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RESEARCH MEMORANDUM

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Comment on 'Microeconometric Demand Systems with Binding Non-Negativity Constraints: The Dual Approach'*

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1. Introduction

In a recent issue of Econometrica Lee and Pitt (1986) advocate the virtual price approach to the problem of binding non-negativity constraints in demand analysis. For a wide range of flexible specifications for demand systems, including the Almost Ideal Demand System and the Indirect Translog model, the direct utility function cannot be derived explicitly, which precludes the direct use of Kuhn-Tucker conditions to derive conditional demand equations when some non-negativity constraints are binding. The apparent advantage of the dual approach is that it does not require a direct utility function to derive conditional demand equations.

To obtain the likelihood function for a sample of independent observations Lee and Pitt derive the likelihood contributions corresponding to all possible demand regimes (defined as the set of positively consumed goods). However, these regimes are only guaranteed to be uniquely characterized if the cost function underlying the demand system is globally concave in prices. In Section 2 we show that for the Indirect Translog specification proposed by Lee and Pitt the lack of global concavity yields the anomaly that the probabilities of observing each of the regimes do generally not sum to unity. Consequently, the likelihood function is not well-defined and parameter estimates will be inconsistent.

2. Derivation of the likelihood function

The share equations of the proposed Indirect Translog specification are:

$$(1) \qquad v_i q_i = \frac{\alpha_i + \sum_{i=1}^{K} \beta_{ij} \ell n\, v_j + \varepsilon_i}{D} \qquad i = 1,\ldots,K$$

where $D = -1 + \sum_{i=1}^{K} \sum_{j=1}^{K} \beta_{ij}\ \ell n\, v_j$. Here v_i is a normalized price, α_i and β_{ij} are parameters and ε_i is a random variable representing stochastic preferences; q_i is the so-called notional demand $(i,j = 1,\ldots,K)$. For normalization Lee and Pitt set $\sum_{i=1}^{K} \alpha_i = -1$, $\sum_{i=1}^{K} \varepsilon_i = 0$.

The demand regime where the first ℓ goods are not consumed is characterized by the conditions

$$\text{(2a)} \qquad \Pi_i(\bar{v}) < v_i \qquad\qquad i = 1,\ldots,\ell$$

$$\text{(2b)} \qquad x_i > 0 \qquad\qquad i = \ell+1,\ldots,K$$

Here $\Pi_i(\bar{v})$ is the virtual price of the i-th good and $\bar{v}$ is the set of market prices of the positively consumed goods; x_i is the demand for the i-th good $(i = \ell+1,\ldots,K)$ given that the first ℓ goods are not consumed. These conditions follow from the Kuhn-Tucker conditions and the concept of virtual prices, assuming that the direct utility function is continuously differentiable, quasi-concave and increasing.

In the following, we only consider three goods, where the third good is always consumed in positive amounts. (These assumptions do not affect the generality of our argument.)

The conditions for observing positive demands for both good 1 and good 2 (regime I) are

$$\text{(3)} \qquad \frac{\varepsilon_i^*}{D} > 0 \qquad\qquad i = 1,2$$

where $\varepsilon_i^* = \alpha_i + \sum_{j=1}^{3} \beta_{ij} \ln v_j + \varepsilon_i$.[1)]

The virtual price of good 1 if $x_1 = 0$ and $x_2 > 0$ is given by

$$\text{(4)} \qquad \ln \Pi_1 = -(\alpha_1 + \sum_{j=2}^{3} \beta_{1j} \ln v_j + \varepsilon_1)/\beta_{11}$$

From (2a) and (2b) it follows that the conditions for observing zero demand for good 1 and positive demand for good 2 (regime II) are

1) Lee and Pitt implicitly assume $D < 0$.

(5a) $$\frac{\varepsilon_1^*}{\beta_{11}} > 0$$

(5b) $$\frac{\varepsilon_2^* - \frac{\beta_{12}}{\beta_{11}} \varepsilon_1^*}{D - \frac{\beta_{.1}}{\beta_{11}} \varepsilon_1^*} > 0$$

where $\beta_{.j} = \sum_{i=1}^{K} \beta_{ij}$. The left hand side of (5b) is obtained by replacing $\ell n\ v_1$ in (1) by $\ell n\ \Pi_1$ given in (4).

