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THE ECONOMIC EXPLANATION OF THE JUMP OF THE CO-STATE VARIABLE

Piet A. Verheyen

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THE ECONOMIC EXPLANATION OF THE JUMP OF THE CO-STATE VARIABLE

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Abstract

Since Dorfman (1969), Shell (1969), and Intrilligator (1971) the dynamic price theory has been based on the economic interpretation of shadow prices in optimal control theory. This interpretation, however, lacks one vital link: it does not offer an economic explanation of the jump of the co-state variables in problems with pure state constraints. This explanation is presented in this article.
Firstly, we describe an economy with irreversible investments following the analysis of Van Hilten (1990). In that case it appears that the jump in the co-state variable is explainable: it is caused by a sudden change in the cost of capital and, therefore, in the net present value. In general it follows that the co-state variables provide the marginal value of the corresponding state variable; the economic interpretation of the co-state variable is, also at the moment of the jump, perfectly clear.

1. Introduction

The general control model is:

$$\max_{u} \int_{0}^{T} F(x,u,t)dt + S(x(T),T)$$

$$x = f(x,u,t) \; ; \; x(0) = x_0$$

$$g(x,u,t) \geq 0$$

$$h(x,t) \geq 0$$

$$a(x(T),T) \geq 0$$

$$b(x(T),T) = 0$$

where u is the control variable and x is the state variable. With suitable assumptions of the functions involved we define

$$L(x,u,\lambda_0,\lambda,\mu,\nu,t) = \lambda_0 F + \lambda f + \mu g + \nu h$$

with constant $\lambda_0 \geq 0$, co-state variables $\lambda(t)$, multipliers $\mu(t)$ and $\nu(t)$ and L jointly concave in x and u.

The usual economic interpretation is as follows:

$\lambda_i(t)$ = the value of the co-state variable i; the rate of change of the maximum attainable value of the objective function as a consequence of the marginal change of the state variable

$\mu_i(t)$, $\nu_j(t)$ = the Lagrange multiplier which indicates how much the optimal value of the objective function improves with a marginal increase of an additional condition $g_i \geq 0$ or $h_j \geq 0$.

Many authors (Sorger 1989, Seierstad and Sydsaeter 1977 and 1987) have considered models with state constraints. At points of time when $h(x,t) \geq 0$ is binding. In principal a discontinuity in $\lambda(t)$ can occur and it can be described by using a jumpparameter $\eta(\tau)$. In the case the discontinuity occurs at $t = \tau$:

$$\lambda(\tau^+) = \lambda(\tau^-) + \eta(\tau)h.$$

Conditions under which such a jump does not occur at all has been described in the literature (Feichtinger and Hartl 1986).

The essential problem is how to explain in economic terms the occurrence of jumps in $\lambda(t)$. At first sight such jumps seem hard to understand because "the co-state variable captures all future effects over change in the state variables ..., the firm "knows" that the constraint will become active: why is that knowledge not incorporated in the value of $\lambda(t)$?" (Van Hilten 1990). So far no economic explanation has been offered and this is a serious lack (Feichtinger and Hartl 1986).

In another important line in the literature the irreversibility of investments and labour are analysed (Johanssen 1959, Arrow en Kurz 1970, Nickell 1974, Leban and Lesourne 1983, Pindyck 1988, Olsen 1989). Because of "the fact that, once a firm has acquired capital stock and "bolted down the machinery", possibilities of subsequent decumulation are severely limited. The optimal policy for the firm will thus consist of segments of positive and zero investments with periodic jumps in capacity, typically corresponding to downward jumps in the marginal opportunity cost" (Nickell 1974). This investment "irreversibility usually arised because capital is industry or firm specific, that is, it cannot be used in a different industry or by a different firm" (Pindyck 1988) with "the impossibility of charging at will the manning of capital equipment once constructed" (Johanssen 1959).

In this paper these two lines are combined. The jump in the co-state variables is explained in a model with irreversible investments, which confirms the development link in dynamic price theory.

