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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

ARBITRAGE AND WALRASIAN EQUILIBRIUM IN ECONOMIES WITH LIMITED INFORMATION

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FEW 531

Arbitrage and Walrasian Equilibrium in Economies with Limited Information*†

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Abstract

In this paper a model of arbitrage in a pure exchange economy with price-setting agents is given. A hierarchically structured trade economy is defined in which a hierarchical relation between two agents is assumed to have the institutional characteristics of a monopolistic relation between the dominating and the dominated agent.

We assume agents can only observe their closest followers in the hierarchical structure. This situation is described by the local information structure. The agents form their conjectures about the consequences of their actions on the basis of their limited knowledge of (the state of) the economy.

We derive a theorem on the existence of equilibrium which states that if the hierarchical structure is sufficiently rich to allow for enough possibilities for arbitrage, then equilibrium exists and each equilibrium is uniformly priced. Furthermore, in equilibrium agents that do not have a direct superior in the hierarchical structure may find themselves being rationed.

We prove a theorem which states that the Walrasian auctioneer can be replaced by a monopolist with neglectable initial endowments. This theorem is then used to prove a theorem on Walrasian equivalence.

1 Introduction

The general equilibrium model, as introduced by Walras (1874) and formulated by Arrow and Debreu (1954) and Debreu (1959), is one of the fundamental models in economics. It is a model in which decentralized selfish decision making leads to outcomes which are efficient for the economy as a whole, as is stated in the First Theorem of Welfare Economics. Unfortunately the model has some weaknesses, two of which will be mentioned here. Firstly, all agents in the economy are assumed to act as price takers, without any agent setting the prices. Secondly it is assumed that each agent can trade with every other agent in the economy only through "the market", so a very particular trade or communication structure of the economy is assumed. In this paper we develop models that aim to tackle both of these problem.

The problem of modelling price setting agents in general equilibrium models has occupied economists for some time and it continues to do so. We refer to Negishi (1961), Arrow and Hahn (1971) and Marschak and Selten (1974), all of whom analyzed general equilibrium models with monopolistic competition. More recently Roberts (1987), Kamiya (1988), and Selten and Wooders (1990) used more sophisticated institutional procedures to model price setting agents. Roberts (1987) incorporated rationing in his model, Kamiya (1988) investigated pricing rules and Selten and Wooders (1990) used a dynamic bargaining model to describe a process of price formation.

Models with restricted communication structures have focussed on spatial economics and theories on intermediaries. In Karmann (1981) a spatial general equilibrium model is given in which transporation technologies play an important rôle. In Grodal and Vind (1989) markets are modelled as exchange institutions with prices. Other models on intermediaries are mostly partial equilibrium models. We refer to Machup and Taber (1960) and Krelle (1976) for models of successive monopolies and vertical integration. More recently Gehrig (1990) introduced a model in which networks of intermediaries in markets with restricted communication, give rise to endogenous product differentiation.

As in Gilles (1990) and Spanjers et al. (1991a,b) our goal is to use a hierarchical structure of a finite economy to build a model with price setting agents. The leader in a hierarchical relation is assumed to behave as a price setter, the follower is assumed to act as a price taker with respect to this relation. In the model of

Gilles (1990) assumptions on the possibilities for retrade are essential. A theorem on the existence of equilibrium is proved through equivalence with Walrasian equilibrium.

In describing the behaviour of economic agents it seems appropriate to make explicit what an agent anticipates to be the consequences of (a change in) his actions. In Negishi (1961) it is assummed that a firm anticipates the demand for the commodity he produces to be a linear function of the current state of the economy and of the prices he charges. In Hahn (1978) and Gale (1978) the conjectures of the agents are functions which for each combination of market signals an agent receives and actions he might take, gives the vector of market prices he anticipates to result. In this context attention is focussed on Walrasian and non-Walrasian equilibria. In Vind (1983) the concept of conjectures is generalized through the model of equilibrium with coordination. Once again the conjectures, now called expectations, are exogenously given functions, although Vind does provide a simple example in which the expectations functions are derived from the set of exchange institutions in the economy.

In Spanjers et al. (1991a,b) the importance of the conjectures of the agents for our type of model is recognized. The conjectures are assumed to depend on the "transparency" of the economy. Two extreme cases, one of high and one of low transparency, called the subgraph information structure and the local information structure, are analyzed in Spanjers et al. (1991a) and Spanjers et al. (1991b), respectively. The analysis, however, is confined to economies in which the hierarchical structure can be represented by a directed graph with a tree structure and one source. The economic consequence of this restriction is that none of the agents can perform arbitrage. The existence theorems for equilibrium are proved under very restrictive assumptions.

In this paper we focus on hierarchically structured trade economies with local information. The case of subgraph information is treated in Spanjers (1991b). In our present paper the existence of equilibrium is proved for economies in which there are sufficient possibilities for arbitrage. Equivalence with Walrasian equilibrium occurs if the equilibrium has uniform prices and the initial endowments of the set of agents who have no leader are neglectible. We find that equilibrium allocations need not be Pareto efficient and that, in equilibrium, agents that do not have a direct leader in the hierarchical structure may find themselves being rationed. Thus in our models with price setting agents the First Theorem of Welfare Economics no longer holds. In our stylised world the invisible hand may fail, even when it establishes uniform prices.

The organization of the paper is as follows. A hierarchically structured trade economy is defined in Section 2. The local information structure is described and equilibrium in a hierarchically structured trade economy is defined. In Section 3 a theorem on the existence for equilibrium and a theorem which states that some agents may be rationed in equilibrium are proved. In Section 4 we prove that the Walrasian auctioneer may be replaced by a monopolist with neglectable initial endowments. This result is then used to prove a theorem on Walrasian equivalence. Some concluding remarks are made in Section 5.

