





A COMPARISON OF TWO LOT SIZING-SEQUENCING HEURISTICS FOR THE PROCESS INDUSTRY

R.M.J. Heuts, H.P. Seidel, W.J. Selen

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FOR THE PROCESS INDUSTRY

R.M.J. Heuts, *) H.P. Seidel, *) W.J. Selen **)

- *) Department of Econometrics
 Tilburg University
 P.O. Box 90153
 5000 LE Tilburg
 The Netherlands
- **)Boston University Brussels Laarbeeklaan 121 1090 Brussels Belgium

Abstract

Two heuristics for operational production planning in a chemical processing environment are compared, characterized by a single bottleneck machine, fixed batch sizes, sequence-dependent setup-times, as well as production and storage constraints. Performance of both heuristics is measured by means of simulation experiments in which the planning horizon is partially frozen and rolled a number of times, as would be the case in real application. Furthermore, demand uncertainty is simulated as well as the variability among setup times. The performance measure used is the total cost for executing a particular production plan over its entire planning horizon.

1. INTRODUCTION

In the process industry one often encounters the following problem: A number of products have to be manufactured on a single bottleneck machine, after which they are stored in pre-assigned tanks of known capacity. Setup-times are very significant and, in some applications, could comprise up to twenty percent and more of total processing time. These setup-times are highly sequence-dependent, influenced by the chemical composition and resulting cleaning operations of the product processed previously in the reactor. Furthermore, because of chemical reaction properties, the reactor has to be completely filled each time when a "batch" of a product is processed: yielding integer production lot sizes. The aim consists of establishing a feasible production plan for a pre-defined planning horizon in which production lot sizes are determined as to minimize setup and inventory holding costs.

Various simple lot sizing heuristics, however without taking into account setup times, are reviewed by Maes and van Wassenhove [3], [4]. Heuristics for lot sizing with setup times and setup costs, are analysed by Trigeiro [10], and Trigeiro et.al. [12]. However, in these algorithms setup times are sequence independent, and so job sequencing is not included within a period. The machine capacitated lot size problem using sequence dependent setup times is e.g. analysed by Smith-Daniels and Smith-Daniels [9], and Smith-Daniels and Ritzman [10], via a mixed integer linear programming model. These procedures may be computationally prohibitive when the number of products and planning periods is large.

In addition to sequence dependent setup times, storage constraints for the respective products as well as the batch-character of the production process are important aspects, which ought to be incorporated.

Two heuristics, which represent the above features, are analysed via simulation experiments to test their performance with respect to certain input factors. Section 2 starts with the construction of a feasible initial production scheme, section 3 examines two simultaneous lot sizing-sequencing heuristics and in section 4 an augmented version of a sequence routine is presented. Then section 5 introduces the concept of a rolling horizon, while in section 6 the experimental design and simulation results are presented. Finally, section 7 ends with the conclusions.

2. THE CONSTRUCTION OF A FEASIBLE INITIAL PRODUCTION SCHEDULE

Using forecasted demand data, a first feasible production schedule is made, which serves as a starting point for the lot sizing-sequencing heuristics described in section 3. First of all the demand scheme is modified for the beginning inventories and denoted as "net"-demand. This net-demand (in batches) is determined per product per period, which then can be used to construct a first production schedule (in batches). The production per period is determined by rounding up to an integer the net-demand per period, taking into account that this rounding creates inventory (difference between net-demand (in real batches) and production (in rounding up to integer batches)). The net-demand is first modified for this inventory before the new production is determined; or:

$$Y_{i1} = Int (X_{i1} + 0.999)$$
 $i=1,...,N$ (1)

$$BI_{it} = Y_{it-1} - X_{it-1}$$

 $\begin{cases} i=1,...,N \\ t=2,...,H \end{cases}$ (2)

$$Y_{it} = Int (X_{it} - BI_{it} + 0.999)$$
 (3)

$$\begin{cases} i=1,\ldots,N \\ t=2,\ldots,H. \end{cases}$$

where

 $\mathrm{BI}_{\mathrm{it}}$ = beginning inventory (in batches) of product i in period t.

