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RESEARCH MEMORANDUM

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ILBURG UNIVERSITY
EPARTMENT OF ECONOMICS
ostbus 90153 - 5000 LE Tilburg
etherlands

NON-CONVEX BUDGET SETS, INSTITUTIONAL CONSTRAINTS AND IMPOSITION OF CONCAVITY IN A FLEXIBLE HOUSEHOLD LABOR SUPPLY MODEL*)

by Arie Kapteyn, Peter Kooreman, Arthur van Soest

Tilburg University
Department of Econometrics
P.O. Box 90153
5000 LE Tilburg
The Netherlands

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Abstract

The model in this paper combines a number of desirable properties of household labor supply models. First, we adopt the Hausman-Ruud specification so that the labor supply functions are second-order flexible in wages. Second, the model takes into account the non-linearity and non-convexity of the budget sets due to the tax system and unemployment compensations. Third, the sample used also provides information on how many hours people would like to work. Using this information we know which household members are restricted in their number of hours worked and employing rationing theory, the effects of institutional constraints on labor supply are modelled. Fourth, in the estimation of the model concavity of the cost function, which is prerequisite for the use of rationing theory, is imposed for certain ranges of exogenous variables.

1. Introduction

This paper is a strictly neoclassical exercise in the modelling of household labor supply. For each of the aspects of family labor supply[1] dealt with, sufficient theory is available. Yet the implementation is often riddled with problems and many compromises have to be made before a workable model is obtained.

First of all we want to use flexible functional forms in our specification of the model. One of the consequences of this is that labor supply functions are derived from cost functions or indirect utility functions and that no closed form for the direct utility function is known[2]. This creates problems in view of the second aspect we want to deal with. We want to allow for non-convexities in the household budget set, and this calls for comparison of the utility of different points in the choice set. If this has to be done without recourse to a direct utility function, one needs to compute virtual wages and virtual non-labor incomes that correspond to the point of which we want to calculate the utility. For the indirect translog and AIDS, the computation of virtual wages ia a cumbersome affair. However, for the flexible system recently introduced by Hausman and Ruud (1984), the computation of shadow wages is fairly easy, and hence we adopt the Hausman-Ruud model for the description of household labor supply.

A third aspect we want to deal with is that the observed number of hours worked is not just the result of the maximization of a household utility function, but is also affected by institutional constraints and demand side factors. In the data used in the empirical analysis, we have information on both the actual number of hours worked by husband and wife and the number of hours they would like to work (to be called preferred hours) at their current wage rate. We assume that it is preferred hours that are the reflection of the household's preferences,

1) "household" and "family" are used a synonyms. In this paper we only deal with complete families, i.e. with at least husband and wife present.

2) The only known flexible form derived from a direct utility function is the direct translog, but this leads to messy expressions.

rather than actual hours. In addition we assume that a respondent in our survey determines his or her preferred hours conditional on the actual number of hours worked by the partner. In other words, when husband or wife tell us how many hours they would like to work, they assume that the number of hours actually worked by their partner does not change. Technically, this means that the number of hours of the partner is rationed, so we need rationing theory to describe the preferred hours of both partners. Once again, this requires the computation of shadow wages.

A fourth aspect is the imposition of concavity, in wages, of the cost function. The Hausman-Ruud cost function cannot be globally concave and at the same time maintain its flexibility. So if we value flexibility, the best thing we can do is to impose concavity in a relevant range of wages. It is shown that this can be done by setting an upper bound to one of the parameters in the model.

A fifth important aspect is to have a consistent stochastic specification. Ours is only consistent under special assumptions. We discuss some aspects of the stochastic specification in Section 4 and we have to conclude that only under very stringent conditions a consistent and tractable stochastic specification is possible.

2. The Model

A household is assumed to maximize a utility function with male leisure, female leisure and total household consumption as its arguments. We assume that the cost function that corresponds to maximization of the utility function under a linear full income constraint[1] is the Gorman polar form type introduced by Hausman and Ruud (1984):

$$c(w_m, w_f, u) = u.\exp(-\beta_m w_m - \beta_f w_f) - \{\theta + \delta_m w_m + \delta_f w_f + \tfrac{1}{2}(\gamma_m w_m^2 + \gamma_f w_f^2) + \alpha w_m w_f\}, \qquad (1)$$

where

w_m, w_f: the husband's and wife's after tax wage rates
u : household utility level
β_m, β_f, θ, δ_m, δ_f, α, γ_m, γ_f: parameters.

Application of Shephard's lemma yields the following labor supply functions:

$$h_m^* = \delta_m + \beta_m \mu^* + \gamma_m w_m + \alpha w_f, \qquad (2a)$$

$$h_f^* = \delta_f + \beta_f \mu^* + \gamma_f w_f + \alpha w_m, \qquad (2b)$$

where

$$\mu^* = \mu + \theta + \delta_m w_m + \delta_f w_f + \tfrac{1}{2}(\gamma_m w_m^2 + \gamma_f w_f^2) + \alpha w_m w_f, \qquad (2c)$$

Here h_m^* and h_f^* are the optimal numbers of working hours (per week) of husband and wife and μ denotes non-labor income of the household.

As noted in the introduction, the respondents in the sample used in our empirical work did not only provide their actual number of hours worked per week but also how many hours they would like to work per week at their current wage rate. The latter concept is called the number of preferred hours of the respondent.

1) Complications are considered below.

Since preferred hours appear to be more of a reflection of the respondent's preferences than actual hours (which are also influenced by demand side factors and institutional constraints), and since we are interested in labor supply, which is a reflection of these preferences, it is preferred hours that we want to explain by means of model (2). Since the male and female partner in a household were asked the questions separately it seems reasonable to assume that a respondent's answer takes the partner's number of hours as given.

From rationing theory (c.f., e.g., Neary and Roberts (1980), Deaton and Muellbauer (1981)) it follows that in this case the equation for the explanation of male preferred hours is

$$h_m^* = \delta_m + \beta_m \bar{\mu}_f^* + \gamma_m w_m + \alpha \bar{w}_f, \tag{3a}$$

where $\bar{w}_f$ and $\bar{\mu}_f^*$ are defined by the equations

$$\bar{\mu}_f^* = \bar{\mu}_f + \theta + \delta_m w_m + \delta_f \bar{w}_f + \tfrac{1}{2}(\gamma_m w_m^2 + \gamma_f \bar{w}_f^2) + \alpha w_m \bar{w}_f, \tag{3b}$$

$$h_f = \delta_f + \beta_f \bar{\mu}_f^* + \gamma_f \bar{w}_f + \alpha w_m \tag{3c}$$

$$\bar{\mu}_f + \bar{w}_f h_f = \mu + w_f h_f, \tag{3d}$$

where h_f denotes the number of hours actually worked by the female. Substitution of (3b) and (3d) in (3c) yields a quadratic equation in $\bar{w}_f$, from which $\bar{w}_f$ can be solved analytically under certain conditions. Having obtained $\bar{w}_f$, computation of $\bar{\mu}_f$ and $\bar{\mu}_f^*$ from (3b) and (3d) is obvious. See the Appendix for details. The corresponding equation for h_f^* is obtained analogously. Note that we implicitly assume the partners to have identical preferences. The preferred hours equations allow h_i^* ($i=m,f$) to be negative. Negative preferred hours h_i^* imply that the i-th partner prefers not to work.

The calculation of wage responses is somewhat less straightforward in the model with rationing than without. For example, in the model without rationing we have

$$\frac{\partial h_m^*}{\partial w_m} = \gamma_m + \beta_m(\delta_m + \gamma_m w_m + \alpha w_f), \tag{4a}$$

whereas with rationing we obtain, after some calculations,

$$\frac{\partial \bar{w}_f}{\partial w_m} = - \frac{\alpha\beta_f \bar{w}_f + \beta_f \gamma_m w_m + \beta_f \delta_m + \alpha}{\gamma_f \beta_f \bar{w}_f + \gamma_f + \beta_f(-h_f+\delta_f+\alpha w_m)} \tag{4b}$$

$$\frac{\partial \bar{\mu}_f}{\partial w_m} = - h_f \frac{\partial \bar{w}_f}{\partial w_m} \tag{4c}$$

$$\frac{\partial h_m^*}{\partial w_m} = \gamma_m + \beta_m(\delta_m+\gamma_m w_m+\alpha\bar{w}_f) + \{\alpha+\beta_m(-h_f+\delta_f+\gamma_f\bar{w}_f+\alpha w_m)\} \frac{\partial \bar{w}_f}{\partial w_m} \tag{4d}$$

Similar expressions can be obtained for other derivatives.

