...IVERSITEIT
BRABANT

POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

DIFFERENCE GAMES AND POLICY EVALUATION: A CONCEPTUAL FRAMEWORK

A.J. de Zeeuw
F. van der Ploeg

FEW 268

DIFFERENCE GAMES AND POLICY EVALUATION: A CONCEPTUAL FRAMEWORK[1)]

A.J. de Zeeuw
Tilburg University, 5000 LE Tilburg, The Netherlands

F. van der Ploeg
London School of Economics, London WC2A 2AE, U.K.
Tilburg University, 5000 LE Tilburg, The Netherlands

ABSTRACT

An overview of policy evaluation based on linear-quadratic non-cooperative difference games is presented. It is argued that such games provide a microeconomic foundation of ad-hoc macroeconomic models with forward-looking behaviour and are therefore immune to the Lucas critique of econometric policy evaluation. When there is a dominant player (such as the Central Bank or Treasury), the Stackelberg difference game (with private sector agents as followers) is relevant. For these non-cooperative games there is a problem of time inconsistency, as the dominant player has an incentive to renege on announced policies, and therefore the dominant player needs to precommit itself. If it cannot, a subgame perfect equilibrium is required. Subgame perfectness is a stronger concept than time consistency, which can be seen from the fact that the Nash difference game with precommitment is time consistent even though it is not subgame perfect. The paper gives on overview of the various solution concepts (open-loop Nash, subgame-perfect Nash, open-loop Stackelberg, subgame-perfect

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1) This paper arose from an earlier paper, entitled "Non-cooperative strategies for dynamic policy games and the problem of time inconsistency: A comment", and has benefited from the comments of three anonymous referees.

Stackelberg) for difference games and stresses that the use of the state space (rather than the final form) representation is essential for a proper evaluation of these concepts. The paper also discusses the use of the conjectural variations equilibrium concept in difference games.

CONTENTS

1. Introduction

The Lucas (1976) critique of econometric policy evaluation has increased the interest in applications of rational expectations and non-cooperative difference/differential game theory to dynamic economic or econometric models, because these techniques take explicitly into account the reaction of the private sector (such as households and firms) to changes in government economic policy. Non-cooperative difference/differential games of the Stackelberg variety, with the government as leader and the private sector agents as followers, provide a microeconomic foundation of ad-hoc macroeconomic models with expectations of future government economic policy affecting the current state of the economy. Obviously, the advantage of difference/differential games over ad-hoc models is that they are immune to the Lucas (1976) critique of policy evaluation as the behaviour of the private sector is no longer invariant to the policy rule adopted by the government. It is well known that economies with rational expectations or open-loop Stackelberg dynamic games are non-causal due to the anticipation of future actions of dominant players (such as the Treasury or the Central Bank). In such models the problem of time inconsistency arises due to the incentive of dominant players to renege on announced strategies (Kydland and Prescott, 1977). If there are no binding commitments, such models are vulnerable to cheating from the side of the dominant player (e.g., the government). A trade-off occurs between cashing in on short-term gains by cheating and building a strong reputation by refraining from cheating (Kreps and Wilson, 1982; Barro and Gordon, 1983; Backus and Driffill, 1985; Meijdam and de Zeeuw, 1986). If the government cannot commit itself and does not have a strong reputation, the private sector cannot be expected to believe time-inconsistent announcements and therefore such economic policies are not credible. The feedback Stackelberg solution concept (Simaan and Cruz, 1973) assumes that the players can change their strategies all the time on the basis of observations on the evolution of the state of the economic system and is therefore by construction time-consistent. This solution concept can be seen as an application of the concept of subgame perfectness (Selten, 1975) or an extension of the principle of optimality (Bellman, 1957) to games. Because it is assumed that the players are ex-ante given the opportunity to renege at each stage of the game, ex-post they will not renege and therefore this concept leads to time-consistent policies by construction. However, subgame perfectness is stronger than time consistency, so that it is possible to formulate a time-consistent open-loop Stackelberg solution which is not subgame perfect (Meijdam and de Zeeuw, 1986). The point is that time consistency implies that there is no incentive to deviate from the equilibrium path and that subgame perfectness implies that there is no incentive to deviate from points that are off the equilibrium path either. Two aspects distinguish the feedback concept from the open-loop concept, namely information structure (Başar and Olsder, 1982) and period of commitment (Reinganum and Stokey, 1985). In the feedback concept it is assumed that the players draw information on the present state of the economic system and have a period of commitment of one, whereas in the open-loop concept there is only information on the initial state of the economic system and the period of commitment is equal to the planning period. When only the information structure is changed but the period of commitment remains the planning period, it is common to use the term closed-loop (memory). When there are even no commitments within each period and the players are supposed to act simultaneously, the Nash solution concept results because the Nash announcement is the only credible announcement (Meijdam and de Zeeuw, 1986).

The Nash solution concept represents the standard approach to non-cooperative dynamic games. The open-loop Nash equilibrium does not suffer from time inconsistency, because there is no dominant player that can manipulate the current actions of the other players by making announcements about its own future actions. The feedback Nash equilibrium again presupposes another information structure and no binding commitments. It is constructed by imposing subgame perfectness. When the conventional "stacking" procedure or final-form model for policy evaluation problems (Theil, 1964) is used, the open-loop model always results which can be compared with the concept of games in normal form. As a consequence typical dynamical aspects of dynamic games and games in extensive form such as the principle of subgame perfectness cannot be discussed within this framework.

