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RESEARCH MEMORANDUM

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ESTIMATION OF A GAME THEORETIC MODEL OF HOUSEHOLD LABOR SUPPLY

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Abstract

Usually, one of the assumptions underlying household labor supply models is that the household's preferences can be represented by a joint utility or cost function. In this paper an empirical household labor sypply model is developed within the framework of two-person co-operative game theory, allowing to test the equality of the utility functions of the spouses.

The model is confined to Pareto optimal allocations (solutions). However, since the actual allocation in a household will depend on the relative power of the spouses we do not explicitly choose one particular Pareto optimal allocation (such as, for example, Nash bargaining) as the preferred solution concept. Instead, using data on both actual and preferred working hours, the Pareto optimal allocation actually chosen by households is estimated.

1. Introduction

Much of the recent empirical work on labor supply has concentrated on the econometric problems associated with non-participation, non-linear and non-convex budget sets, rationing and stochastic specification. In addition, recognizing the interdependence of the labor supply of the spouses in a household, research is directed towards modelling male and female labor supply simultaneously (e.g. Blundell and Walker (1982) and Hausman and Ruud (1984)).

Usually, a maintained assumption underlying these models is that the household is a homogeneous decision makng unit, so that it's preferences can be represented by a joint utility (or cost) function. Although this seems a strong assumption, the literature on empirical models that are based on a more general household decision making framework is limited.

An important theoretical paper on household decision making is the paper by Manser and Brown (1980). Placing the household decision making process in a bargaining framework and applying two-person cooperative game theory, these authors discuss a number of Pareto optimal allocations, including the dictatorial, Nash bargaining and Kalai-Smorodinsky allocations. Each solution implies a different form for the labor supply functions. One of the problems in applied work is that it is not clear which of these solutions should be the basis for the empirical model. McElroy and Horney (1981) concentrate on the theoretical properties of household demand functions based on Nash bargaining. In empirical work using cooperative game theory, Nash bargaining is usually assumed, see Horney and McElroy (1980) and Brown and Manser (1978).

A different approach is followed by Ashworth and Ulph (1981) who estimate a model originally proposed by Leuthold (1968). The Leuthold model assumes that each individual in the household maximizes an independent utility function, given the labor supply of the partner. Both partners then adjust their labor supply until their decisions are mutually consistent. A major difficulty with the Leuthold allocation (which is in fact a non-cooperative Nash equilibrium) is that it is generally not Pareto optimal. The same objection applies to the paper by Bjorn and Vuong (1984), who estimate a labor force participation model employing non-cooperative game theory. As has been argued by Manser and Brown it is more appropriate to employ models which yield Pareto optimal solutions.

A distinctive feature of the model presented in this paper is the close connection between the neoclassical theory of labor supply, game theory and econometric methods. The model describes the set of Pareto optimal allocations of male and female leisure and household consumption. The model is estimated using a data set which does not only contain information on how many hours each partner actually works per week, but also on how many hours they would like to work. Using this extra information, it is possible to explain both preferred and actual working hours, without choosing a priori one particular Pareto optimal allocation as the preferred solution concept. As a matter of fact, the information allows us to estimate the Pareto optimal allocation actually chosen by households. In addition, it allows us to test the equality of the utility functions of both spouses.

The plan of the paper is as follows. In Section 2, we give a discussion of some game theoretic models of household labor supply. In Section 3 we present our specification, and discuss estimation of the model. Section 4 contains empirical results and Section 5 concludes.

2. Household decision making: a graphical exposition

We only consider households where both a male and a female partner are present. The preferences of the i-th (i=m,f; m denotes male, f denotes female) partner can be represented by a well-behaved utility function $U^i(\ell_m,\ell_f,y)$, where ℓ_m is male leisure, ℓ_f is female leisure and y is total household consumption. The dictatorial point $D^i \equiv (\ell_m^i,\ell_f^i,y^i)$ of the i-th partner is defined as the solution of the maximization problem:

$$\max_{\ell_m,\ell_f,y} \quad U^i(\ell_m,\ell_f,y) \qquad i = m,f \tag{2.1}$$

$$\text{s.t. } w_m\ell_m + w_f\ell_f + y = Y \equiv w_mT + w_fT + \mu \tag{2.2}$$

where w_m and w_f are the male and female wage rate, T is total time endowment and μ is non-labor income[1]; Y is full income.

