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RESEARCH MEMORANDUM

OPTIMAL DYNAMIC ENVIRONMENTAL POLICIES OF A PROFIT MAXIMIZING FIRM

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Abstract

In this paper we study the optimal environmental policy of the firm for different scenario's dependent on (costs of) production technologies, financing costs and governmental policy. The governmental measures to be considered are:

- investment grants on cleaner production technologies and on abatement activities;
- taxes imposed on environmental pollution.

The problem will be defined as an optimal control model. In this model, the firm influences its pollution output through the choice of its production technology. Available are a more capital-extensive and dirty activity, a more capital-intensive and clean activity and an abatement activity that eliminates pollution completely or partially.

1. Introduction

Nowadays, in the industrialized world the improvement of environmental quality is one of the most important objectives within the framework of economic and social policy. From the economist's point of view, the environment has become a scarce commodity. Consequently, environmental use is an allocation problem (Siebert [21]) and should be taken into consideration by economic theory. Actually, more and more books are devoted to environmental economics (e.g. [4], [6], [13], [20], [21], [23]).
At the macroeconomic level a lot of attention has been paid to the analysis of the trade off relation between economic growth and environmental quality (e.g. [1], [9], [11], [14], [18], [21], [23], [24]).
The analysis of the effects of governmental regulation on the firm's decision making concerning pollution control spending, employment and investment is an important issue at the microeconomic level (e.g. [8], [16]). The relevant question connected with the strategies of governmental environmental policy deals with the choice of their instrument(s). One class of instruments includes direct controls by setting limits to the amounts of effluent that the factories can discharge into a stream (environmental standards). The problem of optimally selecting this upper bound and of selecting a point of time at which it comes into force is analyzed by Beavis and Dobbs [5]. Another mechanism for the attainment of a given environmental target is the standard-price-approach introduced by Baumol and Oates [3]. The basic idea of this concept is to meet a given quantity of emissions by rationing the demand for emission permits by prices.

The relative efficiency of standard setting and emissions taxation in connection with pollution control is analyzed by Baron [2]. The paper by Margat [19] extends the static comparison of effluent taxes and efficient standards to a dynamic world in which firms invest resources in improving their abatement technology as well as their production technology.

In our paper we deal with the firm's optimal environmental policy when the government uses pollution tax and investment grants in order to encourage the use of cleaner production technologies and/or abatement activities. Under these conditions the policy of the firm consists of decisions about the level of production and the choice of production activities, which in our model not only fix the sales value and production costs, but also the level of pollution.

We assume that management maximizes the shareholders' value of the firm. We present the resulting optimal policies of the firm under different scenarios. Each scenario is characterized by a different set of values for: factor productivities, unit costs, price/demand curve, financing costs, restriction on the capital structure, governmental instruments on pollution and profit tax rate.

The problem will be analyzed by developing a dynamic model of the firm, which is an extended version of the one described by Van Loon [17] (see also [12]). In Section 2 important assumptions are presented, and in Section 3 the dynamic model of the firm is introduced. In Section 4 we present the results and analyze the optimal policies for two different scenarios emphasizing on economic aspects. Section 5 concludes the paper and the technical analysis can be found in the appendices.

2. Preliminaries

In this section we present important assumptions of the model. These assumptions concern the environmental policy of the government, the way the firm can influence its own environmental pollution output, the firm's possibilities to finance its activities and the goal the firm wants to reach in taking its decisions.
As mentioned in the introduction we incorporate the following governmental environmental instruments in our model:

- investment grants on cleaner production technologies and on abatement activities;
- taxes imposed on environmental pollution.

The level of the firm's pollution depends, beside on the level of output, heavily on the choice of the production technology. For simplicity, we assume that the firm produces only one homogeneous product. We also assume that at the start of the planning period the firm uses a more capital-extensive and dirty technology. The firm may consider to replace this technology by a more capital intensive and cleaner one (see e.g. [15]) and/or to build an abatement installation that eliminates pollution completely or partially. This choice will depend on the unit costs of the different combinations of technologies. The amount of time the firm needs to reach its ideal combination is determined by the level of available cash flow (from operations and from funding activities). Funding activities consist of attracting debt money. In this model we assume that there is a maximum debt to equity rate.

The level of internal generated cash flow depends on the return on the invested capital and on the level of corporate profit taxation. We further assume that shareholders do not demand dividend each year, provided that management maximizes the value of the firm. This value of the firm consists of the discounted dividend payments over the planning period and the discounted value of the firm at the horizon date.
The financial situation described in this model holds for many firms: equity can only be increased by retained earnings and subsidies, which are apart from profits, and the total capacity of debt financing depends heavily on the (book) value of equity, thus on the value of total assets.

3. The Model

The firm is able to produce a homogeneous output through two different techniques, a capital-extensive activity and a capital-intensive one:

$$Q(K_1, K_2) = q_1 K_1 + q_2 K_2 \tag{1}$$

$$q_1 > q_2 \tag{2}$$

in which:

K_j : amount of capital goods assigned to activity j

Q : production rate

q_j : capital productivity of activity j.

Production through these two activities causes environmental pollution, where activity 2 is relatively more clean than activity 1. Besides, it is also possible for the firm to invest in a technique that cleans pollution. We assume that pollution is homogeneous by nature:

$$E(K_1, K_2, K_3) = e_1K_1 + e_2K_2 - e_3K_3 \tag{3}$$

$$\frac{e_1}{q_1} > \frac{e_2}{q_2} \tag{4}$$

in which:

E : amount of pollution

K_3 : amount of capital goods assigned to the abatement activity 3

e_j : pollution to capital rate of activity j; j = 1, 2

e_3 : abatement to capital rate of activity 3

There are no unused capital goods, so all capital goods are assigned to any of the three activities:

$$K = K_1 + K_2 + K_3 \tag{5}$$

in which:

K : capital goods stock

Because the labor to capital rate differs among activities, the firm's policy also influences the level of employment:

$$L(K_1,K_2,K_3) = l_1K_1 + l_2K_2 + l_3K_3 \tag{6}$$

$$l_1 \neq l_2 \neq l_3 \tag{7}$$

in which:

L : level of employment of the firm

l_j : labor to capital rate of activity j

The stock of finished products is constant over time, which implies that at each point of time the level of production equals the level of sales. If the firm raises output, its (net) selling price will decrease:

$$S(Q) = P(Q)Q \tag{8}$$

$$S'(Q) > 0; \; S''(Q) < 0; \; S(0) = 0 \tag{9}$$

in which:

S : sales rate

P : (net) selling price

In this model the only asset is capital goods which can be financed by equity and debt. The value per unit of a capital good is fixed at one unit of money. So the balance sheet equation is:

$$X + Y = K_1 + K_2 + K_3 \tag{10}$$

in which:

X : stock of equity

Y : stock of debt

The firm can raise its equity by retained earnings and by acquiring investment grants. However, equity reduces through tax imposed on environmental pollution (which cannot be deducted from profit before taxes):

$$\dot{X} = R + g(I_2+I_3) - f_2E \tag{11}$$

in which:

I_j : investment rate assigned to activity j

R : retained earnings rate

f_2 : pollution tax rate

g : investment grant rate on the cleaner activity 2 and on the abatement activity 3

To construct the profit and loss account-equation we introduce the following assumptions:

- corporate tax is proportional to profit;
- depreciation is proportional to the stock of capital goods;
- borrowing does not carry any transaction costs.

