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PERFECT EQUILIBRIUM IN A MODEL OF COMPETITIVE ARMS ACCUMULATION

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PERFECT EQUILIBRIUM IN A MODEL OF COMPETITIVE ARMS ACCUMULATION

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ABSTRACT

The West is a decentralised market economy whose government uses optimal taxation to provide a public good, defence. The East is a centrally planned economy. Defence is a characteristic, which is an increasing function of the difference between home and foreign weapon stocks. The cooperative outcome leads to a moratorium on investment in weapons. Two non-cooperative solutions to this differential game are considered. The first is an open-loop Nash equilibrium solution, which presumes that countries cannot condition their investment in arms on the rival's weapon stock. The second is a perfect Nash equilibrium solution, which presumes that countries can monitor foreign weapon stocks. The perfect equilibrium solution leads to lower levels of arms and is therefore more efficient, so that a unilateral arms treaty should allow countries to observe their rival's weapon stocks. The perfect equilibrium solution also gives a micro-economic foundation of the Richardson equations.

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1. INTRODUCTION

Since Richardson [1960] introduced a dynamic model for the description of conflict over arms accumulation between two nations, several attempts have been made to find a behavioural micro-foundation for his equations. Brito [1972] reformulated the Richardson model as a differential game, but restricted himself to investment strategies with precommitment. This means that the nations have open-loop information sets and therefore are assumed to be unable to observe their rival's weapon stock. Simaan and Cruz [1975] obtained a feedback (or subgame-perfect) Nash equilibrium solution, which relies on nations being able to monitor their rival's weapon stock. The objectives of this paper are to provide a more satisfactory micro-economic foundation of the Richardson equations and to show that the subgame-perfect Nash equilibrium leads to less weapon accumulation than the open-loop Nash equilibrium. A policy conclusion would be that countries should be encouraged to observe each other's weapon stock.

There are two countries involved in arms conflict. The West is a decentralised market economy whose government maximises the utility of a representative household and levies lump-sum taxes in order to finance investment in arms. The East is a centrally planned economy. Utility depends on consumption, leisure and defence; defence is a characteristic which depends on the difference between home and foreign weapon stocks. When consumption and leisure are normal goods, there is a "guns versus butter" dilemma as more taxes lead to more weapons at the expense of less consumption and leisure. Section 2 formulates this two-country model. Section 3 discusses the cooperative outcome of this differential game and shows that this leads to a moratorium on investment in weapons. Section 4 gives the non-cooperative Nash equilibrium for the case that countries cannot observe their rival's weapon stock. Section 5 gives the perfect equilibrium, which corresponds to the case that countries can monitor their rival's weapon stock. It is shown that this leads to less weapon accumulation and a more satisfactory micro-foundation of the Richardson equations. Section 6 concludes the paper.

2. THE "GUNS VERSUS BUTTER" DILEMMA

Consider a decentralised market economy with a representative household, a representative firm and a government. There are no domestic or foreign financial assets and the economy does not engage in international trade. There is no private capital accumulation, although the government does invest in weapon stocks. There is only one domestically produced commodity, which can be used for both consumption and investment purposes. The government demands goods for investment purposes, the household supplies labour and demands goods for consumption purposes, and the firm demands labour and supplies goods. The real wage adjusts in order to ensure labour market equilibrium. The government finances the provision of public goods, i.e. weapons, by means of non-distortionary taxes and maximises the utility of the representative household.

The household maximises utility, $u(c,\ell,d)$ where c,ℓ and d denote consumption, labour supply and defence, subject to its budget constraint, $0 \leq c \leq w\ell + \pi - \tau$ where w, π and τ denote the real wage rate, profits and lump-sum taxes, respectively. Utility is assumed to be separable in defence. Defence itself is a characteristic (cf. Lancaster [1966]), which is an increasing function of the own weapon stock, a, and a decreasing function of the foreign weapon stock, a^* : $d = D(a,a^*)$. Furthermore, it is assumed that an equal increase in both home and foreign weapon stocks leaves the level of defence or security unaffected so that $D_a(a,a^*) = - D_{a^*}(a,a^*) > 0$. For an interior solution, the marginal rate of substitution between leisure, $1-\ell$, and consumption equals the real opportunity cost of leisure so that $-U_\ell/U_c = w$. The firm maximises profits, $\pi = f(\ell) - w\ell$ where $f(.)$ is a concave production function, which yields $w = f'(\ell)$. Goods market equilibrium implies $f(\ell) = c + g$, where g denotes the level of government spending, and the government budget constraint is $g = \tau$. It follows that the indirect utility function can, without loss of generality, be written as