2. Model formulation

A well-known example of an optimal control model is a dynamic model of the firm (Van Hilten 1990) in which two fundamental issues of the economy are combined, namely:

1. investments are irreversible and
2. the firm operates within an environment of a real business cycle

$$\text{Max} \int_0^T \exp(-it)D(t)dt + \exp(-iT)X(T) \tag{1}$$

$$\dot{K} = I(t) - aK(t) \tag{2}$$

$$K(0) = K_0 \tag{3}$$

$$\dot{X} = (1-f)\{S(Q(t),t) - wL(t) - aK(t) - rY(t)\} - D(t) \tag{4}$$

$$X(0) = X_0 \; , \; Y(0) = Y_0 \tag{5}$$

$$I(t) \geq 0 \tag{6}$$

$$D(t) \geq 0 \tag{7}$$

$$Y(t) \geq 0 \tag{8}$$

$$Y(t) \leq bX(t) \tag{9}$$

in which

$$K(t) = X(t) + Y(t) \tag{10}$$

$$Q(t) = \frac{K(t)}{k} \; ; \; L(t) = lQ(t) = \frac{wl}{k} K(t) \tag{11}$$

The definitions are:

$D(t)$	: dividend rate	a	: depreciation rate
$X(t)$	: stock of equity	b	: maximal debt-equity ratio
$K(t)$	: stock of capital	i	: corporate profit tax rate
$Q(t)$	: production rate	f	: shareholder's time preference
$S(t)$	: sales rate = $Q(t)p(t)$	k	: capital-output ratio
$L(t)$	: labour	l	: labour-output ratio
$Y(t)$	: stock of debt	p	: price of output
T	: horizon date	r	: rate of interest
		w	: wage rate

Furthermore,

$$S(t) = Q(t)p(t) = Q(t)\{\exp(-gt)Q(t)\}\{\exp(-1/\varepsilon)\}, \quad t \le \tau_u \qquad (12)$$

$$= Q(t)\{\exp((m-g)t-m\tau_u)Q(t)\}\{\exp(-1/\varepsilon)\}, \quad \tau_u \le t \le \tau_d \qquad (13)$$

$$= Q(t)\{\exp(-gt)\exp(m(\tau_d-\tau_u))\{Q(t)\{\exp(-1/\varepsilon)\}, \quad t \ge \tau_d \qquad (14)$$

in which:

ε = price elasticity of demand
g = growth rate of the demand function
$m-g$ = decrease rate of the demand function

In this shareholder's value maximizing firm (1) financed by equity and debt (10), the usual formula of net investment applies (2) and given the Leontief technology, production and labour are a linear function of the stock of capital goods (11). The state equation (4) defines, that the sales after deduction of wages, depreciation interest and profit taxation can be used to pay dividends or to increase equity.

The business cycle given by (12), (13), (14), is based on Nickell (1974) and Leban and Lesourne (1983). If we assume that $\varepsilon > 1$ and $m > g$ the grafical form is (Van Hilten 1990):

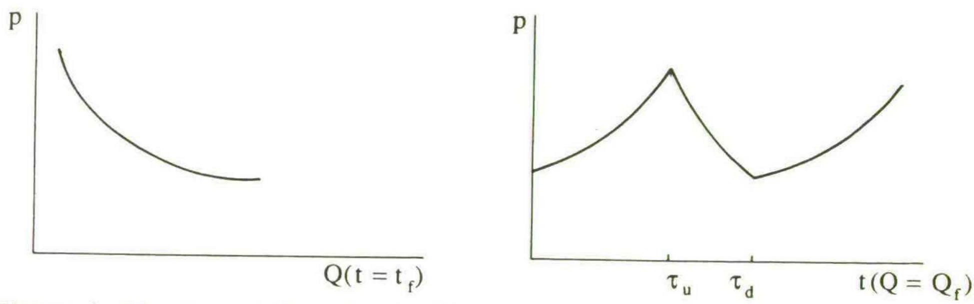


Figure 1. The demand function (suffix f = fixed).

In this article we describe the case of an hidden business cycle ($\varepsilon \ge 1$; $m \gg g$; $(g-a) : \varepsilon-v(1-f) > 0$, $i < (1-f)r$), because we use this model to explain the jumps of the co-state variables and under these conditions jumps exist.

3. The optimal policy

We define

λ_1 = the co-state variable of capital

λ_2 = the co-state variable of equity

μ_1 = the Lagrange multiplier belonging to restriction (6) (investment)

μ_2 = the Lagrange multiplier belonging to restriction (7) (dividend)

ν_1 = the Lagrange multiplier belonging to restriction (8) (minimum conditions of debt)

ν_2 = the Lagrange multiplier belonging to restriction (9) (maximum conditions of debt)

