TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT

POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

THE COVARIANCE MATRIX OF ARMA-ERRORS IN CLOSED FORM

Jan van der Leeuw

R45

FEW 562

Matrices

Communicated by Dr. H.H. Tigelaar

19.06.1992 joe.chl

THE COVARIANCE MATRIX OF ARMA-ERRORS IN CLOSED FORM

Jan van der Leeuw[1]

Dept. of Econometrics, Tilburg University

Tilburg, The Netherlands

Abstract

Several efficient methods to compute the exact ARMA covariance matrix are known. However, a general matrix representation in closed form is lacking. This article presents such a closed form. First a matrix equation, containing the covariance matrix, is derived, next it is solved for the MA, AR and ARMA case. The result is quite, and maybe surprisingly, simple.

---

[1] I am indebted to H.H. Tigelaar, B.B. van de Genugten and F.J.H. Don for their suggestions and comments on an earlier draft.

Present address:
Jan van der Leeuw
Department of Econometrics
Tilburg University
P.O.Box 90153
NL - 5000 LE Tilburg

1. Introduction

Autoregressive, moving average and mixed processes are widely considered in the statistical literature. Several authors have provided efficient methods of calculating the autocovariance functions (McLeod, (1975), Tunnicliffe Wilson, (1979)). But, although the exact covariance matrix and its inverse of several processes, like AR(1), AR(2), MA(1), ARMA(1,1) are known a general and easy to differentiate form is lacking. Only for the MA(q) case a result, due to Diebold (Diebold, 1986), is known. In this article we present a simple form for the general ARMA(p,q) case, which of course includes the AR(p) and the MA(q) as special cases.

The form of the covariance matrix we present is simple enough to be differentiated, which permits analytical expressions for first and higher order differentials. The results can be used both in time series analysis and in the estimation of the linear regression model with ARMA errors. Furthermore our form gives insight into the way the covariance matrix is composed. As can be expected, the MA covariance matrix is simple when not inverted, the AR part is easy when inverted. The core of the inverted matrix consists of a matrix which rank is equal to the highest number of AR or MA parameters.

2. Matrix form for ARMA parameters

The elements of the ARMA(p,q) error vector ε are defined as

$$\varepsilon_t=-\sum_{l=1}^{p}\vartheta_l\varepsilon_{t-l}+v_t+\sum_{l=1}^{q}\alpha_l v_{t-l} \qquad t=1,2,..T \tag{1}$$

where v is a vector of white noise:

$Ev_t=0$, $Ev_t^2=\sigma^2$, $Ev_tv_s=0$ for $t\neq s$. We assume that the ARMA process is stationary over time and that the usual invertibility conditions hold:

$$f(z)=1+\vartheta_1 z+\ldots+\vartheta_p z^p\neq 0 \text{ for } |z|\leq 1 \tag{2}$$

and

$$g(z)=1+\alpha_1 z+\ldots+\alpha_q z^q\neq 0 \text{ for } |z|<1 \tag{2'}$$

while f and g have no zeros in common.

Following Pagan (1974), we introduce two matrices for both the AR parameters and the MA parameters. These are special types of Toeplitz matrices. First we define a (square) lower band matrix, say P, of dimensions TxT as follows:

$$P=\left[\begin{array}{c|c} P_1 & 0 \\ \hline P_2 & P_3 \end{array}\right]=\left[\begin{array}{cccc|cccccc}
1 & & & & & & & & & \\
\vartheta_1 & 1 & & & & & & & & \\
 & \vartheta_1 & \cdot & & & & & & & \\
\cdot & & \cdot & & & & & & & \\
 & & & \cdot & & & & & & \\
\vartheta_{p-1} & & & \cdot \;\; 1 & & & & & & \\
\hline
\vartheta_p & \vartheta_{p-1} & & \vartheta_1 & 1 & & & & & \\
 & \cdot & \cdot & \cdot & \vartheta_1 & \cdot & & & & \\
 & & & & & \cdot \;\; \cdot & & & & \\
 & & \cdot & \cdot & & \cdot & & & & \\
 & & & \cdot \;\; \vartheta_{p-1} & & & & & & \\
 & & & \vartheta_p & \vartheta_{p-1} & & & & & \\
 & & & & \vartheta_p & \cdot & & & & \\
 & & & & & \cdot \;\; \cdot & & & & \\
 & & & & & & \cdot \;\; \cdot & & \cdot \;\; 1 & \\
 & & & & & & \vartheta_p & \vartheta_{p-1} & \vartheta_1 & 1
\end{array}\right]$$