$$u(C(g),L(g),d) = U(g) + D(a,a^*) \qquad (1)$$

where $U' = u_c C' + u_\ell L'$, $C' = (u_c f'' + u_{\ell\ell} + u_{c\ell} f')/\Delta$, $L' = -(u_{\ell c} + u_{cc} f')/\Delta$ and $\Delta = -[u_c f'' + u_{\ell\ell} + 2u_{c\ell} f' + u_{cc}(f')^2] > 0$. It will be assumed that consumption and leisure are normal goods, so that an increase in taxes reduces consumption, leisure and thus utility ($C'<0$, $L'>0$, $U'<0$). A sufficient condition for this assumption is that utility is also separable in consumption and leisure.

The decentralised market economy discussed sofar will be called the West. It is engaged in competitive arms accumulation with a command economy, called the East. The variables in the East are denoted by an asterisk. The East has the same technology and preferences, but its government plans c^*, ℓ^* ang g^* to maximise utility, $u(c^*,\ell^*,d^*)$, subject to the material balance condition, $f(\ell^*) = c^* + g^*$. Since lump-sum taxes are non-distortionary, this yields the same indirect utility as in the West, $U(g^*) + D(a^*,a)$.

In order to obtain an analytical solution for the perfect equilibrium (see Section 5), a second-order Taylor series approximation of indirect utility is adopted:

$$U(g) + D(a,a^*) = \tilde{\theta}_0 + \theta_1 g - \frac{1}{2}\theta_2 g^2 + \theta_3(a-a^*) - \frac{1}{2}\theta_4(a-a^*)^2$$

$$= \theta_0 - \frac{1}{2}\theta_2(g-\bar{g})^2 - \frac{1}{2}\theta_4(a-a^*-\bar{m})^2, \quad \theta_2, \theta_4 > 0 \quad (2)$$

where $\bar{g} \equiv \theta_1/\theta_2$ and $\bar{m} \equiv \theta_3/\theta_4$ can be interpreted as the target level of public spending and the desired lead in weapon stocks, respectively. The assumption of normal goods implies that $g > \bar{g}$ for all $g \geq 0$, so that $\theta_1 \leq 0$. If preferences are quadratic and technology is linear the approximation is exact. The value function for the West corresponding to the problem commencing at time t is defined as

$$V(t,a,a^*) \equiv \int_t^\infty [U(g) + D(a,a^*)] \exp[-r(s-t)]\, ds \quad (3)$$

and similarly for the East, where r is the pure rate of time preference. The West maximises $V(0,a_0,a_0^*)$, where a_0 and a_0^* are the initial weapon stocks, subject to the laws of motion for home weapons,

$$\dot{a} = g - \delta a, \quad a(0) = a_0 \quad (4)$$

where δ is the depreciation rate, and for foreign weapons. The East maximises its intertemporal utility subject to the laws of motion for weapon stocks. The dilemma of "guns versus butter" is that high taxes are required to finance a large build-up of weapons, but this necessarily implies less private consumption and leisure.