After eliminating y from (2.1) using the full income constraint (2.2), the utility functions can be conveniently represented in the (ℓ_m,ℓ_f)-plane; see figure 2.1. The solid curves around the dictatorial point D^f are the indifference curves of the female partner, the dotted curves around D^m are the indifference curves of the male partner. The farther an indifference curve is removed from a dictatorial point, the lower is the utility level corresponding to this curve.

1) The wage rates and non-labor income are all measured after taxes.

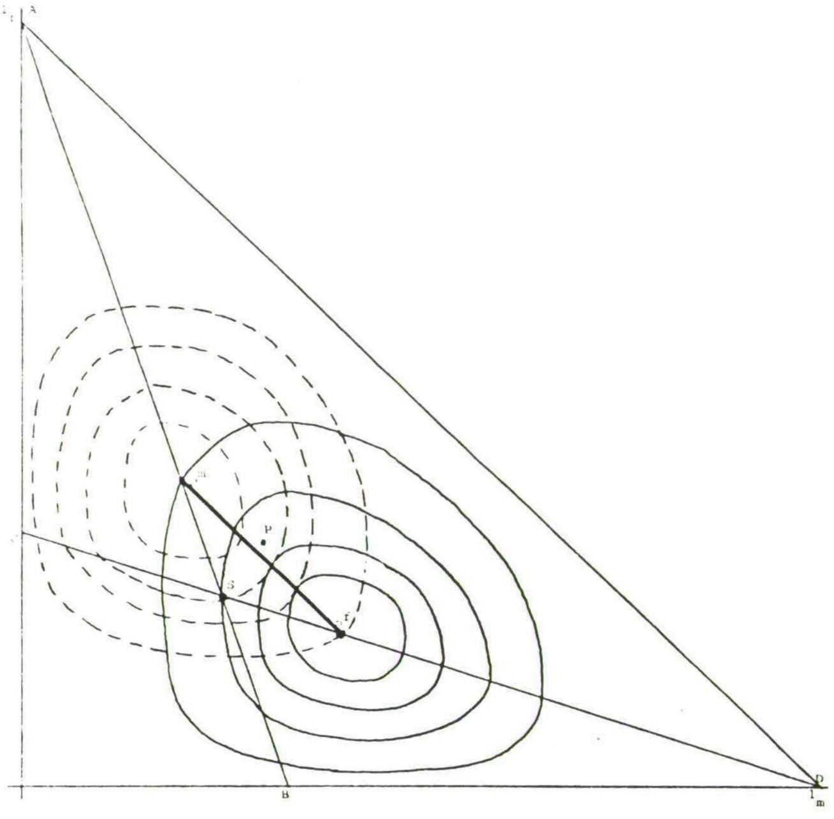

Figure 2.1.[1]

Using figure 2.1., we can also easily represent the allocation implied by the Leuthold model. The rationed (or conditional) leisure demand equation of the male partner, given the leisure demand of the female partner (reaction curve), is the line AB in figure 2.1. It connects the tangency points of horizontal lines with the indifference curves around D^m. Analogously, line CD is the rationed leisure demand equation of the female partner given the leisure demand of the male partner. Graphically, the Leuthold equilibrium is represented as the

1) Figure 2.1. is based on Stone-Geary utility functions (see Section 3), with $w_m=w_f=1$; $\gamma_m=\gamma_f=\gamma_y=\delta_m=\delta_f=\delta_y=0$; $\alpha_m=0.2$; $\alpha_f=0.4$; $\beta_m=0.4$; $\beta_f=0.2$.

point of intersection S of the reaction curves AB and CD. Clearly, S is not Pareto optimal, as both partners can improve by moving from S to, for example, P[1].