A third solution concept for non-cooperative dynamic games, the consistent conjectural variations equilibrium, was introduced in oligopoly theory by Bresnahan (1981) and was recently applied to an open-loop difference game (Hughes Hallett, 1984; Brandsma and Hughes Hallett, 1984) and a feedback difference game (Başar, Turnovsky and d'Orey, 1986). Although it is argued that the concept is logically inconsistent (Daughety, 1985; de Zeeuw and van der Ploeg, 1987), the main importance for the discussion in this paper is that the open-loop consistent conjectural variations equilibrium is time inconsistent.

This paper will give an overview of different solution concepts with their properties and show the results for a standard abstract linear quadratic policy evaluation problem. Special attention is given to the consistent conjectural variations approach, because it is felt that this is not the proper way to go. In section 2 the abstract prototype model is formulated and different decision models or game theoretic solution concepts are discussed. In section 3 properties such as time consistency, subgame perfectness and credibility are defined and evaluated. Section 4 concludes the paper.

2. Linear-quadratic difference games: An evaluation

2.1. Model and solution concepts

In this section some essential concepts for dynamic policy evaluation are discussed and a standard abstract dynamic model is formulated in order to elucidate the conceptual discussion.

The starting point is a linear dynamic economic model in state-space form:

$$y_t = A_t\, y_{t-1} + B_t^1\, x_t^1 + B_t^2\, x_t^2 + s_t\ ,\ y_0 = \bar{y}_0\ . \tag{1}$$

The transition of the state y of the economy from period t-1 to period t is influenced by two players (such as the government and the private sector) who independently control the exogeneous variables x^1 and x^2, respectively. The non-controllable exogeneous variables are denoted by s. The objective of player i, i = 1,2, is to minimize a quadratic welfare loss function over a finite horizon:

$$w^i = \sum_{t=1}^{T} \tfrac{1}{2} \{y_t' Q_t^i y_t + x_t^{i'} R_t^i x_t^i\} , \ i = 1,2, \tag{2}$$

where $Q_t^i \geqq 0$ and $R_t^i > 0$. An extension with linear terms in the welfare loss function is straightforward by redefining the state vector, y_t, and s_t in an appropriate way. The convex linear-quadratic structure is not essential for the discussion but facilitates analytical solutions. It can always be considered as an approximation to the real structure of a specific model. The problem is called an optimal control problem with two decision makers or a difference game.

The traditional approach (Theil, 1964) to an economic optimal control problem is to cast the economic model (1) into a final-form model:

$$y = B^1 x^1 + B^2 x^2 + s, \tag{3}$$

where y, x^1 and x^2 stack the state variables y_t and the policy instruments x_t^1 and x_t^2 for all periods of the finite planning horizon. Consequently, B^1 and B^2 are block-triangular matrices composed of A_t, B_t^1 and B_t^2, and s contains the non-controllable exogeneous variables s_t as well as the influence of the initial state vector $\bar{y}_0$[2]. The corresponding objective functionals become

$$w^i = \tfrac{1}{2} \{y' Q^i y + x^{i'} R^i x^i\} , \ i = 1,2 , \tag{4}$$

where the matrices Q^i and R^i are block-diagonal as the welfare loss functions (2) were assumed to be time separable. In this form the problem cannot be distinguished from a static problem, so that it corresponds to the normal form of the difference game. It explains why after this transformation into final form some crucial dynamical issues disappear. This will become clear in the sequel.

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2) To be precise, $B^i = (B_{jk}^i)$ where $B_{jk}^i = 0$, $j < k$, $B_{jj}^i = B_j^i$, $B_{jk}^i = \prod_{\ell=k}^{j-1} (A_\ell) B_\ell^i$, $j > k$, for $j = 1,\dots T$, $k = 1,\dots T$ and $i = 1,2$.

The by now standard approach to a difference game is to distinguish information patterns and periods of commitment. The decision makers or players announce strategies for the whole planning period but may or may not be committed to stick to these strategies. A strategy is a mapping from the information set and time to the set of available actions. Considering the state of the economic system, this information set can contain only the initial state (open-loop information), only the present state (closed-loop, no memory information) or all the states up to the present state (closed-loop memory information). Memory information complicates matters considerably and can sometimes be excluded on the grounds of bounded rationality (e.g., Rubinstein, 1987). The model with an open-loop information structure and a period of commitment equal to the planning horizon will be called the open-loop model. The model with a closed-loop no memory information structure and a period of commitment of one period will be called the feedback model. In the feedback model the players have access to the current state of the economy and are ex-ante given the opportunity to renege on announced strategies at each stage of the game, so that in equilibrium they have no incentive to renege. The open-loop model is equivalent to the optimal control model based on a final-form economic model, which was described earlier.