 Y_{it} = production (in integer batches) for product i in period t

 X_{it} = net-demand (in real batches) for product i in period t

N = number of products

H = length of planning horizon

When in a particular period the inventory level for a certain product, as determined according the above procedure, is larger than the net-demand, the production for that period is set to zero and the inventory level for the subsequent period is modified. In this way a first production schedule

can be made, for which it is to be verified whether sufficient production capacity exists for each period. In order to do so, the sequencing procedure, which will be briefly described in the next section, will be used interactively to determine an initial feasible production schedule for the entire planning horizon. When the production capacity for the first period would prove insufficient, the forecasted demand scheme can not be satisfied and an infeasible plan would result. Clearly, a feasible production plan can only be constructed if the available production capacity would be adequate or slack production capacity would exist. The slack production capacity can then be determined as:

$$SPC_{t} = PC_{t} - \sum_{i=1}^{N} \{Y_{it}^{*}A_{i} + (\max(Y_{i,t},1)-1)^{*}A_{ii}\} - SW_{t}^{*}, t=1,...,H$$
(4)

where

 SPC_{+} = slack production capacity in period t (in hours)

PC, = production capacity in period t (in hours)

Y; = number of batches of product i produced in period t

A; = production time (in hours) to produce 1 batch of product i

A = setup time (in hours) needed between successive production runs of product i

SW_t = total switch-over time (in hours) between production runs of different products in period t, as determined by the sequence procedure.

When the slack production capacity is negative for any other period t than the first, a "shift procedure" is initiated. As such, preproduction has to take place for one or more products, to reduce the required capacity in the tight production period. That product will first be shifted for which the inventory carrying cost is the smallest, provided that the production for that product in the period to which it is shifted, is possible. The inventory carrying cost is determined per unit for the number of periods shifted. Before shifting takes place, it is checked whether the tank capacity in each intermittent period is sufficient for that product. Furthermore, it is verified whether enough slack machine capacity is present in the period in which the preproduction will take place. The shifting is done batch for batch. Perform the shifting of one batch for that product i

for which Σ H is minimal, where k is the nearest previous period to j=k which shifting can take place, and H is the holding cost for holding one batch of product i in period j. While shifting the following relationships have to be maintained:

$$Y_{ik} > 0$$
, $SPC_k \ge A_i + A_{ii}$, $STC_{ij} \ge BS_i$ (j=k,...,t-1) (5)

where: STC_{ij} = slack tank capacity (in tons) of product i in period j = TC_{ij} - $\{Y_{ij} + B_{ij}\}$ * BS_{i} = batch size (in tons) of product i TC_{ij} = nominal tank capacity (in tons) of product i in period j.

After a batch of product i is shifted, the updating is done as follows:

$$SPC_{t} := SPC_{t} + A_{i} + A_{i}, SPC_{k} := SPC_{k} - A_{i} - A_{i}$$
(6)

$$Y_{it} := Y_{it} - 1, Y_{ik} := Y_{ik} + 1$$
 (7)

$$BI_{ij} = B_{i,j-1} + Y_{i,j-1} - D_{i,j-1}, \quad j=k+1,...,t$$
 (8)

where D_{ij} = demand (in batches) of product i in period j.

$$STC_{ij} = TC_{ij} - (Y_{ij} + BI_{ij}) * BS_{i}, j = k,...,t.$$
 (9)

Preproduction through shifting takes place untill ${\rm SPC}_{\rm t}$ is positive. A feasible production schedule results when for all periods the production capacity is sufficient. This schedule is then used as the start schedule for the procedures in the next section.

Some final remarks should be made with regard to the above procedure for constructing an initial feasible production plan.

a) When, through preproduction and the accumulation of intermittent inventories, the need for production of a particular product in period t no longer exists, new switch-over times will have to be computed by means of the sequencing procedure to be described next. b) When shifting can not be accomplished with products to periods where production for those products is positive, it is checked if eventually a shift can be made to periods where originally no production is planned for that product. Also in this case the sequence routine has to be started to calculate the new switch-over times.

3. THE SIMULTANEOUS LOT SIZING-SEQUENCING HEURISTICS NSHS AND HS

Naidu and Singh [1] [5] [6] developed a lot sizing heuristic, which we have modified by implementing tank storage constraints for each product and a sequencing procedure for determining setup times. This new heuristic will be denoted as Naidu-Singh/Heuts-Selen or NSHS. The NSHS lot sizing-sequencing heuristic is an improvement heuristic, which in effect means that the period to which preproduction is shifted is not known beforehand. This is contrary to the Heuts-Selen (HS)-heuristic, a period by period heuristic, which shifts production to a fixed reference period. The NSHS heuristic is elaborated on next. It consists of the following two steps:

NSHS-step 1

For all products i and periods t > 1 it is verified whether production of product i in period t can be shifted to period p(i) < t.

In order to do so, the following conditions have to be satisfied:

- a) The production in p(i) of i has to be positive.
- b) The slack production capacity in period p(i) has to be sufficient.
- c) The slack tank capacity of period k=p(i),...,t-1 has to be sufficient.

In other words, those shifts are considered for which p(i) is the nearest period, smaller than t, for which the two capacity restrictions are satisfied. Whether a production shift actually takes place depends on the switch-over-cost saved, in relation to the incremental inventory cost. The following incremental cost formula is important in this respect:

$$IC_{i,p(i),t} = (A_{ii} - SW_{it}) * OC + \sum_{j=p(i)}^{t-1} (H_{ij} * Y_{it})$$
 (10)

where

IC $_{i,p(i),t}$ = potential incremental cost of preproducing in period p(i) the production requirement of product i from period t.

