Asymmetric Dependence in US Financial Risk Factors?

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Abstract

When assets exhibit asymmetric dependence or joint downside risk, diversification can fail and financial markets may be prone to systemic risk. We analyze the dependence structure of risk factors in the US economy, using both correlations and a parsimonious set of copulas. We find evidence of downside risk in several risk factors. Interestingly for research on systemic risk, the pairs with downside risk include consumption with the Dow Jones, as well as consumption with market and size factors. Of these pairs, only the size factor exhibits an offsetting upside comovement with consumption during good periods. We also discover significant dynamic behavior in dependence for several risk factors, in particular between consumption and the size factor. Thus, financial markets exhibit time variation in downside risk. Our results provide quantitative evidence on the susceptibility of financial markets to diversification failure and systemic risk.

Keywords: Asymmetric Dependence; Copulas; Diversification Failure; Risk Factors; Systemic Risk; Time-Varying Downside Risk

JEL Classification: C14, E44, G01, G11

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1 Introduction and motivation

Dependence is at the heart of financial theories of risk, since it summarizes diversification benefits.[1] The net benefit of diversification is of great importance in today's economic climate. In general, the balance between diversification's benefits and costs hinges on the degree of dependence across securities, as observed by Samuelson (1967), Veldkamp and Van Nieuwerburgh (2009), Ibragimov, Jaffee, and Walden (2009b), and Shin (2009), among others. Diversification benefits are typically assessed using a measure of dependence, such as correlation.[2] It is therefore vital for investors to have accurate measures of dependence.[3] There are several measures available in finance, including the traditional correlation and copulas. While each approach has advantages and disadvantages, they rarely have been compared in the same empirical study. Such reliance on one dependence measure prevents easy assessment of the degree of diversification opportunities, and how they differ over time or across sectors.

Moreover, from an aggregate perspective, situations of high financial dependence often signal extreme financial fragility, as evidenced by joint down moves of multiple economic sectors and financial asset classes in the US in 2008 and 2009. Such failures of diversification have deep economic and social repercussions.[4] Financial dependence amplifies the effect of surprise events.[5] For example, the collapse of a major lending institution affects many households, and can cause total insurance claims to increase geometrically, since multiple classes are affected, including property loss and job loss.[6] The lack of empirical research on such ”simultaneous hard times” means that individuals and society are not prepared, when such preparation matters most. Historically, financial economists have devoted considerable research effort to examine dependence of key risk factors. Most empirical and theoretical studies consider average dependence, which is appropriate if the true dependence structure is linear. However, when dependence is nonlinear, it is important to

[1]See Samuelson (1967); Solnik (1974); and Ibragimov, Jaffee, and Walden (2009b).

[2]See Solnik (1974); Ingersoll (1987) Chapter 4; and Carrieri, Errunza, and Sarkissian (2008).

[3]Throughout, we use the word dependence as an umbrella to cover any situation where two or more risk factors move together. We adopt this practice because there are numerous words in use (e.g. correlation, concordance, co-dependency, coherence, and comovement), and we wish to use a general term. We do not assume that any dependence measure is ideal, and throughout we indicate advantages and disadvantages as the case may be.

[4]For research on the welfare cost of financial crises, see Chatterjee and Corbae (2007); Reinhart and Rogoff (2009); and the references therein.

[5]See Krishnamurthy (2009) for economic explanations for such amplifications.

[6]For details on insurance during periods of macroeconomic dependence, see Jaffee and Russell (1997); Jaffee (2006); and Ibragimov, Jaffee, and Walden (2009b) .

use robust dependence measures.[7] Recently there have evolved robust tools to study dependence, such as copulas.[8] While such tools have been applied successfully in banking and international economics, there is no comparable research on financial risk factors. In light of the above considerations, we investigate the dependence structure of important US financial risk factors, using both correlations and a parsimonious set of copulas.

The main goal of this paper is to assess the dependence structure of US financial risk factors. This research sheds light on diversification opportunities available in financial markets. The recent history of US markets is interesting in itself, due to the large number of financial crises, increasingly globalized markets, and financial contagion.[9] A secondary focus of our paper is the relation between dependence and systemic stability. In general, systemic instability increases with the degree of market dependence, as observed by Caballero and Krishnamurthy (2008); Ibragimov, Jaffee, and Walden (2009b), and Shin (2009), among others. Systemic instability may also be exacerbated by *correlation complexity*, when different dependence measures give conflicting or inaccurate signals. It is therefore vital for households, banks and policymakers to have accurate estimates of dependence. The importance of this issue is highlighted by both theoretical and applied research.[10] When portfolio distributions have tail dependence, not only do they represent limited diversification, they also suggest a wedge between acceptable individual risk and systemic risk. Thus, there are aggregate ramifications for elevated levels of financial dependence. If systemic costs are too severe, a coordinating agency may be needed to improve the economy's resource allocation.[11] Such policy considerations are absent from previous empirical research on risk factor dependence, and provide a further motivation for our paper.

[7] See Granger (2001); Hamilton (2001); and Embrechts, McNeil, and Straumann (2002).

[8] These tools are drawn from distributional and asymptotic approaches in statistics. For distributional approaches see Embrechts, McNeil, and Straumann (2002); Joe (1997); and Nelsen (1998). For asymptotic approaches see Embrechts, Kluppelberg, and Mikosch (1997); and de Haan and Ferreira (2006).

[9] See Dungey and Tambakis (2005); Reinhart (2008); and Reinhart and Rogoff (2009).

[10] See Rosenberg and Schuermann (2006); Ibragimov, Jaffee, and Walden (2009b); and Shin (2009).

[11] For related work, see Caballero and Krishnamurthy (2008); Ibragimov, Jaffee, and Walden (2009a); and Shin (2009).

1.1 Related empirical research

Previous research generally falls into either correlation or copula frameworks.[12] The literature in each area applied to financial economics is vast and growing, so we summarize only some key contributions.[13] With regard to correlation, a major finding of Longin and Solnik (1995) and Ang and Bekaert (2002) is that international stock correlations tend to increase over time. Moreover, Cappiello, Engle, and Sheppard (2006) document that international stock and bond correlations increase in response to negative returns, although part of this apparent increase may be due to an inherent volatility-induced bias.[14] Regarding copula-based studies of dependence, an early paper by Mashal and Zeevi (2002) shows that the dependence structures of equity returns, currencies and commodities exhibit joint heavy tails. Patton (2004) uses a conditional form of the copula relation (2) to examine dependence between small and large-cap US stocks. He finds evidence of asymmetric dependence in the stock returns. Patton (2004) also documents that knowledge of this asymmetry leads to significant gains for investors who do not face short sales constraints. Patton (2006) uses a conditional copula to assess the structure of dependence in foreign exchange. Using a sample of Deutschemark and Yen series, Patton (2006) finds strong evidence of asymmetric dependence in exchange rates. Jondeau and Rockinger (2006) successfully utilize a model of returns that incorporates skewed-t GARCH for the marginals, along with a dynamic gaussian and student-t copula for the dependence structure. Rosenberg and Schuermann (2006) analyze the distribution of bank losses using copulas to represent, very effectively, the aggregate expected loss from combining market risk, credit risk, and operational risk. Rodriguez (2007) constructs a copula-based model for Latin American and East Asian countries. His model allows for regime switches, and yields enhanced predictive power for international financial contagion. Okimoto (2008) also uses a copula model with regime switching, focusing on the US and UK. Okimoto (2008) finds evidence of asymmetric dependence between stock indices from these countries. Harvey and de Rossi (2009)

[12] There is also a related literature that examines dependence using extreme value theory, as well as threshold correlations or dynamic skewness. These papers all find evidence that dependence is nonlinear, increasing more during market downturns for many countries, and for bank assets as well as stock returns. For extreme value approaches, see Longin and Solnik (2001), Hartmann, Straetmans, and de Vries (2003), and Poon, Rockinger, and Tawn (2004). For threshold correlations, see Ang and Chen (2002). For dynamic skewness, see Harvey and Siddique (1999).

[13] For summaries of copula literature, see Cherubini, Luciano, and Vecchiato (2004), Embrechts, McNeil, and Frey (2005), Jondeau, Poon, and Rockinger (2007), and Patton (2009). For more general information on dependence in finance, see Embrechts, Kluppelberg, and Mikosch (1997), and Cherubini, Luciano, and Vecchiato (2004).

[14] See Forbes and Rigobon (2002).

construct a model of time-varying quantiles, which allow them to focus on the expectation of different parts of the distribution. This model is also general enough to accommodate irregularly spaced data. Harvey and Busetti (2009) devise tests for constancy of copulas. They apply these tests to Korean and Thai stock returns and document that the dependence structure may vary over time. Ning (2008) examines the dependence of stock returns from North America and East Asia. She finds asymmetric, dynamic tail dependence in many countries. Ning (2008) also documents that dependence is higher intra-continent relative to across continents. Ning (2010) analyzes the dependence between stock markets and foreign exchange, and discovers significant upper and lower tail dependence between these two asset classes. Chollete, Heinen, and Valdesogo (2009) use general canonical vines in order to model relatively large portfolios of international stock returns from the G5 and Latin America. They find that the model outperforms dynamic gaussian and student-t copulas, and also does well at modifying the VaR for these international stock returns. These papers all contribute to the mounting evidence on significant asymmetric dependence in joint asset returns.

1.2 Contribution of our paper

Our paper has similarities and differences with the previous literature. The main similarity is that, with the aim of gleaning insight on risk and diversification, we estimate dependence in financial markets. There are two main differences. First, we assess diversification using both correlation and copula techniques, and we are agnostic ex ante about which technique is appropriate. To the best of our knowledge, ours is one of the first papers. Second, our paper builds on specific finance theories of dependence and diversification. Previous empirical research focuses very justifiably on establishing the existence of extreme or asymmetric dependence. Understandably, these empirical studies are generally motivated by implications for individual market participants and risk management benchmarks such as VaR. By contrast, our work builds on theoretical risk research, and discusses both individual and systemic implications of asset dependence structure. Most empirical research assessing market dependence takes it for granted that larger dependence leads to poorer diversification in practice. While this can be true, what is arguably more important from an economic point of view is that there are aggregate ramifications for elevated asset dependence. Therefore, we examine average dependence across risk factors, in order to obtain empirical insight on the possibility of a wedge between individual and social desiderata. Such considerations are absent from most previous empirical copula research.