The standard techniques to solve optimal control problems are Bellman's dynamic programming and Pontryagin's minimum principle. For an optimal control problem with one decision maker the two techniques yield the same optimal actions and performance[3)]. For an optimal control problem with two or more decision makers these techniques lead in general to different solutions. The reason is that dynamic programming solves the feedback model and the minimum principle solves the open-loop model. To put it differently, dynamic programming presupposes information on the present state of the economic system and no commitments, whereas the minimum principle presupposes information on the initial state of the economic system and binding commitments. In the context of a game these assumptions have their influence. In the feedback model the players can observe the

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3) In a stochastic world, dynamic programming leads to policy feedback rules that take account of stochastic shocks and therefore lead to a lower expected welfare loss.

effects of the actions of their opponent and they can react to these observations, whereas in the open-loop model they cannot. Dynamic programming as a solution technique to a one-player optimal control problem is based on Bellman's principle of optimality. Dynamic programming as a solution framework for a difference game presupposes a generalization of the principle of optimality to dynamic games, which is called subgame perfectness and which is treated in more detail in the next section.

The two solution techniques have in common that they transform the dynamic optimization problem into a series of static optimization problems in a dynamic setting. When the minimum principle is applied, the optimization part of the solution is the static optimization of the Hamiltonian. When dynamic programming is applied the optimization part of the solution is the static optimization of the right-hand side of the Hamilton-Jacobi-Bellman equation. As a consequence the game theory involved is limited to static equilibrium concepts.

2.2. Open-loop and feedback Nash equilibrium

The standard non-cooperative equilibrium concept is the Nash concept which is based upon the idea that there should be no individual incentive for any player to deviate from the equilibrium. The Nash equilibrium assumes that the players act simultaneously and form expectations of each other's action, which in equilibrium are fulfilled. This implies that the Nash equilibrium is the intersection of the hypothetical reaction curves which express the optimal decisions of each player conditional on the action of the rival. For the prototype model (1), (2) the first-order conditions of the optimization problem

$$R_t^i x_t^i + B_t^{i'}\{K_t^i y_t + g_t^i\} = 0 , \quad i = 1,2, \tag{5}$$

where y_t is given by (1), lead to the hypothetical reaction functions

$$x_t^i = -(R_t^i + B_t^{i'} K_t^i B_t^i)^{-1} B_t^{i'} K_t^i (A_t y_{t-1} + B_t^j x_t^j + s_t + g_t^i) ,$$

$$j \neq i, \quad i = 1,2 \tag{5'}$$

For the open-loop decision model the terms $\{K_t^i\, y_t + g_t^i\}$ are the so-called co-states (or adjoint variables) of the minimum principle. The parameters K_t^i and g_t^i can be determined from the backward recursive equations:[4)]

$$K_{t-1}^i = Q_{t-1}^i + A_t'\, K_t^i\, [E_t]^{-1}\, A_t\ ,$$
$$K_T^i = Q_T^i\ , \tag{6}$$

$$g_{t-1}^i = A_t'\, \{K_t^i\, [E_t]^{-1}(s_t - B_t^1\, [R_t^1]^{-1}\, B_t^{1'} g_t^1$$
$$- B_t^2\, [R_t^2]^{-1}\, B_t^{2'}\, g_t^2) + g_t^i\}\ ,$$
$$g_T^i = 0\ ,\ i = 1,2, \tag{7}$$

where $E_t = I + B_t^1\, [R_t^1]^{-1}\, B_t^{1'} K_t^1 + B_t^2\, [R_t^2]^{-1}\, B_t^{2'} K_t^2$.

For the feedback decision model the terms K_t^i and g_t^i are the parameters of the quadratic so-called value functions of dynamic programming:

$$\tfrac{1}{2}\, y_{t-1}'\, K_{t-1}^i\, y_{t-1} + g_{t-1}^{i'}\, y_{t-1} + c_{t-1}^i$$
$$= \min_{x_t^i} \{\tfrac{1}{2}\, y_{t-1}'\, Q_{t-1}^i\, y_{t-1} + \tfrac{1}{2}\, x_t^{i'}\, R_t^i\, x_t^i + \tfrac{1}{2}\, y_t'\, K_t^i\, y_t + g_t^{i'} y_t + c_t^i\}.$$

They follow from the backward recursive equations:

$$K_{t-1}^i = Q_{t-1}^i + A_t'\, [E_t']^{-1}\, (I + K_t^i\, B_t^i\, [R_t^i]^{-1}\, B_t^{i'})\, K_t^i\, [E_t]^{-1}\, A_t\ ,$$
$$K_T^i = Q_T^i\ , \tag{8}$$

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4) The proofs of all results are available upon request from the authors.

$$
\begin{aligned}
g^i_{t-1} = A'_t \, [E'_t]^{-1} \, (I + K^i_t \, B^i_t \, [R^i_t]^{-1} \, B^{i'}_t) \\
\{K^i_t \, [E_t]^{-1} \, (s_t - B^1_t \, [R^1_t]^{-1} \, B^{1'}_t \, g^1_t \\
- B^2_t \, [R^2_t]^{-1} \, B^{2'}_t g^2_t) + g^i_t\} \, , \\
g^i_T = 0 \, , \; i = 1,2 \, .
\end{aligned}
\tag{9}
$$

The Nash equilibrium is given by the intersection of the two hypothetical reaction functions, (5'):

$$
x^i_t = G^i_t \, y_{t-1} + h^i_t \, , \; i = 1,2, \tag{10}
$$

where

$$
G^i_t = - [R^i_t]^{-1} \, B^{i'}_t \, K^i_t \, [E_t]^{-1} \, A_t
$$

and

$$
\begin{aligned}
h^i_t = - [R^i_t]^{-1} \, B^{i'}_t \{K^i_t \, [E_t]^{-1} \, (s_t \\
- B^1_t \, [R^1_t]^{-1} \, B^{1'}_t g^1_t - B^2_t \, [R^2_t]^{-1} \, B^{2'}_t \, g^2_t) + g^i_t\} \, .
\end{aligned}
$$