Analogously, the conditions for observing positive demand for good 1 and zero demand for good 2 (regime III) are

(6a) $$\frac{\varepsilon_2^*}{\beta_{22}} > 0$$

(6b) $$\frac{\varepsilon_1^* - \frac{\beta_{12}}{\beta_{22}} \varepsilon_2^*}{D - \frac{\beta_{.2}}{\beta_{22}} \varepsilon_2^*} > 0$$

The virtual prices of goods 1 and 2 if both $x_1 = 0$ and $x_2 = 0$ are given by

(7) $$\begin{pmatrix} \ell n\ \Pi_1 \\ \ell n\ \Pi_2 \end{pmatrix} = -B^{-1} \begin{pmatrix} \alpha_1 + \beta_{13} \ell n\ v_3 \\ \alpha_2 + \beta_{23} \ell n\ v_3 \end{pmatrix} - B^{-1} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

where $B = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{12} & \beta_{22} \end{pmatrix}$. The conditions for observing zero demands for both goods (regime IV) are $\begin{pmatrix} \ell n\ \Pi_1 \\ \ell n\ \Pi_2 \end{pmatrix} < \begin{pmatrix} \ell n\ v_1 \\ \ell n\ v_2 \end{pmatrix}$, which can be written as

(8a) $$\frac{\beta_{22}\varepsilon_1^*}{\beta_{11}\beta_{22} - \beta_{12}^2} > \frac{\beta_{12}\varepsilon_2^*}{\beta_{11}\beta_{22} - \beta_{12}^2}$$

and

$$\text{(8b)} \qquad \frac{\beta_{11}\varepsilon_2^*}{\beta_{11}\beta_{22} - \beta_{12}^2} > \frac{\beta_{12}\varepsilon_1^*}{\beta_{11}\beta_{22} - \beta_{12}^2}$$

For each regime the conditions correspond to a particular region in the $(\varepsilon_1^*, \varepsilon_2^*)$-plane. In figure 1 these regions are indicated for $D < 0$, $\beta_{11} < 0$ and $0 < \beta_{.1} < \beta_{12} < \beta_{22} < \beta_{.2}$.

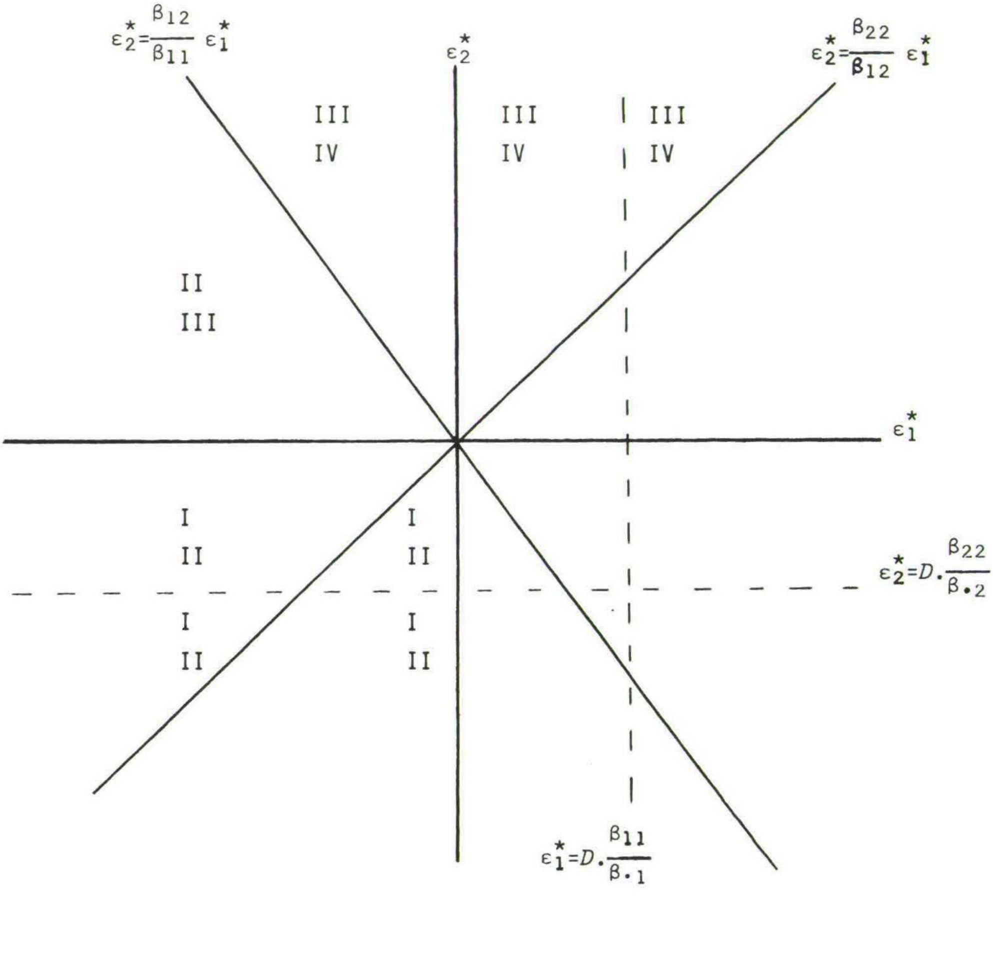

Figure 1. Regimes in $(\varepsilon_1^*, \varepsilon_2^*)$-plane

Clearly, in this example the conditions define sets in the $(\varepsilon_1^*, \varepsilon_2^*)$-plane which are neither mutually exclusive nor exhaustive. As a consequence the probabilities of observing each of the regimes do not add up to unity.