More broadly, our paper can be seen a providing a robust alternative examination of the implications of theoretical research, to see whether their predictions hold differentially in normal and extreme times. This scientific motivation has at least two dimensions. First, existing theoretical models such as that of Lucas (1978) say that assets are priced according to their dependence with consumption. However, dependence during extremes should be more important than dependence at other times, especially for agents that exhibit downside risk aversion.[15] Such nonlinear dependence could not be easily captured by previous studies using correlations. Second, the key insight in most theoretical research is that dependence is to be avoided, and may indicate economic inefficiency. Thus, if dependence during extremes is pronounced, this indicates inefficiency during bad times. In addition to these scientific motivations, there is a strong practical motivation for the financial risk factors. Specifically, one source of confusion in current financial market policy is the lack of robust documentation of dependence in financial risk during extreme periods. Our paper appears to be among the first to examine and document extreme dependence of important risk factors in US financial markets.

The remaining structure of the paper is as follows. In Section 2 we review related theoretical and empirical literature on dependence and diversification in finance. In Section 3 we discuss our data and main results, and Section 4 concludes.

2 Dependence, diversification, and systemic risk

Dependence and diversification are cornerstones of modern finance. The notion that diversification improves portfolio performance is pervasive in financial economics, and appears in asset pricing, insurance, and international finance. A central precept is that, based on the law of large numbers, a group of securities carries a lower variance than any single security.[16] An important caveat, noted as early as Samuelson (1967), concerns the dependence structure of security returns, as we discuss below. This theoretical importance of dependence structure motivates our use of copulas in the empirical analysis.

[15] See Kahneman and Tversky (1979); Barberis, Huang, and Santos (2001); and Polkovnichenko (2005).

[16] Aspects of this precept have been formalized by Markowitz (1952); Sharpe (1964); Lintner (1965); Mossin (1966); and Samuelson (1967).

2.1 Theoretical background

When assets have substantial dependence in their tails, diversification may not be optimal.[17] In an early important paper, Samuelson (1967) examines the restrictive conditions needed to ensure that diversification is optimal.[18] He underscores the need for a general definition of negative dependence, framed in terms of the distribution function of security returns. In a significant development, Brumelle (1974) proves that negative correlation is neither necessary nor sufficient for diversification, except in special cases such as normal distributions or quadratic preferences. Brumelle uses a form of dependence as a sufficient condition for diversification, that involves the shape of the entire distribution. Thus, shortly after the inception of modern portfolio theory, both Brumelle (1974) and Samuelson (1967) realize and discuss the need for restrictions on the joint distribution, in order to obtain diversification. However, that discussion has a gap: it stops short of examining multivariate ($n > 2$) asset returns, and the practical difficulty of imposing dependence restrictions on empirical data. The use of copulas may be one way to fill this gap.[19] The research of Embrechts, McNeil, and Straumann (2002) introduces copulas into risk management. The authors first show that standard Pearson correlations can go dangerously wrong as a risk signal. They then suggest the copula function as a flexible alternative to correlation, which can capture dependence throughout the entire distribution of asset returns. A copula C is by definition a joint distribution with uniform marginals. In the bivariate case, that means

$$C(u, v) = \Pr[U \leq u, V \leq v], \tag{1}$$

where U and V are uniformly distributed.[20]

[17] See Embrechts, McNeil, and Frey (2005); Jondeau, Poon, and Rockinger (2007); and Ibragimov (2009).

[18] Samuelson (1967) discusses several approaches to obtain equal diversification across assets, as well as positive diversification in at least one asset. The distributional assumptions on security returns involve i.i.d. and strict independence of at least one security. Although both utility functions and distributional assumptions are relevant, Samuelson focuses on distributional concerns. A special case of dependence when diversification may be optimal is that of perfect negative correlation. However, if a portfolio consists of more than 2 assets, some of which are negatively correlated, then at least 2 must be positively correlated. This could still result in suboptimality of diversification for at least one asset, when there are short sale constraints. See Ibragimov (2009); and Samuelson (1967), page 7.

[19] Another approach involves extreme value theory, see Embrechts, McNeil, and Frey (2005).

[20] See de la Peña, Ibragimov, and Sharakhmetov (2006), Definition 3.1. It is typical to express the copula in terms of the marginal distributions $F_X(x)$ and $F_Y(y)$. In general, the transformations from X and Y to their distributions F_X and F_Y are known as probability integral transforms, and F_X and F_Y can be shown to be uniformly distributed. See Cherubini, Luciano, and Vecchiato (2004), page 52; and Embrechts (2009).

The intuition behind copulas is that they "couple" or join marginals into a joint distribution. Copulas often have convenient parametric forms, and summarize the dependence structure between variables.[21] Specifically, for any joint distribution $F_{X,Y}(x,y)$ with marginals $F_X(x)$ and $F_Y(y)$, we can write the distribution as

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y)). \tag{2}$$

The usefulness of (2) is that we can simplify analysis of dependence in a return distribution $F_{X,Y}(x,y)$ by studying instead a copula C. Since copulas represent dependence of arbitrary distributions, in principle they allow us to examine diversification effects for heavy-tailed joint distributions, following the logic of Brumelle (1974) and Samuelson (1967).

In order to place the above research in perspective, it is necessary to discuss two aspects of financial risk, namely equilibrium asset pricing and systemic risk. Both aspects revolve around issues of financial dependence. The equilibrium approach says that the price of an asset is an increasing function of its dependence with either the market return or some aggregate risk factor. Regarding the market return, the CAPM model of Sharpe (1964), Lintner (1965) and Mossin (1966) says that under some conditions, for any stock i, its return R_i relates to its dependence (covariance) with the market return R_m:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f], \tag{3}$$

where $\beta = \text{Cov}(R_m, R_i)/\text{Var}(R_m)$. Therefore, the greater its dependence with the market, the higher an asset's own return. Regarding aggregate risk factors, Lucas (1978) constructs a dynamic equilibrium asset pricing model. Under rational expectations and in a stationary environment, the author shows that asset prices are characterized by a stochastic euler equation.[22] This equation may be written as

$$1 = \int_{-\infty}^{\infty} M(1+R_i)dF_{M,R}(M, r_i) = E[M(1+R_i)], \tag{4}$$

where M is a discount factor that prices asset returns R_i, and F is the joint distribution of M and R_i. Equation (4) is typically re-expressed using the covariance decomposition

[21] This result holds for multivariate $(n > 2)$ quantities. It is due to Sklar (1959), who proves that copulas uniquely characterize continuous distributions. For non-continuous distributions, the copula will not necessarily be unique. In such situations, the empirical copula approach of Deheuvels (1979) helps narrow down admissible copulas.

[22] Expression (4) follows from Proposition 2 and Equation 6 of Lucas (1978).

for both a risky asset and a riskless asset with return R_f, as $0 = E[M(1 + R_i - R_f)] = E[M] * E[(R_i - R_f] - Cov[M, R_i - R_f]$, or[23]

$$E[(R_i - R_f] = \frac{-Cov[M, R_i - R_f]}{E[M]}. \tag{5}$$

The intuition for (5) is similar to that of (3): asset $i's$ return is determined by risk, summarized by its dependence (covariance) with the discount factor M. An asset that exhibits large negative dependence with M will provide relatively small returns when the discount factor (and marginal utility) is high. Such an asset does not offer much to investors during bad states of the world. Therefore, in order to entice investors to hold it, the asset requires a large excess return. Although the covariance decomposition is useful in some settings, it does not capture all the dependence in equilibrium, since covariance only measures linear dependence.[24]

A tractable special case of the discount factor M involves linear factor models. Factor models may be interpreted as linear versions of M from above. For example, a 3-factor model of risk is of the form $M = \alpha + \lambda_1 F_1 + \lambda_2 F_2 + \lambda_3 F_3$, where λ_i, $i = 1, 2, 3$, is the risk premium for factor i.[25] Standard risk factors comprise the market, size, book to market and momentum, as well as liquidity, default risk, and volatility.[26] In response to growing research on atheoretical factor models, Campbell (1996) develops a loglinear asset pricing model that allows for changing investment opportunities. In his model, equilibrium expected returns depend on covariances of securities with the market and innovations in the present value of future expected market returns. The author demonstrates that a valid risk factor can be any variable that forecasts market returns. Campbell also asserts that labor income is an important factor to reflect investor wealth in asset pricing studies. Thus, dependence of consumption, the stock market, and labor income are central to the approach of Campbell (1996). Campbell's results are structured around dependence considerations, since the model's testable implications involve correlations of innovations in consumption growth and labor income growth. Furthermore, the loglinear model is obtained by a restriction of distributional dependence, namely the assumption that the joint distribution of

[23]See Campbell, Lo, and MacKinlay (1996); Campbell (2000); and Cochrane (2001).

[24]See Embrechts, McNeil, and Straumann (2002).

[25]See Cochrane (2001).

[26]The market, size and book to market factors are examined by Fama and French (1993). Momentum is examined by Carhart (1997). Liquidity risk is studied by Pastor and Stambaugh (2003); Acharya and Pedersen (2005); and Sadka and Koracjczyk (2008). Default risk is studied by Vassalou and Xing (2004). Volatility risk is examined by Ang, Hodrick, Xing, and Zhang (2006).

asset prices and consumption is lognormal.[27] In a systematic study of major asset pricing models, Hodrick and Zhang (2001) document that the Campbell model is the only one that can plausibly price a standard universe of US stock returns.[28] To summarize, in modern finance the key asset pricing relations center on considerations of dependence, as shown in the CAPM and discount factor approaches of (3) and (4) above.

