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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

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ABOUT TOBIN'S MARGINAL AND AVERAGE q
A NOTE

Raymond Gradus

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Abstract

In this paper we give the relation between Tobin's marginal and average q for the case that the adjustment costs are not linearly homogeneous, but, for example, quadratic in investment. By doing this we give an explanation for the appearing difference between marginal and average q and a better explanation for investment behaviour.

ABOUT TOBIN'S MARGINAL AND AVERAGE q
A Note*)

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1. Introduction

In the last decade the literature on investment has been dominated by the q-theory suggested by Tobin (1969). Starting point of this q theory is the neo-classical theory of the firm (cf. Jorgenson (1963)) and the theory of adjustment costs (cf. Lucas (1967)). By doing this investment becomes a function of the marginal value of capital, i.e. q (see for example Abel and Blanchard (1983)).

This marginal q can be obtained by using optimal control or calculus of variations, where the firm maximizes its present value subject to the capital accumulation equation. However, in practice such a q is not directly observable. Hayashi (1982) showed that under certain assumptions such as constant returns to scale, linear homogeneity of adjustment costs functions and perfect competition this marginal q equals average q, which is defined by the market value of the firm divided by the capital stock. This seems to be an important result, because average q can be easily obtained from data and in this way empirical evidence can be given (see for example Hayashi (1982) and Blundell et al. (1988)). Furthermore, the paper by Hayashi has lead to several attempts in order to provide a better explanation for the actual fluctuations of investment over time (e.g. Hayashi (1984), Schiantarelli and Georgoutos (1987) and Galeotti and Schiantarelli (1988)). Most of these extensions, including monopolistic competition, financial restrictions, multiple capital inputs, invalidate

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the simple and appealing equality between marginal and average q. But, so far less interest has been paid to the structure of adjustment costs, which can also be an explanation for the difference between marginal and average q and because of that for investment behaviour.

The aim of this paper is to derive the relationship between marginal and average q for the case that adjustment costs are no longer linear homogeneous, but instead, a quadratic function of investment.

2. The model

Consider a firm acting to maximize the present value of future after-tax net receipts:

$$V(0) = \int_0^\infty \{(p.f(k,l) - wl)(1-\tau) - i - \varphi(i)\} e^{-\int_0^t r(v)dv} dt,$$

$$\varphi(0) = 0,\ \mathrm{sign}(\varphi') = \mathrm{sign}(i),\ \varphi'' > 0, \tag{1}$$

where p, k, l, i, w, r, f(k,l), $\varphi(i)$, τ denote respectively the price of the firm's output, the level of capital stock, the number of employed workers, the rate of investment, the wage rate, the interest rate, the production function (which is characterised by constant returns to scale), the internal adjustment costs and the proportional tax rate on profits. For the price of the investment goods we assume without loss of generality that it is one.

The strictly convex function $\varphi(.)$ captures that internal adjustment costs increase and are zero only if gross investment is zero. Furthermore, we assume that the adjustment costs are quadratic

$$\varphi(i) = b.i^2 ,\ b > 0. \tag{2}$$

Note that the adjustment costs are not linear homogeneous. The firm maximizes (1) subject to the capital accumulation equation

$$\overset{\circ}{k} = i - \delta k, \tag{3}$$

where δ denotes the rate of depreciation.

The necessary condition for the firm's optimal control problem are:

$$\mathring{q} = (r + \delta)q - p.f_k(1 - \tau), \tag{4}$$

$$\varphi'(i) = q - 1, \tag{5}$$

$$p\ f_\ell = w, \tag{6}$$

$$\mathring{k} = i - \delta k, \tag{7}$$

in which q is the (undiscounted) shadow price of capital.

The dynamics of this problem correspond to a saddle point where the capital stock is a backward-looking variable and the shadow-price of capital is a forward-looking variable.

3. The relation between average and marginal q

In this section we derive the relationship between marginal and average q for the adjustment costs function specified in (2).