2 The Model

In this section we define a hierarchically structured trade economy. We describe such an economy by its hierarchical structure, by the individual characteristics of its agents and by the institutional characteristics of its relations. The hierarchical structure describes between which pairs of agents a hierarchical relation exists and which of the agents in a hierarchical relation dominates the other. Each individual agent is described by his utility function and his initial endowments. Finally each hierarchical relation is assumed to have the insitutional characteristics of a monopolistic relation between the dominating and the dominated agent. We explain what we mean by an information structure which describes the transparency of the economy. We use the information structure to derive the conjectures of the agents about the consequences of their actions for the behaviour of the other agents. The conjectures of the agents are described by the anticipated trade correspondences. The actions an agent anticipates, for a given state of the economy, to lead to outcomes that are feasible for him form his choice set. The correspondence which for each state of the economy has the choice set as its image is called the choice correspondence. Finally an equilibrium is defined to be a state which is (anticipated to be) feasible for each agent is the economy, and which no agent anticipates to be able to improve upon.

We describe the hierarchical structure of an economy by a directed graph which is called its hierarchy graph.

Definition 2.1 *A* **Hierarchy Graph**, $\mathcal{H} := (A, D)$, *is a weakly connected directed simple finite graph.*

The definition of a hierarchy graph implies that for any two agents there exists a chain of relations which connects them. So they are (indirectly) connected within the hierarchical structure.

For each agent $i \in A$ we define $F_i := \{j \in A \mid (i,j) \in D\}$. The set F_i is the set of the direct subordinates or followers of agent i in the hierarchy graph $\mathcal{H}$. We define $L_i := \{h \in A \mid (h,i) \in D\}$ to denote the set of the direct superiors or leaders of agent i in the hierarchy. Clearly L_i may have more or less than one element.

Now we have defined a hierarchy graph we can define a hierarchically structured trade economy or, simply, economy. We consider economies without production.

Definition 2.2 *A* **Hierarchically Structured Trade Economy** *is a tuple* $\mathbf{E} = ((A,D), \{U_a, \omega_a\}_{a \in A}, \{\text{mon}_w\}_{w \in D})$ *where:*

1. (A, D) *is a hierarchy graph.*
2. $U_a : \mathbb{R}^l_+ \to \mathbb{R}$ *is the utility function of agent a which is defined over an l-dimensional commodity space. The utility function is assumed to be strictly monotonic, continuous and strict quasi concave.*
3. $\omega_a \in \mathbb{R}^l_+$ *denotes the initial endowments of agent a.*
4. mon_w *with $w = (a, b)$ is a monopolistic trade relation between the agents a and b where agent a is the price setter and agent b is the price taker.*

An economy consists of a hierarchy graph which describes the social postion of the agents in the economy, a set of agents who have utility functions and initial endowments as their individual characteristics, and a set of relations with their insitutional characteristics. Although we assume every hierarchical relation between two agents to have the institutional characteristics of a monopolistic relation between the leader and the follower, we mention this explicitly in the definition of a hierarchically structured trade economy. The leader in a hierarchical relation sets the prices for the trade on this relation. The prices for buying and selling are assumed to be the same. The follower determines the amounts that are traded, the leader has the obligation to buy or sell whatever amount the follower decides to trade at the given prices. This obligation may be disadvantageous for the leader [1].

Our model of a hierarchically structured trade economy should not be interpreted as a model of spatial economics with a no-costs transportation technology. Superimposing a hierarchical structure on the economy does not change the location of the goods in the economy in any way, it merely restricts the possibilities to transfer ownership of the commodities. In a spatial model the same good at different places may be represtented by different commodities in the corresponding economy in the formulation of Debreu (1959). In our model, representing a good "held" by different agents, by different commodities would amount to representing a good owned by agent i by a different commodity as the same good held by a different agent j. This, however, is something we do not want. Therefore superimposing a hierarchical structure on a set of agents does not (even implicitly) change the set of commodities in the economy.

The trades, the prices and the consumption bundles in the economy are described by the trade-price-allocation system. We use $X_i := \mathbb{R}^{l\times\#L_i} \times S^{(l-1)\times\#F_i}$ to denote the space of trades and prices agent i can choose from.

Definition 2.3 *A* **Trade-Price-Allocation-System** *in the hierarchically structured trade economy* $\mathbf{E}$ *is a tuple* $(d,p,x) \in X \times \mathbb{R}_+^{l\times\#A} := \mathbb{R}^{l\times\#D} \times S^{(l-1)\times\#D} \times \mathbb{R}_+^{l\times\#A}$ *where:*

1. $d_{ji} \in \mathbb{R}^l$ *denotes the trade in the relation* $(i,j) \in D$*. We define* $d_j := (d_{jh})_{h\in L_j}$*.*
2. $p_{ij} \in S^{l-1}$ *is the price vector denoting the prices charged on the trade-relation* $(i,j) \in D$*. We define* $p_i := (p_{ij})_{j\in F_i}$*.*
3. $x_i \subset \mathbb{R}_+^l$ *is the consumption bundle of agent* i*.*

The prices a leader sets in a monopolistic trade relation depend on what he expects to be the consequences of setting these prices. Here the transparency of the economy becomes important. One might assume an agent correctly anticipates the consequences of his actions for the behaviour of the agents of lower echelons in the economy, and that he assumes the actions of the remaining agents not to be influenced by his (change in) actions. This amounts to analyzing the subgame perfect equilibria of a hierarchically structured trade economy, where agents who are of

[1] This follows from Spanjers et al. (1991a, Example 4.1).

the highest hierarchical level move first, the agents of the second level move next etc., if the corresponding game is well defined. The case where the hierarchy graph has a tree structure and only has one source is analysed in Spanjers et al. (1991a). For economies with a different class of hierarchical structures we refer to Spanjers (1991b).

In this paper we assume the economy is not sufficiently transparent to enable each agent to have the conjectures of the consequences of his actions as described above. We assume an agent, say i, knows the utility function of his direct followers, knows their initial endowments, knows the aggregate of the trades between them and their direct followers in the current state of the economy, and knows the prices their other leaders set for them. This specification of knowledge or information is called the **Local Information Structure**. We assume agent i forms his conjectures about the trades that result from a change in the prices he sets by solving the optimization problem of his follower, say j, assuming the prices set by the other leaders of agent j and the trades between agent j and his direct followers do not change. The resulting conjectures of agent i about the consequences of a change in the prices he sets for the behaviour of this follower j are described by the anticipated trade correspondence of agent i for agent $j \in F_i$.

