad k \le t \le \ell;$$

$$x^*(t+1) = A(t)x^*(t) + B(t)u(t) + G(t)d(t) - v(t); \quad x^*(k_0) = x - \Delta, \quad k_0 \le t \le \ell-1,$$

and

$$\sum_{i=k}^{\ell} \| C(i)A(i,k_0)\Delta + \sum_{j=k_0+1}^{i} C(i)A(i,j)v(j-1) + w(i) \| \leq \alpha.$$

Proof:

We note first that from

$$x(t+1) = A(t)x(t) + B(t)u(t) + G(t)d(t);\; x(k_0) = x,$$

$$y(t) = C(t)x(t),$$

we have immediately that, for arbitrary $x^*(.)$ and $y^*(.)$,

$$x(t+1) - x^*(t+1) = A(t)(x(t)-x^*(t)) + A(t)x^*(t) + B(t)u(t) +$$

$$+ G(t)d(t) - x^*(t+1)$$

$$y(t) - y^*(t) = C(t)(x(t)-x^*(t)) + C(t)x^*(t) - y^*(t),$$

$$\text{for all } t \geq k_0.$$

Here, we denote $x^*(k_0)$ by $x-\Delta$.
Now, define the following variables:

$$\Delta(t) := x(t) - x^*(t),$$

$$v(t) := A(t)x^*(t) + B(t)u(t) + G(t)d(t) - x^*(t+1)$$

$$e(t) := y(t) - y^*(t)$$

and

$$w(t) := C(t)x^*(t) - y^*(t)$$

Then, the above relationship can be rewritten as

$$\Delta(t+1) = A(t)\Delta(t) + v(t);\ \Delta(k_0) = \Delta;$$

$$e(t) = C(t)\Delta(t) + w(t) \quad ;\ t \geq k_0.$$

By definition, a reference trajectory $y^*[k,\ell]$ is approximately admissible of level α iff there exists a control sequence $u[k_0,\ell-1]$ such that

$$\|y[k,\ell] - y^*[k,\ell]\| \leq \alpha,$$

or equivalently

$$\sum_{i=k}^{\ell} \|e(i)\| \leq \alpha.$$

Since $\Delta(t) = A(t,k_0)\Delta + \sum_{j=k_0+1}^{t} A(t,j)v(j-1)$, we have that

$$e(t) = C(t)A(t,k_0)\Delta + \sum_{j=k_0+1}^{t} C(t)A(t,j)v(j-1) + w(t).$$

Substitution yields the stated result. □

<u>Theorem 8</u>:

A reference trajectory $y^*[k_0+p,k_0+p+q-1]$ is approximately admissible of level α at k_0 iff the following condition is satisfied:

$$\|(M(k_0;p,q)M^+(k_0;p,q)-I)z_{k_0}[k_0+p,k_0+p+q-1]\| \leq \alpha \tag{1}$$

Moreover, the following control is successful:

$$u[k_0,k_0+p+q-2] = M^+(k_0;p,q)z_{k_0}[k_0+p,k_0+p+q-1]$$ □

The proof of the last theorem is similar to the proof of theorem 6. All we have to remember, in order to get this result, is that the vector $x_0 = A^+b$ yields the best (Euclidean norm) approximate solution of the linear equation $Ax = b$ (see lemma 2). Therefore we omit a proof of this theorem.

It is clear from theorem 8 that for a given lead p a minimal α for which a trajectory is approximately admissible of level α exists. This α is given by (1) where the inequality has to be replaced by an equality.

Finally, we consider the asymptotically admissible target paths. The results are stated in the next two theorems.

Theorem 9:

A reference trajectory $y^*[k_0,.]$ is asymptotically admissible for x at k_0 iff there exist $u[k_0-1,.]$ and $v[k_0,.]$ such that:

$$y^*(k) = C(k)x^*(k) + v(k) \tag{2}$$

$$x^*(k+1) = A(k)x^*(k) + B(k)u(k) + G(k)d(k), \; x^*(k_0) = x,$$

where $v(.) \to 0$.

Proof:

We know that for an arbitrary $y^*(k)$,

$$y(k) - y^*(k) = C(k)x(k) - y^*(k)$$

$$x(k+1) = A(k)x(k) + B(k)u(k) + G(k)d(k); \; x(k_0) = x.$$

Therefore, $y(k) - y^*(k)$ converges to zero when k tends to infinity iff

$$y^*(k) = C(k)x(k) + v(k),$$

with $v(.) \to 0$.

This proves the claim. □

Theorem 10:

A reference trajectory $y^*[k_0,.]$ is asymptotically admissible at k_0 iff the following condition is satisfied:

$$\lim_{k\to\infty} \| \{(C(k)S[k-1,k_0])(C(k)S[k-1,k_0])^+ - I\} z_{k_0}(k) \| = 0.$$

Proof:

Follows directly from the identity

$$y(k) - y^*(k) = C(k)S[k-1,k_0]u[k-1,k_0] - z_{k_0}(k)$$

and lemma 2. □

Notice that in this case the corresponding open loop control $u[k_0-1,k]$ does not necessarily have to converge.
One possibility to obtain a successful control is to rewrite the admissible reference trajectory in the form (2) indicated in theorem 9. The control u(.) is then successful.
That this way of solving the problem is rather unsatisfactory, is obvious.