The above approaches analyze investor decisions or risk, and say little about systemic risk. Evidently investors' decisions, in aggregate, may have an externality effect on financial and economic markets. The existence of externalities related to "excessive" diversification has been emphasized by several recent theoretical papers.[29] We discuss the following articles, since their results relate to distributional dependence.[30] Shin (2009) constructs a model to analyze the relation between asset securitization and financial stability. Shin recognizes that increased securitization, while reducing investor portfolio risk in many instances, may also lower aggregate lending standards. In the author's framework an important role is given to endogeneity in credit, since lenders change credit supply in response to perceived risk. Hence in the model of Shin (2009), when securitization drives down lending standards there is a tradeoff between credit expansion and systemwide stability. Once credit expands too much to include excessively risky borrowers, the entire financial system features larger likelihood of default and there is a financial market downturn. Consequently risk factors related to default should become dependent during market downturns. Skreta and Veldkamp (2009) analyze the driving forces behind recent ratings inflation. The authors build a theoretical model where the dynamics of information production for sophisticated securities is driven by disagreement or *asset complexity*–situations where securities exhibit large cross-sectional variance in value estimates, across the various rating agencies.[31] They demonstrate that even if individual agencies are unbiased, complexity results in an aggregate bias in disclosed ratings. Moreover, if this bias is attempted to be corrected by

[27] For more general tail dependence, the results of Campbell (1996) might not obtain. An alternative method that permits the loglinear approach involves using a second-order Taylor approximation to the euler equation. See Campbell (1996), page 304.

[28] Hodrick and Zhang (2001) find that the Campbell model does not have stable parameters, and is less successful in robustness tests. For a comprehensive study of asset pricing models, see Campbell (2000).

[29] For empirical research on systemic risk, or risk of default, see Vassalou and Xing (2004). The authors find that a factor that summarizes default risk is important for asset prices. Moreover, they show that a default risk factor exhibits dependence with the other risk factors of size and book to market, for portfolios in the extreme quantiles. Furthermore, Campbell, Hilscher, and Szilagyi (2008) show that stocks with large default risk earn anomalously small returns. For other research, see Duffee (1999).

[30] Other theoretical papers include Krishnamurthy (2009); and Danielsson, Shin, and Zigrand (2009).

[31] More generally, the lack of agreement on (beliefs about) asset values has been shown to explain asset bubbles for new technology, see Abreu and Brunnermeier (2003), and Hong, Scheinkman, and Xiong (2008).

using investor-initiated ratings, there is a free-rider problem, which can result in a failure in the market for information. Thus, complexity drives a wedge between optimal investor acquisition of information, and market-wide aggregate production of information. While Skreta and Veldkamp (2009) examine asset complexity in the means, it is also evident that complexity may matter in a similar way for higher moments. The reason is that, as discussed above, standard equilibrium asset pricing is based on correlations of securities, either with consumption or with the market return. Furthermore, correlation complexity is key to market failures in diversifying large risks, as examined by Ibragimov and Walden (2007); Ibragimov, Jaffee, and Walden (2009b) and others. We therefore summarize the results of Shin (2009) and Skreta and Veldkamp (2009) by observing that if financial markets have periods of over-lending, then risk factors related to default risk will exhibit asymmetric dependence. Furthermore, if investors or ratings agencies disagree about securities' values, risk factors will exhibit correlation complexity.

In another line of research, Ibragimov, Jaffee, and Walden (2009b) develop a model of catastrophic risks. They characterize the existence of *non-diversification traps*: situations where insurance providers may not insure catastrophic risks nor participate in reinsurance even though there is a large enough market for complete risk sharing. Conditions for this market failure to occur comprise limited liability or heavy left-tailedness of risk distributions. Economically speaking, if assets have infinite second moments, this represents potentially unbounded downside risk and upside gain. In the face of this, insurers prefer to ration insurance rather than decide coverage unilaterally.[32] The authors go on to say that, if the number of insurance providers is large but finite, then nondiversification traps can arise only with distributions that have moderately heavy left tails. In a related paper, Ibragimov and Walden (2007) examine distributional considerations that limit the optimality of diversification. They show that non-diversification may be optimal when the number of assets is small relative to their distributional support. They suggest that such considerations can explain market failures in markets for assets with possibly large negative outcomes. They also identify theoretical non-diversification regions, where risk-sharing will be difficult to create, and risk premia may appear anomalously large. The authors show that this result holds for many dependent risks as well, in particular convolutions of dependent risk with joint truncated α-symmetric distributions. Since these convolutions exhibit heavy-tailedness and dependence, copula models are potentially useful in empirical applications of this result, by *extracting the dependence structure of portfolio risks.* In economic terms, diversification is

[32]This parallels the credit rationing literature of Jaffee and Russell (1976) and Stiglitz and Weiss (1981).

disadvantageous under some heavy-tailed distributions because they exhibit large downside dependence. Thus, the likelihood and impact of several catastrophes exceed that of a single catastrophe. In a recent working paper, Ibragimov, Jaffee, and Walden (2009a) discuss the importance of characterizing the potential for externalities transmitted from individual bank risks to the distribution of systemic risk. Their model highlights the phenomenon of *diversification disasters*: for some distributions, there is a wedge between the optimal level of diversification for individual agents and for society. This wedge depends crucially on the degree of heavy-tailedness: for very small or very large heavy-tailedness, individual rationality and social optimality agree, and the wedge is small. The wedge is potentially largest for moderately heavy tailed risks.[33] This result continues to hold for risky returns with uncertain dependence or correlation complexity. Economically speaking, when risk distributions are moderately heavy tailed, this represents potentially unbounded downside risk and upside gain. In such a situation, some investors might wish to invest in several asset classes, even though this contributes to an increased fragility of the entire financial system. Thus, individual and social incentives are not aligned. A similar situation exists when the structure of asset correlations is complex and uncertain, a situation which may be termed correlation complexity.[34] The authors provide a calibration illustrating a diversification disaster where society prefers concentration, while individuals prefer diversification. As in Ibragimov, Jaffee, and Walden (2009b), they explain that their results hold for general distributions, all of which exhibit tail dependence.[35]

The research above emphasizes on theoretical and practical grounds the importance of isolating dependence in the joint distribution of risk factors in order to say something concrete about diversification and systemic risk. An additional, very current reason for measuring dependence in a robust way is that most economic measures of systemic risk (Adrian and Brunnermeier (2008); Acharya, Pedersen, Philippon, and Richardson (2010)) involve considerations of tail dependence.[36]

[33]The authors define a distribution $F(x)$ to be moderately heavy-tailed if it satisfies the following relation, for $1 < \alpha < \infty : \lim_{x \to +\infty} F(-x) = \frac{c+o(1)}{x^\alpha} l(x)$. Here c and α are positive constants and $l(x)$ is a slowly varying function at infinity. The parameter α is the tail index, and characterizes the heavy-tailedness of F. α is a parameter in many copula functions. For more details, see de Haan and Ferreira (2006) and Embrechts, Kluppelberg, and Mikosch (1997).

[34] Individuals have an incentive to diversify because they do not bear all the costs in the event of systemic crises. That is, the aggregate risk is an externality, as examined by Caballero and Krishnamurthy (2008), and Shin (2009).

[35]These distributions include the student's t, logistic, and symmetric stable distributions.

[36]See also Hartmann, Straetmans, and de Vries (2003); Cherubini, Luciano, and Vecchiato (2004); and Acharya, Cooley, Richardson, and Walter (2010).

2.2 Consequences of measuring economic dependence by correlation

Most of the above results are originally formulated with some type of covariance. However, if we wish to isolate asymmetric dependence, covariances and correlations are not enough.[37] Covariance measures average linear dependence.[38] However, average dependence differs from dependence of the distribution, in general. Thus, covariance cannot detect dependence in even simple nonlinear relations. Similar reasoning applies to any statistical measure that builds on correlation, such as linear regression.[39]

Such fragility of correlation is of practical importance in financial research and policy. The correlation approach can mask theoretically important nonlinearities, as demonstrated by Granger (2001), Hamilton (2001), and Mogstad and Wiswall (2009). From a policy perspective, it is crucial to understand the dependence patterns of key financial risk factors during upturns versus downturns.[40]

3 Data and Results

Our data comprise personal consumption expenditure (CON), and 4 standard risk factors. These risk factors include the market (MKT) factor (return on the market portfolio in excess of the riskfree rate), a size factor (SMB, the small-stock returns minus the big-stock returns), a book-to-market factor (HML, the high-book-to-market-stock returns minus the low-book-to-market stock returns), and a momentum factor (MOM). In addition, we use the Dow-Jones Industrial Average (DJIA), which is a common proxy for aggregate market behavior. The sample period is January 1959 to June 2008.[41] In order to perform our analysis it requires all variables to be stationary. Specifically, real consumption is not stationary, so we take the first log differences. Moreover, all stationary series show evidence of het-

[37] In the Appendix, we will explain why correlation is misleading as a signal of diversification opportunities and systemic risk. We also explain how copulas can help in estimating extreme dependence, since they are rank based and invariant to common economic transformations. Such research has already been used successfully in international economics and banking. See Okimoto (2008); Ane and Kharoubi (2001); Rosenberg and Schuermann (2006); and Patton (2006).

[38] See Casella and Berger (1990), Chapter 4; Embrechts, McNeil, and Straumann (2002).

[39] Further drawbacks of correlation include non-invariance and volatility bias, as outlined in the Appendix.

[40] For related literature on financial market asymmetries, see De Long and Summers (1986); Veldkamp and Van Nieuwerburgh (2006); and Adrian and Brunnermeier (2008).

[41] The risk factors are downloaded from Kenneth French's website. The Dow-Jones Industrial Average is from WRDS, and aggregate personal consumption expenditure is from the St. Louis Federal Reserve.

eroscedasticity. We therefore remove heteroscedasticity with a GARCH(1,1) filter for all variables except for the consumption variable, which requires an AR(1)-GARCH(1,1) to remove the heteroscedasticity. We now discuss the dependence of these risk factors, using first correlations and then copulas.