It is essential to note that the relationship between x^i_t and y_{t-1} in (10) is only a real functional relationship in the feedback model; it does not represent the policy feedback rule of player i in the open-loop model. Furthermore, the feedback equilibrium strategies $\{G^i_t, h^i_t\}$ are not binding and can be changed whenever one of the players wants to do so. The open-loop equilibrium consists of binding sequences of actions $\{x^i_t\}$ which result from (10) and (1) together with (6) and (7) and which only depend upon the initial state $\bar{y}_0$, so that unexpected state trajectories cannot have their influence. This open-loop outcome coincides with the Nash equilibrium of the static problem (3), (4). The transformation of the economic model into final form implies that issues like information and commitment are disregarded or, to be more precise, static information patterns and periods of commitment equal to the planning horizon are implicitly assumed. It is worth mentioning here that both open-loop and feedback policy rules can be inferior to closed-loop memory policy rules where the players condition their strategies on information on current and past states of the economy (Başar and Olsder, 1982, Section 6.3; de Zeeuw, 1984, Section 4.3).

The open-loop and feedback Nash decision models can have very different economic results. Consider as an example the problem of an oligopoly with restricted entry and exit harvesting a renewable resource with zero extraction costs, iso-elastic demand and serially uncorrelated shocks to the natural replenishment rate. It can then be shown that the open-loop extraction rates obey Hotelling-type arbitrage rules and are therefore efficient whilst the feedback equilibrium leads to excessive extraction rates or even extinction of the resource (van der Ploeg, 1986). The reason is that when an individual firm decides to harvest an additional unit, it realizes that the lower stock increases harvesting costs to the other firms and therefore the other firms will in the feedback model react by harvesting less. This means that the marginal cost of harvesting an additional unit is less than in the absence of such a response from its rivals, hence the feedback model leads to excessive harvesting. (With free entry and exit, the harvesting rates in the feedback model become efficient.) To take another example, in a model of competitive arms accumulation between two countries, where each country has a "guns versus butter" dilemma, the feedback Nash equilibrium proves to be more efficient and leads to less arms accumulation than the open-loop Nash equilibrium (van der Ploeg and de Zeeuw, 1986). The reason is that when one country decides to invest in an additional weapon, it realizes that the security of rival countries is threatened and therefore in the feedback model the rivals respond by investing more in weapons. Obviously, this increases the marginal cost of investment in an additional weapon and therefore the feedback model results in lower weapon stocks. (The policy recommendation is that countries should agree to monitor each other's weapon stocks.)

2.3. Open-loop and feedback Stackelberg equilibrium

Another standard non-cooperative equilibrium concept is the Stackelberg concept. The difference with the Nash concept is the leader/follower structure which means that one of the players (the leader) acts first or, to put it differently, the action or strategy of the leader is part of the information set of the follower. There are again two optimization problems. The first one determines the rational reaction of the follower to the action or strategy of the leader. This rational reaction, which is not a hypothetical reaction as in the Nash concept but a real reaction, is given by the reaction function for the follower (5'). The second optimization problem determines the optimal action or strategy of the leader given the rational reaction of the follower.

For the open-loop decision model this implies that the constraints of this optimization problem consist of the forward recursive system (1), equation (5') for the follower and the backward recursive system for the co-states. The resulting open-loop Stackelberg equilibrium (Kydland, 1975; Başar and Olsder, 1982, Section 7.2; de Zeeuw, 1984, Section 4.5) for the prototype model will not be given here, because it is not immediately relevant for this evaluation.[5)] The backward recursiveness of the so-called adjoint system implies forward-looking behaviour of the follower, which leads to time inconsistency of the optimal actions of the leader. Hence, the leader can by making announcements about its future policy actions manipulate the current policy actions of the follower. However, once the follower has implemented these actions, it may pay the leader to renege and deviate from the previous announcements about its policies. These issues of time inconsistency will be dealt with in the next section.

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5) In any case, one could in principle obtain the open-loop Stackelberg equilibrium as the static Stackelberg equilibrium of the final-form model (3). That is, $x^i = -(R^i + B^{i'} Q^i B^i)^{-1} (B^j x^j + s)$ is the optimal reaction of the follower i to the actions of the leader j. The leader minimizes its welfare loss function subject to the reaction function of the follower, which gives

$$x^j = -(R^j + \bar{B}^{j'} Q^j \bar{B}^j)^{-1} s \text{ where } \bar{B}^j \equiv [I - B^i(R^i + B^{i'}Q^iB^i)^{-1}]B^j.$$