 SW_{it} = switch-over time (in hours) needed to start production of product i in period t

OC = opportunity cost per hour machine (reactor) idle time.

To determine the switch-over time SW_{it} , a sequencing routine is used. This sequencing routine approximates the optimal production sequence for different products, as exact procedures are too time consuming for sequencing many different products. For details we refer to Selen and Heuts [7]. The potential switch-over time saving by not starting up product i in period t is then calculated as the time to switch from the product sequenced before i, to i.

For the above defined potential incremental cost, the minimum of $IC_{i,p(i),t}$ for all $i(i=1,\ldots,N)$ and $t(t=1,\ldots,H)$ is determined. When all $IC_{i,p(i),t}$ are positive (no cost savings are possible), or there exists no $IC_{i,p(i),t}$ (capacity constraints are violated); the existing production plan can not be improved. When the minimum is negative, costs savings are still possible by shifting production.

The actual shifting and sequencing routine is explained in step 2.

NSHS-step 2

The product and period for which $IC_{i,p(i),t}$ is minimal will be denoted by i^* and t^* , respectively. The period where production is shifted to, is denoted by $p(i)^*$. The existing production in period $p(i)^*$ of product i^* is incremented with the production of i^* from period t^* . The slack tank capacity is modified for that product during the periods $p(i)^*,\ldots,t^{*-1}$. After shifting product i^* from period t^* to period $p(i)^*$, sequencing calculations will take place, yielding information to recalculate the slack production capacity, defined as in formula (4), for periods $p(i)^*$ through the end of the planning horizon.

To calculate SW_t in formula (4), sequencing has to be done over multiple periods. For details of this procedure we refer to Selen and Heuts [7].

The actual total costs, after a production shift took place, can be calculated as:

$$TC = \sum_{i=1}^{N} \sum_{t=1}^{H} \{C_{it} * Y_{it} + A_{ii} * (max(1,Y_{it})-1) * OC$$

$$+ H_{it} * (BI_{i1} + \sum_{j=1}^{L} (Y_{ij} - D_{ij})) + \sum_{t=1}^{H} SW_{t} * OC$$
(11)

where

TC = total cost

 C_{i+} = production cost for product i in period t

The NSHS lot sizing-sequencing then proceeds as follows: check if total cost after this production shift is smaller than the minimal total cost which was achieved over all preceding iterations.

- If so, minimal total cost := total cost at this moment, best production plan := production at this moment; return to step 1.
- If not, go to step 1 and restart procedure. Stop when no potential incremental cost is possible, or when there exists no potential incremental cost as any of the capacity constraints is violated.

The simultaneous lot sizing-sequencing heuristic of Heuts-Selen (HS) which uses a different rule for shifting preproduction is described in detail in the literature [7].

Before comparing the NSHS and HS heuristics in an experimental setting, an "augmented" version of the sequencing routine described earlier, is discussed.

4. AN AUGMENTED VERSION OF THE LOTSIZING-SEQUENCING HEURISTIC

The selection criterion for preproduction in the preceding section fails to take into account the consequences of resequencing when computing potential cost savings. This resequencing takes place after each production shift. When a shift is done on a potential cost savings basis, it is still

possible that after resequencing an increase in actual total costs result. Let us assume that the optimal production sequence for five products is: 1-2-3-4-5-, that product 3 is shifted and that the new optimal production sequence is 1-2-4-5. In this case the switch-over time 2-3 is saved, but the switch-over time 2-4 may be much larger than 3-4, and the potential savings may be completely lost. The execution of such a shifting which leads to higher actual costs, also has consequences for the total costs and slack capacities in all following iterations. Thus, there are also consequences for the final production scheme that is eventually produced.

The following augmented version would resolve this potential problem:

- For <u>all</u> shift possibilities with a negative potential incremental cost saving, a <u>preliminary</u> shift is performed with resequencing and actual cost calculations.
- That shift possibility is chosen which realises the largest $\underline{\text{actual}}$ saving.

In this way the consequences of resequencing are taken into account. Previously, the "potential cost saving" was used as an indicative selection criterion to limit the number of shift possibilities. The advantage of the augmented version is that decisions are now based on actual realised savings. Per iteration the total actual costs are guaranteed to decrease. For both heuristics several experiments were done with and without the above modification. All experiments which were done with this modification realised lower actual total costs. This augmented version of the sequencing routine was not implemented, however, because of computation speed. Experiments have shown that the computation time with the augmented version of the heuristic was approximately 30 times higher as compared to using the the heuristic described in the previous section.