3.1 Estimating dependence by correlation

Table 2 shows dependence estimates obtained via correlations. First let us discuss linear correlations. The maximum dependence is between the market factor and Dow Jones returns, at 0.635. The maximal negative dependence is between the market and HML, at -0.3991. The closest to zero is between momentum and SMB, at 0.0151. Rank correlations such as Spearman's ρ and Kendall's τ are generally smaller than the linear correlation. The largest dependence the maximal negative dependence, and the closest to independence pairs are the same as those of the linear correlations. Thus, linear and rank correlations agree with each other.

3.2 Estimating dependence: copulas

We now discuss more general, static dependence between consumption, the market, and other risk factors. Table 3 presents the dependence structure of aggregate consumption and financial risk factors, motivated by the consumption-based models of Lucas (1978) and Campbell (1996). According to the linear correlation ρ, aggregate consumption has significant dependence with all financial risk factors, except for the boo-to-market factor. Specifically, consumption is dependent with the size factor, the market factor, momentum, and the Dow Jones. There is also evidence of downside risk (measured by τ_L between consumption-SMB and consumption-Dow Jones, according to the SJC copula. However, the left tail dependence is not detected by the Clayton copula. There is also strong evidence of upside comovement between aggregate consumption and SMB. In economic terms, aggregate consumption tends to exhibit joint downside risk with the Dow Jones, and there is evidence that consumption comoves with the small firm return premia during both very good and very bad times. Other risk factors have no significant relation to consumption during extreme periods.

Table 4 presents estimates of dependence between the market factor and other factors, inspired by the CAPM models of Sharpe (1964), Lintner (1965), and Mossin (1966). According to both the BIC, the best fitting model is always either the student-t copula or another copula that features tail dependence. According to gaussian and student-t copulas, the market exhibits significant linear dependence ρ with all risk factors besides momentum. Perhaps most interestingly, the market has strong significant tail dependence with SMB and HML. For example, according to the SJC copula, the market's tail dependence with SMB and HML is approximately 27% and 17%, respectively. In economic terms, this means that the likelihood of a joint downturn in the market the size factor is 27% over our sample period. Table 5 shows dependence between the Dow Jones return and financial risk factors. The Dow Jones significant linear dependence with all risk factors, using either the gaussian or student-t copulas. Regarding tail dependence, the Dow Jones exhibits significant τ_L with SMB, as well as with the market and Dow Jones as established above. According to the SJC copula, the probability of a joint down-move in both the Dow Jones and SMB is close to 24% over our sample period.

We now discuss the empirical evidence on *dynamic* dependence across financial risk factors. Our model for estimating dynamic dependence is the DCC model of Engle (2002) for the linear correlation coefficient ρ in the Gaussian and student t copula, and that of Patton (2006) for the tail dependence in the SJC and Clayton copula, described in the Appendix. Table 6 shows dynamic dependence between aggregate consumption and other risk factors. According to the gaussian copula, there are no significant dynamics in linear dependence. Our most striking finding in this table concerns the significant dynamic dependence in the tail of consumption and the size factor. According to the SJC copula, the corresponding β coefficients on upper and lower tail dependence are -0.83 and 0.78, respectively. Since β corresponds to the autoregressive term, this implies strong memory in tail dependence between aggregate consumption and SMB. Another finding is that for the consumption-market pair, the coefficient β_L governing dynamic left tail dependence in the Clayton copula is statistically significant at 92%. This is interesting because, despite the lack of dynamic linear dependence between consumption and the market in the gaussian model, there is strong evidence of dynamic left tail dependence.

Table 7 shows dynamic dependence between the market and other risk factors. In this case, there are significant dynamics in both linear and tail dependence. Regarding tail dependence, the comprehensive SJC copula shows significant autoregressive estimates β_U and β_L close to 90% for both SMB and HML. Thus, periods of joint upturns or downturns have strong likelihood of remaining for the next month, in our sample. Table 8 presents estimates of dynamic dependence between the Dow Jones and financial risk factors. Once again, there are significant dynamics in linear and tail dependence. Focusing on the comprehensive SJC copula, we find β_U estimates of 0.73 and 7.77 for dependence in the Dow Jones-SMB and Dow Jones-HML pairs, respectively. The corresponding estimates for β_L are 0.87 and 0.65. Therefore joint upturns and downturns tend to persist from one period to the next.

A graphical depiction of the dynamics in dependence is presented in figures 1 to 6. The most striking lesson from these graphs is the variety of dynamic dependence between risk factors. First, in all pairs, there exists positive lower tail dependence (joint downside risk). Second, for all pairs except market factor-size, left tail dependence exceeds right tail dependence, indicating a higher probability of joint downside risk than joint booms. While some tail dependence coefficients converge rapidly to zero or a positive constant, others tend to fluctuate widely. An interesting case is that of figure 2, which shows from the SJC copula that downside risk between consumption and the Dow Jones converges to a positive constant. This is quantitative evidence that extremely low consumption is associated with downturns in the stock market. Another interesting case is in figure 4, which shows large variation in tail dependence between the market and size factors. Consequently, investors face a great deal of uncertainty about downside risk from both market and size effects during our sample. Therefore our results indicate that an assumption of constant downside risk across all factors is not reasonable.

To summarize, there is evidence of significant downside risk and upside dependence between many risk factors. Interestingly from the perspective of research on systemic risk, the pairs with downside risk include consumption with the Dow Jones, as well as with market and size factors. Of these pairs, only the size factor exhibits a corresponding comovement with consumption during good periods. Moreover, there are significant dynamics in both linear and downside dependence for several risk factors. Consumption has dynamic downside dependence with the size factor. Both the market factor and the Dow Jones have dynamics in dependence relative to SMB and HML, although these dynamics occur in good and bad times. The existence of time-varying downside risk corroborates theoretical and policy research such as Shin (2009) and Acharya, Cooley, Richardson, and Walter (2010).

4 Conclusions

Dependence summarizes risk in modern finance, yet there are few robust studies of risk factor dependence. When risk factors exhibit tail dependence and correlation complexity, diversification fails and financial markets may be prone to systemic risk. Building on a large body of theoretical research, we analyze the dependence structure of risk factors in the US economy, using both correlations and a parsimonious set of copulas. We find evidence of joint tail dependence in several US risk factors. Interestingly from the perspective of research on systemic risk, the pairs with downside risk include consumption with the Dow Jones, as well as consumption with market and size factors. Of these pairs, only the size factor exhibits a corresponding upside comovement with consumption during good periods.

Moreover, the copula approach allows us to investigate dynamics in tail dependence or downside risk. There are significant dynamics in both linear and downside dependence for several risk factors. Consumption has dynamic downside dependence with the size factor. Both the market factor and the Dow Jones have dynamics in dependence relative to SMB and HML, although these dynamics occur in good and bad times. Thus, our results provide evidence of time varying upside and downside risk. More broadly, the existence of downside risk across factors indicates that financial markets are susceptible to joint extreme events.

Our research is among the first to use distributional techniques to provide quantitative evidence on the exposure of financial markets to diversification failure and systemic risk. Since many of these results would be hidden from traditional correlation-based approaches, a practical implication of this paper is that the copula approach may be a good candidate for risk assessment and financial modelling.

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A Overview of diversification and copulas

Diversification is assessed with various dependence measures. If two assets have relatively lower dependence, they offer better diversification than otherwise. In light of the above discussion, we estimate dependence in two ways, using correlations and copulas.[42] The extent of discrepancy between the two can suggest correlation complexity. It can also be informative if we wish to obtain a sense of possible mistakes from using correlations alone. We now define the dependence measures. Throughout, we consider X and Y to be two random variables, with a joint distribution $F_{X,Y}(x,y)$, and marginals $F_X(x)$ and $F_Y(y)$, respectively.

A.1 Correlations

Correlations are the most familiar measures of dependence in finance. If properly specified, correlations tell us about average diversification opportunities over the entire distribution. The Pearson **correlation** coefficient ρ is the covariance divided by the product of the standard deviations:

$$\rho = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}} \tag{6}$$

The main advantage of correlation is its tractability. There are, however, a number of theoretical shortcomings, especially in finance settings.[43] First, a major shortcoming is that correlation is not invariant to monotonic transformations. Thus, the correlation of two return series may differ from the correlation of the squared returns or log returns. Second, there is substantial evidence of infinite variance in financial data.[44] From equation (6), if either X or Y has infinite variance, the estimated correlation may give little information on dependence, since it will be undefined or close to zero. A third drawback concerns estimation bias: by definition the conditional correlation is biased and spuriously increases during volatile periods.[45] Fourth, correlation is a linear measure and therefore may overlook important nonlinear dependence. It does not distinguish, for example, between dependence during up and down markets.[46] Whether these shortcomings matter in practice is an empirical question that we approach in this paper.

[42] Readers already familiar with dependence and copula concepts may proceed to Section 4.

[43] Disadvantages of correlation are discussed by Embrechts, McNeil, and Straumann (2002).

[44] See Mandelbrot (1963); Fama (1965); Gabaix, Gopikrishnan, Plerou, and Stanley (2003); and Rachev (2003).

[45] See Forbes and Rigobon (2002). After adjusting for such bias, Forbes and Rigobon (2002) document that prior findings of international dependence (contagion) are reversed.

[46] Such nonlinearity may be substantial, as illustrated by Ang and Chen (2002) in the domestic context. These researchers document significant asymmetry in downside and upside correlations of US stock returns.

A related, nonlinear measure is the **rank** (or Spearman) **correlation**, ρ_S. This is more robust than the traditional correlation. ρ_S measures dependence of the ranks, and can be expressed as $\rho_S = \frac{\text{Cov}(F_X(x),F_Y(y))}{\sqrt{\text{Var}(F_X(x))\text{Var}(F_Y(y))}}$.[47] The rank correlation is especially useful when analyzing data with a number of extreme observations, since it is independent of the levels of the variables, and therefore robust to outliers. Another nonlinear correlation measure is one we term **downside risk**,[48] $d(u)$. This function measures the conditional probability of an extreme event beyond some threshold u. For simplicity, normalize variables to the unit interval $[0, 1]$. Hence

$$d(u) \equiv \Pr(F_X(x) \leq u \mid F_Y(y) \leq u). \tag{7}$$

A final nonlinear correlation measure is left **tail dependence**, $\lambda(u)$, which is the limit of downside risk as losses become extreme,

$$\lambda(u) \equiv \lim_{u \downarrow 0} \Pr(F_X(x) \leq u \mid F_Y(y) \leq u). \tag{8}$$

A.2 Copulas

If we knew the entire joint distribution of international returns, we could summarize all relevant dependence and therefore all diversification opportunities. In a portfolio of two assets with returns X and Y, all dependence is contained in the joint density $f_{X,Y}(x, y)$. This information is often not available, especially for large portfolios, because there might be no simple parametric joint density that characterizes the relationship across all variables. Moreover, there is a great deal of estimation and mis-specification error in attempting to find the density parametrically.