Definition 2.4 *The* **Anticipated Trade Correspondence** $t_{ij} : X \times X_i \rightrightarrows \mathbb{R}^l$ *of agent i for $j \in F_i$ for the local information structure is defined to be such that if $q_{ij} = p_{ij}$ then $t_{ij}((d,p),(e_i,q_i)) = d_{ji}$ and if $q_{ij} \neq p_{ij}$ then $t_{ij}((d,p),(e_i,q_i))$ is the set of values of e_{ji} for the solutions of*

$$\max_{(e_j,y_j)\in\mathbb{R}^{l\times\#L_j}\times\mathbb{R}^l} \quad U_j(y_j)$$

such that

$$p_{kj}\cdot e_{jk} \leq 0 \qquad\qquad \forall k \in L_j$$

$$y_j \leq \sum_{k\in L_j} e_{jk} - \sum_{m\in F_j} d_{mj} + \omega_j$$

if this optimization problem has a solution.

If the optimization problem has no solution $t_{ij}((d,p),(e_i,q_i))$ is defined to be the value of e_{ji} for the solutions of the above problem with the additional restriction that y_j is such that $U_j(y_j) \leq U^*(\gamma) := U_j(x_j) + \gamma$ where $x_j := \omega_i + \sum_{h\in L_i} d_{ih} - \sum_{j\in F_i} d_{ji}$

is the consumption of agent j which results from (d,p), $\gamma > 0$ being "sufficiently large" and $U^{-1}(U^*(\gamma)) \neq \emptyset$.

Here γ is said to be "sufficiently large" if one of the following holds for each $\tilde{\gamma} \geq \gamma$ such that $U^{-1}(U^*(\tilde{\gamma})) \neq \emptyset$:

1. *$\nexists\, \tilde{y}_i \in \mathbb{R}^l_+$ which results from an optimizing trade of agent j and which anticipated by agent i to be attainable for him.*
2. *$\forall \tilde{\gamma} > \gamma$: $\exists \tilde{y}_i \in \mathbb{R}^l_+$ which is anticipated by agent i to be attainable for him, and is such that $U_i(\tilde{y}_i) > U_i(x_i)$.*

It should be noted that in the above definition first γ is choosen such that it is sufficiently large, and only then t_{ij} is constructed.

In the case that for a follower j of an agent i it holds that $L_j = \{i\}$, the anticipated trade correspondence of agent i for agent j for the local information structure can be represented by a continuous function [2]. If agent j has more than one leader things get more complicated.

In the case some agent j has, say, two leaders who set the same prices, agent j is indifferent about which of the agents to trade with. This results in anticipated trade correspondences for the leaders of agent j which have a hyperplane in $\mathbb{R}^l$ as their image in the case the prices for trade with agent j are the same.

If the, say two, leaders of agent j set different prices, then j will perform arbitrage. Therefore the optimization problem which defines the anticipated trade correspondence of agent i for agent j has no solution, since agent j will generate a trade flow which is infinitly large in its absolute value. To prevent mathematical difficulties we define the anticipated trade flow in these cases to be "sufficiently large" instead of infinitly large in absolute value. We say an anticipated trade flow is sufficiently large if for agent i who provokes it one of the following holds.

1. The anticipated trade flow, or any such flow which would make agent j still better off, is large enough to make sure agent i can not deliver.
2. The anticipated trade flow is large enough to ensure that agent i can take actions, given this anticipated trade flow, that make him better off than he was before he induced the flow and that any higher utility level agent j may

[2] See Spanjers et al. (1991b).

want to reach can be obtained by a trade flow which makes agent i still better off.

The latter situation may arise if agent i can transfer the anticipated trade flow to one of his leaders or another of his followers at profitable prices.

Despite the difficulties mentioned above we can analyse hierarchically structured trade economies with the local information structure. The reason for this is that the economies we analyse in this paper have rather particular hierarchical structures. These structures are such that the agents effectively only use the strict monotonicity of the utility fuctions of their followers. In economies with a tree structure and only one agent who does not have a (direct) superior in the hierarchical structure, however, the agents need to know more about the utility functions of their followers than that they are strictly monotonic [3].

The set of actions agent i anticipates to be feasible is called the choice set of agent i. Since it depends on the state of the economy as described by the trade-price system and the anticipated trade correspondences of agent i with respect to this followers, it would be more suitable but also more cumbersome to refer to it as the set of anticipated feasible actions of agent i. The choice correspondence of agent i is the correspondence that gives the choice set of agent i as a function of the trade-price system. Once again the term anticipated feasible actions correspondence of agent i would be more appropriate but still more cumbersome.

Definition 2.5 *The* **Choice Correspondence** $B_i : X \rightrightarrows X_i \times \mathbf{R}^l_+$ *of agent i is defined by:*

$$\begin{aligned} B_i(d,p) \quad := \quad & \{(e_i, q_i, y_i) \in X_i \times \mathbf{R}^l_+ \mid e_{ih} \cdot p_{hi} \leq 0 \quad \forall\, h \in L_i \\ & \textit{and} \quad y_i \leq \omega_i + \sum_{h \in L_i} e_{ih} \; - \sum_{j \in F_i} e_{ji} \\ & \textit{with} \quad e_{ji} \in t_{ij}((d,p),(e_i,q_i))\}. \end{aligned}$$

We assume each agent chooses his actions as to maximize his utility over his choice set as it follows from the information structure.

The equilibrium concept we use in this paper comes close to the concept of conjectural equilibrium, and even closer to the concept of equilibrium with co-

[3]See Spanjers et al. (1991a,b).

ordination of Vind (1983), as might be expected from the equivalence result of Spanjers et al. (1991b, Theorem 3.4).