Then the flow of retained earnings can be formulated as:

$$R = (1-f_1)[S - wL - aK - rY] - D \tag{12}$$

in which

D : dividend rate

a : depreciation rate

f_1 : corporate profit tax rate

r : interest rate on debt

w : wage rate

For each technology the usual formula of net investment applies:

$$\dot{K}_j = I_j - aK_j; \; j = 1,2,3 \tag{13}$$

The amount of debt is bounded from above (see [17]), i.e.:

$$Y \le kK \Leftrightarrow Y \le \frac{k}{1-k} X \tag{14}$$

$$0 \le k < 1 \tag{15}$$

in which:

k : maximum debt to capital rate

Assume that the firm behaves so as to maximize the shareholders' value of the firm. This value consists of the sum of the discounted dividend stream over the planning period and the discounted final value of the firm at the end of the planning period. This final value is equal to the difference between the final value of the assets and the sum of the final stock of debt and the amount of investment grants that have to be repaid due to stopping corporate activity:

$$\underset{D, I_1, I_2, I_3}{\text{maximize}} \int_0^z e^{-iT} D(T)dT + e^{-iz}[K(z) - Y(z) - g(K_2(z) + K_3(z))] \tag{16}$$

in which:

T : time, $0 \leq T \leq z$

i : shareholders' time preference rate

z : horizon date

To complete the model, we add some non-negativity conditions:

$$D \geq 0,\ Y \geq 0,\ X \geq 0,\ E \geq 0,\ K_1 \geq 0,\ K_2 \geq 0,\ K_3 \geq 0 \tag{17}$$

$$X(0) = X_0 > 0,\ K(0) = K_1(0) = K_0 > 0 \tag{18}$$

The controls D and I_j (j = 1, 2, 3) do not need to be explicitly bounded from above, because they have an impliciet upperbound induced by the model's financial structure.

As we will show later on, it is convenient to distinguish between different cases, depending on the mode of production, the financial structure and the dividend pay out rate. For each case, we denote the resulting unit cost by:

$$c_{bn},\ b \in \{1,2,12,13,23,123\};\ n \in \{X,Y,YX,XD,YD\} \qquad (19)$$

in which:

b : activity performed by the firm (e.g. b = 123 means that the three activities are performed together)

n : index of financial structure and dividend pay out rate:

- n = X : self-financing case
- n = Y : maximum debt financing case
- n = YX : intermediate debt financing case
- n = XD : self-financing case together with a positive dividend pay out rate
- n = YD : maximum debt financing case together with a positive dividend pay out rate

The firm never performs only activity 3 because of its non-productivity. Due to the following assumption it is not optimal to pay out dividend in the intermediate debt financing case:

$$i \neq (1-f_1)r \qquad (20)$$

This assumption indicates that the capital market is imperfect (see also [17]).

We further assume a sufficiently small initial value of capital goods, so that it is optimal to grow at the start of the planning period while attracting a maximum amount of debt:

$$S'(Q(0)) > \max c_{bn},\ b \in \{1,2,12,13,23,123\};\ n \in \{X,Y,YX,XD,YD\} \tag{21}$$

We exclude solutions that are now well defined by assuming:

$$c_{bn} \neq c_{jm},\ b,\ j \in \{1,2,12,13,23,123\};\ n,\ m \in \{X,Y,YX,XD,YD\} \tag{22}$$

To limit the number of possible solutions we assume that under all circumstances productivity per unit equity of activity 1 is greater than productivity per unit equity of activity 2 (notice that activity 3 is non-productive), which is ensured by the following two inequalities:

$$\frac{e_3}{(1-g)e_1 + e_3} q_1 > \frac{e_3}{(1-g)(e_2+e_3)} q_2;$$

$$\frac{e_3}{(1-k-g)e_1 + (1-k)e_3} q_1 > \frac{e_3}{(1-k-g)(e_2+e_3)} q_2 \tag{23}$$

The first inequality of (23) implies that productivity per unit equity of activity 1 combined with activity 3 exceeds the productivity per unit equity of activity 2 combined with activity 3 in the case of no pollution and zero debt financing. To see this consider first the left hand side of this inequality. If K_1 and K_3 are financed by one unit of equity it holds that $K_1 + (1-g)K_3 = 1$ (gK_3 is paid by the government as investment grants). No pollution in case of a combination between the activities 1 and 3 requires that $e_1K_1 = e_3K_3$ (cf. (3)). From these two equalities we obtain that K_1 equals $\frac{e_3}{(1-g)e_1 + e_3}$ and this amount of K_1 is able to produce $\frac{e_3}{(1-g)e_1 + e_3} q_1$ (cf. (1)). A similar reasoning can be applied to obtain the expression of the right hand side of the first inequality of (23).

The second inequality of (23) has the same meaning as the first one, but it concerns the case of no pollution and maximum debt financing. Due to (4) it is easy to derive that the conditions in (23) imply that productivity per unit equity of activity 1 is greater than productivity per unit equity of activity 2 in the self-financing case and no cleaning activities (i.e. $q_1 > \frac{q_2}{1-g}$) as well as in the case of maximum debt financing, where no capital goods are assigned to the abatement activity ($\frac{q_1}{1-k} > \frac{q_2}{1-k-g}$).

In Appendix 1 we show that the model can be simplified into a model that contains 2 state variables, 4 control variables and 9 restrictions. In Appendix 2 we present the necessary conditions for an optimal solution, which are derived by using Pontryagin's maximumprinciple. We also explain in what way these conditions are transformed into the optimal trajectories of the firm.

4. The firm's optimal trajectories

The optimal policy of the firm depends on the scenario in which it has to operate. From the optimal solution, 16 different scenarios can be discerned, each asking for a different optimal policy of the firm. Such a policy causes an expansion of the firm during which growth and consolidation are alternating stages. If the planning horizon is far enough, these 16 policies lead to 8 different final stages. Which of these final stages is the optimal one depends on 3 characteristics of the scenario: financial costs, technology and environmental policy of the government.