2. Interpretation of the results

The theorems 4, 7 and 9 lead to the, intuitively very appealing, result that any admissible reference trajectory must satisfy a recurrence equation which corresponds to the system. The only difference with the system is that the reference trajectory may posses an additional disturbation. The additional disturbation must satisfy some properties which depend on the kind of admissibility that is considered. They are more stringent for the strongly and approximately than for the asymptotically admissible trajectories. So, it is more difficult to find an strong or approximate admissible target path than an asymptotic admissible one.
Though the remark made at the end of the previous section suggests otherwise, we will see in section IV.1 that this ordering in difficulty carries over to the problem of constructing a successful control.
From the theorems 5 and 8 we have that the successful control we constructed uses all information concerning the deterministic variables and reference trajectories in the time interval $[k_0,k_0+p+q-1]$, and that this information must be available already at time k_0. Engwerda showed in an example in (1988b, pp. 60) that there exist situations in which this information really can not be missed in the design of a successful control. Since information about the future development of deterministic variables

is rarily known, the design of a successful control becomes a delicate matter in those cases.

In the next section we shall see that for any asymptotically admissible target paths there exists a successful control which only uses the information concerning the deterministic variables and the reference trajectories of one timestep ahead. So, this control is much preferable.

So far about some general conclusions that can be drawn from the previous section. In the remainder of this section we shall concentrate on the implications of these results for economic regulation.

From mathematics we have that many economic processes are approximately described by a model of the form Σ_{yd}. Let us assume that this is also the case for the development of the economy of a country. According to the above developed theory, policy makers then are very restricted in their choice of admissible target paths. These paths depend severely on the structural parameters of the system. If the current development of the economy does not fit with the policy makers goals, they can influence the development in two ways. One way is, by altering the policy instruments. They can be raised or lowered, just what the policy makers like. From the theorems 4, 7 and 9 it is, however, obvious that the existence of a policy which will yield admissibility of the policy makers goals is not guaranteed. In general the policy objectives will conflict whatever the choice is of the instruments. If the policy makers stick to their goals in such a situation, the only way left open to realize them is to pursue a policy that is aimed at changing the structural (system) parameters.

In both cases they have to design a suitable robust control. As we stressed at the beginning of this section a prerequisite to design such a successful control is, however, that mostly as well the structural parameters as a good estimate of the future development of the deterministic variables is available. Now, the economy of a country consists in particular of a private sector. It is well known that this sector reacts relatively fast on data, which often hardly can be modelled. But, since this sector also influences the structural parameters of the economy, it is clear that these parameters are difficult to forcast. A similar problem in this context arises when calamities occur, the current structural parameters are then also hardly predictable. This, since the sampled data from the past is related to the past values of these parameters.

So, the policy makers are confronted with an almost unsolvable problem. At those moments that a change in the control strategy is really needed there is no-information available concerning the value of the current structural parameters. In practice, policy makers mostly design in such situations their control strategy based on the passed values of these parameters. A policy which may yield a destabilizing effect instead of a stabilizing effect w.r.t. the goals of the target variables!
So, the design of an adequate policy is a rather delicate matter. The conclusion of this reasoning is that if a government likes to pursue an effective short term stabilization policy, then she should minimize her number of political goals and the number of policy instruments which she wants to manipulate. In that way it may be possible to obtain in a short time a small reliable model of the changed economy. A necessary condition to pursue such a policy is, however, that the instruments are flexible. That is, the time gap between execution and decision for a policy is short (Avoiding bureaucracy is a must!), and that there is enough elbow-room for change of instruments.
So, the policy makers must take care that they pursue a structural buffer policy. In other words, they must take care that in the long run there are enough financial resources to enhance a short term stabilization policy. Maybe it is a good idea to split up these tasks. That is, to introduce an economic regulation department, which acts independently of the government, and provides for a short run stabilization policy concerning goals from target variables that are commonly agreed on. The government itself would then be able to concentrate on the problem of how the short run measurements can be translated into an effective long run stabilization policy.
An example which fits into this context would be the introduction of unemployment projects to lower unemployment rates (by the regulation department) which are then as quickly as possible either privatized or financed by (e.g.) raising taxes (a governmental decision).

We conclude this discussion on economic regulation by the (trivial) remark that a permanent high level of the instruments has a direct influence on the current economic variables (via B(k)) and consequently a permanent effect on the future values (via A(k+1) etc.).

IV. Characterization of admissible minimum variance controlled target paths

1. The admissible reference trajectories

In this subsection we extend some results concerning minimum variance control obtained by Engwerda in (1988c) (see also 1988b).
In these papers the problem was studied of which target paths are asymptotically admissible if a control strategy is used based on minimizing the minimum variance (MV) cost criterion.

This criterion expresses that positive and negative deviations of target variables from desired levels are weighted equally and that they are increasingly costly. The major reasons to concentrate on this criterion are:

1) Because of the uncertainty in the real-life macro-economic situation there is a constant need for short period adaptation of control with respect to new information. A regulator which is based on minimizing a cost criterion with a short planning horizon makes such an adaptation possible. The MV-controller is a typical example of a regulator which satisfies this requirement (see Aalders et al. (1983) and Åström et al. (1983, 1984)).
2) The computational ease and relatively simple formulas of the control algorithm.