Definition 2.6 *A trade-price-allocation-system* $(d^*, p^*, x^*) \in X \times \mathbb{R}_+^{l \times \#A}$ *is an* **Equilibrium** *in the economy* **E** *if for each agent* $i \in A$*:*

1. $(d_i^*, p_i^*, x_i^*) \in B_i(d^*, p^*)$.
2. $x_i^* \leq \omega_i + \sum_{h \in L_i} d_{ih}^* - \sum_{j \in F_i} d_{ji}^*$.
3. $(d_i^*, p_i^*, x_i^*) \in \operatorname{argmax}_{(e_i, q_i, y_i) \in B_i(d^*, p^*)} U_i(y_i)$.

So an equilibrium tuple is a tuple of actions of the agents in the economy such that:

1. *[Anticipated Feasibility]* The equilibrium tuple is anticipated to be feasible by each agent in the economy.
2. *[Actual Feasibility]* The consumption bundle agent i anticipates to end up with in equilibrium is attainable at equilibrium.
3. *[Stability]* The equilibrium actions of each agent are maximal with respect to the set of actions this agent anticipates to be feasible for the equilibrium trade-price-system.

It should be noted that in the present paper the condition of actual feasibility is always satisfied if the two other equilibrium conditions hold. This is a consequence of the definition of the anticipated trade correspondences. In Spanjers et al. (1991a) we have a different definition of the anticipated trade correspondences which is such that anticipated feasibility and stability no longer imply actual feasibility.

In the construction of our model we followed an amended version of the methode of relational modelling of Gilles and Ruys (1988). This method states that an economic model should satisfy two main principles, the separation principle and the interdependency principle. The separation principle states that the individual characteristics and the social characteristics of (the agents in) the economy are to be described separately. The interdependency principle states that, once the individual and social characteristics of (the agents in) the economy are modelled separately, the interaction between them must be described in order to properly describe the behaviour of the agents.

In our model we applied a modified version of the separation principle in separately describing the hierarchical structure, the agents and their individual characteristics and the hierarchical relations with their institutional characteristics. We established the interdependence between the individual characteristics of the agents and the institutional characteristics of the relations in the economy through the conjectures of the agents. Therefore the interdependence principle for relational modelling is also satisfied.

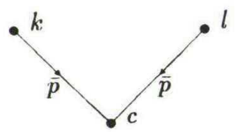
3 The Existence Theorem

In this section we prove a theorem on the existence of equilibrium. As a corollary we prove that in equilibrium some agents in $S_1 := \{i \in A \mid L_i \neq \emptyset\}$, the set of agents dat are not dominated in the hierarchical structure, may be rationed.

For the case that $\#S_1 = 1$ a theorem on the existence of equilibrium in an economy in which the hierarchy graph has a tree structure and only one source can be proved under very restrictive conditions by using a fixed point theorem [4].

To begin with we prove a lemma which states that if two agents have the same follower they will, in equilibrium, set the same prices for this follower. The intuition is that if they do not, then their common follower could improve his allocation by performing arbitrage between these two leaders.

We use $\overleftarrow{\rho}\,(a) := \#\{(b,a) \in D\}$ to denote the indegree of agent a in the hierarchy graph $\mathcal{H}$. The indegree of a node in a directed graph is the number of ingoing arrows of that node in the graph.



Lemma 3.1 *Suppose (d^*, p^*, x^*) is an equilibrium in* **E** *under local information. Let $c \in A : \overleftarrow{\rho}\,(c) \geq 2$. Then $\forall\, k, l \in L_c :\ p^*_{kc} = p^*_{lc}$.*

Proof

Let (d^*, p^*, x^*) be an equilibrium in **E**. Let $c \in A$ such that $\overleftarrow{\rho}\,(c) \geq 2$. Suppose

[4] Spanjers et al. (1991b, Theorem 4.1).

$k, l \in A$ with $k \neq l$ and $p^*_{kc} \neq p^*_{lc}$.
Since $p^*_{kc}, p^*_{lc} \in$ int S^{l-1} there exist commodities, say $r, s \in \{1, \ldots, l\}$, such that

$$\begin{array}{rcl} p^*_{kcr} & > & p^*_{lcr}. \\ p^*_{kcs} & < & p^*_{lcs}. \end{array}$$

The optimization problem of agent c is

$$\max_{(e_c, q_c, y_c) \in \mathbf{R}^{l \times \# L_c} \times S^{(l-1) \times \# F_i} \times \mathbf{R}^l_+} \quad U_c(y_c)$$

such that

$$p^*_{hc} \cdot e_{ch} \leq 0, \qquad \forall\, h \in L_c.$$

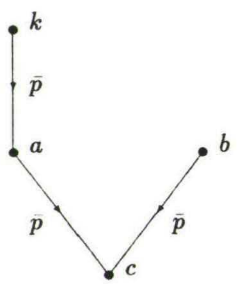
$$y_c \leq \omega_c + \sum_{h \in L_c} e_{ch} - \sum_{j \in F_c} e_{jc},$$

$$\text{where} \quad e_{jc} \in t_{cj}((d^*, p^*), (e_c, q_c)).$$

Now $\exists (d_{ckr}, d_{cks}, d_{clr}, d_{cls}) \in \mathbf{R}^4$ such that e_c, being d^*_c with d^*_{cab} replaced by d_{cab} for $a \in \{k, l\}$ and $b \in \{r, s\}$, is such that $(e_c, p^*_c, y_c) \in B_c(d^*, p^*)$ with $y_c := \omega_c + \sum_{h \in L_c} e_{ch} - \sum_{j \in F_c} e^*_{jc}$, and furthermore

$$\begin{array}{rcl} e_{ckr} + e_{clr} & > & d^*_{ckr} + d^*_{clr}. \\ e_{cks} + e_{cls} & = & d^*_{cks} + d^*_{cls}. \end{array}$$

The thus constructed trades d_c lead to a consumption bundle y_c which is weakly larger than x^*_c which results from (d^*, p^*). By the strict monotonicity of the preferences of agent c this implies that y_c is preferred by c to x^*_c. This contradicts (d^*, p^*, x^*) being an equilibrium.
$\mathcal{Q.E.D.}$



Lemma 3.2 *Suppose (d^*, p^*, x^*) is an equilibrium in* **E**. *Let $a \in A$ such that $\exists\, k \in L_a$. Let $c \in F_a$ such that $b \in L_c, b \neq a$. Then $p^*_{ka} = p^*_{ac} = p^*_{bc}$.*

Proof

By Lemma 3.1 it follows that $p^*_{ac} = p^*_{bc}$.