The upper triangular of a lower band matrix consists of zeros and the lower part has off-diagonals with the same elements. As is well-known its inverse can be obtained by a simple algorithm. An other important characteristic of these matrices is the fact that they commute and that their product is a matrix of the same type. It is useful to partition P in P_1 of dimensions pxp, P_2 of dimensions (T-p)xp px(T-p) NE-part (all zero's) and P_3 of dimensions (T-p)x(T-p).

$$Q=\left[\begin{array}{c} Q_1 \\ \hline 0 \end{array}\right]=\left[\begin{array}{cccc}
\vartheta_p & \vartheta_{p-1} & \cdot & \vartheta_1 \\
0 & \cdot & \cdot & \cdot \\
\cdot & \cdot & & \\
0 & & & \vartheta_p \\
\hline
0 & & & 0 \\
\cdot & & & \cdot \\
 & & & \\
0 & \cdot & & 0
\end{array}\right]$$

Q has the dimensions Txp and is partitioned in an upper pxp part and a lower (T-p)xp part, which consists of only zeros.

Observe, that P_1 and P_3 have the same structure as P itself. Moreover, P_2 has the same structure as Q, while Q_1 is the transpose of a lower band matrix. In the sequel we will also use the matrices M and N. M and N have the same structure as P and Q with ϑ replaced by α and p replaced by q. Observe that $|P|$ and $|M|$ are equal to 1.

To relate the invertibility condition to these matrices we give the following theorem:

Theorem 1

Let P_1 and Q_1 be defined as above. The invertibility condition is equivalent to the condition that all solutions to $|\lambda P_1+Q_1|=0$ satisfy $-1<\lambda<1$.

Proof

Observe, that λP_1+Q_1 is a circulant matrix. Its eigenvalues μ_k are (see *e.g.* Davies, 1979):

$$\mu_k=\lambda+\vartheta_p+\vartheta_{p-1}z_k+\ldots+\vartheta_1 z_k^{p-1} \quad (k=1,\ldots,p)$$

where $z_k=\lambda^{1/p}e^{2ki\pi/p}$, which implies $|z_k|=|\lambda|^{1/p}$. As $z_k^p=\lambda$, we can also write

$$\begin{aligned}\mu_k&=z_k^p\ (1+\vartheta_1/z_k+\ldots+\vartheta_p/z_k^p)\\&=\lambda f(1/z_k).\end{aligned}$$

f is as defined in (1), where the AR-invertibility condition is stated.

Let λ_0 be a solution to $|\lambda P_1+Q_1|=0$ and suppose that the invertibility condition holds. Since λ_0 is never zero we have $\mu_k\neq 0$ for $|1/z_k|\leq 1$ or $|z_k|\geq 1$ or $|\lambda_0|\geq 1$. But $|\lambda_0 P_1+Q_1|$ can only be zero if at least one of the eigenvalues is zero, which can never be the case for $|\lambda_0|\geq 1$. Therefore we conclude $|\lambda_0|<1$.

For the second part of the proof, suppose $|\lambda_0 P_1+Q_1|=0$ implies $0<|\lambda_0|<1$. Then $|\lambda_0 P_1+Q_1|=0$ means that at least one of the eigenvalues μ_k is zero or $f(1/z_k)=0$, while $|\lambda_0|<1$ means $|1/z_k|>1$. Hence $f(1/z_k)=0$ implies $|1/z_k|>1$, and this is equivalent to $f(1/z_k)\neq 0$ for $|1/z_k|\leq 1$. *Q.E.D.*

3. Covariance equation

In this section we will derive an equation from which the exact covariance matrix can be solved. First we rewrite the errorvector in matrix form. As done by several other authors (de Gooijer, 1978 or Galbraith and Galbraith, 1974) we form an equation for the covariance matrix. But there is one difference as our equation involves only one unknown matrix. The solution to this covariance equation will be given in the next section.

Denoting the covariance matrix by V and using the symbol $-T$ for the inverse of a transposed matrix, we state

Theorem 2

The covariance matrix V corresponding to the ARMA(p,q) error specification is a solution to the equation

$$PVP^T=NN^T+MM^T+\begin{bmatrix}Q & O\end{bmatrix}V\begin{bmatrix}Q & O\end{bmatrix}^T-\begin{bmatrix}N & O\end{bmatrix}M^TP^{-T}\begin{bmatrix}Q & O\end{bmatrix}^T-\begin{bmatrix}Q & O\end{bmatrix}P^{-1}M\begin{bmatrix}N & O\end{bmatrix}^T \tag{3}$$

where P, Q, M and N are defined as above and O is a matrix consisting of zeros.