Using either the NSHS or HS heuristic in operational production planning, a quick response time is important because of the following reasons:

- a) Often a sensitivity analysis has to be performed with regard to:
 - changing demand forecasts
 - changing opportunity costs
 - reallocation of the tank assignments

b) Superimposing an expert system on the existing simultaneous lot sizingsequencing heuristic, is an interesting feature which the authors are currently studying. However, such an approach needs flexibility and quick response time for operational planning purposes.

5. PRODUCTION PLANNING USING A ROLLING HORIZON

In making an operational production plan, one should note the following:

- In business practice the production plan is periodically updated when new information on clients and/or technical constraints becomes available.
- Demand information was until yet assumed deterministic. However, in practice one has to rely on a forecasting procedure which inherently produces growing forecasting errors as the uncertainty increases.
- It is further assumed that the production plan is executed over the entire planning horizon. In practice, however, if one uses for example a horizon of 10 weeks, only a few weeks of that plan are "frozen" and executed according to the production plan, while the rest of the plan is updated when new information comes available.

The rolling horizon concept attemps to overcome the shortcomings of the deterministic modelling of the lot sizing-sequencing problem. The steps which will be executed under a rolling horizon concept during the simulation experiments, are briefly outlined below:

Step 1:

In period 1 a demand forecast for all products is made over all periods of the horizon. On the basis of these forecasts a production schedule is made for all periods of the horzion.

Step 2:

The production schedule is executed for a part of the horzion. We will call this the frozen part of the horizon. In executing the plan for these frozen periods we will to some extent take into account the differences between actual and forecasted demands.

Step 3:

After executing a part of the plan, the planning horizon is rolled. From the new reference period till the end of the planning horizon new forecasts of the demand are made and a new production scheme is made using the chosen heuristic. Then we return to step 2. When the maximum number of simulated periods is reached, the last production plan is made and executed, with or without modifications as a result of forecast errors.

Next, the above steps are further elaborated on.

Step 1: Generating demand forecasts and production plans

First, the length of the planning horizon is chosen, taking into account that:

- When the planning horizon is too short, the possibilities of preproduction of the lot sizing methods can not be used in an optimal way and long term demand forecasts are not taken into account properly.
- When the planning horzion is too long, the computation time of the lot sizing-sequencing heuristics becomes a problem, as well as the quality of the demand forecasts.

In most cases, the demand forecasts will deviate from the actual demand figures, where the resulting forecast errors will increase with the number of periods forecasted. The actual demand, d(i,j) ($i=1,\ldots,N$; $j=1,\ldots,H$) in tons, is for simulation purposes generated from a normal distribution with known mean and standard deviation.

To generate the forecasts, f(i,j); a forecast error, error (i,j), is added to the generated demand d(i,j), or:

$$f(i,j) = d(i,j) + error(i,j), i=1,...,N; j=1,...,H$$
 (12)

The forecast error is generated from a normal distribution with the following properties:

- The average forecast error is in first instance taken as zero (unbiased forecasts). Later on systematic under- and overestimations are simulated. The same average demand is generated for all products and periods.
- 2) The standard deviation of the forecast errors is assumed equal for all products and satisfies the following equation:

$$SE_{t} = SE_{1} + a * (t-1), t=1,...,H$$
 (13)

where

 ${\rm SE}_{\rm t}$ = standard deviation of the forecast errors when forecasting t periods in the future

 $a = growth\ factor\ of\ the\ standard\ deviation\ of\ the\ forecast\ errors.$ When a = 0, the standard deviation of the forecast errors is held constant, and grows linear in time when a > 0. In this way uncertainty can be simulated.

Step 2: Freezing a part of the planning horizon

The scheduled production plan will only be executed for a limited number of periods of the panning horizon, denoted as "freeze". The choice of freeze depends on the following:

- a) A small freeze (for example 25% of the planning horizon) does not use the preproduction possibilities in an optimal way (as 75% of the plan will not be executed).
- b) A large freeze gives little flexibility in the short term, as the plan is fixed for a large number of periods. Adjustments in the short term for modified circumstances are not possible.
- c) A large freeze increases the likelihood of backorders or excess inventory, as the realisation of the demand forecasts in the more distant future becomes increasingly uncertain.
- d) A large freeze, however, provides a more stable planning environment as new production plans are called for less frequently.

Production is effectively performed according to schedule till the end of the freeze. Each period, planned production, actual demand, and beginning inventory are monitored and updated if necessary. The ending inventory for product i in period j (in tons) is determined as:

inventory level
$$(i,j)$$
 = inventory level $(i,j-1)$ + production level (i,j) * batchsize (i) - $d(i,j)$ (14)

The inventory level (i,j) can be positive because of two reasons:

- 1) In period j production already takes place for future periods.
- 2) The actual demand, d(i,j), in period j appears to be lower than the forecast, f(i,j).