Formally, the MV-controller is defined as the controller which minimizes, for a given initial state x and reference trajectories $x^*(.)$, the following cost criterion, J, subject to the system Σ_d:

$$\Sigma_d : x(k+1) = A(k)x(k) + B(k)u(k) + G(k)d(k); \; x(0) = x,$$

$$J := (x(k+1)-x^*(k+1))^T Q(k+1)(x(k+1)-x^*(k+1)),$$

where $Q(k)$ is a positive definite matrix, for any k.
If we perform this minimization we get the result stated in lemma 11. In this lemma the notation $P(k,Q)$ is introduced to denote the matrix
$I - B(k)(S(k+1)B(k))^+ S(k+1)$, where S comes from the factorization of Q into S^TS.

This notation will be used throughout this section.

Lemma 11:

The MV-controller is given by:

$$u(k) = -(S(k+1)B(k))^{+}S(k+1)(A(k)x(k)+G(k)d(k)-x^{*}(k+1)). \qquad (3)$$

Proof:

Follows straightforward from lemma 2. □

In the next theorem we give a characterization of all admissible trajectories $y^{*}(.)$ when MV-control is used to regulate the system Σ_{yd}. To this extent we first define what we mean by admissibility in this context.

Definition 12:

Let Q(.) be given. A reference trajectory $y^{*}[0,\ell]$ is called

i) MV-strongly admissible with respect to Σ_{yd} and x_0, if there exists a sequence $x^{*}[0,\ell-1]$ such that the MV-control (3) yields $y[0,\ell] = y^{*}[0,\ell]$ and $x[0,\ell-1] = x^{*}[0,\ell-1]$.

ii) MV-asymptotically admissible with respect to Σ_{yd} and x_0, if there exists a sequence $x^{*}[0,.]$ such that the MV-control (3) yields $y(.) - y^{*}(.) \to 0$ and $x(.) - x^{*}(.) \to 0$.

Note: A similar definition can be given for MV-approximately admissible trajectories. However, since this concept does not play any role in the rest of this paper we omit it. Readers interested in this concept are referred to Engwerda (1988b, pp. 121).

Theorem 13:

A trajectory $y^{*}[0,\ell]$ is

i) MV-strongly admissible w.r.t. Σ_{yd} and x_0 iff $\exists\, z[0,\ell-1]$ such that:

$$x^{*}(k+1) = A(k)x^{*}(k) + B(k)z(k) + G(k)d(k),\ x^{*}(0) = x_0,$$

$$y^{*}(k) = C(k)x^{*}(k),\ 0 \le k \le \ell.$$

ii) MV-asymptotically admissible w.r.t. Σ_{yd} and x_0 iff $\exists\ \bar{x}^*(0)$, $z[0,.]$, $v[0,.]$ and $w[0,.]$ such that:

1) $x^*(k+1) = A(k)x^*(k) + B(k)z(k) + G(k)d(k) - P^T(k)v(k), x^*(0) = \bar{x}^*(0)$
2) $y^*(k) = C(k)x^*(k) + w(k)$, where $w(.) \to 0$, and
3) $\bar{e}(0) := x - \bar{x}^*(0)$ is controlled to zero by means of $v(.)$ in:

$$e(k+1) = P(k)A(k)e(k) + P(k)P^T(k)v(k),\ e(0) = \bar{e}(0).$$

Proof:

Let $x^*[0,.]$ be any state trajectory. Then, application of the MV-controller yields, with $e(k) = x(k) - x^*(k)$, the closed-loop error equation:

$$e(k+1) = P(k)A(k)e(k) + P(k)\{A(k)x^*(k) + G(k)d(k) - x^*(k+1)\}.$$

Consequently, $x(k) - x^*(k)$ is zero on $0 \le t \le \ell-1$ iff $x^*(0) = x_0$ and $P(k)\{A(k)x^*(k) + G(k)d(k) - x^*(k+1)\}$ equals zero on this interval. Since the kernel of $P(k)$ equals $\operatorname{Im} B(k)$, we can rewrite this last condition as:

$$x^*(k+1) = A(k)x^*(k) + B(k)z(k) + G(k)d(k) \text{ for some } z(k).$$

As $y(k) = C(k)x^*(k)$, we have that an additional condition for strong admissibility for $y^*[0,\ell]$ is that $y^*(k) = C(k)x^*(k)$. This proves result numer i).

To prove result number ii) we first recall from Engwerda (1988c, theorem 1) that an arbitrary MV-admissible state trajectory $x^*[0,.]$ has to satisfy relation 1 and 3. Moreover, since both $y(k) - y^*(k) = C(k)x(k) - y^*(k)$ and $x(k) - x^*(k)$ converge to zero, we have that $y^*(k)$ equals $C(k)x^*(k) + w(k)$ for some $w(k)$, where $w(.) \to 0$. □

From this theorem, together with the theorems 4 and 9, we immediately deduce the following fundamental result.

Theorem 14:

i) The set of MV-strongly admissible trajectories equals the set of strongly admissible trajectories.

ii) If the system $x(k+1) = P(k)A(k)x(k)$ is asymptotically stable, with $P(k)$ as defined above, then the same property as in i) holds for the asymptotically admissible target paths. □

Another phraseology of theorem 14 is to say that the MV-controller is a successful controller for strongly admissible target paths which start at initial time zero, and that it is also successful for asymptotically admissible target trajectories provided it stabilizes the closed-loop system. Since we can influence the stability of the closed-loop matrix $P(k)A(k)$ by choosing an appropriate weight matrix $Q(k+1)$, we discuss in the next subsection some alternative choices for it.