In figure 2 the regions are indicated for $D < 0 < \beta_{12} < \beta_{11} = \beta_{22} < \beta_{.1} = \beta_{.2}$.

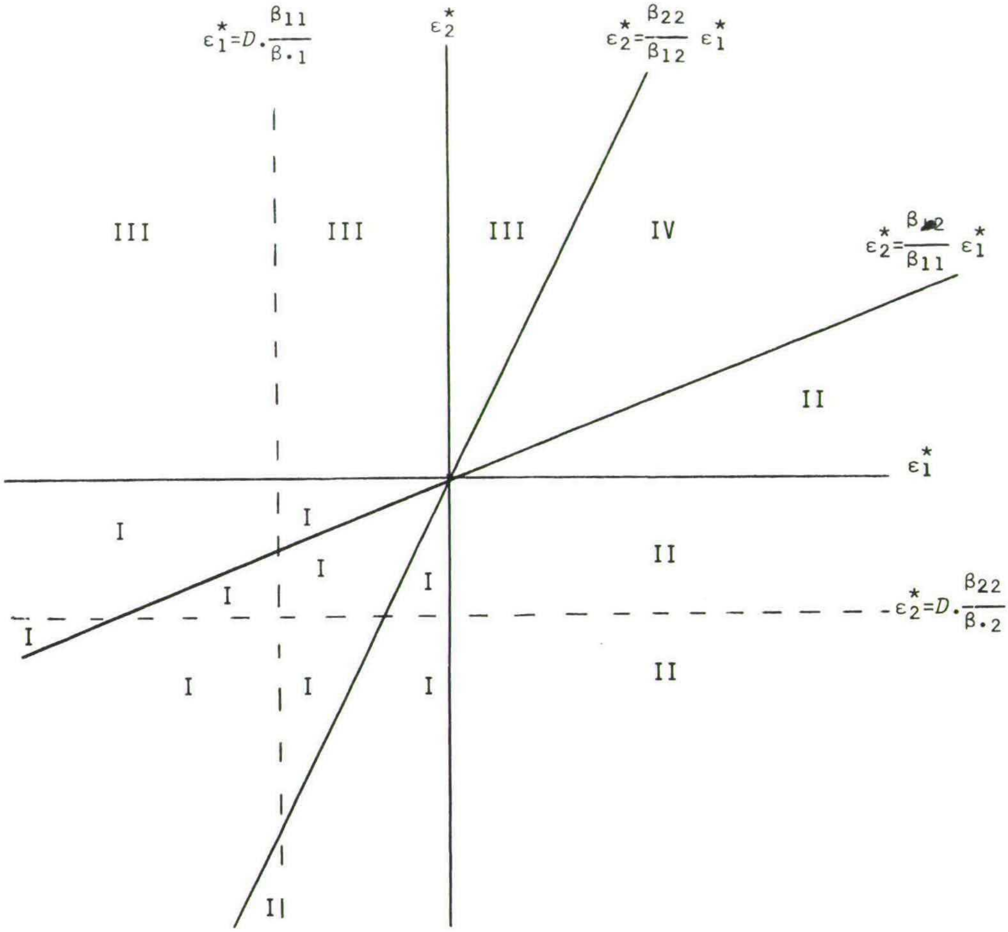

Figure 2.

In this case the regimes are properly defined in the sense that the conditions define mutually disjoint and exhaustive sets.

It is straightforward to show that, for $D < 0$, the demand regimes are uniquely characterized if and only if the matrix B is positive definite and $\beta_{.1} > 0$ and $\beta_{.2} > 0$.

Apparently, the likelihood function is only well-defined for a restricted subset of the parameter space and a restricted range of exogenous variables (note that D depends on the normalized prices, which differ per observation).

A unique characterization of the regimes for all possible values of the exogenous variables will be guaranteed if the cost function underlying the demand system is globally concave. For the translog model concavity is satisfied if the matrix defined by

$$(9) \qquad c_{ij} = \frac{1}{D} \{-\delta_{ij}v_i q_i D + \beta_{ij} - v_i q_i \beta_{.j} - v_j q_j \beta_{.i} + v_i q_i v_j q_j (\sum_j \beta_{.j} + D)\}$$

is negative semi-definite ($\delta_{ij} = 1$ if $i = j$ and 0 otherwise). Note that for $v_1 q_1 = v_2 q_2 = 0$ (regime IV) this requires the matrix B to be positive semi-definite if $D < 0$. As is easily seen from (9) global concavity cannot be imposed by restricting parameters without completely sacrificing flexibility.

3. Discussion

The advantage of the approach proposed by Lee and Pitt is that it in principle allows for consistent estimation using all observations. Moreover, in contrast to the approach based on the direct use of the Kuhn-Tucker conditions, Lee and Pitt's framework allows for the use of flexible functional forms.

However, as we have shown in section 2, restrictions on the parameters and exogenous variables are required for the model to make sense. Since these restrictions also are a condition sine qua non for the likelihood function to be well-defined, they have to be imposed during estimation.

Reference

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