Suppose $p^*_{ka} \neq p^*_{ac}$. This implies there exist commodities $r, s \in \{1, \ldots, l\}$ such that:

$$\begin{aligned} p^*_{kar} &> p^*_{acr}. \\ p^*_{kas} &< p^*_{acs}. \end{aligned}$$

Choose $\overline{p} \in S^{l-1}$ such that $\overline{p}_t = p^*_{kat}$ for all $t \in \{1, \ldots, l\} \setminus \{r, s\}$ and

$$\begin{aligned} p^*_{kar} &> \overline{p}_r > p^*_{acr}. \\ p^*_{kas} &< \overline{p}_s < p^*_{acs}. \end{aligned}$$

Define $U^*(\gamma) := U_c(x^*_c) + \gamma$ with $\gamma > 0$ and such that $U^{-1}(U^*(\gamma)) \neq \emptyset$. The optimization problem of agent c which defines $t_{ac}((d^*, p^*), (e_i, q_i))$ is

$$\max_{(e_c, y_c) \in \mathbf{R}^{l \times \# L_c} \times \mathbf{R}^l \text{ such that } U_c(y_c) \leq U^*(\gamma)} U_c(y_c)$$

such that

$$p^*_{hc} \cdot e_{ch} \leq 0 \qquad , \forall h \in L_c.$$

$$y_c \leq \omega_c + \sum_{h \in L_c} e_{ch} - \sum_{j \in F_c} d^*_{jc}.$$

The solution of this problem can be obtained by trades e_c such that

$$\begin{aligned} e_{car} &> d^*_{car} \\ e_{cas} &< d^*_{cas} \end{aligned}$$

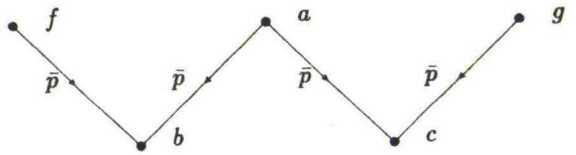
and $e_{cat} = d^*_{cat}$ for all other commodities. This implies agent c is assumed to make all other changes in his consumption through his trade with agent b.

Furthermore there exist trades e_a such that for e_{ak} we have that

$$\begin{aligned} e_{akr} + e_{car} &> d^*_{akr} + d^*_{car} \\ e_{aks} + e_{cas} &= d^*_{aks} + d^*_{cas} \end{aligned}$$

and $e_{aht} := d^*_{aht}$ if either $h \in L_a \setminus \{k\}$ or $t \in \{1, \ldots, l\} \setminus \{r, s\}$. Clearly $(e_a, \overline{p}_a, y_a) \in B_a(d^*, p^*)$ for $y_a := \omega_a + \sum_{h \in L_a} e_{ah} - \sum_{j \in F_a \setminus \{c\}} d^*_{ja} - e_{ca}$. The resulting consumption bundle y_a for agent a is weakly larger than the bundle x^*_a and is therefore preferred by a to x^*_a. This contradicts (d^*, p^*, x^*) being an equilibrium.

$\mathcal{Q.E.D.}$



Lemma 3.3 *Suppose (d^*, p^*, x^*) is an equilibrium in $\mathbf{E}$. Let $a \in A$, let $b, c \in F_a$, $b \neq c$. Suppose $a \neq f \in L_b$ and $a \neq g \in L_c$. Then $p^*_{fb} = p^*_{ab} = p^*_{ac} = p^*_{gc}$.*

Proof

By Lemma 3.1 it holds that $p^*_{fb} = p^*_{ab}$ and $p^*_{ac} = p^*_{gc}$.

Suppose $\bar{p} = p^*_{fb} \neq p^*_{gc} = \hat{p}$. Then there exist commodities $r, s \in \{1, \ldots, l\}$ such that

$$\begin{aligned} \bar{p}_r &> \hat{p}_r. \\ \bar{p}_s &< \hat{p}_s. \end{aligned}$$

Define q^b, q^c such that $q^b_i = q^c_i = \bar{p}_i$ for $i \in \{1, \ldots, l\} \setminus \{r, s\}$ and $q^b_r, q^b_s, q^c_r, q^c_s$ such that

$$\begin{aligned} \bar{p}_r &> q^b_r > q^c_r > \hat{p}_r. \\ \bar{p}_s &< q^b_s < q^c_s < \hat{p}_s. \end{aligned}$$

Define e such that

$$\begin{aligned} e_{ji} &= d^*_{ji} && \text{if} \quad (i,j) \in D \setminus \{(f,b),(a,b),(a,c),(g,c)\}. \\ e_{ji} &= \tilde{e}_{ji} && \text{if} \quad (i,j) \in \{(f,b),(a,b),(a,c),(g,c)\}. \end{aligned}$$

Now there exist $\tilde{e}_{bf}, \tilde{e}_{ba}, \tilde{e}_{ca}, \tilde{e}_{cg}$, induced by some γ sufficiently large, such that for the allocation y, which results from e such that for each $a \in A$ we have that $y_a := \omega_a + \sum_{h \in L_a} e_{ah} - \sum_{j \in F_a} e_{ja}$, it holds that

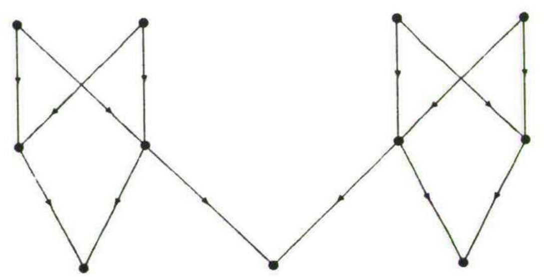
$$\begin{aligned} U_b(y_b) &> U_b(x^*_b). \\ U_c(y_c) &> U_c(x^*_c). \\ U_a(y_a) &> U_a(x^*_a). \end{aligned}$$

By the definition of $t_{ab}((d^*, p^*), (e_a, q_a))$, $t_{ac}((d^*, p^*), (e_a, q_a))$ and $B_a(d^*, p^*)$ it follows that $(e_a, q_a, y_a) \in B_a(d^*, p^*)$.