2. Choice of weight matrix Q(k+1)

In the previous subsection we pointed out that a prerequisite to obtain a successful MV controller, tracking any asymptotically admissible target path, is that the matrix $P(k)A(k)$ is asymptotically stable. Moreover, we indicated that this can be accomplished by choosing an appropriate weight matrix $Q(k+1)$.

In this section, we shall go into detail about the pros and cons of several choices of this matrix. We will show that the pros and cons closely are related to the available amount of information concerning the future development of the structural parameters $A(.)$ and $B(.)$ of the system Σ_{yd}. Thereto, we first discuss the construction of a stabilizing weight matrix if we know very much about the structural parameters and, later on, constructions if less information concerning these parameters is available.

The first special case we consider is a time-invariant controllable system, i.e. a system with the property that the structural parameters do not change in time and, moreover, the matrix $S[0,n-1]$ (which consequently is time-invariant too) has full row-rank. Then, from Engwerda et al. (1988e) we have the following result.

Construction 1:

Let (A,B) be controllable. Then there exists a weight matrix Q such that PA is nilpotent with index κ_1 (κ_1 is the controllability index; a natural number which is smaller than n). That is, there exists a weight matrix Q such that $(PA)^{\kappa_1} = 0$. Moreover, this weight matrix can be constructed by using Luenberger's phase canonical form (see Luenberger (1967)). □

If the system Σ_{yd} has really no noise components, then this choice of the weight matrix seems to be the best one. For, any initial error $\bar{e}(0)$ is controlled within n steps to zero. Note, however, that the components of matrix Q may be very big. Consequently, the applied control is very sensitive to small system noise disturbations; a highly undesirable property. Therefore, one has to be very careful if this weighting matrix is used to force a stabilizing MV-control.
A less sensitive stabilizing weighting matrix Q can be obtained by using the positive definite solution of an algebraic riccati equation. An additional advantage of this approach is that the considered system only has to be stabilizable, i.e. that rank$[A-\lambda I, B] = n$, for all complex λ. The result reads as follows:

<u>Construction 2</u>:
Let (A,B) be stabilizable, B injective and Q the positive definite solution of the next algebraic riccati equation.

$$Q = A^T\{Q - QB(B^TQB)^{-1}B^TQ\}A + I$$

Then, PA is asymptotically stable. □

A proof of this conjecture can be found e.g. in Engwerda et al. (1988e). Another advantage of this choice is, that many reliable computerpackages are available to calculate it.
A similar result holds for time-varying systems. However, this result uses the notion of stabilizability of a time-varying system. Since this is a rather difficult concept and goes beyond the scope of this paper we just state here the final result. This result can be found in Engwerda (1988b).

<u>Construction 3</u>:
Let (A(.),B(.)) be uniformly periodically smoothly exponentially stabilizable and assume, moreover, that there exists an $\beta > 0$ such that $B^T(k)B(k) \geq \beta I$ for all $k \geq 0$.

Then, there exists a sequence of matrices Q(.) such that for all k $\lim_{N\to\infty} (PA)(k,N) = 0$. Here, at any point k in time Q(k) is obtained as $\lim_{N\to\infty} Q_N(k)$, where $Q_N(k)$ is the solution to the recurrence equation:

$$Q_N(i) = A^T(i)\{Q_N(i+1) -$$

$$- Q_N(i+1)B(i)(B^T(i)Q_N(i+1)B(i))^{-1}B^T(i)Q_N(i+1)\}A^T(i) + I$$

$$Q_N(N) = I.$$ □

The disadvantage of the previous derived weighting matrices is that they all are designed using information concerning the future development of the structural parameters. The last weight matrix we design is motivated by the fact that usually in economies this information is lacking. Therefore, we assume in the sequel that the evolution of these parameters in the future is unknown and, moreover, information about the state of the system is not available in time. All we know, are the current system parameters. Based on this information, Engwerda showed in (1988a) that, with Q = I, the spectral norm of matrix P(k)A(k) (i.e. the largest singular value of this matrix) becomes as small as possible. Since the spectral norm measures the smallest norm (compared with any other matrixnorm) that can be obtained for normal matrices, this approach is optimal in case P(k)A(k) is normal.

Now, in general, the spectral norm of P(k)A(k) will be not smaller than one. This is a direct consequence of the fact that at most m singular values of matrix A(k) can be reduced (Eckart Young theorem). So, in general it is doubtfull whether the system e(k+1) = P(k)A(k)e(k) will be stabilized. But, since we assume that really no information is available concerning the future development of the system parameters, there is no alternative. Of course one can guess at the parameter developments. But from this point of view, the judgment that the system parameters e.g. will be time-invariant is as good (or bad) as the judgment that the system has some predescribed time-varying behaviour. Therefore the policy of minimizing the norm of the current closed-loop system matrix, P(k)A(k), is given the circumstances the most rational one. This in spite of the fact that, when performing this policy, it may happen that we obtain an unstable closed-loop system though the system is stabilizable (as turns out after all). So, summarizing we have the next result.

Construction 4:

It at time k only A(k) and B(k) are known to the designer, then the best choice (in the sense of optimal stabilization policy) for Q is the identity matrix I. In that case $\|P(k)A(k)\|_s$ is as small as possible, although in general not smaller than one. □

In the succeeding section, we shall illustrate the effect on the closed-loop stability of some of the above proposed weighting matrices.

3. A simulation study

In this final paragraph we illustrate some of the obtained theoretical results from this section by means of a simulation study.