The introduction of non-linearities in the budget constraint due to a progressive income tax ioes not create any fundamental difficulties. Hausman's algorithm (cf. Hausman (1979) or Blomquist (1983)) can be applied to find the preferred number of working hours for both partners in a household. A more thorny problem arises as a result of the non-convexities created by the operation of the social security system.

3. Non-convex Budget Sets

In view of the complexity of the social security system in The Netherlands, and the limited amount of information in our sample on tax rates and deductions, we will only pay attention to the system of unemployment insurance. In The Netherlands, employees who are laid off usually receive unemployment compensation, whereas people who quit their job voluntarily do not receive compensation.

In the estimation of the model we only take into account the non-convexity of the budget set of individuals who receive unemployment benefits. We assume that an individual who is presently employed is not entitled to employment benefits if he or she quits his or her job and that, as a result, the budget set of such an individual is convex.[1)]

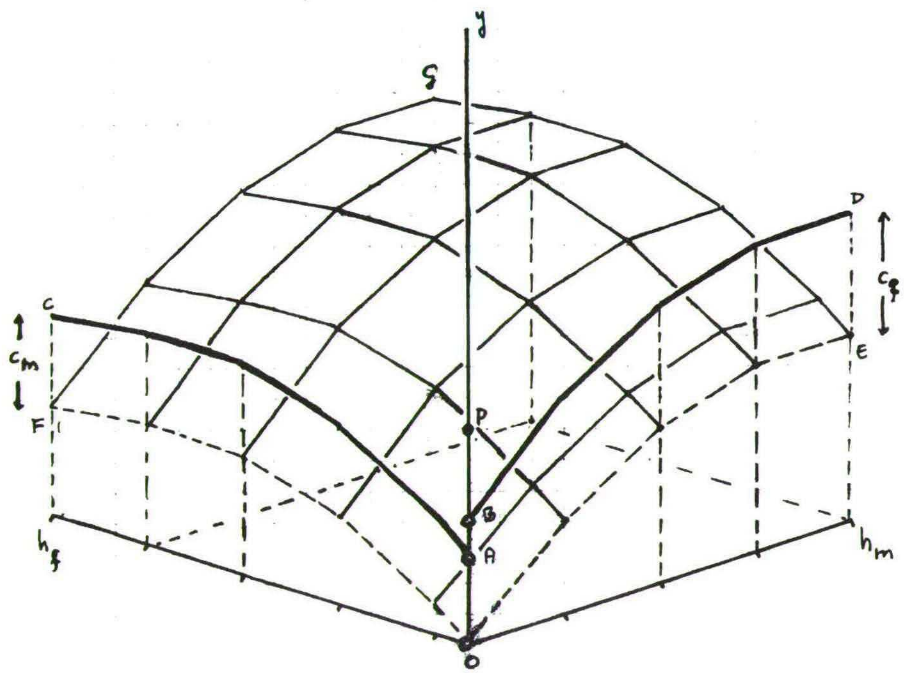


Figure 1. Non-convex household budget set

1) We ignore minor non-convexities caused by some pay-roll taxes.

Figure 1 shows the budget set of a household if both partners receive an unemployment compensation when unemployed. Here y denotes the household's total after-tax income, including labor income, unemployment benefits and other sources of non-labor income, c_m and c_f denote the husband's and wife's unemployment compensation respectively. The household's budget set consists of the single point P (h_m=h_f=0, $y-\mu=c_m+c_f$), the curves BD (h_f=0) and AC(h_m=0) and the "manifold" OEGF.

The assumption that an individual looses all benefits at the moment that he or she works slightly more than zero hours is incorrect. However, the marginal tax on increased earnings for someone on unemployment compensation is close to 100%. Thus, for most recipients of unemployment compensation the choice will be between working zero hours or at least so many hours that one is in the convex part of the budget set. From a practical point of view, our assumption therefore appears to be harmless.

As a consequence of the non-convexity of the budget set of recipients of unemployment benefits, the maximum utility (U_1) in employment without receiving a benefit has to be compared to the utility of being unemployed (U_0) and receiving an unemployment benefit. The details of the calculation of U_0 and U_1 are relegated to the Appendix.

4. Stochastic Specification, Data, and Likelihood Function

The stochastic specification in utility consistent rationing models is a delicate problem, even in the case of a convex budget set[1]. In these types of models it is important to distinguish between different sources of random errors. Let us first consider the case where stochastic variation in preferred hours arises from differences in preferences across households. One way to allow for random preferences is to allow some parameters to vary stochastically, e.g.

$$\delta_i = \sum_{j=1}^{m} x_j \xi_{ij} + \varepsilon_i, \qquad i = m, f \tag{5a}$$

where x_j $(j=1,...,m)$ are observed characteristics that are thought to influence the household's preferences (like family composition) and the error term ε_i represents unobserved sources of preference variation.

If the δ s are random, then (3) implies that $\bar{w}_m$, $\bar{w}_f$, $\bar{\mu}_m$ and $\bar{\mu}_f$ are random as well. Assume, for instance, that ε_m and ε_f (and consequently δ_m and δ_f) are normally distributed. Since $\bar{w}_m$, $\bar{w}_f$, $\bar{\mu}_{m_*}$ and $\bar{\mu}_{f_*}$ are nonlinear functions of the δ's, the joint distribution of h_m and h_f is then non-normal and computationally intractable. The δ's also appear in the expressions for U_0 and U_1 (cf. Appendix) in a non-linear way, so that it is basically impossible to derive the joint distribution of U_0 and U_1. Even if we would content ourselves to obtain densities of observable variables entirely through numerical methods, we face the problem that a prerequisite for these densities to be proper ones is the existence and uniqueness of the shadow wages defined by equations such as (3b)-(3d).This can only be guaranteed if the cost function is globally concave. The Hausman-Ruud cost function can only be globally concave if $\beta_m = \beta_f = 0$, in which case flexibility is lost.

Similar problems arise in the case where random preferences are introduced through shifts in hours, starting from a direct utility function of the form

1) Some problems have been discussed before by Kooreman and Kapteyn (1986).

$$U = u(h_m-\varepsilon_m, h_f-\varepsilon_f, y)$$

(see McElroy (1985)). A specification like this would make it necessary to use $h_i-\varepsilon_i$ instead of actual hours h_i (i=m,f) in computing shadow wages and thus would yield stochastic shadow wages. The same holds if actual hours are measured with error.

In view of these problems we have adopted a pragmatic solution. We allow the δ's to depend on observed characteristics only, i.e.

$$\delta_i = \sum_{j=1}^{m} x_j \xi_{ij} \qquad i = m, f$$

so that shadow wages are non-random.

Let U_{1i} be the utility of working max $(0,h_i^*)$ hours by the i-th partner without unemployment compensation and let $U_{0i}(c_i)$ be the utility if the person does not work, but continues to receive an amount c_i as unemployment compensation. We have $U_{1i}-U_{0i}(0) > 0$ so that we can omit the utility comparison for people without unemployment compensation. For these people, we write the stochastic version of the model as

$$h_i^* = \delta_i + \beta_i \bar{\mu}_j^* + \gamma_i w_i + \alpha \bar{w}_j, \tag{6a}$$

$$h_i^p = h_i^* + \varepsilon_i, \quad \text{if } h_i^* + \varepsilon_i > 0, \tag{6b}$$

$$h_i^p = 0, \qquad \text{if } h_i^* + \varepsilon_i < 0, \tag{6c}$$

$$\bar{\mu}_j^* = \bar{\mu}_j + \theta + \delta_i w_i + \delta_j \bar{w}_j + \tfrac{1}{2}(\gamma_i w_i^2 + \gamma_j \bar{w}_j^2) + \alpha w_i \bar{w}_j, \tag{6d}$$

$$h_j = \beta_j \bar{\mu}_j^* + \gamma_j \bar{w}_j + \delta_j + \alpha w_i, \tag{6e}$$

$$\bar{\mu}_j + h_j \bar{w}_j = \mu + h_j w_j, \tag{6f}$$

$$i,j \in \{m,f\};\ j \neq i,$$

where h_i^p is the preferred number of hours reported by respondent i. The error terms can either represent measurement errors or optimization errors on the part of the individual.