Financial costs.

Main issue here is whether cost of equity is larger than cost of debt (including its tax advantage), so:

$$i \gtrless (1-f_1).r \tag{24}$$

If debt is cheaper in the relevant scenario, the firm will finance its activities in the final stage with as much debt as possible. If equity is cheaper, which scenario is not purely hypothetical due to the assumption of the capital market being imperfect (see equation (11)), the firm will pay back all its debt before entering the final stage.

Technology.

To characterize a scenario it is important to know the relation between the unit costs of both technologies:

$$c_{1XD} \gtrless c_{2XD} \tag{25}$$

$$c_{1XD} = \frac{1}{q_1}\left[wl_1 + a + \frac{i}{1-f_1} + \frac{f_2}{1-f_1}\, e_1\right]$$

$$c_{2XD} = \frac{1}{q_2} \left[wl_2 + (1-g- \frac{f_1}{1-f_1}g)a + (1-g)\frac{i}{1-f_1} + \frac{f_2}{1-f_2} e_2 \right]$$

These unit cost formulas, derived from the optimality conditions, thus include such costs as: wages, depreciation (adjusted for tax and investment grants), financing costs (in a situation of financing with equity only) and environmental taxation costs. All these kinds of costs affect the proportion between both unit costs. In that way these costs determine whether it is more profitable, in the final stage, still to produce by means of the old, less clean production technology 1 or to switch before that stage to production technology 2. Notice also that the environmental policy of the government (i.e. fixing f_2 and g) influences the relationship of c_{1XD} and c_{2XD}. A more rigorous interpretation of such unit cost formulas will be presented in the next two subsections.

Environmental policy.

The impact of the governmental policy on the pollution of the firm in the final stage of its development is described in the optimality conditions through the next inequality:

$$c_3 \gtrless \frac{f_2}{1-f_1} e_3 \qquad (26)$$

in which:

$$c_3 = wl_3 + (1-g- \frac{f_1}{1-f_1} g)a + (1-g)\frac{i}{1-f_1}$$

The left part of (26) are the costs per dollar invested in the cleaning technology 3. Given the technological possibilities, government may decrease these cleaning costs by raising the investment grant rate g. The right part of (26) is the decrease in environmental tax due to lower pollution of e_3 per dollar invested in technology 3. In a scenario with a government stressing on environmental features such as a high investment grant rate g and/or a high environmental tax rate f_2, the < sign may hold for (26). In that case, it is worth while for the firm to install a cleaning technology in the final stage of its development.

As stated in the beginning of this section, the signs of (24), (25) and (26) fix the final stage towards which will lead the optimal policy of the firm. Different stages of growth and consolidation may precede this final stage. In the next subsections we describe two patterns towards two different final stages. In that way we are able to demonstrate some more interesting features of the optimal solution. In Subsection 4.3 the total set of the firm's optimal trajectories is presented.

4.1. The firm's optimal policy under a weak environmental policy of the government.

Here we analyze a scenario, for which the following conditions hold:

financing costs: $i < (1-f_1)r$ (27)

technology: $c_{1XD} < c_{2XD}$ (28)

environmental policy: $c_3 > \frac{f_2}{1-f_1} e_3$ (29)

The firm's optimal policy to be studied in this subsection is depicted in Figure 1. This figure shows that the firm starts with maximum borrowing in spite of the fact that debt is the expensive way of financing. The reason is that marginal sales exceed the unit cost, even if capital stock is financed by debt money, and so each additional capital good, bought by means of debt money, yields a positive income. This can be shown as follows: in the beginning of the planning horizon it holds that Q is less than Q_{1YX} (cf. (21)), where:

$$S'(Q_{1YX}) = c_{1YX} \tag{30}$$

in which:

$$c_{1YX} = \frac{1}{q_1}\left[wl_1 + a + \frac{f_2}{1-f_1} e_1 + r\right]$$

[Place Figure 1 about here]

We now discuss the above formulation of c_{1YX} in more detail. The part between brackets is the cost per capital good assigned to activity 1, when this capital good is financed by debt money only. It is divided by the output per capital good, q_1, in order to obtain the unit cost of activity 1. The cost per capital good consists of four parts:

wages : wl_1

depreciation : a

cost of pollution : $f_2 e_1/(1-f_1)$

interest on debt : r

The components that contain the costs of wages, depreciation and debt are obvious, so they do not need any further explanation. About the cost of pollution component we can argue that e_1 is equal to the amount of pollution per capital good. The pollution is taxed with rate f_2, but it is not allowed to subtract this tax payment from the firm's profit before paying profit tax. Therefore the tax payment due to the pollution per capital good assigned to activity 1 ($f_2 e_1$) has to be multiplied by the factor $1/(1-f_1)$.

Having explained that c_{1YX} equals the unit cost, where the firm uses activity 1 and the relevant capital good is financed by debt money, we can conclude from (30), from the concavity of S(Q) and from the fact that Q is less than Q_{1YX}, that on the first expansion path marginal sales exceed the unit cost, where capital stock is financed by debt money:

$$S'(Q) > c_{1YX} \tag{31}$$

As soon as Q reaches Q_{1YX} we get an equality between marginal sales and c_{1YX} (cf. (30)). Now, due to the concavity of S(Q), further expansion would imply that marginal sales fall below marginal cost, where capital goods are financed by debt only, and therefore it is optimal for the firm to pay off debt first before growing any further. After all debt is paid off a new expansion phase begins, but now growth is financed by equity only. At the end of the planning period the firm pays out dividend, while reducing investment to replacement level. This phase begins when Q equals Q_{1XD}, for which it holds that:

$$S'(Q_{1XD}) = c_{1XD} \tag{32}$$

in which:

$$c_{1XD} = \frac{1}{q_1}\left[wl_1 + a + \frac{f_2}{1-f_1}e_1 + \frac{i}{1-f_1}\right]$$

c_{1XD} is the same as c_{1YX}, except that the term $i/(1-f_1)$ has replaced r. $i/(1-f_1)$ is the desired marginal rate of return to equity before paying profit tax. From (32) we can conclude that the firm starts paying out dividend when the marginal rate of return to equity exactly equals its desired rate. On the expansion path before this dividend path the marginal rate of return to equity is higher than $i/(1-f_1)$ and therefore it is optimal for the firm to grow at its maximum on this phase.
It is clear that this solution can only occur if: $c_{1XD} < c_{1YX}$, and it is not difficult to derive that this inequality equals the financing costs condition (27).