A small macro-economic model, estimated by Kendrick for the U.S. economy in (1982), serves as a starting point for the various experiments. The reduced form reads as follows:

$$\begin{bmatrix} C(k) \\ I(k) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} C(k-1) \\ I(k-1) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} u_1(k-1) \\ u_2(k-1) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} d(k) + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix}$$

where $C(k)$ = private consumption;
$I(k)$ = gross private investment;
$u_1(k)$ = governmental expenditures;
$u_2(k)$ = money supply;
$d(k)$ = (non-controllable) deterministic variable;
$V^T(k) = (v_1^T(k)\ v_2^T(k))$ is a white noise vector with $\text{cov}\{V(k)V^T(s)\} = \Sigma_v \delta_{ks}$.

Here, δ_{ks}, the Kronecker delta, equals 1 if k=s and 0 otherwise.
The parameters he obtained are respectively:

$$A = \begin{bmatrix} 0.914 & -0.016 \\ 0.097 & 0.424 \end{bmatrix} \; ; \; B = \begin{bmatrix} 0.305 & 0.424 \\ -0.101 & 1.459 \end{bmatrix} \; ;$$

$$\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} d(k) = \begin{bmatrix} -59.437 \\ -184.766 \end{bmatrix} \; ; \; \Sigma_v = \begin{bmatrix} 3.73 & 0 \\ 0 & 8.58 \end{bmatrix}$$

with inital values $C(0) = 387.9$ and $I(0) = 85.3$ and a weighting matrix $Q = \begin{bmatrix} 0.0625 & 0 \\ 0 & 1 \end{bmatrix}$.

Engwerda showed in (1988c) that the reference trajectories for the target paths chosen by Kendrick are in general not MV-asymptotically admissible. Therefore he proposed the next growth paths for the reference trajectories:

$$\begin{bmatrix} C^*(k+1) \\ I^*(k+1) \end{bmatrix} = \begin{bmatrix} 0.914 & -0.016 \\ 0.097 & 0.424 \end{bmatrix} \begin{bmatrix} C^*(k) \\ I^*(k) \end{bmatrix} + \begin{bmatrix} 0.305 & 0.424 \\ -0.101 & 1.459 \end{bmatrix} \begin{bmatrix} u_1^*(k) \\ u_2^*(k) \end{bmatrix} + \begin{bmatrix} -59.437 \\ -184.766 \end{bmatrix}$$

with $C^*(0) = 387.0$, $I^*(0) = 85.3$, $u_1^*(0) = 110.4$, $u_2^*(0) = 157.3$ and $u_i^*(k+1) = 1.0075\, u_i^*(k)$, $i = 1,2$.

The simulation results with this model, together with a discussion on several related admissibility problems can be found in the above mentioned paper.

Here, we illustrate the influence of the weighting matrix Q on the MV-admissibility properties. Since the influence is nil as long as the matrix B is invertible, we only consider one control variable in the next experiments. The chosen input is the money supply. The growth paths for consumption and investment are chosen such that they are asymptotically admissible.

For experiment 1 (= construction 4) the following data are used:

$$A, \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} d(k), \Sigma_v \text{ and initial values as before; } B = \begin{bmatrix} 0.424 \\ 1.459 \end{bmatrix} \; ;$$

$$\begin{bmatrix} C^*(k+1) \\ I^*(k+1) \end{bmatrix} = A \begin{bmatrix} C^*(k) \\ I^*(k) \end{bmatrix} + Bu_2^*(k) + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} d(k),$$

with $C^*(0) = 480$, $I^*(0) = 100$, $u_2^*(k+1) = 1.015\, u_2^*(k)$, $u_2^*(0) = 157.3$, and $Q = I$.

The same data are used for experiment 2 (= construction 1). The only difference between both experiments is, that in experiment 2 we use $Q = \begin{bmatrix} 60.0 & -13.9 \\ -13.9 & 3.5 \end{bmatrix}$. This matrix is obtained by using Luenberger's canonical decomposition for this system.

For both choices of the Q matrix, the closed-loop matrix PA becomes asymptotically stable. This is also clear from the figures, for in both cases the initial command error converges to zero despite white noise influences. Figures 1.iii and 2.iii show, moreover, that the tracking properties of the second controller are indeed better than those of the first (minimum norm) controller. Furthermore, we see in figures 1.iv and 2.iv that this is obtained at the expense of a controller which is more sensitive to white noise terms. This last mentioned property is due to the large components which appear in the Q matrix. So, one can say that the choice of weighting matrices must be a well considered choice between tracking speed and disturbance sensitivity of the controller.

In the last two experiments 3 and 4, we show that for time-varying systems, which at any time aim at a reduction of the command error to zero as quickly as possible, does not always result in an asymptotically stable closed-loop system.

To this end, reconsider Kendrick's model. Assume that alternately the government expenditures and money supply are solely used in time to regulate the system. That is, let

$$B(2k) := \begin{bmatrix} 0.305 & 0 \\ -0.101 & 0 \end{bmatrix} \text{ and } B(2k+1) := \begin{bmatrix} 0 & 0.424 \\ 0 & 1.459 \end{bmatrix}, \; k = 0,1,\ldots$$

Furthermore, assume that the reference trajectories are given by

$$\begin{bmatrix} C^*(k+1) \\ I^*(k+1) \end{bmatrix} = A\begin{bmatrix} C^*(k) \\ I^*(k) \end{bmatrix} + \begin{bmatrix} 59.437 \\ 184.766 \end{bmatrix},$$

$C^*(0) = 400$, $I^*(0) = 100$, and that in the model equation the sign of the deterministic variables is also reversed (this for purely illustrative purposes).