3. COOPERATIVE BEHAVIOUR

Pareto efficient outcomes for the West and the East may be found from:

$$\underset{g,g^*}{\text{Max}} \int_0^\infty \{\theta[U(g) + D(a,a^*)] + (1-\theta)[U(g^*) + D(a^*,a)]\} \exp(-rt)dt \quad (5)$$

subject to (4) and $\dot{a}^* = g^* - \delta a^*$, $a^*(0) = a_0^*$, where $0 < \theta < 1$. This yields $-\theta U'(g) = \lambda$ and $-(1-\theta)U'(g^*) = \lambda^*$, so that the marginal disutility of government investment in arms, in terms of foregone consumption and leisure, should equal the marginal value of weapon stocks (λ and λ^* for the West and East, respectively). However, if the marginal disutility of government spending exceeds the marginal value of weapons, the complementary slackness conditions say that no investment in weapons takes place ($g = 0$ if $-\theta U'(g) > \lambda$). The marginal values of the weapon stocks follow from

$$\dot{\lambda} = (r+\delta)\lambda - \theta D_a(a,a^*) - (1-\theta)D_{a^*}(a^*,a), \ \lim_{t\to\infty} \exp(-rt)\lambda(t)a(t) = 0 \quad (6)$$

and

$$\dot{\lambda}^* = (r+\delta)\lambda^* - \theta D_{a^*}(a,a^*) - (1-\theta)D_{a^*}(a^*,a),$$

$$\lim_{t\to\infty} \exp(-rt)\lambda^*(t)a^*(t) = 0, \quad (7)$$

so that the "rental" charge plus the depreciation charge minus the capital gains term defines the user cost of weapons and should match the marginal utility of weapons to the world. If the "world peace authority" attaches equal weights to the West and East ($\theta = \frac{1}{2}$), it follows that $\lambda(t) = \lambda^*(t) = 0, \forall t > 0$ and therefore $g(t) = g^*(t) = 0, \forall t > 0$.[1)] Since the game between the two economies is zero-sum at the margin, the cooperative outcome is to have a moratorium on investment in arms and to run down weapon stocks (via wear and tear) until they have fallen to zero. The cooperative outcome is not sustainable, as each country has an incentive to deviate from a multilateral arms treaty by increasing its security at the expense of its rival, if the desired lead in weapons is greater than zero (i.e. $\theta_3 > 0$). Furthermore, a moratorium on investment in arms is seldom observed in the real world. Hence, Sections 4 and 5 consider non-cooperative outcomes.

4. NASH EQUILIBRIUM WITH PRE-COMMITMENT

Consider a situation where the West and East do not cooperate and where neither country dominates the arms race, so that a Nash equilibrium is appropriate. Many Nash equilibrium concepts have been defined for differential games (e.g. Starr and Ho [1969a,b]). The most common one is perhaps the <u>open-loop</u> Nash equilibrium solution (OLNES), which has been used in Brito [1972]. It presumes that the optimal investments in arms at each point of time are only conditioned on the initial weapon stocks, a_0 and a_0^*, and therefore the expected investments of the rival do not depend on past or current weapon stocks or on past or current

1) For $\theta_1 < 0$ the cooperative outcome is a corner solution, but for $\theta_1 = 0$ the corner solution coincides with the unconstrained solutions. To avoid corner solutions, both in this Section and in later Sections, the value of θ_1 can be taken to be zero as this avoids outcomes with negative weapon stocks in the unconstrained solution.

investments of the country under consideration. It follows that the OLNES requires that each country pre-commits itself to a path of investment in arms. The first-order conditions of the OLNES give rise to:

$$\dot{a} = G(\lambda) - \delta a = (\lambda+\theta_1)/\theta_2 - \delta a, \; a(0) = a_0 \tag{8}$$

$$\dot{a}^* = G(\lambda^*) - \delta a^* = (\lambda^*+\theta_1)/\theta_2 - \delta a^*, \; a^*(0) = a_0^* \tag{9}$$

$$\dot{\lambda} = (r+\delta)\lambda - D_a(a,a^*) = (r+\delta)\lambda - \theta_3 + \theta_4(a-a^*),$$

$$\lim_{t\to\infty} \exp(-rt)\lambda(t)a(t) = 0 \tag{10}$$

$$\dot{\lambda}^* = (r+\delta)\lambda^* - D_{a^*}(a^*,a) = (r+\delta)\lambda^* - \theta_3 + \theta_4(a^*-a),$$