Another striking characteristic is that during the whole planning period the firm keeps on producing by using the most dirty activity. Obviously, the government's environmental instruments, i.e. the pollution tax rate f_2 and the investment grant rate g on cleaner investments, are not sufficiently strong that it is optimal for the firm to exchange a part of its growth for producing output by using cleaner production activities. This is confirmed by the environmental policy condition (29) and also by the technology condition (28).

4.2. The firm's optimal policy under strong environmental measures of the government.

In the scenario to be studied in this subsection the following conditions are satisfied:

financing costs: $i < (1-f_1)r$ (33)

technology: $c_{1XD} > c_{2XD}$ (34)

environmental policy: $c_3 < \frac{f_2}{1-f_1} e_3$ (35)

The solution in this case is presented in Figure 2. Due to (21), here it is also optimal to start growing by using the capital-extensive dirty activity 1, while attracting maximum debt. When time proceeds, marginal sales decrease due to concavity (Q increases so S'(Q) decreases), and, therefore, at a certain point of time it could be the case that the higher capital productivity of activity 1 does not counterbalance anymore the higher costs per capital good due to pollution of activity 1.

[Place Figure 2 about here]

One of the possibilities to reduce the costs is to replace the capital goods of activity 1 by those of the cleaner capital-intensive activity 2. This will happen as soon as the marginal rate of return to equity of activity 1 becomes equal to the marginal rate of return to equity of activity 2. The expression of the marginal rate of return to equity of activity 1 under maximum debt financing is:

$$\frac{1}{1-k}\left[q_1\ S'(Q) - wl_1 - a - \frac{f_2}{1-f_1}\,e_1 - kr\right] \tag{36}$$

Within brackets we have the marginal rate of return to capital goods. A part of the capital goods is financed by debt, i.e. $Y = kK$ (cf. (14)), and therefore the interest cost per capital good equals kr. To transform the marginal rate of return to capital goods into the marginal rate of return to equity we have to divide the whole thing by 1-k, because it holds that $X = (1-k)K$.

The marginal rate of return to equity of activity 2 under maximum debt financing equals:

$$\frac{1}{1-k-g}\left[q_2\ S'(Q) - wl_2 - \left(1 - \frac{g}{1-f_1}\right)a - \frac{f_2}{1-f_1}\,e_2 - kr\right] \tag{37}$$

If the firm invests in the cleaner production activity 2, it receives an investment grant g from the government. Between the main brackets of expression (36) depreciation appears net from investment grants. These subsidies may be considered as diminishing the price of capital goods at a rate g, resulting in a decrease of depreciation of ag in the case of absence of corporation profit tax. When corporation profit tax is introduced, we have to reckon with the fact that investment grants are free from corporation profit tax, so the relevant decrease of ag is then after tax payments and this equals a change of depreciation before taxes of $ag/(1-f_1)$. Due to maximum debt financing and the investment grants only (1-g-k) per unit capital is financed by equity, so we have to divide the marginal rate of return to capital goods by 1-g-k to obtain the marginal rate of return to equity.

As mentioned before the replacement of the capital goods of activity 1 by those of activity 2 will happen when the marginal rates of return to equity are equal. This holds for $Q = Q_{12Y}$, and this value can be obtained by equalizing (36) and (37):

$$S'(Q_{12Y}) = c_{12Y} \tag{38}$$

in which:

$$c_{12Y} = \frac{1}{(1-k)q_2 - (1-k-g)q_1}\left[((1-k)l_2-(1-k-g)l_1)w + \frac{k-f_1}{1-f_1}ag + \right.$$

$$\left. + gkr + ((1-k)e_2-(1-k-g)e_1)\frac{f_2}{1-f_1}\right].$$

After the capital goods of activity 1 have been replaced by those of activity 2, the firm starts growing again but now by using the relatively clean activity 2. When time proceeds marginal sales again decrease and therefore the marginal rate of return to equity will also decrease (cf. (37)). If the pollution tax rate f_2 is relatively high, after some time it may be worthwhile to stop further expansion (and thus more pollution) and to start investing in the non-productive abatement activity 3, while keeping the investment in capital goods of activity 2 at replacement level. This policy stops as soon as the abatement capacity is that high that the pollution, caused by production through activity 2, is eliminated. The marginal rate of return to equity under maximum debt financing and where the activities 2 and 3 are combined such that there is no pollution, can be expressed as:

$$\frac{1}{1-k-g}\left[q_2 S'(Q)\,\frac{e_3}{e_2+e_3} - \left(\frac{l_2 e_3}{e_2+e_3} + \frac{l_3 e_2}{e_2+e_3}\right) w - \left(1 - \frac{g}{1-f_1}\right) a - kr\right] \quad (39)$$

Due to the absence of pollution, the marginal rate of return to equity does not contain any pollution costs. From (3) we obtain that the elimination of pollution implies that $e_2K_2 = e_3K_3$. Within the main brackets we have the marginal rate of return to capital, which implies that this is the extra profit that arises due to the application of an additional capital good. From this capital good $e_3/(e_2+e_3)$ is assigned to activity 2 and $e_2/(e_2+e_3)$ to activity 3.

The investment in the abatement activity starts as soon as the marginal rate of return to equity of activity 2 (cf. (37)) equals the marginal rate of return to equity, where the activities 2 and 3 are combined such that there is no pollution (cf. (39)). Hence, the value of Q for which these rates are equal can be obtained by equalizing (37) and (39) and is denoted by Q_{23}:

$$S'(Q_{23}) = c_{23} \tag{40}$$

in which

$$c_{23} = \frac{1}{q_2}\left[w(l_2-l_3) + \frac{f_2}{1-f_1}(e_2+e_3)\right]$$

Notice that the amount of debt financing does not have any influence on the value of c_{23}, because c_{23} does not contain an interest component. Therefore, the argument that indicates the way of financing is dropped. After the abatement capacity has reached such a level that all pollution is eliminated, a new expansion phase starts in which a part of the retained earnings is invested in the abatement activity so that the amount of pollution remains zero. The continued expansion leads to a further decrease of the marginal sales. Therefore, after a while it will be optimal for the firm to reduce its costs by paying off the expensive debt (cf. (33)). This will happen as soon as the marginal rate of return to equity (cf. (39)) equals the interest rate on debt:

$$\frac{1}{1-k-g}\left[q_2S'(Q)\ \frac{e_3}{e_2+e_3} - \left(\frac{l_2e_3}{e_2+e_3} + \frac{l_3e_2}{e_2+e_3}\right)w - \left(1 - \frac{g}{1-f_1}\right)a-kr\right] = r \tag{41}$$

