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RESEARCH MEMORANDUM

AXIOMATIZATIONS OF THE CONJUNCTIVE PERMISSION VALUE FOR GAMES WITH PERMISSION STRUCTURES

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Axiomatizations of the Conjunctive Permission Value for Games with Permission Structures*

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Abstract

A situation in which players can generate certain pay-offs by cooperating can be described by a *cooperative game with transferable utilities*. In this paper we assume that the players who are participating in such a game, are part of some *permission structure*. This means that there are players who need permission from one or more other players before they can act or cooperate with other players to generate some pay-off. It is clear that such a permission structure limits the possibilities of cooperation. We derive a modified game that takes account of these limited cooperation possibilities. We then give axiomatic characterizations of the *Shapley value* of this modified game.

1 Introduction

Players in a finite player set N who are participating in a cooperative game with transferable utilities – or simply a TU-game – are mostly assumed to be socially symmetric. They only differ with respect to their abilities to let coalitions obtain certain pay-offs as represented by a *characteristic function* $v: 2^N \rightarrow \mathbf{R}$ with $v(\emptyset) = 0$, where $\mathbf{R}$ denotes the set of real numbers. In the sequel the collection of all TU-games represented by their characteristic function is denoted by $\mathcal{G}^N$. Many authors have developed models that introduce social asymmetries between players in such TU-games. In, e.g., Aumann and Drèze (1974), Owen (1977), and Winter (1989), the players are assumed to be part of some *coalition structure* which influences the possibilities of cooperation and coalition formation. Another social difference that can be considered is the introduction of *limited communication* possibilities of the players. For this we refer to Myerson (1977), Kalai, Postlewaite and Roberts (1978), Owen (1986), and Borm, Owen and Tijs (1990), where the limited communication structure is represented by an undirected graph.

In this paper we consider another social feature of players which influences the possibilities of cooperation in a TU-game. We assume that there are players who have veto power over the actions undertaken by certain other players. Such a social organisation is represented by a mapping which assigns to every player the collection of players, whose actions he can veto, i.e., the players that require his permission for their actions. A situation in which players can obtain certain pay-offs by cooperation but in which some players need permission from their *superiors* before they can cooperate is described by what we call a *game with a permission structure*. These games are introduced in Gilles, Owen and van den Brink (1991). We assume that every player needs permission from *all* of his superiors. This interpretation of the permission structure is referred to as the *Conjunctive approach**.

In section 2 we describe games with permission structures. We derive a modified TU-game from such a game with a permission structure in which we take account of the limited possibilities of cooperation determined by the permission structure. Furthermore we introduce an *allocation rule* for these games with permission structures. An allocation rule is a mapping which assigns to every game with a permission structure a distribution of the pay-offs over the players. The allocation rule that we consider assigns to every game with permission structure the Shapley value (Shapley (1953)) of the modified game. This allocation rule is referred to as the *Conjunctive permission value* and is shown to be an extension of the Shapley value to the collection of games with permission structures.

In section 3 we give an axiomatization of this Conjunctive permission value. In this axiomatization an important role is played by the class of *monotone* TU-games. A TU-game v is called monotone if for all $E \subset F \subset N$ it holds that $v(E) \leq v(F)$. The collection of all monotone TU-games is denoted by $\mathcal{G}_M^N$.

We remark that there can occur 'domination cycles' in permission structures. Such a 'domination cycle' is a group of players that can be ordered such that each player, except the first one, needs permission from the previous player, while the first player needs permission from the last one. In section 4 we concentrate on the subclass of permission structures in which these 'domination cycles' do not occur.

2 Games with a permission structure

In this paper we assume that the cooperation possibilities of a finite group of players are limited because some players might need permission from one or more other players. Formally such a *permission structure* is described as follows.

Definition 2.1 *Let N be a finite player set. A mapping $S\colon N \to 2^N$ is a* **permission structure** *on N if it is asymmetric on N, i.e., for every $i, j \in N$ it holds that*

if $j \in S(i)$ then $i \notin S(j)$.

The collection of all permission structures on N is denoted by $\mathcal{S}^N$.

*In Gilles and Owen (1991) it is assumed that every player needs permission from *at least one* of his direct superiors. This is referred to as the *Disjunctive approach*.

The players $j \in S(i)$ are called the *successors* of $i \in N$ in $S \in \mathcal{S}^N$. (Note that asymmetry of S implies that $i \notin S(i)$ for all $i \in N$.) A permission structure $S \in \mathcal{S}^N$ can be represented alternatively by the pair (N, R_S) where R_S is the ***binary relation*** given by

$$R_S := \{(i,j) \in N \times N \mid j \in S(i)\}.$$

For each permission structure $S \in \mathcal{S}^N$ we now define the mapping $\widehat{S}: N \to 2^N$ as follows:

$$\widehat{S}(i) := \{j \in N \mid (i,j) \in tr(R_S)\},$$

where $tr(R_S)$ indicates the *transitive closure* of R_S†. The players in $\widehat{S}(i)$, $i \in N$, are called the *subordinates* of i in S and the players in $\widehat{S}^{-1}(i) := \{j \in N \mid i \in \widehat{S}(j)\}$ are called the *superiors* of i in S. Furthermore we define for every $E \subset N$, $S(E) := \bigcup_{i \in E} S(i)$ and $\widehat{S}(E) := \bigcup_{i \in E} \widehat{S}(i)$. Although a player cannot be a successor of himself he can be a subordinate of himself. If we want to exclude these kind of 'domination cycles' then we must add another condition with respect to the mapping S.

Definition 2.2 *Let N be a finite player set. A permission structure $S \in \mathcal{S}^N$ is* **acyclic** *on N if for every $i \in N$ it holds that $i \notin \widehat{S}(i)$.*
The collection of all acyclic permission structures on N is denoted by $\mathcal{S}_A^N$.

In the Conjunctive approach with respect to permission structures we assume that a player needs permission from *all* his superiors before he can act. This means that a coalition $E \subset N$ is *formable* if and only if all superiors of the players in E are also part of E, i.e., $\widehat{S}^{-1}(E) \subset E$.