An alternative to measuring diversification in this setting is the **copula function** $C(u, v)$. From expression (1) above, a copula is a joint distribution with uniform marginals U and V, $C(u, v) = \Pr[U \leq u, V \leq v]$. As shown in (2), any joint distribution $F_{X,Y}(x, y)$ with continuous marginals is characterized by a copula distribution C such that $F_{X,Y}(x, y) = C(F_X(x), F_Y(y))$. It is often convenient to differentiate equation (2) and use a corresponding ”canonical” density version

$$f(x, y) = c(F_X(x), F_Y(y)) \cdot f_X(x) \cdot f_Y(y), \tag{9}$$

[47] See Cherubini, Luciano, and Vecchiato (2004), page 100.

[48] The concept of downside risk appears in a number of settings without being explicitly named. It is the basis for many measures of systemic risk, see Cherubini, Luciano, and Vecchiato (2004) page 43; Hartmann, Straetmans, and de Vries (2003); and Adrian and Brunnermeier (2008).

where $f(x, y)$ and $c(F_X, F_Y)$ are the joint and copula densities, respectively.[49] Equation (9) is interesting because it empowers us to separate out the joint distribution from the marginals. For example, if we are interested in why heavy tailedness increases risk in a US-UK portfolio, this could come from either the fact that the marginals are heavy-tailed, or their dependence is heavy-tailed, or both. This distinction is relevant whenever we are interested in the downside risk of the entire portfolio, more than the heavy tailedness of each security in the portfolio. We estimate (9) in Section 5, for different copula specifications.

There are a number of parametric copula specifications. We focus on three types, the normal, the student-t, and the Clayton copulas, for several reasons.[50] The normal specification is a natural benchmark, as the most common distributional assumption in finance, with zero tail dependence.[51] The student-t is useful since it has symmetric but nonzero tail dependence and nests the normal copula. The Clayton copula is useful because it has nonlinear dependence and asymmetric tail dependence–the mass in its right tail greatly exceeds the mass in its left tail. Moreover, the Clayton is a member of an important family, Archimedean copulas.[52] Practically, these copulas represent the most important shapes for finance, and are a subset of those frequently used in recent empirical papers.[53] Table 1 provides functional forms of the copulas. They are estimated by maximum likelihood.

There are several main advantages of using copulas in finance. First, they are a convenient choice for modeling potentially nonlinear portfolio dependence, such as correlated defaults. This aspect of copulas is especially attractive since they nest some important forms of dependence, as described in Section 3.3. A second advantage is that copulas can aggregate portfolio risk from disparate sources, such as credit and operational risk. This is possible even for risk distributions that are subjective and objective, as in Rosenberg and Schuermann (2006). In a related sense, copulas permit one to model *joint* dependence in a portfolio without specifying the distribution of individual assets in the portfolio.[54] A third advantage is invariance. Since the copula is based on ranks, it is invariant under

[49]Specifically, $f(x, y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$, and similarly $c(F_X(x), F_Y(y)) = \frac{\partial^2 C(F_X(x), F_Y(y))}{\partial x \partial y}$. The terms $f_X(x)$ and $f_Y(y)$ are the marginal densities.

[50]Since we wish to investigate left dependence or downside risk, we also utilize the survivor function of the Clayton copula, denoted Survival Clayton.

[51]Tail dependence refers to dependence at the extreme quantiles as in expression (8). See de Haan and Ferreira (2006).

[52]Archimedean copulas represent a convenient bridge to gaussian copulas since the former have dependence parameters that can be defined through a correlation measure, Kendall's tau. Extreme value copulas are important since they can be used to model joint behavior of the distribution's extremes.

[53]See for example, Embrechts, McNeil, and Straumann (2002), Patton (2004) and Rosenberg and Schuermann (2006).

[54]This is usually expressed by saying that copulas do not constrain the choice of individual or marginal asset distributions. For example, if we model asset returns of the US and UK as bivariate normal, this automatically restricts both the individual (marginal) US and UK returns to be univariate normal. Our semi-

strictly increasing transforms. That is, the copula extracts the way in which x and y comove, regardless of the scale used to measure them.[55] Fourth, since copulas are rank-based and can incorporate asymmetry, they are also natural dependence measures from a theoretical perspective. The reason is that a growing body of research recognizes that investors care a great deal about the ranks and downside performance of their investment returns.[56] More generally, since copulas are joint distributions they are naturally well-suited to discussions of a vast array of research issues in economics. These issues include optimal commodity bundling, income inequality, expected utility and parsimonious modelling of dependent multivariate time series.[57] In addition, copulas are directly relevant to the practice of business, in the context of portfolio risk assessment. In an increasingly globalized economy, security returns seem to exhibit unexpectedly greater dependence during certain periods, as evidenced by recent international contagion episodes and US subprime mortgage spillovers. In light of these unexpected events, copula-based stress testing methods can help explain, forecast, and hedge extreme dependence in financial markets. Development of such copula-based methods is relevant for many market actors, including institutional investors, hedge funds, regulatory authorities and central banks.[58]

There are two drawbacks to using copulas. First, from a finance perspective, a potential disadvantage is that many copulas do not have moments that are directly related to Pearson correlation. It may therefore be difficult to compare copula results to those of financial models based on correlations or variances. This is not an issue for our study, since our model selection chooses a *t* copula, which contains a correlation parameter. Second, from a statistical perspective, it is not easy to say which parametric copula best fits the data, since some copulas may fit better near the center and others near the tails. This issue is not strongly relevant to our paper, since the theoretical background research from Section 2 focuses on asymmetry and tail dependence. Thus the emphasis is on the shape of copulas, rather than on a specific copula. Further, we use several specification checks, namely AIC, BIC, and a mixture model.

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parametric approach avoids restricting the marginals by using empirical marginal distributions, based on ranks of the data. Specifically, first the data for each marginal are ranked to form empirical distributions. These distributions are then used in estimating the parametric copula.

[55]See Schweizer and Wolff (1981). For more details on copula properties, see Nelsen (1998), Chapter 2.

[56] See Polkovnichenko (2005) and Barberis, Huang, and Santos (2001).

[57] For research on some of these disparate topics, see the work of Ibragimov (2009) and Patton (2006).

[58]For example, it is well known that many pricing relations such as the CAPM and option-pricing formulae do not function well outside of the elliptical world, see Chamberlain (1983). Copulas inherently capture such complex dependence structures. Since the dependence structure of financial markets is dynamic, we also use a conditional copula model of Patton (2006), see the Appendix. For research on the inherent dynamism of capitalistic markets, see Phelps (2007).

A.3 Relationship of diversification measures

We briefly outline the relationship of the diversification measures.[59] If the true joint distribution is bivariate normal, then the copula and traditional correlation give the same information. Once we move far away from normality, there is no clear relation between correlation and the other measures. However, all the other, more robust measures of dependence are pure copula properties, and do not depend on the marginals. We describe relationships for rank correlation ρ_S, downside risk $d(u)$, and tail dependence $\lambda(u)$ in turn. The relation between copulas and rank correlation is given by

$$\rho_S = 12 \int_0^1 \int_0^1 C(u,v) dC(u,v) - 3. \tag{10}$$

This means that if we know the correct copula, we can recover rank correlation, and vice versa. Therefore, rank correlation is a pure copula property. Regarding downside risk, it can be shown that $d(u)$ satisfies

$$\begin{aligned} d(u) &\equiv \Pr(F_X(x) \le u \mid F_Y(y) \le u) \\ &= \frac{\Pr(F_X(x) \le u, F_Y(y) \le u)}{\Pr(F_Y(y) \le u)} \\ &= \frac{C(u,u)}{u}, \end{aligned} \tag{11}$$

where the third line uses definition (1) and the fact since $F_Y(y)$ is uniform, $\Pr[F_Y(y) \le u] = u$. Thus downside risk is also a pure copula property and does not depend on the marginals at all. Since tail dependence is the limit of downside risk, it follows from (8) and (11) that $\lambda(u) = \lim_{u \downarrow 0} \frac{C(u,u)}{u}$. To summarize, the nonlinear measures that we consider are directly related to the copula, and ρ and the normal copula give the same information when the data are jointly normal. While the above discussion describes how to link the various concepts in theory, there is little empirical work comparing the different diversification measures. This provides a rationale for our empirical study.

For our economic applications below, we will also use an important notion related to the copula, namely **tail dependence**. Intuitively, left tail dependence λ_L refers to the relative amount of mass in the lower quantile. Formally we define left tail dependence of a copula $C(u,v)$ as

$$\lambda_L \equiv \lim_{u \downarrow 0} \frac{C(u,u)}{u}.[60] \tag{12}$$

[59] For background and proofs on the relations between dependence measures, see Cherubini, Luciano, and Vecchiato (2004) Chapter 3; Embrechts, McNeil, and Frey (2005); and Jondeau, Poon, and Rockinger (2007).

[60] Right tail dependence is defined similarly, as

$$\lambda_R = \lim_{u \uparrow 1} \frac{\bar{C}(u,u)}{1-u},$$

B The Methodology

B.1 Estimation method for copulas

One advantage of copula approach is that it can separate the dependence structure from the marginals, with dependence completely captured in the copula function.[61] Since our focus is on the dependence between financial variables, rather than their marginals, we specify a parametric copula function but make no assumptions on the marginal distributions of the macro variables. Therefore, the approach is free of specification errors for the marginals.[62] The estimation procedure comprises two steps. In the first step, the marginal distribution function $G(.)$ is estimated non-parametrically via its rescaled empirical cumulative distribution function (ECDF)

$$\widehat{F}(x_t) = \frac{1}{T+1}\sum_{t=1}^{T} 1\{X_t < x\}. \tag{13}$$

The ECDF is rescaled to ensure that the first order condition of the copula's log-likelihood function is well defined for all finite T.[63] By the Glivenko–Cantelli theorem, $\widehat{F}_X(x_t)$ converges to its theoretical counterpart $F(y_t)$ uniformly.