Growing any further, while still using maximum debt financing, would result in a fall of the marginal rate of return to equity below r. This implies that it is better for the firm to use the marginal dollar for paying off debt than for expansion investments. Therefore it is optimal to pay off debt first before growing any further. If we write Q_{23YX} for Q, expression (41) can be rewritten into:

$$S'(Q_{23YX}) = c_{23YX} \tag{42}$$

in which:

$$c_{23YX} = \frac{1}{q_2}\left[w\left(l_2 + \frac{e_2}{e_3} l_3\right) + \left(\left(1 - \frac{g}{1-f_1}\right)a + (1-g)r\right)\frac{e_2 + e_3}{e_3}\right].$$

After all debt is paid off, a last expansion phase begins which lasts until the marginal rate of return to equity equals the marginal rate of return desired by the shareholders:

$$\frac{1}{1-g}\left[q_2 S'(Q)\,\frac{e_3}{e_2+e_3} - \left(\frac{l_2 e_3}{e_2+e_3} + \frac{l_3 e_2}{e_2+e_3}\right)w - \left(1 - \frac{g}{1-f_1}\right)a\right] = \frac{i}{1-f_1} \tag{43}$$

From (43) we can obtain that for the optimal production rate, which we denote by Q_{23XD}, it holds that:

$$S'(Q_{23XD}) = c_{23XD} \tag{44}$$

in which:

$$c_{23XD} = \frac{1}{q_2}\left[w\left(l_2 + \frac{e_2}{e_3} l_3\right) + \left(\left(1 - \frac{g}{1-f_1}\right)a + (1-g)\frac{i}{1-f_1}\right)\frac{e_2+e_3}{e_3}\right].$$

During this final stage the retained earnings are used for replacement investment and for paying dividend to the shareholders.

In this subsection we described a situation in which the government's environmental policy is strong enough to force the firm to replace first the capital goods of the dirty activity, and second to eliminate the remaining amount of pollution, still caused by production through the cleaner activity, by investing in a non-productive abatement activity.

The technology condition (34) and the environmental policy condition (35) indicate that it is possible for such a solution to be optimal. However, to avoid any confusion we repeat that the conditions (33), (34) and (35) are only useful to determine the optimal policy in the final interval. They do not provide any information about the way this final interval is reached. To state this differently, the final policy of investing in activities 2 and 3, only financed by equity, can be preceded through several patterns of intermediate stages. This is shown explicitly in Figure 3 of the next subsection.

4.3. The total set of optimal trajectories of the firm.

The optimal trajectory of the firm depends on the values of the parameters such as the tax rates, investment grant rate, the labor to capital rates, etc. Each set of parameter values fixes a ranking of the unit costs. In Figure 3 it is shown in what way such rankings correspond to the firm's optimal trajectories.

[Place Figure 3 about here]

Due to (21) the firm starts in each trajectory with growing at its maximum by using activity 1 and maximum debt financing. In Figure 3 this feature is pointed out by stating "1Y" in the upper square. The optimal policy in the next phase depends on the relationship between the unit costs c_{1YD}, c_{1YX}, c_{12Y} and c_{13Y}. This is pointed out by stating "max(c_{1YD}, c_{1YX}, c_{12Y}, c_{13Y})" in the diamond below the upper square (see Figure 3). If c_{1YD} has the maximum value of these four unit costs it is optimal for the firm to pay out dividend, while keeping investment at replacement level, as soon as the production rate is such that it holds that:

$$S'(Q) = c_{1YD} \tag{45}$$

If, in stead of paying out dividend when the production rate satisfies (45), the firm would go on with expansion investment, the marginal rate of return to equity then falls below the rate desired by the shareholders, so this is not optimal.

In a similar way we can argue that after a while it is optimal to pay off debt if c_{1YX} has the largest value, to replace the capital goods of activity 1 by those of activity 2 if c_{12Y} has the largest value and to start investing in the non-productive abatement activity 3 if c_{13Y} has the largest value. Now, it is not difficult anymore for the reader to interpret the rest of this figure by himself. The trajectories treated in the Subsections 4.1 and 4.2 are pointed out by the solid lines. From "the bottom of the tree" it can be derived that there are sixteen different optimal trajectories, each of which ends with a phase where the firm pays dividend. Of course it must be assumed here that the planning period is sufficiently long so that the final phases can be reached.

The expressions of the unit costs, that appear in Figure 3 and which are not presented in the paper, can be obtained from the authors upon request.

5. Conclusions

In this paper the optimal policy of a profit maximizing firm is studied for different scenarios, depending on the costs of available production and cleaning activities, financing costs and governmental policy. The governmental instruments consist of a tax rate on pollution and investment grants that reward investments in capital goods by which the production process leads to less pollution. The problem is analyzed by developing a deterministic dynamic model of the firm which is solved by applying standard control theory. The firm's production process is described by activity analysis (e.g.[17], [22]).

As in Van Loon [17] the firm's optimal trajectories consist of different phases. Each growth phase is followed by a stationary phase on which the firm replaces capital goods of one production activity by those of another, the firm pays off debt or the firm pays out dividend. On such a stationary phase the production rate is fixed by an equality between marginal sales and the unit cost. The explicit formulation of such a unit cost shows how its value depends on the investment grant rate and the pollution tax rate. Hence, by knowing the explicit formulations of the unit costs the government can easily derive in what way a particular change in its environmental policy influences the firm's optimal trajectory and thus the amount of pollution caused by the firm, the firm's employment capacity, etc.

As a topic of future research we can think of developing a differential game between the government and a representative firm where the government's objective consists of maximizing a utility function over time, where utility depends on the amount of pollution and the employment capacity of the firm. In this way the pollution tax rate and investment grant rate can be determined endogenously. A similar kind of research is carried out by Gradus [10], who studies the influence of the government's taxation policy on the optimal dynamic firm behavior within the framework of a differential game.

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Appendix 1. Reconstruction of the model.