Definition 2.3 *Let $S \in \mathcal{S}^N$. The* **sovereign part** *of $E \subset N$ according to S is the coalition given by*

$$\sigma(E) := E \setminus \widehat{S}(N \setminus E).$$

The **authorizing set** *of $E \subset N$ according to S is the coalition given by*

$$\alpha(E) := E \cup \widehat{S}^{-1}(E).$$

†The *transitive closure* $tr(R)$ of a binary relation $R \subset N \times N$ is given by: $(i,j) \in tr(R)$ if and only if there exists a sequence $\{h_k\}_{1 \le k \le m}$ such that $h_1 = i$, $(h_k, h_{k+1}) \in R$ for all $1 \le k \le m-1$ and $h_m = j$.

The sovereign part of E consists of those players in E whose superiors are all part of E. This means that $\sigma(E)$ is the largest subcoalition of E that is formable. The authorizing set of E consists of E together with all its superiors. Thus $\alpha(E)$ is the smallest formable coalition that contains E. Using the notion of sovereign part we can transform the game $v \in \mathcal{G}^N$ so that we take account of the permission structure S in the following way.

Definition 2.4 *Let $v \in \mathcal{G}^N$ and $S \in \mathcal{S}^N$. The* **Conjunctive restriction** *of v on S is the game $\mathcal{R}_S(v) \in \mathcal{G}^N$ that is given by*

$$\mathcal{R}_S(v)(E) := v(\sigma(E)) \quad \textit{for all } E \subset N.$$

For properties of the mapping $\mathcal{R}_S: \mathcal{G}^N \to \mathcal{G}^N$ we refer to Gilles, Owen and van den Brink (1991).

In the sequel a triple (N, v, S) with $v \in \mathcal{G}^N$ and $S \in \mathcal{S}^N$ will be indicated as a *game with a permission structure*. An *allocation rule* for games with a permission structure is a mapping that assigns to every game with a permission structure (N, v, S) a distribution of the pay-offs that are attainable in the restricted game $\mathcal{R}_S(v)$. In the following sections we concentrate on the allocation rule $\varphi: N \times \mathcal{G}^N \times \mathcal{S}^N \to \mathbf{R}$ that is given by

$$\varphi(i, v, S) := Sh_i(\mathcal{R}_S(v)) \text{ for all } i \in N, \ v \in \mathcal{G}^N \text{ and } S \in \mathcal{S}^N,$$

where for every $i \in N$, $Sh_i(v)$ is the Shapley value of player i in game $v \in \mathcal{G}^N$, i.e,

$$Sh_i(v) = \sum_{E \ni i} \frac{(\#N - \#E)!(\#E-1)!}{(\#N)!} \, (v(E) - v(E \setminus \{i\})).$$

The allocation rule φ is referred to as the *Conjunctive permission value*. If we take the *trivial* mapping $S_\emptyset \in \mathcal{S}^N$ which is given by $S_\emptyset(i) = \emptyset$ for all $i \in N$, then it is easy to see that the restriction $\mathcal{R}_{S_\emptyset}(v)$ is equal to the original game v. Thus the Conjunctive permission value φ is a generalization of the Shapley value for TU-games. For computing the Conjunctive permission value of a game with permission structure we derive the following formula.

Proposition 2.5 *Let $v \in \mathcal{G}^N$, $S \in \mathcal{S}^N$ and for every $i \in N$*

$$\Gamma_i := \{E \subset N \mid E \cap [\hat{S}(i) \cup \{i\}] \neq \emptyset\}.$$

Then

$$\varphi(i,v,S)=\sum_{E\in\Gamma_i}\frac{\Delta_v(E)}{\#\alpha(E)},$$

where the dividends $\Delta_v(E)$ *are given by (see Harsanyi (1959))*

$$\Delta_v(E):=\sum_{F\subset E}(-1)^{\#E-\#F}v(F).$$

PROOF

Let $v \in \mathcal{G}^N$ and $S \in \mathcal{S}^N$. In Gilles, Owen and van den Brink (1991) it is proved that the restriction $\mathcal{R}_S(v)$ can be written as

$$\mathcal{R}_S(v)=\sum_{\substack{F\subset N\\ F=\alpha(F)}}\left\{\sum_{\substack{E\subset N\\ \alpha(E)=F}}\Delta_v(E)\right\}\cdot u_F,$$

where u_F is the unanimity game of $F \subset N$, i.e.,

$$u_F(E)=\begin{cases}1 & \text{if } E\supset F\\ 0 & \text{else}\end{cases}$$

From the additivity property of the Shapley value it then follows that

$$Sh_i(\mathcal{R}_S(v))=\sum_{\substack{F\ni i\\ F=\alpha(F)}}\ \sum_{\substack{E\subset N\\ \alpha(E)=F}}\frac{\Delta_v(E)}{\#F}.$$

Since

(i) $F = \alpha(F)$ if and only if there is an $E \subset N$ such that $\alpha(E) = F$, and

(ii) $i \in \alpha(E)$ if and only if $E \in \Gamma_i$

it holds that

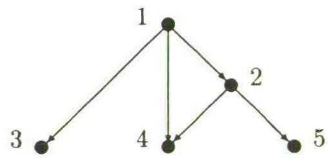
$$\varphi(i,v,S)=Sh_i(\mathcal{R}_S(v))=\sum_{E\in\Gamma_i}\frac{\Delta_v(E)}{\#\alpha(E)}.$$

Q.E.D.