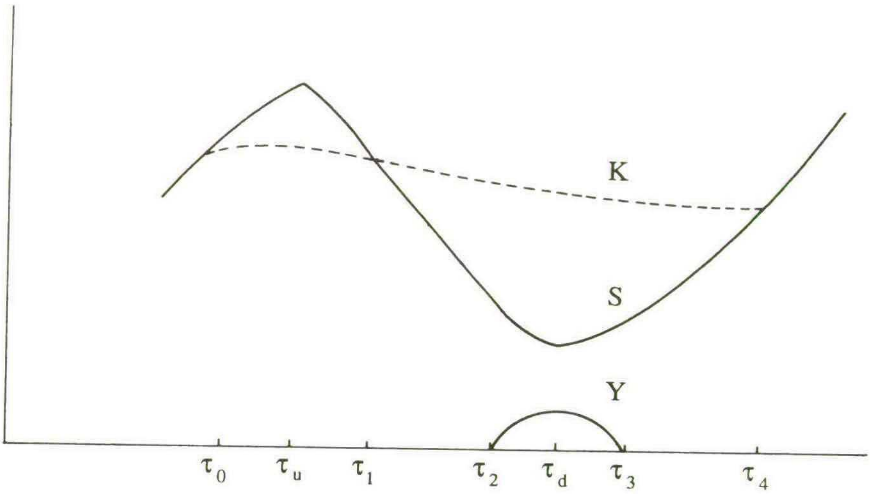
Hence the Lagrangian is represented by

$$L = \lambda_0 D + \lambda_1 (I-aK) + \lambda_2 \{\{(1-f)S(K,t) - \frac{wl}{k} K - aK - rK + rX\} - D\}$$

$$+ \mu_1 I + \mu_2 D + \nu_1 (K-X) + \nu_2 \{(1+b)X-K\}. \tag{15}$$

In appendix 1 the optimality conditions are described. According to Van Hilten (1990) the solution in the case of a severe recession is presented in figure 2.

Figure 2. Policy of the firm during the hidden business cycle.



Investment (I)	+	0	0	0	+
Dividend (D)	+	+	0	+	+
λ_1	+	-	-	-	+
λ_2	1	1	>1	1	1
μ_1	0	+	+	+	0
μ_2	0	0	+	0	0
ν_1	+	+	0	+	+

During the period τ_1, τ_4 the firm does not invest because an extra capital good causes a decrease of the Hamiltonian, i.e. $\lambda_1 < 0$. In the lowest point of the recession the firm has to borrow money ($\nu_1 = 0$) to pay the wages because of the negative accounting cash flow. If $D = 0$, $I = 0$ and $Y > 0$ the latter can be expressed as: $(1-f)\{S(Q,t) - wL - rY\}$.
The development of the total path proceeds as follows: at the top of a boom period, investments are reduced as there is an expectation of lower sales in subsequent periods. Because of the higher prices, sales increase. After the peak (τ_u), a turning point is reached at τ_1.

The marginal return of capital (λ_1) is

$$\lambda_1(\tau_1) = \int_{\tau_1}^{T} \{\exp - (i+a)(t-\tau_1)\}(1-f)\{\frac{\partial S}{\partial K} - (\frac{wl}{k} + a + \frac{i}{1-f})\}dt = 0 \tag{16}$$

So firms invest until the marginal costs of investment (wages wl/k, depreciation a and net capital cost i/(1-f)) is equal to the shadow price of installed capital ($\partial S/\partial K$) and therefore net present value in terms of revenues and costs of the invested dollar is zero (Kort 1990 defined the net present value in terms of cash flows).

From that time-point on the capital goods surplus decreases by aK; $\lambda_1(t)$ being negative indicates that a large decrease of K yields advantages, but such a drop ($\mu > 0$) is not possible because of the irreversibility constraint (6) (Nickell 1975). A negative marginal price is associated with this investment-irreversibility constraint (Olsen 1989).
The accounting cash flow surplus will be distributed as dividends ($D > 0$; $\mu_2 = 0$), since lending is not possible ($Y \geq 0$; $\nu_1 = (1-f)r - i > 0$).
After τ_2 the accounting cash flow is negative because the cash inflow of sales is lower than the cash outflow of the wages. This liquidity shortage can be eliminated only by borrowing. After the lowest point in the recession has been reached (τ_d) the accounting cash flow is positive and so the firm can pay off its debts. The τ_3, τ_4 period has the same characteristics as τ_1, τ_2, while after τ_4 the boom period with positive net investments follows.

This description has been given by many authors. Nickell, among others, points to "the downward jump in the marginal opportunity cost" (see also Katayama 1989) and Bertola and Caballero 1990 indicate "the wedge ... between the cost of capital and the marginal contribution to profit".

At τ_2 there are jumps in the co-state variables from which the graphic representation is presented in Figure 3.