We introduce the following new variables:

$$\bar{K} := K_1 + (1-g)(K_2+K_3) \tag{A.1}$$

$$\bar{I} := I_1 + (1-g)(I_2+I_3) \tag{A.2}$$

$$C := (1-f_1)[S - \sum_{j=1}^{3} wl_j K_j - rY] + f_1 \sum_{i=1}^{3} aK_i - f_2 E \tag{A.3}$$

in which:

- $\bar{K}$: The value of the capital goods stock financed by the firm's own profit
- $\bar{I}$: rate of investment financed by the firm's own funds
- C : cash flow after interest and taxes

After substitution of the above variables in the model, given by equations (1) through (18) in Section 3, we can obtain the following simplified model:

$$\underset{K_2, K_3, D, \bar{I}}{\text{maximize}} \int_0^z e^{-iT} D(T) + e^{-iz}[\bar{K}(z) - Y(z)] \tag{A.4}$$

subject to

$$\dot{\bar{K}} = \bar{I} - a\bar{K} \tag{A.5}$$

$$\dot{Y} = \bar{I} + D - C \tag{A.6}$$

$$e_1\bar{K} + [e_2 - (1-g)e_1]K_2 - [e_3 + (1-g)e_1]K_3 \geq 0 \tag{A.7}$$

$$k[\bar{K} + gK_2 + gK_3] - Y \geq 0 \tag{A.8}$$

$$\bar{K} - (1-g)(K_2+K_3) \geq 0 \tag{A.9}$$

$$K_2 \geq 0;\ K_3 \geq 0;\ D \geq 0 \tag{A.10}$$

$$Y \geq 0 \tag{A.11}$$

$$Y(0) - \bar{K}(0) - g[K_2(0) + K_3(0)] = X_0 > 0;\ \bar{K}(0) = K_1(0) =$$

$$K_0 > 0 \tag{A.12}$$

in which:

$$Q(\bar{K},K_2,K_3) = q_1\bar{K} + [q_2 - (1-g)q_1]K_2 - q_1(1-g)K_3 \tag{A.13}$$

$$L(\bar{K},K_2,K_3) = l_1\bar{K} + [l_2 - l_1(1-g)]K_2 + (l_3 - l_1(1-g)]K_3 \tag{A.14}$$

$$E(\bar{K},K_2,K_3) = e_1\bar{K} + [e_2 - (1-g)e_1]K_2 + [e_3 + (1-g)e_1]K_3 \tag{A.15}$$

$$C(\bar{K},K_2,K_3,Y) = (1-f_1)[S - wl_1\bar{K} +$$

$$- w[l_2 - (1-g)l_1]K_2 +$$

$$- w[l_3 - (1-g)l_1]K_3 - rY] +$$

$$+ f_1 a[\bar{K} + gK_2 + gK_3] - f_2 E \tag{A.16}$$

$$\frac{e_3}{(1-g)e_1 + e_3} q_1 > \frac{e_3}{(1-g)(e_2+e_3)} q_2;$$

$$\frac{e_3}{(1-k-g)e_1 + (1-k)e_3} q_1 > \frac{e_3}{(1-k-g)(e_2+e_3)} q_2 \tag{A.17}$$

$$S := P(Q)Q;\ S'(Q) \geq 0 \quad S''(Q) < 0 \tag{A.18}$$

a, f_1, f_2, g, i, k, r : constants with values between zero and one (A.19)

e_j, l_j, q_j, w, z : constants which are greater than zero (A.20)

The simplified model contains two state variables, $\bar{K}$ and Y, four control variables, K_2, K_3, D and $\bar{I}$, one pure state constraint, and six constraints that each contain at least one control variable. Finally, we have two initial conditions represented by (A.12).

Appendix 2. Solution procedure.

We can derive the necessary conditions for an optimal solution by using Pontryagin's Maximum Principle. After applying the direct adjoining approach (see e.g. [7]) the Lagrangian becomes:

$$L = e^{-iT}D + \psi_1(\bar{I}-a\bar{K}) + \psi_2(\bar{I}+D-C) + \lambda_1(e_1\bar{K}+[e_2-(1-g)e_1]K_2+$$

$$-[e_3+(1-g)e_1]K_3) + \lambda_2(k[\bar{K}+gK_2+gK_3] - Y) + \lambda_3(\bar{K}-(1-g)(K_2+K_3)) +$$

$$+ \lambda_4K_2 + \lambda_5K_3 + \lambda_6D + \lambda_7Y \quad (A.21)$$

in which:

ψ_i : co-state variable belonging to the i-th state variable, which is continuously differentiable

λ_j : dynamic Lagrange multiplier belonging to the j-th restriction, which is piecewise continuous.

From Corollary 6.3b of Feichtinger and Hartl [7] it can be derived that the co-state variables really are continuous, because due to the properties of the paths treated later on it will turn out that entry to/exit from a boundary arc of the state constraint always occurs in a non-tangential way.

After some rearranging, the Lagrangian leads to the following conditions:

$$\psi_1 + \psi_2 = 0 \quad (A.22)$$

$$e^{-iT} + \psi_2 + \lambda_6 = 0 \tag{A.23}$$

$$\dot{\psi}_1 = -(e^{-iT}+\lambda_6)(1-f_1)\left[q_1 S'(Q) - wl_1 - a - \frac{f_2}{1-f_1}e_1\right] - \lambda_1 e_1 - \lambda_2 k - \lambda_3 \tag{A.24}$$

$$\dot{\psi}_2 = (e^{-iT}+\lambda_6)(1-f_1)r + \lambda_2 - \lambda_7 \tag{A.25}$$

$$(e^{-iT}+\lambda_6)(1-f_1)\Bigg[(q_2-(1-g)q_1)S'(Q) - w(l_2-(1-g)l_1) + \frac{f_1 ag}{1-f_1} +$$

$$- \frac{f_2(e_2-(1-g)e_1)}{1 - f_1}\Bigg] = -\lambda_1(e_2-(1-g)e_1) - \lambda_2 kg + \lambda_3(1-g) - \lambda_4 \tag{A.26}$$

$$(e^{-iT}+\lambda_6)(1-f_1)[-q_1(1-g)S'(Q) - w(l_3-(1-g)l_1) + \frac{f_1 ag}{1-f_1} +$$

$$\frac{f_2(e_3+(1-g)e_1)}{1 - f_1}\Bigg] = \lambda_1(e_3 + (1-g)e_1) - \lambda_2 gk + \lambda_3(1-g) - \lambda_5 \tag{A.27}$$

$$\lambda_1 \geqq 0,\ \lambda_1(e_1\bar{K}+(e_2-(1-g)e_1)K_2 - (e_3+(1-g)e_1)K_3) = 0 \tag{A.28}$$

$$\lambda_2 \geqq 0,\ \lambda_2(k[\bar{K} + gK_2 + gK_3]-Y) = 0 \tag{A.29}$$

$$\lambda_3 \geqq 0,\ \lambda_3(\bar{K}-(1-g)(K_2+K_3)) = 0 \tag{A.30}$$

$$\lambda_4 \geqq 0,\ \lambda_4 K_2 = 0 \tag{A.31}$$

$$\lambda_5 \geqq 0, \ \lambda_5 K_3 = 0 \tag{A.32}$$

$$\lambda_6 \geqq 0, \ \lambda_6 D = 0 \tag{A.33}$$

$$\lambda_7 \geqq 0, \ \lambda_7 Y = 0 \tag{A.34}$$

$$\psi_1(z) = e^{-iz} \tag{A.35}$$

$$\psi_2(z) = -e^{-iz} \tag{A.36}$$

We can transform the conditions into the optimal trajectories of the firm by applying the "iterative path connecting"-procedure designed by Van Loon [17]. The procedure starts with determining the feasible paths. Based on the fact that the Lagrange multipliers $\lambda_j (j = 1,7)$ can be positive or zero, each path is characterized by a combination of positive λ's. However, some of these combinations are infeasible, e.g. λ_2 and λ_7 cannot be positive at the same time, for this would imply that the value of Y equals its upper- and lower-bound at the same time (cf. (A.29) and (A.34)) and this is not possible. In Table 1 we present the feasible paths and their economic features. The mathematical derivations of these features and the expressions of the c's are available from the authors upon request.