In the second step, given the non-parametrically estimated ECDF, $\widehat{F(x_t)}$ and $\widehat{G}(y_t)$, we estimate the copula parameters θ_c parametrically by maximum likelihood, with

$$\begin{aligned} \widehat{\theta}_c &= \arg\max_{\theta_c} \widetilde{L}, \\ \text{where } \widetilde{L}(\theta_c) &= \frac{1}{T}\sum \log c(\widehat{F}(x_t), \widehat{G}(y_t); \theta_c), \end{aligned}$$

where c(.) is the copula density function. Joe (1997) proves that under a set of regularity conditions, the two-step estimator is consistent and asymptotically normal. Joe (1997) also demonstrates that the two-step method is highly efficient. In addition, as indicated in Patton (2006), this method has the benefit of being computationally tractable. Chen and Fan (2006) establish asymptotic properties for this semi-parametric estimator.

where $\bar{C}(u,u) = P(U > u, U > u)$ is the survival function of $C(u,u)$.

[61] See Sklar (1959); and Embrechts, McNeil, and Frey (2005); and Patton (2006).

[62] Our approach is therefore semi-parametric. For further details, see Joe (1997), and Cherubini, Luciano, and Vecchiato (2004). Statistical properties of this approach are highlighted in the simulation studies of Fermanian and Scaillet (2003).

[63] See Genest, Ghoudi, and Rivest (1995), and Chen and Fan (2006) for further discussion on this methodology.

Copula estimation requires that the series be i.i.d. Since many of our macro series are not i.i.d., thus we filter the variables with various ARMA-GARCH models.[64] We then compute the ECDFs of the filtered variables, which are used in the second-stage maximum likelihood estimation.

B.2 Static dependence model

The SJC copula model

To examine thedegree of dependence, we adopt the Symmetrised Joe Clayton (SJC) copula used in Patton (2006). The SJC copula is a modification of the so called "BB7" copula of Joe (1997). It is defined as

$$\begin{aligned} & C_{SJC}(u, v|\lambda r, \lambda_l) && (14) \\ = \quad & 0.5 \times (C_{JC}(u, v|\lambda r, \lambda_l) + C_{JC}(1-u, 1-v|\lambda_l, \lambda r) + u + v - 1), && (15) \end{aligned}$$

where $C_{JC}(u, v|\lambda r, \lambda_l)$ is the BB7 copula (also called Joe-Clayton copula), which is in turn defined as

$$\begin{aligned} & C_{JC}(u, v|\lambda r, \lambda_l) && (16) \\ = \quad & 1 - (1 - \left\{ \left[1-(1-u)^k\right]^{-r} + \left[1-(1-v)^k\right]^{-r} - 1 \right\}^{-1/r})^{1/k}, && (17) \\ \text{with } k = \quad & 1/log_2(2-\lambda r) \text{ and } r = -1/log_2(\lambda_l) && (18) \end{aligned}$$

where λ_l and $\lambda r \in (0, 1)$. By construction, the SJC copula is symmetric when λ_l=λr. This copula is very flexible since it allows for both asymmetric upper and lower tail dependence and symmetric dependence as a special case.

B.3 Dynamic dependence model

In order to examine the possibility of dynamic or time varying tail dependence in the data, we follow the approach of Patton (2006). We estimate the following ARMA-type process for the tail dependence parameters $\tau_{L,t}$ and $\tau_{U,t}$:

$$\begin{aligned} \tau_{L,t} &= (1 + \exp(-h_{L,t}))^{-1}, \quad \tau_{U,t} = (1 + \exp(-h_{U,t}))^{-1}, && (19) \\ h_{L,t} &= w_L + \beta_L h_{L,t-1} + \alpha_L \sum_{j=1}^{p} |u_{t-j} - v_{t-j}|, && (20) \end{aligned}$$

[64] Details of the filtering procedure for the macro variables are available from the authors, upon request.

$$h_{U,t} \quad = \quad w_U + \beta_U h_{U,t-1} + \alpha_U \sum_{j=1}^{p} |u_{t-j} - v_{t-j}|. \tag{21}$$

The dynamic models contain an autoregressive term designed to capture persistence in dependence, and a variable which is a mean absolute difference between u and v. The latter variable is positive when the two probability integral transforms are on the opposite side of the extremes of the joint distribution and close to zero when they are on the same side of the extremes. The logistic transformation of the ARMA process guarantees that the weight and tail dependence parameters lie in the [0,1] interval.

Table 1: Distribution of various copulas

Copula	Distribution	Parameter Range	Complete Dependence	Independence
Normal	$C_N(u,v;\rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v))$	$\rho \in (-1,1)$	$\rho = 1$, or-1	$\rho = 0$
Student-t	$C_t(u,v;\rho,d) = t_{d,\rho}(t_d^{-1}(u), t_d^{-1}(v))$	$\rho \in (-1,1)$	$\rho = 1$,or-1	$\rho = 0$
Clayton	$C_C(u,v) = (u^{-\alpha_c} + v^{-\alpha_c} - 1)^{-1/\alpha_c}$	$\alpha_c > 0$	α_c	$\alpha_c =$

$\Phi_\rho(x,y)$ and $t_{\nu,\rho}(x,y)$ denote the standard bivariate normal and Student-*t* cumulative distributions, respectively: $\Phi_\rho(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} \frac{1}{2\pi|\Sigma|} \exp\{-\frac{1}{2}(x\ \ y)\Sigma^{-1}(x\ \ y)'\}dxdy$, and $t_{\nu,\rho}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} \frac{\Gamma(\frac{\nu+2}{2})}{\Gamma(\nu/2)(\nu\pi)|\Sigma|^{1/2}}\{1+(s\ \ t)\Sigma^{-1}(s\ \ t)'/\nu\}^{\frac{-(\nu+2)}{2}} dsdt$. The correlation matrix is given by $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$.

Table 2: Dependence Structure of Financial Risk Factors: Correlations

The table presents linear (Pearson) and rank correlations of financial variables. The frequency is monthly, and the time period is from January 1959 to June 2008.

Variables	MKT	DJIA	SMB	HML	MOM	CONS
MEAN	0.0045	0.0051	0.0021	0.0043	0.0088	0.0008
STD	0.0431	0.0348	0.0309	0.028	0.0389	0.002
Correlations	MKT	DJIA	SMB	HML	MOM	CONS
MKT	1	**0.635**	**0.2991**	**-0.3991**	**-0.0737**	**0.1004**
DJIA		1	**0.3274**	**-0.1144**	**-0.1627**	**0.1852**
SMB			1	**-0.2692**	0.0151	**0.1902**
HML				1	**-0.1307**	-0.0253
MOM					1	**-0.0995**
CON						1
Spearman ρ	MKT	DJIA	SMB	HML	MOM	CONS
MKT	1	**0.5799**	**0.2773**	**-0.3664**	-0.0483	**0.0987**
DJIA		1	**0.329**	**-0.1362**	**-0.1314**	**0.129**
SMB			1	**-0.1924**	0.0051	**0.1617**
HML				1	**-0.1161**	0.0149
MOM					1	**-0.0713**
CON						1
Kendall's τ	MKT	DJIA	SMB	HML	MOM	CONS
MKT	1	**0.4174**	**0.1957**	**-0.2538**	-0.0317	**0.0663**
DJIA		1	**0.2269**	**-0.0925**	**-0.0908**	**0.0864**
SMB			1	**-0.1335**	0.0042	**0.109**
HML				1	**-0.0829**	0.0091
MOM					1	-0.0462
CON						1

Table 3: Dependence Structure of Consumption and Financial Risk Factors

The table presents the dependence between consumption and various financial risk factors. The frequency is monthly, and the time period is January 1959 to June 2008. The parameters in each column represent the estimated dependence between the variable at the head of each column and consumption. For example, the parameters in the column entitled SMB show, for various copulas, dependence estimates for consumption and the size factor SMB. The term SJC refers to the symmetrized Joe-Clayton copula of Patton (2006). ρ is the correlation coefficient. τ_L and τ_U represent lower and upper tail dependence, respectively. T-statistics are in parentheses underneath parameter estimates.

	SMB	-HML	DJIA	MKT	MOM
Gaussian copula					
ρ	**0.2016**	$<$0.0001	**0.1882**	**0.1174**	**0.0936**
	(5.135)	($<$0.0001)	(4.7543)	(2.8673)	(2.2681)
AIC	-21.7471	2.0000	-18.6357	-5.9337	-3.0292
BIC	-17.3619	6.3852	-14.2505	-1.5485	1.3560
t copula					
ρ	**0.2003**	-0.0112	**0.1815**	**0.1170**	**0.0917**
	(5.0484)	(-0.2634)	(4.4217)	(2.7659)	(2.1536)
ν	99.0000	51.0021	**27.0000**	**27.0000**	**27.0000**
	(0.6406)	(1977.2773)	(15410.6549)	(9534.5321)	(15223.9041)
AIC	-19.5076	3.9094	-17.8603	-5.2375	-1.8606
BIC	-10.7372	12.6798	-9.0899	3.5329	6.9098
SJC copula					
τ_L	**0.1019**	$<$0.0001	**0.1153**	0.1010	0.1653
	(570.9851)	($<$0.0001)	(2.6199)	(0.1836)	(0.1734)
τ_U	**0.1022**	0.0001	0.0022	$<$0.0001	$<$0.0001
	(1573.1403)	($<$0.0001)	(0.2279)	($<$0.0001)	($<$0.0001)
AIC	-14.9234	4.2242	-24.5306	-6.6858	-0.1062
BIC	-6.1530	12.9946	-15.7602	2.0846	8.6642
Clayton copula					
τ_L	0.0276	0.0010	0.0607	0.0138	0.0114
	(0.9803)	(0.0001)	(1.5707)	(0.7219)	(0.6642)
AIC	-13.1673	9.8151	-24.0552	-9.9944	-8.9177
BIC	-8.7821	14.2003	-19.6700	-5.6092	-4.5325

Table 4: Dependence Structure of Market Return and Financial Risk Factors

The table presents the dependence between the market return and various financial risk factors. The frequency is monthly, and the time period is January 1959 to June 2008. The parameters in each column represent the estimated dependence between the variable at the head of each column and the market. For example, the parameters in the column entitled SMB show, for various copulas, dependence estimates for the market and the size factor SMB. The term SJC refers to the symmetrized Joe-Clayton copula of Patton (2006). ρ is the correlation coefficient. τ_L and τ_U represent lower and upper tail dependence, respectively. T-statistics are in parentheses underneath parameter estimates.