Example 2.6 Consider the permission structure $S: N \rightarrow 2^N$ on $N = \{1,\ldots,5\}$ which is given by:

$$S(1) = \{2,3,4\},\ S(2) = \{4,5\},\ S(3) = S(4) = S(5) = \emptyset.$$

This acyclic permission structure can be represented by the following directed graph



Consider the coalition $\{1,3,4\}$. The sovereign part and authorizing set of this coalition respectively are given by $\sigma(\{1,3,4\}) = \{1,3\}$ and $\alpha(\{1,3,4\}) = \{1,2,3,4\}$.
Let $v \in \mathcal{G}^N$ be the additive game given by: $v(E) = \#E$ for all $E \subset N$.
It is easy to see that the Shapley value of this game is given by $Sh_i(v) = 1$ for all $i \in N$.
The restriction of v on S is the game $\mathcal{R}_S(v)$ given by

$$\mathcal{R}_S(v)(E) = v(\sigma(E)) = \#\sigma(E).$$

So, for example, $\mathcal{R}_S(v)(\{1,3,4\}) = v(\{1,3\}) = 2$.
The Conjunctive permission value of the game with permission structure (N, v, S) is given by

$$\varphi(\cdot, v, S) = \frac{1}{6}(16, 7, 3, 2, 2).$$

Comparing φ with the Shapley value of game v we see that a substantial shift in the distribution of the pay-offs occurs. Especially the 'topman', player 1, gets a lot more because of his strong position in the permission structure S.

We conclude this section by introducing some concepts that will be used in the axiomatizations that are discussed in the following sections.

Definition 2.7 *The allocation rule* $f: N \times \mathcal{G}^N \times \mathcal{S}^N \to \mathbf{R}$ *is* **efficient** *if for every* $v \in \mathcal{G}^N$ *and every* $S \in \mathcal{S}^N$ *it holds that*

$$\sum_{i \in N} f(i, v, S) = v(N).$$

The allocation rule $f\colon N \times \mathcal{G}^N \times \mathcal{S}^N \to \mathbf{R}$ *is* **additive** *if for every* $i \in N$, $v, w \in \mathcal{G}^N$ *and* $S \in \mathcal{S}^N$ *it holds that*

$$f(i,v,S) + f(i,w,S) = f(i,v+w,S).$$

Finally we indicate a special class of players that play an important role in all axiomatizations that we give in the following sections.

Definition 2.8 *Let* $v \in \mathcal{G}^N$. *A player* $j \in N$ *is* **necessary** *in* v *if for every* $E \subset N \setminus \{j\}$ *it holds that* $v(E) = 0$.

The next result shows that a necessary player in a monotone game is assigned the highest pay-off in the Shapley value of that monotone game.

Proposition 2.9 *Let* $v \in \mathcal{G}_M^N$ *and let* $j \in N$ *be a necessary player in* v. *Then it holds that*

$$Sh_j(v) \geq Sh_i(v) \quad \text{for all } i \in N.$$

PROOF

Let $j \in N$ be a necessary player in the monotone game v. Then for every $i \in N$ the following properties hold:

(i) Let $E \supset \{i,j\}$. Since $v(E \setminus \{j\}) = 0$ by j being a necessary player in v and $v(E \setminus \{i\}) \geq v(\emptyset) = 0$ by monotonicity of v it holds that $v(E) - v(E \setminus \{j\}) = v(E) \geq v(E) - v(E \setminus \{i\})$.

(ii) If $i \in E$ and $j \notin E$ then $v(E) - v(E \setminus \{i\}) = 0$.

(iii) If $i \notin E$ and $j \in E$ then $v(E) - v(E \setminus \{j\}) = v(E) \geq 0$.

Using the fact that $g(E) := \frac{(\#N-\#E)!(\#E-1)!}{(\#N)!} > 0$ for all $E \subset N$ we can deduce that:

$$\begin{aligned}
Sh_j(v) &= \sum_{E \supset \{i,j\}} g(E)(v(E) - v(E \setminus \{j\})) + \sum_{\substack{E \not\ni i \\ E \ni j}} g(E)(v(E) - v(E \setminus \{j\})) \\
&\geq \sum_{E \supset \{i,j\}} g(E)(v(E) - v(E \setminus \{j\})) \\
&\geq \sum_{E \supset \{i,j\}} g(E)(v(E) - v(E \setminus \{i\})) \\
&= \sum_{E \supset \{i,j\}} g(E)(v(E) - v(E \setminus \{i\})) + \sum_{\substack{E \ni i \\ E \not\ni j}} g(E)(v(E) - v(E \setminus \{i\})) = Sh_i(v).
\end{aligned}$$

Q.E.D.

3 An axiomatization of the Conjunctive permission value

In this section we give a set of axioms that uniquely determine the Conjunctive permission value for games with permission structures. First we introduce a special type of player in games with a permission structure.

Definition 3.1 *Let $v \in \mathcal{G}^N$ and $S \in \mathcal{S}^N$. Player $j \in N$ is* **weakly inessential** *in (N, v, S) if every $h \in \widehat{S}(j) \cup \{j\}$ is a dummy player in game v, i.e., for every $h \in \widehat{S}(j) \cup \{j\}$ it holds that*

$$v(E) = v(E \setminus \{h\}) \text{ for all } E \subset N.$$

A player thus is weakly inessential if he himself as well as all his subordinates have no individual abilities because they are dummy players in the original game v. The following lemma will be used in Theorem 3.3 as well as in Theorem 4.4.

Lemma 3.2 *Let $S \in \mathcal{S}^N$ and let $\sigma(E)$ be the sovereign part of $E \subset N$ according to S. Then*

$$\sigma(E) \setminus [\widehat{S}(j) \cup \{j\}] = \sigma(E \setminus \{j\}) \text{ for all } j \in N.$$

Proof

Let $S \in \mathcal{S}^N$ and $j \in N$. Then

$$\begin{aligned} \sigma(E) \setminus [\widehat{S}(j) \cup \{j\}] &= [E \setminus \widehat{S}(N \setminus E)] \setminus [\widehat{S}(j) \cup \{j\}] \\ &= [E \setminus \{j\}] \setminus [\widehat{S}(N \setminus E) \cup \widehat{S}(j)] \\ &= [E \setminus \{j\}] \setminus \widehat{S}(N \setminus [E \setminus \{j\}]) = \sigma(E \setminus \{j\}) \end{aligned}$$

Q.E.D.