For the feedback decision model the first-order conditions of the two optimization problems are

$$R_t^i\, x_t^i + B_t^{i'}\{K_t^i\, y_t + g_t^i\} = 0$$

$$R_t^j\, x_t^j + \{B_t^{j'} + (\partial x_t^i/\partial x_t^j) B_t^{i'}\}\{K_t^j\, y_t + g_t^j\} = 0 \tag{11}$$

where i is the follower and j is the leader. The crucial difference with the Nash concept is the reaction coefficient $\partial x_t^i/\partial x_t^j = -(R_t^i + B_t^{i'} K_t^i B_t^i)^{-1} B_t^{i'} K_t^i B_t^j$. The feedback Stackelberg equilibrium is given by

$$x_t^i = F_t^i\, \{A_t\, y_{t-1} + B_t^j\, x_t^j + s_t\} + F_t^{ii}\, g_t^i$$

$$x_t^j = F_t^j\, \{A_t\, y_{t-1} + s_t\} + F_t^{ji}\, g_t^i + F_t^{jj}\, g_t^j \tag{12}$$

where

$$F_t^{ii} = -\,[R_t^i + B_t^{i'}\, K_t^i\, B_t^i]^{-1}\, B_t^{i'}$$

$$F_t^i = F_t^{ii}\, K_t^i$$

$$F_t^{jj} = -\,[R_t^j + B_t^{j'}(I+B_t^iF_t^i)'\, K_t^j\, (I+B_t^iF_t^i)\, B_t^j]^{-1}\, B_t^{j'}(I+B_t^iF_t^i)'$$

$$F_t^{ji} = F_t^{jj}\, K_t^j\, B_t^i\, F_t^{ii}$$

$$F_t^j = F_t^{jj}\, K_t^j\, (I+B_t^iF_t^i)$$

and the backward recursions by

$$K_{t-1}^i = Q_{t-1}^i + A_t'\, \{(I+B_t^jF_t^j)'\, F_t^{i'}\, R_t^i\, F_t^i\, (I+B_t^jF_t^j) + (I+B_t^jF_t^j)'\, (I+B_t^iF_t^i)'\, K_t^i\, (I+B_t^iF_t^i)\, (I+B_t^jF_t^j)\}\, A_t$$

$$K_T^i = Q_T^i$$

$$K^j_{t-1} = Q^j_{t-1} + A'_t \{F^{j'}_t R^j_t F^j_t +$$

$$(I+B^j_tF^j_t)' (I+B^i_tF^i_t)' K^j_t (I+B^i_tF^i_t) (I+B^j_tF^j_t)\} A_t$$

$$K^j_T = Q^j_T$$

$$g^i_{t-1} = A'_t \{(I+B^j_tF^j_t)' F^{i'}_t R^i_t [F^i_t (I+B^j_tF^j_t) s_t +$$

$$F^{ii}_t g^i_t + F^i_t B^j_t F^{ji}_t g^i_t + F^i_t B^j_t F^{jj}_t g^j_t] +$$

$$(I+B^j_tF^j_t)' (I+B^i_tF^i_t)' \{K^i_t [(I+B^i_tF^i_t) (I+B^j_tF^j_t) s_t +$$

$$B^i_t F^{ii}_t g^i_t + (I+B^i_tF^i_t) B^j_t (F^{ji}_t g^i_t + F^{jj}_t g^j_t)] + g^i_t\}\}$$

$$g^i_T = 0$$

$$g^j_{t-1} = A'_t \{F^{j'}_t R^j_t [F^j_t s_t + F^{ji}_t g^i_t + F^{jj}_t g^j_t] +$$

$$(I+B^j_tF^j_t)' (I+B^i_tF^i_t)' \{K^j_t [(I+B^i_tF^i_t) (I+B^j_tF^j_t) s_t +$$

$$B^i_t F^{ii}_t g^i_t + (I+B^i_tF^i_t) B^j_t (F^{ji}_t g^i_t + F^{jj}_t g^j_t)] + g^j_t\}\}$$

$$g^j_T = 0.$$

Given the parameters, the action x^i_t of the follower i is a function of the state y_{t-1} and the action x^j_t of the leader j which both belong to the follower's information set. The action x^j_t of the leader j is only a function of the state y_{t-1}. It is essential to note that for logical reasons the players cannot have each other's action in their information set at the same time. Either player i acts first, so that the action x^i_t is part of the information set of player j, or it is the other way around. The follower just plays optimally given the state of the economy and the action of the leader. In the Stackelberg equilibrium the leader expects the follower to react rationally and the action is chosen accordingly. The rational reaction is determined by the first equation of (11) and influences the reaction coefficient $\partial x^i_t/\partial x^j_t$ as well as the state transition y_t in the second equation of (11). After substitution of this rational reaction the second equation of (11) determines the optimal action of the leader and not an optimal reaction, because the leader is not reacting to the follower. These considerations are essentially of a static nature within each subgame and they apply also to the open-loop decision model, especially when the final-form representation (3), (4) is used. The only difference is that in the feedback decision model the leader reacts indirectly to past actions of the follower through observations on the state of the economy. The feedback model is obtained from dynamamic prgramming and therefore satisfies subgame perfectness. This means that the leader has no incentive to deviate and therefore its policies are time consistent by construction.