$$\lim_{t\to\infty} \exp(-rt)\lambda^*(t)a^*(t) = 0 \tag{11}$$

where $G' = -1/U'' > 0$ and λ (or λ^*) denotes the marginal value to the West (or East) of its weapon stock. The marginal disutility of public spending has to match the marginal value of weapons, $-u'(g) = \lambda$, which gives investment in weapons as an increasing function of its marginal value, $g = G(\lambda)$. The steady state of (8)-(11) follows from $(r+\delta)u'(\delta a) + D_a(a,a) = 0$, which gives for the approximation (2):

$$g(\infty) = g^*(\infty) = \delta a(\infty) = \delta a^*(\infty) = \frac{\theta_1}{\theta_2} + \frac{\theta_3}{\theta_2(r+\delta)} \equiv g^0 > 0. \tag{12}$$

Hence, the steady-state levels of weapon stocks increase when the discount rate or depreciation rate decreases and when the desired lead in weapon stocks over the rival country increases. The steady state is a saddlepoint, since there are two stable eigenvalues ($-\delta$ and $\frac{1}{2}r - \frac{1}{2}\sqrt{(r+2\delta)^2+8\theta_4/\theta_2}$) associated with the backward-looking variables, a and a^*, and two unstable eigenvalues ($r+\delta$ and $\frac{1}{2}r + \frac{1}{2}\sqrt{(r+2\delta)^2+8\theta_4/\theta_2}$) associated with the forward-looking variables, λ and λ^*. Since (8)-(11) is effectively a perfect-foresight system, Buiter's [1984] method of spectral decomposition or the method of undetermined coefficients can be used to solve it. If can be shown that the stable manifold is given by $\lambda = \psi^0\theta_2(a^*-a) + \theta_3/(r+\delta)$ where

$$\psi^0 \equiv -\tfrac{1}{4}[r+2\delta - \sqrt{(r+2\delta)^2+8\theta_4/\theta_2}\,] > 0,$$

so that $g = g^0 + \psi^0(a^*-a)$. Hence, investment in weapons increases over and above its steady-state level when foreign weapon stocks exceed home weapons stocks. Upon substitution, one obtains:

$$\dot{a} = g^0 + \psi^0(a^*-a) - \delta a, \ a(0) = a_0 \tag{13}$$

$$\dot{a}^* = g^0 + \psi^0(a-a^*) - \delta a^*, \ a^*(0) = a_0^* \tag{14}$$

which is a stable system as the eigenvalues associated with (13)-(14), i.e. $-\delta$ and $-2\psi^0-\delta$, are both negative. Equations (13)-(14) can be interpreted as Richardson's [1960] equations: ψ^0 is the "defence coefficient", $\psi^0+\delta$ is the "fatigue coefficient" and g^0 is the "grievance or hatred coefficient". However, this interpretation seems inappropriate in view of the open-loop or pre-commitment nature of the solution concept. In the OLNES the countries cannot condition their investments on current weapon stocks, so that $g = g^0 + \psi^0(a^*-a)$ should be interpreted as a feedback realisation of an open-loop sequence of levels of investments.

Olsder [1977] calls (13)-(14) the "open-loop, open-eye" solution and (8)-(11) the "open-loop, closed-eye" solution, but when monitoring foreign weapon stocks is feasible the "closed-loop, open-eye" solution (see Section 5) seems more appropriate.

Since the marginal value of Eastern (or Western) weapons to the West (or East), say λ_* (or λ_*^*), does not affect the OLNES, it does not matter whether the countries can observe their own weapon stocks. This means that the OLNES also describes the situation where each country can monitor its own weapon stocks but not monitor foreign weapon stocks. The next Section considers the situation where both countries can monitor the weapon stocks of their rival.