Theorem 3.3 *The allocation rule $f\colon N \times \mathcal{G}^N \times \mathcal{S}^N \to \mathbf{R}$ is equal to the Conjunctive permission value φ if and only if it is efficient, additive, and satisfies the following three conditions:*

1. *If* $v \in \mathcal{G}^N$, $S \in \mathcal{S}^N$ *and* $j \in N$ *is a weakly inessential player in* (N, v, S), *then*

   (i) $f(j, v, S) = 0$;

   (ii) *for every player* $i \in N$ *it holds that*

   $$f(i, v, S) = f(i, v, S_{-j}),$$

   where $S_{-j} \in \mathcal{S}^N$ *is given by:* $S_{-j}(i) := S(i) \setminus \{j\}$ *for all* $i \in N$

2. *If* $S \in \mathcal{S}^N$ *and player* $j \in N$ *is such that* $S(j) \neq \emptyset$, *then for every* $v \in \mathcal{G}_M^N$ *it holds that*

   $$f(j, v, S) \geq \max_{i \in S(j)} f(i, v, S)$$

3. *If* $v \in \mathcal{G}_M^N$ *and* $j \in N$ *is a necessary player in* v, *then for every* $S \in \mathcal{S}^N$ *it holds that*

   $$f(j, v, S) \geq f(i, v, S) \quad \textit{for all } i \in N.$$

PROOF

We first prove that φ is efficient, additive and satisfies the three conditions. Let S be a given permission structure.

- Let $v \in \mathcal{G}^N$. It is easy to see that $N = \sigma(N)$. Thus $\mathcal{R}_S(v)(N) = v(\sigma(N)) = v(N)$. From the efficiency property of the Shapley value for TU-games it then follows that $\sum_{i \in N} \varphi(i, v, S) = \sum_{i \in N} Sh_i(\mathcal{R}_S(v)) = v(N)$. Thus φ is efficient.

- Let $v, w \in \mathcal{G}^N$. Then for all $E \subset N$ it holds that

  $$\mathcal{R}_S(v)(E) + \mathcal{R}_S(w)(E) = v(\sigma(E)) + w(\sigma(E)) =$$

  $$= (v + w)(\sigma(E)) = \mathcal{R}_S(v + w)(E).$$

  Additivity of φ then follows from additivity of the Shapley value for TU-games.

- Let $v \in \mathcal{G}^N$ and let $j \in N$ be a weakly inessential player in (N, v, S). With Lemma 3.2 and the fact that every $h \in \widehat{S}(j) \cup \{j\}$ is a dummy player in game v it then follows that

$$\begin{aligned}\mathcal{R}_S(v)(E) &= v(\sigma(E)) = v(\sigma(E) \setminus [\widehat{S}(j) \cup \{j\}]) \\ &= v(\sigma(E \setminus \{j\})) = \mathcal{R}_S(v)(E \setminus \{j\}) \text{ for all } E \subset N.\end{aligned}$$

  This implies that

  (i) $\varphi(j, v, S) = Sh_j(\mathcal{R}_S(v)) = 0$;

  (ii) $\mathcal{R}_S(v)(E) = \mathcal{R}_{S_{-j}}(v)(E)$ for all $E \subset N$, and thus $\varphi(i, v, S) = \varphi(i, v, S_{-j})$ for all $i \in N$.

  Thus φ satisfies condition 1.

- Let $v \in \mathcal{G}_M^N$, $S \in \mathcal{S}^N$, $j \in N$ and $i \in S(j)$. Then we can state that

  (i) Let $E \supset \{i, j\}$. Since $\sigma(E \setminus \{j\}) \subset \sigma(E \setminus \{i\})$ and v is monotone it follows that

$$\begin{aligned}\mathcal{R}_S(v)(E) - \mathcal{R}_S(v)(E \setminus \{j\}) &= v(\sigma(E)) - v(\sigma(E \setminus \{j\})) \\ &\geq v(\sigma(E) - v(\sigma(E \setminus \{i\})) \\ &= \mathcal{R}_S(v)(E) - \mathcal{R}_S(v)(E \setminus \{i\}),\end{aligned}$$

  (ii) If $i \in E$ and $j \notin E$, then $\mathcal{R}_S(v)(E) - \mathcal{R}_S(v)(E \setminus \{i\}) = 0$.

  (iii) If $i \notin E$ and $j \in E$ then $\mathcal{R}_S(v)(E) - \mathcal{R}_S(v)(E \setminus \{j\}) \geq 0$ by $\mathcal{R}_S(v)$ being monotone.

  With this and taking $w = \mathcal{R}_S(v) \in \mathcal{G}_M^N$ it follows that

$$\begin{aligned}Sh_j(w) &= \sum_{E \supset \{i,j\}} g(E)(w(E) - w(E \setminus \{j\})) + \sum_{\substack{E \not\ni i \\ E \ni j}} g(E)(w(E) - w(E \setminus \{j\})) \\ &\geq \sum_{E \supset \{i,j\}} g(E)(w(E) - w(E \setminus \{i\})) + \sum_{\substack{E \ni i \\ E \not\ni j}} g(E)(w(E) - w(E \setminus \{i\})) \\ &= Sh_i(w).\end{aligned}$$

Thus φ satisfies condition 2.

- Let $v \in \mathcal{G}_M^N$ and let $j \in N$ be a necessary player in v. Because $\sigma(E) \subset E$ it holds that $\mathcal{R}_S(v)(E) = v(\sigma(E)) = 0$ for all $E \subset N \setminus \{j\}$. Thus j is a necessary player in $\mathcal{R}_S(v)$. Since $\mathcal{R}_S(v) \in \mathcal{G}_M^N$ as shown in Gilles, Owen and van den Brink (1991) it follows from Proposition 2.9 that $\varphi(j, v, S) = Sh_j(\mathcal{R}_S(v)) \geq Sh_i(\mathcal{R}_S(v)) = \varphi(i, v, S)$ for all $i \in N$.