2.4. Consistent conjectural variations equilibrium

Recently, a third non-cooperative equilibrium concept for difference games was developed: a consistent conjectural variations equilibrium (for the open-loop case from the final-form representation: Hughes Hallett, 1984; Brandsma and Hughes Hallett, 1984; for the feedback case: Başar, Turnovsky and d'Orey, 1986). The equilibrium was introduced in the context of oligopoly theory (Bresnahan, 1981) and is based upon the concept of conjectural variation (Bowley, 1924). A conjectural variation in this context is a coefficient $\partial x_t^i/\partial x_t^j$ as in (11), which comes from a conjecture of player j with respect to the reaction of player i. In the Stackelberg equilibrium the leader conjectures a rational reaction function of the follower. In the Nash equilibrium the two players conjecture the action of the other player and they are right in their conjecture (consistency argument). The idea behind the consistent conjectural variations equilibrium is that the two players conjecture the reaction of the other player and that they are right in their conjecture. The equilibrium is determined by introducting conjectural variations for both players and requiring consistency of conjectural variations and reaction coefficients. This seems a natural extension in the line of Nash and Stackelberg, but one ends up with logical inconsistencies (Daughety, 1985; de Zeeuw and van der Ploeg, 1987). The reason is simply, as stated before, that the players cannot react to each other at the same time, so that a fortiori conjectures and reactions cannot be consistent. The reaction functions that show up in the calculation of the Nash equilibrium are hypothetical or represent a possible reaction process in notional time, which is not conceivable for the consistent conjectural variations concept. It is certainly not true that the proposed equilibrium is a Nash equilibrium or, worse, a superior one (as stated in Hughes Hallett, 1984; Brandsma and Hughes Hallett, 1984).

The idea of conjectures and conjectural variations is all right, but the consistency argument should be different. There are two ways out. The first one is to formulate an infinite regress decision model of the type "player i conjectures that player j conjectures that player i conjectures...." (Daughety, 1985). The other way out is to start with conjectures and corresponding conjectural variations and to require consistency of conjectures and actions. To stress the difference the resulting equilibrium will be called <u>consistent conjectures equilibrium</u>. The first-order conditions with conjectures are:

$$\begin{aligned} R_t^i\, x_t^i + \{B_t^{i'} + (\partial x_t^j/\partial x_t^i)\, B_t^{j'}\}\, \{K_t^i\, y_t + g_t^i\} &= 0 \\ R_t^j\, x_t^j + \{B_t^{j'} + (\partial x_t^i/\partial x_t^j)\, B_t^{i'}\}\, \{K_t^j\, y_t + g_t^j\} &= 0 \end{aligned} \tag{13}$$

where

$$y_t = A_t\, y_{t-1} + B_t^i\, x_t^i + B_t^j\, x_t^j + s_t.$$

The conjectures $x_t^j(x_t^i)$ and $x_t^i(x_t^j)$ account for the conjectural variations $\partial x_t^j/\partial x_t^i$ and $\partial x_t^i/\partial x_t^j$. The first equation becomes an equation in x_t^i determining the optimal action of player i and the second one becomes an equation in x_t^j determining the optimal action of player j. Consistency of conjectures and actions requires that these optimal actions fit the conjectures, which yields restrictions on the parameters of the conjectures. This weaker concept of consistency usually leads to multiple equilibria. How can the idea of consistency of conjectures and reactions arise? In that literature reaction functions are created by not substituting the conjectures in the state transition y_t. The resulting "reaction functions" lead to reaction coefficients which are required to match the conjectural variations. There are, however, not only these logical difficulties. The consistent conjectural variations equilibrium for the open-loop model in final form (3), (4) suffers from more problems. Firstly, the result is time inconsistent for the same reason as the Stackelberg open-loop equilibrium is (see section 3). Secondly, the informational requirements seem particularly unrealistic. Finally, the outcome is typically worse for the players than the Nash outcome and it suffers from non-uniqueness and instability. Two examples will clarify these statements.

Example 1 (Hughes Hallett, 1984, pp. 389-390)

Consider the game with objectives $w^i = y^2 + x^{i2}$ where $y = x^1 + x^2 - 1$. The Nash equilibrium is $x^i = 1/3$ with outcome $w^i = 2/9$.

Hughes Hallett argues that $x^i = 2/5$ is a better solution, because the associated outcome $w^i = 1/5$ implies an improvement for both players. This is not surprising, since it is well known that it is possible to find Pareto improvements over the Nash outcome even though there is no unilateral incentive for any player to deviate from the Nash equilibrium. In fact, $x^i = 2/5$ is what is generally called the Nash bargaining solution. However, the solution is not sustained as a consistent conjectural variations equilibrium. To find one, Hughes Hallett describes an iterative procedure and searches for a fixed point in the conjectured and actual "reaction coefficients". This procedure starts from an initial pair (d^1, d^2) of conjectural variations, where $d^i \equiv \partial x^i / \partial x^j$, and yields new pairs being the corresponding "reaction coefficients" $(-(1+d^2)/(2+d^2),$ $-(1+d^1)/(2+d^1))$. There are two fixed points here, namely $d^i = -3/2 \pm 1/2 \sqrt{5}$ with corresponding actions $x^i = 1/2 \mp 1/10 \sqrt{5}$ and outcome $w^i = 1/2 \mp 1/10 \sqrt{5}$. Both consistent conjectural variations equilibria produce worse results for both players as compared to the Nash equilibrium. Furthermore, they do not satisfy the Nash property since each player can unilaterally improve by playing, for example, $x^i = 1/4 \pm 1/20 \sqrt{5}$. Finally, it follows from the derivative of the fixed point mapping, $-1/(2+d^i)^2 = -3/2 \pm 1/2 \sqrt{5}$, that one of the fixed points ($d^i = -3/2 + 1/2 \sqrt{5}$) is stable whilst the other is unstable.