In figure 2.1. we can also clearly visualize the set of Pareto optimal allocations (i.e. the contract curve). Obviously, all tangency points of indifference curves around D^m with indifference curves around D^f, between (and including) both dictatorial points, represent Pareto optimal allocations. Hence, the contract curve satisfies:

$$\frac{\partial V^m/\partial \ell_m}{\partial V^m/\partial \ell_f} = \frac{\partial V^f/\partial \ell_m}{\partial V^f/\partial \ell_f} , \tag{2.3}$$

where $V^i = U^i(\ell_m, \ell_f, Y - w_m \ell_m - w_f \ell_f) \qquad i = m, f.$

Equation (2.3) follows immediately from the first order conditions for maximizing $V^i\ (\ell_m, \ell_f, y)$ subject to $V^j\ (\ell_m, \ell_f, y) = V_0^j$ ($i=m,f$; $j=f,m$). Alternatively, it can be obtained by maximizing a convex combination of V^m and V^f, i.e. by maximizing

$$\tilde{V} = (1-\lambda)V^m + \lambda V^f \tag{2.4}$$

for a given λ $(0 \leq \lambda \leq 1)$

The contract curve can also be characterized using the concept of a <u>shadow wage</u> set forth in, for example, Neary and Roberts (1980).
In terms of U^i (2.3) can be written as

$$\frac{\dfrac{\partial U^m}{\partial \ell_m} \Big/ \dfrac{\partial U^m}{\partial y} - w_m}{\dfrac{\partial U^m}{\partial \ell_f} \Big/ \dfrac{\partial U^m}{\partial y} - w_f} = \frac{\dfrac{\partial U^f}{\partial \ell_m} \Big/ \dfrac{\partial U^f}{\partial y} - w_m}{\dfrac{\partial U^f}{\partial \ell_f} \Big/ \dfrac{\partial U^f}{\partial y} - w_f} \tag{2.5}$$

1) Neither Leuthold nor Ashworth and Ulph pay attention to the game theoretic properties of the equilibrium implied by their model. A thorough discussion of this kind of equilibrium is provided in Basar and Olsder (1982).

The shadow wages $\bar{w}^i_j$ (i,j=m,f) at which a particular point on the contract curve would be optimal for the i-th partner (i.e. would coincide with his or her dictatorial point) is defined by

$$\frac{\partial V^i}{\partial \ell_j} = \frac{\partial U}{\partial \ell_j} - \frac{\partial U^i}{\partial y} . \bar{w}^i_j = 0 \qquad (2.6)$$

or equivalently

$$\bar{w}^i_j = \frac{\partial U^i}{\partial \ell_j} \Big/ \frac{\partial U^i}{\partial y} \qquad (2.7)$$

Using (2.7) we can rewrite (2.5) as

$$\frac{\bar{w}^m_m - w_m}{\bar{w}^m_f - w_f} = \frac{\bar{w}^f_m - w_m}{\bar{w}^f_f - w_f} \qquad (2.8)$$

In addition, the shadow wages satisfy

$$g^m_i \; (\bar{w}^m_m, \; \bar{w}^m_f, \; \mu) = g^f_i \; (\bar{w}^f_m, \; \bar{w}^f_f, \; \mu) \qquad i = m,f \qquad (2.9)$$

where g^j_i (w_m, w_f, μ) is the demand equation for ℓ_i of the j-th partner in the case of dictatorship.

The characterization of the contract curve in terms of shadow wages is particularly useful in the case where preferences are represented by a cost function or an indirect utility function and no explicit form for the corresponding direct utility function is known (as in the case of, for example, the Almost Ideal Demand System).