When the inventory level (i,j) is negative, a backorder situation occurs for product i in period j. This backorder can only be caused by the fact that the actual demand in period j appeared to be larger than the forecast f(i,j). These two inventory positions are elaborated on next. It is assumed that, when the inventory level is positive; the planned production will be executed, provided enough tank capacity is available for that product. When the tank capacity is insufficient, the production plan will be corrected downwards to avoid inflexibilities. When the inventory level is negative (a backorder position), the production plan for period j will be modified in as far as machine and tank capacities permit this.

First, the number of batches to be produced will be determined by rounding up the backorder position to an integer batchsize.

The following situations can occur:

- 1) Product i is planned for production in period j. It is checked whether sufficient machine and tank capacity exists to produce the number of batches backordered. If one of these capacities is violated, as many batches as possible, given the constraints, are produced, and the relevant parameter values updated.
- 2) Product i is not planned for production in period j. Scheduling for product j is repeated, adding the backorder situation for product i, and checking on available machine and tank capacities. If capacity restrictions are violated, again as many batches as possible are produced, as far as the capacity constraints permit, and relevant parameter values are updated.

This procedure is repeated for all products. In case of many backorders, the available slack production capacity in period j, SPC(j), will first be allocated to product 1, then to product 2, etc. As such, products should be numbered in descending order of importance. The above procedure is summarized in Figure 1. The actual realised costs during the frozen part of the horizon are determined as follows:

$$CF = \sum_{i=1}^{n} \sum_{j=1}^{freeze} \{A_{ii} * (max(1,Y_{ij})-1) * OC + C_{ij} Y_{ij} + SW_{ij} * OC + H_{ij} * \frac{excess(i,j)}{batchsize(i)} + penalty(i) * backorder(i,j)\}$$

$$(15)$$

where,

CF = cost during frozen part of planning horizon

backorder (i,j) = the backorder (in tons) of product i in period j

excess (i,j) = the excess (in tons) of product i in period j

batchsize (i) = the batchsize (in tons) of product i

penalty (i) = the backorder cost of 1 ton of product i.

freeze = the number of time periods that are "frozen" for planning purposes.

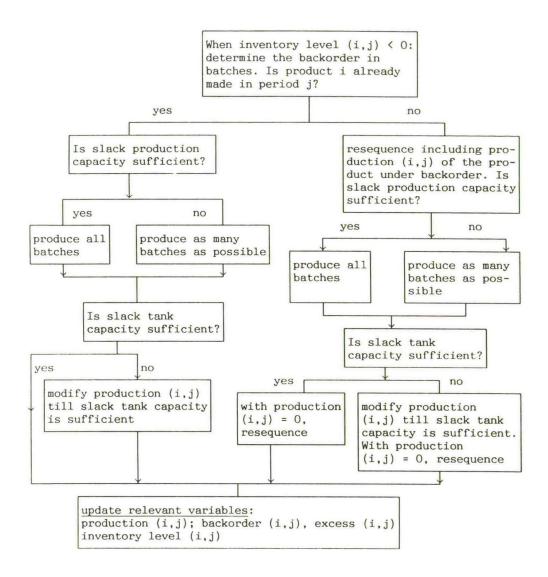


Figure 1: Freezing a part of the planning horizon

Step 3: Rolling the horizon

After production has taken place according to schedule during the frozen part of the planning horizon, the planning horizon is rolled. As such, the new reference period is rolled to the "end of the freeze period + 1" and new forecasts are generated.

We generate the actual demands d(i,j) (i=1,...,N,j=H+1,...,H+freeze) and forecast error(i,j) (i=1,...,N,j=freeze+1,...,freeze+H). Note that a complete new error-matrix is generated as new forecast information becomes available.

New forecasts are determined as:

$$f(i,j) := d(i,j) + error(i,j) i=1,...,N, j=freeze+1,...,freeze+H.$$
(16)

Based on these forecasts a new production plan is made, which is partially executed according to the freeze period, until the horizon is rolled again. The number of times the planning horizon is rolled is determined in the simulation experiment. For each freeze period the total actual cost is calculated, and eventually accumulated over the number of times the planning horizon is rolled. This cumulative cost figure is then used as the performance measure for evaluating both heuristics.

The experimental design used for comparing both heuristics, as well as the most important findings of this simulation experiment, are discussed next.

6. EXPERIMENTAL DESIGN AND SIMULATION RESULTS

Only experiments with a rolling horizon are described, as they are most relevant in practice. With a rolling horizon the actual total cost (for one replication) is calculated for a number (the number of times the horizon is rolled) of partially executed production schedules, which are based on forecasts, and hence on necessary modifications, resulting from forecast errors. For a single replication and a horizon which is rolled three times the idea is visualized as in Figure 2.