The Pareto improvement $x^i = 2/5$ is, however, sustained as a consistent conjectures equilibrium. The conjectures "my rival mimicks what I do", $x^i = x^j$, with conjectural variations 1 lead to optimal actions $x^i = 2/5$, which are consistent with the conjectures. The Nash equilibrium is also sustained as a consistent conjectures equilibrium. The consistent conjectures are in this case $x^i = 1/3$ with conjectural variations 0. A final example of such an equilibrium is the solution $x^i = 0$ with outcome $w^i = 1$, which results from the conjectures $x^i = -x^j$ with conjectural variations -1. However, this outcome is obviously unattractive for the players.

Example 2

Consider the game with objectives $w^i = 1/2(y'y+x^{i'}x^i)$ where $y = x^1+x^2+s$, $s = [1,1]'$. The Nash equilibrium is $x^i = [-1/3, -1/3]'$ with outcome $w^i = 2/9$.

The consistent conjectural variations (D^1, D^2) are characterized by

$$(D^i)^2 + 3\, D^i + I = 0\ . \tag{14}$$

There are an infinite number of solutions to (14), which can be found analytically after some tedious calculations. Hughes Hallett's iterative scheme is

$$D^i_{s+1} = -\ (2I + D^j_s)^{-1}\ (I + D^j_s) = (2I + D^j_s)^{-1} - I.$$

The local stability of the iterative scheme in the neighbourhood of the fixed points follows if all of the eigenvalues of the Jacobian

$$\partial \text{ vec } D^i_{s+1} \;/\; \partial \text{ vec } D^j_s = -\ \{(2I + D^j_s)^{-1'} \otimes (2I + D^j_s)^{-1}\}\ ,$$

evaluated at D^j, are inside the unit circle. It can be shown after considerable manipulation that $D^i = (-3/2 + 1/2\sqrt{5})\ I$ is the only stable fixed point. Again the corresponding welfare loss, $w^i = 1/2 - 1/10\sqrt{5}$, is higher than the welfare loss which can be obtained under the Nash concept. Finally, the Nash equilibrium is not sustained as a conjectural variations equilibrium, because $x^i = -1/3\ s$, $x^i = D^i(x^i+s)$ and (14) are inconsistent.

3. Time inconsistency, subgame perfectness and credibility

In section 2 several decision models for dynamic policy evaluation problems have been discussed. This section discusses properties of these dynamic decision models, such as time inconsistency, subgame perfectness and credibility.

A strategy is time inconsistent if there is an incentive for the player to renege on this strategy in the future (Kydland and Prescott, 1977). A decision model which typically has time-inconsistent strategies is the open-loop Stackelberg equilibrium. In this equilibrium the leader's strategy $\{x_1^j,\dots,x_T^j\}$ is optimal given the follower's rational reaction $\{x_1^i,\dots,x_T^i\}$. However, at time s > 1 the remaining strategy $\{x_s^j,\dots,x_T^j\}$ of the leader is typically not optimal anymore. The reason is that at time s the actions $\{x_s^j,\dots,x_T^j)$ have done the job of influencing the past actions of the follower and can now be solely employed to influence the present and future actions of the follower. The strategy announcement $\{x_s^j,\dots,x_T^j\}$ can be considered as some sort of threat which helped to have the follower play $\{x_1^i,\dots,x_{s-1}^i\}$. For example, a benevolent government, who maximizes the gross consumers' surplus of the representative household, may announce taxation of the supply of labour rather than of capital tomorrow in order to induce agents to accumulate capital today. Once the capital stock is in place, it pays the government (improves economic welfare) to renege by taxing capital, instead of labour, tomorrow, despite the fact that the government has the same preferences as the representative household (Fischer, 1980). The crucial point is the forward-looking behaviour of the follower which looses its impact as soon as actions are performed. In the example, once the investment has occurred, the government can extract the quasi-rent on it. The same phenomenon occurs in other models with forward-looking variables such as models with rational expectations. For example, the optimal taxation of a monetary economy with a Cagan-type money demand function is time inconsistent (Calvo, 1978). The reason is that the government finds it optimal to announce a low monetary growth rate in order to induce large holdings of real money balances and low inflation, but once the real money balances have been accumulated it pays the government to renege and impose a surprise inflation tax. Alternatively, in an economy with nominal wage rigidity the government might announce a low money supply in order to induce workers to lock themselves into low nominal wage contracts. Once they have done this, the government has an incentive to renege and implement a high money supply and thus gain more employment without pushing up the price level (e.g., Kydland and Prescott, 1977; Barro and Gordon, 1983).

This type of forward-looking behaviour can easily be derived for the prototype model in section 2. For the follower the strategy x_t^j of the leader has the same role as the exogeneous input s_t. The rational reaction x_t^i of the follower is given by equation (5') which means that it is a function of x_t^j and g_t^i. According to (7) g_t^i is a function of all the future exogeneous inputs, so that x_t^i is a function of present and future actions of the leader:

$$x_t^i = \varphi(x_t^j, \ldots, x_T^j).$$

There is one logical difficulty in this analysis. Strictly speaking the players cannot renege in an open-loop decision model. However, time inconsistency remains a possible property of a decision model which can be regarded as undesirable or unrealistic. The feedback Stackelberg equilibrium, on the other hand, is time consistent by construction, because it is based on the idea that the players are ex-ante constantly given the opportunity to renege and therefore ex-post have no incentive to renege. But there is more. Feedback decision models have the stronger property of subgame perfectness. A game equilibrium is <u>subgame perfect</u> if it remains an equilibrium for any subgame. A subgame in this respect is a game with the same players, objectives and system dynamics, but starting from an arbitrary state y_s at time s, $1 \leq s \leq T$. This concept reveals precisely the structure of dynamic programming and thus of the feedback decision model. It can be said that subgame perfectness is time consistency on the equilibrium path as well as off the equilibrium path. A subgame perfect equilibrium is robust against mistakes or other unexpected events (Selten, 1975). Because subgame perfectness is stronger than time consistency, it is possible to formulate a Stackelberg equilibrium which is time consistent but not subgame perfect (Meijdam and de Zeeuw, 1986). The idea is to keep the open-loop decision structure but to cut off the forward-looking behaviour of the follower. It is also possible to achieve time consistency by requiring that the leader follows a feedback decision model, whereas the follower still has the open-loop decision model (Cohen and Michel, 1985).

The open-loop Nash equilibrium is time consistent. As long as the state of the economic system follows the open-loop Nash equilibrium path none of the players has an incentive to renege. The open-loop consistent conjectural variations equilibrium, however, is time inconsistent for the same reason as the open-loop Stackelberg equilibrium is. Time inconsistency is a property of an equilibrium and can only be avoided by changing the equilibrium concept. It is not possible to solve the problem technically (as stated in Hughes Hallet, 1984; Brandsma and Hughes Hallett, 1984)[6].

A strategy is <u>credible</u> if it contains announcements on future actions and if these announcements are believed by the other players. Announcements are believed if they are considered to be optimal at the time of action or, alternatively, if there is no incentive to deviate from the announcements. The open-loop decision model typically contains announcements on future actions. Time-inconsistent strategies are an example of strategies that are not credible. The credibility problem can also occur in a static context where players are in principle of equal strength and act simultaneously. One of the players can try to become a Stackelberg leader by announcing his action beforehand. If the announcement has effect the Stackelberg equilibrium may result. In this case, however, the "leader" can do even better, because generally there will be an incentive to deviate from the announcement under the assumption that the other player expects it to be true. In this simple framework the only credible announcement is the Nash action, because this is the only announcement that is

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6) These problems also feature in Hillier's (1987) comment on Hughes Hallett (1987).

at the same time the optimal reaction to the optimal reaction to the announcement (Meijdam and de Zeeuw, 1986). However, in a more advanced framework with imperfect (or incomplete) information, reputational effects can lead to credible strategies which are not Nash (Kreps and Wilson, 1982). For example, with incomplete information it is possible to have the private sector believing announcements of the government that it will fight inflation, whereas with complete information the Nash announcement of high inflation is the only credible one (Backus and Driffill, 1985). Alternatively, reputational effects can occur when the game is repeated indefinitely (e.g., Barro and Gordon, 1983). When the discount rate is small enough, this punishments from reneging are relatively large and therefore there may be no temptation to renege even though the policy actions may be time inconsistent in the absence of such reputational effects.

4. Conclusion

In this paper several methods to analyse policy problems with two or more decision makers are evaluated. These methods employ decision models which are distinguished according to different non-cooperative game theoretic solution concepts (Nash, Stackelberg, consistent conjectural variations), different information structures (open-loop, closed-loop) and different periods of commitment. The decision models are evaluated by considering properties such as time consistency, subgame perfectness and credibility, and the links with solution techniques like dynamic programming and the minimum principle are precisely described.

The formulation of the problem on the basis of economic models in final form is rejected, because typical dynamical issues disappear in this formulation. The consistent conjectural variations equilibrium is rejected on principles of logic and the alternative consistent conjectures equilibrium is presented. The decision model with open-loop information structure and binding commitments can suffer from time inconsistency, although there is some logical contradiction here with the assumption of binding commitments. The decision models with closed-loop information structure and without commitments are subgame perfect and thus time consistent, and therefore they deserve more attention. When the Stackelberg leader/follower structure is based upon announcements and not upon sequential actions, the requirement of credibility leads back to Nash. Credible strategies which are not Nash can arise when it is possible to evoke reputational effects.

The prototype model used in the paper is based on quadratic preferences and linear models, which keeps matters tractable. Although it is relatively straightforward to develope iterative Gauss-Newton algorithms to derive <u>open-loop</u> Stackelberg or Nash equilibrium solutions, it is very difficult to calculate <u>feedback</u> Stackelberg or Nash equilibrium solutions for non-linear models or non-quadratic preferences. The reason is that it is usually impossible to find analytic expressions for the functional forms of the value functions. All that one can do in such cases is to discretize the space of control variables of the players and calculate the subgame-perfect solution numerically by dynamic programming. This procedure rapidly runs in to combinatorial problems and is thus very expensive in terms of computer requirements of storage and time. This problem is particularly severe when there are externalities or market imperfections present, because it is then not possible to invoke the fundamental theorem of welfare economics which says that the market (read difference/differential game) outcome is the same as the outcome of a centrally planned economy. When there are no externalities or market imperfections, Kydland and Prescott (1982) calculate the outcome of a centrally planned economy and thus avoid the derivation of value functions for the market outcome. Unfortunately, for most interesting policy problems, this trick cannot be used and future research must be concerned with the technical difficulties of calculating subgame-perfect solutions for non-linear models (cf., Lucas, 1987).

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