But this contradicts (d^*, p^*, x^*) being an equilibrium.

$\mathcal{Q.E.D.}$

Lemma 3.1, Lemma 3.2 and Lemma 3.3 enable us to prove a theorem on the existence of equilibrium in hierarchically structured trade economies. The intuition behind the existence theorem is that if there are enough possibilities for arbitrage in the economy and if there is a uniform price which leads to a total net demand from the agents from $A \setminus S_1$ such that the agents from S_1 can meet this total net demand, then this uniform price is an equilibrium price. The reason for this is that no single agent dares to deviate from the uniform price since he anticipates such a deviation to result in arbitrage which is disadvantageous for him.



Theorem 3.4 [Existence Theorem]
Let $\mathcal{H} = (A, D)$ be a hierarchy graph. Define $S_1 := \{a \in A \mid \overleftarrow{\rho}\,(a) = 0\}$. Suppose $\forall c \in A \setminus S_1 : \overleftarrow{\rho}\,(c) \geq 2$. Now there exists a uniform price equilibrium in the economy **E** *which has $\mathcal{H}$ as its hierarchy graph. Furthermore every equilibrium in* **E** *is an uniform price equilibrium.*

Proof
Let $\bar{p} \in S^{l-1}$ be the Walrasian equilibrium price for the market consisting of the agents A. By the assumptions on the individual characteristics of the agents the some Walrasian equilibrium $(\bar{p}, x^*)$ exists for the market consisting of the agents A. Let the trades d^* be such that the corresponding Walrasian allocations x^* result. Clearly (d^*, p^*, x^*) with $p^* := (\bar{p})_{w \in D}$ is feasible. It remains to show it is stable.
Suppose $\exists\, b \in A \setminus \{S_1\} : \exists\, (e_b, q_b, y_b) \in B_b(d^*, p^*)$ such that the resulting consumption bundle for agent b, $y_b := \omega_b + \sum_{h \in L_b} e_{bh} - \sum_{j \in F_b} e_{jb}$ with $e_{jb} \in t_{bj}((d^*, p^*), (e_b, q_b))$, is preferred to x^*_b by him.
This implies that $q_b \neq p^*_b$. So $\exists\, c \in F_b : \; q_{bc} \neq p^*$. So there exist commodities

$r, s \in \{1, \ldots, l\}$ such that

$$\begin{aligned} q_{bcr} - \overline{p}_r &> 0. \\ q_{bcs} - \overline{p}_s &< 0. \end{aligned}$$

Arbitrage by agent c with another of his leaders inclines agent b to anticipate

$$\begin{aligned} e_{bcr} - d^*_r &< 0. \\ e_{bcs} - d^*_s &> 0. \end{aligned}$$

For the profits from trade for agent b with respect to agent c, this means that

$$(q_{bcr} - \overline{p}_r)(e_{cbr} - d^*_{cbr}) + (q_{cbs} - \overline{p}^*_s)(e_{cbs} - d^*_{cbs}) < 0.$$

This implies that the value of the consumption y_b, at given prices $\overline{p}$, is less than that of the bundle x^*_b, therefore y_b is worse for agent b than x^*_b.

Suppose $\exists b \in S_1 : \exists (q_b, y_b) \in B_b(d^*, p^*)$ such that the resulting consumption bundle for agent b, $y_b := \omega_b - \sum_{j \in F_b} e_{jb}$ with $e_{jb} \in t_{bj}((d^*, p^*), (q_b))$, is preferred to x^*_b by him.

Once again this implies that $q_b \neq p^*_b$. So $\exists c \in F_b : q_{bc} \neq \overline{p}$. By the same line of reasoning as before it follows that for e_{cb}, induced by γ sufficiently large, this results in a consumption bundle for agent b such that $y_b \notin \mathbf{R}^l_+$ which contradicts $(q_b, y_b) \in B_b(d^*, p^*)$.

Therefore $(\overline{p})_{w \in D}$ is a uniform price equilibrium in $\mathbf{E}$.

Furthermore suppose p^* is not an uniform price system. Now one of the following three cases holds:

Case 1.

$\exists a \in A : \exists b, c \in L_a : p^*_{ba} \neq p^*_{ca}$. This contradicts Lemma 3.1.

Case 2.

$\exists a, b, c, k \in A$ as in Lemma 3.2 with $p^*_{ka} \neq p^*_{ac} = p^*_{bc}$. This obviously contradicts Lemma 3.2.

Case 3.

$\exists a \in A : \exists b, c \in F_a$ as in Lemma 3.3 and $p^*_{ab} \neq p^*_{ac}$. This contradicts Lemma 3.3.

$\mathcal{Q.E.D.}$

Theorem 3.4 shows that even if the individual agents in the economy know almost nothing about the economy they participate in, equilbrium still exists. The weak point, of course, is that this theorem does not give any indication of how this equilibrium could be attained. To attain an equilibrium, actions between agents who may not even know about each others existence have to be co-ordinated, one way or another.