Proof

First define the auxiliary vectors $\bar{\varepsilon}$ and $\bar{v}$:

$\bar{\varepsilon}^T= (\varepsilon_{-p+1},\varepsilon_{-p+2},\dots,\varepsilon_{-1},\varepsilon_0)^T$

$\bar{v}^T= (v_{-q+1},v_{-q+2},\dots,v_{-1},v_0)^T$

Then we can write (1) in matrix form:

$$\begin{bmatrix}Q & P\end{bmatrix}\begin{bmatrix}\bar{\varepsilon}\\ \varepsilon\end{bmatrix}=\begin{bmatrix}N & M\end{bmatrix}\begin{bmatrix}\bar{v}\\ v\end{bmatrix}$$

or $P\varepsilon=N\bar{v}+Mv-Q\bar{\varepsilon}$. Post multiplying both sides by its transpose and taking expectations gives

$PE(\varepsilon\varepsilon^T)P^T=E(N\bar{v}+Mv-Q\bar{\varepsilon})(N\bar{v}+Mv-Q\bar{\varepsilon})^T$.

The right hand side contains the expressions $E\bar{v}\bar{v}^T$, $E\bar{v}v^T$, $E\bar{v}\bar{\varepsilon}^T$, Evv^T, $Ev\bar{\varepsilon}^T$ and $E\bar{\varepsilon}\bar{\varepsilon}^T$. These can all be expressed in matrix form or are zero. v is an independently distributed variable which implies $Evv^T=\sigma_v^2 I_T$,$E\bar{v}\bar{v}^T=\sigma_v^2 I_p$ and $Ev\bar{v}^T=0$. Because we assume that the ARMA(p,q) process is stationary over time we have the same structure for $E\bar{\varepsilon}\bar{\varepsilon}^T$ as for $E\varepsilon\varepsilon^T$, *i.e.* V. As the vector $\bar{\varepsilon}$ depends only on v_0, v_{-1}, ... (which are by assumption uncorrelated with v_1, v_2, ...), we conclude $Ev\bar{\varepsilon}=0$. The resulting equation can be found in *e.g.* Galbraith and Galbraith, 1974 or de Gooijer, 1978. But we can go one step further, for the covariances of $\bar{\varepsilon}$ and $\bar{v}$ have - supposing stationarity - the same structure as the covariances of ε and v. This covariance can be derived as follows:

$$E(P\varepsilon v^T)=E(N\bar{v}+Mv-Q\bar{\varepsilon})v^T$$

$$=NE(\bar{v}v^T)+ME(vv^T)-QE(\bar{\varepsilon}v^T)$$

$$=M$$

which gives $E(\varepsilon v^T)=P^{-1}M$. For $E(\bar{\varepsilon}\bar{v}^T)$ we get the first p rows and the first q columns of $P^{-1}M$. Using O as the matrix which consists of only zeros gives equation (3).

The problem of finding V is thus reduced to the problem of finding a solution to (3). We will show that this is possible if the invertibility condition holds.

4. Solution of the covariance equation

Theorem 3

The covariance equation (1) has an unique solution if the invertibility condition for the AR-part is fulfilled. The solution is

$$V=[N\ M][\bar{P}^T\bar{P}-\bar{Q}\bar{Q}^T]^{-1}[N\ M]^T \tag{4}$$

where M and N are as defined in section **2** and $\bar{P}$ and $\bar{Q}$ have the same structure as in section **2**, but are of order (T+p)x(T+p) and (T+p)xp.

Corollary 1

The covariance matrix for the MA(q) model is

$$V=NN^T+MM^T \tag{4a}$$

Corollary 2

The covariance matrix for the AR(p) model is

$$V=[P^TP-QQ^T]^{-1} \tag{4b}$$

Proof

To prove uniqueness we proceed as follows. Writing (3) in *vec*-notation and rearranging terms we see that uniqueness is guaranteed if $P\otimes P-[Q\ O]\otimes[Q\ O]$ is not singular. Its determinant, D, is:

$D=|P\otimes P-[Q\ O]\otimes[Q\ O]|$

$\quad=|I-[Q\ O]\otimes[Q\ O][P\otimes P]^{-1}|\,|P\otimes P|$

$\quad=|I-[Q\ O]P^{-1}\otimes[Q\ O]P^{-1}|.$

Hence, a sufficient condition for nonsingularity is that all eigenvalues of $[Q\ O]P^{-1}$ are less than one in absolute value. These eigenvalues are zero or equal to those of $Q_1P_1^{-1}$. As Theorem 1 states that $|\lambda|$ is less than one when the invertibility condition holds, we conclude that D is nonzero, and thus that (3) has an unique solution.