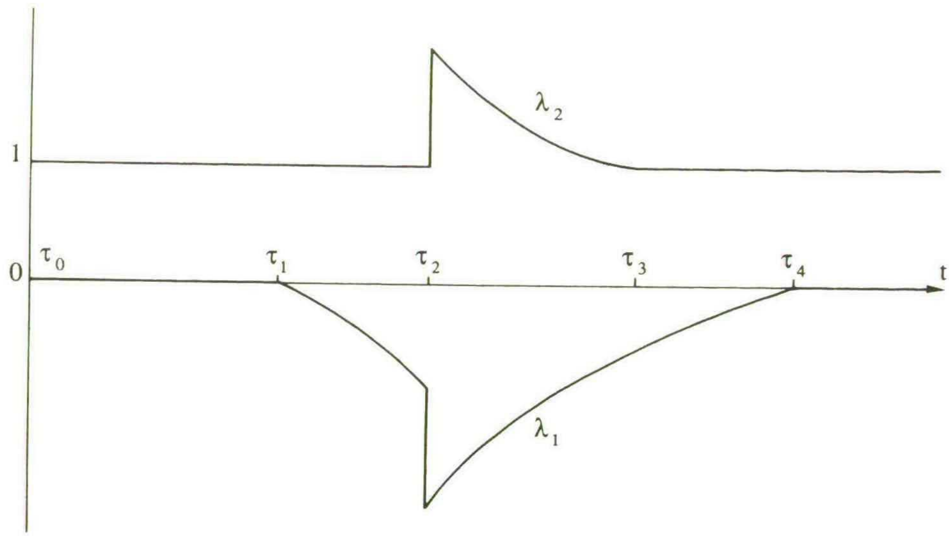


Figure 3. The jump of the co-state variables.

The jump is

$$-\eta_1(\tau_2) = \eta_2(\tau_2) = \exp\{(1-f)r - i)(\tau_3-\tau_2)\} - 1 = \mu_2(\tau_2) \qquad (17)$$

This equation will be economically explained in the next section.

4. The economic explanation of the jump

Let us study the development over time of the co-state variable $\lambda_2(t)$. In general, the value of the co-state variable is equal to the change of the value function arising from a marginal increase of the corresponding state variable. In this model λ_2 corresponds to the state variable $X(t)$. Hence, in order to interpret the development of λ_2 we have to establish the change of the value function due to an additional increase of the value of equity, $X(t)$, at each moment in time, which is done below:

$\tau_0 \le t < \tau_2 : \lambda_2 = 1$

One extra unit of equity will be used to pay out dividend. Therefore the objective increases by one (cf. (1) and notice that $\lambda_2(t)$ is a current value co-state variable).

$\tau_2 \le t < \tau_3 : \lambda_2 = \exp\{((1-f)r-i)(\tau_3-t)\}$

When the firm has one extra unit of equity at its disposal, it can diminish its amount of debt money, needed to finance the expenses of the firm (i.e. paying wages and interest), by one dollar. Hence, according to continuous corresponding, the amount of interest the firm has to pay diminishes by $\exp\{(1-f)r(\tau_3-t)\}$ (notice that $Y(t) = 0$ for $t > \tau_3$). Because the value of the co-state variable is measured at time point t we have to discount the interest payments back to t, so $\lambda_2(t) = \exp\{(1-f)r-i)(\tau_3-t)\}$.

$\tau_3 \le t < \tau_4 : \lambda_2 = 1$

Again the extra unit of equity will be used to pay out dividend.

From the above we can conclude that there are two possible time-points at which λ_2 can jump: τ_2 and τ_3.

$$\left.\begin{aligned} \tau_2 : \lambda_2(\tau_2^-) &= 1 \\ \lambda_2(\tau_2^+) &= \exp\{(1-f)r - i\}(\tau_3-\tau_2) \end{aligned}\right\} \begin{aligned} &\text{the jump is} \\ &\exp\{(1-f)r - i\}(\tau_3-\tau_2) - 1 \end{aligned}$$

$$\left.\begin{aligned} \tau_3 : \lambda_1(\tau_3^-) &= \exp\{(1-f)r - i\}(\tau_3-\tau_3) = 1 \\ \lambda_2(\tau_3^+) &= 1 \end{aligned}\right\} \text{no jump}$$

In the example used in Section 3 the model is determined at the moment of the jump by the constraints: the state constraints allow the model no further freedom. The control variable which influences the state variable X and so the $F(x,u,t)$, is then totally fixed as a result of the investments and the accounting cash flow $F(x,u,t) = F_0(t)$ (Sorger 1989).

"The restriction on the control variables force the firm to leave or to enter the boundary. At time τ_2 investments and dividends have reached their lower bound. Thus the firm has no choice" (Van Hilten 1990) "At this extreme there may be no real choice at all if the constraints are so restrictive that only one reasonable option is really available to the planner" (Dreze 1990). There is therefore no "competitive path" (Sorger 1989b) or trajectory. Only afterwards the firm can use the positive cash inflow for dividends or investment so that there will be other possible paths.