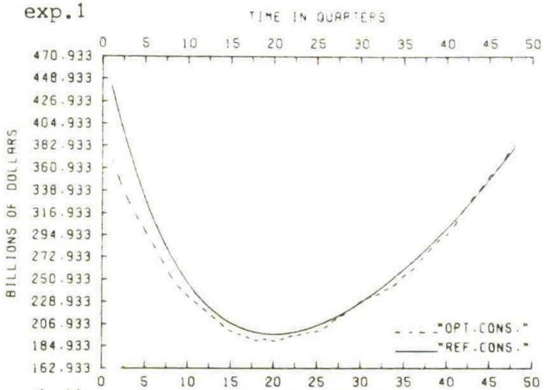
In figure 3 the simulation results are shown if we use the minimum norm controller (i.e. $Q = I$) for regulating this model. We see that the reference trajectories are tracked in this case.

Figure 4 shows the simulation results for the first ten time periods if the model is controlled at any point in time by means of the corresponding construction 1 regulator.

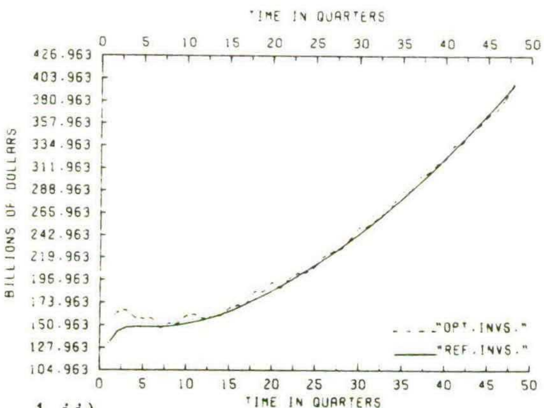
In contrast with experiment 3 we see that now a highly unstable closed-loop system is obtained. It is easily verified that this is due to the fact that matrix PA(0,2) is not stable.

Concluding, we can say that these four experiments sustain the claim that, if the choice Q = I in the MV-regulator gives rise to a closed-loop matrix PA that is in norm smaller than one, then this controller is a good sample among the class of successful, disturbance rejecting controllers.

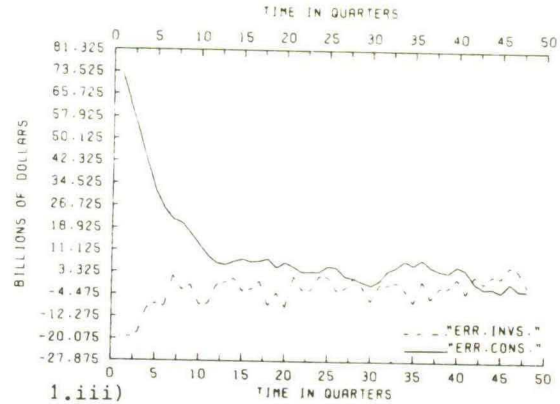
exp.1



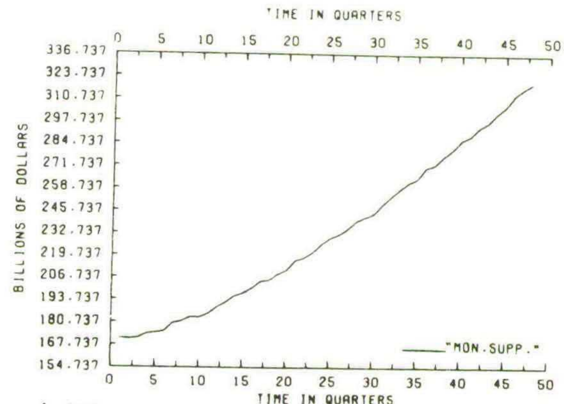
1.i)