Now suppose that $f: N \times \mathcal{G}^N \times \mathcal{S}^N \to \mathbf{R}$ is efficient, additive and satisfies the three conditions stated in the theorem.
Consider the game $w_T = c_T u_T$ where u_T is the unanimity game of $T \subset N$ and $c_T \geq 0$ is some non-negative constant, i.e.,

$$w_T(E) = \begin{cases} c_T & \text{if } E \supset T \\ 0 & \text{else} \end{cases}$$

and let S be a permission structure on N. Clearly w_T is a monotone game. Then

$$\mathcal{R}_S(w_T)(E) = \begin{cases} c_T & \text{if } E \supset \alpha(T) \\ 0 & \text{else} \end{cases}$$

Thus

$$\varphi(i, w_T, S) = \begin{cases} \frac{c_T}{\#\alpha(T)} & \text{if } i \in \alpha(T) \\ 0 & \text{else} \end{cases}$$

We now show that f must be equal to φ. Consider the permission structure $S^*: N \to 2^N$ that is given by

$$S^*(i) := S(i) \cap \alpha(T) \quad \text{for all } i \in N.$$

Clearly $S^*(i) \subset S(i)$ for all $i \in N$. Suppose that $j \in S(i) \setminus S^*(i)$ for some $i \in N$. Then $j \notin \alpha(T)$. This implies that

(i) $j \notin T$ and thus j is a dummy player in the game w_T;

(ii) $\widehat{S}(j) \cap T = \emptyset$. This means that for every $h \in \widehat{S}(j)$ it holds that $h \notin T$ and thus h is a dummy player in w_T.

Each player $j \in N$ for which there exists an $i \in N$ such that $j \in S(i) \setminus S^*(i)$ thus is weakly inessential in (N, w_T, S). According to condition 1 (ii) it then holds that

$$f(i, w_T, S^*) = f(i, w_T, S) \text{ for all } i \in N. \tag{1}$$

Next we determine $f(i, w_T, S^*)$. Therefore we distinguish exactly three cases with respect to player $j \in N$.

1. Suppose that $j \in T$.

   Then j is necessary in the monotone game w_T and thus, according to condition 3, $f(j, w_T, S^*) \geq f(i, w_T, S^*)$ for all $i \in N$. Thus there exists a constant $c \geq 0$ such that

$$\begin{aligned} f(j, w_T, S^*) = c \qquad & \text{for all } j \in T \\ \text{and} \qquad & \\ f(i, w_T, S^*) \leq c \qquad & \text{for all } i \in N \setminus T \end{aligned} \tag{2}$$

2. Suppose that $j \in \alpha(T) \setminus T$.

   Then $S^*(j) \neq \emptyset$. By definition of $\widehat{S}^*$ it holds that $\max_{i \in S^*(j)} f(i, v, S^*) = \max_{i \in \widehat{S}^*(j)} f(i, v, S^*)$. Because w_T is monotone it follows with condition 2 that

$$f(j, w_T, S^*) \geq \max_{i \in \widehat{S}^*(j)} f(i, w_T, S^*).$$

   From (2) and the fact that $\widehat{S}^*(j) \cap T \neq \emptyset$ it then follows that $f(j, u_T, S^*) = c$ for all $j \in \alpha(T) \setminus T$.

3. Suppose that $j \in N \setminus \alpha(T)$.

   Then j is a dummy player in w_T and $S^*(j) = \emptyset$. Thus j is weakly inessential in (N, w_T, S^*) and thus it follows from condition 1 (i) that $f(j, w_T, S^*) = 0$ for all $j \in N \setminus \alpha(T)$.

With (1) it then follows that

$$f(i, w_T, S) = f(i, w_T, S^*) = \begin{cases} c & \text{for all } i \in \alpha(T) \\ 0 & \text{else} \end{cases}$$

Efficiency implies that $c = \frac{c_T}{\#\alpha(T)}$ and thus $f(i, w_T, S) = \varphi(i, w_T, S)$ for all $i \in N$.

Now let $v \in \mathcal{G}^N$ be an arbitrary game. As is known, v can be expressed as a weighted sum of unanimity games:

$$v = \sum_{T \subset N} c_T u_T. \tag{3}$$

If $c_T \geq 0$ then $c_T u_T$ is monotone and as we proved above it holds that $f(i, c_T u_T, S) = \varphi(i, c_T u_T, S)$ for all $i \in N$.
If $c_T < 0$ then $c_T u_T$ is not monotone. In that case, by defining $c_T^+ := -c_T > 0$, we have that $c_T u_T = v_0 - c_T^+ u_T$, where v_0 is the null-game, i.e., $v_0(E) = 0$ for all $E \subset N$. Both v_0 and $c_T^+ u_T$ are monotone games and thus as proved above

$$f(i, v_0, S) = \varphi(i, v_0, S) = 0 \text{ and } f(i, c_T^+ u_T, S) = \varphi(i, c_T^+ u_T, S) \text{ for all } i \in N.$$

From additivity of f, $\mathcal{R}_S$, and the Shapley value it then follows that for every $i \in N$

$$\begin{aligned} f(i, c_T u_T, S) &= f(i, v_0, S) - f(i, c_T^+ u_T, S) = -\varphi(i, c_T^+ u_T, S) \\ &= -Sh_i(\mathcal{R}_S(c_T^+ u_T)) = Sh_i(\mathcal{R}_S(c_T u_T)) = \varphi(i, c_T u_T, S). \end{aligned}$$

With (3) and additivity of f and the Shapley value it then follows that

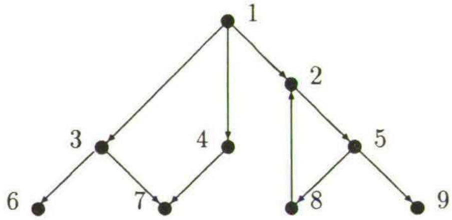
$$f(i, v, S) = \sum_{T \subset N} f(i, c_T u_T, S) = \sum_{T \subset N} \varphi(i, c_T u_T, S) = \varphi(i, v, S) \text{ for all } i \in N.$$

Q.E.D.