In economic terms the jump is perfectly clear: at τ_2 the disadvantage of the extra loan for this marginal investment must of course be fully ameliorated by this investment, but only for the period that the investment is financed by expensive money (similar to jump, described by Turnovsky (1990) as an accumulation effect in response of the increase in taxes) and the cost of capital is higher because of the high interest rate.

5. The general explanation

Optimal control theory, in addition to the well-known mathematical derivations, also allows an economic interpretation: first of all $\lambda_j(t)$ always gives the cumulative value of the marginal change, thus, the jump in $\lambda_j(t)$ also gives a cumulative assessment. Furthermore, $\lambda_j(t)$ is equal to the shadow price for the current $X_j(t)$. If a condition if now placed on X_j at τ_j, $\lambda_j(\tau)$ will also adjust to the changed situation and remain so for the entire adjustment period.
To arrive at the correct economic explanation of the jump in the co-state variables in optimal control problems with state constraints, it is necessary explain the well known definition of these co-state variables namely: $\lambda_j(\tau_i)$ is the rate of change of the maximum attainable value of the objective function as a consequence of a marginal change of the state variable. When there is a break in the current state, a jump can occur in $\lambda_j(\tau_i)$, which can be explained in economic terms by the cumulative (dis)advantages of the marginal adjustment of the state constraint.

6. An addendum: the economic ridge

Developments in the values of the co-state variables and the parameters are also based on the comparison of the given state with and without the marginal changes. To explain the shadow prices, the additional conditions are related by a marginal increase and the effect on the objective function is measured. To use an analogy, if in climbing a mountain the minimalization of the distance to the top is the objective, a measure of how much the distance to the top increases is made for every deviation from the optimal path.

The economic analysis is fundamentally different if no alternatives are available. In that case there is only one criterion, i.e. a direct confrontation with the objective. To extend the mountainclimbing analogy: a ridge has no alternative. The ridge has to be taken to reach the top. The distance of the ridge is a given for the objective function and there is no alternative.

In this example: at τ_2 the firm reaches the ridge and remains on the ridge until τ_3. From the very first step, he can see the entire length of the path: the longer the time interval starting at τ_2 and ending at τ_3 the greater the influence at the ridge on the value of the objective. The value of $\lambda_j(\tau_i)$ depends on the current state $X_j(\tau_i)$. Crossing the ridge the co-state variable can jump.

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Appendix 1. Optimality conditions

$$L = \lambda_0 D + \lambda_1(I-aK) + \lambda_2\{(1-f)(S(K,t) - \frac{wl}{k} K - aK - rK + rX\} - D\} +$$

$$+ \mu_1 I + \mu_2 D + \nu_1(K-X) + \nu_2((1+b)X-K)$$

$$\frac{\delta L}{\delta I} = 0 \; ; \; \lambda_1 + \mu_1 = 0$$

$$\frac{\delta L}{\delta D} = 0 \; ; \; 1 - \lambda_2 + \mu_2 = 0$$

$$\dot{\lambda}_1 = -\frac{\delta L}{\delta K} + i\lambda_1 = (i+a)\lambda_1 + \lambda_2(1-f)\{-\frac{\delta S}{\delta K} + \frac{wl}{k} + a + r\} - \nu_1 + \nu_2$$

$$\dot{\lambda}_2 = \frac{\delta L}{\delta X} + i\lambda_2 = (i-(1-f)r)\lambda_2 + \nu_1 - (1+b)\nu_2$$

$$\mu_1 I = 0 \; , \; \mu_2 D = 0$$

$$\nu_1(K-X) = 0 \; , \; \nu_2\{(1+b)X - K\} = 0$$

$$\lambda_1(T) = -\gamma_2 + \gamma_1 \; , \; \lambda_2(T) = 1 + (1+b)\gamma_2 - \gamma_1$$

$$\gamma_1\{K(T) - X(T)\} = 0 \; , \; \gamma_2\{(1+b)X(T) - K(T)\} = 0$$

$$\lambda_1(\tau_i^+) = \lambda_1(\tau_i^-) - \eta_1(\tau_i) + \eta_2(\tau_i)$$

$$\lambda_2(\tau_i^+) = \lambda_2(\tau_i^-) + \eta_1(\tau_i) - (1+b)\eta_2(\tau_i)$$

$$\eta_1(\tau_i)\{K(\tau_i) - X(\tau_i)\} = 0 \; , \; \eta_2(\tau_i)\{(1+b)X(\tau_i) - K(\tau_i)\} = 0$$

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