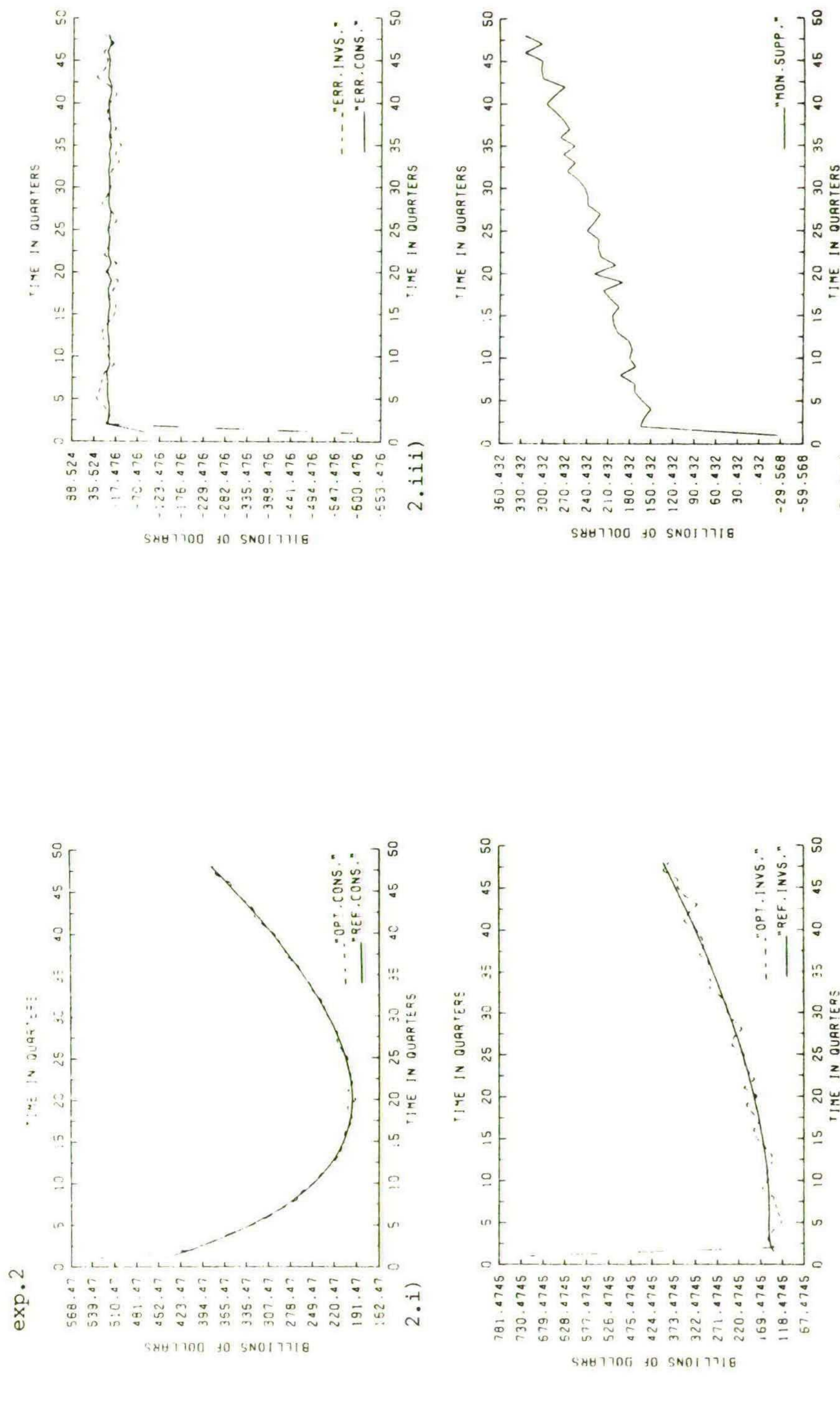
1.ii)



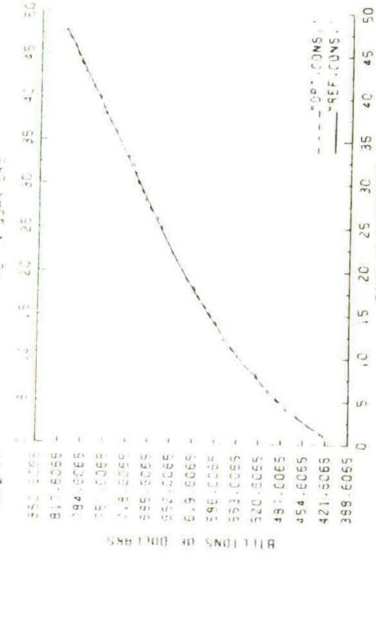
1.iii)



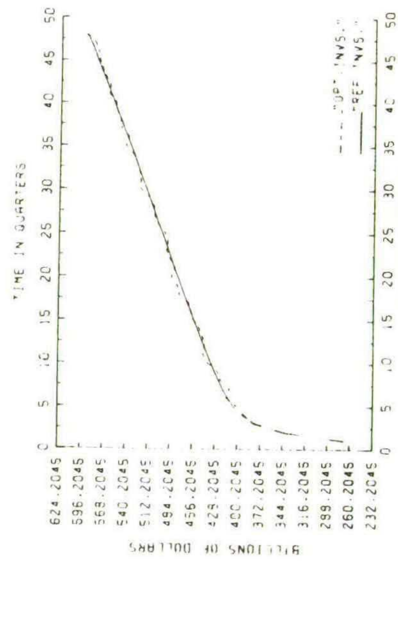
1.iv)


exp.2
2.i)
2.ii)
2.iii)
2.iv)
TIME IN QUARTERS
BILLIONS OF DOLLARS
"OPT.CONS."
"REF.CONS."
"OPT.INVS."
"REF.INVS."
"ERR.INVS."
"ERR.CONS."
"MON.SUPP."

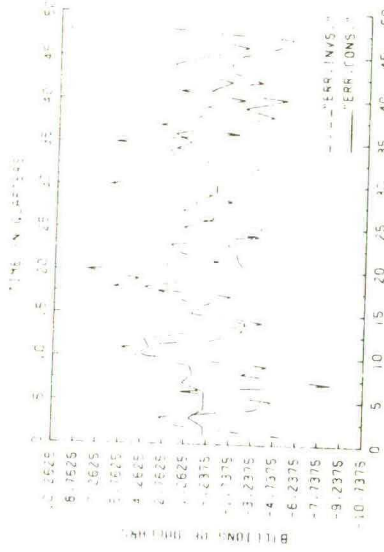
exp.3



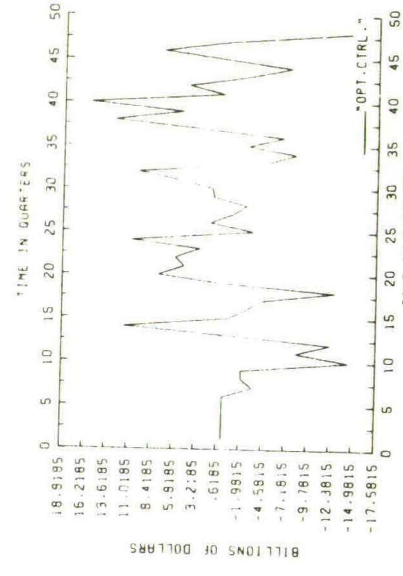
3.i)