The market value of the firm is given by

$$\begin{aligned} V(t) &= \int_t^\infty \{(p\ f(k,l) - wl)(1 - \tau) - i - \varphi(i)\} e^{-\int_t^s r(v)dv} ds \\ &= \int_t^\infty \pi(s) e^{-\int_t^s r(v)dv} ds. \end{aligned} \tag{8}$$

Differentiating (8) with respect to time gives

$$\mathring{V} = rV - (p\ f(k,l) - wl)(1 - \tau) + i + \varphi(i). \tag{9}$$

Since f(k,l) is a constant returns to scale production function, which implies $f = f_k.k + f_l.l$, we can rewrite (9) into:

$$\mathring{V} = rV - (p(f_k.k + f_l.l) - wl)(1 - \tau) + i + \varphi(i). \tag{10}$$

Substituting for $f_k(.)$, $f_1(.)$ from (4) and (6) gives

$$\overset{\circ}{V} - rV = -k((r + \delta)q - \overset{\circ}{q}) + i + \varphi(i). \tag{11}$$

For the adjustment costs function we can derive

$$\varphi(i) = \varphi'(i).\frac{i}{2}. \tag{12}$$

Substituting (5), (7) and (12) into (11) gives

$$\begin{aligned}\overset{\circ}{V} - rV &= -k((r + \delta)q - \overset{\circ}{q}) + \overset{\circ}{k} + \delta k + (q - 1).\frac{i}{2} \\ &= -k(r + \delta)q + k\overset{\circ}{q} + \overset{\circ}{k} + \delta k + (\overset{\circ}{k} + \delta k)(q - 1) - \varphi(i) \\ &= -krq + k\overset{\circ}{q} + q\overset{\circ}{k} - \varphi(i). \end{aligned} \tag{13}$$

Therefore

$$(V \overset{\circ}{-} qk) = r(V - qk) - \varphi(i), \tag{14}$$

which can be rewritten as

$$V(t) - q(t)\ k(t) = \int_t^{\infty} \varphi(i(s))e^{-\int_t^s r(v)dv}ds. \tag{15}$$

Hence,

$$q(t) = \frac{V(t)}{k(t)} - \frac{1}{k(t)} \int_t^{\infty} \varphi(i(s))e^{-\int_t^s r(v)dv}ds. \tag{16}$$

So, marginal q is equal to average q corrected for the stream of discounted adjustment costs per capital stock.
In the steady-state it holds that:

$$V^* = q^*k^* + \frac{\varphi(i^*)}{r^*}, \tag{17}$$

where r^* is interest rate in the steady-state.
In the Abel and Blanchard model r^* equals the social discount rate.
Furthermore, define the modified market value by

$$\tilde{V}(t) = \int_t^{\infty}(\pi(s) - \varphi(i(s)))e^{-\int_t^s r(v)dv}ds. \tag{18}$$

Then it is not difficult to see that for this modified market value it holds that

$$\tilde{V} = qk. \tag{19}$$

So, for the corrected market value as defined in equation (18) we get a equality between marginal and "modified" average q.

The crucial step in the derivation is equation (12), where we write the adjustment costs as the product of its derivative and the rate of investment divided through 2. Because of that it is possible to obtain equation (13). To apply this method for other adjustment costs functions we have to derive an equation such as (12). This can be done for a third power and exponential function. An interesting field of future research could be to obtain a relationship between average and marginal q for all kinds of adjustment cost functions.

4. Conclusions

Although the formulation of q-models has become standard in economic theory, a quite satisfactory empirical evidence has not been given and people have been searching for an explation. Some authors have brought forward that the assumptions made by Hayashi (1982) are to restrictive and they extended the model by incorporating financial restrictions and monopolistic competition, which leads to a better explanation for investment behaviour. However, less interest has been paid at the structure of adjustment costs.

In this paper we reject the assumption of linearly homogeneous adjustment costs and give for a special adjustment cost function, which is often used in the literature, a relation between average and marginal q. In that case marginal q is less than average q, because we have to correct for adjustment costs.

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