Assuming absence of optimization errors, the choice to work or not (or rather to look for a job or not) for an individual receiving an unemployment benefit c_i is determined by $U_{1i}-U_{0i}(c_i)$; if $U_{1i}-U_{0i}(c_i) > 0$, the number of hours these people would like to work is described by (6a) and (6d-6f). For these people the data set we use only tells us whether they are looking for a job or not, and not their preferred number of hours. Therefore, for an individual receiving an unemployment benefit we omit the h_i^p equation and write the stochastic version of the model as

$$v_i = U_{1i}-U_{0i}(c_i) + \eta_i$$

$$\begin{aligned} z_i &= 1 \qquad \text{if } v_i > 0 \\ z_i &= 0 \qquad \text{if } v_i < 0. \end{aligned}$$

where z_i takes the value 1 if the i-th partner is seriously looking for a job and 0 otherwise. The random variable η_i can represent measurement error or optimization error on the part of the individual.

This means, that we have not only ignored the possibility of stochastic preferences, but we have also assumed that the individual is completely certain about his or her hourly wage rate if he or she decides to accept a job. Assuming a non-trivial distribution of offered wage rates would again lead to stochastic shadow wages and is computationally intractable.

Due to the design of the questionnaire used in the survey, the information on preferred numbers of hours h_m^p and h_f^p has only been collected for individuals who actually work at least 15 hours per week. Individuals who have a job, but are working less than 15 hours are only asked whether or not they are seriously looking for a job. If the answer is no, preferred hours are taken equal to actual hours. If the answer is in the affirmative, it is assumed that preferred hours exceed actual hours (This only applies to four cases in the sample. The assumption that these respondents want to work more hours is based on other questions in the questionnaire).

We assume $(\varepsilon_m, \varepsilon_f, \eta_m, \eta_f)$ to follow a multivariate normal distribution with zero mean and covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_m^2 & \cdot & \cdot & \cdot \\ \rho\,\sigma_m\,\sigma_f & \sigma_f^2 & \cdot & \cdot \\ * & 0 & \sigma_v^2 & \cdot \\ 0 & * & * & \sigma_v^2 \end{bmatrix}$$

An asterisk indicates that the covariance does not appear in the likelihood function for the present sample, so that it cannot be estimated. Because of the small number of observations on people receiving an unemployment benefit (see below) we impose cov (ε_m, η_f) = cov $(\varepsilon_f, \eta_m) = 0$ and var (η_m) = var (η_f).

The contributions L to the likelihood by households in the different groups as well as the sample composition are given in Table 1. Here f is the joint density of ε_1 and ε_2, Φ and ϕ are the standard normal distribution and density function respectively.

Table 1. Likelihood contributions

husband wife	489 $h_m > 15$ $(b_m = 0)$	0 $0 < h_m < 15$ $z_m = 0$ $(b_m = 0)$	0 $0 < h_m < 15$ $z_m = 1$ $(b_m = 0)$
133 $h_f > 15$ $(b_f = 0)$	129 $f(h_m^P - h_m^*,\ h_f^P - h_f^*)$	0 $f(h_m - h_m^*,\ h_f^P - h_f^*)$	0 $\int_{h_m - h_m^*}^{\infty} f(\varepsilon_m,\ h_f^P - h_f^*) d\varepsilon_m$
56 $0 < h_f < 15$ $z_f = 0$ $(b_f = 0)$	51 $f(h_m^P - h_m^*,\ h_f - h_f^*)$	0 $f(h_m - h_m^*,\ h_f - h_f^*)$	0 $\int_{h_m - h_m^*}^{\infty} f(\varepsilon_m,\ h_f - h_f^*) d\varepsilon_m$
5 $0 < h_f < 15$ $z_f = 1$ $(b_f = 0)$	5 $\int_{h_f - h_f^*}^{\infty} f(h_m^P - h_m^*, \varepsilon_f) d\varepsilon_f$	0 $\int_{h_f - h_f^*}^{\infty} f(h_m - h_m^*, \varepsilon_f) d\varepsilon_f$	0 $\int_{h_m - h_m^*}^{\infty} \int_{h_f - h_f^*}^{\infty} f(\varepsilon_m, \varepsilon_f) d\varepsilon_f d\varepsilon_m$
298 $h_f = 0$ $z_f = 0$ $b_f = 0$	280 $\int_{-\infty}^{-h_f^*} f(h_m^P - h_m^*, \varepsilon_f) d\varepsilon_f$	0 $\int_{-\infty}^{-h_f^*} f(h_m - h_m^*, \varepsilon_f) d\varepsilon_f$	0 $\int_{h_m - h_m^*}^{\infty} \int_{-\infty}^{-h_f^*} f(\varepsilon_m, \varepsilon_f) d\varepsilon_f d\varepsilon_m$
0 $h_f = 0$ $z_f = 0$ $b_f = 1$	0 $\frac{1}{\sigma_m} \phi(\frac{h_m^P - h_m^*}{\sigma_m}) \Phi(-\frac{v_f^*}{\sigma_v})$	0 $\frac{1}{\sigma_m} \phi(\frac{h_m - h_m^*}{\sigma_m}) \Phi(-\frac{v_f^*}{\sigma_v})$	0 $\Phi(\frac{h_m^* - h_m}{\sigma_m}) \Phi(-\frac{v_f^*}{\sigma_v})$
25 $h_f = 0$ $z_f = 1$ $b_f = 0$	21 $\int_{-h_f^*}^{\infty} f(h_m^P - h_m^*, \varepsilon_f) d\varepsilon_f$	0 $\int_{-h_f^*}^{\infty} f(h_m - h_m^*, \varepsilon_f) d\varepsilon_f$	0 $\int_{h_m - h_m^*}^{\infty} \int_{-h_f^*}^{\infty} f(\varepsilon_m, \varepsilon_f) d\varepsilon_f d\varepsilon_m$
3 $h_f = 0$ $z_f = 1$ $b_f = 1$	3 $\frac{1}{\sigma_m} \phi(\frac{h_m^P - h_m^*}{\sigma_m}) \Phi(\frac{v_f^*}{\sigma_v})$	0 $\frac{1}{\sigma_m} \phi(\frac{h_m - h_m^*}{\sigma_m}) \Phi(\frac{v_f^*}{\sigma_v})$	0 $\Phi(\frac{h_m^* - h_m}{\sigma_m}) \Phi(\frac{v_f^*}{\sigma_v})$