[Place Table 1 about here]

To find the optimal trajectories, we start at the horizon date z, and work backwards in time. Hence, we first select those paths that may be final paths. From substitution of (A.35) and (A.36) into (A.22) and (A.23) we obtain that $\lambda_6 = 0$ at the end of the planning period. From this we derive that the paths 4, 5, 9, 10, 18, 19, 25 and 26 may be a final path (cf. Table 1).

Next, we have to start the coupling procedure to construct the optimal trajectories. To see if two paths can be coupled we test whether the following conditions hold:

- continuity of the state variables $\bar{K}$ and Y;
- continuity of the co-state variables ψ_1 and ψ_2;
- continuity of the stock of equity X.

The coupling procedure starts by selecting paths which can preceed the final path and proceeds backwards in time. It stops when the set of feasible paths is empty.

Finally, we check if the sequence of paths satisfies the initial conditions. In this case they consist of (A.12) and the assumption that the firms starts producing by using the capital-extensive dirty activity (see Section 2).

Application of the above described procedure leads to sixteen different feasible solutions, from which some of them are treated in Section 4. A survey of the complete solution and its mathematical derivation can again be obtained from the authors upon request.

List of Symbols

K_j : amount of capital goods assigned to activity j

Q : production rate

q_j : capital productivity of activity j

E : amount of pollution

e_j : pollution of capital rate of activity j; j = 1,2

e_3 : abatement to capital rate of activity 3

K : capital goods stock

L : level of employment of the firm

ℓ_j : labor to capital rate of activity j

S : sales rate

P : (net) selling price

X : stock of equity

Y : stock of debt

I_j : investment rate assigned to activity j

R : retained earnings rate

f_2 : pollution tax rate

g : investment grant rate on the cleaner activity 2 and on the abatement activity 3

D : dividend rate

a : depreciation rate

f_1 : corporate profit tax rate

r : interest rate on debt

w : wage rate

k : maximum debt to capital rate

T : time

i : shareholders' time preference rate

z : horizon date

$\bar{K}$: the value of capital goods stock financed by the firm's own profit

$\bar{I}$: rate of investment financed by the firm's own funds

C : cash flow after interest and taxes

ψ_i : co-state variable belonging to the i-th state variable, which is continuously differentiable

λ_j : dynamic Lagrange multiplier belonging to the j-th restriction, which is piecewise continuous

Figure and table captions.

Figure 1. The firm's optimal policy when debt money is expensive ($i < (1-f_1)r$) and the government's environmental measures are weak.

Figure 2. The firm's optimal policy when debt money is expensive ($i < (1-f_1)r$) and the government's environmental measures are strong.

Figure 3. The firm's optimal trajectories depending on the unit costs.

Table 1. The feasible paths.

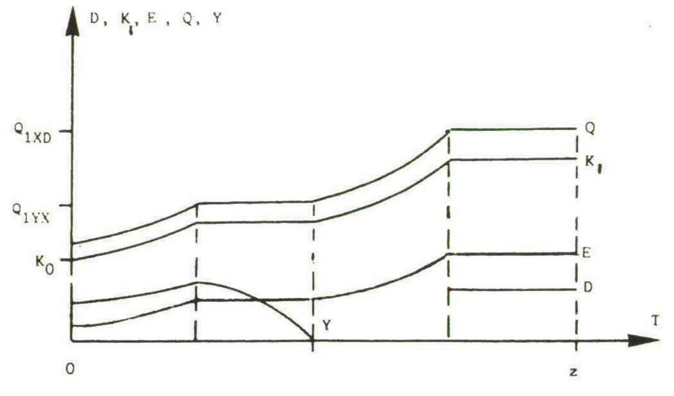


Figure 1

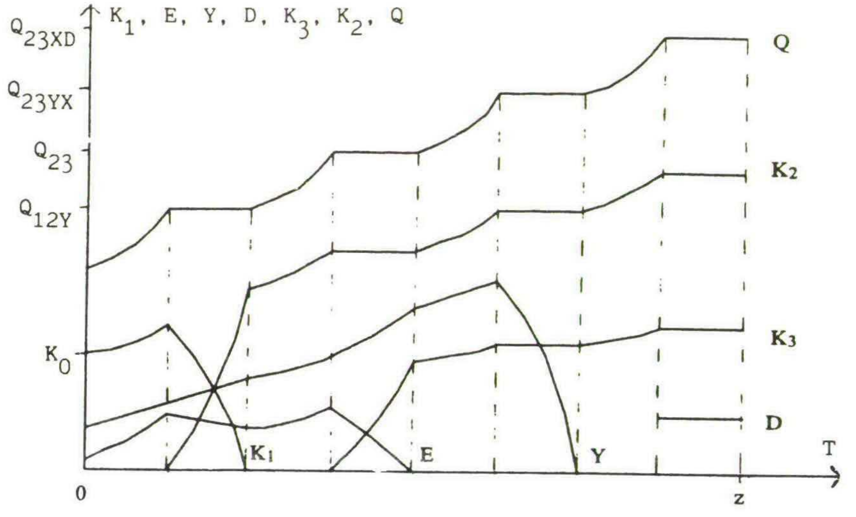
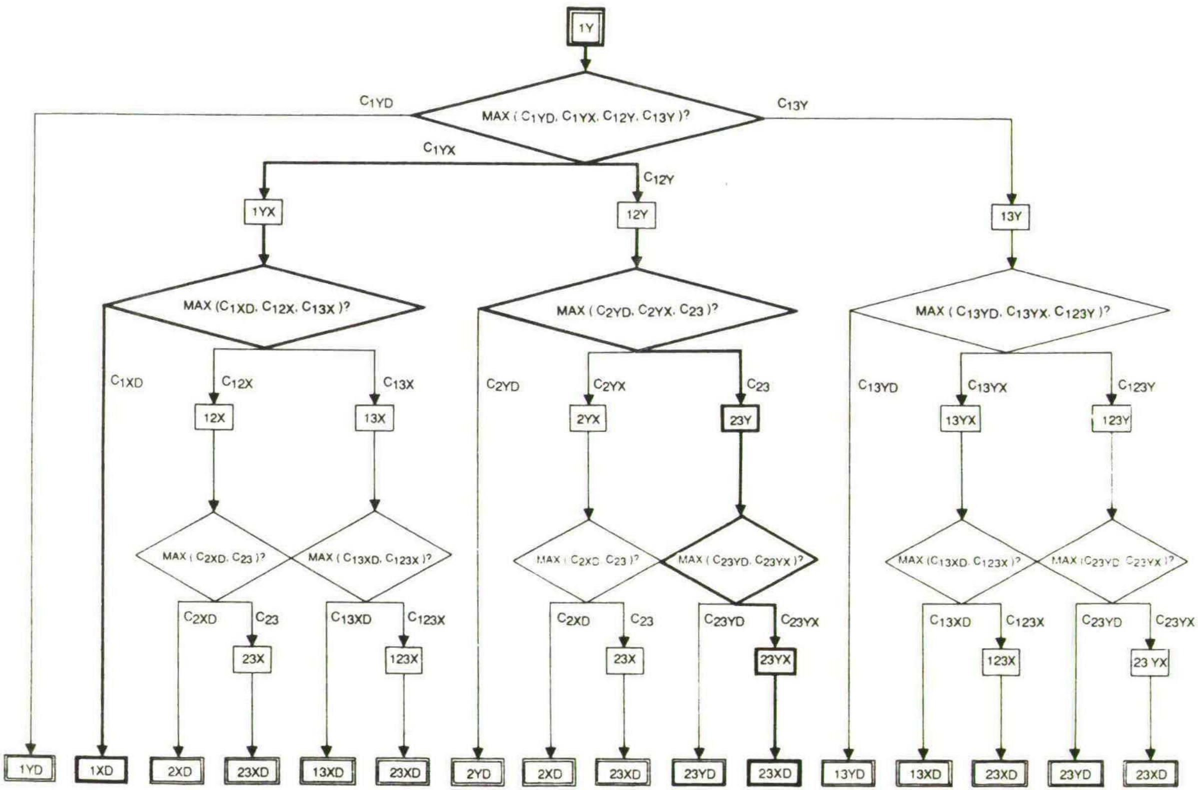