Following the arguments in Manser and Brown (1980) we will impose Pareto optimality of the actual allocatons in our model. Assuming that partners know each other so well that they cannot hide their true preferences, the Pareto optimality property follows directly from the assumption of utility maximizing behavior. An important advantage of the approach followed in this paper is that it is not necessary to choose a priori one particular Pareto optimal allocation, such as Kalai-Smorodinsky, Nash bargaining or male or female dictatorship as the preferred solution concept. The actual allocation in a household (i.e. the point on the contract curve) will depend on the relative power of the spouses in the household labor supply decision. If the influence of the male partner is relatively large, the actual allocation is likely to be close to the male dictatorial point D^m; if the female influence is relatively large, we expect the actual allocation to be close to the female dictatorial point D^f. We leave the relative influence of the partners as an empirical matter by estimating a parameter representing the point on the contract curve corresponding with the actual allocation.

3. Specification of the model and the estimation method.

As a specification for the utility functions (2.1) we choose the familiar Stone-Geary utility function:

$$U^m = \alpha_m \log(\ell_m-\gamma_m) + \alpha_f \log(\ell_f-\gamma_f) + \alpha_y \log(y-\gamma_y) \qquad (3.1)$$

$$U^f = \beta_m \log(\ell_m-\delta_m) + \beta_f \log(\ell_f-\delta_f) + \beta_y \log(y-\delta_y) \qquad (3.2)$$

with $\alpha_y = 1 - \alpha_m - \alpha_f$

$\beta_y = 1 - \beta_m - \beta_f$

The demand equations in the case of male or female dictatorship are given by:

$$w_i\ell_i^m = w_i\gamma_i + \alpha_i(Y-w_m\gamma_m-w_f\gamma_f-\gamma_y) \qquad (i = m,f) \qquad (3.3)$$

and

$$w_i\ell_i^f = w_i\delta_i + \beta_i(Y-w_m\delta_m-w_f\delta_f-\delta_y) \qquad (i = m,f) \qquad (3.4)$$

respectively.

It is important to note that the number of unknown utility parameters is twice as large as in household labor supply models where preferences are represented by a single utility function. Therefore, extra information is needed to be able to estimate all parameters. A particular feature of the data we use is that it does not only contain information on the number of hours partners actually work (actual hours), but it also tells us how many hours each partner would like to work at the going wage rate (preferred hours). With respect to the latter variable we assume that it is determined exclusively on the basis of the respondent's own preferences. This data interpretation is supported by the fact that in the phrasing of the question there is no reference at all to the partner's behavior or preferences. Moreover, partners answer these questions separately, without a possibility of joint decision making.

As a result, we have the following preferred hours equations:

$$w_m \ell_m^p = w_m \gamma_m + \alpha_m (Y - w_m \gamma_m - w_f \gamma_f - \gamma_y) + \varepsilon_m^p \tag{3.5}$$

$$w_f \ell_f^p = w_f \delta_f + \beta_f (Y - w_m \delta_m - w_f \delta_f - \delta_y) + \varepsilon_f^p \tag{3.6}$$

where the supercript p refers to preferred hours.

Turning to the interpretation of the actual hours observed in the data, we first note that the utility functions (3.1) and (3.2) are only well-defined if

$$\ell_m > \gamma_m, \ \ell_f > \gamma_f, \ y > \gamma_y \tag{3.7}$$

$$\ell_m > \delta_m, \ \ell_f > \delta_f, \ y > \delta_y \tag{3.8}$$

As is well-known, the γ's and δ's can be interpreted as subsistence quantities. We assume that in the household decision making process the partners respect each other's subsistence quantities, so that the contract curve is based on

$$\hat{U}^m = \alpha_m \ \ell og(\ell_m - \eta_m) + \alpha_f \ \ell og(\ell_f - \eta_f) + \alpha_y \ \ell og(y - \eta_y) \tag{3.9}$$

$$\hat{U}^f = \beta_m \ \ell og(\ell_m - \eta_m) + \beta_f \ \ell og(\ell_f - \eta_f) + \beta_y \ \ell og(y - \eta_y) \tag{3.10}$$

where $\eta_m = \max(\gamma_m, \delta_m)$, $\eta_f = \max(\gamma_f, \delta_f)$ and $\eta_y = \max(\gamma_y, \delta_y)$.