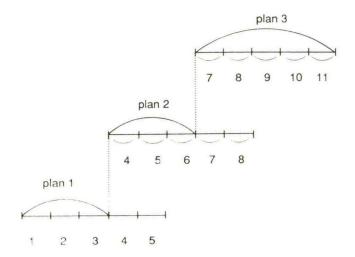


Figure 2. A single replication with a horizon which is rolled three times.

The actual cost related to the situation described in Figure 2 is the accumulation of the cost of period 1-3 of plan 1, period 4-6 of plan 2, and finally period 7-11 plan 3.

At it is impossible from a practical point of view to vary all the input data, a preselection was made. The data which are held constant in the study are:

- the number of times that the horizon is rolled per replication (3)
- the shortage cost per ton (\$ 30)
- the number of products (15)
- the batch-size per product (60 tons)
- the batch production time in hours (with different values for the 15 products)
- beginning inventory for the products (0)
- batch production cost per hour (\$ 500)
- inventory holding cost per batch (\$ 1000)
- opportunity cost per hour (\$ 2000)
- the freeze period of the rolling horizon experiments is chosen as 60% of the planning horizon, so when e.g. the length of the planning horizon is 5 periods, the freeze period is 3 periods.

The following input data are varied during the experiments as they may have an important impact on the performance of the two heuristics:

a) The demand distribution.

The demand, d(i,j), is generated for all products and all periods from normal, uniform and gamma distributions. The mean and standard deviation of all demand data are taken to be equal for all products and periods.

b) The structure of the setup time matrix.

The structure of the setup time matrix, representing the time (in hours) to switch from product i to product j, is in first instance varied on two levels. These levels represent the increasing variation of the switch-over time in hours in the setup time matrix.

c) The machine production capacity.

The machine production capacity, representing the effective machine hours per period which can be used for production, is varied on three levels.

d) The tank capacity.

The tank capacity, representing the number of batches which can be stocked per product per period, is varied on three levels.

- e) The <u>number of replications</u> per experiment is varied on three levels (10,15,30).
- f) The <u>length of the planning horizon</u> is varied on two levels (5 and 10 periods), together with the number of times the horizon is rolled, and this determines the number of simulation-periods per replication.
- g) Two <u>lot sizing-sequencing heuristics</u> are used: Naidu-Singh/Heuts-Selen (NSHS) and Heuts/Selen (HS).

When the demand forecast matrix for all products and periods is generated and all other data are fixed, a production scheme for a specified lot

sizing-sequencing heuristic is calculated, together with the total cost of implementing this production scheme.

These production schemes are executed for "freeze" periods with possible modifications due to forecast errors. The total cost of the executed schemes are calculated for both heuristics NSHS and HS. After these manipulations a new demand forecast matrix is generated, new production schemes are determined and partly executed during freeze periods. Total costs are updated for both heuristics. The chosen number of simulation periods determines how often the above process is repeated. When the end of the horizon is reached, the given levels of the input data, the lot sizing-sequencing heuristic used, and the actual total costs over all simulation periods are stored in a file. In this way, one replication is generated for both heuristics using the rolling horizon concept. We repeat this for the required number of replications in the experiment. Each time the horizon is rolled, a new production scheme is determined. During the initialisation stage, a first feasible solution for a given demand matrix is generated. However, infeasibilities may occur when the generated demand is too large given the available machine and tank capacities. In that case no lot sizing-sequencing heuristic is applied. In our analysis, such a replication is removed for both heuristics. The files which are created in this way are then used for variance analysis, using the computer package SAS. Two treatments (the two heuristics) are studied. For the i-th treatment the response (the total costs for that treatment) is a random variable that varies around an unknown treatment mean μ_i , i=1,2. For each treatment group the response is assumed to be normally distributed with both groups having equal variances. The ANOVA procedure then calls for F-test to test the null hypothesis: $H_0: \mu_1 = \mu_2$, against $H_1: \mu_2 \neq \mu_2$. Furthermore, a confidence interval for the difference between the two treatment means is obtained. For a detailed discussion of ANOVA, the reader is referred to the literature (e.g. [2]). Both the normality assumption and the equality of the variances of both treatment groups was tested, and no statistical evidence was found to reject both assumptions. As, even on a VAX mainframe computer the computation time for one experiment on a rolling horizon basis with 20 replications is still 10 minutes. several trial runs were performed to gather preliminary information on the more important factors affecting the performance of the heuristics; resulting in the following additional information on the variation of the input factors:

- For any additional experiments, it was decided to generate the demand matrix only from normal distributions, as several runs with gamma and uniform distributions showed no important influence on the performance of the heuristics.
- The machine and tank capacity levels had a logical influence on the total costs of both heuristics: higher levels of effective capacity give more preproduction possibilities for both heuristics, so more savings, and thus lower total costs. To make the preproduction possibilities as large as possible, it was decided in all additional experiments to set the available machine and tank capacities at their highest level.
- The type of structure of the setup matrix seemed to have different impacts on the performance of each heuristic. As, such it was decided in any additional experiments to study the type of structure of the setup matrix at four, rather than two, levels. To set these four levels, a matrix of switch-over times is generated from uniform distributions with different but given average switch-over times per product, where the upper and lower bounds of the uniform distribution are determined as: upper bound = average switch-over time +k hours

lower bound = average switch-over time -k hours,

where,

k=1; 2; 3; 3.8.