3.ii)

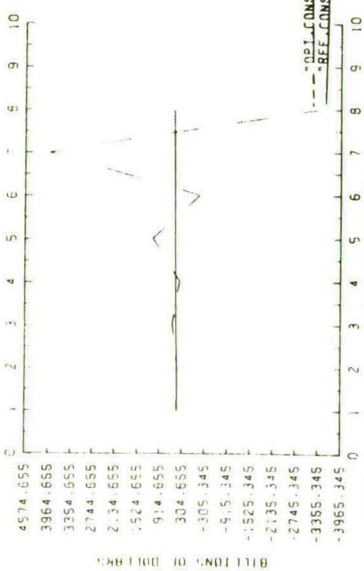


3.iii)

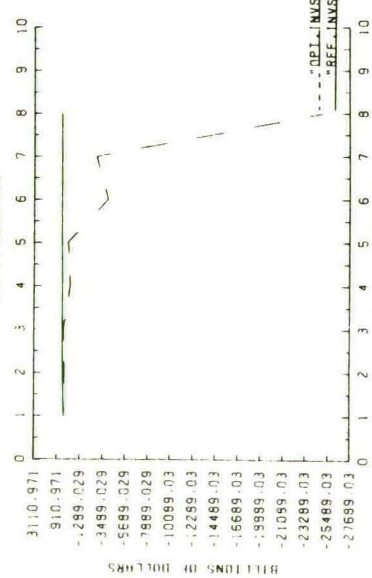


3.iv)

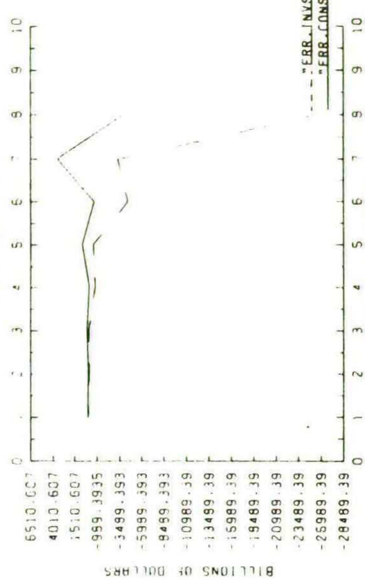
exp.4



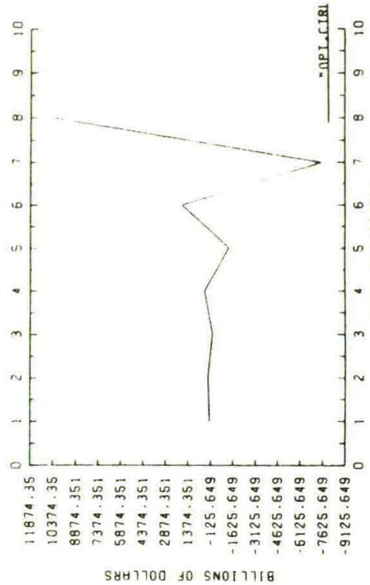
4.i)



4.ii)



4.iii)



4.iv)

V. Conclusion

In this paper we discussed the possibilities to control an economy, under the supposition that the economy is described by a linear time-varying difference equation. The discussion was splitted up into two parts.
First of all we investigated the admissibility of prescribed target paths. Apart from giving conditions on the dynamic evolution of an admissible target path, we also presented a method for checking the admissibility of a trajectory and constructed a controller which provides for the desired behaviour of the system if a trajectory is admissible. Since the proposed controllers are rather unfit for practical use, we studied the target paths that can be tracked by a more appropriate controller, the MV controller, in the second part of the paper. It turned out that both an important class of the strongly admissible target paths and the asymptotically admissible target paths can be tracked by this controller. For asymptotically admissible target paths we saw that the weighting matrix in the MV controller plays a crucial role. If this weighting matrix is chosen such that the corresponding MV controller stabilizes the closed-loop system matrix, then even every asymptotically admissible target path can be tracked. Therefore, we paid special attention to the construction of weighting matrices yielding a stable closed-loop system. We argued that in general the decision for choosing a certain matrix depends on the available amount of information concerning the future development of the system parameters. We illustrated in a simulation study the pros and cons of various proposed choices. In particular, we saw that if the future structural parameters of an economy are unpredictable than the best weighting matrix one can choose is the identity matrix. Moreover, we argued that in case the structural parameters change in time, short term fine tuning of the economy is practically impossible. In those cases we suggested to concentrate on a small number of targets and to introduce an economic regulation department which looks after the realization of these targets. This department should act independent of the government and have its own budget in order to facilitate quick interventions in the economy. How far these ideas are fruitfull and realizable remains, however, a topic for future research.

Concluding, we might say that the paper treats various limitations that exist in achieving prescribed goals from both a theoretical point of view as well as from a more practical point of view.

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