A practical advantage of this assumptions is that the contract curve now reduces to a straight line trough both dictatorial points[1]:

$$\ell_f = c_0 + c_1 \ell_m \tag{3.11}$$

with

1) There are no conceptual problems with using more flexible specifications. A practical complication however is that in general it is not possible to derive an explicit closed form for the contract curve, so that in the estimation procedure numerical methods have to be used.

$$c_1 = \frac{\alpha_f - \beta_f}{\alpha_m - \beta_m} \cdot \frac{w_m}{w_f} \tag{3.12}$$

$$c_0 = (\gamma_f - c_1\gamma_m) + Y^*(\frac{\alpha_f}{w_f} - c_1 \cdot \frac{\alpha_m}{w_m}) \tag{3.13}$$

$$Y^* = Y - w_m\eta_n - w_f\eta_f - \eta_y \tag{3.14}$$

(see Appendix A for details)

Now we can write the actual hours equations as

$$w_m\ell_m^a = w_m\eta_m + Y^* \{\alpha_m + \lambda(\beta_m - \alpha_m)\} + \varepsilon_m^a \tag{3.15}$$

$$w_f\ell_f^a = w_f\eta_f + Y^* \{\alpha_f + \lambda(\beta_f - \alpha_f)\} + \varepsilon_f^a \tag{3.16}$$

$$0 \leqslant \lambda \leqslant 1$$

where the superscript a refers to actual hours. So, for the present specification, the marginal budget share in the actual hours equation is a convex combination of the marginal budget shares in the corresponding male and female dictatorial equations.

If $\lambda = 0$ we have male dictatorship, if $\lambda = 1$ we have female dictatorship.

It is easily seen that (3.15) and (3.16) can also be obtained using (2.7) and maximizing

$$\tilde{U} = (1-\lambda)\hat{U}^m + \lambda\hat{U}^f$$

$$= \nu_m \,\ell og(\ell_m - \eta_m) + \nu_f \,\ell og(\ell_f - \eta_f) + \nu_y \,\ell og(y - \eta_y) \tag{3.17}$$

$$\text{s.t. } w_m\ell_m + w_f\ell_f + y = Y \tag{2.2}$$

where $$\nu_m = \alpha_m + \lambda(\beta_m - \alpha_m) \tag{3.18}$$

$$\nu_f = \alpha_f + \lambda(\beta_f - \alpha_f) \tag{3.19}$$

$$\nu_y = \alpha_y + \lambda(\beta_y - \alpha_y) \tag{3.20}$$

which implies that for this specification it would be legitimate in explaining actual hours to assume that the household's preferences can be represented by one Stone-Geary utility function, even if the spouses have different preferences. However, in general the functional form of the actual hours equations obtained by maximizing (2.4) will differ from the functional form of the preferred hours equations obtained by maximizing V^m and V^f seperately. In that case, employing the traditional model (based on one utility function) is generally a misspecification.

The error terms ε_m^a, ε_f^a, ε_m^p en ε_f^p are introduced to account for omitted variables, optimization errors, etc. In the actual hours equations the ε's may also comprise the effects of institutional restrictions.

In order to identify the parameters of both U^m and U^f, as well as λ, one extra restriction on the marginal budget share parameters is needed. We choose $\alpha_y = \beta_y$.

Assuming the error terms to be normally distributed with zero mean and (unrestricted) covariance matrix Σ, we estimate equations (3.5), (3.6), (3.15) and (3.16) jointly using Full Information Maximum Likelihood.