It should be noted that by the line of proof of Theorem 3.4 only shows that the Walrasian equilibrium is one of the possibly many equilibria of this economy. This implies the Second Theorem of Welfare Economics holds under the assumptions of the theorem. However, not every equilibrium need to be Pareto efficient. For instance, if there are at least two agents in S_1 who have initial endowments that are not zero, then there exist equilibria with the Walrasian equilibrium prices in which all agents except the agents in S_1 end up with the Walrasian allocations. This may happen because there is no way for the agents in S_1 to co-ordinate their trades in such a way that they too end up with their Walrasian allocations. This is example also indicates there may be a continuum of equilibrium allocations.

Another example of an equilibrium which is not Pareto efficient is the following. Suppose $\exists\, a \in S_1 : \ \omega_a \neq 0$. Now the monopoly price of agent a for the market consisting of the agents of $A \setminus \{a\}$ is also an equilibrium price. Summarizing we conclude that our specification of price setting behaviour and arbitrage destroys the First Theorem of Welfare Economics for the economies under consideration.

In fact, one may interpret these examples as examples in which the agents in S_1 that do not end up with their "price taking" consumption bundles for the uniform price p^* are being rationed in equilibrium. Because they are obliged to meet the trades of their direct followers they end up with a consumption bundle which differ from the best bundle at prices p^*, which is the "price taking" bundle at prices p^*. This is formalized by the following corollary.

Corollary 3.5 [Rationing in Equilibrium]
Let $\mathbf{E}$ *be as in Theorem 3.4 and assume* $\#S_1 \geq 2$. *Now a tuple* (d^*, p^*, x^*) *is an equilibrium if and only if the following holds:*

1. $\exists\, \overline{p} \in S^{l-1}$ *such that* $p^* := (\overline{p})_{w \in D}$.
2. $\forall i \in A \setminus S_1$ *the consumption bundle* x_i^* *is the optimal consumption of a price taking agent* i *at prices* $\overline{p}$.
3. $\forall i \in S_1$ *we have that* $x_i^* \in \mathbb{R}_+^l$ *and* $\overline{p} \cdot x_i^* = \overline{p} \cdot \omega_i$.
4. *The trades* d^* *are such that* $\forall i \in A$ *we have that* $x^* = \omega_i + \sum_{h \in L_i} d_{ih}^* - \sum_{j \in F_i} d_{ji}^*$.

Proof

The proof of this statement follows the line of proof of Theorem 3.4 and uses the definition of equilibrium and the strict monotonicity of the utility functions of the agents in $\mathbf{E}$.

$\mathcal{Q.E.D.}$

4 The Equilvalence Theorem

In this section we prove a theorem on Walrasian equivalence. In order to do so we first prove that the Walrasian auctioneer is equivalent to a monopolist with neglectable initial endowments. To be more precise, we show that the Walrasian auctioneer is equilvalent to a monopolist with initial endowments that equal zero and who is forced to set the same prices for all his followers. This theorem is inspired by Gilles (1989, Theorem 4.3).

Consider the set of agents $A := \{0, 1, \ldots, n\}$ with strict quasi-concave, stricly monotonic and continuous utility functions. We take the agents $a \in \{1, \ldots, n\}$ to be price-taking consumers with initial endowments $\omega_a \in \mathbf{R}^l_+$. Agent 0 is assumed to have initial endowments $\omega_0 = 0$. Agent 0 sets the prices in the economy and we assume he cannot discriminate in the prices for the other consumers in the economy. Since the agents in the trade economy have strictly monotonic utility functions, there exists some $\varepsilon > 0$ such that without loss of generality we can restrict the set of prices to S^{l-1}_ε.

The optimization problem for agent $a \in \{1, \ldots, n\}$ for a given $p \in S^{l-1}_\varepsilon$ is

$$\max_{(d_a, x_a) \in \mathbf{R} \times \mathbf{R}^l_+} U_a(x_a)$$

subject to

$$p \cdot d_a \geq 0.$$

$$x_a \leq \omega_a + d_a.$$

This is the familiar optimization problem for a price-taking consumer. We define $t_a : S_\epsilon^{l-1} \to \mathbb{R}^l$ to be a function that for each $p \in S_\epsilon^{l-1}$ we have that $t_a(p)$ is the trade d_a that is belongs to the solution of the above maximization problem for prices p.

The optimization problem for agent 0, the trader, is

$$\max_{(x_0,p)\in \mathbb{R}_+^l \times S_\epsilon^{l-1}} U_0(x_0)$$

subject to

$$x_0 \;\le\; 0 - \sum_{a=1}^{n} d_a(p).$$

Theorem 4.1 [Walrasian Monopolist]
Any equilibrium in the trade economy $\mathbf{E}$ *corresponds to a Walrasian equilibrium in the pure exchange economy* $\bar{\mathbf{E}} = \{U_a, \omega_a\}_{a\in A}$ *and vice versa.*

Proof

Only if
Let (x^*, p^*) be an equilibrium in the trade economy $\mathbf{E}$.
Furthermore let $a \in \{1, \ldots, n\}$.
Given the optimization problem of agent a it follows that the trades $t_a(p)$ of agent a will be the "Walrasian trade" of a price-taking consumer. So it remains to be shown that if (x^*, p^*) is an equilibrium in the trade economy $\mathbf{E}$, then p^* is a Walrasian equilibrium price. For in the case p^* is an Walrasian equilibrium prices the trades of agent $a \in \{1, \ldots, n\}$ with agent 0 as specified above just are the (net) trades of agent a on the Walrasian market.
The inequality $p \cdot t_a(p) \le 0$ for every $a \in \{1, \ldots, n\}$ implies that

$$p \cdot (\sum_{a=1}^{n} t_a(p)) \;\le\; 0.$$

From $p \in S_\epsilon^{l-1}$ and $x_0 \le \omega_0 - \sum_{a=1}^{n} t_a(p)$ for $x_0 \in \mathbb{R}_+^l$ it follows that $\sum_{a=1}^{n} t_a(p) \ge 0$.
Therefore we have for each $p \in S_\epsilon^{l-1}$ that

$$\sum_{a=1}^{n} t_a(p) = 0.$$

Consequently we have that $x_0 = 0$. So the Walrasian equilibrium conditions hold for the exchange economy $\bar{\mathbf{E}}$.