As is proven in the appendix we can write (4) as

$$V=P^{-1}[MM^T+(PN-MQ)(P_1^TP_1-Q_1Q_1^T)^{-1}(PN-MQ)^T]P^{-T} \tag{4'}$$

That the right hand side of (4') is a solution to (3) is established by direct verification. The essence of the proof is the fact that lower band matrices commute. The proof can be found in the Appendix.

The proof of Corollary 1 is trivial. Substituting P=I and Q=0 in (1), we get the MA(q) expression for V.

To prove Corollary 2 substitute M=I and N=0, next premultiply both sides of (4') by P and postmultiply by its transpose. The resulting equation is equal to the corresponding covariance equation if V_1 (the NW-part of V) is equal to $(P_1^TP_1-Q_1Q_1^T)^{-1}$, which is proven in the appendix. *Q.E.D.*

It is clear that the second term of (4') within brackets is of order p. Because of the commuting property PN-MQ can be written as $\begin{bmatrix} P_1N_1-M_1Q_1 \\ O \end{bmatrix}$, which makes clear that the main part of V consists of $P^{-1}MM^TP^{-T}$, the rest being a correction matrix of which the rank is p. Furthermore (4') is easy to invert: the core of the inverse consists of a (p×p) matrix, which can be triangulized. Use an expression for the inverse of the sum of two matrices (see *e.g.* Rao, 1973, p. 33), which gives

$$V^{-1}=P^TM^{-T}\{I_T-R(R^TR+P_1^TP_1-Q_1Q_1^T)^{-1}R^T\}M^{-1}P \tag{5}$$

with $R=M^{-1}PN-Q$.

It is not clear whether it is possible to write (5) in a form similar to (4), where the MA part and the AR part are separated.

The determinant of V can be obtained in the following way. Observing that the value of the determinant of $M^{-1}P$ is equal to one we have

$$|V|=|I_T+M^{-1}\begin{bmatrix} P_1N_1-M_1Q_1 \\ O \end{bmatrix}(P_1^TP_1-Q_1Q_1^T)^{-1}\begin{bmatrix} P_1N_1-M_1Q_1 \\ O \end{bmatrix}^T M^{-T}|$$

$$=|I_p+(P_1^TP_1-Q_1Q_1^T)^{-1}(P_1N_1-M_1Q_1)^T\mathcal{M}_1^T\mathcal{M}_1(P_1N_1-M_1Q_1)^T|.$$

where $\mathcal{M}_1$ is the (T×p) matrix, consisting of the first p columns of M^{-1}. The equality is due to the fact, that the second term of the sum in both equations has the same nonzero characteristic roots. The evaluation of the determinant can thus be reduced from a (T×T) matrix to one of order (p×p), the highest number of AR or MA parameters.

5. Concluding remarks

In this article we present a compact matrix expression for the covariance matrix of ARMA distributed errors. While the individual elements of the covariance matrix are very complicated, this form is charmingly simple. For the AR case and the MA case the forms are even more simple as can be expected. Expressions for the inverse and the determinant are given.

Furthermore it is shown how the invertibility condition and the positive definiteness of the covariance matrix are interconnected.

Appendix

Because of the structure of the matrices we partition after p rows and columns. We shall use p, the number of AR parameters instead of max(p,q), because we may suppose p to be equal to q. This gives no loss of generality as it is possible to fill up the shorter vector by zeros. First we will prove the following lemma, next we will show, that (4') is a solution to the covariance equation.

Lemma

$$V=[N\ M][\bar{P}^T\bar{P}-\bar{Q}\bar{Q}^T]^{-1}[N\ M]^T \tag{4}$$

can also be written as

$$V=P^{-1}[MM^T+(PN-MQ)\Delta^{-1}(PN-MN)^T]P^{-T} \tag{4'}$$

with $\Delta=P_1^TP_1-Q_1Q_1^T$.

Δ is positive definite if the invertibility condition holds.

