


# A $(49,16,3,6)$ STRONGLY REGULAR GRAPH 

 DOES NOT EXISTF.C. Bussemaker, W.H. Haemers, R. Mathon and H.A. Wilbrink

## FEW 355

A $(49,16,3,6)$ STRONGLY REGULAR GRAPH DOES NOT EXIST ${ }^{+}$
by
F.C. BUSSEMAKER,

Technological University Eindhoven, The Netherlands,
W.H. HAEMERS, University of Tilburg, The Netherlands,
R. MATHON, University of Toronto, Canada, and
H.A. WILBRINK,

Technological University Eindhoven, The Netherlands.

ABSTRACT.
We prove the non-existence of a strongly regular graph with 49 vertices and degree 16.

[^0]1. INTRODUCTION.

A strongly regular graph with 49 vertices and degree 16 has parameters $(v, k, \lambda, \mu)=(49,16,3,6)$. In this paper we show that such a graph cannot exist. Up till now it was the smallest (with respect to the number of vertices) feasible strongly regular graph for which existence was not settled. Our result is the second "ad hoc" non-existence result for strongly regular graphs. Earlier Wilbrink and Brouwer [2] proved that (57,14,1,4) cannot be the parameter set of a strongly regular graph. At the moment the smallest unsettled case is $(65,32,15,16)$. See Brouwer and Van Lint [1] for a survey of recent results on strongly regular graphs.

The present proof involves counting techniques, enumeration, linear algebra, and the use of a computer. Although only little computing time was needed, we could not manage without a computer.

## 2. COUNTING

Let $\Gamma$ denote a $(49,16,3,6)$ strongly regular graph, that is, $\Gamma$ has 49 vertices, each vertex has 16 neighbours, any two adjacent vertices have 3 common neighbours and any two distinct non-adjacent vertices have 6 commom neighbours. For a vertex $\infty$ of $\Gamma$, let $\Gamma_{\infty}$ denote the subgraph of $\Gamma$ induced by the 16 neighbours of $\infty$. Claerly, $\Gamma_{\infty}$ is regular of degree 3 , but we have more restrictions for $\Gamma_{\infty}$.

LEMMA 1.
i. The girth of $\Gamma_{\infty}$ is at least 5.
ii. Any two distict pentagons of $\Gamma_{\infty}$ have at most one edge in common.

PROOF. For a subgraph $H$ and a vertex $\alpha$ of $\Gamma$, let $h_{1}$ denote the number of vertices outside $H$ adjacent to exactly $i$ vertices of $H$, and let $h_{i}^{\alpha}$ denote the number of vertices outside $H$ adjacent to $\alpha$ and to exactly i vertices of $H$. Then for $H=K_{4}$ we have

$$
\sum_{i=0}^{4} h_{i}=45, \quad \sum_{i=1}^{4} i h_{i}=52, \quad \sum_{i=2}^{4}\left[\begin{array}{l}
i \\
2
\end{array}\right] h_{i}=6
$$

$$
-1=\sum_{i=0}^{4}\left(h_{i}-i h_{i}+\left[\begin{array}{l}
i \\
2
\end{array}\right] h_{i}\right)=h_{0}+h_{3}+3 h_{4} \geq 0
$$

a contradiction proving that $\Gamma_{\infty}$ has no triangles. Next suppose that $\Gamma_{\infty}$ contains a 4 -gon. Let $H$ be the subgraph of $\Gamma$ induced by $\infty$ and the $4-g o n$ (i.e. H is the wheel $W_{5}$ ). Then $h_{5}=0$, since $\Gamma$ contains no $K_{4}$, and we have

$$
\sum_{i=0}^{4} h_{i}=44, \quad \sum_{i=1}^{4} i h_{i}=64, \sum_{i=2}^{4}\left[\begin{array}{l}
i \\
2
\end{array}\right] h_{i}=18
$$

This implies

$$
-2=\sum_{i=0}^{4}\left(h_{i}-i h_{i}+\left[\begin{array}{l}
i \\
2
\end{array}\right] h_{i}\right)=h_{0}+h_{3}+3 h_{4} \geq 0
$$