In the estimation only households are used where both the male and the female partner work in a paid job for at least 15 hours per week. The 15 hours cut-off point is dictated by the survey design by which certain items of information are not collected for people who work less than 15 hours per week. This sample selection rule can be taken into account appropriately by maximizing the likelihood function.

$$L = \prod_n \frac{f_1^n(\ell_m^a, \ell_f^a, \ell_m^p, \ell_f^p)}{\int_0^{T-15} \int_0^{T-15} f_2^n(\ell_m^a, \ell_f^a)\, d\ell_m^a\, d\ell_f^a} \tag{3.21}$$

Here $f_1^n(\ell_m^a, \ell_f^a, \ell_m^p, \ell_f^p)$ is the joint density function of ℓ_m^a, ℓ_f^a, ℓ_m^p and ℓ_f^p implied by (3.5), (3.6), (3.15) and (3.16) for the n-th household and $f_2^n(\ell_m^a, \ell_f^a)$ is the joint marginal density function of ℓ_m^a and ℓ_f^a for the n-th household.

The likelihood (3.21) is maximized using a quasi-Newton algorithm which requires no (analytical) derivatives, as provided by the NAG-Library (EØ4JBF). The (asymptotic) covariance matrix of the maximum likelihood estimators is estimated by the inverse of the (numerically calculated) Hessian of the min-ℓoglikelihood function:

$$\text{var}\,(\hat{\theta}) = \left(\frac{\partial^2 \ell nL(\theta)}{\partial\theta\partial\theta'}\right)^{-1}_{\theta\,=\,\hat{\theta}} \tag{3.22}$$

where θ denotes the parameters of the model and $\hat{\theta}$ is the maximum likelihood estimate of θ.

Although the estimation of the type of models developed in this paper requires information on both actual and preferred hours, it is not necessary to know the 'exact' number of preferred hours. It is sufficient to know whether the respondent is content with his or her working time and, if not, whether he or she wants to work fewer or more hours per week (assuming that the net wage per hour does not change). Ham (1982) uses this type of data. In the case, for example, where both partners want to work fewer hours than they actually do, the numerator of the expression for the contribution of this household to the likelihood function becomes

$$\int_{\ell_f^a}^{\infty} \int_{\ell_m^a}^{\infty} f_1(\ell_m^a, \ell_f^a, x, y)\,dx\,dy \tag{3.23}$$

4. Empirical results

The results of the FIML-estimation of the model are summarized in table 4.1.

Table 4.1. Estimation results

parameter	estimate	standard error
α_m	0.29	0.10
α_f	0.02	0.13
β_m	0.17	0.07
β_f	0.14	0.05
γ_m 1)	119.2	8.1
γ_f 1)	140.8	22.4
γ_y	348.1	249.7
δ_m 1)	123.4	3.3
δ_f 1)	141.3	2.6
δ_y	644.6	91.9
λ	0.64	0.74

$$\tilde{\Sigma}\ ^{2)} = \begin{bmatrix} 5436 & . & . & . \\ 0.08 & 10170 & . & . \\ -0.05 & -0.08 & 14170 & . \\ -0.13 & 0.04 & 0.66 & 3961 \end{bmatrix}$$

In the first place, we note that the estimated α's, β's and λ fall between zero and one, as they should. As has been noted before, the utility functions (3.1) and (3.2) are only well-defined if the observed quantities exceed the subsistence quantities. We have checked per observation point whether these conditions are satisfied; see table 4.2.

1) T is set equal to 168 hours per week. The estimates of $(T-\gamma_i)$ and $(T-\delta_i)$ (i=m,f), however, are independent of the choice of T.

2) Diagonal elements are variances, off-diagonal elements are correlation coefficients.

Table 4.2. Percentage observations satisfying regularity conditions

	γ_m	γ_f	γ_y	δ_m	δ_f	δ_y
Preferred hours	98	55	98	96	55	70
Actual hours	89	41	100	82	41	89

In view of the restrictive functional forms of the labor supply equations implied by the additive utility functions (3.1) and (3.2), the numbers in table 4.2. are rather encouraging.

Although it is tempting to interpret the estimated λ as an indication of the relative power of the spouses in the decision regarding joint labor supply, it should be borne in mind that the results might be affected by (for male and female partner possibly different) institutional constraints. Moreover, the estimated standard error of λ is relatively large. Therefore we shall abstain from an interpretation of λ in terms of relative power.