5. PERFECT NASH EQUILIBRIUM

The closed-loop Nash equilibrium solution (CLNES) allows each country to condition its investment in weapons on the current and, possibly, past stocks of weapons, so that each country should be able to monitor its own as well as foreign weapon stocks. This type of information structure admits, among others, memory and threat strategies, so that the CLNES set is non-unique (see Başar and Olsder [1982]). However, if the principle of subgame perfection (see Selten [1975]) is imposed, uniqueness typically results. The resulting outcome will be called the subgame-perfect Nash equilibrium solution (SPNES), although Starr and Ho [1969b] and Simaan and Cruz [1975] refer to this outcome as the feedback Nash equilibrium solution. An equilibrium solution is subgame-perfect, if for each subgame the relevant part of the solution is also a Nash equilibrium. A subgame in this context is a game over a remainder of the planning period, say over $[\bar{t},\infty)$ rather than over $[0,\infty)$. The restriction of the solution to a subgame must be a Nash equilibrium for all $\bar{t} \in [0,\infty)$ and for all $\{a(\bar{t}),a^*(\bar{t})\}$. Subgame perfection rules out threat equilibria, which rely on information patterns with memory, and equilibria which imply future investments that are not rational to carry out if called upon to do so in the future. It is clear that the SPNES can be found with the aid of dynamic programming.

The Hamilton-Jacobi-Bellman equation for the West yields $g = G(V_a) = (V_a+\theta_1)/\theta_2$, and similarly for the East. The SPNES follows from the following coupled system of partial differential equations:

$$rV-V_t = U(G(V_a))+D(a,a^*)+V_a[G(V_a)-\delta a]+V_{a^*}[G(V^*_{a^*})-\delta a^*] \quad (15)$$

$$rV^*-V^*_t = U(G(V^*_{a^*}))+D(a^*,a)+V^*_a[G(V_a)-\delta a]+V^*_{a^*}[G(V^*_{a^*})-\delta a^*]. \quad (16)$$

Although (16)-(17) can in principle be solved for the value functions, $V(t,a,a^*)$ and $V^*(t,a^*,a)$, an analytical solution is usually impossible. However, with quadratic objective functions and linear laws of motion, an analytical solution can be found. Hence, presume that the value functions are given by $V(t,a,a^*) = p_0 + (p_1,p_2)(a,a^*)' - \frac{1}{2}(a,a^*)P(a,a^*)'$ and $V^*(t,a^*,a) = p^*_0 + (p^*_1,p^*_2)(a^*,a)' - \frac{1}{2}(a^*,a)P^*(a^*,a)'$, where P and P^* are positive semi-definite symmetric matrices. Upon substitution of the value functions into (15)-(16) and equating coefficients on a, a^*, a^2, a^{*2} and aa^*, one finds $p^*_1 = p_1$, $p^*_2 = p_2$, $P^* = P$,

$$\dot{p}_1 = (r+\delta)p_1 - \theta_3 + (\theta_1+p_1)P_{11}/\theta_2 + (\theta_1+p_1+p_2)P_{12}/\theta_2 = 0 \quad (17)$$

$$\dot{p}_2 = (r+\delta)p_2 + \theta_3 + [(\theta_1+p_1)(P_{12}+P_{22}) + P_{11}p_2]/\theta_2 = 0 \tag{18}$$

$$\dot{P}_{11} = (r+2\delta)P_{11} + (P_{11}^2+2P_{12}^2)/\theta_2 - \theta_4 = 0 \tag{19}$$

$$\dot{P}_{22} = (r+2\delta)P_{22} + (P_{12}^2+2P_{11}P_{22})/\theta_2 - \theta_4 = 0 \tag{20}$$

and

$$\dot{P}_{12} = [r+2\delta+(2P_{11}+P_{22})/\theta_2]P_{12} + \theta_4 = 0. \tag{21}$$

There are a number of solutions that satisfy the equations (19)-(21), but only one of them ensures that the Hessians of the value functions, $-P$ and $-P^*$, are negative semi-definite.[1)] This solution is given by

$$P_{11} = P_{22} = -P_{12} = \tfrac{1}{6}\sqrt{[(r+2\delta)\theta_2]^2+12\theta_2\theta_4} - \tfrac{1}{6}(r+2\delta)\theta_2 > 0 \tag{22}$$

and

$$p_1 = -p_2 = \theta_2\theta_3/[(r+\delta)\theta_2+P_{11}]. \tag{23}$$

It follows that $g = g^P + \psi^P(a^*-a)$ and $g^* = g^P + \psi^P(a-a^*)$, where $\psi^P \equiv P_{11}/\theta_2 > 0$ and the steady-state levels of investments in arms are given by

$$0 < g(\infty) = g^*(\infty) = \delta a(\infty) = \delta a^*(\infty) = \frac{\theta_1}{\theta_2} + \frac{\theta_3}{\theta_2(r+2\delta) + P_{11}} \equiv g^P < g^0. \tag{24}$$