Proof

First we prove that Δ is positive definite, if the invertibility condition is fulfilled. Observe, that $\begin{bmatrix} P_1 & O \\ Q_1 & P_1 \end{bmatrix}$ and $\begin{bmatrix} Q_1^T & O \\ P_1^T & Q_1^T \end{bmatrix}$ are both lower band matrices. As they commute we have $\Delta=P_1^TP_1-Q_1Q_1^T=P_1P_1^T-Q_1^TQ_1$ or $\Delta=1/2(P_1P_1^T-Q_1Q_1^T)+1/2(P_1^TP_1-Q_1^TQ_1)$. Both parts of the right hand side are symmetric, implying that they have real eigenvalues. Next we show that they are positive. For the first part we have $P_1P_1^T-Q_1Q_1^T=P_1(I-P_1^{-1}Q_1Q_1^TP_1^{-T})P_1^T$. The eigenvalues of the expression between brackets at the right hand side are equal to one minus the square of the eigenvalues of $P_1^{-1}Q_1$. But from Theorem 1 we know that $|\lambda P_1+Q_1|=0$ implies $|\lambda|<1$, which means that an eigenvalue of $P_1P_1^T-Q_1Q_1^T$ is equal to $1-\lambda^2$. In the same way we can prove that the second part is positive.

To prove that (4) is equivalent to (4') we partition $\bar{P}$ and $\bar{Q}$ as before. For

$\bar{P}^T\bar{P}-\bar{Q}\bar{Q}^T\bar{P}^T\bar{P}-\bar{Q}\bar{Q}^T$ we get $\begin{bmatrix} P_1P_1^T & Q^TP \\ P^TQ & P^TP \end{bmatrix}$ because $P_1^TP_1+P_2^TP_2-Q_1Q_1^T=P_1P_1^T$. As is easily verified, its inverse is $\begin{bmatrix} \Delta^{-1} & -\Delta^{-1}Q^TP^{-T} \\ -P^{-1}Q\Delta^{-1} & P^{-1}P^{-T}+P^{-1}Q\Delta^{-1}Q^TP^{-T} \end{bmatrix}$. Premultiplying by [N M] and postmultiplying by its inverse gives (4'), because P (and thus P^{-1}) and M commute. *Q.E.D.*

To prove **Theorem 3** substitute the right hand side of (4') for V in (3) and partition as before[2]. Observe, that all parts, apart of MM^T, on both sides are zero except the NW part. This means that we have to demonstrate that

$(P_1N_1-M_1Q_1)\Delta^{-1}(P_1N_1-M_1Q_1)^T=N_1N_1^T+Q_1V_1Q_1^T-N_1M_1^TP_1^{-T}Q_1^T-Q_1P_1^{-1}M_1N_1^T$

with $V_1=P_1^{-1}[M_1M_1^T+(P_1N_1-M_1Q_1)\Delta^{-1}(P_1N_1-M_1Q_1)^T]P_1^{-T}$.

Insert the expression for V_1, rearrange terms and make use of the commuting property to get $P_1\Delta^{-1}P_1^T=I+Q_1\Delta^{-1}Q_1^T$. But this is the NW-part of the covariance-equation in the pure AR-case.

To show this equality, use $\Delta=P_1^TP_1-Q_1Q_1^T=P_1P_1^T-Q_1^TQ_1$ and thus

$\Delta^{-1}=P_1^{-1}(I-P_1^{-T}Q_1Q_1^TP_1^{-1})^{-1}P^{-T}=Q_1^{-1}(Q_1^{-T}P_1P_1^TQ_1^{-1}-I)^{-1}Q_1^{-T}$.

Here we have $P_1^{-T}Q_1Q_1^TP_1^{-1}=(Q_1^{-T}P_1P_1^TQ_1^{-1})^{-1}$ and straightforward algebra completes the proof. *Q.E.D.*

---

2 A detailed proof can be obtained from the author upon request.

References

Davies, J.Ph. (1979), *Circulant Matrices*, New York.

Diebold, F.X. (1986), The exact initial matrix of the state vector of a general MA(q) process. *Economics Letters*, 22.

Galbraith, R.F. and Galbraith, J.I (1974), On the Inverse of Some Patterned Matrices Arising in the Theory of Stationary Time Series. *J. Appl. Prob.*, 11.

Gooijer, J.G. de (1978), On the Inverse of the Autocovariance Matrix for a General Mixed Autoregressive Moving Average Process. *Statistische Hefte*, 19/2.

McLeod, I. (1975), Derivation of the Theoretical Autocovariance Function of Autoregressive-Moving Average Time Series. *Applied Statistics*, 24.

