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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

A SILENT DUEL OVER A CAKE

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A SILENT DUEL OVER A CAKE

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Abstract

The division of a cake by two players is modelled by means of a silent game of timing. It is shown that this game has a unique Nash equilibrium. The strategies of the Nash equilibrium are explicitly given.

KEYWORDS: Nash equilibrium, Game of timing.

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1 Introduction

We consider the situation that two players divide a cake of size 1. At time 0 player 1 has the initial right to receive the amount $\alpha_1 > 0$ and player 2 has the initial right to receive the amount $\alpha_2 > 0$. Here, it is assumed that $\alpha_1 + \alpha_2 < 1$. Player i must choose a point in time $t_i \in [0, \infty)$ to claim his piece of the cake. If $t_1 < t_2$, then player 1 gets the discounted part $\alpha_1 \delta^{t_1}$ of the cake, while player 2 receives the discounted remaining part $(1 - \alpha_1)\delta^{t_2}$, with $0 < \delta < 1$. So, in particular, we assume that both players have identical discount factors. For $t_1 > t_2$ the cake is divided in an analogous way and if $t_1 = t_2$, then each player receives his discounted initial right and they share the remaining part equally.

Note that the above described procedure results in a non-zero sum silent duel, which is a special case of a game of timing first analysed by Karlin (1959) in the zero sum context. In a silent duel both participants can not observe the execution of the action of their opponent. Noisy duels, in which a player observes the action of his opponent at the moment of their execution, were investigated e.g. in Hendricks, Weiss and Wilson (1988).

Further it can be noted that the above model does not fit within the extensive literature on bargaining models like in Rubinstein (1982). In these models the strategies of the players are a combination of proposals and reactions on proposals. This is not the case here, the players simultaneously choose a time point at which they want to get their part of the cake.

The main result of this paper is that the non-cooperative silent game of timing as described above has a unique Nash equilibrium in mixed strategies. The strategies of this equilibrium are explicitly given and it is found that the corresponding equilibrium payoffs do not depend on the discount factor δ.

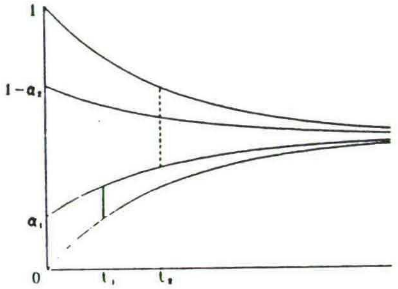
2 The model

Consider a cake of size 1. Let $\alpha_1 > 0$ and $\alpha_2 > 0, \alpha_1 + \alpha_2 < 1$, be the initial right of player 1 and player 2 and let $\delta \in (0, 1)$ be the common discount factor. With player i choosing a point in time $t_i \in [0, \infty)$ to claim his piece of cake, the payoff of player 1 is defined by

$$\pi_1(t_1,t_2) = \begin{cases} \alpha_1 \delta^{t_1} & t_1 < t_2 \\ \{\alpha_1 + \frac{1}{2}(1-\alpha_1-\alpha_2)\}\delta^{t_1} & t_1 = t_2 \\ (1-\alpha_2)\delta^{t_1} & t_1 > t_2 \end{cases}$$

and of player 2 by

$$\pi_2(t_1,t_2) = \begin{cases} (1-\alpha_1)\delta^{t_2} & t_1 < t_2 \\ \{\alpha_2 + \frac{1}{2}(1-\alpha_1-\alpha_2)\}\delta^{t_2} & t_1 = t_2 \\ \alpha_2 \delta^{t_2} & t_1 > t_2 \end{cases}$$



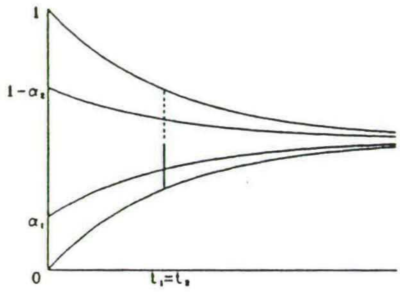


Figure 1 Payoffs if $t_1 < t_2$ **Figure 2** Payoffs if $t_1 = t_2$

A mixed strategy of player i is a probability measure P_i on $[0,\infty)$. Let F_i be the corresponding distribution function defined by $F_i(x) = P_i\{(-\infty, x]\}$. Note that F_i is right continuous and $\lim_{x\to\infty} F_i(x) = 1$. We will use both P_i and F_i to denote a mixed strategy of player i. The probability in a point we denote for convienence by $q_i(x) = F_i(x) - F_i(x^-)$, where $F_i(x^-) = P_i\{(-\infty, x)\}$. The Lebesgue-Stieltjes integral is used to calculate the payoff of the players if both players play a mixed strategy.