1) For example, $P_{12} = -(r+2\delta)\theta_2-P_{11} = -P_{22} = -\tfrac{1}{6}(r+2\delta)\theta_2 \pm \tfrac{1}{6}\sqrt{[(r+2\delta)\theta_2]^2+12\theta_2\theta_4}$ are two additional solutions but neither of them satisfies trace $(\nabla^2 V) < 0$ and det $(\nabla^2 V) \geq 0$.

Upon substitution of the investment rule into (4), one obtains

$$\dot{a} = g^P + \psi^P(a^* - a) - \delta a, \; a(0) = a_0 \tag{25}$$

and similarly for the East. It seems appropriate to view these as a micro-economic underpinning of the Richardson's [1960] equations. In fact, equation (25) corresponds to a "closed-loop, open-eye" solution so that the implied information on the monitoring of the foreign weapon stock permits a sensible interpretation in terms of defence, fatigue and grievance coefficients.

The main result is that monitoring of foreign weapon stocks leads to less weapon stocks than in the absence of monitoring, since $g^0 > g^P > 0$. The intuition behind this result is that, when one country considers the purchase of one additional unit of weapons, it considers the direct marginal contribution to security and welfare, d_a, but it also considers the reaction of its rival. That is, it takes account of the fact that its rival's security is worsened and therefore it will purchase more weapons. This therefore reduces the direct marginal contribution to security and welfare, $D_a + \psi^P V_{a^*} < D_a$, instead of D_a, so that there is less incentive to invest in weapons than when countries cannot observe their rival's weapon stock. The obvious policy implication is that countries should be encouraged to monitor each other's weapon stocks as this will lead to some unilateral disarmament.

Note that, when defence is a linear function of the lead in weapon stocks ($\theta_4 = 0$), the defence coefficient is zero ($\psi^P = 0$) and the grievance coefficient is independent of whether countries can monitor their rival's weapon stock or not ($g^P = g^0$). Since for this case the defence coefficient for the OLNES is also zero ($\psi^0 = 0$), it follows that for this special case the OLNES and SPNES coincide and therefore whether countries can monitor their rival's weapon stock or not is irrelevant. Finally, note that, when neither country attempts to establish a lead in weapon stocks ($\theta_3 = 0$), both grievance coefficients are zero ($g^P = g^0 = 0$). Hence, for this special case the non-cooperative outcome (either with or without monitoring the rival's weapon stock) reduces to the cooperative outcome with a moratorium in weapons.

6. CONCLUSIONS

The conflict over arms accumulation between a decentralised market economy, the West, and a centrally planned economy, the East can be modelled as a differential game. A multilateral arms treaty leads to a moratorium on investment in weapons. The open-loop Nash equilibrium solution presumes that countries cannot condition their investment in arms on the rival's weapon stock, whilst the perfect Nash equilibrium solution presumes that countries can monitor foreign weapon stocks. The perfect equilibrium solution leads to lower levels of arms accumulation and is therefore more efficient. It follows that a unilateral arms treaty should enable countries to observe their rival's weapon stocks. The perfect equilibrium solution also gives a more satisfactory micro-economic foundation of the Richardson equations where the desired lead of weapons over the rival country increases the grievance coefficients, the concavity of the defence function determines the defence coefficients and the sum of the defence coefficients and depreciation rates gives the fatigue coefficients. It follows that the weapon stocks increase proportionately to the level of weapon stocks of the rival nation ("defence"), decrease proportionately to the economic burden of its own weapon stock ("fatigue") and increase due to the desired weapon lead over the rival nation ("grievance" or "hatred").

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