It is of interest to test whether the utility functions of both spouses are significantly different. So, we test the joint hypothesis:

$$\alpha_i = \beta_i \qquad i = m, f \tag{4.1}$$

$$\gamma_i = \delta_i \qquad i = m, f, y \tag{4.2}$$

An appropriate test in this case is the Wald test[1]. On the basis of the value of the Wald statistic (47.7), the null hypothesis of equal utility functions is rejected decisively. Subject to the qualifications of the model, this result indicates that the traditional neo-

1) Writing (4.1) and (4.2) as $h(\theta_0) = 0$, the Wald statistic is

$$W = h(\hat{\theta}_0)' \left[\mathrm{cov}\ h(\hat{\theta}_0)\right]^{-1} h(\hat{\theta}_0) \tag{4.3}$$

where cov $h(\hat{\theta}_0)$ is obtained from the covariance matrix of the ML estimator of θ. W has (asymptotically) a χ^2-distribution with 5 degrees of freedom. The critical levels for 5% and 2.5% are 11.1 and 12.8, respectively.

classical framework, where preferences are represented by a single utility function, is too limited as a description of the household's labor supply decision. Horney and McElroy (1980) and Brown and Manser (1978) come to similar conclusions.

5. Concluding remarks

Models of household labor supply are usually based on the assumption that the household's preferences can be represented by a single utility function. In this paper we have developed an empirical model which starts from a more general household decision making framework. It is based on the assumption that partners cooperate and confine themselves to Pareto optimal bargains.

The estimation results are plausible and a test of the more general model against the traditional model supports the more general model.

Although the specifications used in the empirical part of the paper are rather simple, the extension to using more flexible specifications is straightforward. Indeed, using the concept of shadow wages (see Section 2), it is in principle possible to employ specifications for which no explicit form for the direct utility function exists. However, the computational burden of such models will be substantially higher.

Appendix A. Derivation of the contract curve

Applying (2.3) to (3.9) and (3.10) we find that the contract curve satisfies

$$(\ell_m \;\ell_f \;1)\begin{pmatrix} A & B & D \\ B & C & E \\ D & E & F \end{pmatrix}\begin{pmatrix} \ell_m \\ \ell_f \\ 1 \end{pmatrix} = 0 \tag{A. 1}$$

where

$$A = w_m^2(\beta_f-\alpha_f) \tag{A. 2}$$

$$B = \tfrac{1}{2}w_m w_f\{(\alpha_m-\beta_m) + (\beta_f-\alpha_f)\} \tag{A. 3}$$

$$C = w_f^2(\alpha_m-\beta_m) \tag{A. 4}$$

$$D = \tfrac{1}{2}w_m\{-(1-\alpha_f)Q_1 - \beta_f Q_2 + \alpha_f Q_3 + (1-\beta_f)Q_4\} \tag{A. 5}$$

$$E = \tfrac{1}{2}w_f\{-\alpha_m Q_1-(1-\beta_m)Q_2 + (1-\alpha_m)Q_3 + \beta_4 Q_4\} \tag{A. 6}$$

$$F = Q_1Q_2 - Q_3Q_4 \tag{A. 7}$$

with

$$Q_1 = \beta_f(Y-\gamma_y) + \beta_y w_f \gamma_f \tag{A. 8}$$

$$Q_2 = \alpha_m(Y-\gamma_y) + \alpha_y w_m \gamma_m \tag{A. 9}$$

$$Q_3 = \beta_m(Y-\gamma_y) + \beta_y w_m \gamma_m \tag{A.10}$$

$$Q_4 = \alpha_f(Y-\gamma_y) + \alpha_y w_f \gamma_f \tag{A.11}$$

Assume that

$$\begin{vmatrix} A & B \\ B & C \end{vmatrix} = -\tfrac{1}{4}w_m^2\, w_f^2\, [(\alpha_m-\beta_m) - (\beta_f-\alpha_f)]^2 \tag{A.12}$$

is nonzero.