Figure 2


1Y
MAX (C1YD, C1YX, C12Y, C13Y)?
C1YD
C1YX
C12Y
C13Y
1YX
12Y
13Y
MAX (C1XD, C12X, C13X)?
MAX (C2YD, C2YX, C23)?
MAX (C13YD, C13YX, C123Y)?
C1XD
C12X
C13X
C2YD
C2YX
C23
C13YD
C13YX
C123Y
12X
13X
2YX
23Y
13YX
123Y
MAX (C2XD, C23)?
MAX (C13XD, C123X)?
MAX (C2XD, C23)?
MAX (C23YD, C23YX)?
MAX (C13XD, C123X)?
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23XD
13YD
13XD
23XD
23YD
23XD

Figure 3

path	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	K_1	K_2	K_3	E	Y	D	Q[a]
1	0	0	0	+	+	+	+	+	0	0	+	0	0	
2	0	+	0	+	+	+	0	+	0	0	+	kK	0	
3	0	0	0	+	+	+	0	+	0	0	+	+	0	1YX
4	0	0	0	+	+	0	+	+	0	0	+	0	+	1XD
5	0	+	0	+	+	0	0	+	0	0	+	kK	+	1YD
6	0	0	+	0	+	+	+	0	+	0	+	0	0	
7	0	+	+	0	+	+	0	0	+	0	+	kK	0	
8	0	0	+	0	+	+	0	0	+	0	+	+	0	2YX
9	0	0	+	0	+	0	+	0	+	0	+	0	+	2XD
10	0	+	+	0	+	0	0	0	+	0	+	kK	+	2YD
11	0	0	0	0	+	+	+	+	+	0	+	0	0	12X
12	0	+	0	0	+	+	0	+	+	0	+	kK	0	12Y
13	0	0	0	+	0	+	+	+	0	+	+	0	0	13X
14	+	0	0	+	0	+	+	+	0	+	0	0	0	
15	0	+	0	+	0	+	0	+	0	+	+	kK	0	13Y
16	+	+	0	+	0	+	0	+	0	+	0	kK	0	
17	+	0	0	+	0	+	0	+	0	+	0	+	0	13YX
18	+	0	0	+	0	0	+	+	0	+	0	0	+	13XD
19	+	+	0	+	0	0	0	+	0	+	0	kK	+	13YD
20	0	0	+	0	0	+	+	0	+	+	+	0	0	23
21	+	0	+	0	0	+	+	0	+	+	0	0	0	
22	0	+	+	0	0	+	0	0	+	+	+	kK	0	23
23	+	+	+	0	0	+	0	0	+	+	0	kK	0	
24	+	0	+	0	0	+	0	0	+	+	0	+	0	23YX
25	+	0	+	0	0	0	+	0	+	+	0	0	+	23XD
26	+	+	+	0	0	0	0	0	+	+	0	kK	+	23YD
27	+	0	0	0	0	+	+	+	+	+	0	0	0	123X
28	+	+	0	0	0	+	0	+	+	+	0	kK	0	123Y

Table 1

[a] 1YX in the column below Q means: $S'(Q) = c_{1YX}$

IN 1989 REEDS VERSCHENEN

368 Ed Nijssen, Will Reijnders
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369 Raymond Gradus
Optimal dynamic taxation with respect to firms

370 Theo Nijman
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371 Robert P. Gilles, Pieter H.M. Ruys
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372 F.A. van der Duyn Schouten, S.G. Vanneste
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Het company ranking model: a means for evaluating the competition

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Infogame Final Report

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422 J.P.C. Blanc
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423 M.H.C. Paardekooper
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424 Hans Gremmen
On the political (ir)relevance of classical customs union theory

425 Ed Nijssen
Marketingstrategie in Machtsperspectief

426 Jack P.C. Kleijnen
Regression Metamodels for Simulation with Common Random Numbers: Comparison of Techniques

427 Harry H. Tigelaar
The correlation structure of stationary bilinear processes

428 Drs. C.H. Veld en Drs. A.H.F. Verboven
De waardering van aandelenwarrants en langlopende call-opties

429 Theo van de Klundert en Anton B. van Schaik
Liquidity Constraints and the Keynesian Corridor

430 Gert Nieuwenhuis
Central limit theorems for sequences with m(n)-dependent main part

431 Hans J. Gremmen
Macro-Economic Implications of Profit Optimizing Investment Behaviour

432 J.M. Schumacher
System-Theoretic Trends in Econometrics