Pagan, A (1974), A Generalised Approach to the Treatment of Autocorrelation. *Australian Economic Papers*, 13.

Rao, C.R. (1973), *Linear Statistical Inference and Its Applications*, New York.

Tunnicliffe Wilson, G. (1979), Some Efficient Computational Procedures for High Order ARMA Models, *J. Statistical Computation and Simulation*, 8.

IN 1991 REEDS VERSCHENEN

466 Prof.Dr. Th.C.M.J. van de Klundert - Prof.Dr. A.B.T.M. van Schaik
Economische groei in Nederland in een internationaal perspectief

467 Dr. Sylvester C.W. Eijffinger
The convergence of monetary policy - Germany and France as an example

468 E. Nijssen
Strategisch gedrag, planning en prestatie. Een inductieve studie binnen de computerbranche

469 Anne van den Nouweland, Peter Borm, Guillermo Owen and Stef Tijs
Cost allocation and communication

470 Drs. J. Grazell en Drs. C.H. Veld
Motieven voor de uitgifte van converteerbare obligatieleningen en warrant-obligatieleningen: een agency-theoretische benadering

471 P.C. van Batenburg, J. Kriens, W.M. Lammerts van Bueren and R.H. Veenstra
Audit Assurance Model and Bayesian Discovery Sampling

472 Marcel Kerkhofs
Identification and Estimation of Household Production Models

473 Robert P. Gilles, Guillermo Owen, René van den Brink
Games with Permission Structures: The Conjunctive Approach

474 Jack P.C. Kleijnen
Sensitivity Analysis of Simulation Experiments: Tutorial on Regression Analysis and Statistical Design

475 C.P.M. van Hoesel
An $O(nlogn)$ algorithm for the two-machine flow shop problem with controllable machine speeds

476 Stephan G. Vanneste
A Markov Model for Opportunity Maintenance

477 F.A. van der Duyn Schouten, M.J.G. van Eijs, R.M.J. Heuts
Coordinated replenishment systems with discount opportunities

478 A. van den Nouweland, J. Potters, S. Tijs and J. Zarzuelo
Cores and related solution concepts for multi-choice games

479 Drs. C.H. Veld
Warrant pricing: a review of theoretical and empirical research

480 E. Nijssen
De Miles and Snow-typologie: Een exploratieve studie in de meubelbranche

481 Harry G. Barkema
Are managers indeed motivated by their bonuses?

482 Jacob C. Engwerda, André C.M. Ran, Arie L. Rijkeboer
Necessary and sufficient conditions for the existence of a positive definite solution of the matrix equation $X + A^T X^{-1} A = I$

483 Peter M. Kort
A dynamic model of the firm with uncertain earnings and adjustment costs

484 Raymond H.J.M. Gradus, Peter M. Kort
Optimal taxation on profit and pollution within a macroeconomic framework

485 René van den Brink, Robert P. Gilles
Axiomatizations of the Conjunctive Permission Value for Games with Permission Structures

486 A.E. Brouwer & W.H. Haemers
The Gewirtz graph - an exercise in the theory of graph spectra

487 Pim Adang, Bertrand Melenberg
Intratemporal uncertainty in the multi-good life cycle consumption model: motivation and application

488 J.H.J. Roemen
The long term elasticity of the milk supply with respect to the milk price in the Netherlands in the period 1969-1984

489 Herbert Hamers
The Shapley-Entrance Game

490 Rezaul Kabir and Theo Vermaelen
Insider trading restrictions and the stock market

491 Piet A. Verheyen
The economic explanation of the jump of the co-state variable

492 Drs. F.L.J.W. Manders en Dr. J.A.C. de Haan
De organisatorische aspecten bij systeemontwikkeling
een beschouwing op besturing en verandering

493 Paul C. van Batenburg and J. Kriens
Applications of statistical methods and techniques to auditing and accounting

494 Ruud T. Frambach
The diffusion of innovations: the influence of supply-side factors

495 J.H.J. Roemen
A decision rule for the (des)investments in the dairy cow stock

496 Hans Kremers and Dolf Talman
An SLSPP-algorithm to compute an equilibrium in an economy with linear production technologies

497 L.W.G. Strijbosch and R.M.J. Heuts
Investigating several alternatives for estimating the compound lead time demand in an (s,Q) inventory model