Example 3.4 Consider the game with permission structure (N, v, S), where $N = \{1, \ldots, 9\}$, $v = u_{\{6,7\}}$ being the monotone unanimity game of $\{6, 7\}$ and $S: N \to 2^N$ is given by:

$$S(1) = \{2, 3, 4\},\ S(2) = \{5\},\ S(3) = \{6, 7\},\ S(4) = \{7\},$$

$$S(5) = \{8, 9\},\ S(6) = S(7) = \emptyset,\ S(8) = \{2\},\ S(9) = \emptyset.$$

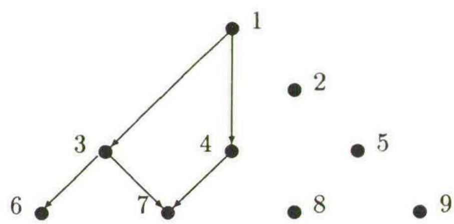


It is easy to see that

$$\varphi(i,v,S) = \begin{cases} \frac{1}{5} & \text{for all } i \in \{1,3,4,6,7\} = \alpha(\{6,7\}) \\ 0 & \text{else} \end{cases}$$

Let $f: N \times \mathcal{G}^N \times \mathcal{S}^N \to \mathbf{R}$ be efficient, additive and satisfy the three conditions stated in Theorem 3.3.

Every $h \in \widehat{S}(i) \cup \{i\}$, $i \in \{2,5,8,9\}$ is a dummy player in the game v and thus every $i \in \{2,5,8,9\}$ is weakly inessential in (N,v,S). Therefore, according to condition 1, $f(i,v,S) = 0$ for all $i \in \{2,5,8,9\}$ and the relations $(1,2)$, $(2,5)$, $(5,8)$, $(5,9)$ and $(8,2)$ can be deleted without influencing the distribution of the pay-offs. Thus $f(i,v,S) = f(i,v,S^*)$ for all $i \in N$ where S^* is represented by



Players 6 and 7 are necessary in the game v. From condition 3 it then follows that there is a constant $c \geq 0$ such that

$$f(i,v,S^*) = c \text{ for } i \in \{6,7\}$$

and

$$f(i,v,S^*) \leq c \text{ for } i \in \{1,3,4\}.$$

By condition 2 it holds that $f(i,v,S^*) \geq c$ for $i \in \{3,4\}$. From conditions 2 and 3 it thus follows that $f(i,v,S^*) = c$, for $i \in \{3,4\}$.

By the same conditions it then holds that $f(1,v,S^*) = c$.

With condition 1 (ii) and efficiency it then follows that

$$f(i,v,S) = f(i,v,S^*) = \begin{cases} \frac{1}{5} & \text{for all } i \in \{1,3,4,6,7\} \\ 0 & \text{else} \end{cases}$$

$$= \varphi(i,v,S).$$

4 Games with acyclic permission structures

In this section we focus on games with acyclic permission structures. The conditions stated in Theorem 3.3 restricted to the class $\mathcal{S}_A^N$ axiomatize the Conjunctive permission value on $\mathcal{S}_A^N$. We will see that we still can axiomatize the Conjunctive permission value for the class of acyclic permission structures by weakening the condition with respect to inessential players.

Definition 4.1 *Let $v \in \mathcal{G}^N$ and $S \in \mathcal{S}^N$. Player $j \in N$ is* **strongly inessential** *in (N, v, S) if j is a dummy player in the game v and $S(j) = \emptyset$.*

Note that in determining whether a player is weakly inessential in a game with permission structure we have to look at the game and the permission structure simultaneously. In determining whether a player is strongly inessential we can look at the game and the permission structure separately. Also we remark that if a player is strongly inessential then he also is weakly inessential.

Theorem 4.2 *The allocation rule $f\colon N \times \mathcal{G}^N \times \mathcal{S}_A^N \to \mathbf{R}$ is equal to the Conjunctive permission value φ restricted to the class $N \times \mathcal{G}^N \times \mathcal{S}_A^N$ if and only if it is efficient, additive and satisfies the following three conditions*

1. *If $v \in \mathcal{G}^N$, $S \in \mathcal{S}_A^N$ and $j \in N$ is a strongly inessential player in (N, v, S), then*

   (i) $f(j, v, S) = 0$;

   (ii) *for every player $i \in N$ it holds that*

   $$f(i, v, S) = f(i, v, S_{-j}).$$

2. *If $S \in \mathcal{S}_A^N$ and player $j \in N$ is such that $S(j) \neq \emptyset$, then for every $v \in \mathcal{G}_M^N$ it holds that*

   $$f(j, v, S) \geq \max_{i \in S(j)} f(i, v, S)$$

3. *If $v \in \mathcal{G}_M^N$ and $j \in N$ is a necessary player in v, then for every $S \in \mathcal{S}_A^N$ it holds that*

   $$f(j, v, S) \geq f(i, v, S) \quad \text{for all } i \in N.$$

PROOF

That φ is efficient, additive and satisfies conditions 2 and 3 follows directly from Theorem 3.3.

Let $v \in \mathcal{G}^N$, $S \in \mathcal{S}_A^N$ and let $j \in N$ be a strongly inessential player in (N, v, S). Because $S(j) = \emptyset$ it holds that $\sigma(E \setminus \{j\}) = \sigma(E) \setminus \{j\}$ for all $E \subset N$. But then $\mathcal{R}_S(v)(E) = v(\sigma(E)) = v(\sigma(E) \setminus \{j\}) = v(\sigma(E \setminus \{j\}) = \mathcal{R}_S(v)(E \setminus \{j\})$ for all $E \subset N$. This implies that

(i) $\varphi(j, v, S) = Sh_j(\mathcal{R}_S(v)) = 0$;

(ii) $\mathcal{R}_S(v)(E) = \mathcal{R}_{S_{-j}}(v)(E)$ for all $E \subset N$, and thus $\varphi(i, v, S) = \varphi(i, v, S_{-j})$ for all $i \in N$.

Thus φ satisfies condition 1.

Now suppose that $f: N \times \mathcal{G}^N \times \mathcal{S}_A^N \to \mathbf{R}$ is efficient, additive and satisfies the three conditions stated in the theorem.