1 $h_m = 0$ $z_m = 0$ $b_m = 0$	6 $h_m = 0$ $z_m = 0$ $b_m = 1$	4 $h_m = 0$ $z_m = 1$ $b_m = 0$	20 $h_m = 0$ $z_m = 1$ $b_m = 1$
0 $\int_{-\infty}^{-h_m^*} f(\epsilon_m, h_f^p - h_f^*)d\epsilon_m$	1 $\Phi(-\frac{v_m^*}{\sigma_v})\frac{1}{\sigma_f}\phi(\frac{h_f^p - h_f^*}{\sigma_f})$	1 $\int_{-h_m^*}^{\infty} f(\epsilon_m, h_f^p - h_f^*)d\epsilon_m$	2 $\Phi(\frac{v_m^*}{\sigma_v})\frac{1}{\sigma_f}\phi(\frac{h_f^p - h_f^*}{\sigma_f})$
0 $\int_{-\infty}^{-h_m^*} f(\epsilon_m, h_f - h_f^*)d\epsilon_m$	0 $\Phi(-\frac{v_m^*}{\sigma_v})\frac{1}{\sigma_f}\phi(\frac{h_f - h_f^*}{\sigma_f})$	1 $\int_{-h_m^*}^{\infty} f(\epsilon_m, h_f - h_f^*)d\epsilon_m$	4 $\Phi(\frac{v_m^*}{\sigma_v})\frac{1}{\sigma_f}\phi(\frac{h_f^* - h_f}{\sigma_f})$
0 $\int_{-\infty}^{-h_m^*}\int_{h_f - h_f^*}^{\infty} f(\epsilon_m, \epsilon_f)d\epsilon_f d\epsilon_m$	0 $\Phi(-\frac{v_m^*}{\sigma_v})\Phi(\frac{h_f^* - h_f}{\sigma_f})$	0 $\int_{-h_m^*}^{\infty}\int_{h_f - h_f^*}^{\infty} f(\epsilon_m, \epsilon_f)d\epsilon_f d\epsilon_m$	0 $\Phi(\frac{v_m^*}{\sigma_v})\Phi(\frac{h_f^* - h_f}{\sigma_f})$
1 $\int_{-\infty}^{-h_m^*}\int_{-\infty}^{-h_f^*} f(\epsilon_m, \epsilon_f)d\epsilon_f d\epsilon_m$	5 $\Phi(-\frac{v_m^*}{\sigma_v})\Phi(-\frac{h_f^*}{\sigma_f})$	2 $\int_{-h_m^*}^{\infty}\int_{-\infty}^{-h_f^*} f(\epsilon_m, \epsilon_f)d\epsilon_f d\epsilon_m$	10 $\Phi(\frac{v_m^*}{\sigma_v})\Phi(-\frac{h_f^*}{\sigma_f})$
0 $\Phi(-\frac{h_m^*}{\sigma_m})\Phi(-\frac{v_f^*}{\sigma_v})$	0 $\Phi(-\frac{v_m^*}{\sigma_v})\Phi(-\frac{v_f^*}{\sigma_v})$	0 $\Phi(\frac{h_m^*}{\sigma_m})\Phi(-\frac{v_f^*}{\sigma_v})$	0 $\Phi(\frac{v_m^*}{\sigma_v})\Phi(-\frac{v_f^*}{\sigma_v})$
0 $\int_{-\infty}^{-h_m^*}\int_{-h_f^*}^{\infty} f(\epsilon_m, \epsilon_f)d\epsilon_f d\epsilon_m$	0 $\Phi(-\frac{v_m^*}{\sigma_v})\Phi(\frac{h_f^*}{\sigma_f})$	0 $\int_{-h_m^*}^{\infty}\int_{-h_f^*}^{\infty} f(\epsilon_m, \epsilon_f)d\epsilon_f d\epsilon_m$	4 $\Phi(\frac{v_m^*}{\sigma_v})\Phi(\frac{h_f^*}{\sigma_f})$
0 $\Phi(-\frac{h_m^*}{\sigma_m})\Phi(\frac{v_f^*}{\sigma_v})$	0 $\Phi(-\frac{v_m^*}{\sigma_v})\Phi(\frac{v_f^*}{\sigma_v})$	0 $\Phi(\frac{h_m^*}{\sigma_m})\Phi(\frac{v_f^*}{\sigma_v})$	0 $\Phi(\frac{v_m^*}{\sigma_v})\Phi(\frac{v_f^*}{\sigma_v})$

Explanation: h_i : actual hours of work.

$v_i = U_{1i} - U_{0i}$.

z_i = 1 if the individual is seriously looking for a job,
z_i = 0 otherwise.

b_i = 1 if the individual receives an unemployment compensation,
b_i = 0 otherwise.

Table 2. Sample Statistics

	mean	standard deviation	min	max	number of observations
male preferred hours	37.60	6.62	15	70	489
actual hours (all males)	39.77	11.78	0	70	520
actual hours (working males only)	42.29	6.38	20	70	489
before tax wage rate	14.97	7.66	4.98	55.87	520
after tax wage rate	11.74	3.72	4.95	29.48	520
unemployment benefit (recipients only)	357.70	96.40	228.99	644.38	26
female preferred hours	24.49	8.52	8	50	133
actual hours (all females)	8.44	13.24	0	42	510
actual hours (working females only)	22.62	12.20	2	42	194
before tax wage rate	14.18	5.72	3.41	23.26	520
after tax wage rate	12.71	5.12	3.06	21.82	520
unemployment benefits (recipients only)	132.33	71.59	50.83	184.11	3
non-labor household income	80.62	122.54	0	927.41	520
log (family size)	1.200	0.349	0.693	2.303	520
dummy children < 6 years old	0.317	0.466	0	1	520

Explanation:
hours: working hours per week
wage rates: in Dfl. per hour worked
benefits: in Dfl. per week
non-labor income: in Dfl. per week, not including unemployment benefits.

The data used stem from a labor mobility survey conducted in The Netherlands in 1982. In the estimation, data on 520 households have been employed. Some sample statistics are given in Table 2.

According to Table 1, there are only six males who receive unemployment compensation and state that they are not looking for a job. Most male recipients of unemployment compensation claim to be looking for a job. Since job hunting is a prerequisite for getting unemployment compensation, it may very well be that some respondents who would rather not work have not revealed their true preferences. Consequently the job-search activities of the unemployed may be overestimated. As a result our estimation results may be biased in favor of a preference for work.

The before tax wage rates in Table 2 are *predicted* wages on the basis of a wage equation with log(age), log(age)-squared and education as predictors. For males and females separate wage equations have been estimated, using Heckman's two-stage procedure (Heckman (1979)).

5. Empirical results

Table 3 presents the parameter estimates. With respect to the statistical significance of the parameters, we observe that β_m ("the male non-labor income effect") and γ_f (representing (the largest part of) the female own wage effect) are significantly different from zero, whereas β_f, α and γ_m are not. The variables concerning family composition play a significant role in the female hours equation but not in the male hours equation.

The estimates of γ_m, γ_f and α yield a positive definite matrix $A = \begin{bmatrix} \gamma_m & \alpha \\ \alpha & \gamma_m \end{bmatrix}$. The positive definiteness of the matrix A is crucial in our derivation of conditions for concavity of the cost function (see Appendix).

To impose concavity of the cost function in wages in a relevant region of the (h_m,h_f,y) space and the (w_m,w_f,μ) space, the parameter θ has been restricted, i.e. an upper bound in terms of other parameters in the model has been set to θ (see Appendix, inequalities (A.15) and (A.30); it turns out that condition (A.30) is binding in estimation) such that concavity is guaranteed in all data points.
To illustrate the resulting region of concavity of the cost function, Figure 2a presents, for a family without children, the region in the space of w_m, w_f and μ where the cost function is concave (other family compositions yield only minor changes in the area of concavity). The region of concavity is bounded from above by the paraboloid (see Appendix) drawn in the figure. It appears that the cost function is concave for almost any reasonable value of w_m, w_f and μ. To fix ideas: a weekly after tax unearned income of 2,500 guilders amounts to an annual income of approximately U.S. \$ 52,000 (2.5 guilders is taken as approximately 1 U.S. dollar); an after tax wage rate of 60 guilders per hour amounts to about \$ 24 per hour. Figure 2b shows the corresponding region of concavity in (h_m,h_f,y)-space. This region (outside the paraboloid, see Appendix, eq. (A.9) and (A.6)) appears to contain all reasonable values of h_m, h_f and y.

The estimates of β_m and β_f have the expected sign, indicating that both male and female leisure are normal goods. As far as the other parameters are concerned, a direct economic interpretation is mostly hard to give.

Table 3. Estimation results

Parameter	Estimate	Standard error[1)]
α	0.88×10^{-3}	0.13×10^{-2}
β_m	-0.20×10^{-2}	0.10×10^{-2}
β_f	-0.47×10^{-3}	0.47×10^{-3}
γ_m	0.86×10^{-2}	0.93×10^{-2}
γ_f	[illegible]	0.20
δ_{m0}	32.3	2.2
δ_{f0}	24.0	4.1
δ_{m1}	3.9	1.0
δ_{f1}	-24.0	3.5
δ_{m2}	[illegible]	0.82
δ_{f2}	-13.9	2.7
σ_m	6.7	0.12
σ_f	19.3	1.7
σ_v	21.4×10^{10}	32.5×10^{11}
ρ	-0.21	0.07
θ	-390.18	- [2)]

Explanation: the parameters δ_m and δ_f have been made dependent upon additional exogenous variables as follows:

$$\delta_i = \sum_{j=0}^{2} x_j \, \delta_{ij} \qquad (i=m,f)$$

$x_0 = 1$

$x_1 = \log$ (family size)

$x_2 = \begin{cases} 1 & \text{if there are children in the family younger than six} \\ 0 & \text{otherwise} \end{cases}$

1) Covariance matrix of the parameter estimates is estimated as
$$\left\{\sum_i \frac{\partial \log L_i}{\partial \theta}(\theta) \frac{\partial \log L_i}{\partial \theta}(\theta)'\right\}^{-1}$$

2) The estimate of θ attains its upper bound (due to the imposition of concavity) so no standard error could be computed.