Define

$$\begin{pmatrix} \tilde{\ell}_m \\ \tilde{\ell}_f \end{pmatrix} = \begin{pmatrix} \ell_m \\ \ell_f \end{pmatrix} - \begin{pmatrix} A & B \\ B & C \end{pmatrix}^{-1} \begin{pmatrix} D \\ E \end{pmatrix} \tag{A.13}$$

and

$$k = F - (D\ E) \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \begin{pmatrix} D \\ E \end{pmatrix} \tag{A.14}$$

Then (A.1) can be rewritten as

$$(\tilde{\ell}_m\ \tilde{\ell}_f) \begin{pmatrix} A & B \\ B & C \end{pmatrix} \begin{pmatrix} \tilde{\ell}_m \\ \tilde{\ell}_f \end{pmatrix} = k \tag{A.15}$$

After substitutions of (A.2) - (A.11) into (A.14) and some straight forward manipulations it turns out that $k = 0$. Next, solving $\tilde{\ell}_f$ from

$$(\tilde{\ell}_m\ \tilde{\ell}_f) \begin{pmatrix} A & B \\ B & C \end{pmatrix} \begin{pmatrix} \tilde{\ell}_m \\ \tilde{\ell}_f \end{pmatrix} = 0 \tag{A.16}$$

we find

$$\tilde{\ell}_f = \tilde{\ell}_m \left\{- \frac{B}{C} \pm \sqrt{\frac{B^2-AC}{C^2}}\right\} \tag{A.17}$$

So, (A.1) - (A.11) define two straight lines. Their slopes are

$$c_1 = \frac{-B}{C} + \sqrt{\frac{B^2-AC}{C^2}} = -\frac{w_m}{w_f}\cdot \frac{\beta_f-\alpha_f}{\alpha_m-\beta_m} \tag{A.18}$$

and

$$d_1 = \frac{-B}{C} - \sqrt{\frac{B^2-AC}{C^2}} = -\frac{w_m}{w_f} \tag{A.19}$$

Their intercepts are

$$c_0 = (\gamma_f - c_1\gamma_m) + Y^*(\frac{\alpha_f}{w_f} - c_1 \cdot \frac{\alpha_m}{w_m}) \quad \text{(A.20)}$$

and

$$d_0 = \frac{Y-\gamma_y}{w_f} \quad \text{(A.21)}$$

respectively.

The line $\ell_f = c_0 + c_1\ell_m$ is the line through both dictatorial points. All points between (and including) both dictatorial points represent Pareto optimal allocations. The line $\ell_f = d_0 + d_1\ell_m$ is the line AD in figure 2.1. at which the indifference contours of the male and the female partner coincide. Clearly, the points on this line are not Pareto optimal, as the utility level of both partners is minus infinity.

Appendix B. The data

The model has been estimated for data from a labor mobility survey in the Netherlands, conducted in the fall of 1982 by the Netherlands Central Bureau of Statistics and the Institute for Social Research of Tilburg University. The sample has been drawn randomly from the population of all households in the Netherlands whose head is between 18 and 65 years of age. The sample contains 1315 households. Within each household each member of 18 years or over has been interviewed. As a result the sample contains 2677 respondents.

For our empirical analysis we only consider households where both the male and the female partner work in a paid job for at least 15 hours per week. The 15 hours cut-off point is dictated by the survey design by which certain items of information are not collected for people who work less than 15 hours per week. As a result, we analyse a sample of 139 households for whom a sufficient amount of information has been collected to be able to estimate the model. Some sample statistics are presented in table B.1.

Table B.1. Sample statistics[a)]

		mean	s.d.	min	max
Male	preferred hours	36.0	8.2	0	70
	actual hours	41.9	6.2	20	70
	wage rate	13.2	3.8	7.6	28.9
Female	preferred hours	25.8	8.2	12	50
	actual hours	30.0	9.1	15	50
	wage rate	9.5	1.5	6.3	14.4
Family non labor income		17.2	30.1	0	140.6

a) Hours are per week, wage rates are in Dfl. per hour, non labor income is in Dfl. per week.

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