By this contradiction $\Gamma_{\infty}$ has no 4 -gons. This completes the proof of (i). Suppose $\Gamma_{\infty}$ contains a pentagon. Let $H$ be the subgraph of $\Gamma$ induced by $\infty$ and the pentagon (i.e. $H=W_{6}$ ). By (i) $h_{i}=0$ if $i \geq 4$, hence

$$
\sum_{i=0}^{3} h_{i}=43, \quad \sum_{i=1}^{3} i h_{i}=76, \quad \sum_{i=2}^{3}\left[\begin{array}{l}
i \\
2
\end{array}\right] h_{i}=35
$$

This implies that $h_{0}+h_{3}=2$. Suppose $h_{0}>0$, that is, there exists a vertex $\omega$ adjacent to no vertex of $H$. Then

$$
\sum_{i=0}^{3} h_{i}^{\omega}=16, \quad \sum_{i=1}^{3} i h_{i}^{\omega}=36
$$

This gives $2 h_{0}^{\omega}+h_{1}^{\omega}-h_{3}^{\omega}=-4$, hence $h_{3} \geq h_{3}^{\omega} \geq 4$, a contradiction to $h_{0}+h_{3}=2$. So $h_{0}=0$, and we find $h_{1}=12, h_{2}=29$ and $h_{3}=2$. We easily have $h_{i}^{\infty}=0$ for $i \geq 3, h_{0}^{\infty}=0, h_{2}^{\infty}=5, h_{1}^{\infty}+h_{2}^{\infty}=11$, so $h_{1}^{\infty}=6$. Let $X$ be the set of vertices of $\Gamma$ not adjacent to $\infty$ and adjacent to exactly one vertex of $H$. Then $|X|=h_{1}-h_{1}^{\infty}=6$. Next suppose that there exist vertices $\alpha$ and $\beta$ in $\Gamma_{\infty}$, such that together with the vertices of $H$ they induce the following graph $H^{*}$.


Then

$$
\sum_{i=0}^{3} h_{i}^{\alpha}=14, \quad \sum_{i=1}^{3} i h_{i}^{\alpha}=22
$$

Because $h_{0}^{\alpha} \leq h_{0}=0$, we have $h_{1}^{\alpha}=6+h_{3}^{\alpha}$. There is just one vertex outside $H^{*}$ adjacent to both $\alpha$ and $\infty$. Therefore, since $h_{1}^{\alpha} \geq 6$, at least five of the vertices adjacent to $\alpha$ are contained in the set $X$. The same is true for $\beta$. Since $|x|=6$, at least four vertices of $X$ are adjacent to both $\alpha$ and $\beta$. This is a contradiction. Therefore $\Gamma$ does not contain $H^{*}$, and (ii) follows.

## 3. ENUMERATION.

The conditions of Lemma 1 are strong enough to enumerate (by hand) all feasible candidates for $\Gamma_{\infty}$.
If $\Gamma_{\infty}$ contains no pentagon, then the girth is at least six. It is easily seen that the girth cannot be bigger than six and that there is a unique 3-regular graph on 16 vertices with girth six, being:


CANDIDATE 1.

Suppose $\Gamma_{\infty}$ contains a pentagon $P$. Then each vertex of $P$ has just one neighbour outside P. By Lemma 1, these five neighbours are mutually distinct and non-adjacent. Thus $\Gamma_{\infty}$ contains the following graph $P^{*}$ as an induced subgraph.


Consider the subgraph $F$ of $\Gamma_{\infty}$, induced by the remaining six vertices. One easily sees that $F$ has just four edges and therefore, by Lemma 1, no cycles. Hence $F$ is one of the following graphs:

1

2

3

4

5

In the case that $F$ is graph number 1 or graph number 2, the isolated vertex in $F$ is adjacent to three distinct vertices of $P^{*}$, so two vertices of $P^{*}$ are adjacent to two vertices of the larger component of $F$. It is easily seen that this cannot be realised without violating Lemma 1. If F is graph number 3 or 4, the two vertices of the isolated edge in $F$ are adjacent to four distinct vertices of $P^{*}$, so one vertex of $P^{*}$ is adjacent to two vertices of the larger component of $F$. For number 3 this is clearly impossible, and for number 4 we find that $\Gamma_{\infty}$ can be obtained from the following graph by adding eight edges.