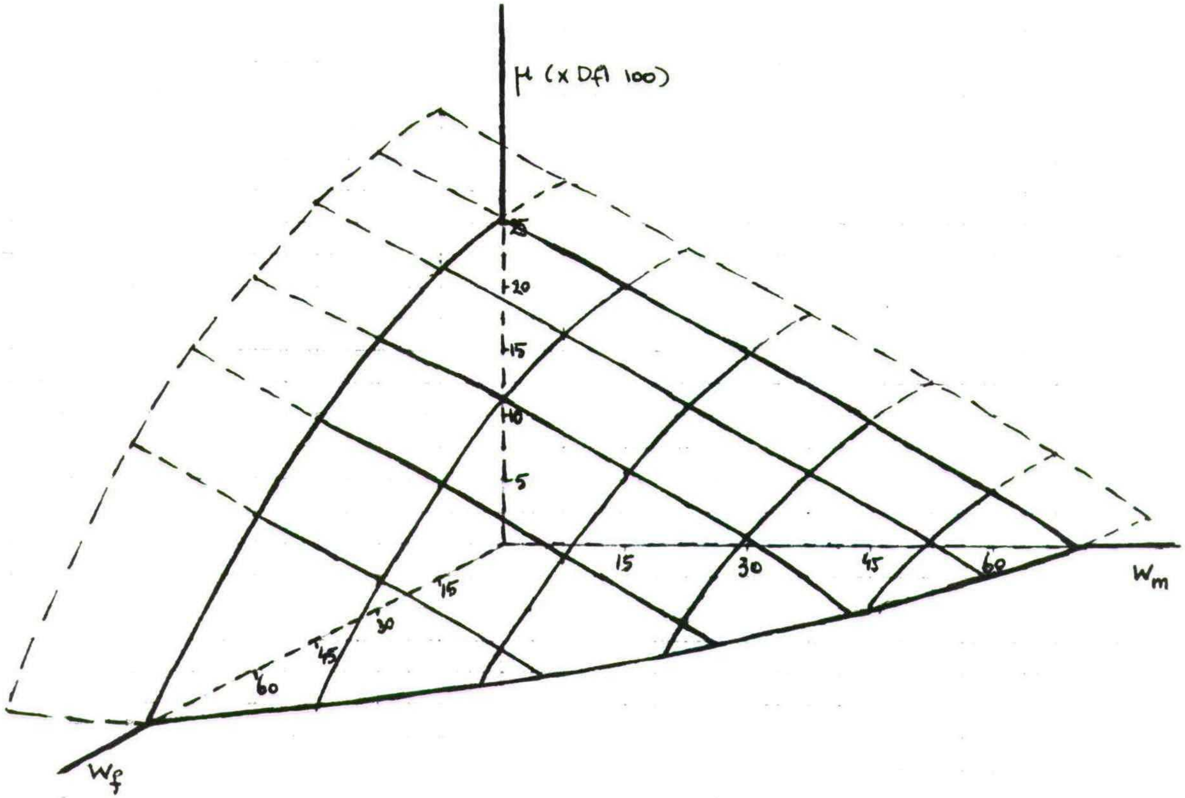


Figure 2a. The region of concavity of the estimated cost function in (w_m, w_f, μ)-space

The region inside (i.e. below) the paraboloid is the region of concavity in the (w_m, w_f, μ) space.