498 Bert Bettonvil and Jack P.C. Kleijnen
Identifying the important factors in simulation models with many factors

499 Drs. H.C.A. Roest, Drs. F.L. Tijssen
Beheersing van het kwaliteitsperceptieproces bij diensten door middel van keurmerken

500 B.B. van der Genugten
Density of the F-statistic in the linear model with arbitrarily normal distributed errors

501 Harry Barkema and Sytse Douma
The direction, mode and location of corporate expansions

502 Gert Nieuwenhuis
Bridging the gap between a stationary point process and its Palm distribution

503 Chris Veld
Motives for the use of equity-warrants by Dutch companies

504 Pieter K. Jagersma
Een etiologie van horizontale internationale ondernemingsexpansie

505 B. Kaper
On M-functions and their application to input-output models

506 A.B.T.M. van Schaik
Produktiviteit en Arbeidsparticipatie

507 Peter Borm, Anne van den Nouweland and Stef Tijs
Cooperation and communication restrictions: a survey

508 Willy Spanjers, Robert P. Gilles, Pieter H.M. Ruys
Hierarchical trade and downstream information

509 Martijn P. Tummers
The Effect of Systematic Misperception of Income on the Subjective Poverty Line

510 A.G. de Kok
Basics of Inventory Management: Part 1
Renewal theoretic background

511 J.P.C. Blanc, F.A. van der Duyn Schouten, B. Pourbabai
Optimizing flow rates in a queueing network with side constraints

512 R. Peeters
On Coloring j-Unit Sphere Graphs

513 Drs. J. Dagevos, Drs. L. Oerlemans, Dr. F. Boekema
Regional economic policy, economic technological innovation and networks

514 Erwin van der Krabben
Het functioneren van stedelijke onroerend-goed-markten in Nederland - een theoretisch kader

515 Drs. E. Schaling
European central bank independence and inflation persistence

516 Peter M. Kort
Optimal abatement policies within a stochastic dynamic model of the firm

517 Pim Adang
Expenditure versus consumption in the multi-good life cycle consumption model

518 Pim Adang
Large, infrequent consumption in the multi-good life cycle consumption model

519 Raymond Gradus, Sjak Smulders
Pollution and Endogenous Growth

520 Raymond Gradus en Hugo Keuzenkamp
Arbeidsongeschiktheid, subjectief ziektegevoel en collectief belang

521 A.G. de Kok
Basics of inventory management: Part 2
The (R,S)-model

522 A.G. de Kok
Basics of inventory management: Part 3
The (b,Q)-model

523 A.G. de Kok
Basics of inventory management: Part 4
The (s,S)-model

524 A.G. de Kok
Basics of inventory management: Part 5
The (R,b,Q)-model

525 A.G. de Kok
Basics of inventory management: Part 6
The (R,s,S)-model

526 Rob de Groof and Martin van Tuijl
Financial integration and fiscal policy in interdependent two-sector economies with real and nominal wage rigidity

527 A.G.M. van Eijs, M.J.G. van Eijs, R.M.J. Heuts
Gecoördineerde bestelsystemen
een management-georiënteerde benadering

528 M.J.G. van Eijs
Multi-item inventory systems with joint ordering and transportation decisions

529 Stephan G. Vanneste
Maintenance optimization of a production system with buffercapacity

530 Michel R.R. van Bremen, Jeroen C.G. Zijlstra
Het stochastische variantie optiewaarderingsmodel

531 Willy Spanjers
Arbitrage and Walrasian Equilibrium in Economies with Limited Information

IN 1992 REEDS VERSCHENEN

532 F.G. van den Heuvel en M.R.M. Turlings
Privatisering van arbeidsongeschiktheidsregelingen
Refereed by Prof.Dr. H. Verbon

533 J.C. Engwerda, L.G. van Willigenburg
LQ-control of sampled continuous-time systems
Refereed by Prof.dr. J.M. Schumacher

534 J.C. Engwerda, A.C.M. Ran & A.L. Rijkeboer
Necessary and sufficient conditions for the existence of a positive definite solution of the matrix equation $X + A^*X^{-1}A = Q$.
Refereed by Prof.dr. J.M. Schumacher

535 Jacob C. Engwerda
The indefinite LQ-problem: the finite planning horizon case
Refereed by Prof.dr. J.M. Schumacher

536 Gert-Jan Otten, Peter Borm, Ton Storcken, Stef Tijs
Effectivity functions and associated claim game correspondences
Refereed by Prof.dr. P.H.M. Ruys