Like in the proof of Theorem 3.3 consider the games $w_T = c_T u_T$, $T \subset N$. Again consider the permission structure $S^*: N \to 2^N$ which is given by

$$S^*(i) := S(i) \cap \alpha(T) \text{ for all } i \in N.$$

First we show that $f(i, w_T, S^*) = f(i, w_T, S)$ for all $i \in N$.

We claim that if $S(N) \setminus \alpha(T) \neq \emptyset$ then $S(N) \setminus \alpha(T)$ contains at least one player who is strongly inessential in (N, w_T, S). Suppose to the contrary that $S(N) \setminus \alpha(T)$ does not contain a strongly inessential player. From the fact that each player in $S(N) \setminus \alpha(T)$ is a dummy player in the game w_T and by assumption is not strongly inessential, it follows that $S(j) \neq \emptyset$ for all $j \in S(N) \setminus \alpha(T)$. Furthermore $j \notin \alpha(T)$ implies that $S(j) \cap \alpha(T) = \emptyset$. This means that for every $j \in S(N) \setminus \alpha(T)$ there is an $h \in S(N) \setminus \alpha(T)$ such that $h \in S(j)$. Thus there exists an infinite sequence of players $(h_k)_{k \in \mathbf{N}}$ such that $h_1 \in S(N) \setminus \alpha(T)$ and $h_{k+1} \in S(h_k) \setminus \alpha(T)$ for all $k \in \mathbf{N}$. Acyclicity of S implies that all h_k's in this sequence are distinct. But then $S(N) \setminus \alpha(T)$ must consist of an infinite number of players which contradicts the finiteness of N. Thus $S(N) \setminus \alpha(T)$ contains at least one strongly inessential player.

Let $\mathrm{SI}(N, w_T, S)$ denote the collection of all strongly inessential players in (N, w_T, S). Now we recursively define the following sequence of permission structures $S^k: N \to 2^N$, $k \in \mathbf{N}$

$$S^1(i) := S(i)$$
$$\text{for all } i \in N$$
$$S^k(i) := S(i) \setminus \mathrm{SI}(N, w_T, S^{k-1})$$

From condition 1 (ii) it follows that for every $k \in \mathbf{N}$ and every $i \in N$ there is some constant $c_i \in \mathbf{R}$ such that $f(i, w_T, S^k) = c_i$. From the discussion above and finiteness of N it follows that there is some $M < \infty$ such that $S^k(N) \setminus \alpha(T) = \emptyset$ for every $k \geq M$. Furthermore, if $i \in \alpha(T)$ then $i \notin \mathrm{SI}(N, w_T, S^k)$ because $S^k(i) \neq \emptyset$ for every $i \in \alpha(T)$ and every $k \in \mathbf{N}$. Therefore $S^k = S^*$ for all $k \geq M$ and thus

$$f(i, w_T, S) = f(i, w_T, S^*) \text{ for all } i \in N. \tag{4}$$

Similarly as in the proof of Theorem 3.3 it follows from conditions 2 and 3 that there is some constant $c \geq 0$ such that $f(i, w_T, S^*) = c$ for all $i \in \alpha(T)$.
If $i \in N \setminus \alpha(T)$ then i is strongly inessential in (N, w_T, S^*) and thus by condition 1 (i) $f(i, w_T, S^*) = 0$ for all $i \in N \setminus \alpha(T)$.
With (4) and efficiency it then follows that

$$f(i, w_T, S) = f(i, w_T, S^*) = \begin{cases} \frac{c_T}{\#\alpha(T)} & \text{for all } i \in \alpha(T) \\ 0 & \text{else.} \end{cases}$$

Hence, $f(i, w_T, S) = \varphi(i, w_T, S)$ for all $i \in N$.

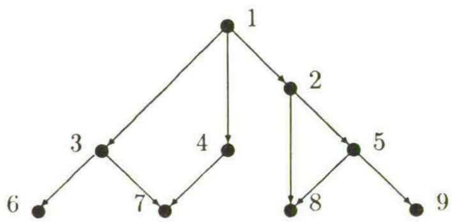
If $v \in \mathcal{G}^N$ then we can prove that $f(i, v, S) = \varphi(i, v, S)$ for all $i \in N$ using additivity in the same way as in the proof of Theorem 3.3.

Q.E.D.

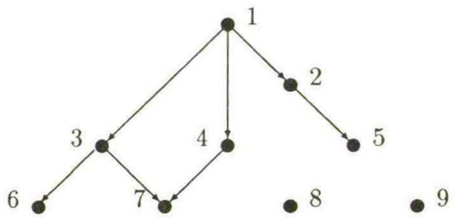
Example 4.3 Consider the game with acyclic permission structure (N, v, S) where $N = \{1, \ldots, 9\}$, $v = u_{\{6,7\}}$ and $S: N \to 2^N$ is given by

$$S(1) = \{2,3,4\},\ S(2) = \{5,8\},\ S(3) = \{6,7\},\ S(4) = \{7\},$$

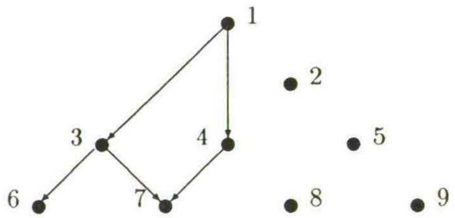
$$S(5) = \{8,9\},\ S(6) = S(7) = S(8) = S(9) = \emptyset.$$



This is the same permission structure as in Example 3.4, except that the dominance relation between players 2 and 8 is in the opposite direction. Suppose that the allocation rule $f: N \times \mathcal{G}^N \times \mathcal{S}_A^N \to \mathbf{R}$ is efficient, additive and satisfies the three conditions stated in Theorem 4.2. Players 8 and 9 are strongly inessential in (N, v, S). Deleting the relations with them results in the following permission structure S^2



In the new game with permission structure (N, v, S^2), besides players 8 and 9, player 5 also is strongly inessential and deleting the relations with player 5 results in the permission structure S^3. In (N, v, S^3) player 2 also is inessential. If the relations with player 2 also are deleted then no player has a strongly inessential player as a successor anymore. The resulting permission structure S^* is



This is the same S^* as in Example 3.4 and the players 2, 5, 8 and 9 all are strongly inessential in (N, v, S^*). Condition 1 (i) now implies that $f(i, v, S^*) = 0$ for all $i \in \{2, 5, 8, 9\}$.