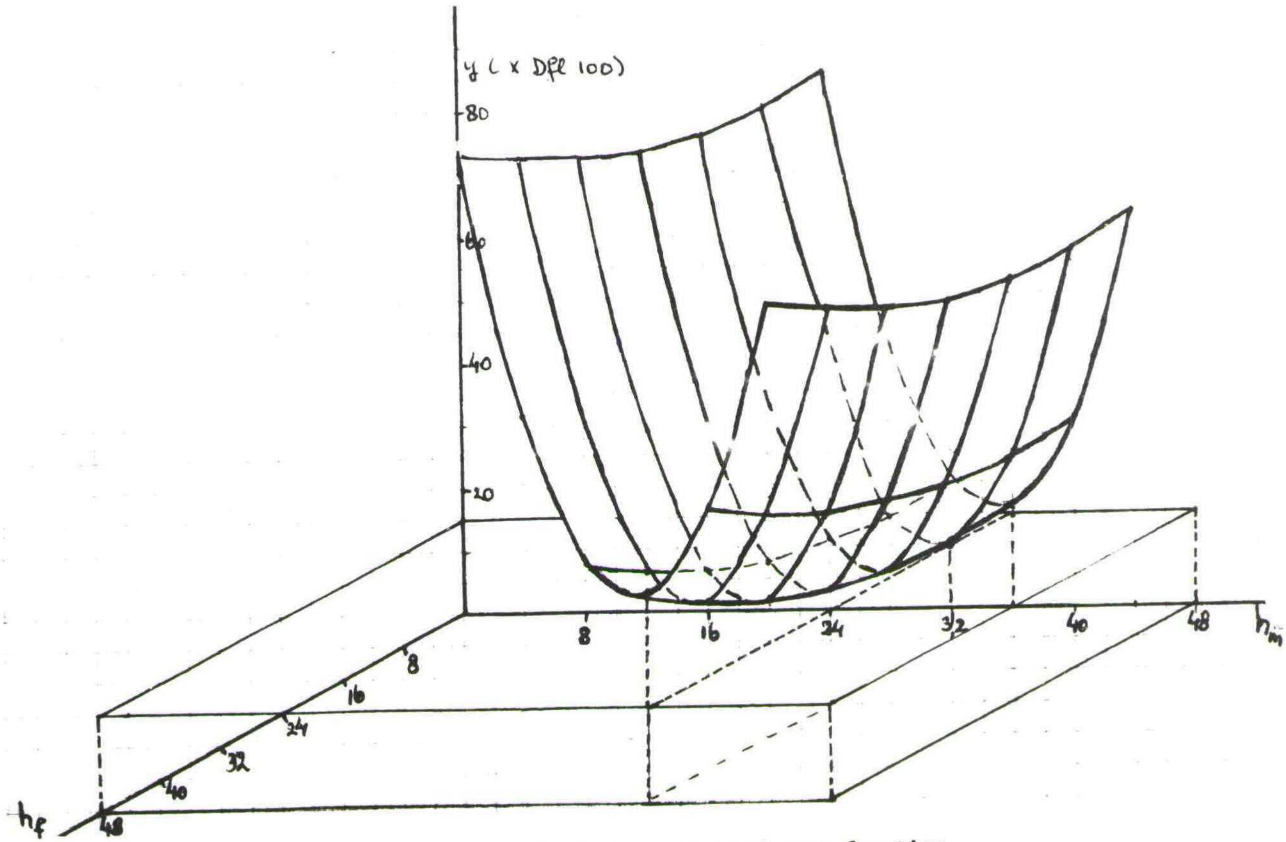


Figure 2b. <u>The region of concavity of the estimated cost function</u>

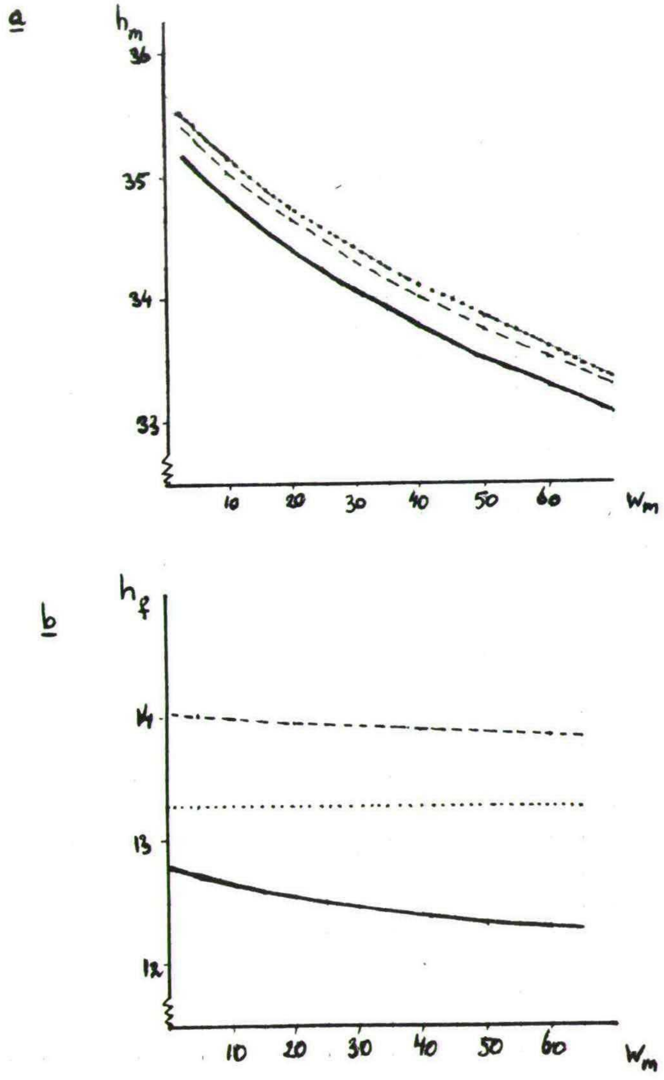
<u>(h_m, h_f, y)- space</u>

The region outside (i.e. below) the paraboloid is the region of concavity in (h_m, h_f, y)-space.

The economic meaning of the estimates will be brought out more clearly by the presentation of graphs and elasticities.

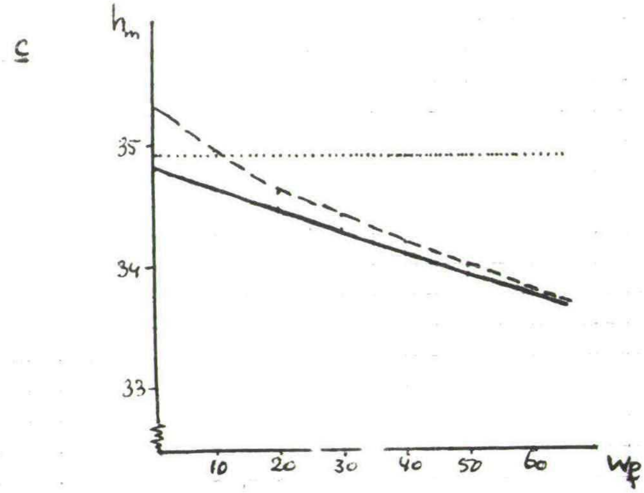
In Figure 3 a through d family labor supply functions have been drawn for a family without children as a function of before tax wage rates. In each case the remaining variables, are set at their sample means. We distinguish between short run (the partner is rationed at his or her current number of hours) and long run (the partner is not rationed) labor supply functions. In each of the four figures two short run labor supply functions are drawn: one for the case where the actual number of hours worked by the partner equals the sample mean ($\bar{h}_f$= 22.62 or $\bar{h}_m$=42.29) and one for the case where the partner does not work at all. It is remarkable, that long run labor supply is always (a little bit) lower than short run labor supply.

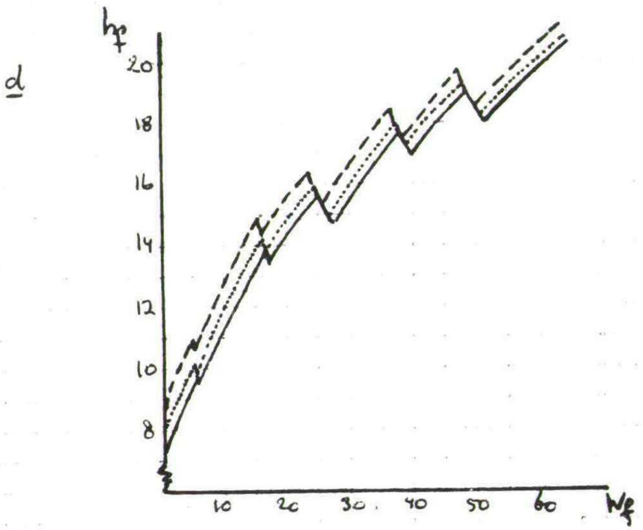
Figure 3a shows a backward bending male labor supply function implying that the negative income effect dominates the positive own wage effect. Figures 3b and 3c demonstrate the expected negative relationship between one's preferred number of hours and the partner's wage rate, but the effects are small. Figure 3d shows that female labor supply is forward bending. The own wage impact is much larger for the wife than for the husband. Figure 3d also reveals the working of the tax system. The piece-wise linear progressive tax system leads to jig-sawed responses of preferred hours to the own before-tax wage rate. The reason for this is that each time an individual is at a kink in the budget constraint, she wants to stay there if we change the before-tax wage rate a little bit. To stay at a kink with an increasing before-tax wage rate entails a reduction of work effort. The downward sloping parts in Fig. 3d are hence hyperbolas. The same kind of non-differentiabilities is in principle also present in Figure 3a, but in this case the hyperbola parts are so small that the drawing cannot reveal them. This is caused by the very small male own wage effect. The difference in own wage elasticities is borne out by Figure 4 where some indifference curves are depicted. Figure 4a shows a few indifference curves upon which the husband's decision is based if his wife works $\bar{h}_f$ = 22.62 hours; it is easy to see that a change in the male wage rate only has a very small impact on the optimal number of male working hours. In Figure 4b, where the wife's indifference curves are drawn if the husband works $\bar{h}_m$ = 42.29 hours a week, the (own) wage impact is much larger. Similar figures could be drawn for different family compositions. The main difference would be a strong downward shift in all female labor supply functions (due to the negative estimates for δ_{f1} and δ_{f2}, the parameters that represent the impact of family size and the presence of children younger than six respectively on the wife's labor supply). As a result (predicted) preferred hours of the wife are then only non-zero for very high female wage rates.



——— long run
\- - - partner works $\bar{h}_j$ hours
.... partner does not work

Figure 3. Preferred hours as a function of before tax hourly wage rates for a couple without children.