The above described silent game of timing will be shortly denoted by Γ.

3 The Nash equilibrium

In this section we show that the game Γ introduced in section 2 has a unique Nash equilibrium in mixed strategies.

It is not difficult to see that there is no Nash equilibrium in pure strategies. Suppose (t_1, t_2) is a Nash equilibrium. If $t_1 < t_2$ $(t_2 < t_1)$then player 2 (1) has an incentive to claim his part of the cake earlier then he did but still later then player 1 (2). If $t_1 = t_2$ then each player has the incentive to make his claim a fraction later. Hence a Nash equilibrium in the game of timing, if it exists, will be one in mixed strategies.

In the following we assume that (F_1, F_2) is a Nash equilibrium with payoff (η_1, η_2). Note that each player i can guarantee himself at least α_i by playing the pure strategy $t_i = 0$. This implies that $\eta_i \geq \alpha_i$.
We first introduce two functions that will play an important role. The functions $g_1^{\eta_2} : [0, \infty) \to [0, \infty)$ and $g_2^{\eta_1} : [0, \infty) \to [0, \infty)$ are defined by

$$g_1^{\eta_2}(t) = \frac{\eta_2 - \alpha_2 \delta^t}{(1 - \alpha_1 - \alpha_2)\delta^t} \qquad \text{for all } t \in [0, \infty) \tag{1}$$

and

$$g_2^{\eta_1}(t) = \frac{\eta_1 - \alpha_1 \delta^t}{(1 - \alpha_1 - \alpha_2)\delta^t} \qquad \text{for all } t \in [0, \infty) \tag{2}$$

A relation between these functions and the equilibrium strategies (F_1, F_2) is given in the following argument. The definition of a Nash equilibrium implies that for all $t \in [0, \infty)$, $\eta_1 \geq \pi_1(t, F_2)$ and $\eta_2 \geq \pi_2(F_1, t)$.

The payoff of player 1 playing t when player 2 plays F_2 is given by

$$\pi_1(t, F_2) = \alpha_1 \delta^t (1 - F_2(t)) + (1 - \alpha_2)\delta^t F_2(t^-)$$

$$+\delta^t \{\alpha_1 + \frac{1}{2}(1 - \alpha_1 - \alpha_2)\}(F_2(t) - F_2(t^-))$$

$$= \delta^t \{\alpha_1 + (1 - \alpha_1 - \alpha_2) F_2(t)\} - \delta^t \frac{1}{2}(1 - \alpha_1 - \alpha_2) q_2(t) \tag{3}$$

Analogously we find for player 2 playing t when player 1 plays F_1 that

$$\pi_2(F_1, t) = \delta^t \{\alpha_2 + (1 - \alpha_1 - \alpha_2) F_1(t)\} - \delta^t \frac{1}{2}(1 - \alpha_1 - \alpha_2) q_1(t) \tag{4}$$

By the equilibrium condition and the right-continuity of F_i we obtain the following inequalities

$$F_1 \leq g_1^{\eta_2} \qquad \text{and} \qquad F_2 \leq g_2^{\eta_1} \tag{5}$$

In the following lemma we show that F_1 and F_2 do not have a masspoint at the same time moment. The argument is similar to the non-existence of a Nash equilibrium in pure strategies.

Lemma 1 *If* (F_1, F_2) *is a Nash equilibrium of* Γ *then* $q_1(t).q_2(t) = 0$ *for all* $t \in [0, \infty)$.