If

Let (x^*, p^*) be a Walrasian equilibrium in the exchange economy $\bar{\mathbf{E}}$, where x denotes the allocation. Clearly x^*, with $x_0^* = 0$, is attainable for every agent in the trade economy $\mathbf{E}$.

Suppose (x^*, p^*) is not an equilibrium in $\mathbf{E}$.

Then there exists a price $\tilde{p}$ and an allocation $\tilde{x}$ such that $(\tilde{x}, \tilde{p})$ is attainable for every agent $a \in A$ and $U_0(\tilde{x}) > U_0(0)$. Because of the strict monotonicity of the utility function of agent 0 this implies that $\tilde{x} > 0$. Now $\tilde{p} \in S_\epsilon^{l-1}$ implies that:

$$\tilde{p} \cdot (\sum_{a=1}^{n} t_a(\tilde{p})) < 0.$$

On the other hand the budget condition and the strict monotonicity of the utility function for every $a \in \{1, \ldots, n\}$ implies

$$\tilde{p} \cdot t_a(\tilde{p}) = 0.$$

Therefore

$$\sum_{a=1}^{n} \tilde{p} \cdot t_a(\tilde{p}) = 0,$$

and we find that

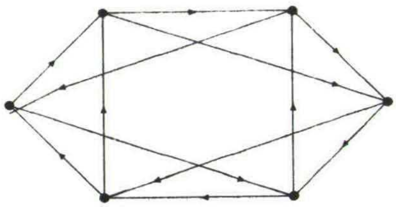
$$\tilde{p} \cdot (\sum_{a=1}^{n} t_a(\tilde{p})) = 0.$$

This contradicts $\tilde{p} \cdot (\sum_{a=1}^{n} t_a(\tilde{p})) < 0$.

Therefore (x^*, p^*) is an equilibrium in $\mathbf{E}$.

$\mathcal{Q.E.D.}$

To end this section we prove a theorem on the equivalence of equilibrium in a hierarchically structured trade economy with Walrasian equilibrium. It states that if the agents of S_1 have neglectible initial endowments in an economy with a local information structure and with sufficient possibilities for arbitrage, then every equilibrium in the hierarchically structured trade economy is a Walrasian equilibrium and vice versa. Note that this in particular is the case if $S_1 = \emptyset$.



Theorem 4.2 [Walrasian Equivalence]
Let $\mathbf{E}$ *be as in Theorem 3.4. Assume additionally that* $\forall\, a \in S_1 : \ \omega_a = 0$, *which is equivalent to assuming* $\sum_{a \in S_1} \omega_a = 0$. *Then* p^* *is an (uniform) equilibrium price for* $\mathbf{E}$ *if and only if it is a Walrasian equilibrium price in* $\mathbf{E}$. *Furthermore the equilibrium allocations in* $\mathbf{E}$ *for* p^* *are the Walrasian allocations for* p^* *and vice versa.*

Proof
Case 1. $S_1 \neq \emptyset$.
The statement follows directly from Theorem 3.4 and Theorem 4.1.
Case 2. $S_1 = \emptyset$.
Every Walrasian equilibrium is an equilibrium in the economy $\mathbf{E}$ by the proof of Theorem 3.4.
Let (d^*, p^*, x^*) be an equilibrium in $\mathbf{E}$. By Theorem 3.4 it follows that it is a uniform price equilibrium. So no agent can gain additional income from his position as an intermediary. Therefore every agent solves the problem of maximizing his utility given his initial endowments and the equilibrim prices as set for him by his direct leaders. But this implies the equilibrium is a Walrasian equilibrium.
$\mathcal{Q}.\mathcal{E}.\mathcal{D}.$

The Walrasian Equivalence of Theorem 4.2 is attained by assuming the initial endowments of the agents in S_1 to be such that the equilibrium condition of feasibility "solves" the coordination problem of those agents. There is, however, another way to solve this coordination problem. The definition of the anticipated trade correspondences may be changed by dropping the condition "*if* $q_{ij} = p_{ij}$ *then*

$t_{ij}((d,p),(e_i,q_i)) = d_{ji}$". In this case we find that the equilibrium condition of stability in the economy **E** "solves" the coordination problem, because for each uniform price system the optimization problem of each agent in S_1 is solved by trades that yield him his Walrasian allocation for the given price vector. So any tuple that does not give each agent in S_1 his Walrasian allocation for the equilibrium price system does not satisfy the equilibrium condition of stability and therefore is no longer an equilibrium.

5 Conclusions

The aim of this paper was to enrich the general equilibrium model of an exchange economy with price setting agents by making use of a hierarchical structure on the set of agents of the economy. We incorporated a notion of arbitrage in our models.

In Section 2 we defined a hierarchically structured trade economy. We assumed agents to be embedded in a hierarchical structure. A hierarchical relation between two agents was assumed to have the institutional form of a monopolistic relation, i.e. the dominating agent acts as a price setter, the dominated agent acts as a price taker with respect to this relation. We described what the local information structure is and how the conjectures of the agents are derived from the model of the economy for this specific information structure. Finally equilibrium in a hierarchically structured trade economy was defined.

A theorem on the existence of equilbrium was proved in Section 3. Furthermore it was shown that our models allow for agents in S_1, the set of agents who do not have any direct leaders in the hierarchical structure, to be rationed in equilibrium. In Section 4 a theorem was proved which shows that the Walrasian auctioneer can be replaced by a Walrasian monopolist who has to set a uniform price for the whole market and who has neglectible initial endowments. This result was used to prove a theorem on Walrasian equivalence.

In the models of Section 3 we found that even if an equilibrium with uniform prices exists in the hierarchically structured trade economy, this by no means implies these equilibria to result in Walrasian allocations. It was argued that adding monopolistic price setting to a pure exchange economy may lead to equilibria which are not Pareto efficient. Furthermore we also found that in equilibrium some agents may be rationed. Therefore in our stylized world with price setting agents the invisible hand may fail.

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