	SMB	-HML	-MOM	CON	SMB and -HML
Gaussian copula					
ρ	**0.2785**	**0.3599**	$<$0.0001	**0.1174**	**0.1870**
	(7.5144)	(10.5619)	($<$0.0001)	(2.8673)	(4.7201)
AIC	-44.2833	-77.7437	2.0000	-5.9337	-18.3633
BIC	-39.8981	-73.3585	6.3852	-1.5485	-13.9781
t copula					
ρ	**0.2931**	**0.3673**	-0.0059	**0.1170**	**0.2009**
	(6.8996)	(10.1576)	(-0.122)	(2.7659)	(4.6191)
ν	**4.5000**	**15.0000**	2.7525	**27.0000**	**4.7102**
	(7926.733)	(3157.4595)	(12580.6745)	(9534.5321)	(4.2859)
AIC	-74.0044	-84.6966	-41.6819	-5.2375	-37.3301
BIC	-65.2340	-75.9262	-32.9115	3.5329	-28.5597
SJC copula					
τ_L	**0.2675**	**0.1738**	$<$0.0001	0.1010	0.0418
	(5.3774)	(3.1382)	($<$0.0001)	(0.1836)	(0.8584)
τ_U	0.0084	**0.1990**	0.1307	$<$0.0001	0.0969
	(0.2429)	(3.7071)	(0.0472)	($<$0.0001)	(1.8694)
AIC	-69.1283	-83.4471	-1.2329	-6.6858	-25.9061
BIC	-60.3580	-74.6767	7.5375	2.0846	-17.1357
Clayton copula					
τ_L	**0.2217**	**0.2125**	0.0010	0.0138	0.0384
	(4.6208)	(4.4848)	(0.0001)	(0.7219)	(1.1229)
AIC	-65.5398	-60.7852	10.8127	-9.9944	-14.9109
BIC	-61.1546	-56.4000	15.1979	-5.6092	-10.5257

Table 5: Dependence Structure of the Dow Jones and Financial Risk Factors

The table presents the dependence between the Dow Jones Industrial Average and various financial risk factors. The frequency is monthly, and the time period is January 1959 to June 2008. The parameters in each column represent the estimated dependence between the variable at the head of each column and the Dow Jones. For example, the parameters in the column entitled SMB show, for various copulas, dependence estimates for the Dow Jones and the size factor SMB. The term SJC refers to the symmetrized Joe-Clayton copula of Patton (2006). ρ is the correlation coefficient. τ_L and τ_U represent lower and upper tail dependence, respectively. T-statistics are in parentheses underneath parameter estimates.

	SMB	-HML	MOM	CON
Gaussian copula				
ρ	**0.3458**	**0.1263**	¡0.0001	**0.1882**
	9.9821	3.0949	<0.0001	4.7542
AIC	-71.1580	-7.1911	2.0000	-18.6357
BIC	-66.7729	-2.8059	6.3852	-14.2505
t copula				
ρ	**0.3348**	**0.1306**	**-0.1074**	**0.1815**
	(8.8163)	(3.0903)	(-2.3987)	(4.4217)
ν	**9.0000**	**27.0000**	**7.5031**	**27.0000**
	(8630.0822)	(14114.1807)	(2.9468)	(15410.6548)
AIC	-72.9861	-8.5301	-11.9683	-17.8603
BIC	-64.2157	0.2403	-3.1979	-9.0899
SJC copula				
τ_L	**0.2356**	0.1344	<0.0001	**0.1153**
	(4.9032)	(0.2130)	(<0.0001)	(2.6208)
τ_U	0.0895	<0.0001	<0.0001	0.0022
	(1.6722)	(<0.0001)	(<0.0001)	(0.2278)
AIC	-78.3496	-1.3303	5.9835	-24.5306
BIC	-69.5792	7.4401	14.7539	-15.7602
Clayton copula				
τ_L	**0.2308**	0.0085	0.0010	0.0607
	(4.9980)	(0.5726)	(0.0001)	(1.5707)
AIC	-70.7514	-7.1240	9.9067	-24.0552
BIC	-66.3662	-2.7388	14.2919	-19.6700

Table 6: Dynamic Dependence of Consumption and Financial Risk Factors

The table presents the dynamic dependence between consumption and various financial risk factors. The frequency is monthly, and the time period is January 1959 to June 2008. The parameters in each column represent the estimated dependence between the variable at the head of each column and consumption. For example, the parameters in the column entitled SMB show, for various copulas, dependence estimates for consumption and the size factor SMB. The term SJC refers to the symmetrized Joe-Clayton copula of Patton (2006). For the gaussian and student-t copulas, a and b represent the coefficients on the autoregressive and variance terms in the DCC(1,1) model of Engle (2002). For the SJC and Clayton copulas, w, α and β represent the intercept, coefficient on the past 10 periods of cdf differences, and the autoregressive term, all from the dynamic copula model of Patton (2006). T-statistics are in parentheses underneath parameter estimates.

	DJIA	MKT	SMB	-HML
Gaussian copula				
α	0.0001	0.0233	0.0375	0.0001
	(0.0001)	(0.4802)	(0.4683)	(0.0001)
β	0.0001	0.0001	0.0001	0.0001
	(0.0001)	(0.0001)	(<0.0001)	(<0.0001)
AIC	-16.6265	-4.3219	-20.5319	3.9628
BIC	-7.8561	4.4485	-11.7615	12.7332
t copula				
α	0.0001	0.0216	0.0372	0.0001
	(0.0001)	(0.3603)	(0.4774)	(0.0001)
β	0.0001	0.0001	0.0001	0.0001
	(0.0001)	(0.0001)	(<0.0001)	(<0.0001)
ν	**22.0983**	**22.0827**	200.0000	83.3797
	(22.0891)	(21.7349)	(0.5096)	(0.2941)
AIC	-15.9063	-3.6405	-18.4056	5.8935
BIC	-2.7507	9.5150	-5.2500	19.0490
SJC copula				
w_U	3.6317	-18.4926	-8.4052	-283.1822
	(0.3947)	(<0.0001)	(-1.6753)	(<0.0001)
α_U	-41.4127	-8.5843	13.4829	-148.0826
	-(1.0115)	(<0.0001)	(1.0192)	(<0.0001)
β_U	**-0.5891**	6.2576	**-0.8314**	186.8784
	(-2.2088)	(<0.0001)	(-6.3542)	(<0.0001)
w_L	-0.7358	-3.1499	0.1233	-60.1164
	(-0.6003)	(-0.4637)	(0.0714)	(<0.0001)
α_L	0.1962	1.0630	-3.7292	-21.6583
	(0.0985)	(0.1592)	(-0.5372)	(<0.0001)
β_L	0.6406	-0.0220	**0.7326**	73.2998
	(1.5097)	(-0.0107)	(2.1864)	(<0.0001)
AIC	-17.0049	1.3379	-13.3086	14.4839
BIC	9.3063	27.6490	13.0025	40.7951
Clayton copula				
w_L	-1.1483	0.3940	-0.1743	-49.4655
	(-0.5496)	(0.6939)	(-0.1188)	(<0.0001)
α_L	0.4614	-2.4162	-2.6286	86.6726
	(0.1574)	(-1.3092)	(-0.524)	(<0.0001)
α_L	0.6087	**0.9197**	**0.7215**	1479.6782
	(1.0627)	(10.6877)	(1.8764)	(<0.0001)
AIC	-20.2532	-6.5682	-9.7721	10.8346
BIC	-7.0977	6.5874	3.3834	23.9902

Table 7: Dynamic Dependence of Market Return and Financial Risk Factors

The table presents the dynamic dependence between the Market and various financial risk factors. The frequency is monthly, and the time period is January 1959 to June 2008. The parameters in each column represent the estimated dependence between the variable at the head of each column and the market. For example, the parameters in the column entitled SMB show, for various copulas, dependence estimates for the market and the size factor SMB. The term SJC refers to the symmetrized Joe-Clayton copula of Patton (2006). For the gaussian and student-t copulas, a and b represent the coefficients on the autoregressive and variance terms in the DCC(1,1) model of Engle (2002). For the SJC and Clayton copulas, w, α and β represent the intercept, coefficient on the past 10 periods of cdf differences, and the autoregressive term, all from the dynamic copula model of Patton (2006). T-statistics are in parentheses underneath parameter estimates.