The switch-over times between batches of the same product is set at 1/2 hour for all products. In this way the variation in switch-over times is varied on four levels, with k=1 exhibiting a low variation and k=3.8 resulting in a relative large variation in switch-over times.

- It was decided to run all experiments under a single cost structure to make efficient use of the available resources. The average switch-over time for all matrices was 4.8 hours, yielding an average switch-over cost of 4.8 * \$ 2000 = \$ 9,600 per product. The inventory holding cost was set at \$ 1000 per batch. This relative high setup/inventory holding cost relationship is often encountered in the process industry.

	setup matrices					
	k=1	k=2	k=3	k=3.8		
number of replications	10	10	15	10		
number of periods in						
planning horizon	10	10	10	10		
number of freeze periods	6	6	6	6		
number of times the						
planning horizon is rolled	3	3	3	3		
average demand	150	150	150	150		
standard deviation demand	20	20	20	20		
average forecast error	+100	+100	+100	+100		
standard deviation of forecast errors (SE ₁)	0	0	0	0		
growth factor a	7	7	7	7		
F value ANOVA-analysis	4.16	8.14	4.56	4.07		
PR > F	0.05	0.01	0.04	0.06		
average cost NSHS	26,504,862.4	24,390,267.4	20,133,698.1	18,870,158.8		
average cost HS	26,246,343.6	24,114,208.0	20,377,353.7	19,183,608.4		
90% confidence interval:	[38,709;478,329]	[108,226;443,893]	[-49,647;-437,664]	[-44,127;-582,772]		
Which heuristic is better?	HS	HS	NSHS	NSHS		

TABLE 1: First group of experiments with varying setup time matrix.

A first group of experiments was conducted with the following characteristics:

- Planning is done with a rolling horizon and 60% freeze of the production plan.
- The forecasting system produces forecasts with a systematic over-estimation (100 tons on average).
- The standard deviation of the forecast errors is increasing in time.

The empirical information from table 1 can be summarized as follows. For a rolling horizon with a 60% freeze and increasing forecast errors over time (growing uncertainty in more distant future), the heuristic of Heuts-Selen peforms better than the amended version of Naidu-Singh for small variations in the setup matrix. The opposite is true for large variations in the setup matrix. This could be explained by the fact that the assumption of demand over-estimation, together with the rolling horizon concept with a partly freeze and possible production modifications, result in overproduction and hence growning inventory levels. However, the HS heuristic leads to more excess inventories than NSHS, as the HS algorithm shifts relatively more production to earlier periods.

For small variations in the switch-over times, the difference (during an iteration) between the maximal savings of the NSHS heuristic as compared to HS for reference period k, will be small. Nevertheless, the HS heuristic (which is a period-by-period algorithm) realises on average more iterations and as such a possible cheaper production plan. However, as was pointed out earlier, the calculated maximal savings and actual savings during an iteration need not be identical. Still, for small differences between switch-over times, the consequences for rescheduling are minimal and hence actual and calculated savings are nearly equal. Moreover, the HS heuristic realises more effective iterations than NSHS when the setup matrix exhibits small variations in switch-over times. An iteration is called effective when it yields an overall lower cost for the production plan. The above is illustrated in example 1; which is characterized by:

- same basic input data as in table 1: rolling horizon (3 times), systematic over-estimation of demand, increasing demand uncertainty over time
- small variations in the setup matrix (k=1)

- two experiments with different generated demand data from the <u>same</u> normal distribution with parameters as indicated in table 1.
- for each heuristic, the number of iterations and actual cost per production plan are given; as well as the average actual savings per iteration.

The following conclusions can be drawn from example 1. For small variations in the setup matrix the average savings per iteration is nearly the same for both heuristics. As the HS heuristic has more iterations, it produces a lower total cost level. The consequences of rescheduling were small for this type of setup matrix. In both methods, each subsequent iteration results in a lower actual cost level. As such, each iteration was effective in reducing the total cost of the production plan.