(Figure 3)
continued

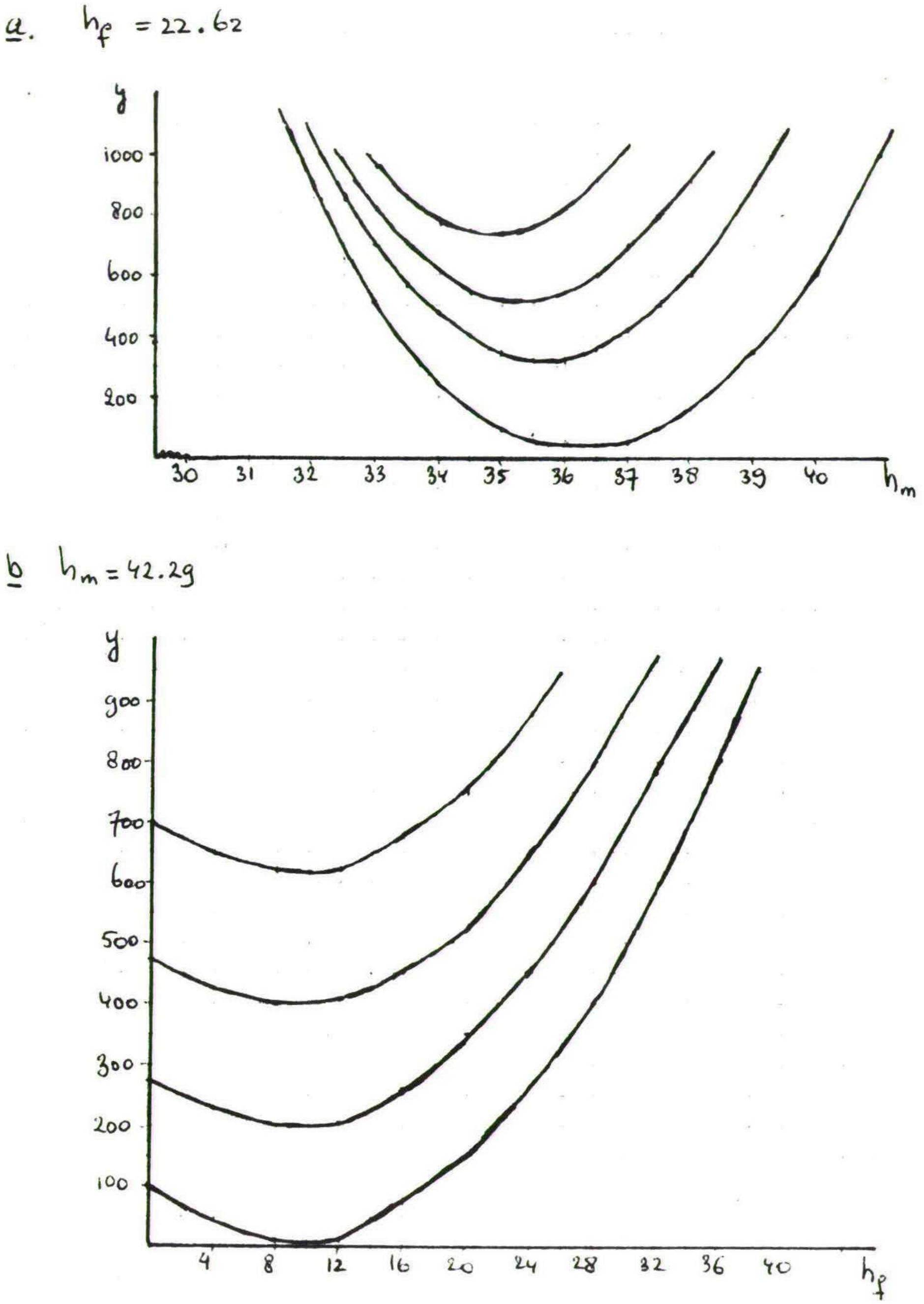


Figure 4. Some indifference curves for a family without children if the number of working hours of one spouse are fixed.

The rooting of our model in neoclassical theory allows us to calculate the compensating variation (cv) and the deadweight loss (dwl) that results from the taxes on labor income. The compensating variation is the lump-sum the household needs in the presence of taxes to be as well off as it would have been without taxes. The deadweight loss is the difference between the compensating variation and the taxes paid by the household after being restored to the original utility level. We calculate this as a percentage of the taxes paid (cf, e.g., Hausman (1981)). We calculate the compensating variation and the deadweight loss for different wages, setting all other variables equal to their sample mean. Table 4 shows the familiar pattern of an increase in deadweight loss if market wages rise, which is a result of the progressivity of the tax system. The average magnitude of the deadweight loss is modest. For comparison,

Table 4. Compensating variation and deadweight loss

Exogenous variables				compensating variation	Tax	Deadweight loss
x_1	x_2	w_m	w_f			
2	0	14.97	14.18	108.80	106.68	1.99
2	0	29.94	28.36	430.10	392.38	9.61
4	0	14.97	14.18	98.01	97.76	0.26
4	0	29.94	28.36	357.19	354.04	0.89
4	1	14.97	14.18	101.30	101.02	0.28
4	1	29.94	28.36	359.21	358.95	0.07

Explanation:

x_1 and x_2 are defined in Table 3.

w_m and w_f are the husband's and wife's before tax hourly wage rates. We have used the sample means of these values (see table 2) and twice the sample means.

Compensating variation and tax are expressed in Dfl per week.

Deadweight loss = (Compensating variation-tax)/tax × 100%.

Blomquist (1983) reports an average deadweight loss of approximately 20% of the income tax in Sweden. The explanation for the difference may lie primarily in the differences in wage elasticities. In our model, behavioral responses to wage changes tend to be modest, so that taxes do not influence behavior very much.

6. Concluding remarks

The basic idea of neoclassical models is, of course, that decision making units maximize a utility function under constraints. Much of recent work in labor supply has been directed towards a better modelling of these constraints. In order to deal with certain kinds of constraints (like corner solutions and rationing), integrability conditions have to be satisfied. If, for example, the cost function is not concave in a certain region of (w_m, w_f, μ) space, a calculated female shadow wage $\bar{w}_f$ that corresponds to a point in this region does not have a theoretically well-defined meaning. Thus imposition of theoretical restrictions on the model is a conditio sine qua non for the use of shadow wages.

It is not possible to test the validity of the concavity restriction imposed, simply because if the restriction does not hold the model does not make sense.

Perhaps the most important problem left is to find a realistic and consistent stochastic specification. Our stochastic specification is only consistent with the assumption that preferred hours suffer from measurement or optimization errors. Since the possibility of measurement errors in actual hours or random preferences is ruled out by our specification, it can hardly be called realistic. Perhaps the size of measurement errors in actual hours will be small, so that we are justified in ignoring this source of randomness. But it is quite unlikely that all variation in preferences is correctly described by the equation in Table 3 and our choice of x_j's. As it appears almost impossible to model random preferences consistently within a flexible specification, the best thing to do may be to invest more time in an investigation of determinants of systematic variation of preferences across households; in other words to extend the list of x_j's in Table 3.

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Appendix: DIRECT UTILITY, SHADOW WAGES, AND THE IMPOSITION OF CONCAVITY

Direct utility

The non-convexity of the budget set makes it necessary to compare the values of the direct utility function in different points. We shall determine the utility level in some given point (h_m, h_f, y), where y is the household's income:

(A.1) $$y = \mu + w_m h_m + w_f h_f.$$

We use the following notation:

$$\beta := (\beta_m, \beta_f)'$$

$$A := \begin{bmatrix} \gamma_m & \alpha \\ \alpha & \gamma_f \end{bmatrix}$$

$$k := (h_m - \delta_m, h_f - \delta_f)'$$

$$w := (w_m, w_f)'$$

Given h_m, h_f and y, we first want to find (shadow-)wages w and corresponding (virtual) non-labor income $\mu := y - w_m h_m - w_f h_f$ satisfying

(A.2) $$k = \mu^* \beta + Aw$$

(A.3) $$\mu^* = \mu + \theta + (\delta_m, \delta_f)\, w + \tfrac{1}{2} w'Aw = \tfrac{1}{2} w'Aw - w'k + y + \theta$$

Inserting w and μ in the indirect utility function then yields the utility of the point (h_m, h_f, y).

From now on, we assume that the matrix A is positive definite. Equations (A.2) and (A.3) can then be rewritten as

(A.4) $$w - A^{-1}k = -\mu^* A^{-1}\beta$$

(A.5) $$\mu^* = \tfrac{1}{2}(w - A^{-1}k)'\, A(w - A^{-1}k) - \tfrac{1}{2} k'\, A^{-1}k + y + \theta.$$

Substituting (A.4) into (A.5) yields a quadratic equation in μ^*:

$$\text{(A.6)} \qquad \mu^* = \tfrac{1}{2}\, \mu^{*2}\ \beta'\ A^{-1}\beta - \tfrac{1}{2}\, k'\ A^{-1}k + y + \theta$$

and if μ^* is known (A.4) implies that w is given by

$$\text{(A.7)} \qquad w = A^{-1}(k-\mu^*\beta).$$

So, shadow wages w and corresponding non-labor income μ can be determined iff the quadratic equation (A.6) has a real solution. This is the case if

$$\text{(A.8)} \qquad D:= 1 + \beta'\ A^{-1}\beta\ (k'A^{-1}\ k-2(y+\theta)) > 0$$