537 Jack P.C. Kleijnen, Gustav A. Alink
Validation of simulation models: mine-hunting case-study
Refereed by Prof.dr.ir. C.A.T. Takkenberg

538 V. Feltkamp and A. van den Nouweland
Controlled Communication Networks
Refereed by Prof.dr. S.H. Tijs

539 A. van Schaik
Productivity, Labour Force Participation and the Solow Growth Model
Refereed by Prof.dr. Th.C.M.J. van de Klundert

540 J.J.G. Lemmen and S.C.W. Eijffinger
The Degree of Financial Integration in the European Community
Refereed by Prof.dr. A.B.T.M. van Schaik

541 J. Bell, P.K. Jagersma
Internationale Joint Ventures
Refereed by Prof.dr. H.G. Barkema

542 Jack P.C. Kleijnen
Verification and validation of simulation models
Refereed by Prof.dr.ir. C.A.T. Takkenberg

543 Gert Nieuwenhuis
Uniform Approximations of the Stationary and Palm Distributions of Marked Point Processes
Refereed by Prof.dr. B.B. van der Genugten

544 R. Heuts, P. Nederstigt, W. Roebroek, W. Selen
Multi-Product Cycling with Packaging in the Process Industry
Refereed by Prof.dr. F.A. van der Duyn Schouten

545 J.C. Engwerda
Calculation of an approximate solution of the infinite time-varying LQ-problem
Refereed by Prof.dr. J.M. Schumacher

546 Raymond H.J.M. Gradus and Peter M. Kort
On time-inconsistency and pollution control: a macroeconomic approach
Refereed by Prof.dr. A.J. de Zeeuw

547 Drs. Dolph Cantrijn en Dr. Rezaul Kabir
De Invloed van de Invoering van Preferente Beschermingsaandelen op Aandelenkoersen van Nederlandse Beursgenoteerde Ondernemingen
Refereed by Prof.dr. P.W. Moerland

548 Sylvester Eijffinger and Eric Schaling
Central bank independence: criteria and indices
Refereed by Prof.dr. J.J. Sijben

549 Drs. A. Schmeits
Geïntegreerde investerings- en financieringsbeslissingen; Implicaties voor Capital Budgeting
Refereed by Prof.dr. P.W. Moerland

550 Peter M. Kort
Standards versus standards: the effects of different pollution restrictions on the firm's dynamic investment policy
Refereed by Prof.dr. F.A. van der Duyn Schouten

551 Niels G. Noorderhaven, Bart Nooteboom and Johannes Berger
Temporal, cognitive and behavioral dimensions of transaction costs; to an understanding of hybrid vertical inter-firm relations
Refereed by Prof.dr. S.W. Douma

552 Ton Storcken and Harrie de Swart
Towards an axiomatization of orderings
Refereed by Prof.dr. P.H.M. Ruys

553 J.H.J. Roemen
The derivation of a long term milk supply model from an optimization model
Refereed by Prof.dr. F.A. van der Duyn Schouten

554 Geert J. Almekinders and Sylvester C.W. Eijffinger
Daily Bundesbank and Federal Reserve Intervention and the Conditional Variance Tale in DM/$-Returns
Refereed by Prof.dr. A.B.T.M. van Schaik

555 Dr. M. Hetebrij, Drs. B.F.L. Jonker, Prof.dr. W.H.J. de Freytas
"Tussen achterstand en voorsprong" de scholings- en personeelsvoorzieningsproblematiek van bedrijven in de procesindustrie
Refereed by Prof.dr. Th.M.M. Verhallen

556 Ton Geerts
Regularity and singularity in linear-quadratic control subject to implicit continuous-time systems
Communicated by Prof.dr. J. Schumacher

557 Ton Geerts
Invariant subspaces and invertibility properties for singular systems: the general case
Communicated by Prof.dr. J. Schumacher

558 Ton Geerts
Solvability conditions, consistency and weak consistency for linear differential-algebraic equations and time-invariant singular systems: the general case
Communicated by Prof.dr. J. Schumacher

559 C. Fricker and M.R. Jaïbi
Monotonicity and stability of periodic polling models
Communicated by Prof.dr.ir. O.J. Boxma

560 Ton Geerts
Free end-point linear-quadratic control subject to implicit continuous-time systems: necessary and sufficient conditions for solvability
Communicated by Prof.dr. J. Schumacher

561 Paul G.H. Mulder and Anton L. Hempenius
Expected Utility of Life Time in the Presence of a Chronic Noncommunicable Disease State
Communicated by Prof.dr. B.B. van der Genugten