Thus, if $F$ is graph number 4, we find in a straight forward way the following ten feasible structures for $\Gamma_{\infty}$, where the adjacencies between the labelled vertices are given in the table below.


| $\begin{aligned} & \alpha \\ & \beta \\ & \gamma \\ & \delta \end{aligned}$ | $\begin{aligned} & \mathrm{a} \mathrm{a}^{\prime} \\ & \mathrm{bb}{ }^{\prime} \\ & \mathrm{cc}^{\prime} \\ & \mathrm{d} \mathrm{~d}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{aa}{ }^{\prime} \\ & \mathrm{bb}^{\prime} \\ & \mathrm{cd}^{\prime} \\ & \mathrm{d}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{aa}{ }^{\prime} \\ & \mathrm{bc}{ }^{\prime} \\ & \mathrm{cd}^{\prime} \\ & \mathrm{db}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{aa} \mathrm{a}^{\prime} \\ & \mathrm{bd}{ }^{\prime} \\ & \mathrm{cc}^{\prime} \\ & \mathrm{d}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{ad} \mathrm{~d}^{\prime} \\ & \mathrm{bc}{ }^{\prime} \\ & \mathrm{ca}{ }^{\prime} \\ & \mathrm{db}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{ad}^{\prime} \\ & \mathrm{bc} \mathrm{c}^{\prime} \\ & \mathrm{cb}^{\prime} \\ & \mathrm{da}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{a} \mathrm{a}^{\prime} \\ & \mathrm{cb}{ }^{\prime} \\ & \mathrm{bc} \mathrm{c}^{\prime} \\ & \mathrm{d} d^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{aa} \mathrm{a}^{\prime} \\ & \mathrm{cc}{ }^{\prime} \\ & \mathrm{bd} \\ & \mathrm{~d}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{aa}{ }^{\prime} \\ & \mathrm{cd}^{\prime} \\ & \mathrm{bc}{ }^{\prime} \\ & \mathrm{db}^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{ad}^{\prime} \\ & \mathrm{ca} \\ & \mathrm{bc} \mathrm{c}^{\prime} \\ & \mathrm{d} \mathrm{~b}^{\prime} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

Finally we consider the case when $F$ is graph number 5. Again it is a matter of straigt forward checking that this leads to just two new condidates:


CANDIDATE 12


CANDIDATE 13

This completes the enumeration of the thirteen candidates for $\Gamma_{\infty}$.

## 4. LINEAR ALGEBRA

In terms of the adjacency matrix $A$ the definition of a $(49,16,3,6)$ strongly regular graph reads
(*) $\quad A^{2}=-3 A+10 I+6 J$.
(We use $I$ for the identity matrix, $J$ for the all-one matrix and $j$ for the all-one vector.) The eigenvalues of $A$ are 16,2 and -5 with multiplicities 1,32 and 16 , respectively. We say that a graph $\Gamma_{1}$ is extendable to a graph $\Gamma$, whenever $\Gamma_{1}$ is a subgraph of $\Gamma$ induced by the neighbours of some vertex of $\Gamma$.

## LEMMA 2.

Let $\Gamma_{1}$ be a 3-regular graph on 16 vertices with adjacency matrix $A_{1}$, and let 2 not be an eigenvalue of $A_{1}$. Then $\Gamma_{1}$ is extendable to $a(49,16,3,6)$ strongly regular graph if and only if the vertex set of $\Gamma_{1}$ admits 32 distinct 6 -subsets with charateristic vectors $x_{1}, \ldots, x_{32}$ (say), such that
i.
ii.

$$
\begin{aligned}
& X X^{T}+A_{1}^{2}=-3 A_{1}+10 I+5 \mathrm{~J} \text {, where } X=\left[x_{1}, \ldots, x_{32}\right] \text {, }
\end{aligned}
$$

$$
\begin{aligned}
& \text { where } \tilde{x}_{i}=7 x_{i}-2 j(i=1, \ldots, 32) \text { and } \widetilde{A}_{1}=7 A_{1}-14 I-2 J \text {. }
\end{aligned}
$$

PROOF. First suppose we have 32 such subsets. Put $\widetilde{X}=7 X-2 J, \widetilde{A}_{2}=\widetilde{X}^{T} \widetilde{A}_{1}^{-1} \tilde{X}$ and

$$
\widetilde{A}=\left[\begin{array}{rcc}
-16 & 5 j^{T} & -2 j^{T} \\
5 j & \widetilde{A}_{1}^{1} & \widetilde{X} \\
-2 j & \widetilde{X}^{\mathrm{T}} & \widetilde{A}_{2}
\end{array}\right],
$$