Proof: Suppose that both $q_1(t) > 0$ and $q_2(t) > 0$.
Since $q_1(t) > 0$ we have that $\pi_1(t, F_2) = \pi_1(F_1, F_2)$.
Since $q_2(t) > 0$ and F_2 is a right continuous and monotone there exists an $\epsilon > 0$ (small enough) such that both $q_2(t+\epsilon) = 0$ and $\pi_1(t, F_2) < \pi_1(t+\epsilon, F_2)$ (cf (3)). This contradicts the fact that (F_1, F_2) is a Nash equilibrium of Γ. □

The payoffvector (η_1, η_2) of the Nash equilibrium (F_1, F_2) satisfies the following conditions:

$$\eta_1 \leq 1 - \eta_2 \leq 1 - \alpha_2 \text{ and } \eta_2 \leq 1 - \eta_1 \leq 1 - \alpha_1$$

Hence, there exists a time moment c in which the piece of cake that player 2 will leave is equal to the equilibrium payoff that player 1 receives, i.e. $c \in [0, \infty)$ such that $\delta^c(1 - \alpha_2) = \eta_1$. Analogously we can define $d \in [0, \infty)$ such that $\delta^d(1 - \alpha_1) = \eta_2$. Lemma 2 shows that c and d coincide and that both equilibrium strategies put no probability on the interval (c, ∞).

Lemma 2 *Let* (F_1, F_2) *be a Nash equilibrium of* Γ *with payoff vector* (η_1, η_2) *and let* $c, d \in [0, \infty)$ *such that* $\delta^c(1 - \alpha_2) = \eta_1$ *and* $\delta^d(1 - \alpha_1) = \eta_2$. *Then* $c = \inf\{x \mid F_1(x) = 1\} = \inf\{x \mid F_2(x) = 1\} = d$.

Proof: First we show that $F_1(c) = 1$.
Suppose $1 - F_1(c) > 0$. In the following calculation the first equality is obtained by integrating (3) and lemma 1, the first inequality by using the fact $F_2 \leq g_2^{\eta_1}$ (cf (5)), the second inequality holds since $F_2(x) \leq 1$ for all $x \in [0, \infty)$ and the strict inequality since $\delta^x(1 - \alpha_2)$ is a strictly decreasing function in x and $1 - F_1(c) > 0$.

$$\pi_1(F_1, F_2)$$

$$= \int_{[0,\infty)} \delta^x\{\alpha_1 + (1-\alpha_1-\alpha_2)F_2(x)\}dP_1(x)$$

$$= \int_{[0,c]} \delta^x\{\alpha_1 + (1-\alpha_1-\alpha_2)F_2(x)\}dP_1(x)$$

$$+ \int_{(c,\infty)} \delta^x\{\alpha_1 + (1-\alpha_1-\alpha_2)F_2(x)\}dP_1(x)$$

$$\leq \eta_1 F_1(c) + \int_{(c,\infty)} \delta^x\{\alpha_1 + (1-\alpha_1-\alpha_2)F_2(x)\}dP_1(x)$$

$$\leq \eta_1 F_1(c) + \int_{(c,\infty)} \delta^x(1-\alpha_2)dP_1(x)$$

$$< \eta_1 F_1(c) + \int_{(c,\infty)} \eta_1 dP_1(x) = \eta_1$$

Contradiction. Analogously one can show that $F_2(d) = 1$.

Let $c^* = \inf\{x \mid F_1(x) = 1\}$ and $d^* = \inf\{x \mid F_2(x) = 1\}$. Then $c^* \leq c$ and $d^* \leq d$.

Suppose $c^* < d^*$. Then for all $x \in (c^*, d^*)$, (3) and the definition of d imply that

$$\pi_2(F_1, x) = \delta^x(1-\alpha_1) > \delta^d(1-\alpha_1) = \eta_2 \tag{6}$$

Contradition since (F_1, F_2) is a Nash equilibrium.

Hence, we may conclude that $c^* = d^*$.

Finally, using the same line of argument one can prove that $c = c^*$ and $d = d^*$. □

Lemma 2 immediately implies

Corollary 1 *If (F_1, F_2) is a Nash equilibrium of Γ with payoff vector (η_1, η_2), then*

$$\frac{\eta_1}{\eta_2} = \frac{1-\alpha_2}{1-\alpha_1}$$

The following lemma shows that on the interval $[0,c]$ the equilibrium strategy F_1 coincides with the function $g_1^{\eta_2}$ as defined in (1). Here one should note that, since $g_1^{\eta_2}$ is strictly increasing, $g_1^{\eta_2}(0) \geq 0$ and $g_2^{\eta_1}(c) = 1$, $g_1^{\eta_2}$ is a distribution function on $[0,c]$. Similary, we can find that $F_2(x) = g_2^{\eta_1}(x)$ for all $x \in [0,c]$.