	SMB	-HML	SMB and -HML
Gaussian copula			
α	0.0115	0.0159	0.0712
	(1.2404)	(1.7115)	(0.0001)
β	**0.9705**	**0.9720**	1.6656
	(35.919)	(54.887)	(0.0001)
AIC	-45.8209	-84.1221	-20.2690
BIC	-37.0505	-75.3517	-11.4986
t copula			
α	0.0129	0.0247	0.0605
	(1.4795)	(1.2053)	(1.2074)
β	**0.9732**	**0.9514**	0.5332
	(53.557)	(19.492)	(1.0633)
ν	**3.6407**	**7.2723**	**4.7666**
	(5.7379)	(2.7935)	(4.7113)
AIC	-76.6909	-92.7498	-38.8826
BIC	-63.5353	-79.5942	-25.7270
SJC copula			
w_U	3.3520	-0.3091	-3.7033
	(0.853)	(-0.2416)	(-1.324)
α_U	-46.6246	-7.6505	9.5705
	(-1.4008)	(-1.507)	(1.3386)
β_U	**-0.9435**	**-0.9893**	**0.8807**
	(-15.8674)	(-117.8088)	(11.3119)
w_L	0.4743	-2.6992	-3.3596
	(0.3077)	(-1.1766)	(-0.6156)
α_L	-7.4310	-0.1873	5.4305
	(-1.2838)	(-0.0242)	(0.4889)
β_L	**-0.8322**	**-0.9652**	0.2285
	(-3.4429)	(-27.0395)	(0.222)
AIC	-68.1397	-81.2792	-22.7918
BIC	-41.8285	-54.9681	3.5193
Clayton copula			
w_L	0.5459	0.3747	-6.3707
	(1.472)	(1.1078)	(-1.0446)
α_L	-3.2783	-2.3767	13.1113
	(-1.6013)	(-1.2226)	(0.9476)
β_L	**0.6988**	**0.8165**	0.2699
	(4.1099)	(6.1705)	(0.4722)
AIC	-68.5692	-62.3835	-12.8767
BIC	-55.4136	-49.2280	0.2789

Table 8: Dynamic Dependence of Dow Jones and Financial Risk Factors

The table presents the dynamic dependence between the Dow Jones Industrial Average and various financial risk factors. The frequency is monthly, and the time period is January 1959 to June 2008. The parameters in each column represent the estimated dependence between the variable at the head of each column and the Dow Jones. For example, the parameters in the column entitled SMB show, for various copulas, dependence estimates for the Dow Jones and the size factor SMB. The term SJC refers to the symmetrized Joe-Clayton copula of Patton (2006). For the gaussian and student-t copulas, a and b represent the coefficients on the autoregressive and variance terms in the DCC(1,1) model of Engle (2002). For the SJC and Clayton copulas, w, α and β represent the intercept, coefficient on the past 10 periods of cdf differences, and the autoregressive term, all from the dynamic copula model of Patton (2006). T-statistics are in parentheses underneath parameter estimates.

	SMB	-HML
Gaussian copula		
α	0.0217	0.0181
	(1.509)	(1.5724)
β	**0.9488**	**0.9587**
	(23.7413)	(32.336)
AIC	-74.4660	-8.7754
BIC	-65.6956	-0.0050
t copula		
α	**0.0293**	0.0184
	(1.9402)	(1.2102)
β	**0.9306**	**0.9509**
	(26.0231)	(22.3211)
ν	**11.3348**	**12.2976**
	(1.9592)	(1.8681)
AIC	-76.9670	-10.7770
BIC	-63.8114	2.3786
SJC copula		
w_U	0.2198	-8.8370
	(0.3455)	(<0.0001)
α_U	-3.3499	51.8283
	(-0.7964)	(<0.0001)
β_U	**0.7318**	**7.7728**
	(2.7833)	(278588.67)
w_L	0.2034	1.2280
	(0.1368)	(0.6646)
α_L	-6.9025	-8.4236
	(-1.2548)	(-0.7298)
β_L	**-0.8722**	**0.6480**
	(-7.6033)	(1.639)
AIC	-74.7615	-1.0962
BIC	-48.4503	25.2149
Clayton copula		
w_L	0.2603	-0.2394
	(0.8403)	(<0.0001)
α_L	-1.9686	**3.7994**
	(-1.1078)	(278448.06)
β_L	**0.7085**	**2.7395**
	(2.8795)	(1145.9393)
AIC	-70.6273	-2.3391
BIC	-57.4717	10.8164

Figure 1: Dynamics of Dependence between Consumption and the Market

The figure plots the dynamic behavior of dependence parameters for consumption and the market return.

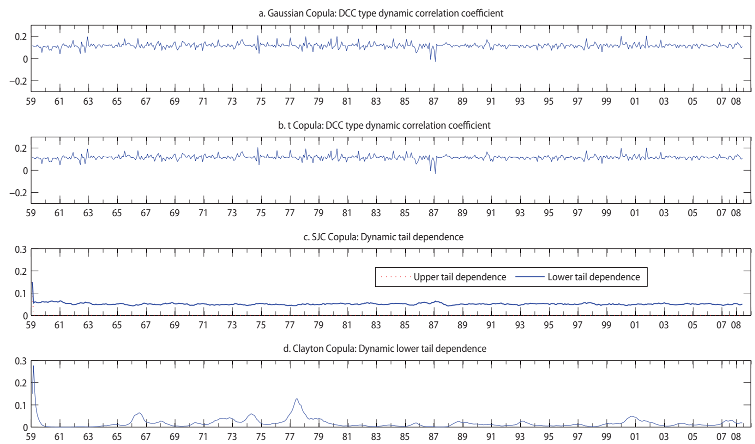

Figure 2: Dynamics of Dependence between Consumption and the Dow Jones

The figure plots the dynamic behavior of dependence parameters for consumption and the Dow Jones Industrial Average.

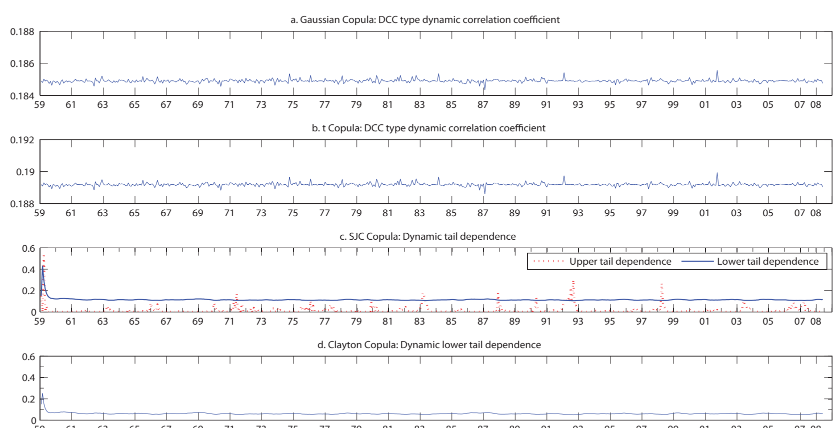

Figure 3: Dynamics of Dependence between Consumption and the Size Factor

The figure plots the dynamic behavior of dependence parameters for consumption and the Size Factor.

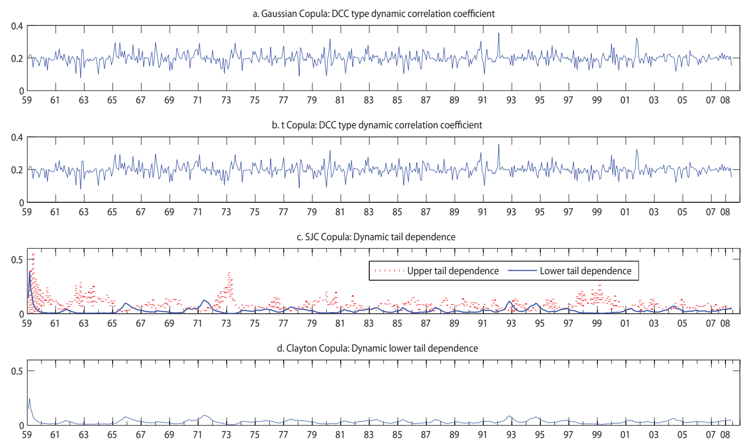

Figure 4: Dynamics of Dependence between the Market and the Size Factor

The figure plots the dynamic behavior of dependence parameters for the Market and the Size Factor.

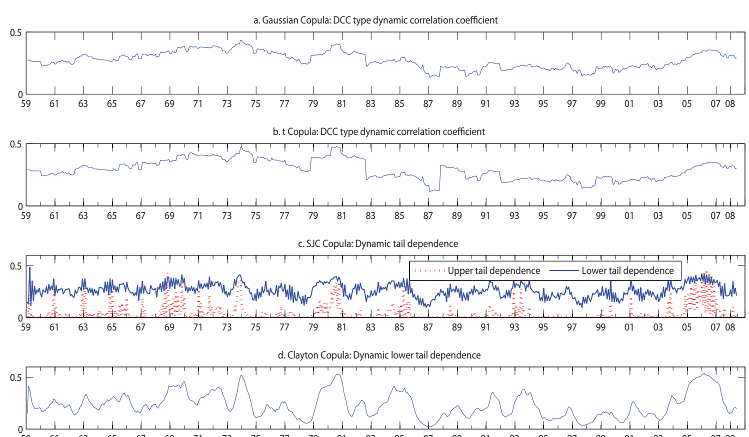

Figure 5: Dynamics of Dependence between the Dow Jones and the Size Factor

The figure plots the dynamic behavior of dependence parameters for the Dow Jones and the Size Factor.

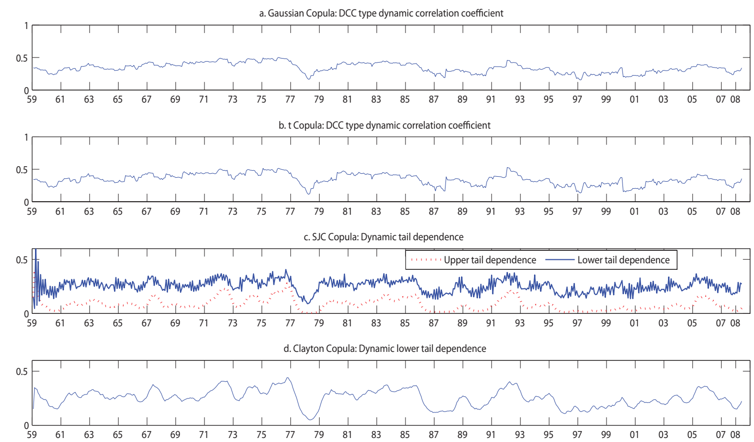

Figure 6: Dynamics of Dependence between the Book to Market Factor and the Size Factor

The figure plots the dynamic behavior of dependence parameters for the Book to Market Factor and the Size Factor.