then it follows by straight forward verification that $\widetilde{A}^{2}=49 \widetilde{A}$. Hence the $(0,1)$ matrix $A=\frac{1}{7}(\widetilde{A}+14 I+2 J)$ satisfies $\left(^{*}\right)$ and therefore $A$ is the adjacency matrix of a $(49,16,3,6)$ strongly regular graph.
Next suppose $\Gamma_{1}$ is extendable, that is, there exist matrices $X$ and $A_{2}$ such that the matrix

$$
A=\left[\begin{array}{ccc}
0 & j^{T} & 0^{T} \\
j & A_{1} & X \\
0 & X^{T} & A_{2}
\end{array}\right]
$$

is the adjacency matrix of a $(49,16,3,6)$ strongly regular graph. Let $x_{1}, \ldots, x_{32}$ be the columns of $X$. Then clearly the $x_{i}^{\prime} s$ consist of 6 ones and 10 zeros, and (i) follows from (*). To prove (ii) define $\widetilde{A}=7 \mathrm{~A}-14 \mathrm{I}-2 \mathrm{~J}$. (In terms of association schemes, $\widetilde{A}$ is a multiple of a minimal idempotent in the Bose-Messner algebra.) Since A has eigenvalue 16 with eigenvector $j$ and eigenvalue 2 with multiplicity 32 , and since A, I and J have a common full set of eigenvectors, $\tilde{A}$ must have an eigenvalue 0 of multiplicity 33 , so $\operatorname{rank}(\widetilde{A})=16$. Also $A_{1}, I$ and $J$ have a common full set of eigenvectors and therefore, because $A_{1}$ has eigenvalue 3 with eigenvector $j$ and no eigenvalue 2, $\widetilde{\mathrm{A}}_{1}$ has no eigenvalue 0 , so $\widetilde{\mathrm{A}}_{1}$ is non-singular. Thus the following submatrix of $\widetilde{A}$ has rank 16.

$$
\left[\begin{array}{cc}
\widetilde{\mathrm{A}}_{1}^{1} & \tilde{\mathrm{x}}_{\mathrm{j}} \\
\widetilde{\mathrm{x}}_{\mathrm{i}}^{\mathrm{T}} & \alpha
\end{array}\right]
$$

where $\alpha=-16$ if $i=j$, and $\alpha=-2$ or 5 otherwise. Therefore the last column is a linear combination of the other columns, that is, there exists a vector $v$ such that $\tilde{A}_{1} v=\tilde{x}_{j}$ and $\widetilde{x}_{i}^{T} v=\alpha$. Hence $\alpha=\widetilde{x}_{i}^{T} \widetilde{A}_{i}^{-1} \tilde{x}_{j}$. This proves (ii). Finally, if $x_{i}=x_{j}$, then (ii) implies $i=j$, so all 32 vectors $x_{i}$ are distinct.

We remark that the above lemma generalizes to strongly regular graphs for which the multiplicity of one of the eigenvalues equals the degree. These are precisely the so called latin square graphs, negative latin square graphs and conference graphs.

## 5. COMPUTER RESULTS

For all 13 candidates the eigenvalues of the adjacency matrix have been computed. None has an eigenvalue equal to 2 , so Lemma 2 applies in all cases. For each candidate we computed the matrix $\tilde{\mathrm{A}}_{1}^{-1}$, and made a list of all 6 -subsets for which the characteristic vector $x_{i}$ satisfies $\widetilde{x}_{i}^{T} \widetilde{A}_{1}^{-1} \tilde{x}_{i}=-16$ (we use the notation of Lemma 2). For each candidate we searched by computer for 32 vectors in the list that also satisfy the other conditions of Lemma 2. For no candidate 32 suitable 6 -subsets were found. Hence, we have:

THEOREM.
There exists no $(49,16,3,6)$ strongly regular graph.

The mentioned computer search is not necessary. All candidates can be ruled out by ad hoc arguments ones the lists of feasible 6-subsets are computed. For instance, candidates number. 3, 4 and 10 do not work because the list is smaller than 32 , and candidates number 5 and 8 do not work because an edge (edge $\{a, d\}$ and $\{a, \alpha\}$ respectively) is contained in just one set of the list, whilst by Lemma 2.1 two sets are needed. For the other candidates the ad hoc arguments are more complicated. But it doesn't seem useful to spend a lot of place and effort to treat all these arguments, since a computer search is needed anyway to generate the lists of 6 -subsets.

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[^0]:    *) The research for this paper was done in 1979 at the Technological University of Eindhoven.