Lemma 3 *Let (F_1, F_2) be a Nash equilibrium of Γ with corresponding payoffvector (η_1, η_2). Then for all $x \in [0,c]$ it holds that*

$$F_1(x) = g_1^{\eta_2}(x) \qquad \textit{and} \qquad F_2(x) = g_2^{\eta_1}(x)$$

PROOF:
It suffices to prove the first equality.

(i) First we show that $P_2(\{x \mid F_1(x) < g_1^{\eta_2}(x)\}) = 0$.
Integration of (4) with respect to P_2 and using lemma 1 and lemma 2 gives

$$\eta_2 = \pi_2(F_1, F_2) = \int_{[0,c]} \delta^x \{\alpha_2 + (1-\alpha_1-\alpha_2)F_1(x)\} dP_2(x) \tag{7}$$

and by straightforward calculation

$$\int_{[0,c]} \delta^x \{\alpha_2 + (1-\alpha_1-\alpha_2)g_1^{\eta_2}(x)\} dP_2(x) = \eta_2 \tag{8}$$

From (7) and (8) it follows that

$$\int_{[0,c]} \delta^x F_1(x) dP_2(x) = \int_{[0,c]} \delta^x g_1^{\eta_2}(x) dP_2(x)$$

Since (5) gives that $F_1(x) \leq g_1^{\eta_2}(x)$ for all $x \in [0,c]$ the proof is completed.
Analogously one can show that $P_1(\{x \mid F_2(x) < g_2^{\eta_1}(x)\}) = 0$.

(ii) Secondly we prove that F_1 is continuous on $(0,c]$.
Suppose F_1 is not continuous on $(0,c]$. Then there exists a $z \in (0,c]$ such that $q_1(z) = \epsilon > 0$. Since $g_1^{\eta_2}$ is continuous on $[0,c]$, there exists a $\delta > 0$ such that $\forall x \in (z-\delta, z]$ it holds that

$$g_1^{\eta_2}(z) - g_1^{\eta_2}(x) < \epsilon$$

Then for all $x \in (z-\delta, z)$ we have

$$F_1(x) \le F_1(z^-) = F_1(z) - q_1(z) = F_1(z) - \epsilon$$

$$\le g_1^{\eta_2}(z) - \epsilon < g_1^{\eta_2}(x)$$

Hence, part (i) implies that $P_2(\{(z-\delta, z)\}) = 0$. So F_2 is constant on $(z-\delta, z)$. Since $q_1(z) > 0$ Lemma 1 implies that $q_2(z) = 0$. Hence, F_2 is constant on $(z-\delta, z]$ and, since $g_2^{\eta_1}$ is strictly increasing, we find that $F_2(z) < g_2^{\eta_1}(z)$. However, using (i) this should imply that $q_1(z) = P_1(\{z\}) = 0$.
From (i) and (ii) it follows that $F_1(x) = g_1^{\eta_2}(x)$ for all $x \in [0, c]$. □

Until now we only have shown some properties a possible Nash equilibrium of the game Γ does satisfy. The following theorem gives the strategies of the unique Nash equilibrium and its payoff.

Theorem 1 *The game Γ has a unique Nash equilibrium (F_1^*, F_2^*) in mixed strategies, given by*

$$F_1^*(x) = \begin{cases} g_1^{\eta_2^*}(x) & \text{if } 0 \le x \le c \\ 1 & \text{if } x > c \end{cases}$$

and

$$F_2^*(x) = \begin{cases} g_2^{\eta_1^*}(x) & \text{if } 0 \le x \le c \\ 1 & \text{if } x > c \end{cases}$$

with c such that $g_1^{\eta_2^*}(c) = g_2^{\eta_1^*}(c) = 1$. The equilibrium payoff is (η_1^*, η_2^*) where $\eta_1^* = \frac{\alpha_2(1-\alpha_2)}{1-\alpha_1}$ and $\eta_2^* = \alpha_2$ in case $\alpha_1 \le \alpha_2$
and
$\eta_1^* = \alpha_1$ and $\eta_2^* = \frac{\alpha_1(1-\alpha_1)}{1-\alpha_2}$ in case $\alpha_1 \ge \alpha_2$