Similarly as in Example 3.4 it follows that there is some constant $c \in \mathbf{R}$ such that $f(i, v, S^*) = c$ for all $i \in \{1, 3, 4, 6, 7\}$.

From condition 1 (ii) it then follows that

$$f(i, v, S) = f(i, v, S^2) = f(i, v, S^3) = f(i, v, S^*) \text{ for all } i \in N.$$

Efficiency then implies that

$$f(i,v,S) = f(i,v,S^*) = \begin{cases} \frac{1}{5} & \text{for all } i \in \{1,3,4,6,7\} = \alpha(\{6,7\}) \\ 0 & \text{else} \end{cases}$$

$$= \varphi(i,v,S).$$

In the following theorem we axiomatize the Conjunctive permission value for games with acyclic permission structures by replacing the condition with respect to inessential players by some boundary condition.

Theorem 4.4 *The allocation rule $f\colon N \times \mathcal{G}^N \times \mathcal{S}^N_A \to \mathbf{R}$ is equal to the Conjunctive permission value φ restricted to the class $N \times \mathcal{G}^N \times \mathcal{S}^N_A$ if and only if it is efficient, additive and satisfies the following three conditions for every $v \in \mathcal{G}^N_M$ and $S \in \mathcal{S}^N_A$:*

1. *For every player $i \in N$ it holds that*

   $$0 \leq f(i,v,S) \leq C(\widehat{S}(i) \cup \{i\}, v),$$

   where for every $E \subset N$, $C(E,v) := \max_{F \subset N} (v(F) - v(F \setminus E))$.

2. *For every player $j \in N$ such that $S(j) \neq \emptyset$ it holds that*

   $$f(j,v,S) \geq \max_{i \in S(j)} f(i,v,S)$$

3. *If $j \in N$ is a necessary player in v then*

   $$f(j,v,S) \geq f(i,v,S) \text{ for all } i \in N.$$

PROOF
That φ is efficient, additive and satisfies conditions 2 and 3 follows directly from Theorem 3.3.
Let $v \in \mathcal{G}^N_M$ and $S \in \mathcal{S}^N_A$. Since $\mathcal{R}_S(v) \in \mathcal{G}^N_M$ it holds that $\varphi(i,v,S) = Sh_i(\mathcal{R}_S(v)) \geq 0$. From the definition of the Shapley value it follows that $Sh_i(v) \leq C(\{i\},v)$ for all $i \in N$. But then it follows from Proposition 3.2 that

$$\varphi(i,v,S) = Sh_i(\mathcal{R}_S(v)) \leq C(\{i\}, \mathcal{R}_S(v))$$

$$= \max_{E \ni i} (\mathcal{R}_S(v)(E) - \mathcal{R}_S(v)(E \setminus \{i\}))$$

$$= \max_{E \ni i} \left(v(\sigma(E)) - v(\sigma(E \setminus \{i\})) \right)$$

$$= \max_{E \ni i} \left(v(\sigma(E)) - v(\sigma(E) \setminus [\widehat{S}(i) \cup \{i\}]) \right)$$

$$= \max_{\substack{E \ni i \\ E=\sigma(E)}} \left(v(E) - v(E \setminus \widehat{S}(i) \cup \{i\}) \right)$$

$$\leq \max_{E \ni i} \left(v(E) - v(E \setminus [\widehat{S}(i) \cup \{i\}]) \right) = C(\widehat{S}(i) \cup \{i\}, v) \text{ for all } i \in N.$$

Thus φ satisfies condition 1.

Now suppose that $f: N \times \mathcal{G}^N \times \mathcal{S}_A^N \to \mathbf{R}$ is efficient, additive and satisfies the three conditions stated in the theorem.
Again consider the monotone game $w_T = c_T u_T$, $T \subset N$, $c_T \geq 0$, and let S be an acyclic permission structure on N.
Similarly as in the previous proves it follows from conditions 2 and 3 that there is some constant $c \geq 0$ such that $f(i, w_T, S) = c$ for all $i \in \alpha(T)$.
If $i \in N \setminus \alpha(T)$ then $[\widehat{S}(i) \cup \{i\}] \cap T = \emptyset$. This implies that $w_T(E) - w_T(E \setminus [\widehat{S}(i) \cup \{i\}]) = 0$ for all $E \subset N$ and thus $C(\widehat{S}(i) \cup \{i\}, w_T) = 0$. With condition 1 it then follows that $f(i, w_T, S) = 0$ for all $i \in N \setminus \alpha(T)$.
With efficiency it follows that

$$f(i, w_T, S) = \begin{cases} \frac{c_T}{\#\alpha(T)} & \text{for all } i \in \alpha(T) \\ 0 & \text{else} \end{cases}$$

Hence, $f(i, w_T, S) = \varphi(i, w_T, S)$ for all $i \in N$.

If $v \in \mathcal{G}^N$ then we can prove that $f(i, v, S) = \varphi(i, v, S)$ for all $i \in N$ using additivity in the same way as in the proof of Theorem 3.3.

Q.E.D.

Example 4.5 Consider the same game with permission structure as in Example 4.3. Application of condition 1 directly yields that

$$f(i, v, S) = 0 \text{ for all } i \in \{2, 5, 8, 9\}$$

Conditions 2 and 3 then lead to

$f(i, v, S) = c$, for $i \in \{1, 3, 4, 6, 7\}$

With efficiency it then follows that

$$f(i, v, S) = \begin{cases} \frac{1}{5} & \text{for all } i \in \{1, 3, 4, 6, 7\} = \alpha(\{6, 7\}) \\ 0 & \text{else} \end{cases}$$

$$= \varphi(i, v, S).$$

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