	# iterations	total cost of the production plan	average actual savings per iteration
EXPERIMENT 1: first production plan NSHS first production plan HS	38 42	\$ 15,030,542.0 \$ 14,831,543.0	\$ 62,394.0 \$ 61,190.0
second production plan NSHS second production plan HS	37	\$ 13,344,695.0	\$ 60,081.0
	40	\$ 13,240,897.0	\$ 58,170.0
third production plan NSHS	39	\$ 13,148,170.0	\$ 61,974.0
third production plan HS	42	\$ 12,944,568.0	\$ 62,395.0
EXPERIMENT 2: first production plan NSHS first production plan HS	39 44	\$ 14,640,437.0 \$ 14,345,439.0	\$ 64,538.0 \$ 63,908.0
second production plan NSHS second production plan HS	36	\$ 13,372,855.0	\$ 64,138.0
	44	\$ 13,004,058.0	\$ 62,795.0
third production plan NSHS	42	\$ 12,707,449.0	\$ 63,785.0
third production plan HS	42	\$ 12,549,650.0	\$ 62,510.0

EXAMPLE 1: Detailed cost information in a rolling horizon environment with small variations in the setup matrix.

Referring back to the set of experiments of table 1, it is noted that for large variations in the setup matrix, the maximal savings for the NSHS heuristic is in general larger as compared to the HS heuristic. Hence,

NSHS yields lower costs per effective iteration. In this case, with large variations in setup times, the consequences of rescheduling may be much more severe, as is illustrated in example 2. Example 2 is idential to example 1, except that a setup matrix is used which exhibits large variations in switch-over times (k=3.8). In this example not every single iteration yields an overall lower cost level.

Both the average savings per iteration, as well as the the average savings per effective iteration, are calculated. It is seen that, although the HS algorithm leads to more iterations than NSHS, the number of effective iterations is approximately equal. Per effective iteration the algorithm of NSHS yields larger savings than HS, hence producing a cheaper production plan. In comparing both examples, we notice that the average actual savings per iteration are much lower in example 2. This could be explained by the larger consequences of rescheduling, adversely affecting the total cost of the production plan.

EXPERIMENT 1:	# iterations	total cost of the production plan	average actual savings per iteration
first production plan NSHS first production plan HS	46 16(e) 52 22(e)	\$ 10,663,159.0 \$ 10,573,161.0	\$ 11,630.0 \$ 35,666.0 \$ 12,019.0 \$ 28,408.0
second production plan NSHS second production plan HS	45 17(e) 48 9(e)	\$ 9,160,892.0 \$ 9,913,694.0	\$ 14,688.0 \$ 38,882.0 \$ 4,500.0 \$ 24,000.0
third production plan NSHS third production plan HS	42 15(e) 45 10(e)	\$ 9,679,659.0 \$ 10,208,257.0	\$ 12,666.0 \$ 35,466.0 \$ 7,022.0 \$ 31,599.0
EXPERIMENT 2: first production plan NSHS first production plan HS	38 16(e) 42 15(e)	\$ 10,936,838.0 \$ 11,125,836.0	\$ 15,789.0 \$ 37,498.0 \$ 9,785.0 \$ 27,398.0
second production plan NSHS second production plan HS	41 14(e) 43 16(e)	\$ 9,370,886.0 \$ 9,686,986.0	\$ 14,048.0 \$ 41,053.0 \$ 6,044.0 \$ 16,243.0
third production plan NSHS third production plan HS	33 14(e) 39 16(e)	\$ 9,862,591.0 \$ 9,780,988.0	\$ 12,545.0 \$ 29,571.0 \$ 12,179.0 \$ 29,687.0

EXAMPLE 2: Detailed information in a rolling horizon with <u>large</u> variations in the setup matrix (e = effective iteration).

Several additional experiments where performed, with the following results briefly outlined below.

a) For the same set of experiments as described in table 1, but using unbiased forecasts, the HS heuristic outperformed NSHS for all types of setup matrices.

- b) A similar set of experiments was performed, characterized by:
 - rolling horizon and 60% freeze
 - increasing standard deviation of the forecast errors over time
 - systematic under-estimation of the demand
 - several types of setup matrices.

Also in these cases, the HS heuristic outperformed the NSHS heuristic.

CONCLUSIONS

Two lot sizing-sequencing heuristics, HS and NSHS, were compared in a set of experimental studies, characterized by a partially frozen rolling planning horizon, demand uncertainty and varying setup time matrices, where setup times are sequence-dependent. The HS heuristic, a period-by-period heuristic, performed better when demand was overestimated, demand uncertainty grew over time, and setup times did not vary too much; where NSHS, an improvement heuristic, did better when a large variation in setup times was present. In similar experiments, but with unbiased demand estimates, the HS heuristic always outperformed the NSHS heuristic. Also when demand was consistently underestimated, HS produced lower cost production plans as compared to NSHS. Both heuristics always produced significantly lower cost-final production plans as compared to the initial feasible production schedule.

Although the simulated environment is far too complex for obtaining true optimal solutions because of its combinatorial nature, these experiments indicate that both heuristics could be used effectively in a variety of chemical processing environments, yielding lower cost production plans.

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