To have theoretical significance a solution $(w,\bar{\mu})$ should satisfy the additional requirement that the cost function is concave in wages at (w,μ), i.e. the matrix

$$B:= \mu^*\ \beta\ \beta' - A$$

must be negative semi-definite. Since A is assumed to be positive definite, it is easy to see, that this condition is equivalent to

$$\text{(A.9)} \qquad \mu^*\ \beta'\ A^{-1}\beta < 1.$$

Clearly, if $\beta = 0$, (A.6) and (A.7) yield a unique solution (w,μ^*) that satisfies (A.9). If $\beta \neq 0$ and inequality (A.8) holds, (A.6) and (A.7) yield (at most) two solutions (w,μ^*) and only the smallest of them satisfies (A.9):

$$\text{(A.10)} \qquad \mu^* = (\beta'A^{-1}\beta)^{-1} - [(\beta'A^{-1}\beta)^{-2} + (\beta'A^{-1}\beta)^{-1}\ \{k'A^{-1}k-2(y+\theta)\}]^{\frac{1}{2}}$$

$$\text{(A.11)} \qquad w = A^{-1}(k-\mu^*\beta)$$

$$\text{(A.12)} \qquad \mu = y - w_m\ h_m - w_f\ h_f$$

The utility level is given by

$$U(h_m,h_f,y) = V(w_m,w_f,\mu) = \mu^* \exp(\beta'w). \qquad (A.13)$$

Concavity in (h_m,h_f,y)-space

Given that $A > 0$, (A.9) gives a necessary and sufficient condition for concavity of the cost function at a point in (w_m,w_f,μ)- space. In order to arrive at an equivalent condition in (h_m,h_f,y)-space one simply substitutes (A.10) into (A.9). It is easy to see that this yields (A.8). Thus the necessary and sufficient condition for a real solution for the shadow wages is simply the concavity condition in (h_m,h_f,y)-space. If one wants to impose concavity for a certain region S of (h_m,h_f,y)-space, this can be done conveniently by restricting the parameter θ,
such that

$$\theta < \min_S \{\tfrac{1}{2}(\beta'A^{-1}\beta)^{-1} + \tfrac{1}{2}\, k'\, A^{-1}k - y\}, \qquad (A.14)$$

assuming $\beta \neq 0$ (for β=0 guarantees concavity). To give one particularly simple example: (A.14) holds for all h_m and h_f and for all incomes y not exceeding a given y_0 if

$$\theta < \tfrac{1}{2}(\beta'A^{-1}\beta)^{-1} - y_0. \qquad (A.15)$$

Shadow wages if one spouse is rationed

The shadow wage and corresponding virtual non-labor income necessary to determine an individual's preferred hours conditional on the actual number of hours the partner works can be derived in a way that is similar to the derivation of (A.10)-(A.12), although a lack of symmetry makes vector notation less fruitful.

Corresponding to given h_j, w_i, w_j and μ $(i,j \in \{m,f\}; i \neq j)$ we must find $\bar{w}_j$, $\bar{\mu}_j$, and $\bar{h}_i$ such that:

(A.16) $h_j = \beta_j \bar{\mu}_j^* + \gamma_j \bar{w}_j + \alpha w_i + \delta_j$

(A.17) $\bar{\mu}_j + h_j \bar{w}_j = \mu + h_j w_j$

(A.18) $\bar{\mu}_j^* \equiv \bar{\mu}_j + \theta + \delta_i w_i + \delta_j \bar{w}_j + \frac{1}{2}(\gamma_i w_i^2 + \gamma_j \bar{w}_j^2) + \alpha w_i \bar{w}_j$

(A.19) $\bar{h}_i = \beta_i \bar{\mu}_j^* + \gamma_i w_i + \alpha \bar{w}_j + \delta_i$

Equations (A.16)-(A.18) imply

(A.20) $a_2 \bar{w}_j^2 + a_1 \bar{w}_j + a_0 = 0$

where

$$a_0 = -h_j + \beta_j(h_j w_j + \mu + \theta + \delta_i w_i + \tfrac{1}{2} \gamma_i w_i^2) + \alpha w_i + \delta_j$$

$$a_1 = \gamma_j + \beta_j(-h_j + \delta_j + \alpha w_i)$$

$$a_2 = \tfrac{1}{2} \beta_j \gamma_j.$$

If (A.20) admits no real solution for $\bar{w}_j$, no shadow wage can be determined and $\bar{h}_i$ cannot be found. Equation (A.20) is solvable iff

$$D := \beta_j^2(-h_j + \delta_j - \alpha w_i)^2 + \gamma_j^2 - 2\beta_j^2 \gamma_j(h_j w_j + \mu + \theta + \delta_i w_i + \tfrac{1}{2} \gamma_i w_i^2) > 0$$

Given a solution $\bar{w}_j$ of (A.20), one immediately finds values of $\bar{\mu}_j$, $\bar{\mu}_j^*$ and $\bar{h}_i$ by using (A.17), (A.18) and (A.19). The solution is feasible only if the cost function is concave in wages at $(w_i, \bar{w}_j, v(w_i, \bar{w}_j, \bar{\mu}_j))$. Assuming A is positive definite, this concavity condition can be written as in equation (A.9):

(A.21) $\bar{\mu}_j^* \beta' A^{-1} \beta < 1$

If $\beta_j = 0$, the unique solution of (A.20) is given by

(A.22) $\bar{w}_j = (h_j - \alpha w_i - \delta_j)/\gamma_j$

and substitution of (A.22) and (A.17) into (A.18) yields

(A.23) $$\bar{\mu}_j^* = \theta + \delta_i w_i + \tfrac{1}{2}\gamma_i w_i^2 - \tfrac{1}{2}(h_j - \alpha w_i - \delta_j)^2/\gamma_j$$

From (A.23) it is easy to check concavity condition (A.21). Note that global concavity is only guaranteed if $\beta_i = 0$ as well.

Now let us consider the case $\beta_j \neq 0$. If $D > 0$, (A.20) yields (at most) 2 real solutions for $\bar{w}_j$, given by

(A.24) $$\bar{w}_j = -\beta_j^{-1} + (h_j - \delta_j - \alpha w_i)/\gamma_i \pm (\beta_j\gamma_j)^{-1}\sqrt{D}.$$

The corresponding values of $\bar{\mu}_j^*$ are

(A.25) $$\bar{\mu}_j^* = \beta_j^{-2}\gamma_j \mp \beta_j^{-2}\sqrt{D}.$$

It is straightforward to show that the positive definiteness of A implies

(A.26) $$\beta_j^{-2}\gamma_j(\beta' A^{-1}\beta) > 1$$

From (A.21), (A.25) and (A.26) it follows that a feasible solution, satisfying concavity exists if

(A.27) $$\sqrt{D} > \gamma_j - \beta_j^2(\beta' A^{-1}\beta)^{-1}$$

In this case, the feasible solution is unique:

(A.28) $$\bar{w}_j = -\beta_j^{-1} + (h_j - \delta_j - \alpha w_i)/\gamma_j + (\beta_j\gamma_j)^{-1}\sqrt{D}.$$

and $\bar{\mu}_j$ and $\bar{h}_i$ can be found from (A.17)-(A.19)

Concavity in (h_j, w_i, w_j, μ)-space

As before, concavity for a certain range of values of h_j, w_i, w_j and μ, can be imposed by setting an upper bound to the parameter θ.

If $\beta_j = 0$, the upper bound immediately follows from (A.21) and (A.23). If $\beta_j \neq 0$, we can rewrite condition (A.27):

$$\text{(A.29)} \qquad \theta < (\mu+h_j w_j+\delta_i w_i+\delta_i w_i+\tfrac{1}{2}\gamma_i w_i^2) + \tfrac{1}{2}\gamma_j^{-1}(-h_j+\delta_j+\alpha w_i)^2 + \psi,$$

where

$$\psi = \tfrac{1}{2}\gamma_j^{-1}\beta_j^2(\beta' A^{-1}\beta)^{-2} + (\beta' A^{-1}\beta)^{-1}.$$

In estimating the model, we want to set an upperbound to θ (in terms of the other parameters in the model) which guarantees (A.29) to hold at all data-points. There are two things we have to take into account. Firstly, δ_i $(i=1,2)$ is allowed to vary across households, as are the observed marginal wage rate w_j and actual hours h_j. We write $\delta_i^{(k)}$, $w_j^{(k)}$ and $h_j^{(k)}$ $(k=1,\ldots,520)$ to indicate household k. Secondly, due to the progressive tax system, w_i is not a priori known and μ contains an unknown component (virtual non-labor income due to the tax system for spouse i). Now let $\mu_0 = 1064.41$ (the maximum sample-value of non-labor income including virtual income due to taxes) and let $w_m^0 = 4.94$, $w_m^1 = 20.41$, $w_f^0 = 2.84$ and $w_f^1 = 21.82$ (the sample minima and maxima of male and female marginal wage rates). A sufficient condition for (A.29) to hold at all data-points is now given by

$$\text{(A.30)} \qquad \theta < \max\{\mu_0 + h_j^{(k)}\, w_j^{(k)} + \delta_i^{(k)}\, w_i^g + \tfrac{1}{2}\gamma_i(w_i^g)^2,\ i,j \in \{m,f\},\ i\neq j,$$
$$k \in \{1,\ldots 520\},\ g \in \{0,1\}\} + \psi.$$

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