PROOF: We only consider the case $\alpha_1 \le \alpha_2$.
Clearly $\pi_1(F_1^*, F_2^*) = \eta_1^*$ and $\pi_2(F_1^*, F_2^*) = \eta_2^*$.
First we show that (F_1^*, F_2^*) is a Nash equilibrium.
Let G_1 be a mixed strategy of player 1. Then

$$\pi_1(G_1, F_2^*) \le \int_{[0,\infty)} \delta^x \{\alpha_1 + (1 - \alpha_1 - \alpha_2) F_2^*(x)\} dG_1(x)$$

$$= \int_{[0,c]} \eta_1^* dG_1(x) + \int_{(c,\infty)} \delta^x (1 - \alpha_2) dG_1(x)$$

$\leq \eta_1^* = \pi_1(F_1^*, F_2^*)$

The first inequality follows by (3), the equality by substitution of F_2^*. The last inequality follows since for $x \in (c, \infty)$ it holds that $\delta^x(1-\alpha_2) \leq \eta_1^*$.
Analogously it can be shown that $\pi_2(F_1^*, F_2^*) \geq \pi_2(F_1^*, G_2)$ for all mixed strategies G_2 of player 2.
Secondly we show uniqueness.
From lemma 3 it follows that if Γ has a Nash equilibrium (F_1, F_2) with payoff (η_1, η_2) then (F_1, F_2) is the unique Nash equilibrium with payoff (η_1, η_2). Suppose there exists a Nash equilibrium (F_1, F_2) with a payoff $(\eta_1, \eta_2) \neq (\eta_1^*, \eta_2^*)$. Since $\eta_2 \geq \alpha_2$, corollary 1 implies that $\eta_1 = \frac{1-\alpha_2}{1-\alpha_1}\eta_2 \geq \frac{1-\alpha_2}{1-\alpha_1}\alpha_2 = \eta_1^*$. Similarly one can show that $\eta_2 \geq \eta_2^*$. Using corollary 1 again gives $\frac{\eta_1}{\eta_2} = \frac{\eta_1^*}{\eta_2^*}$ and hence $\eta_1 > \eta_1^*$ and $\eta_2 > \eta_2^*$. This yields that $\eta_i > \alpha_i$. This implies that $g_1^{\eta_2}(0) > 0$ and $g_2^{\eta_1}(0) > 0$ (cf (1) and (2)). However, from lemma 3 follows that $F_1(0) = g_1^{\eta_2}(0)$ and $F_2(0) = g_2^{\eta_1}(0)$. This contradicts with lemma 1. □

Note that the the payoff of the unique Nash equilibrium is independent of the discount factor δ. In fact, one could say that δ only influences the duration of the game. If δ becomes larger the players will become more patient, i.e. the interval $[0, c]$ will become larger.

We conclude this paper with three additional remarks with respect to some slight changes of the model.

In this paper we studied the case that when the pure strategies of both players coincide each player obtains his discounted initial right while the remaining part is split equally. In stead of a half-half division of the remainder in case of a tie one could divide the remaining part in any other fixed proportion to the players, i.e. if both players claim on time t the payoff of player 1 is $\delta^t(\alpha_1 + p(1-\alpha_1-\alpha_2))$ and the payoff of player 2 is $\delta^t(\alpha_2 + (1-p)(1-\alpha_1-\alpha_2))$ with $p \in [0, 1]$. This modification does not affect the results of this paper. Moreover, the expressions stated in theorem 1 will be independent of the parameter p.

Secondly we can consider the case when the initial rights of the players constitute a division of the whole cake, i.e. $\alpha_1 + \alpha_2 = 1$. Then obviously each player will claim his initial right at time $t = 0$. Hence, in this case the strategy $(0, 0)$ is the unique Nash equilibrium with payoff (α_1, α_2).

Finally, in case there is no discounting, i.e. $\delta = 1$, it will be obvious that there is no Nash equilibrium in mixed strategies.

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